Filter Banks II

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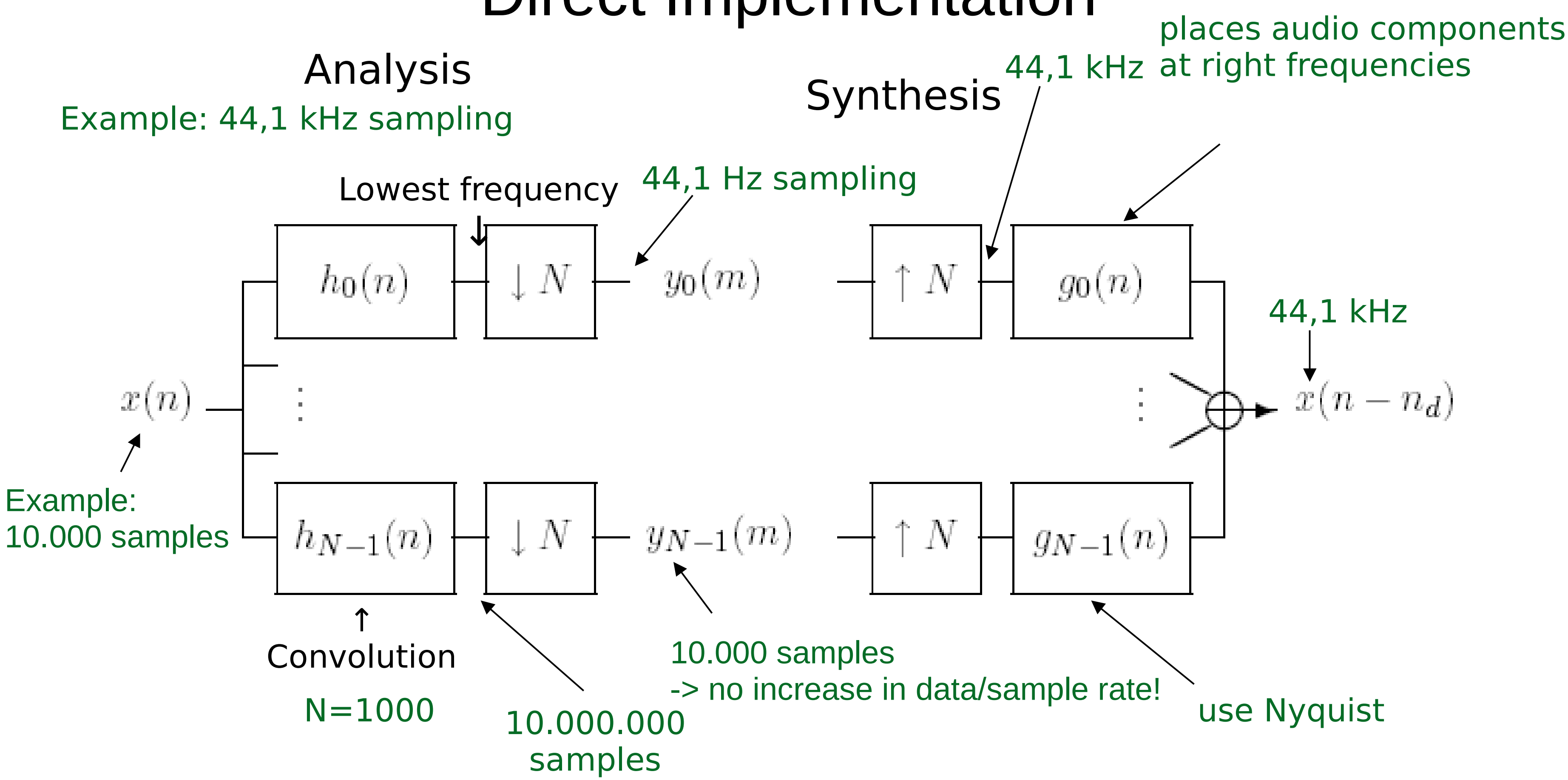
Critically sampled Analysis and Synthesis Filter Bank, Direct Implementation



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Modulated Filter Banks – Extending the DCT

Last time we saw that the DCT4 corresponds to a filter bank with impulse responses for the analysis here in time reversed form to simplify the right hand side:

$$h_k(N-1-n)=\cos\left(\frac{\pi}{N}\left(k+\frac{1}{2}\right)\left(n+\frac{1}{2}\right)\right)$$

For subband k and time index n both in the range of 0,…N-1.

With the help of a "baseband prototype" or "window" *h(n)* (independent of k):

window function allows to improve filter`s parameters like stopband attenuation and transition band width

$$h(n)=\begin{cases}1 & n=0\ldots N-1\\ 0 & else\end{cases}$$

We can now re-write this as a "**modulated filter**",

$$h_k(N-1-n)=h(n)\cdot\cos\left(\frac{\pi}{N}\left(k+\frac{1}{2}\right)\left(n+\frac{1}{2}\right)\right)$$

With k=0,…N-1, but now with $-\infty<n<\infty$

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Modulated Filter Banks

So called **Modulated Filters** as part of a **Modulated Filter Bank** are defined to have the following general form:

$$h_k(n) = h(n) \cdot \Phi_k(n)$$

$h(n)$	window function (not necessarily limited in length)
$\Phi_k(n)$	modulation function, for instance the cosine function

frequency index

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Modulated Filter Banks

- Another example of filters for so-called **Cosine Modulated Filter Banks**:

$$h_k(n) = h(n) \cdot \cos\left(\frac{\pi}{N}(k+0.5)(n+0.5)\right)$$

- With the cosine modulation, the resulting frequency responses (using the DTFT) of the filters in the filter bank are:

$$H_k(\omega) = H(\omega) \star \frac{1}{2}\left[\delta\left(\omega - \frac{\pi}{N}\left(k+\frac{1}{2}\right)\right) + \delta\left(\omega + \frac{\pi}{N}\left(k+\frac{1}{2}\right)\right)\right]$$

Multiplication in time becomes convolution in frequency

Delta functions from cosine term

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Modulated Filter Banks

$$= H\left(\omega - \frac{\pi}{N}\left(k + \frac{1}{2}\right)\right) + H\left(\omega + \frac{\pi}{N}\left(k + \frac{1}{2}\right)\right)$$

Shift in frequency

- Hence: Modulated filter banks obtain their filters by shifting a „baseband filter“ *h(n)* in frequency $-\pi < \omega < \pi$.
- As a result, we need to design only h(n) with high stopband attenuation and perfect reconstruction.

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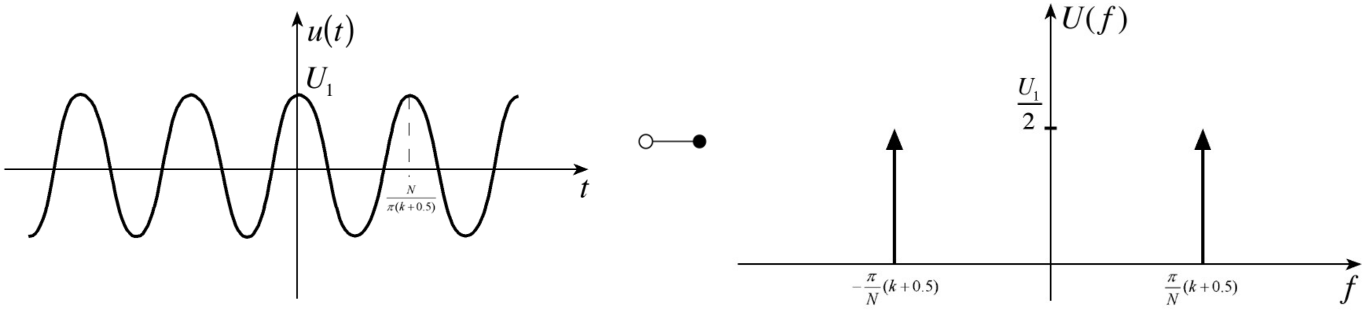
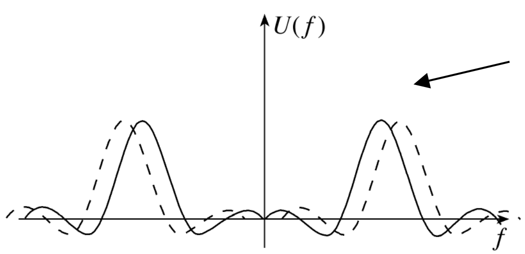
Modulated Filter Banks: Frequency Shifts



The subbands of the filter bank are frequency-shifted versions of the window frequency response:

$\Rightarrow$



Place passband in frequency, depending on the modulation function, for Subbands *k* and *k+1*.

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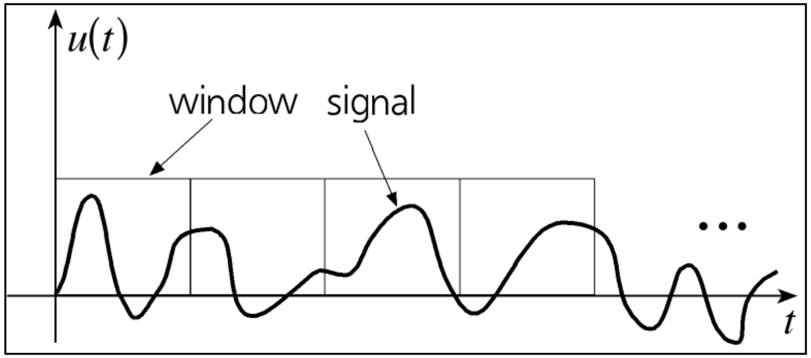
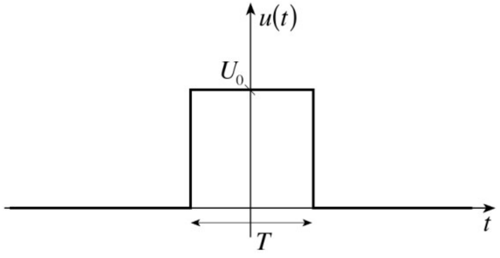
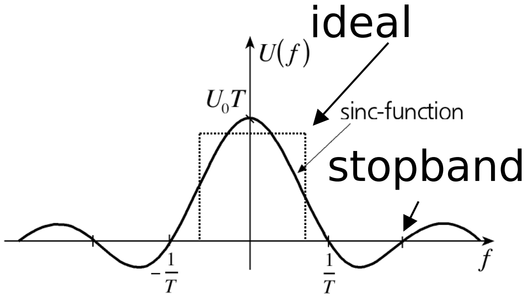
Modulated Filter Banks: The Window Function



Frequency response of the rectangular window function of the DCT:





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Modulated Filter Banks

Improve filter banks:

- make window longer
- different window shape

Examples (all have the same principle):

- TDAC (time domain aliasing cancellation) (Princen and Bradley 1986&1987)

- LOT (lapped orthogonal transform) (Malvar 1989)

- MDCT (modified DCT) (Bernd Edler 1988)

Fast Implementation: Analysis Polyphase Matrix

- Remember: the analysis polyphase matrix is:

$$\underline{\underline{H}}(z) = \begin{bmatrix} H_{0,0}(z) & H_{0,1}(z) & \cdots & \\ H_{1,0}(z) & H_{1,1}(z) & & \\ \vdots & & \ddots & \\ & & & H_{N-1,N-1}(z) \end{bmatrix}$$

with the analysis polyphase components

$$H_{k,n}(z) = \sum_{m=0}^{\infty} h_k(n+mN) z^{-m}$$

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The MDCT Filter Bank

- The so-called MDCT filter bank has a prototype or window length of L=2N, and is defined with its filter impulse responses in the direct implementation as,
- Analysis filters:

$$h_k(L-1-n)=h(n)\cdot\cos\left(\frac{\pi}{N}\cdot\left(k+\frac{1}{2}\right)\left(n+\frac{1}{2}-\frac{N}{2}\right)\right)\cdot\sqrt{\frac{2}{N}}$$

- Synthesis filters:

$$g_k(n)=g(n)\cdot\cos\left(\frac{\pi}{N}\cdot\left(k+\frac{1}{2}\right)\left(n+\frac{1}{2}-\frac{N}{2}\right)\right)\cdot\sqrt{\frac{2}{N}}$$

for n=0,…,2N-1; k=0,…,N-1.

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The MDCT Filter Bank

- The resulting **Analysis Polyphase** matrix is

$$\underline{\underline{H}}(z) = \begin{bmatrix} h_0(0) + z^{-1} h_0(N) & h_1(0) + z^{-1} h_1(N) & \cdots & \\ h_0(1) + z^{-1} h_0(N+1) & h_1(1) + z^{-1} h_1(N+1) & & \\ \vdots & & \ddots & \\ & & & h_{N-1}(N-1) + z^{-1} h_{N-1}(2N-1) \end{bmatrix}$$

Still square matrix, still invertible, *NxN*

- observe: this $h_k(n)$ has length 2N, and is more general than the rectangular window (not just 1 or 0)
- $\underline{\underline{H}}(z)$ is composed of **1st order polynomials**
- Goal: find "good" *h(n)*

MDCT, Fast Implementation

- Fortunately, the MDCT polyphase matrix can be decomposed into a **product of simpler matrices**, hence easier to invert to obtain perfect reconstruction:

$$\underline{\underline{H}}(z) = \underbrace{\begin{bmatrix} 0 & h(0) & h(N) & 0 \\ \iddots & & & \ddots \\ h(\frac{N}{2}-1) & 0 & 0 & h(1.5N-1) \\ h(\frac{N}{2}) & 0 & 0 & -h(1.5N) \\ \ddots & & & \iddots \\ 0 & h(N-1) & -h(2N-1) & 0 \end{bmatrix}}_{F_a\text{, real valued}} \cdot \underbrace{\begin{bmatrix} z^{-1} & & & & & \\ & \ddots & & & 0 & \\ & & z^{-1} & & & \\ & & & 1 & & \\ & 0 & & & \ddots & \\ & & & & & 1 \end{bmatrix}}_{\text{Delay matrix } D(z)} \cdot T$$

matrix inverse is inverse of each entry!

DCT4-Matrix

$$\begin{bmatrix} 1 & & 0 \\ & 2 & \\ 0 & & 3 \end{bmatrix} \begin{bmatrix} 1 & & 0 \\ & 2 & \\ 0 & & 3 \end{bmatrix} = \begin{bmatrix} 1 & & 0 \\ & 4 & \\ 0 & & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & & 0 \\ & 2 & \\ 0 & & 3 \end{bmatrix} \begin{bmatrix} 1 & & 0 \\ & 1/2 & \\ 0 & & 1/3 \end{bmatrix} = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$$

MDCT, Fast Implementation

- Observe the diamond shaped form of the matrix $F_a(z)$ and the sparse structure
- Beneficial for an efficient implementation

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MDCT synthesis, Fast Implementation

- The MDCT synthesis Polyphase matrix can be similarly decomposed into a product of matrices. Needs to be the inverse and a delay for Perfect Reconstruction (PR).

$$\underline{\underline{G}}(z) = T^{-1} \cdot \underbrace{\begin{bmatrix} 1 & & & & & 0 \\ & \ddots & & & & \\ & & 1 & & & \\ & & & z^{-1} & & \\ & & & & \ddots & \\ 0 & & & & & z^{-1} \end{bmatrix}}_{D^{-1}(z)\, z^{-1}} \cdot \underbrace{\begin{bmatrix} 0 & & g(\frac{N}{2}-1) & g(\frac{N}{2}) & & 0 \\ & \iddots & & & \ddots & \\ g(0) & & 0 & 0 & & g(N-1) \\ g(N) & & 0 & 0 & & -g(2N-1) \\ & \ddots & & & \iddots & \\ 0 & & g(1.5N-1) & -g(1.5N) & & 0 \end{bmatrix}}_{F_s}$$

z^{-1}: Delay by one time step (past)

z: Looking into the future $\rightarrow$ non-causal $\rightarrow$ not practical hence mult. with z^{-1} (delay!) $\rightarrow$ cause of signal delay

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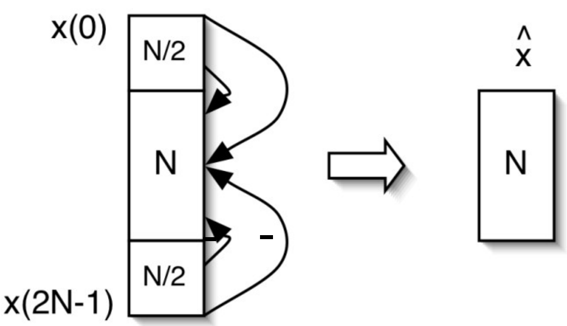
Graphical Interpretation of Analysis Matrix F_a

$$\text{time} \quad [x] \cdot \underbrace{\begin{bmatrix} 0 & h(0) & h(N) & 0 \\ & \iddots & \ddots & \\ h(0) & 0 & 0 & h(N-1) \\ h(N) & 0 & 0 & -h(2N-1) \\ & \ddots & \iddots & \\ 0 & h(N-1) & -h(2N-1) & 0 \end{bmatrix}}_{F_a} \cdot \underbrace{\begin{bmatrix} z^{-1} & & & 0 \\ & \ddots & & \\ & & z^{-1} & \\ & & & 1 \\ & & & & \ddots \\ 0 & & & & 1 \end{bmatrix}}_{D(z)} \cdot [DCT_4] = \underset{\text{subbands}}{[y]}$$



- „Folding“ the upper and lower quarter of the signal into a length *N* block (aliasing components)
- Invertible by matrix inversion containing overlap-add

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MDCT, Perfect Reconstruction

- DCT matrix T and the delay matrix *D(z)* are easily invertible for perfect reconstruction.
- System Delay results from making inverse of *D(z)* causal (one block), and the blocking delay of *N-1 samples*.
- F_a is also easily invertible, with some simple matrix algebra:

$$g(n) = \frac{h(n)}{h(n)h(2N-1-n) + h(N+n)h(N-1-n)}$$

$$g(N+n) = \frac{h(N+n)}{h(n)h(2N-1-n) + h(N+n)h(N-1-n)}$$

Determinant in the denominator

with $n=0,\dots,N-1$

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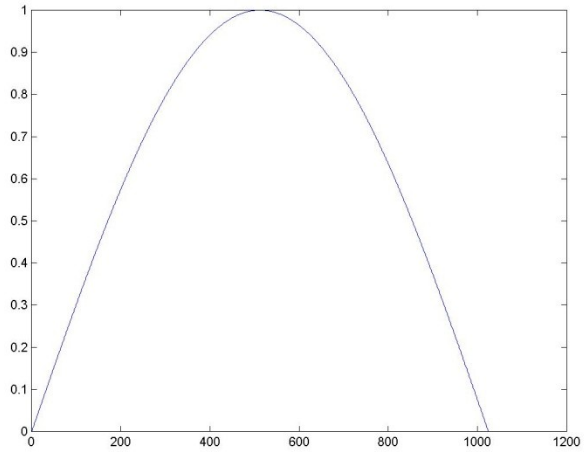
MDCT Filter Banks, Sine Window

- Modified Discrete Cosine Transform (MDCT): *g(n)=h(n)* ⇒ Denominator=1
- Example which fulfils this condition: Sine window



$$h(n) = \sin\left(\frac{\pi}{2N}(n+0.5)\right)$$ for *n=0,…,2N-1*

System delay=*2N-1*=1023 for *N*=512
(from the delay matrices, 1 block of *N*, and the blocking delay of *N-1*)

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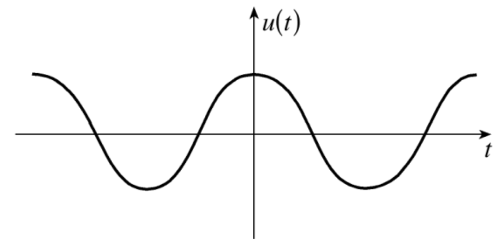
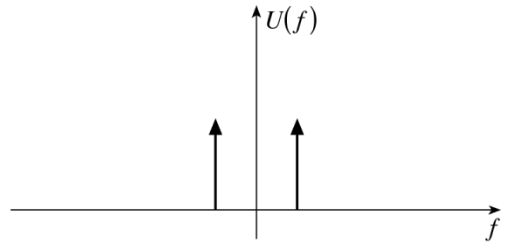
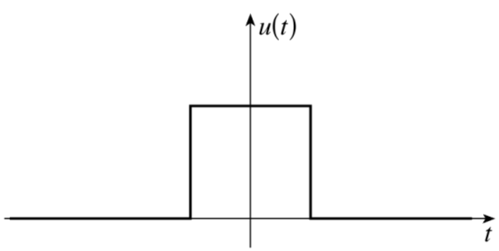
Sine-Window Frequency Response

- Modulation of a rectangular window of length *2N*

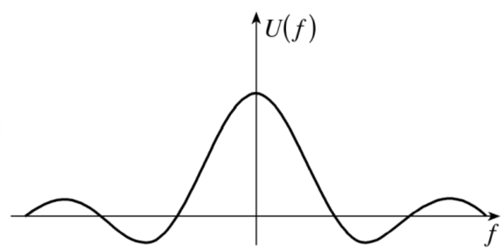
sinusoidal function





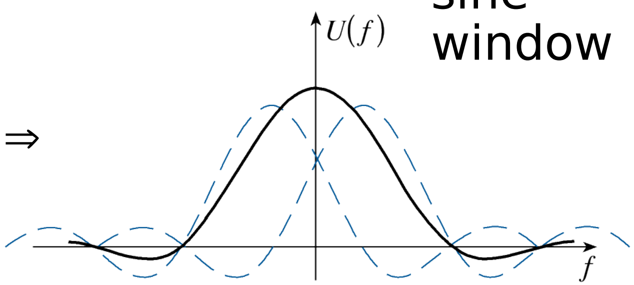


↑
length *2N*



⇒

sine window



↑
Better attenuation outside of passband than with simple sinc

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MDCT, Advantages

- Improved frequency responses, higher stopband attenuation

- Easy to design filter banks with many subbands
  (for instance $N=1024$ for audio coding)

- Efficient implementation with the shown sparse matrices and a fast DCT. Important for large number of subbands, as in audio coding.

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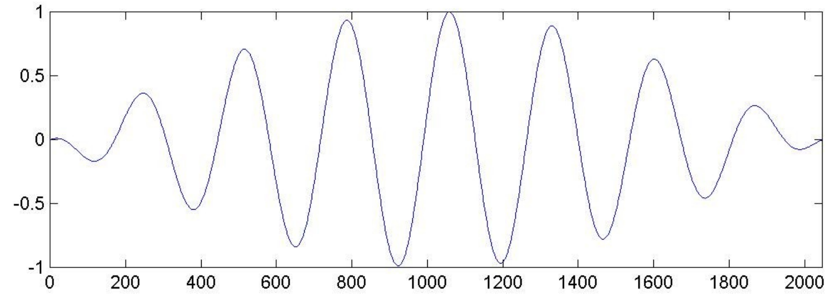
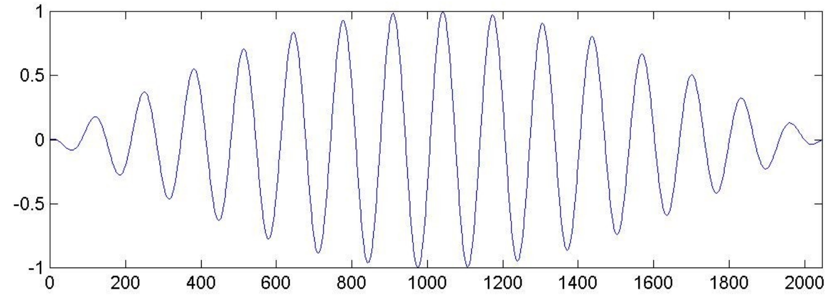
MDCT Filter Banks, Impulse Responses

Examples: filter impulse responses
$h_7(n)$, $h_{15}(n)$, $N=1024$ bands, sine window.





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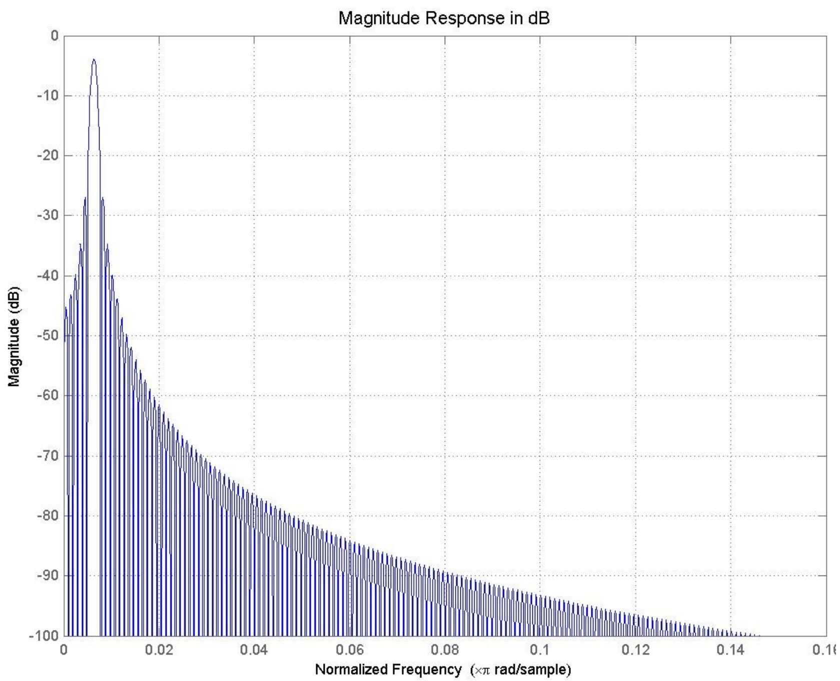
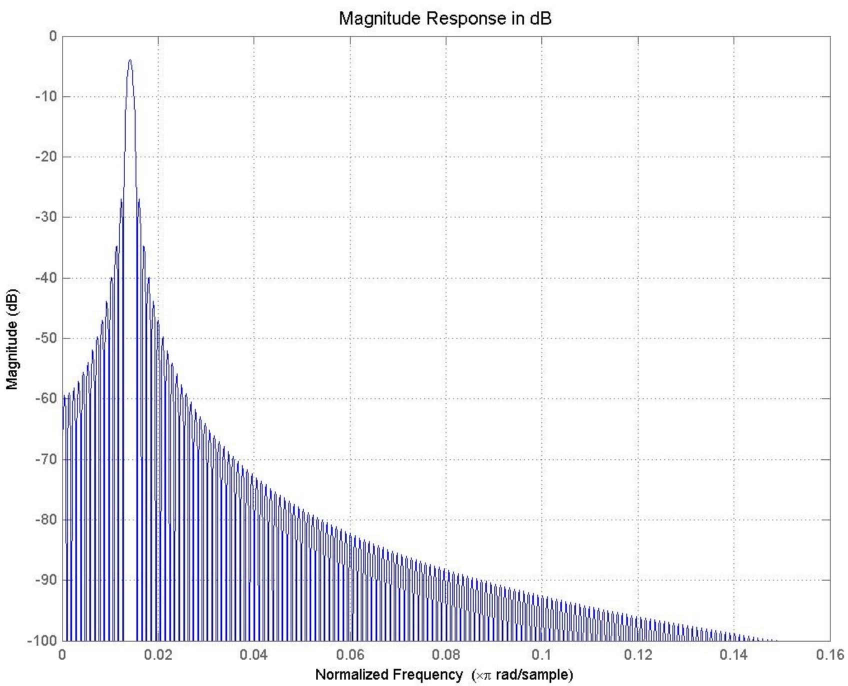
MDCT Filter Banks, Frequency Responses



Magnitude response 7th band



Magnitude response 15th band

indeed better filters!

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Python Examples

- Next is a time-frequency representation, a spectrogram, which displays time on the vertical axis, and which shows the magnitude of the FFT coefficients as different colors:

  ```
  Python pyrecspecwaterfall.py
  ```

- **Observe**: This shows the time-frequency nature of filter banks (of which the FFT is a special example). You have both, time and frequency dependencies.
- Next improved, with the MDCT

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Python Examples

- This is an example for the MDCT filter bank. You see a decomposition of the audio signal into MDCT subbands. These subbands can then be processed, for instance we set every subband except for a few to zero. Then we display the result as a spectrogram waterfall diagramm, and use the inverse/synthesis MDCT for reconstrution and play the resulting sound back:

  ```
  python pyrecplayMDCT.py
  python pyrecplayfastMDCT.py
  ```
- **Observe:** The MDCT does not have those symmetric 2 sides, it only has one side of the spectrum, with the lowest frequencies on the left side, and the hightest on the right.
- If we only keep a few subbands, it sounds muffled or „narrowband".

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Extending the Length of the MDCT

- **Longer filters** are obtained with **higher order polynomials** in the polyphase matrix
- Approach to obtain easily invertible polyphase matrices
- multiply MDCT polyphase matrix with more **easily invertible matrices** with polynomials of 1st order
- To control the resulting system delay: design different matrices with different needs for delay to make them causal

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Extending the Length

- Take the MDCT Polyphase matrix with a general window function h(n) (not nec. Sine window):

$$\underline{\underline{H}}_{MDCT}(z)$$

- This matrix contains polynomials of first order. Multiply it with another matrix with polynomials of first order (Schuller, 1996, 2000):

$$L(z) \cdot \underline{\underline{H}}_{MDCT}(z)$$

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Extending the Length

- This matrix **L**(z) needs to have a form such that again a modulated filter bank results.
- Diamond shaped form needs to be maintained

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Extending the Length, Zero-Delay Matrix

- This matrix fulfills the conditions
- *Zero-Delay Matrix*:

$$L(z) = \begin{bmatrix} z^{-1} l_0 & & & & & 1 \\ & \ddots & & & \iddots & \\ & & z^{-1} l_{N/2-1} & 1 & & \\ & & 1 & 0 & & \\ & \iddots & & & \ddots & \\ 1 & & & & & 0 \end{bmatrix}$$

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Extending the Length, Zero-Delay Matrix

- Its inverse is <u>causal</u>, hence does not need a delay to make it causal:

$$L^{-1}(z) = \begin{bmatrix} 0 & & & & & 1 \\ & \ddots & & & \iddots & \\ & & 0 & 1 & & \\ & & 1 & -z^{-1} l_{N/2-1} & & \\ & \iddots & & & \ddots & \\ 1 & & & & & -z^{-1} l_0 \end{bmatrix}$$

still increases filter length!

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Extending the Length, Zero-Delay Matrix

- Observe: Since the matrix has a causal inverse, it can increase the filter length of the resulting filter bank without increasing the system delay!
- Hence adds zeros inside unit circle
- The coefficients $h(n)$ and l_n don‘t affect the delay or the PR property, but the frequency response of the resulting filter bank
- Coefficients need to be found by numerical optimization.

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Extending the Length, Maximum-Delay Matrix

- Consider the following matrix
- *Maximum-Delay Matrix*:

$$H(z) = z^{-1} L(z^{-1})$$

- Its inverse and delay for causality is

$$H^{-1}(z) \cdot z^{-2} = z^{-1} L^{-1}(z^{-1})$$

- Observe: This matrix and its inverse need a delay of 2 blocks to make it causal.
- Hence adds zeros outside the unit circle

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Extending the Length, Design Method

- Determine the total number of Zero-Delay Matrices and Maximum-Delay Matrices according to the desired filter length

- Determine the number of Maximum-Delay Matrices according to the desired system Delay

- Determine the coefficients of the matrices with numerical optimization to optimize the frequency response

Example: `python pyrecplayfastLDFB.py`

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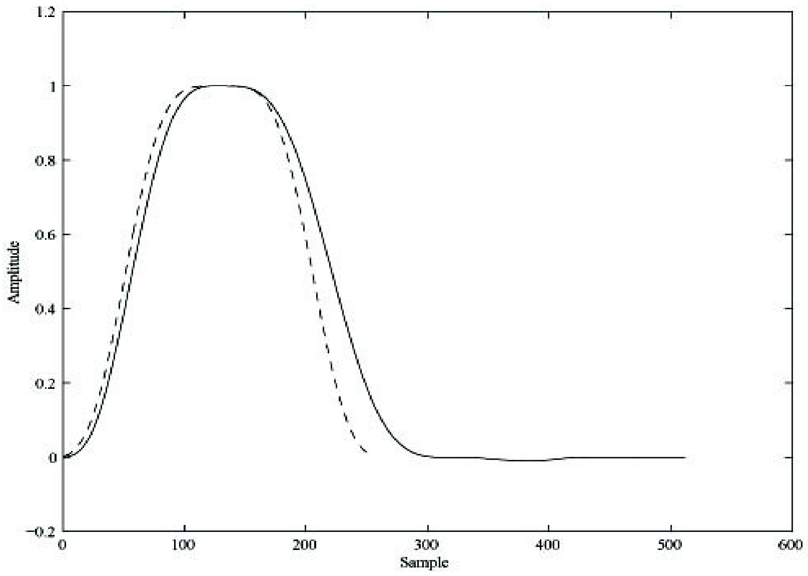
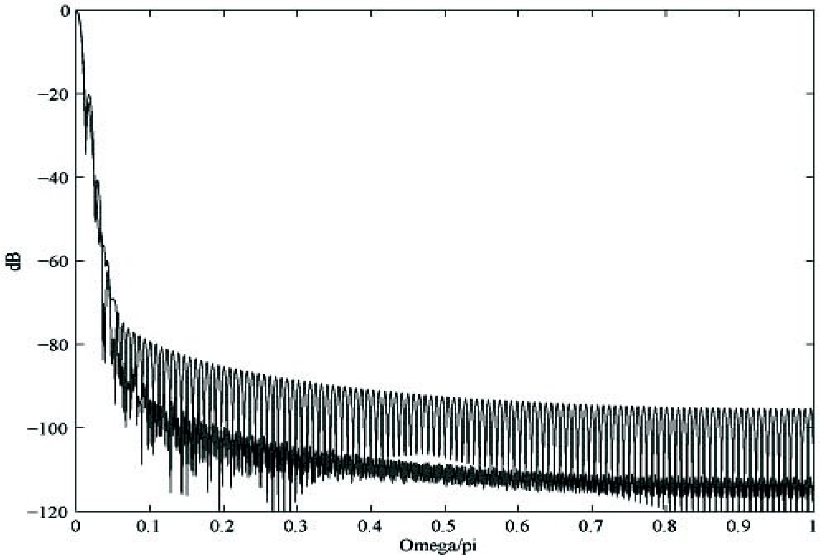
Example

- Comparison for 128 subbands.
- Dashed line: Orthogonal filter bank, filter length 256, system delay 255 samples.
- Solid line: Low delay filter bank, length 512, delay 255





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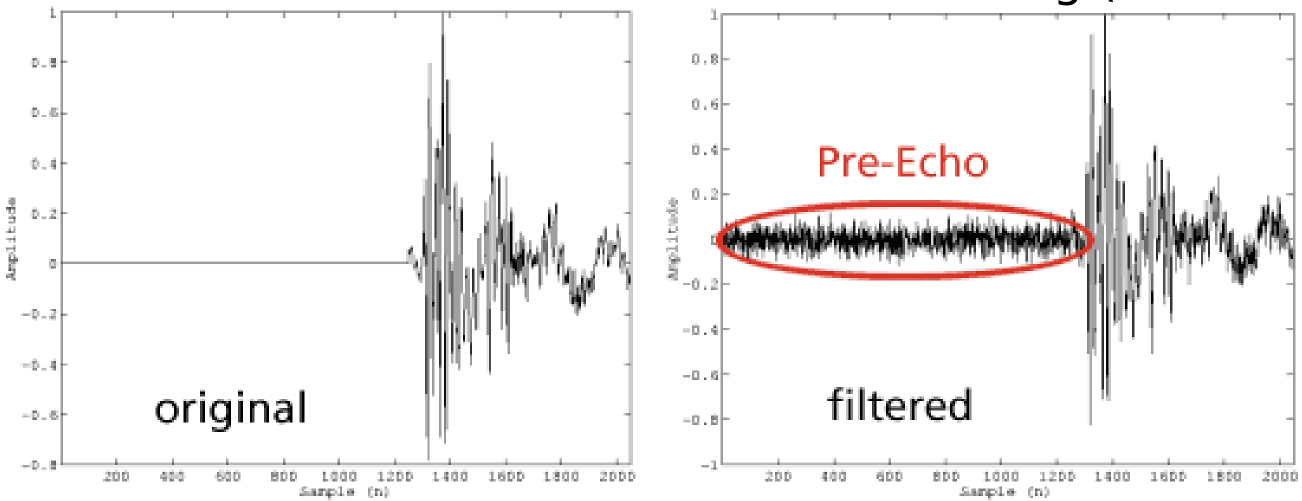
Block Switching

- Problem: In audio coding, Pre-echoes appear
  before transcients
  - reason: blocks too long (too many



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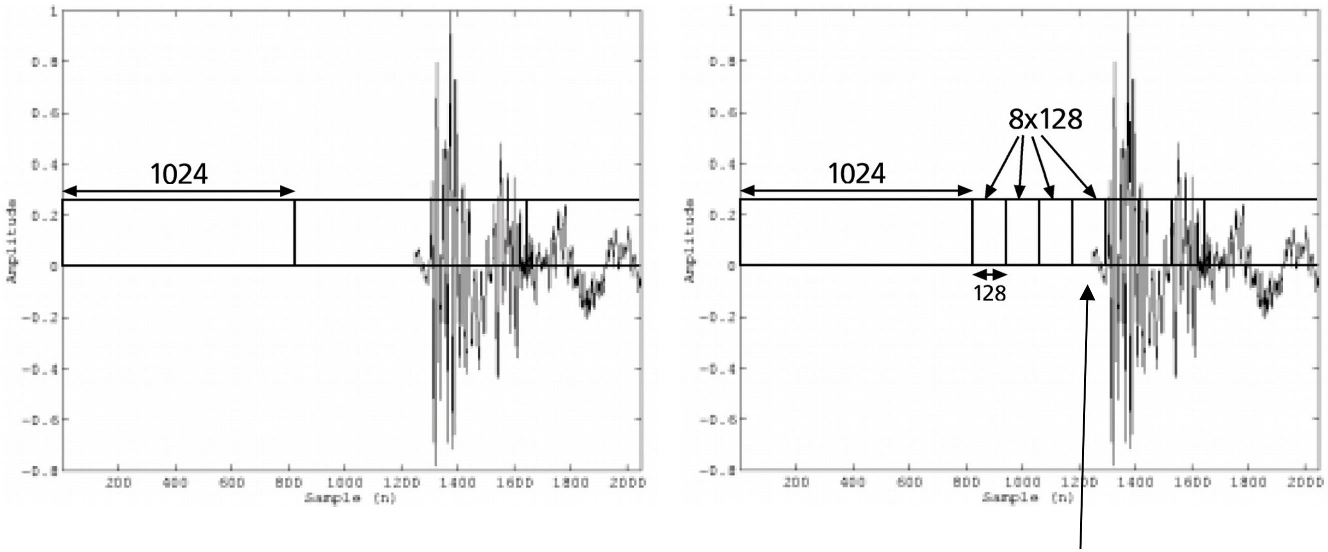
Block Switching

- Approach: for fast changing signals use block switching to lower number of subbands



less noise spread in time!

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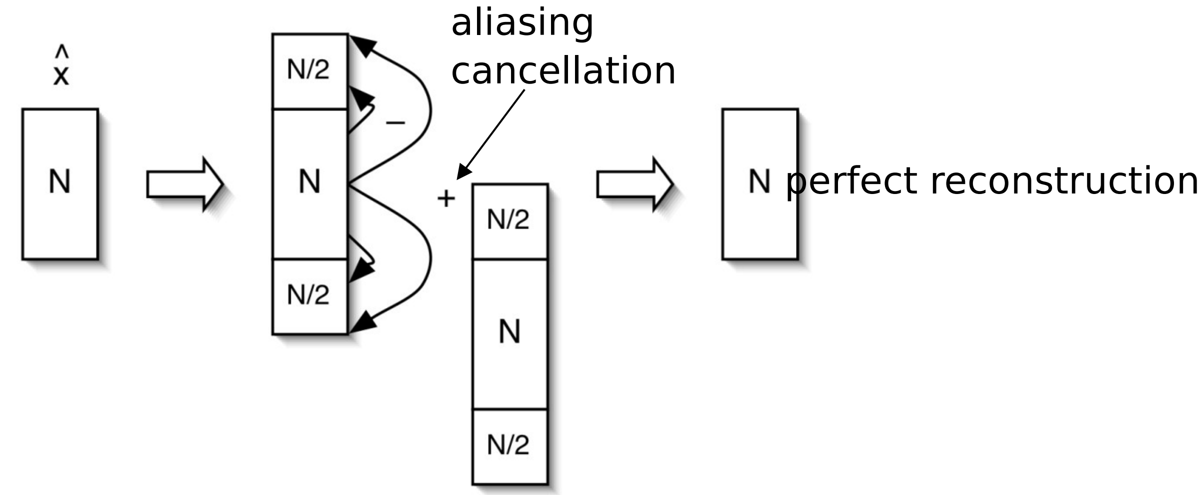
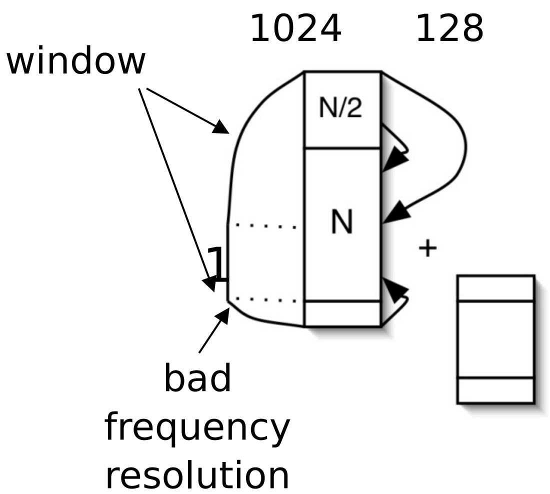
Accommodate Overlap-Add for Block Switching





⇒ Analysis switching

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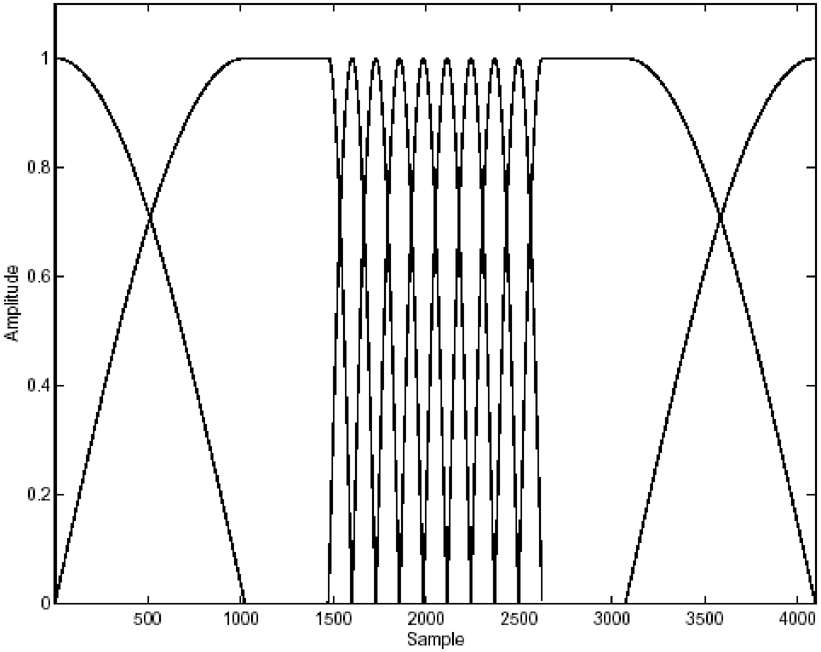
Block Switching

- Sequence of windows for switching the number of sub-bands
- Shorter windows → better resolution

window value *h(n)*



both, analysis and synthesis

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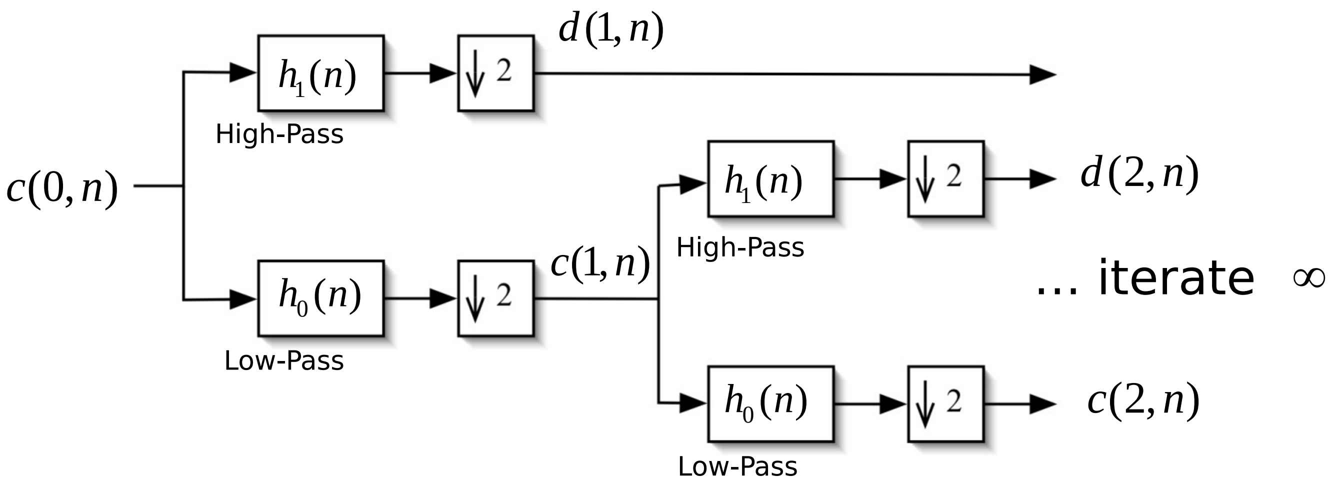
Wavelets, QMF Filter Banks

- Iterate 2-band system
- See also: Wavelet Packets (more general)
- Problem: Aliasing propagation reduces frequency selectivity!
- Important in image coding, but no big role in Audio Coding



How to Obtain a Two Band Filter Bank

- Application: QMF filter banks, Wavelets,...
- Analysis polyphase for a 2-band filter bank:

$$\underline{\underline{H}}(z) = \begin{bmatrix} H_{0,0}(z) & H_{0,1}(z) \\ H_{1,0}(z) & H_{1,1}(z) \end{bmatrix}$$

- Observe: $H_{0,0}(z)$ contains the even coefficients of the low pass filter, and $H_{1,0}(z)$ its odd coefficients.
- Accordingly for the high pass filter

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How to Obtain a Two Band Filter Bank

- Given the analysis filters, the synthesis filters can be obtained by inverting the analysis polyphase matrix,

$$\underline{\underline{H}}^{-1}(z) = \frac{1}{Det\left(\underline{\underline{H}}(z)\right)} \begin{bmatrix} H_{1,1}(z) & -H_{0,1}(z) \\ -H_{1,0}(z) & H_{0,0}(z) \end{bmatrix}$$

- Observe: If the analysis filters have a finite impulse response (FIR), and the synthesis is desired to also be FIR, the **determinant** of the polyphase matrix needs to be a **constant or a delay**!

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How to Obtain a Two Band Filter Bank

$$\det\left(\underline{\underline{H}}(z)\right) = H_{1,1}(z)H_{0,0}(z) - H_{0,1}(z)H_{1,0}(z)$$

= const or a delay

- Observe: This is the output of the lower band of the filter bank if the input signal is

$$\underline{x}(z) = \left[H_{1,1}(z), \quad -H_{0,1}(z)\right]$$

- Hence the determinant can be formulated as a **convolution**
- This input is the high band filter coefficients, with the sign of the even coefficients flipped and switched places with the odd coefficients.

How to Obtain a Two Band Filter Bank

- Since this represents a critically sampled filter bank, the result represents **every second sample** of the convolution of the low band filter with the correspondingly modified high band filter.
- This modified high band filter is a low band filter (every second sample sign flipped).
- The desired output of this downsampled convolution is a single pulse (corresponding to a constant or a delay), hence flat in frequency
- Another interpretation: correlation of the 2 signals, where the even lags that appear after downsampling are zero, except for the one pulse

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QMF (Quadrature Mirror Filter)

- This suggests a simple design strategy:
  - Design a low pass filter for analysis and synthesis
  - Obtain the high pass filters by flipping the low pass filters every second coefficient

analysis FB: $h_1(n) = (-1)^n h_0(n) \quad n = 0,1,\ldots,N-1$

Synth. low pass: $g_0(n) = h_0(n)$

synth. FB high pass: $g_1(n) = -h_1(n)$

- This is an early two band filter bank: QMF, Quadrature Mirror Filter (Croisier, Esteban, Galand, 1976)

- For more than 2 bands: GQMF (Cox, 1986), PQMF

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QMF (2)

- Sign flipping to obtain the high band filter leads to the polyphase components:

$$H_{0,1}(z) = H_{0,0}(z)$$

$$H_{1,1}(z) = -H_{1,0}(z)$$

- The resulting determinant is:

$$\det\left(\underline{\underline{H}}(z)\right) = H_{1,1}(z)H_{0,0}(z) - H_{0,1}(z)H_{1,0}(z)$$

$$= -2H_{0,1}(z)H_{0,0}(z)$$

- Observe: This cannot be made a constant or delay for finite polynomials of order 1 or greater, hence no PR for finite length filters!

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QMF (3)

- The QMF accounts for the sign flipping in the determinant equation.
- But not for the trading places of even and odd coefficients
- Hence: **No Perfect Reconstruction** (only for simple Haar and IIR filters)
- High stopband attenuation needed to keep reconstruction error small
- Numerical optimization to obtain

$$\left|H_0\left(e^{j\omega}\right)\right|^2 + \left|H_1\left(e^{j\omega}\right)\right|^2 \approx 1$$

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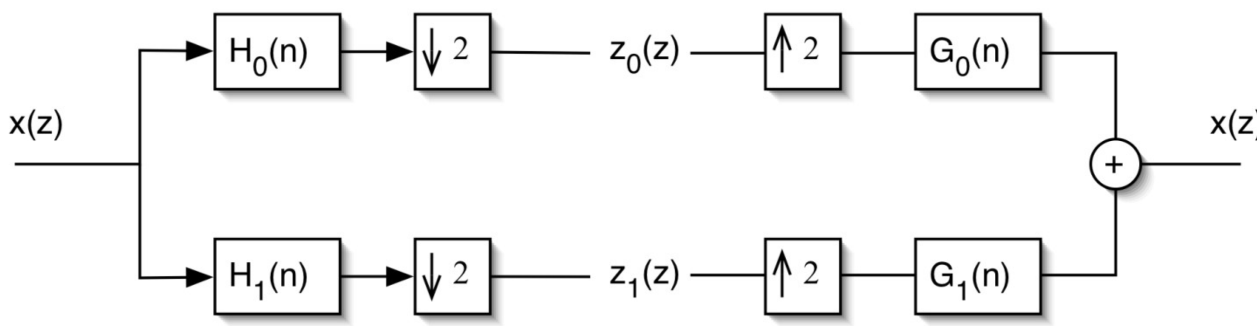
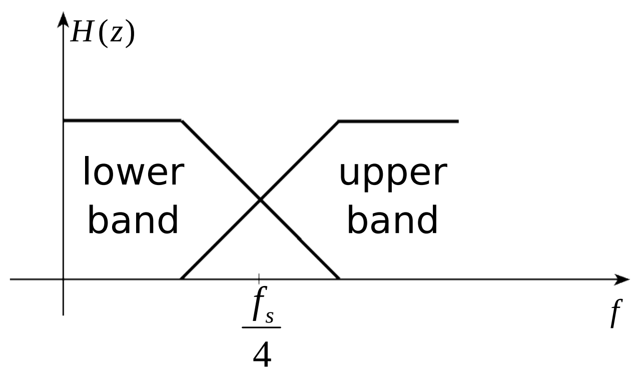
QMF (4)





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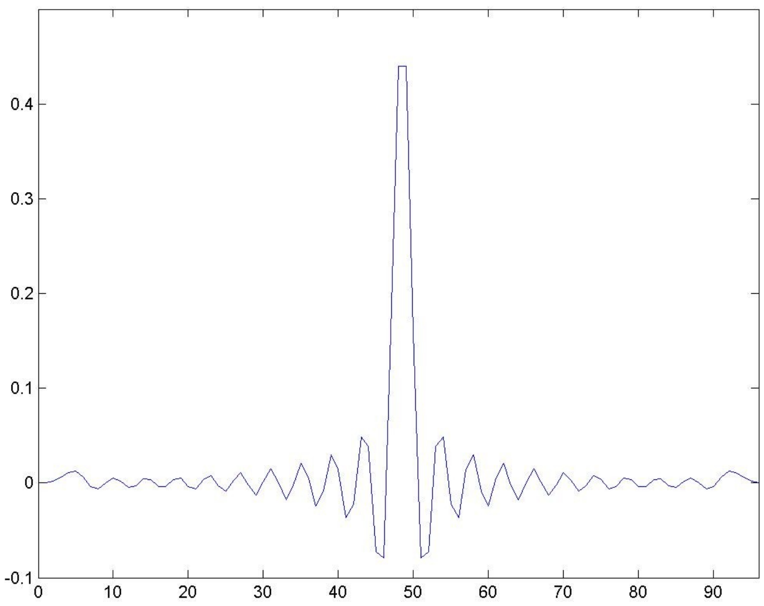
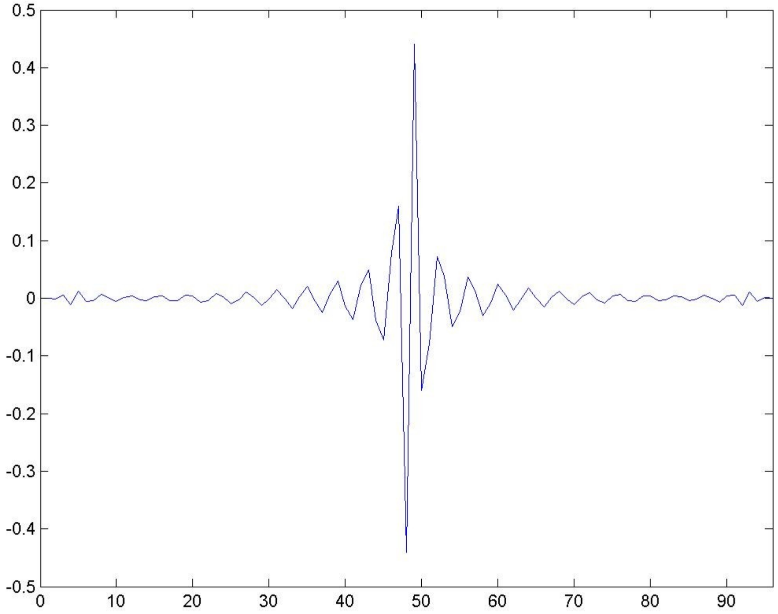
QMF: Example with Impulse Response of Length 96



Low-Pass



High-Pass

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CQMF (1): Conjugate QMF

- To also accommodate for the trading places of odd and even coefficients, a natural choice is to also reverse the temporal order of the synthesis filter.

$$h_1(n) = -h_0(L-1-n)(-1)^n$$

- With *L*: filter length, and

$$g_0(n) = h_0(n)$$

$$g_1(n) = -h_1(n)$$

- Introduced e.g. by Smith, Barnwell, 1984

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CQMF (2)

- For the polyphase components this means

$$H_{0,1}(z) = -z^{-L/2} H_{0,0}(z^{-1})$$

$$H_{1,1}(z) = z^{-L/2} H_{1,0}(z^{-1})$$

- And the input for our determinant calculation is

$$\underline{x}(z) = z^{-L/2} \left[H_{1,0}(z^{-1}), \quad H_{0,0}(z^{-1}) \right]$$

- This corresponds exactly to the time reversed low band filter!

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CQMF

- Let's define

$$A(z) = H_{1,0}(z^{-1}) H_{0,0}(z)$$

- The determinant is now

$$\det\left(\underline{\underline{H}}(z)\right) = H_{1,1}(z) H_{0,0}(z) - H_{0,1}(z) H_{1,0}(z)$$

$$= z^{-L/2} \left(A(z) + A(z^{-1})\right)$$

- Observe: This can be a constant if all even coefficients of *A(z)* are zero, except for the center coefficient!

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CQMF (3)

- Remember: the determinant was the output of the low band with this input

- Hence: Every second sample of the convolution of the low band filter with its time reversed version.

- This is equal to **every second value** of the **autocorrelation** function of the **low band filter**!

- Determinant is a constant or a delay: only one sample of this downsampled autocorrelation function (all even coefficients) can be unequal zero (most even coefficients are zero)

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CQMF (4)

- The Determinant is a constant means:
  - The zeroth autocorrelation coefficient is a constant (unequal 0), and all other even coefficients must be zero.
  - Called Nyquist filter property
  - -> Design method

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CQMF (5)

z-transform of
ACF of low pass
filter
-> power spectrum

- In other terms: Define *P(z)* as the z-transform of this autocorrelation function, the **Power Spectrum**:

$$P(z) := H_0(z) \cdot H_0(z^{-1})$$

- Then all nonzero coefficients of *P(z)* are the zeroth coefficient and the odd coefficients.
- As a result:

$$P(z) + \underbrace{P(-z)}_{\text{Frequency reversal}} \overset{\text{The odd coefficients cancel}}{=} const$$

This is also called the halfband filter property.

Design approach: Design *P(z)* accordingly, then *H(z)*

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Pseudo-QMF (PQMF)

- So far we only had 2 subband QMF filter banks
- Only for the 2-band case we get perfect reconstruction (in the CQMF case)
- The PQMF extents the QMF approach to *N>2* subbands
- But it has only „Near Perfect Reconstruction“, meaning a reconstruction error by the filter bank
- It is modulated filter band (like the MDCT), using a baseband prototype filter *h(n)* (a lowpass)

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PQMF

- Its analysis filters are given by the impulse responses (L being the length of the impulse response)

$$h_k(n)=h(n)\cos\left(\frac{\pi}{N}\cdot(k+0.5)(n+0.5-\frac{L}{2}+(-1)^k\frac{\pi}{4})\right)$$

- It is an (almost) **orthogonal filter bank**, which means that the synthesis filter impulse responses are simply the time inverses of the analysis impulse responses,

$$g_k(n)=h_k(L-1-n)$$

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PQMF

- Its baseband prototype filters h(n) are now designed such that aliasing cancels between adjacent neighbouring bands,

$$\left|H(e^{j\Omega})\right|^2 + \left|H(e^{j(\pi/N-\Omega)})\right|^2 = 1, \text{for } 0 < |\Omega| < \frac{\pi}{2N}$$

- beyond the adjacent bands, the attenuation should go towards infinity,

$$\left|H(e^{j\Omega})\right|^2 = 0, \text{for} |\Omega| > \frac{\pi}{N}$$

This leads to “Near Perfect Reconstruction” (there is a reconstruction error)

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PQMF

- The PQMF filter bank is used in MPEG1/2 Layer I and II and III. There it has *N=32* subbands and filter length *L=512*

- Also used in MPEG 4 for so-called SBR (Spectral Band Replication) and for parametric sourround coding. There it has *N=32* or *N=64* subbands, and filter length *L=320* or *L=640*

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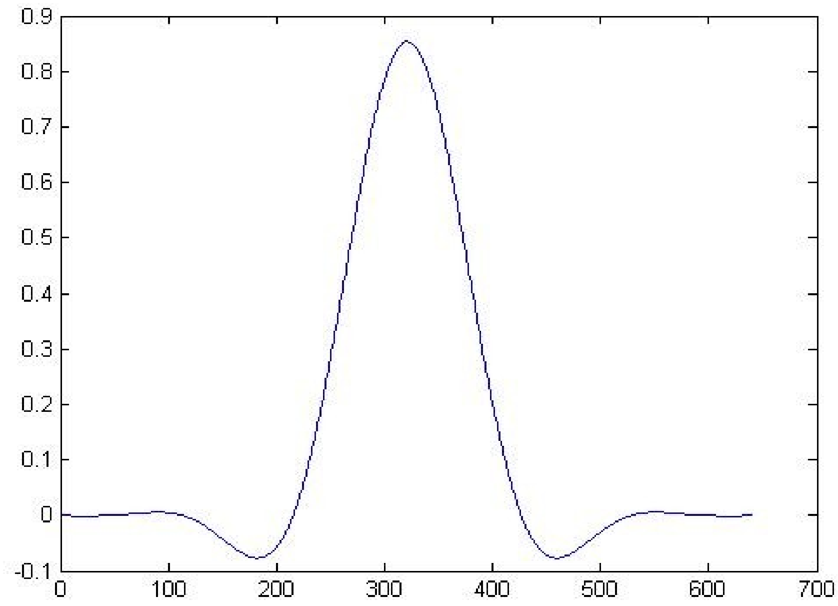
PQMF used in MPEG4

- Impulse response of the baseband prototype (the window), with *N=64* and *L=640*



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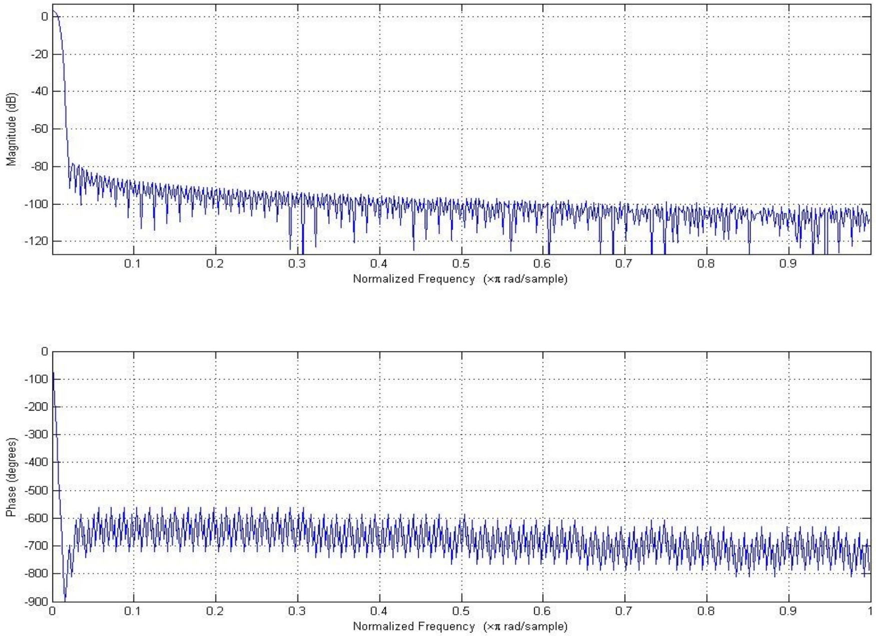
PQMF used in MPEG4

- Frequency response of the baseband prototype (the window)



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