



G

Generating Boxes from Formulas

People who define new math fonts and/or macros sometimes need to know exactly how \TeX manipulates the constituents of formulas. The purpose of this appendix is to explain the precise positioning rules by which \TeX converts a math list into a horizontal list. (It is a good idea to review the introduction to math lists in Chapter 17 before reading further; “double dangerous bends” are implied throughout this appendix.)

\TeX relies on lots of parameters when it typesets formulas, and you have the option of changing any or all of them. But of course you will want to know what each parameter means, before you change it. Therefore each rule below is numbered, and a table appears at the end to show which rules depend on which parameters.

The most important parameters appear in the symbol fonts (family 2) and the extension fonts (family 3). \TeX will not typeset a formula unless \textfont2 , \scriptfont2 , and $\text{\scriptscriptfont2}$ each contain at least 22 \fontdimen parameters. For brevity we shall call these parameters σ_1 to σ_{22} , where the parameter is taken from \textfont2 if the current style is display or text (D or D' or T or T'), from \scriptfont2 if the current style is S or S' , and from $\text{\scriptscriptfont2}$ otherwise. Similarly, the three fonts in family 3 must each have at least 13 \fontdimen parameters, and we will denote them by ξ_1 to ξ_{13} . The notation ξ_9 , for example, stands for the ninth parameter of \scriptfont3 , if \TeX is typesetting something in \scriptstyle .

A math list is a sequence of items of the various kinds listed in Chapter 17, and \TeX typesets a formula by converting a math list to a horizontal list. When such typesetting begins, \TeX has two other pieces of information in addition to the math list itself. (a) The starting style tells what style should be used for the math list, unless another style is specified by a style item. For example, the starting style for a displayed formula is D , but for an equation in the text or an equation number it is T ; and for a subformula it can be any one of the eight styles defined in Chapter 17. We shall use C to stand for the current style, and we shall say that the math list is being typeset in style C . (b) The typesetting is done either with or without penalties. Formulas in the text of a paragraph are converted to horizontal lists in which additional penalty items are inserted after binary operations and relations, in order to aid in line breaking. Such penalties are not inserted in other cases, because they would serve no useful function.

The eight styles are considered to be $D > D' > T > T' > S > S' > SS > SS'$, in decreasing order. Thus, $C \leq S$ means that the current style is S , S' , SS , or SS' . Style C' means the current style with a prime added if one isn't there; for example, we have $C' = T'$ if and only if $C = T$ or $C = T'$. Style $C\uparrow$ is the superscript style for C ; this means style S if C is D or T , style S' if C is D' or T' , style SS if C is S or SS , and style SS' if C is S' or SS' . Finally, style $C\downarrow$ is the subscript style, which is $(C\uparrow)'$.

Chapter 17 stated that the most important components of math lists are called atoms, and that each atom has three fields called its nucleus, subscript, and superscript. We frequently need to execute a subroutine called “Set box x to the so-and-so field in style such-and-such.” This means (a) if the specified field is empty, x is set equal to a null box; (b) if the field contains a symbol, x is set to an hbox containing that symbol in the appropriate size, and the italic correction for the character is included in the width of the box; (c) if the field contains a math list or horizontal list, x is set to an hbox containing the result of typesetting that list with the specified starting style. In case (c), the glue is set with no stretching or shrinking, and an additional level of boxing is omitted if it turns out to be redundant.

Another subroutine sets box x to a specified variable delimiter, having a specified minimum height plus depth. This means that a search is conducted as follows:

math lists
symbol fonts
extension fonts
textfont
scriptfont
scriptscriptfont
fontdimen
styles
italic correction
delimiter

The delimiter is defined by two symbols, a “small character” a in family f and a “large character” b in family g . The search looks first at character a in scriptscriptfont f , if $C \leq SS$; then it looks at a in scriptfont f , if $C \leq S$; then it looks at a in textfont f . If nothing suitable is found from a and f , the larger alternative b and g is examined in the same way. Either (a, f) or (b, g) may be $(0, 0)$, which means that the corresponding part of the search is to be bypassed. When looking at a character in a font, the search stops immediately if that character has sufficient height plus depth, or if the character is extensible; furthermore, if the character does not stop the search, and if it has a successor in the font, the successor is looked at next. (See the METAFONT manual or the system documentation of **tfm** files for further information about successors and extensible characters.) If the search runs all the way to completion without finding a suitable character, the one with greatest height plus depth is chosen. If no characters at all were found (either because $a = f = b = g = 0$ or because the characters did not exist in the fonts), x is set to an empty box whose width is `\nulldelimiterspace`. If an extensible character was found, x is set to a vbox containing enough pieces to build up a character of sufficient size; the height of this vbox is the height of the topmost piece, and the width is the width of the repeatable piece. Otherwise x is set to an hbox containing the character that was found; the italic correction of the character is included in the width of this box.

There’s also a subroutine that “reboxes” a given box to a given width. If the box doesn’t already have the desired width, \TeX unpackages it (unless it was a vbox), then adds a kern for an italic correction if one was implied, and inserts `\hss` glue at both left and right; the resulting horizontal list is packaged into an hbox. This process is used, for example, to give a common width to the numerator and denominator of a fraction; it centers whichever is smaller, unless infinite glue is present in addition to the newly added `\hss`.

If x is a box, we shall use the abbreviations $h(x)$, $d(x)$, and $w(x)$ for its height, depth, and width, respectively.

Here now are the rules for typesetting a given math list in starting style C . The process applies from left to right, translating each item in turn. Two passes are made over the list; most of the work is done by the first pass, which compiles individual translations of the math items. We shall consider this part of the task first:

1. If the current item is a rule or discretionary or penalty or “whatsit” or boundary item, simply leave it unchanged and move to the next item.
2. If the current item is glue or a kern, translate it as follows: If it is glue from `\nonscript`, check if the immediately following item is glue or a kern; and if so, remove that item if $C \leq S$. Otherwise, if the current item is from `\mskip` or `\mkern`, convert from `mu` to absolute units by multiplying each finite dimension by $\frac{1}{18}\sigma_6$. Then move on to the next item.
3. If the current item is a style change, set C to the specified style. Delete the current item from the list and move on to the next.
4. If the current item is a four-way choice, it contains four math lists for the four main styles. Replace it by the math list that corresponds to the current style C , then move to the first unprocessed item.
5. If the current item is a Bin atom, and if this was the first atom in the list, or if the most recent previous atom was Bin, Op, Rel, Open, or Punct, change the current Bin to Ord and continue with Rule 14. Otherwise continue with Rule 17.

extensible
successor
METAFONT
nulldelimiterspace
built-up characters
hss
nonscript
mskip
mkern

6. If the current item is a Rel or Close or Punct atom, and if the most recent previous atom was Bin, change that previous Bin to Ord. Continue with Rule 17.

7. If the current item is an Open or Inner atom, go directly to Rule 17.

8. If the current item is a Vcent atom (from `\vcenter`), let its nucleus be a vbox of height-plus-depth v . Change the height to $\frac{1}{2}v + a$ and the depth to $\frac{1}{2}v - a$, where a is the axis height, σ_{22} . Change this atom to type Ord and continue with Rule 17.

9. If the current item is an Over atom (from `\overline`), set box x to the nucleus in style C' . Then replace the nucleus by a vbox containing kern θ , hrule of height θ , kern 3θ , and box x , from top to bottom, where $\theta = \xi_8$ is the default rule thickness. (This puts a rule over the nucleus, with 3θ clearance, and with θ units of extra white space assumed to be present above the rule.) Continue with Rule 16.

10. If the current item is an Under atom (from `\underline`), set box x to the nucleus in style C . Then replace the nucleus by a vbox made from box x , kern 3θ , and hrule of height θ , where $\theta = \xi_8$ is the default rule thickness; and add θ to the depth of the box. (This puts a rule under the nucleus, with 3θ clearance, and with θ units of extra white space assumed to be present below the rule.) Continue with Rule 16.

11. If the current item is a Rad atom (from `\radical`, e.g., `\sqrt`), set box x to the nucleus in style C' . Let $\theta = \xi_8$; and let $\varphi = \sigma_5$ if $C > T$, otherwise $\varphi = \theta$. Set $\psi = \theta + \frac{1}{4}|\varphi|$; this is the minimum clearance that will be allowed between box x and the rule that will go above it. Set box y to a variable delimiter for this radical atom, having height plus depth $h(x) + d(x) + \psi + \theta$ or more. Then set $\theta \leftarrow h(y)$; this is the thickness of the rule to be used in the radical construction. (Note that the font designer specifies the thickness of the rule by making it the height of the radical character; the baseline of the character should be precisely at the bottom of the rule.) If $d(y) > h(x) + d(x) + \psi$, increase ψ by half of the excess; i.e., set $\psi \leftarrow \frac{1}{2}(\psi + d(y) - h(x) - d(x))$. Construct a vbox consisting of kern θ , hrule of height θ , kern ψ , and box x , from top to bottom. The nucleus of the radical atom is now replaced by box y raised by $h(x) + \psi$, followed by the new vbox. Continue with Rule 16.

12. If the current item is an Acc atom (from `\mathaccent`), just go to Rule 16 if the accent character doesn't exist in the current size. Otherwise set box x to the nucleus in style C' , and set u to the width of this box. If the nucleus is not a single character, let $s = 0$; otherwise set s to the kern amount for the nucleus followed by the `\skewchar` of its font. If the accent character has a successor in its font whose width is $\leq u$, change it to the successor and repeat this sentence. Now set $\delta \leftarrow \min(h(x), \chi)$, where χ is `\fontdimen5` (the x-height) in the accent font. If the nucleus is a single character, replace box x by a box containing the nucleus together with the superscript and subscript of the Acc atom, in style C , and make the sub/superscripts of the Acc atom empty; also increase δ by the difference between the new and old values of $h(x)$. Put the accent into a new box y , including the italic correction. Let z be a vbox consisting of: box y moved right $s + \frac{1}{2}(u - w(y))$, kern $-\delta$, and box x . If $h(z) < h(x)$, add a kern of $h(x) - h(z)$ above box y and set $h(z) \leftarrow h(x)$. Finally set $w(z) \leftarrow w(x)$, replace the nucleus of the Acc atom by box z , and continue with Rule 16.

13. If the current item is an Op atom, mark this atom as having limits if it has been marked with `\limits`, or if it has been marked with `\displaylimits` and $C > T$. If the nucleus is not a symbol, set $\delta \leftarrow 0$ and go to Rule 13a. Otherwise if $C > T$ and if the nucleus symbol has a successor in its font, move to the successor. (This is where

`vcenter`
`overline`
 default rule thickness
`underline`
`radical`
`sqrt`
`mathaccent`
`skewchar`
`x-height`
`limits`
`displaylimits`

operators like \sum and \int change to a larger size in display styles.) Put the symbol into a new box x , in the current size, and set δ to the italic correction for the character; include δ in the width of box x if and only if limits are to be set or there is no subscript. Shift box x down by $\frac{1}{2}(h(x) - d(x)) - a$, where $a = \sigma_{22}$, so that the operator character is centered vertically on the axis; this shifted box becomes the nucleus of the Op atom.

over
overwithdelims
atop
atopwithdelims
above
abovewithdelims

13a. If limits are not to be typeset for this Op atom, go to Rule 17; otherwise the limits are attached as follows: Set box x to the superscript field in style $C\uparrow$; set box y to the nucleus field in style C ; and set box z to the subscript field in style $C\downarrow$. Rebox all three of these boxes to width $\max(w(x), w(y), w(z))$. If the superscript field was not empty, attach box x above box y , separated by a kern of size $\max(\xi_9, \xi_{11} - d(x))$, and shift box x right by $\frac{1}{2}\delta$; also put a kern of size ξ_{13} above box x . If the subscript field was not empty, attach box z below box y , separated by a kern of size $\max(\xi_{10}, \xi_{12} - h(z))$, and shift box z left by $\frac{1}{2}\delta$; also put a kern of size ξ_{13} below box z . The resulting vbox becomes the nucleus of the current Op atom; move to the next item.

14. If the current item is an Ord atom, go directly to Rule 17 unless all of the following are true: The nucleus is a symbol; the subscript and superscript are both empty; the very next item in the math list is an atom of type Ord, Op, Bin, Rel, Open, Close, or Punct; and the nucleus of the next item is a symbol whose family is the same as the family in the present Ord atom. In such cases the present symbol is marked as a text symbol. If the font information shows a ligature between this symbol and the following one, using the specified family and the current size, then insert the ligature character and continue as specified by the font; in this process, two characters may collapse into a single Ord text symbol, and/or new Ord text characters may appear. If the font information shows a kern between the current symbol and the next, insert a kern item following the current atom. As soon as an Ord atom has been fully processed for ligatures and kerns, go to Rule 17.

15. If the current item is a generalized fraction (and it had better be, because that's the only possibility left if Rules 1–14 don't apply), let θ be the thickness of the bar line and let (λ, ρ) be the left and right delimiters. If this fraction was generated by `\over` or `\overwithdelims`, then $\theta = \xi_8$; if it was generated by `\atop` or `\atopwithdelims`, $\theta = 0$; otherwise it was generated by `\above` or `\abovewithdelims`, and a specific value of θ was given at that time. The values of λ and ρ are null unless the fraction is “with delims.”

15a. Put the numerator into box x , using style T or T' if C is D or D' , otherwise using style $C\uparrow$. Put the denominator into box z , using style T' if $C > T$, otherwise using $C\downarrow$. If $w(x) < w(z)$, rebox x to width $w(z)$; if $w(z) < w(x)$, rebox z to width $w(x)$.

15b. If $C > T$, set $u \leftarrow \sigma_8$ and $v \leftarrow \sigma_{11}$. Otherwise set $u \leftarrow \sigma_9$ or σ_{10} , according as $\theta \neq 0$ or $\theta = 0$, and set $v \leftarrow \sigma_{12}$. (The fraction will be typeset with its numerator shifted up by an amount u with respect to the current baseline, and with the denominator shifted down by v , unless the boxes are unusually large.)

15c. If $\theta = 0$ (`\atop`), the numerator and denominator are combined as follows: Set $\varphi \leftarrow 7\xi_8$ or $3\xi_8$, according as $C > T$ or $C \leq T$; φ is the minimum clearance that will be tolerated between numerator and denominator. Let $\psi = (u - d(x)) - (h(z) - v)$ be the actual clearance that would be obtained with the current values of u and v ; if $\psi < \varphi$, add $\frac{1}{2}(\varphi - \psi)$ to both u and v . Then construct a vbox of height $h(x) + u$ and depth $d(z) + v$, consisting of box x followed by an appropriate kern followed by box z .

15d. If $\theta \neq 0$ (`\over`), the numerator and denominator are combined as follows: Set $\varphi \leftarrow 3\theta$ or θ , according as $C > T$ or $C \leq T$; φ is the minimum clearance that will be tolerated between numerator or denominator and the bar line. Let $a = \sigma_{22}$ be the current axis height; the middle of the bar line will be placed at this height. If $(u - d(x)) - (a + \frac{1}{2}\theta) < \varphi$, increase u by the difference between these quantities; and if $(a - \frac{1}{2}\theta) - (h(z) - v) < \varphi$, increase v by the difference. Finally construct a vbox of height $h(x) + u$ and depth $d(z) + v$, consisting of box x followed by a kern followed by an hrule of height θ followed by another kern followed by box z , where the kerns are figured so that the bottom of the hrule occurs at $a - \frac{1}{2}\theta$ above the baseline.

scriptspace

15e. Enclose the vbox that was constructed in Rule 15c or 15d by delimiters whose height plus depth is at least σ_{20} , if $C > T$, and at least σ_{21} otherwise. Shift the delimiters up or down so that they are vertically centered with respect to the axis. Replace the generalized fraction by an Inner atom whose nucleus is the resulting sequence of three boxes (left delimiter, vbox, right delimiter).

Rules 1–15 account for the preliminary processing of math list items; but we still haven’t specified how subscripts and superscripts are to be typeset. Therefore some of those rules lead to the following post-process:

16. Change the current item to an Ord atom, and continue with Rule 17.

17. If the nucleus of the current item is a math list, replace it by a box obtained by typesetting that list in the current style. Then if the nucleus is not simply a symbol, go on to Rule 18. Otherwise we are in the common case that a math symbol is to be translated to its horizontal-list equivalent: Convert the symbol to a character box for the specified family in the current size. If the symbol was not marked by Rule 14 above as a text symbol, or if `\fontdimen` parameter number 2 of its font is zero, set δ to the italic correction; otherwise set δ to zero. If δ is nonzero and if the subscript field of the current atom is empty, insert a kern of width δ after the character box, and set δ to zero. Continue with Rule 18.

18. (The remaining task for the current atom is to attach a possible subscript and superscript.) If both subscript and superscript fields are empty, move to the next item. Otherwise continue with the following subrules:

18a. If the translation of the nucleus is a character box, possibly followed by a kern, set u and v equal to zero; otherwise set $u \leftarrow h - q$ and $v \leftarrow d + r$, where h and d are the height and depth of the translated nucleus, and where q and r are the values of σ_{18} and σ_{19} in the font corresponding to styles $C\uparrow$ and $C\downarrow$. (The quantities u and v represent minimum amounts by which the superscript and subscript will be shifted up and down; these preliminary values of u and v may be increased later.)

18b. If the superscript field is empty (so that there is a subscript only), set box x to the subscript in style $C\downarrow$, and add `\scriptspace` to $w(x)$. Append this box to the translation of the current item, shifting it down by $\max(v, \sigma_{16}, h(x) - \frac{4}{5}|\sigma_5|)$, and move to the next item. (The idea is to make sure that the subscript is shifted by at least v and by at least σ_{16} ; furthermore, the top of the subscript should not extend above $\frac{4}{5}$ of the current x-height.)

18c. Set box x to the superscript field in style $C\uparrow$, and add `\scriptspace` to $w(x)$. Then set $u \leftarrow \max(u, p, d(x) + \frac{1}{4}|\sigma_5|)$, where $p = \sigma_{13}$ if $C = D$, $p = \sigma_{15}$ if $C = C'$, and $p = \sigma_{14}$ otherwise; this gives a tentative position for the superscript.

18d. If the subscript field is empty (so that there is a superscript only), append box x to the translation of the current atom, shifting it up by u , and move to the next item. Otherwise (i.e., both subscript and superscript are present), set box y to the subscript in style $C\downarrow$, add `\scriptspace` to $w(y)$, and set $v \leftarrow \max(v, \sigma_{17})$.

18e. (The remaining task is to position a joint subscript/superscript combination.) Let $\theta = \xi_8$ be the default rule thickness. If $(u - d(x)) - (h(y) - v) \geq 4\theta$, go to Rule 18f. (This means that the white space between subscript and superscript is at least 4θ .) Otherwise reset v so that $(u - d(x)) - (h(y) - v) = 4\theta$. Let $\psi = \frac{4}{5}|\sigma_5| - (u - d(x))$. If $\psi > 0$, increase u by ψ and decrease v by ψ . (This means that the bottom of the superscript will be at least as high above the baseline as $\frac{4}{5}$ of the x-height.)

18f. Finally, let δ be zero unless it was set to a nonzero value by Rules 13 or 17. (This is the amount of horizontal displacement between subscript and superscript.) Make a vbox of height $h(x) + u$ and depth $d(y) + v$, consisting of box x shifted right by δ , followed by an appropriate kern, followed by box y . Append this vbox to the translation of the current item and move to the next.

`delimiterfactor`
`delimitershortfall`
`thinmuskip`
`medmuskip`
`thickmuskip`
`nonscript`
`binoppenalty`
`relpenalty`

After the entire math list has been processed by Rules 1–18, \TeX looks at the last atom (if there was one), and changes its type from Bin to Ord (if it was of type Bin). Then the following rule is performed:

19. If the math list begins and ends with boundary items, compute the maximum height h and depth d of the boxes in the translation of the math list that was made on the first pass, taking into account the fact that some boxes may be raised or lowered. Let $a = \sigma_{22}$ be the axis height, and let $\delta = \max(h - a, d + a)$ be the amount by which the formula extends away from the axis. Replace the boundary items by delimiters whose height plus depth is at least $\max(\lfloor \delta/500 \rfloor f, 2\delta - l)$, where f is the `\delimiterfactor` and l is the `\delimitershortfall`. Shift the delimiters up or down so that they are vertically centered with respect to the axis. Change the left boundary item to an Open atom and the right boundary item to a Close atom. (All of the calculations in this step are done with C equal to the starting style of the math list; style items in the middle of the list do not affect the style of the right boundary item.)

20. Rules 1–19 convert the math list into a sequence of items in which the only remaining atoms are of types Ord, Op, Bin, Rel, Open, Close, Punct, and Inner. After that conversion is complete, a second pass is made through the entire list, replacing all of the atoms by the boxes and kerns in their translations. Furthermore, additional inter-element spacing is inserted just before each atom except the first, based on the type of that atom and the preceding one. Inter-element spacing is defined by the three parameters `\thinmuskip`, `\medmuskip`, and `\thickmuskip`; the μ units are converted to absolute units as in Rule 2 above. Chapter 18 has a chart that defines the inter-element spacing, some of which is `\nonscript`, i.e., it is inserted only in styles $> S$. The list might also contain style items, which are removed during the second pass; they are used to change the current style just as in the first pass, so that both passes have the same value of C when they work on any particular atom.

21. Besides the inter-element spacing, penalties are placed after the translation of each atom of type Bin or Rel, if the math list was part of a paragraph. The penalty after a Bin is `\binoppenalty`, and the penalty after a Rel is `\relpenalty`. However, the penalty is not inserted after the final item in the entire list, or if it has a numeric

value ≥ 10000 , or if the very next item in the list is already a penalty item, or after a Rel atom that is immediately followed by another Rel atom.

22. After all of the preceding actions have been performed, the math list has been totally converted to a horizontal list. If the result is being inserted into a larger horizontal list, in horizontal mode or restricted horizontal mode, it is enclosed by “math-on” and “math-off” items that each record the current value of `\mathsurround`. Or if this list is a displayed formula, it is processed further as explained in Chapter 19.

math-on
math-off
mathsurround
VOLTAIRE
SMITH

Summary of parameter usage. Here is the promised index that refers to everything affected by the mysterious parameters in the symbol fonts. Careful study of the rules allows you to get the best results by appropriately setting the parameters for new fonts that you may wish to use in mathematical typesetting. Each font parameter has an external name that is used in supporting software packages; for example, σ_{14} is generally referred to as ‘sup2’ and ξ_8 as ‘default_rule_thickness’. These external names are indicated in the table.

<i>Parameter</i>	<i>Used in</i>	<i>Parameter</i>	<i>Used in</i>
σ_5 x_height	11, 18b, 18c, 18e	σ_{18} sup_drop	18a
σ_6 quad	2, 20	σ_{19} sub_drop	18a
σ_8 num1	15b	σ_{20} delim1	15e
σ_9 num2	15b	σ_{21} delim2	15e
σ_{10} num3	15b	σ_{22} axis_height	8, 13, 15d, 19
σ_{11} denom1	15b	ξ_8 default_rule_thickness	9, 10, 11, 15, 15c, 18e
σ_{12} denom2	15b	ξ_9 big_op_spacing1	13a
σ_{13} sup1	18c	ξ_{10} big_op_spacing2	13a
σ_{14} sup2	18c	ξ_{11} big_op_spacing3	13a
σ_{15} sup3	18c	ξ_{12} big_op_spacing4	13a
σ_{16} sub1	18b	ξ_{13} big_op_spacing5	13a
σ_{17} sub2	18d		

Besides the symbol and extension fonts (families 2 and 3), the rules above also refer to parameters in arbitrary families: Rule 17 uses `\fontdimen` parameter 2 (space) to determine whether to insert an italic correction between adjacent letters, and Rule 12 uses parameter 5 (x_height) to position an accent character. Several non-font parameters also affect mathematical typesetting: dimension parameters `\delimitershortfall` (Rule 19), `\nulldelimiterspace` (in the construction of variable delimiters for Rules 11, 15e, 19), `\mathsurround` (Rule 22), and `\scriptspace` (Rules 18bcd); integer parameters `\delimiterfactor` (Rule 19), `\binoppenalty` (Rule 21), and `\relpenalty` (Rule 21); muglue parameters `\thinmuskup`, `\medmuskup`, and `\thickmuskup` (Rule 20).

*Woe to the author who always wants to teach!
The secret of being a bore is to tell everything.*
— VOLTAIRE, *De la Nature de l'Homme* (1737)

*Very few Compositors are fond of Algebra,
and rather chuse to be employed upon plain work.*
— JOHN SMITH, *The Printer's Grammar* (1755)