Multirate Signal Processing, DSV2
Introduction

Lecture: Mi., 9-10:30 HU 010

Seminar: Do. 9-10:30, K2032 (bi-weekly)

Our Website contains the slides

www.tu-ilmenau.de/mt → Lehrveranstaltungen → Master → Multirate Signal Processing

We also have a **Moodle 2** website, at

https://moodle2.tu-ilmenau.de/course/view.php?id=395

which will contain the newest slides and the weekly **quiz** assignments (to be done individually).

There will be **bi-weekly homework** assignments, which will count 30% towards the final grade. The quizzes are part of the homework, and will count as 25% of the homework. To pass the course you need to pass the final exam.

We have Python Homework, which can be done in groups of max. 3 people.

The course has 5 LP (Credit Points), each LP is about 2-3 hours per week, which means we have 10-15 hours total per week or about 20 **hours** for 2 weeks (the homework cycle).

We will use the system **Moodle2** for information exchange and for our quiz assignments, at:

https://moodle2.tu-ilmenau.de/login/index.php

where you need to sign up.

Prerequisite: the course **Advanced Digital Signal Processing or DSP2**, or the corresponding knowledge.

Python is a very high level programming language, which contains the convenience and capabilities of Matlab/Octave, but is a full fledged programming language with many more capabilities and libraries. It is Open Source and hence free to download and use. It can be installed under Windows, but under **Linux** it is already installed, and additional libraries are more easily installed under Linux. In general programming is much easier under Linux, because that is an operating system made for scientists, engineers, and programmers, unlike Windows, which is made for office applications.

Hence we will use Linux, and only support Linux (you can use Windows but then you are on your own).

Linux is also Open Source, and can be freely downloaded and used (unlike Windows, which is closed source and costs about 100 eur per copy!) Linux can be easily installed as a dual boot system or on “VirtualBox” as a virtual system on your Laptop or PC, for instance by downloading the system from www.ubuntu.com – Download -Overview. Download either “**Desktop**” for laptops or desktop computers, or Ubuntu-Flavours – **Ubuntu Mate** for Netbooks or the

Raspberry Pi. Then burn it to a USB stick or SD card, and then start the installation for instance by restarting the PC, keep F2 or F12 or so constantly pressed to get into the Boot menu, and then choose “start from USB” or such. From there you can install Ubuntu as a dual boot.

Alternatively, you can also install Linux under “**Virtual Box”** on your PC or Laptop or Netbook, which is a virtual machine (a program which acts like a different computer).

Here is a short installation guide:

-Install in Windows:

Install VirtualBox and VirtualBox-Extensions from virtualbox.org

Download .iso image from www.ubuntu.com or https://ubuntu-mate.org/download/ (for lightweight laptops) to usb stick or the PC hard drive

-Start VirtualBox, Setup "New Machine" using the iso image. After finishing, remove the iso image (if it is on the hard drive, rename it to avoid the installation to start again)

-Before starting the virtual machine in VirtualBox:

Set RAM and Number of CPU's high, in Settings-System-Processor

(http://askubuntu.com/questions/365615/how-do-i-enable-multiple-cores-in-my-virtual-enviroment)

-After starting the virtual machine, install guest additions. Open a terminal Window, e.g. with keyboard shortcut Strg-Alt-T. Then type:

sudo apt install virtualbox-guest-additions-iso

cd /media/schuller/VBOXADDITIONS...

sudo sh ./VBoxLinuxAdditions.run

(See: wiki.ubuntuusers.de/VirtualBox/Installation)

-Then restart the virtual machine.

Press (right)Strg-F for full screen mode

-Activate external Devices for the virtual machine:

USB: On top menu bar of the VirtualBox go to Devices-USB click for checkmark

Webcam: on the VirtualBox menu bar go to Devices-Webcam, click for checkmark.

Scientific Literature Work

When reading papers, check each information you deem as important by reading the corresponding **references** and **check them** by checking their references, until you reach the **source** of the information.

For information that is claimed as original, do an internet search on **similar results**.

For **formulas**, check them by **programming** them (e.g. in Python), and see if you can reproduce the results presented in the paper. If there is not enough information to reproduce the results, or the results differ significantly, that is a bad sign.

For paper writing, reference every information that you deem important or relevant to the source where you‘ve got your information from. These references are important for reproducible research, and they are also the „**currency**“ of science, because scientists get their reputation from other people referencing them. Omitting them is like „stealing“ or cheating.

If you are presenting algorithms or programs, present them in formulas or pseudo-code in such a way that other people can **reproduce** your results (again, **reproducable research**). **Check your formulas by programming** them yourself, and see if they work. If you get error messages or wrong results, this shows that your formula has a problem.

-**Goal of the course**: To be able to **solve problems** in the area of Multirate Signal Processing, like how to design sampling rate converters or filter banks.

-**Approach**: Memorize only the most basic facts or properties, like the definition of the z-transform, then **know how** to use them (like how to derive the z-transform of a delayed signal). This is a **skill**.

As an Engineer, we need to get something to **work or to function** in the end. Engineering is also like applied physics, we need the **theory** and then the **experimental verification.**

It is recommended to **read** the slides **before** a lecture, such that we can answer **questions** during the lecture, and include the answers in the slides. Hence the slides will be kept in **editing** mode.

I will read each paragraph, and then ask for questions.

I will also show simple Python examples. **Bring your Laptops, such that you can**

try out these examples at the same time!

Course Book: Gilbert Strang, Truong Nguyen: “Wavelets and Filter Banks”

Ein deutsches Buch:

N. Fliege, "Multiraten-Signalverarbeitung: Theorie und Anwendungen", Teubner, Stuttgart, 1993

Lectures (as opposed to pure electronic learning) have the advantage that we can talk to each other, and that in this way the content can be tailored to your background. So you should ask questions, and also short discussions are beneficial in this way.

What is Multirate Signal Processing?

Multirate: meaning different sampling rates, as from using downsampling or upsampling. In filter banks, we reduce the sampling rate after filtering a signal, which reduces the bandwidth. For reconstruction and obtaining the original sampling rate, we need to up-sample and filter (for interpolation) the signal.

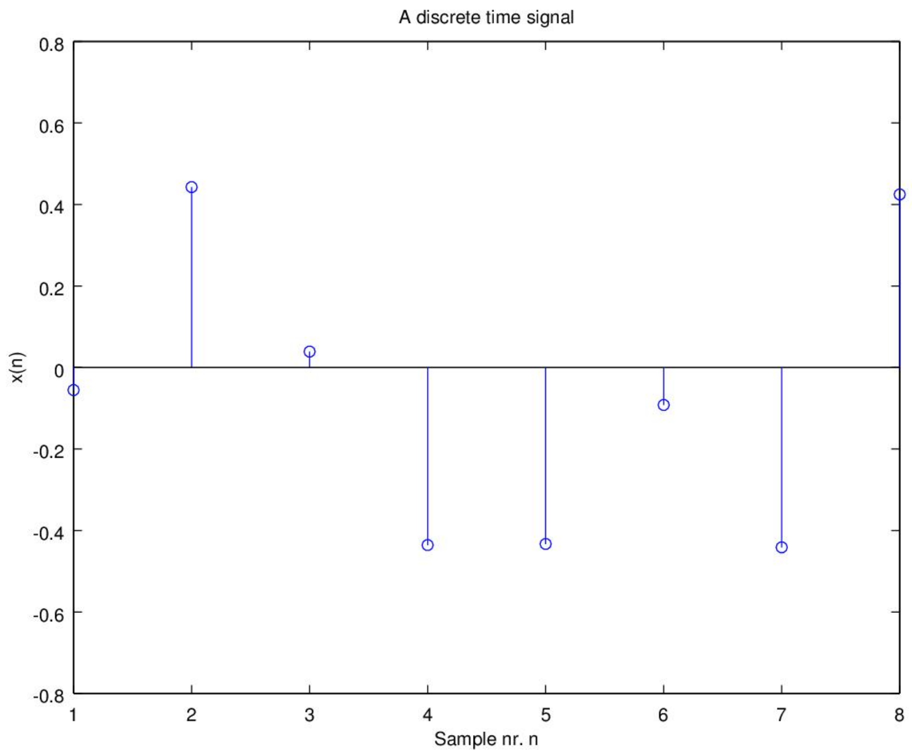
Where is Multirate Signal Processing used?

For instance in coding and compression algorithms, like the

- so-called Modified Discrete Cosine Transform (MDCT) filter bank in audio coding, or the
- Discrete Cosine Transform (DCT) in image or video coding
- Channel coding (OFDM), where a channel is divided into many narrower channels with lower data rates and hence longer

symbol duration, to reduce problems with multipath/reflections

- Example of a discrete time signal:



Typical sampling rate of audio from a CD:

44100 samples/second, or

44.1 Khz sampling rate.

Python example for a live plot of a microphone signal:

```
python pyrecplotanimation.py
```

- The Nyquist theorem tells us: Our signal needs to be **band limited** to

less than half the sampling frequency, here: less than 22.05 kHz. **Half the sampling frequency** is also called the **Nyquist frequency.**

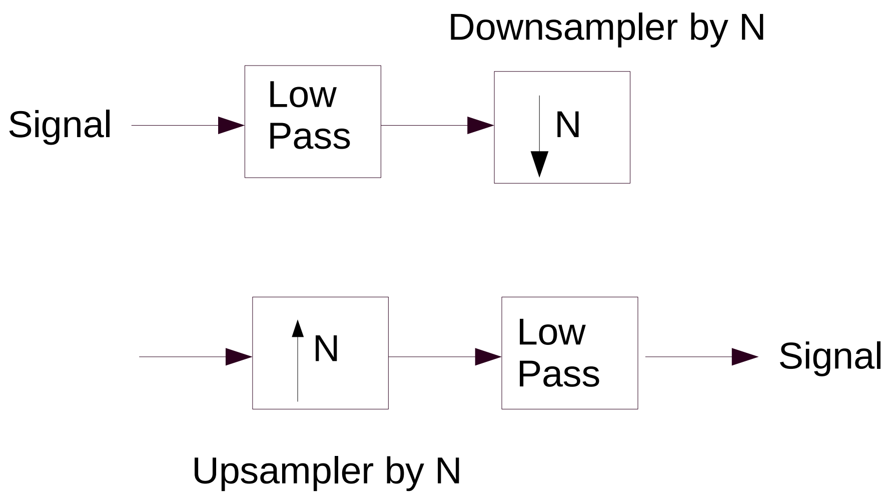
- For time discrete signals we only use **normalized frequencies**, normalized to the **sampling frequency** or the **Nyquist frequency**. For the latter, the normalized frequency of 1 would be the Nyquist frequency. Often you also find π as the Nyquist Frequency.

- Simple sample rate conversion example: Sampling rate conversion of an audio signal from 44.1 kHz (from a CD) down to 32 kHz on the computer. The signal at 44.1 kHz sampling rate has all frequencies strictly below 22.05 kHz (because of the Nyquist Theorem). A signal at 32 Khz sampling rate needs all frequencies strictly below 16 kHz. **Observe**: here we lose the highest frequency components (16kHz-22 kHz, which is basically okay since human hearing is usually only up to about 16 kHz). Before down-sampling we have to remove these high-frequency components by low pass filtering.

- Upsampling: The other way around, from 32 kHz to 44.1 kHz sampling rate. Observe: here we obtain a new frequency range from 16kHz to 22 kHz, which should contain no signal components. Here we have to low pass the up-sampled signal to these 16 kHz.

- This up- and down-sampling are the basic building blocks of multirate signal processing.

- The following picture shows the basic building blocks for low-pass filtering and down-sampling by a factor of N, and up-sampling by a factor of N followed by low pass filtering:



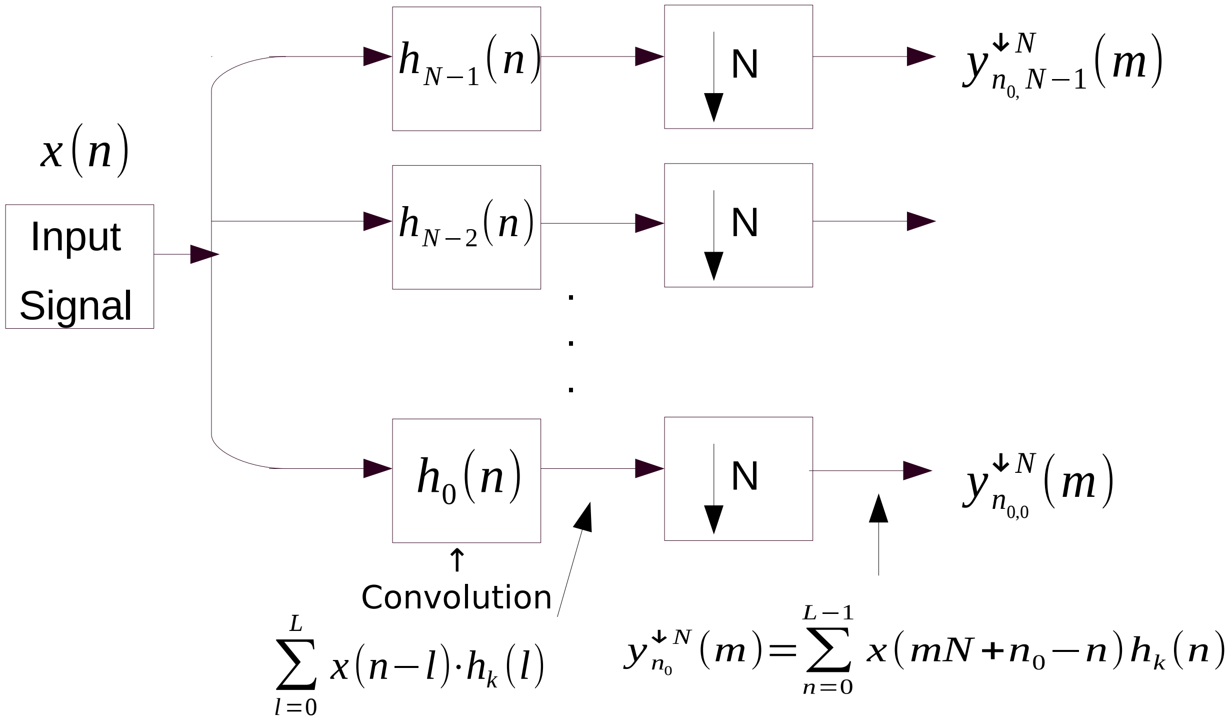
Observe: We can do this downsampling and upsampling without loss of information (meaning the reconstructed signal is identical to the lowpass signal), if we obey the **Shannon-Nyquist** law. This means the low pass (LP) needs to be an ideal low pass with (normalized to the Nyquist frequency at the higher sampling rate) cutoff frequency 1/N.

In this way we can perfectly reconstruct the lowest 1/N th of our signals spectrum.

Example: We have an audio signal with a sampling rate of 32 kHz and hence an audio bandwidth of less than 16 kHz. For N=2 we low pass filter it to 8 kHz, to remove the frequencies at and above the new Nyquist frequency of 8 kHz. Then we can downsample it by a factor of N=2 (by dropping every second sample), to obtain an audio signal at a sampling rate of 16 kHz.

We can the upsample the audio signal back to 32 kHz, by a factor of 2, by insterting a 0 after each sample, which produces “alias” components above 8 kHz, and then low pass filter the signal, again with our lowpass with cutoff frequency of 8 kHz, to remove the alias components. This results in the same audio signal with bandwidth of 8 kHz, but now at 32 kHz sampling rate.

The following picture shows a **filter bank** with **critical sampling**, which means its downsampling rate N is identical to the number of subbands**.** It is the principal tool for multirate signal processing, first the analysis filter bank:



Observe: For each time step m we obtain a spectrum, and for each subband we obtain a narrow bandwidth time signal. So depending on our perspective, we have a set of spectra, or a set of narrow bandwidth time signal.

This is why we also call this a **time/frequency** representation.

Remember: The **filter** boxes symbolize a **convolution** of the signal $x(n)$ with the impulse response $h(n)$ of length L of each filter, before downsampling:

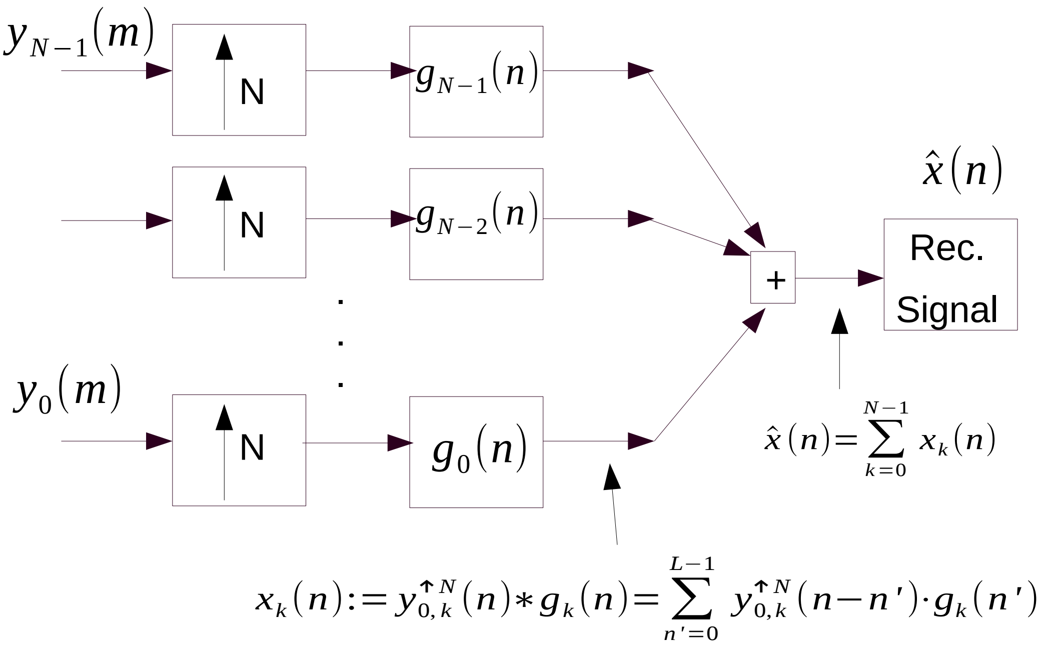
$$x(n)*h(n)=\sum_{l=0}^{L-1} x(n-l)\cdot h(l)$$

where the sum is assumed to go over only the parts where $x(n)$ and $h(n)$ are defined.

Downsampling by N means we replace n by $mN+n_0$, with m the index m at the **lower sampling rate**, and n_0 the phase index,

$$y_{n_0}^{\downarrow N}(m)=\sum_{n=0}^{L-1} x(mN+n_0-n)h_k(n)$$

The **analysis filter bank** decomposes the signal into different frequency bands. Observe that each frequency band has a lower sampling rate, which is possible because they have a lower bandwidth. Using the “bandpass Nyquist” theorem we can reconstruct the original signal from the subbands!



To reconstruct the original from the different frequency bands, we need the **synthesis filter bank**:

To simplify notation we dropped the downarrow and phase index for the subband signals $y_k(m)$.

Observe: The filters after the up-sampling take on the role of the lowpass filter in the conventional Nyquist theorem, to block the alias components. They “fish out” the correct frequency image out of the aliased images for that subband. All the subbands are then added up to reconstruct the original signal.

Example for our 2-band system: We have N=2, original sampling rate 32 kHz. Then the low pass branch corresponds to our low pass example above, which reconstructs the signal from 0 to 8 kHz. We now also have a high pass branch, which in addition reconstructs the frequency from 8kHz to 16 kHz. We add the two subbands in the synthesis filter bank to obtain the full bandwidth signal from 0 to 16 kHz.

For the playback of sound (with the function “sound”), also the software package “sox” needs to be installed from the software package manager under Linux.

In the command line with

```
sudo apt-get install sox
```