DiffStream: Differential Output Testing for Stream Processing Programs

ANONYMOUS AUTHOR(S)

High performance architectures for processing distributed data streams, such as Flink, Spark, and Storm, are increasingly deployed in emerging data-driven computing systems. Exploiting the parallelism afforded by such platforms, while preserving the semantics of the desired computation, is prone to errors, and motivates the development of tools for specification, testing, and verification. We focus on the problem of differential output testing for distributed stream processing systems, that is, checking whether two implementations produce equivalent output streams in response to a given input stream. The notion of equivalence allows reordering of logically independent data items, and the main technical contribution of the paper is an optimal online algorithm for checking this equivalence. Our testing framework is implemented as a library called DiffStream in Flink. We present four case studies to illustrate how our framework can be used to (1) correctly identify bugs in a set of benchmark MapReduce programs, (2) facilitate the development of difficult-to-parallelize high performance applications, and (3) monitor an application for a long period of time using minimal computational resources.

1 INTRODUCTION

In recent years, a dramatic increase in real-time data processing needs has led to the popularity of high performance distributed stream processing systems, such as Flink [Carbone et al. 2015], Samza [Noghabi et al. 2017], Spark Streaming [Zaharia et al. 2013], Storm [Apache 2019], and Twitter Heron [Kulkarni et al. 2015]. While these systems achieve high performance through aggressive parallelization of data operators, programs written in such frameworks are nevertheless inherently concurrent and consequently prone to errors. As a result, there is a growing need for tools that make it easier to write correct programs in such systems, through specification, testing, and verification.

In a broader context, the problem of ensuring correctness of distributed programs is long established, as can be seen by the significant amount of past and ongoing work on the correctness of distributed protocols (e.g., Chand et al. [2016]; Padon et al. [2017, 2016]), concurrent data structures (e.g., Burckhardt et al. [2007, 2014]), and distributed systems (e.g., Hawblitzel et al. [2015]; Ozkan et al. [2018]; Wilcox et al. [2015]). In general, because this body of work addresses the challenges of verification of distributed protocols and low-level primitives, it targets experts who design and implement complex distributed systems. However, data-parallel programming frameworks, such as stream processing systems, aim to offer a simplified view of distributed programming where low level coordination and protocols are hidden from the programmer. This has the advantage of bringing distributed programming to a wider audience of end-users, and at the same time it requires new tools that *can be used by such end-users* (rather than just experts) to automatically test for correctness.

Unfortunately, there is limited work in testing for stream processing programs; in fact, the state of the art in practice is unit and integration testing [Vianna et al. 2019]. In order to bridge this gap and provide support for checking correctness to end-users, we focus on the problem of *differential testing* for distributed stream processing systems, that is, checking whether the outputs produced by two implementations in response to an input stream are equivalent. Differential testing [Evans and Savoia 2007; Groce et al. 2007; McKeeman 1998], allows for a simple specification of the correct program behavior, in contrast to more primitive testing techniques, where the

2020. 2475-1421/2020/1-ART1 $15.00
https://doi.org/

specification is either very coarse (i.e. the application doesn't crash) or very limited (i.e. on a given input, the application should produce a specific output). More precisely, differential testing allows for a reference implementation to be the specification. This is especially useful in the context of distributed stream processing systems, since bugs introduced due to distribution can be caught by comparing a sequential and a distributed implementation. In addition, having a reference implementation as a specification allows testing with random inputs, since there is no need to specify the expected output.

We identify two critical challenges in testing stream processing programs. The first challenge is dealing with output events that are out-of-order due to parallelism. In particular, for differential testing, two implementations might produce events in different rates, asynchronously, and out-of-order. However, the order between specific output events might not affect the downstream consumer (e.g. events with timestamp less than t can arrive in any order, as long as they arrive before the watermark t), therefore requiring the notion of equivalent streams to be relaxed to allow for out-of-order events. In fact, lack of order is often desirable, since it enables parallelism.

The second challenge is that stream processing systems, in contrast to batch processing systems, are designed to process input data that would not fit in memory. As best practice, it is recommended that applications written in these systems are tested under heavy load for long periods of time, to match the conditions that are expected after deployment [Vianna et al. 2019]. Achieving this requires that the testing framework itself is an online algorithm, in the sense that it processes output events as they arrive, and that the computational overhead is minimal.

To address these challenges, we propose a matching algorithm that incrementally compares two streams for equivalence. Following the approach of Mamouras et al. [2019], in our solution, ordering requirements between pairs of events are abstracted in a *dependence relation* that indicates when the ordering of two specific events is of significance to the output consumer. Given *any* dependence relation provided by the user, the algorithm determines, in an online fashion, whether the streams are equivalent up to the reorderings allowed by the dependence relation. We show that the algorithm is correct and that it reaches a verdict at the earliest possible time (Theorem 4.8). We also prove that the algorithm is optimal, in the sense that it keeps a minimal amount of space: any correct online algorithm must store at least as much information (Theorem 4.9).

We have implemented DiffStream, a differential testing library for Apache Flink that incorporates our algorithm. DiffStream is implemented in Java, and it can be used alongside existing testing frameworks such as JUnit [2019], or in stand-alone Flink programs. In order to evaluate the effectiveness and usability of the proposed testing framework we have conducted a series of case studies.

First, we evaluate the effectiveness of the framework on a set of nondeterministic MapReduce programs adapted from Xiao et al. [2014]. For some of these programs nondeterminism constitutes a bug, while for others it is acceptable, depending on input assumptions and application requirements. Using our framework, we demonstrate that tests can be written to successfully detect 5 out of 5 bugs (true positives), and to avoid flagging 5 out of 7 bug-free programs (false positives). This improves on previous work [Xu et al. 2013b], which would generally flag all nondeterministic programs as buggy, thus suffering from false positives.

Second, we design two specific use cases to illustrate the benefits of using DiffStream to design and implement parallel Flink applications. We consider a difficult-to-parallelize application which requires event-based windowing: we show that it is significantly more difficult (requiring twice as many lines of code) to effectively parallelize this application using Flink, and we show how our framework can be used to test and correctly implement such an application. We also evaluate the effort needed to write tests for an example computation with a subtle bug. The choice of specific programs we consider are explained in more details in Section 2.

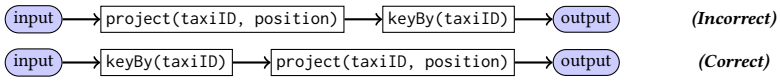


Fig. 1. A subtle consequence of implicit parallelism.

Finally, we demonstrate that the matching algorithm is efficient in practice, and can be used in an online monitoring setting, by monitoring two implementations of the Yahoo Streaming Benchmark [Chintapalli et al. 2016] over the span of two hours. After a short initial adaptation, the number of unmatched items that the matcher has at any point is less than 600 (out of 30K items that are output every second).

In total, the main contributions of this work are:

- A new testing methodology for specifying ordering requirements in stream processing programs. (Section 3)
- An optimal online matching algorithm for differential testing of stream processing programs which uniformly handles data with differing ordering requirements. (Section 4)
- DiffStream, a differential testing library for testing Apache Flink applications based on the online matching algorithm, which is being made available as an open source repository, together with a series of case studies to evaluate its usability and effectiveness. (Section 5)

2 OVERVIEW

2.1 Motivating Examples

Programs written in distributed stream processing frameworks exhibit implicit parallelism, which can lead to subtle bugs. Programs in such frameworks are usually written as *dataflow graphs*, where the edges are data streams and the nodes are streaming operators or transformations. Common operators include stateless transformations (*map*), and operations that group events based on the value of some field (*key-by*). For example, suppose that we have a single input stream which contains information about rides of a taxi service: each input event (`id`, `pos`, `meta`) consists of a taxi identifier, the taxi position, and some metadata. In the first stage, we want to discard the metadata component (*map*) and partition the data by taxi ID (*key-by*). In the second stage, we want to aggregate this data to report the total distance traveled by each taxi. Notice that the second stage is order-dependent (events for each taxi need to arrive in order), so it is important that the first stage does not disrupt the ordering of events for a particular taxi ID.

To make a program for the first stage of this computation in a distributed stream processing framework such as Flink or Storm, we need to build a dataflow graph representing a sequence of transformations on data streams. A first (natural) attempt to write the program is given in Fig. 1 (top). Here, the `project` node projects the data to only the fields we are interested in; in this case, `taxiID` and `position`. And `keyBy` (also known as "group by" in SQL-like languages, or the concept of a "stream grouping" in Storm) partitions the data stream into substreams by `taxiID`.

The first attempt is incorrect, however, because it fails to preserve the order of data for a particular key (taxi ID), which is required for the second stage of the computation. The problem is that dataflow graph operators are implicitly parallelized—here, the stateless map `project` is internally replicated into several copies, and the events of the input stream are divided among the copies. Because input events of the same key may get split across substreams, when the operator `keyBy` reassigns each item to a new partition based on its key, if items of a particular key were previously split up, then they might get reassembled in the wrong order.

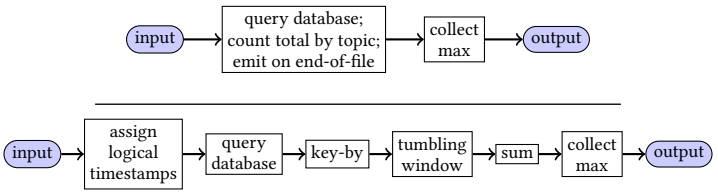


Fig. 2. A difficult-to-parallelize computation (top), and the parallel version.

This issue can be addressed by ensuring that parallelization is done only on the basis of `taxiID` *from the beginning of the pipeline*. For instance, this can be accomplished in Flink simply by reversing the `project` and `keyBy` transformations, as in Fig. 1 (bottom). In Storm, instead of an explicit `keyBy` operator one would implicitly construct such an operator by setting the input stream to grouped by key. Although the two programs are equivalent when the `project` operation is not parallelized, the second lacks the undesirable behavior in the presence of parallelism: assuming the `project` operation has the same level of parallelism as `keyBy`, Flink will continue to use the same partition of the stream to compute the projection, so data for each key will be kept in-order. The same fix will work in any other framework which guarantees that the same key-based partitioning is used between stages.

We have seen that even simple programs can exhibit counterintuitive behavior. In practice, programs written to exploit parallelism are often much more complex. To illustrate this, consider a single input stream consisting of very large documents, where we want to assign a topic to each document. The documents are streamed word by word and delineated by end-of-file markers. The topic of each word is specified in a precomputed database, and the topic of a document is defined to be the most frequent topic among words in that document.

In this second example querying the database is a costly operation, so it is desirable to parallelize by partitioning the words within each document into substreams. However, the challenge is to do so in a way that allows for the end-of-file markers to act as *barriers*, so that we re-group and output the summary at the end of each document. Although a sequential solution for this problem is easy, the simplest solution we have found in Flink that exploits parallelism uses about twice as many lines of code (Fig. 2). We also consulted with Flink users on the Flink mailing list, and we were not able to come up with a simpler solution. This example is explored in detail in Section 5.2. The additional complexity in developing the parallel solution, which requires changing the dataflow structure and not simply tuning some parameter, further motivates the need for differential testing.

2.2 Solution Architecture

These examples motivate the need for some form of testing to determine the correctness of distributed stream processing applications. We propose *differential testing* of the sequential and parallel versions: as the parallel solution might be much more involved, this helps validate that parallelization was done correctly and did not introduce bugs.

In the example of Fig. 1, the programmer begins with either the correct program P_1 (bottom), or the incorrect program P_1' (top), and wishes to test it for correctness. To do so, they write a correct reference implementation P_2; this can be done by explicitly disallowing parallelism. Most frameworks allow the level of parallelism to be customized; e.g. in Flink, it can be disabled by calling `.setParallelism(1)` on the stream. However, this specification alone is not enough, because we need to know whether the output data produced by a program should be considered unordered, ordered, or a mixture of both. A naive differential testing algorithm might assume that output streams are out-of-order, checking for multiset equivalence; but in this case, the two possible

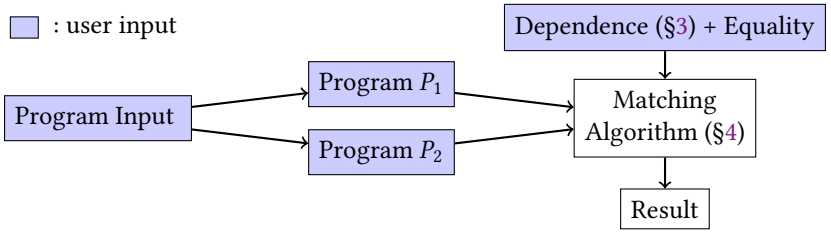


Fig. 3. Architecture for our testing framework.

programs P_1 will both be equivalent to P_2. Alternatively, it might assume that output streams are in-order; but in this case, neither P_1 nor P_1' will be equivalent to P_2, because data for different taxi IDs will be out of order in the parallel solution.

To solve this, the programmer additionally specifies a *dependence relation*: given two events of the output stream, it returns *true* if the order between them should be considered significant. For this example, output events are dependent if they have the same taxi ID. In general, the dependence relation can be used to describe a flexible combination of ordered, unordered, or partially ordered data.

The end-to-end testing architecture is shown in Fig. 3. In summary, the programmer provides: (1) a program (i.e., streaming dataflow graph) P_1 which they wish to test for correctness; (2) a correct reference implementation P_2; (3) a *dependence relation* which tells the tester which data items are allowed to arrive out-of-order; (4) if needed, overriding the definition of equality for output stream events (for example, this can be useful if the output items may contain timestamps or metadata that is not relevant for the correctness of the computation); and (5) optionally, a custom generator of input data streams, or a custom input stream—otherwise, the default generator is used to generate random input streams. The two programs are then connected to our differential testing algorithm, which consumes the output data, monitors whether the output streams so far are equivalent, and reports a mismatch in the outputs as soon as possible.

3 SPECIFYING ORDERING REQUIREMENTS

In this section we describe how the programmer writes specifications in DiffStream. Let's look back at the taxi example from Section 2.1. The second stage of the program computes the total distance travelled by each taxi by computing the distance between the current and the previous location, and adding that to a sum. For this computation to return correct results, location events for each taxi should arrive in order in its input—a requirement that must be checked if we want to test the first stage of the program. We propose expressing this ordering requirement using a *dependence relation* D. The concept of dependence relations was first introduced in research on concurrency theory, where it was used to define Mazurkiewicz traces, i.e. partially ordered sequences of events in distributed systems [Mazurkiewicz 1986], and has previously been used to give semantics for stream processing programs [Mamouras et al. 2019].

A dependence relation is a symmetric binary relation on events of a stream with the following semantics. If x D y, then the order of x and y in a stream is significant and reordering them gives us two streams that are not equivalent. This could be the case if the consumer of an output stream produces different results depending on the order of x and y. Thus, the dependence relation can be thought of as encoding the pairwise ordering requirements of the downstream consumer.

It is often helpful to visualize dependence relations as unordered graphs, where nodes are equivalence classes of the dependence relation. For the taxi example, the dependence relation is

```
(ev1, ev2) ->
    ev1.taxiID == ev2.taxiID
```

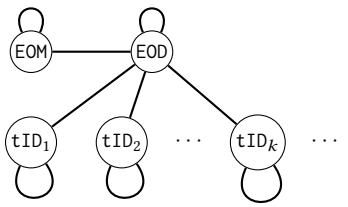
(a) Specification in DiffStream

(b) Dependence visualized as a graph

Fig. 4. Example specification in DiffStream for the taxi example. Taxi events with the same `taxiID` are dependent.

```
(ev1, ev2) ->
    ev1.isEOD() ||
    ev2.isEOD() ||
    (ev1.isEOM() && ev2.isEOM()) ||
    (ev1.isTaxiEv() &&
     ev2.isTaxiEv() &&
     ev1.taxiID == ev2.taxiID)
```

(a) Specification in DiffStream



(b) Dependence visualized as a graph

Fig. 5. Example specification in DiffStream for the extended taxi example. Taxi events with the same `taxiID` are dependent and all events are dependent with end-of-day (EOD) events.

```
(ev1, ev2) -> distance(ev1.loc, ev2.loc) < 1
```

Fig. 6. Example specification in DiffStream where events are dependent if their locations are close.

visualized in Fig. 4b, and it indicates that events with the same taxi identifier are dependent. In DiffStream, dependence relations can be specified using a boolean function on a pair of events. These functions should be pure and should only depend on the fields of the two events. The DiffStream specification of the dependence relation from Fig. 4b is shown in Fig. 4a.

Now let's consider an extension of the above example where the downstream consumer computes the total distance travelled by each taxi *per day*, and also computes the average daily distance by each taxi every month. To make this possible, the output of the program under test is now extended with special EOD (*end-of-day*) and EOM (*end-of-month*) events. The ordering requirements on this output, while more subtle, can still be precisely specified using a dependence relation. For example, EOD events are dependent with taxi events since all events of a specific day have to occur before the EOD event of that day for the total daily distance to be correctly computed. On the other hand, EOM events do not have to be dependent with taxi events since daily distances are computed on EOD events. Therefore, an EOM event can occur anywhere between the last EOD event of the month and the first EOD event of the next month. The DiffStream specification of the dependence relation and its visualization are both shown in Fig. 5.

Several frequently occurring dependence relations can be specified using a combination of the predicates seen in the above examples. This includes predicates that check if an event is of a specific type (e.g. `isEOD()`, `isTaxiEv()`), and predicates that check a field (possibly denoting a key or identifier) of the two events for equality (e.g. `ev1.taxiID == ev2.taxiID`). However, it is conceivable that the dependence of two events is determined based on a complex predicate on their fields. Figure 6 shows an example where the proximity of two taxi location events determines if they are dependent.

```
(ev1, ev2) -> (ev1.isPunctuation() &&
                ev2.timestamp < ev1.timestamp) ||
               (ev2.isPunctuation() &&
                ev1.timestamp < ev2.timestamp)
```

Fig. 7. Example specification in DiffStream where punctuation events, used to enforce progress, depend on other events only if the punctuation timestamp is larger.

Algorithm DiffStream Checking equivalence of two streams

Input: Equality relation $\equiv$, dependence relation D
Input: Connected stream s with $\pi_1(s) = s_1$ and $\pi_2(s) = s_2$
Require: Relations $\equiv$ and D are compatible

1: **function** StreamsEquivalent(s)
2: $u_1, u_2 \leftarrow$ empty logically ordered sets
3: Ghost state: $p_1, p_2 \leftarrow$ empty logically ordered sets
4: Ghost state: $f \leftarrow$ empty function $p_1 \to p_2$
5: **for** (x, i) in s **do**
6: $j \leftarrow 3 - i$
7: **if** x is minimal in u_i and $\exists y \in \min u_j : x \equiv y$ **then**
8: $u_j \leftarrow u_j \setminus \{y\}$
9: $p_i \leftarrow p_i \cup \{x\}$; $p_j \leftarrow p_j \cup \{y\}$
10: $f \leftarrow f[x \mapsto y]$ **if** $i = 1$ **else** $f[y \mapsto x]$
11: **else if** $\exists y \in u_j : x$ D y **then**
12: **return false**
13: **else**
14: $u_i \leftarrow u_i \cup \{x\}$
15: **return** $(u_1 = \emptyset$ and $u_2 = \emptyset)$

Another interesting dependence relation occurs in cases where output streams contain punctuation events. Punctuations are periodic events that contain a timestamp and indicate that all events up to that timestamp, i.e. all events `ev` such that `ev.timestamp < punc.timestamp`, have *most likely* already occured. Punctuation events allow programs to make progress, completing any computation that was waiting for events with earlier timestamps. However, since events could be arbitrarily delayed, some of them could arrive after the punctuation. Consider as an example a taxi that briefly disconnects from the network and sends the events produced while disconnected after it reconnects with the network. These events are usually processed with a custom out-of-order handler, or are completely dropped. Therefore, punctuation events are dependent with events that have an earlier timestamp, since reordering them alters the result of the computation, while they are independent of events with later timestamps. This can be specified in DiffStream as shown in Fig. 7.

4 ALGORITHM

In this section we present Algorithm DiffStream, our algorithm for checking equivalence of two streams. As described in Section 1, the algorithm has two main features: (i) it can check for equivalence up to any reordering dictated by a given dependence relation, and (ii) it is online—it

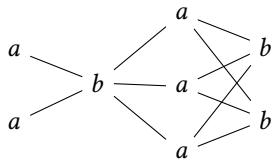


Fig. 8. The logical order of the stream from Example 4.1. Vertically aligned elements are logically unordered, and for two elements that are not aligned, the left one logically precedes the right one. The two leftmost elements are minimal.

processes elements of the stream one at a time. We prove that our algorithm is correct, and we show that it is optimal in the amount of state it stores during execution.

Before getting to the algorithm itself, we need to introduce some terminology. A *stream* s is a bounded or unbounded sequence of elements: $s = \langle x_1, x_2, \ldots \rangle$. We write $x \in s$ to denote that x is an element of s, we write $s[n]$ for the nth element of s, and we write $s[{:}n]$ for the bounded substream of elements up to and including the nth element. Two streams s_1, s_2 are given as input to the algorithm as a *connected stream* s, which is a stream obtained by arbitrarily interleaving the elements of s_1 and s_2. More precisely, the elements of the connected stream s are of the form (x, i) such that $i \in \{1, 2\}$ and $x \in s_i$. We can recover the original streams by using *projections* π_1 and π_2: $\pi_1(s) = s_1$ and $\pi_2(s) = s_2$. Conversely, given a stream s, we can form connected streams using *injections* ι_1 and ι_2: $\iota_1(s)$ is obtained by mapping each $x \in s$ to $(x, 1)$, and analogously, $\iota_2(s)$ is obtained by mapping each $x \in s$ to $(x, 2)$. Thus, $\iota_1(s)$ and $\iota_2(s)$ are characterized by $\pi_1(\iota_1(s)) = \pi_2(\iota_2(s)) = s$ and $\pi_1(\iota_2(s)) = \pi_2(\iota_1(s)) = \emptyset$. The motivation for connected streams comes from the fact that the streams s_1 and s_2 are produced by the stream processing system asynchronously.

Next, we need to describe what it means for two streams to be equivalent. Our notion of equivalence relies on two user-specified relations on the elements of the streams: an *equality relation*, denoted by $\equiv$, and a *dependence relation*, denoted by D. The equality relation is required to be an equivalence relation, that is, it should be reflexive, transitive, and symmetric. For elements x and y, we write $x \equiv y$ instead of $x = y$ in order to emphasize that the equality is provided by the user, e.g., in Java by overriding the method equals(). The dependence relation is required to be symmetric, that is, for elements x and y, x D y implies y D x. Finally, the equality and the dependence are required to be *compatible*: if x D y and $x \equiv x'$, then x' D y. The three requirements—the equality being an equivalence relation, the dependence being a symmetric relation, and the equality and dependence being compatible—need to be ensured by the user.

Given a stream s, a dependence relation D gives rise to a *logical order* on the elements in s: for elements $x, y \in s$, x logically precedes y, denoted by $x < y$, if x precedes y in the stream and either x and y are dependent or they are transitively dependent—there are intermediate elements $x_1, \ldots, x_n \in s$ given in their order of occurrence in s such that x D x_1 D $\ldots$ D x_n D y. It can be shown that the logical order is irreflexive and transitive, that is, it is a partial order on the elements of the stream s.

Example 4.1. Consider a stream $s = \langle a, a, b, a, a, a, b, b \rangle$. The equality relation $\equiv$ is given by $a \equiv a$ and $b \equiv b$, and the dependence relation D is given by a D b (and b D a). The logical order arising from D is shown in Fig. 8. The logical orderings between elements include $s[1] < s[3]$, $s[3] < s[4]$, and $s[5] < s[7]$. Also $s[4] \parallel s[5]$, $s[4] \parallel s[6]$, and $s[5] \parallel s[6]$. Note that $s[1] < s[4]$ even though $s[1] \not\mathrel{D} s[4]$ (both elements are a). This is because they both depend on $s[3] = b$, which is in between.

Given two streams s and s', an equality relation $\equiv$, and a dependence relation D, we say that s and s' are *equivalent* if they give rise to the same logical order. More precisely, we say they are equivalent if there exists a bijective mapping $f : s \to s'$, called a *matching*, that matches equal elements and preserves the logical order, that is, for every $x, y \in s$, $f(x) \equiv x$ and $f(x) < f(y)$ if and only if $x < y$. In case the streams are equivalent, we write $s \equiv_{\mathrm{D}} s'$, or simply $s \equiv s'$ if the dependence relation is clear from the context. We call two streams that are not equivalent *distinguishable*.

If the two streams s and s' are bounded, one way to think about them being equivalent is as follows: we can get from s to s' in finitely many steps by either swapping two adjacent logically unordered elements or by replacing an element with another equal element. In particular, bounded equivalent streams have the same length.

Example 4.2. Streams $s_1 = \langle a, c, b\rangle$ and $s_2 = \langle c, a, b\rangle$ are equivalent with respect to a dependence relation given by a D b and c D b. A (unique) matching is given by $f : s_1[1] \mapsto s_2[2]$, $s_1[2] \mapsto s_2[1]$, and $s_1[3] \mapsto s_2[3]$. Note that the same streams are not equivalent with respect to a dependence relation where additionally a D c.

When it comes to unbounded streams, it may be impossible to algorithmically decide whether they are equivalent or not. For example, consider $s_1 = \langle a, a, a, \ldots\rangle$ and $s_2 = \langle b, b, b, \ldots\rangle$ with $a \not\mathrm{D}\, b$. Clearly, $s_1 \not\equiv s_2$, but an algorithm processing a connected stream s with $\pi_1(s) = s_1$ and $\pi_2(s) = s_2$ one element at a time can never reach a conclusion: perhaps eventually b's will start arriving on the first stream, and a's will start arriving on the second stream. However, there are situations when an algorithm can reach a definite decision even if the streams are unbounded. We say that a connected stream s is *finitely distinguishable* if there is a position n such that for every continuation s' of $s[{:}n]$, the projected streams $\pi_1(s[{:}n] \cdot s')$ and $\pi_2(s[{:}n] \cdot s')$ are distinguishable.

Example 4.3. If s is a connected stream such that $\pi_1(s) = \langle a, a, b\rangle$ and $\pi_2(s) = \langle a, b\rangle$, and a D b, then for no continuation of s will the two projections ever be equivalent. Thus, s is finitely distinguishable.

Given a partial order p, we say that an element $x \in p$ is *minimal* if no other element is less than x. There can be multiple minimal elements in p; we denote the set of minimal elements in p by $\min p$. Given two partial orders p and q such that $p \subseteq q$, we say that p is a *prefix* of q if for every element $x \in p$, p also contains all the elements that are less than x in q.

We now give a general specification of an online equivalence-checking algorithm. The algorithm's inputs are an equality relation $\equiv$, a dependence relation D, and a connected stream s with the projections $s_1 = \pi_1(s)$ and $s_2 = \pi_2(s)$. We require the equality and the dependence relations to be compatible. The algorithm provides a function StreamsEquivalent that returns return **true** or **false** to report whether or not $s_1 \equiv s_2$. The function is allowed to iterate over s exactly once.

The algorithm is correct if it has the following behavior:

(I) StreamsEquivalent returns **true** if and only if s is bounded and s_1, s_2 are equivalent.

(II) StreamsEquivalent returns **false** if and only if either s is finitely distinguishable or it is bounded and the streams s_1, s_2 are distinguishable. Additionally, if s is finitely distinguishable, it returns **false** after processing $s[n]$ for the first position n such that $s[{:}n]$ is finitely distinguishable.

Algorithm DiffStream achieves the required behavior in the following way. Intuitively, it tries to construct a matching to demonstrate equivalence of s_1 and s_2. In doing so, as part of its state it maintains two logically ordered sets u_1 and u_2, both initially empty. Their role is to keep track of the unmatched elements from s_1 and s_2, respectively. In addition to u_1 and u_2, which constitute the physical state, the algorithm maintains the so-called "ghost state," written in gray in Algorithm DiffStream. The ghost state need not exist in any real implementation of the algorithm; its sole purpose

is to aid in proving correctness. As part of the ghost state, the algorithm maintains two additional logically ordered sets p_1 and p_2, whose role is to keep track of the successfully matched prefixes of s_1 and s_2. In the ghost state, the algorithm also explicitly keeps track of the matching $f : p_1 \to p_2$.

When processing a new element x from s_i (lines 5–14 in Algorithm DiffStream), there are three distinguished cases:

(1) The element x is minimal in u_i and there is a corresponding unmatched minimal element $y \in u_j$ such that $x \equiv y$ (line 7). In this case we remove y from u_j. In the ghost state, we add x to p_i and y to p_j, and we extend the matching f to map x to y or y to x, depending on whether $x \in s_1$ or $y \in s_1$ (lines 9–10).
(2) The element x depends on some unmatched element $y \in u_j$ (line 11). If this is the case, then we have detected finite distinguishability and the function returns **false** (line 12).
(3) If neither of the previous cases holds (line 13), then for every $y \in u_j$, x and y are unequal and independent. We add x to u_i as an unmatched element (line 14).

If the whole connected stream has been processed and the function StreamsEquivalent did not return **false** in line 12, it returns **true** in line 15 if and only if both sets of unmatched elements are empty.

Example 4.4. Let us demonstrate the execution of Algorithm DiffStream on streams s_1 and s_2 from Example 4.2. Suppose the connected stream given as input is

$$s = \langle (a,1), (c,2), (c,1), (b,1), (a,2), (b,2) \rangle .$$

At the time of processing the element $s[3] = (c,1)$, the first two elements have already been processed, and both of them are unmatched: $u_1 = \{a\}$ and $u_2 = \{c\}$. The algorithm detects that the new element c is minimal in u_1 and it can be matched with the element $c \in u_2$, so it updates the matching f with $f(s_1[2]) = s_2[1]$ and removes c from u_2. Next, it processes $s[4] = (b,1)$: the element b is not minimal in u_1 as it depends on $a \in u_1$. It is also not dependent on any element in u_2, as u_2 is empty. Therefore, it is added to u_1, which now contains the ordering $a < b$. Finally, the two elements $s[5] = (a,2)$ and $s[6] = (b,2)$ arrive precisely in the right order to be matched with the elements in u_1, and the algorithm concludes that the streams are equivalent.

If the dependence relation contained the additional dependence $a \mathrel{D} c$, the processing would have stopped at the element $s[2] = (c,2)$, since the element c from s_2 would have been dependent on an unmatched element $a \in u_1$. And indeed, the connected stream $s[:2] = \langle (a,1), (c,2) \rangle$ would have been finitely distinguishable.

4.1 Correctness

In order to show that the algorithm is correct, we will show that the loop in StreamsEquivalent (lines 5–14) maintains the following invariants.

(I0) For every $x \in u_1$ and $y \in u_2$, x and y are unequal and independent: $\forall x \in u_1, \forall y \in u_2 : x \neq y \wedge x \not\mathrel{D} y$.
(I1) p_1 is a prefix of s_1: $\forall x \in p_1, \forall y \in s_1 : y < x \Rightarrow y \in p_1$.
(I2) p_2 is a prefix of s_2: $\forall x \in p_2, \forall y \in s_2 : y < x \Rightarrow y \in p_2$.
(I3) $f : p_1 \to p_2$ is a maximal matching, that is, it is a matching and no extension $f' : p_1' \to p_2'$ to proper supersets $p_1' \supset p_1$ and $p_2' \supset p_2$ is a matching. We also say that p_1 and p_2 are maximally matched prefixes.

Lemma 4.5. *The loop in* StreamsEquivalent *(lines 5–14) maintains invariants (I0)–(I3).*

Proof. Of the four invariants, the one that is least straightforward is (I3), so let us show that it holds. In particular, let us show that after the update in lines 9–10 of Algorithm DiffStream, the

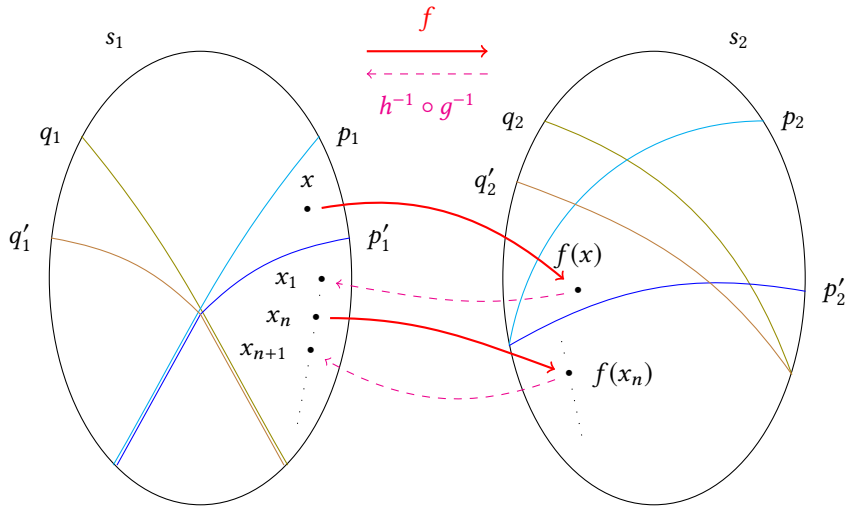


Fig. 9. Illustration of the argument showing that $p_1' \neq p_1$ is impossible in the proof of Lemma 4.6.

function f remains a matching between p_1 and p_2. Clearly it is still a bijection and it maps elements in p_1 to equal elements in p_2. In order to show that it still preserves the logical order, we only need to show that for every $x' \in p_1$, $x' < x$ if and only if $f(x') < f(x) = y$. Let us show this claim in one direction (the other one is analogous). Assume $x' < x$. By definition, there exists $n \geq 1$ and elements $x_0, \ldots, x_n \in p_1$ such that $x' = x_0$ D x_1 D $\ldots$ D $x_n = x$. By the compatibility of $\equiv$ and D, we also have $f(x') = f(x_0)$ D $f(x_1)$ D $\ldots$ D $f(x_n) = y$. By the invariant (I2), y does not logically precede $f(x_{n-1})$, so it must be $f(x_{n-1}) < y$, and finally by transitivity $f(x') < y$. As for the maximality of f, since (I3) holds at the start of the loop, any extension to f must involve the element x that is being processed. Thus, if f can be extended, it is extended by the ghost statements in lines 9–10. □

Lemma 4.6. *In a bounded connected stream s, all maximally matched prefixes of $s_1 = \pi_1(s)$ and $s_2 = \pi_2(s)$ are equivalent.*

Proof. Let $f\colon p_1 \to p_2$ and $g\colon q_1 \to q_2$ be two maximal matchings, where p_1 and q_1 are prefixes of s_1, and p_2 and q_2 are prefixes of s_2. Let $u_1 = s_1 \setminus p_1, u_2 = s_2 \setminus p_2$, and $v_1 = s_1 \setminus q_1, v_2 = s_2 \setminus q_2$ be the corresponding unmatched elements. To show the claim, it suffices to show that $p_1 \equiv q_1$.

Clearly the identity function $\mathrm{id}\colon p_1 \cap q_1 \to p_1 \cap q_1$ is a matching. Let $h\colon p_1' \to q_1'$ be a maximal matching between p_1 and q_1 that extends id; thus, p_1' and q_1' are prefixes such that $p_1 \cap q_1 \subseteq p_1' \subseteq p_1$ and $p_1 \cap q_1 \subseteq q_1' \subseteq q_1$. To show $p_1 \equiv q_1$, it suffices to show that $p_1' = p_1$ and $q_1' = q_1$.

First we note that for the matchings f, g, h, an analog of the invariant (I0) holds. In particular, the sets $p_1 \setminus p_1'$ and $q_1 \setminus q_1'$, as well as v_1 and v_2 are pairwise independent and unequal. Moreover, let us set $p_2' = f(p_1')$ and $q_2' = g(q_1')$. The sets $p_2 \setminus p_2'$ and $q_2 \setminus q_2'$ are also pairwise independent and unequal. Let us show the claim for $p_1 \setminus p_1'$ and $q_1 \setminus q_1'$. Suppose first that $x \in p_1 \setminus p_1'$ and $y \in q_1 \setminus q_1'$ are elements such that x D y. Since x and y are both in s_1, we either have $x < y$ or $y < x$. If $x < y$, then we have $x \in q_1$ since q_1 is a prefix, and consequently $x \in p_1 \cap q_1 \subseteq p_1'$, which is a contradiction. Likewise, $y < x$ leads to $y \in q_1'$, which is also a contradiction. Suppose now that $x \in p_1 \setminus p_1'$ and $y \in q_1 \setminus q_1'$ are elements such that $x \equiv y$. They cannot both be minimal, for we would be able to extend h with $h(x) = y$. Thus, one of x and y has a logical predecessor; say $x' \in p_1 \setminus p_1'$ is such that

$x' < x$. Without loss of generality, x' D x. Since $\equiv$ and D are compatible, this leads to x' D y, which we have just shown to be impossible.

We are now ready to prove $p_1' = p_1$ ($q_1' = q_1$ is analogous). An illustration of this proof is shown in Fig. 9. For the sake of contradiction, suppose that $p_1' \neq p_1$, that is, there exists an element $x \in p_1 \setminus p_1'$. Note that $x \notin q_1$, that is, $x \in v_1$. We send x to s_2 via f; we have $f(x) \in p_2 \setminus p_2'$. The element $f(x)$ cannot be in v_2 since $x \in v_1$ and $x \equiv f(x)$. The element $f(x)$ also cannot be in $q_2 \setminus q_2'$, since it is in $p_2 \setminus p_2'$ and $f(x) \equiv f(x)$. Hence, $f(x) \in q_2'$. Now we pull $f(x)$ back to s_1 via g and h. Set $x_1 = h^{-1}(g^{-1}(f(x)))$; we have $x_1 \in p_1'$. Since $x \notin p_1'$, x_1 and x are distinct elements such that $x_1 \equiv x$. We can continue iterating the described process. Suppose we have defined distinct elements $x_1, \ldots, x_n \in p_1'$ for some $n \geq 1$ such that $x \equiv x_1 \equiv \ldots \equiv x_n$. We send x_n to s_2 via f and note that neither $f(x_n) \notin v_2$ (otherwise we would have $x \in v_1$, $f(x_n) \in v_2$, and $x \equiv f(x_n)$) nor $f(x_n) \in q_2 \setminus q_2'$ (otherwise we would have $f(x) \in p_2 \setminus p_2'$, $f(x_n) \in q_2 \setminus q_2'$, and $f(x) \equiv f(x_n)$). Therefore, $f(x_n) \in q_2'$ and we can bring it back to s_1 via g and h to get a well-defined element $x_{n+1} = h^{-1}(g^{-1}(f(x_n))) \in p_1'$. Suppose $x_{n+1} = x_k$ for some k with $1 \leq k \leq n$. By removing k layers of application of $h^{-1} \circ g^{-1} \circ f$, we conclude that $x_{n+1-k} = x$, which cannot be since $x_{n+1-k} \in p_1'$ and $x \notin p_1'$. Therefore, x_{n+1} is distinct from all previously defined elements.

By defining the described process, we have shown that there are infinitely many distinct elements in p_1', which cannot be since p_1' is a finite set. Hence, $x \in p_1 \setminus p_1'$ cannot exist in the first place, and we have established that $p_1 = p_1'$. Analogously, we have $q_1 = q_1'$, and since $p_1' \equiv q_1'$, we also have $p_1 \equiv q_1$. Finally, using $p_1 \equiv q_1$ as a link, we establish that $p_2 \equiv p_1 \equiv q_1 \equiv q_2$, that is, all the maximally matched prefixes in s are equivalent. □

Lemma 4.7. *If* StreamsEquivalent *returns* ***false*** *in line 12 while processing $s[n]$ for some $n \geq 1$, then the connected stream $s[:n]$ is finitely distinguishable.*

Proof. Let $s[n] = (x, 1)$ and assume that ProcessElement returns **false** in line 12 when processing $s[n]$. Since the condition in line 11 was satisfied, there exists $y \in u_2$ such that x D y. Without loss of generality, let y be a minimal such element in u_2.

The challenge here is that even though f can never be extended to match either x or y, it is plausible that $s[:n]$ can nevertheless be extended to s' with a completely different matching that somehow accommodates both elements. To show that such a scenario is impossible, suppose s' is an extension of $s[:n]$ such that $\pi_1(s') = s_1' \supseteq s_1$, $\pi_2(s') = s_2' \supseteq s_2$, and suppose $g\colon s_2' \to s_1'$ is a matching (it helps to view g in the direction opposite of f). Since x D y and $g(y) \equiv y$, we have x D $g(y)$, so x and $g(y)$ are logically ordered in s_1'. There are three possibilities: $g(y) = x$ (the elements are identical), $g(y) < x$, or $x < g(y)$.

The first possibility can easily be discarded. From $g(y) = x$ it follows that $x \equiv y$. The elements x and y cannot both be minimal elements unmatched by f, otherwise x would have been matched in lines 7–10. Therefore, either there is $x' \in u_1$ such that $x' < x$ and x' D x, or there is $y' \in u_2$ such that $y' < y$ and y' D y. In the former case we have x' D y, contradicting the invariant (I0), and in the latter case we have x D y' and $y' < y$, contradicting the choice of y as a minimal unmatched element such that x D y.

The second possibility, that $g(y) < x$, can be discarded as follows. Set $y_0 = y$. The element $g(y_0)$ cannot be in u_1 as that would break the invariant (I0); therefore it has to be in p_1. We can send it to p_2 via f; let $y_1 = f(g(y_0))$. Since $y_0 \notin p_2$, y_0 and y_1 are distinct elements such that $y_0 \equiv y_1$. We iterate the process: suppose we have defined the elements $y_1, \ldots, y_n \in p_2$ for some $n \geq 1$ such that all of them are equal to y_0, and all of them together with y_0 are distinct. Since $g(y_n)$ D x, $g(y_n)$ and x are logically ordered. It cannot be $g(y_n) \geq x$, as that would imply $g(y_n) > g(y_0)$ and consequently $y_n > y_0$. But then, since $y_n \in p_2$ and p_2 is a prefix, we would have $y_0 \in p_2$, which is a contradiction. Thus, $g(y_n) < x$ and consequently $g(y_n) \in p_1$ due to the invariant (I0). Hence, $y_{n+1} = f(g(y_n))$ is a

well-defined element such that $y_{n+1} \in p_2$. Clearly y_{n+1} is distinct from y_0, and $y_{n+1} \equiv y_0$. Suppose $y_{n+1} = y_k$ for some k with $1 \leq k \leq n$. By "pealing off" k layers of applications of f and g, we would get $y_{n+1-k} = y_0$, which is a contradiction. Therefore, the new element is distinct from every previously defined element. We have thus defined infinitely many distinct elements in the finite set p_2, which is a contradiction. Hence, $g(y) \not< x$.

The third possibility, that $x < g(y)$, is discarded by a similar argument. The idea is again to start from $x_0 = x$ and define an infinite sequence of distinct elements $x_1, x_2, \ldots$ in p_1, all of which are equal to and distinct from x_0. However, the argument that shows the sequence is well-defined is slightly different. Similarly as before, given x_n for $n \geq 1$ we start by arguing that $g^{-1}(x_n) \in p_2$. We first establish that $g^{-1}(x_n) < y$, as otherwise $g^{-1}(x_n) \geq y > g^{-1}(x_0)$ would imply $x_n > x_0$ and consequently $x_0 \in p_1$. Next, if $g^{-1}(x_n) \in u_2$, then we argue as in the first possibility: either x and $g^{-1}(x_n)$ can be matched, or there is $x' < x$ in u_1 such that $x' \mathrel{D} g^{-1}(x_n)$, contradicting the invariant (I0), or there is $y' < g^{-1}(x_n) < y$ in u_2 such that $x \mathrel{D} y'$, contradicting the minimality of y. Hence, $g^{-1}(x_n) \in p_2$ and $x_{n+1} = f^{-1}(g^{-1}(x_n))$ is well-defined. Showing that it is distinct from previously defined elements is done using the same argument as before. Thus, we again reach a contradiction, and $x \not< g(y)$.

By discarding all three possibilities, we conclude that the extension of $s[{:}n]$ to a connected stream s' such that $s'_1 \equiv s'_2$ does not exist. Hence, $s[{:}n]$ is finitely distinguishable. □

Theorem 4.8. *Algorithm DiffStream is correct.*

Proof. We first show that Algorithm DiffStream satisfies the correctness condition (I). If StreamsEquivalent returns **true** in line 15, then the whole connected stream s has been processed. Hence, s is bounded. Moreover, in this case both u_1 and u_2 are empty, implying $p_1 = s_1$ and $p_2 = s_2$. Therefore, $s_1 \equiv s_2$ follows from the invariant (I3). Conversely, if s is bounded and s_1, s_2 are equivalent, by Lemma 4.7, StreamsEquivalent does not return **false** on line 12, and the loop in lines 5–14 finishes. By the invariant (I3) and Lemma 4.6, f must fully match s_1 and s_2. Hence, u_1 and u_2 are empty and StreamsEquivalent returns **true**.

Next, we show the correctness condition (II). If StreamsEquivalent returns **false**, it either returns **false** in line 12 and s is finitely distinguishable by Lemma 4.7, or it returns **false** in line 15. In the latter case, s is bounded and one of u_1 and u_2 is not empty. Hence, s_1 and s_2 are distinguishable by the invariant (I3) and Lemma 4.6. Conversely, if s is bounded and s_1, s_2 are distinguishable, by (I) the function StreamsEquivalent does not return **true**, so it returns **false**.

It remains to show that if s is finitely distinguishable, then StreamsEquivalent returns **false** in line 12 for the first position n such that $s[{:}n]$ is finitely distinguishable. Clearly StreamsEquivalent does not return **false** on line 12 when processing $s[k]$ for $k < n$, since in that case by Lemma 4.7 already $s[{:}k]$ would be finitely distinguishable. Let $s[n] = (x, 1)$ and let u_1 and u_2 be the logically ordered sets of unmatched elements at the start of the loop when processing $s[n]$. If x can be matched with an element $y \in \min u_2$, then we extend $s[{:}n]$ with $\iota_1(u_2 \setminus \{y\})$ followed by $\iota_2(u_1)$, with the elements of $u_i, i \in \{1, 2\}$ given in the order in which they appear in $s[{:}n-1]$. Likewise, if $x \not\mathrel{D} y$ for every $y \in u_2$, then we extend $s[{:}n]$ with $\iota_1(u_2)$ followed by $\iota_2(u_1 \cup \{x\})$. In either case, from the invariants (I0)–(I3) it follows that Algorithm DiffStream would decide that the two streams in the extension are equivalent, and since the algorithm is correct for bounded equivalent streams, the streams would indeed be equivalent. Therefore, in both cases the connected stream $s[{:}n]$ would not be finitely distinguishable. Since $s[{:}n]$ is finitely distinguishable, the only remaining option is that there exists $y \in u_2$ such that $x \mathrel{D} y$, and hence StreamsEquivalent returns **false** in line 12 when processing $s[n]$. □

4.2 Optimality

When it comes to stateful stream processing programs, space usage is an important topic. If a stateful stream processing program inadvertently stores too much of the stream's history, since the stream is potentially unbounded, the program's space usage may grow unboundedly as well. In case of Algorithm DiffStream, its space usage may indeed grow unboundedly. Since Algorithm DiffStream stores sets of unmatched elements, it can be forced to keep the complete history of the connected stream it takes as input. For example, this would happen on a connected stream s such that $\pi_1(s) = \langle a, a, \ldots \rangle$, $\pi_2(s) = \langle b, b, \ldots \rangle$, with $a \not\equiv b$ and $a \not\mathbb{D} b$. However, it turns out that for a correct equivalence-matching algorithm there is no way around this: a correct equivalence-checking algorithm must store a certain amount of unmatched elements in one way or another. In each step, Algorithm DiffStream stores minimal sets of unmatched elements (the complements of maximally matched prefixes), and as we show in this subsection, in this sense it is optimal.

Recall that by Lemma 4.6, in a bounded connected stream s with $s_1 = \pi_1(s)$ and $s_2 = \pi_2(s)$, all maximally matched prefixes of s_1 and s_2 are equivalent. It follows that their complements—minimal sets of unmatched elements—are equivalent as well. More precisely, let (p_1, p_2) and (q_1, q_2) be two pairs of maximally matched prefixes such that $p_i, q_i \subseteq s_i$ for $i \in \{1, 2\}$, and let $u_i = s_i \setminus p_i$ and $v_i = s_i \setminus q_i$ for $i \in \{1, 2\}$. Then $u_1 \equiv v_1$ and $u_2 \equiv v_2$. This allows us to define up to equivalence a function $u(s) = (u_1, u_2)$, where u_1 and u_2 are any minimal sets of unmatched elements in s_1 and s_2. We write $(u_1, u_2) \equiv (v_1, v_2)$ to mean $u_1 \equiv v_1$ and $u_2 \equiv v_2$.

If a bounded connected stream s with $u(s) = (u_1, u_2)$ is not finitely distinguishable, then an analog of the invariant (I0) holds for u_1 and u_2: for every $x \in u_1$ and $y \in u_2$, $x \not\equiv y$ and $x \not\mathbb{D} y$.

Theorem 4.9. *Algorithm DiffStream is optimal. More precisely, let A be any other* correct *algorithm for the equivalence-checking problem. If s and s' are two bounded connected streams that are not finitely distinguishable, and if Algorithm DiffStream reaches a different state after processing s and s', then A reaches a different state after processing s and s'.*

Proof. The state stored by Algorithm DiffStream on processing a string s is $u(s)$. Suppose a correct equivalence-checking algorithm reaches the same state after processing s and s'. Let $u(s) = (u_1, u_2)$ and $u(s') = (u'_1, u'_2)$. We extend both s and s' with $\iota_1(u_2)$ followed by $\iota_2(u_1)$. It is not difficult to see that the two connected streams in the extension of s are equivalent, and the streams in the extension of s' are not equivalent. However, since the algorithm is deterministic, it would come to the same decision for both extensions, which contradicts its correctness. □

4.3 Practical Bounds on Space Usage

While space used by Algorithm DiffStream can grow with the input stream in the worst case, Theorem 4.9 shows that it is impossible to write an algorithm which uses a smaller amount of space. In addition to this result, it is possible to give concrete bounds on the space usage in certain cases. Here we discuss some patterns that we have encountered, including the examples we implemented in Section 5, and how bounds on the space usage can be derived for these patterns.

The simplest pattern is differential testing of sequential outputs: both streams are completely ordered, i.e., any two events are dependent. In this case, any differential testing algorithm must at least keep track of the difference of the two streams seen so far (assuming their prefixes are equal). For example, suppose both streams are sequences of integers, and one stream has seen m integers and the other has seen n integers, where $m < n$. Then if the first m integers are equal, any matching algorithm must keep track of the remaining $(n - m)$ integers. Thus, the space usage of the algorithm is bounded by the maximum *drift* between the two streams, defined as the difference in the number of events produced. In practice, such drift is typically bounded since inputs arrive at the same rate for both the implementations, and most systems try to ensure that no operator

in the stream processing dataflow graph lags behind the others by accumulating a large queue of unprocessed inputs. This dependence relation pattern (and the resulting bound) occurs in the case studies of Sections 5.2 and 5.3.

A second common pattern is *key-based parallelism*, where events with the same key are dependent, but events with different keys are independent. In such a case, considering the drift between the two streams is not enough. For example, suppose there are only two keys, a and b, and one stream produces n as, but the other stream produces n bs. Then although the two streams are producing the same number of items, because stream 1 never produces an a and stream 2 never produces a b, any algorithm for correct matching must keep at least the as and the bs so far until they are matched on the other stream. To address this, we can obtain a bound on the space by considering the drift *per key*. In general, "keys" can be generalized as dependent subsets of events, and a bound can be obtained by taking the maximum drift on any dependent subset, together with the number of independent keys in the input. This dependence relation pattern (and the resulting bound) occur in the case study of Section 5.1. Related to key-based parallelism, *fully independent parallelism* (where all input events are independent) occurs in the case studies of Sections 5.3 and 5.4.

Finally, we have encountered cases where the input includes special synchronization events, such as punctuation marks and end-of-day markers, as described in Section 3. These events are dependent on all other events. If both input programs to the differential testing algorithm produce these events at regular intervals, then the space usage becomes bounded. In particular, suppose that the *frequency* of such events is at least one in every k events, and the *drift* restricted only to such events is d. Then the space usage of our algorithm is at most $k \times d$.

5 IMPLEMENTATION AND CASE STUDIES

We implemented the matcher algorithm in DiffStream, a differential testing library written in Java. The matcher can be used to test programs in any distributed stream processing system given an output interface. For our case studies we chose Flink as the target platform because it is one of the most widely used distributed stream processing frameworks [Stack Overflow 2020]. We integrated DiffStream with JUnit-Quickcheck [Holser 2013] to support generation of streams of random input values.

Our first case study (Section 5.1) is used to qualitatively measure the developer effort needed to test an application with non-trivial ordering dependency in its output. In the second case study (Section 5.2) we demonstrate that getting performance benefits from parallelization in Flink might require an elaborate implementation and we illustrate how our tool can be used to streamline that process. In the third case study (Section 5.2), we show that our framework is successful in finding real bugs while largely avoiding false positives by adapting a set of non-deterministic MapReduce programs from the literature [Xiao et al. 2014]. The final case study (Section 5.4) empirically establishes that our algorithm has a small memory footprint and can be used for online monitoring of long-running applications.

5.1 Taxi Distance

This case study illustrates the process that one has to follow in order to test their implementation using our tool. Recall the taxi distance example in Section 2.1. This example shows two seemingly equivalent implementations of the same query that produce different results in the presence of parallelism. Here is an instantiation of that example in Flink; the first implementation preserves the order of events for each key, while the second one does not:

```
inStream.keyBy("taxiID").project("taxiID", "position");

inStream.project("taxiID", "position").keyBy("taxiID");
```

```
public void testKeyBy() throws Exception {
    StreamExecutionEnvironment env = ...;

    DataStream input = generateInput(env);

    StreamEquivalenceMatcher matcher =
        StreamEquivalenceMatcher.createMatcher(
            sequentialImpl(input), parallelImpl(input),
            (ev1, ev2) -> ev1.taxiID == ev2.taxiID);

    env.execute();
    matcher.assertStreamsAreEquivalent();
}
```

Fig. 10. An example test in DiffStream.

Note that both implementations preserve the order when executed sequentially. Since such subtle differences are difficult to spot manually, we would like to be able to test a parallel implementation against a sequential one, before deploying it. A slightly simplified example of a test that can exercise this bug using our framework is shown in Fig. 10. The complete test is shown in Fig. 13 in Appendix A. First, the Flink execution environment is initialized and the dataflow graph is setup. Then, a random input stream is generated and fed to both implementations, that are finally compared using the matcher for output equivalence.

Notice that the final argument of the matcher is a lambda expression representing the dependence relation that is expected from the consumer of the output. This specific instantiation represents the dependence that was shown in Fig. 4a—i.e., that two items are dependent (and thus must be ordered) if they have the same `taxiID`. If the user did not want to test differences in the ordering of the output, they can use `(ev1, ev2) -> false` as the dependence relation.

In order to compare the effort required to write a test with and without using our framework, we manually implemented a test that exercises this bug. The manually implemented matcher spans two Java classes, totalling around 100 LoC (in contrast to the 13 LoC of the test in our framework shown in Fig. 10). This does not include input generation, for which we used JUnit Quickcheck. The manually implemented matcher keeps two hashmaps—one for each implementation—that map keys to lists, in order to encode the dependence of events of the same key. It appends each output item to the list associated with its key. After the two implementations stop executing, it checks that the two hashmaps represent equivalent output. Note that this manual matcher is not online, in the sense that the two implementations have to stop producing outputs for it to make a decision. We also implemented an online version of it by extending it with 30 more lines of code. The code for both the offline and online manual matchers can be found in Appendix A.

The important point is that the dependence relation abstraction enables the design of a reusable testing framework that can be used for testing applications with different ordering requirements on their outputs. In contrast, the main drawback of the manual matcher is that it is tied to a specific ordering requirement on the output; whenever a user wants to write a test that requires a different output dependence relation, they would have to implement a new matcher that maintains the output in a data structure suitable for the specific dependence. This can quickly become an overhead if the user wants to write tens or hundreds of tests for different parts of their application.

In summary, we have shown that using our tool to write a test for a stream processing application is significantly easier than writing custom tests (~10 LoC vs. ~100 LoC). In addition, our tool offers additional flexibility, as it can be used to test any two implementations just by changing the dependence relation given to it. This flexibility reduces the effort needed to implement tests for

an application. It also exposes ordering requirements, forcing the developer to think about them explicitly.

5.2 Topic Count

The main goal of our second case study is to show that achieving parallelism in distributed stream processing programs can be very difficult and require a drastically more complicated solution than the sequential code. In particular, we consider the example introduced in Section 2.1, that involves counting topics associated with words in a long document and outputting the most frequent topic as the overall topic of the document. The documents are streamed word by word, with end-of-file markers delineating words in different documents. In the sequential solution (Fig. 2, top), we process each word by querying the topic for that word, then updating the total count for that topic; when we get end-of-file, we emit the counts. We feed this output to a second operator *count max*, which finds the maximum over all topics of the count.

At first, one may think that going from a sequential to a parallel program is simply a matter of setting the Flink's parallelism parameter to more than 1. Unfortunately, this is not the case. Consider the first operator in the sequential dataflow shown in Fig. 2 (top). The problem with parallelizing this operator is that an end-of-file marker for a particular document would only be processed by a single suboperator; thus the other suboperators would not be able to properly delineate words from this document and the next one. Another way of stating the problem is to say that even though the words themselves are independent, they are dependent on the end-of-file markers, and thus the obvious parallelization is not possible. A differential test that compares the sequential dataflow to the same dataflow with parallelism set to more than 1 quickly discovers that the two versions are indeed not equivalent.

Instead, a correct parallel solution (Fig. 2, bottom) works as follows. We first attach logical timestamps to each word in the input, corresponding to the number of the document that we are currently processing. We then replace end-of-file markers, which act as explicit punctuation in the stream, with punctuated watermarks—a mechanism in Flink that informs the dataflow operators about the passage of logical time. Unlike the explicit end-of-file markers that cannot be shared by multiple suboperators, the watermarks are seamlessly propagated by the system. In effect, they allow us to break the explicit dependence between words and end-of-file markers in the input stream, allowing the later stages of the dataflow to be parallelized. We parallelize with *key-by* (keys can be assigned arbitrarily to words), and query the database for each word. The next operator is a tumbling window which uses the logical timestamps from earlier to form the window of events *for a single document*, still parallelized. Finally, we sum up the values in each window by topic, and in the last stage *count max* we find the maximum over all topics of the count.

In our search for a correct parallel solution, we consulted with Flink users on the Flink mailing list. Several iterations of feedback were needed to find a correct parallel implementation, and this shows that it is not obvious. Having the differential testing framework helped guide the search by quickly dismissing wrong implementations. The final solution is the dataflow shown in Fig. 2 (bottom), consisting of 6 dataflow operators and twice as many lines of code as the sequential solution.

It is not necessarily true that a solution that seems parallel achieves a speed-up in practice. We therefore finally need to measure the performance of the parallel solution to show that it indeed takes advantage of parallelism and scales performance with the level of parallelism. We evaluated our parallel solution on an input stream of 5 documents, each consisting of 500,000 words randomly selected from a list of 10,000 most common English words. Each word had previously been randomly assigned one of 20 topics, and the association had been stored in a standalone Redis key-value store. The purpose of having the Flink program query the Redis store was to simulate a series of

Solution	Lines of Code	Speedup due to parallelism (par.) par. 2	par. 4	par. 6	par. 8
Sequential	68	–	–	–	–
Correct Parallel	133	2.01	4.33	5.0	4.99

Table 1. Results of the second case study on difficulty of writing parallel code.

non-trivial operations that would benefit by being parallelized. We executed this experiment on a server with an Intel Xeon Gold 6154 processor and 384 GB of memory. The results of the evaluation are shown in Table 1. By increasing parallelism from 1 to 6, the execution time decreases from 80 s to 16 s (on our setup, the benefits taper off after 6).

In summary, although there is a clear performance benefit in having a parallel solution (Table 1), the correct solution is difficult to find, as evidenced qualitatively by our search and discussions on the Flink mailing list, and quantitatively because it has about twice as many lines of code as the sequential solution. Applying differential testing to the parallel solution ensures that bugs were not introduced in the process.

5.3 Real-World MapReduce Programs

To determine whether our testing framework can successfully find bugs in real-world programs, we surveyed the literature for empirical studies which have collected and categorized bugs in stream- and batch-processing programs. We excluded works that focus on job failures and performance issues [Kavulya et al. 2010; Li et al. 2013; Schroeder and Gibson 2009; Zhou et al. 2015], as they do not provide examples of semantic bugs where the output might be incorrect. In contrast, Xiao et al. [2014] study nondeterminism due to parallelism in MapReduce jobs in production workflows, identifying both real bugs as well as several nondeterministic code patterns which are bug-free. This empirical study provides a good starting point to evaluate whether our framework can successfully identify the bugs, while not falsely flagging the bug-free examples.

By examining 507 custom (i.e., user-written) reduce functions, the study identifies 5 common reducer patterns (spanning 292 custom reducers) which are *non-commutative* on general input data, meaning that there is potential for nondeterminism in the output due to parallelism. However, the study notes that there are two reasons why code written using these patterns might not be erroneous. First, with certain input assumptions, the nondeterminism may disappear (this is possible in 4 out of 5 patterns). Second, nondeterminism may be acceptable for the application requirements (this is the case in 3 out of 5 patterns). We therefore evaluate our testing framework for each of the five patterns, considering three possibilities for the application-specific requirements: determinism (nondeterminism is not acceptable), determinism under certain input assumptions, and no determinism required. We summarize the success of our framework in these cases in Fig. 11.

We implemented each of the 5 reducer patterns in Flink. To adapt them to the streaming setting, a tumbling window is applied to the input stream, and the reducer (implemented as an `AggregateFunction` in Flink) is applied to each window to get a result for that window. Before constructing the tumbling window, we used an identity map operator to shuffle the data for each key, so that the order is nondeterministic (thus exposing bugs due to parallelism). We then compared the parallel version of this pipeline with the sequential one using our matcher.

We first evaluate whether our framework can successfully identify unwanted nondeterminism by generating arbitrary input data and feeding it to the sequential and parallel versions. With enough input data (3000 input data items is sufficient), using a small number of keys and possible input data items, our tester consistently detects the incorrectly parallelized program for all 5 patterns.

Code pattern	Application-Specific Requirements		
	Determinism	Determinism under input assumptions	None (nondeterminism)
`SingleItem`	✓	✓	n/a
`IndexValuePair`	✓	✓	n/a
`MaxRow`	✓	✓	✗
`FirstN`	✓	✓	✗
`StrConcat`	✓	n/a	✓

Fig. 11. Results of the MapReduce case study.

Concretely, of the 5 confirmed bugs found in production code by the previous study, 4 of the 5 are of this nature, so our test cases would have successfully identified these 4 bugs. On the other hand, we evaluate whether our framework can be tuned to *not* detect bugs in cases where the application requirements include input assumptions, or tolerance to nondeterminism. For all patterns except one (called `StrConcat`), the output is deterministic if certain assumptions are made on the input. We write a custom input data generator for these cases, and we show that in 4 out of 4 patterns, the output successfully passes our tester.

There are three patterns where it is conceivable that nondeterminism in the output is acceptable. The `MaxRow` pattern involves finding the value of one field such that another field is maximized; it is nondeterministic because there may be multiple values which achieve the maximum. In such a case, differential testing results in a false positive because the two programs may return different values, even though they are both correct. Similarly, the `FirstN` pattern is a reducer which discards all but the first N elements that are seen; on inputs with more than N elements, differential testing results in a false positive for a bug. However, we can avoid the false positive in the last `StrConcat` pattern. This reducer consumes a sequence of input items and concatenates them into a single string separated by a special character (say, `@`). It is nondeterministic because the concatenation is noncommutative, but it is likely that the application requirements consider this acceptable. For this pattern, we implement a custom-defined equality on output data items to define when strings separated by `@` are equal; using this, the pattern is able to pass our tester. Additionally, we implement a second version of `StrConcat` which is more suited to the streaming setting: instead of collecting all items in a single `@`-separated string, we output the items as a data stream. In this case, our tester successfully reports a bug when nondeterminism is undesirable; but when the dependence relation is used to indicate that nondeterminism is acceptable, the test passes.

We summarize the results as follows: when nondeterminism is undesirable, we have written tests to successfully identify it (if it exists) for 5 out of 5 reducer patterns. Of the reducer patterns where nondeterminism might not be present due to input data assumptions, we show how a generator can be used to cause the programs to pass our tester for 4 out of 4 patterns. Finally, in the reducer patterns where it is conceivable that nondeterminism in the output might be acceptable, we show how using the dependence relation *or* custom equality with our tester can successfully make the test pass for 1 of the 3 patterns (`StrConcat`). In total, out of the scenarios where the reducer might conceivably not be buggy (the 4 and 3 patterns just mentioned), we avoid a false positive in 5 out of 7.

5.4 Online Monitoring

In this section we show that our framework can be used for online monitoring by measuring its space usage on large input streams. This can be particularly useful when testing an application

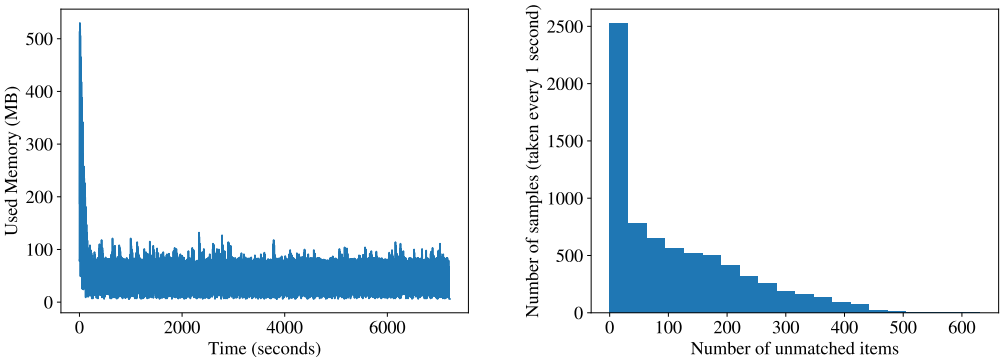


Fig. 12. Results of the fourth case study: performance measurements of our matcher on the Yahoo streaming benchmark over a span of 2 hours.

under production load before deploying it, as well as for multi-version execution [Hosek and Cadar 2013; Maurer and Brumley 2012; Tucek et al. 2009], which is a method commonly used to safely update production software.

We use our tool to compare the outputs of two similar implementations of the Yahoo streaming benchmark [Chintapalli et al. 2016], which is a well-established benchmark for stream processing systems. It consists of an advertisement aggregation query that receives advertisement events as inputs, aggregates them into campaigns, and finally stores them in a key-value store. Since the output events are stored based on unique identifiers, there is no ordering dependence on the output. We executed two versions of the query while our tool was checking for output equivalence. The total computation lasted 2 hours. Every second of execution, we sampled the memory usage of the matcher, as well as the number of unmatched items.

The results are shown in Fig. 12. Figure 12 (left) shows the memory usage of our matcher over a span of 2 hours. After an initial short period of adaptation, the matcher stabilizes and its memory usage ranges from 8 to 140 MBs. The memory usage mostly oscillates due to the garbage collection passes, and so the low values indicate the memory that the matcher really needs. Figure 12 (right) shows a histogram of the number of total unmatched items. The x-axis shows the number of unmatched items, and is separated into bins of approximately 30 items. The y-axis indicates the number of samples for each bin. In almost half of the samples the matcher had less than 100 unmatched items and it never had more than 600 unmatched items (less than 2% of the events processed per second).

The results illustrate that the theoretical optimality of our algorithm is observable in practice. Its memory usage is low (8–140MBs) and does not increase over time, and the number of unmatched elements that it keeps is very small ($< 2\%$) compared to the number events that it processes.

6 RELATED WORK

Mazurkiewicz traces. We build on foundational work in concurrency theory dating back to Mazurkiewicz [1986], where partially ordered sets of events are called *traces*. Mazurkiewicz traces have been studied from the viewpoint of algebra, combinatorics, formal languages and automata, and logic [Diekert and Rozenberg 1995]. In practical applications to verification and testing of concurrent systems, they appear in relation to *partial order reduction* [Godefroid 1996; Peled 1994], a technique for pruning the search space of possible execution sequences. The idea of a dependence

relation to specify output ordering originally comes from Mazurkiewicz traces; however, the core algorithmic problem in our work corresponds to checking *equivalence* of two Mazurkiewicz traces, and to our knowledge this particular testing problem has not been studied in any of the mentioned contexts. Furthermore, in the theory of Mazurkiewicz traces, one usually assumes a finite, symmetric, and reflexive dependence relation. In contrast, the only assumption we require is that the relation is symmetric.

Testing of data-parallel programs. Many previous works focus on batch programs written in the MapReduce framework [Chen et al. 2016; Csallner et al. 2011; Marynowski et al. 2012; Xu et al. 2013a] (see also the recent survey by Morán et al. [2019]). Going beyond batch processing, Xu et al. [2013b] study testing semantic properties of operators in general dataflow or stream-processing programs. One problem with many of these works [Chen et al. 2016; Csallner et al. 2011; Xu et al. 2013a,b] is that real-world MapReduce programs (and, by extension, aggregators in stream processing programs) can be non-commutative: the empirical study at Microsoft [Xiao et al. 2014] reports that about 58% of 507 user-written reduce jobs are non-commutative, and that most of these are most likely not buggy. The previous work on testing would erroneously flag such programs as containing bugs due to nondeterminism, which would generate a large number of false positives. We adopt a black-box differential testing approach with the goal of avoiding this problem. Concretely, we have shown in Section 5.3 how to avoid a false positive for most cases where the application requirements imply that the nondeterminism is acceptable.

Testing can also be applied to check functional correctness of a stream processing system implementation, rather than user-written programs. A framework in this direction has been designed for Microsoft StreamInsight [Raizman et al. 2010].

Correctness by design in distributed stream processing. In contrast to the dynamic approach of testing, there are static approaches to achieve correct (i.e., semantics-preserving) parallelization in stream processing programs. The language StreamIt [Thies et al. 2002] leverages Synchronous Dataflow [Lee and Messerschmitt 1987] to achieve correct parallelization; however, this requires a restriction on dataflow graphs where all operators must have a static *selectivity* (number of output items produced per input item), so it is not appropriate for general stream processing where operators often lack static selectivity. For general stream processing, Schneider et al. [2013] and Mamouras et al. [2019] have proposed and implemented different approaches to ensure correct parallelization: the first is based on categorizing operators for properties such as statefulness and selectivity, while the second is based on a type discipline where streams are annotated with types. The idea of using dependence relations to specify partially-ordered output streams is originally proposed by Mamouras et al. [2019], and here we apply that idea to testing and online monitoring.

Runtime verification. Our work contributes to the large body of work on runtime verification [Havelund and Roşu 2004; Leucker and Schallhart 2009], a lightweight verification paradigm which aims to identify bugs in the output of a program as it is executed, using minimal computational resources. Most work in runtime verification focuses on detecting violations of a property written in a logical specification language (e.g., the temporal logic LTL and its extensions), whereas we consider differential testing of a program against a reference implementation, and we model program execution traces as partially rather than totally ordered.

Differential testing. Differential testing [Groce et al. 2007; McKeeman 1998] is a well-established, lightweight, and scalable way to detect bugs in complex programs (for instance, in C compilers [Yang et al. 2011]), by simply comparing two programs that are supposed to be equivalent. We tackle some specific problems that arise in the stream processing domain, specifically, output comparison in the presence of out-of-order data.

7 CONCLUSION

We have presented an algorithm and a library for differential testing of distributed stream processing applications. The input to our library, implemented in Flink, consists of the two programs as well as a *dependence relation* which is used to describe which output events must be produced in order, and an optional custom equality which is used to compare output events. Our four case studies demonstrate that (1) our framework can be used to successfully find bugs in MapReduce programs that were previously identified in the literature, while at the same time it avoids false positives in most scenarios; (2) it can be used to ease the development of Flink applications, particularly programs that are difficult to parallelize; and (3) it can run in an online fashion on large input streams, using a small and empirically non-increasing amount of memory.

The core algorithmic problem is testing output streams from two implementations for equivalence, when the output may contain both ordered and unordered events. Formally, we justified that our algorithm is *correct* in an online sense, meaning that it produces a verdict of whether the streams are inequivalent as early as possible; and that it is *optimal*, meaning that for any other correct online algorithm, that algorithm must maintain at least as much state information as our algorithm does. These theorems translate to concrete performance benefits (Section 5.4).

The current work focuses on bugs due to parallelism. In future work, we would like to extend our framework to target other classes of bugs, including those due to node or network faults. Additionally, we would like to generalize the definition of correct behavior, which currently assumes that the output should be determined up to allowed reordering and equality of data items. There are cases where this is too strong, for instance when operations are approximate or randomized. Finally, we hope to extend our framework to consider input generation and input equivalence as well as output equivalence, and use this for a better strategy for selecting inputs.

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A MANUAL MATCHER FOR THE TAXI EXAMPLE

Figure 13 contains the complete code of the simplified test harness shown in Fig. 10. Figure 14 contains the code of the manual matcher for testing the Taxi distance example. Figure 15 contains the extensions to the matcher so that it does not keep unneccessary events. Figure 16 contains the test harness for the manual test. Note that it is almost the same as the one in DiffStream, the difference is that for the manual test we had to implement the matcher in Figs. 14 and 15. The class `KeyByParallelismManualSink` is not shown since it is a straightforward Flink sink node that calls the matcher methods whenever an output event is produced.

```
public void testByKey() throws Exception {

    StreamExecutionEnvironment env = StreamExecutionEnvironment.getExecutionEnvironment();

    DataStream<Tuple3<Long, Tuple2<Long, Long>, Integer>> input =
        generateInput(env);

    KeyedStream<Tuple2<Long, Tuple2<Long, Long>>, Tuple> seqOutput =
        sequentialComputation(input);
    KeyedStream<Tuple2<Long, Tuple2<Long, Long>>, Tuple> parallelOutput =
        correctParallelComputation(input);

    StreamEquivalenceMatcher<Tuple2<Long, Tuple2<Long, Long>>> matcher =
        StreamEquivalenceMatcher.createMatcher(new KeyByParallelismDependence());
    seqOutput.addSink(matcher.getSinkLeft()).setParallelism(1);
    parallelOutput.addSink(matcher.getSinkRight()).setParallelism(1);

    env.execute();

    matcher.assertStreamsAreEquivalent();
}
```

Fig. 13. The complete test harness that was shown in Fig. 10.

```
public class KeyByParallelismManualMatcher {
    public volatile static HashMap<Long, ArrayList<Tuple2<Long, Tuple2<Long, Long>>>>
         leftOutput = new HashMap<>();
    public volatile static HashMap<Long, ArrayList<Tuple2<Long, Tuple2<Long, Long>>>>
         rightOutput = new HashMap<>();

    private static final Object lock = new Object();

    public static void newLeftOutput(Tuple2<Long, Tuple2<Long, Long>> item) {
        Long key = item.f0;
        synchronized (lock) {
            ArrayList oldItems = leftOutput.getOrDefault(key, new ArrayList<>());
            oldItems.add(item);
            leftOutput.put(key, oldItems);
        }
    }

    public static void newRightOutput(Tuple2<Long, Tuple2<Long, Long>> item) {
        Long key = item.f0;
        synchronized (lock) {
            ArrayList oldItems = rightOutput.getOrDefault(key, new ArrayList<>());
            oldItems.add(item);
            rightOutput.put(key, oldItems);
        }
    }

    public static boolean allMatched() {
        boolean areAllMatched = false;
        synchronized (lock) {
            areAllMatched = leftOutput.equals(rightOutput);
            leftOutput.clear();
            rightOutput.clear();
        }
        return areAllMatched;
    }
}
```

Fig. 14. The manually written matcher for the Taxi Distance example.

```
public static boolean newLeftOutputOnline(Tuple2<Long, Tuple2<Long, Long>> item) {
    newLeftOutput(item);
    Long key = item.f0;
    return cleanUpPrefixes(key);
}

public static boolean newRightOutputOnline(Tuple2<Long, Tuple2<Long, Long>> item) {
    newRightOutput(item);
    Long key = item.f0;
    return cleanUpPrefixes(key);
}

public static boolean cleanUpPrefixes(Long key) {
    synchronized (lock) {
        ArrayList oldLeft = leftOutput.getOrDefault(key, new ArrayList<>());
        ArrayList oldRight = rightOutput.getOrDefault(key, new ArrayList<>());

        boolean unmatched = false;
        while(oldLeft.size() > 0 && oldRight.size() > 0) {
            if(oldLeft.get(0) == oldRight.get(0)) {
                oldLeft.remove(0);
                oldRight.remove(0);
            } else {
                unmatched = true;
                break;
            }
        }
        leftOutput.put(key, oldLeft);
        rightOutput.put(key, oldRight);
        return unmatched;
    }
}
```

Fig. 15. The online extension of the manual matcher.

```
public void manualTestByKey() throws Exception {
    StreamExecutionEnvironment env = StreamExecutionEnvironment.getExecutionEnvironment();

    DataStream<Tuple3<Long, Tuple2<Long, Long>, Integer>> input =
        generateInput(env);

    KeyedStream<Tuple2<Long, Tuple2<Long, Long>>, Tuple> seqOutput =
        sequentialComputation(input);
    KeyedStream<Tuple2<Long, Tuple2<Long, Long>>, Tuple> parallelOutput =
        correctParallelComputation(input);

    seqOutput.addSink(new KeyByParallelismManualSink(true, false)).setParallelism(1);
    parallelOutput.addSink(new KeyByParallelismManualSink(false, false)).setParallelism(1);

    env.execute();

    assert(KeyByParallelismManualMatcher.allMatched());
}
```

Fig. 16. The test harness that uses the manual matcher.