Simplex Algorithm - Simple Tableau ex.

max $3x_1 + 2x_2 + 3x_3$

sub to: $-x_1 - 2x_2 - 3x_3 \leq 5$

$2x_1 + x_2 + x_3 \leq 3$ with $x_1, x_2, x_3 \geq 0$

$-x_1 - x_2 + x_3 \leq 1$

- Verify it is basic feasible (with all x_i set to 0, equation is valid) : It is
- Create slack variables (one per equation):

$x_4 = 5 + x_1 + 2x_2 + 3x_3$

$x_5 = 3 - 2x_1 - x_2 - x_3$

$x_6 = 1 + x_1 + x_2 - x_3$

- Create tableau based on slack equations:

$-x_1 - 2x_2 - 3x_3 + x_4 = 5$

$2x_1 + x_2 + x_3 + x_5 = 3$

$-x_1 - x_2 + x_3 + x_6 = 1$

(equation to maximize has inverted coefficients because magic)

basic var	x_1	x_2	x_3	x_4	x_5	x_6	Value	
X	-3	-2	-3	0	0	0	0	
x_4	-1	-2	-3	1	0	0	5	Line 1
x_5	2	1	1	0	1	0	3	Line 2
x_6	-1	-1	1	0	0	1	1	Line 3

(basic variables are originally set to be the slack variables)

- Pick a positive variable (meaning it is constrained/bounded: increasing its value in an equation gets the equation closer to its value) where said variable is neg. in obj function
  - Here available values are x_1, x_2 and x_3 in L_2 or x_3 in L_3
  - We see using x_1, it would be difficult to cancel out x_1 in the objective function (we would need to divide L_2 by 2 before doing our pivot, creating halves basically everywhere). Here, using x_2 seems easy enough. The variable must use its most constraining equation (cf next step)

- Every line must have, at the end of this step,
  - 0 in the row corresponding to the chosen var.
  - 1 in that row, if the line is the line used as pivot.
- Since L_2 is using x_2 as chosen variable, the basic variable of L_2 (used to get obtained point at the end) becomes x_2

	basic var	x_1	x_2	x_3	x_4	x_5	x_6	value	
	X	1	0	-1	0	2	0	6	$L_0 = L_0 + 2L_2$
	x_4	3	0	-1	1	2	0	11	$L_1 = L_1 + 2L_2$
$x_5 \to x_2$	$\boxed{x_2}$	2	1	1	0	1	0	3	
	x_6	1	0	2	0	1	1	4	$L_3 = L_3 + L_2$

- repeat until every variable is positive in obj. func.
  - Only variable available is x_3.
    - Pick its most constraining equation because it is positive in L_2 and L_3.
    - $L_2 \Leftrightarrow 2x_1 + x_2 + x_3 + x_5 = 3$

      $L_3 \Leftrightarrow x_1 + 2x_3 + x_5 + x_6 = 4$

      setting every x_i to 0, we see L_3 is more constraining : $L_2 \Leftrightarrow 0 + 0 + 1 \times 3 + 0 = 3$

      $L_3 \Leftrightarrow 0 + 2 \times 2 + 0 + 0 = 4$

      because $2 < 3$

	basic var	x_1	x_2	x_3	x_4	x_5	x_6	value	
	X	$\frac{3}{2}$	0	0	0	$\frac{5}{2}$	$\frac{1}{2}$	8	$L_0 \leftarrow L_0 + L_3$
	x_4	$\frac{7}{2}$	0	0	1	$\frac{5}{2}$	$\frac{1}{2}$	13	$L_1 \leftarrow L_1 + L_3$
	x_2	$\frac{3}{2}$	1	0	0	$1/2$	$-\frac{1}{2}$	1	$L_2 \leftarrow L_2 - L_3$
$x_6 \to x_3$	$\boxed{x_3}$	$1/2$	0	1	0	1	1	2	$L_3 \leftarrow L_3 \div 2$

- No more variable is negative in obj function !

  Obj value is reached in point (x_1, x_2, x_3) replacing basic variables with their values, others by 0. So here $(0, 1, 2)$