Esercizio 1

Si consideri il segnale:

$$s(t) = rect\left(\frac{t-2}{4}\right)e^{-2t}$$

e si risponda alle seguenti domande:

a) rappresentare graficamente il segnale;

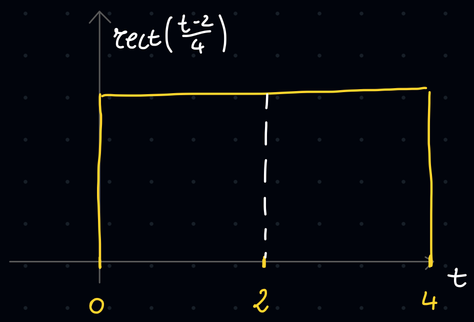
b) calcolare l'energia e la potenza media del segnale e discutere se $s(t)$ è un segnale a energia finita o a potenza media finita (NOTA: da ora in avanti per potenza si intenderà sempre la potenza media e non la potenza istantanea);

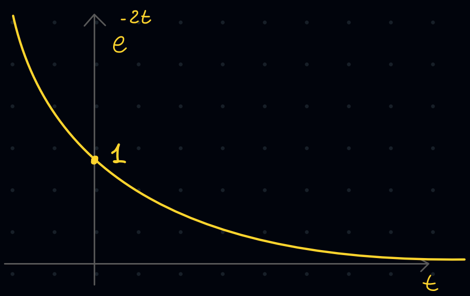
c) scrivere l'espressione analitica e rappresentare graficamente i segnali:

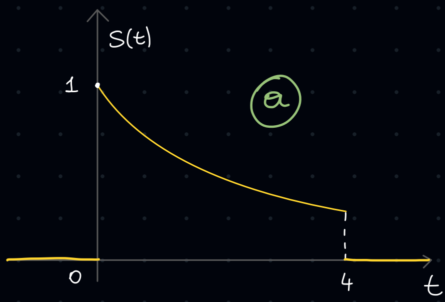
$$z(t) = -s(-t)$$

$$v(t) = s(t+4)$$

$$s(t) = rect\left(\frac{t-2}{4}\right)\cdot e^{-2t}$$







a

b $\mathcal{E}_s$ e $\mathcal{P}_s$

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} \|s(t)\|^2 dt = \int_0^4 \left(e^{-2t}\right)^2 dt = \int_0^4 e^{-4t} dt = \frac{1}{4}\left[-e^{-4t}\right]_0^4 = \frac{1}{4}\left[-e^{-16} + e^{0}\right]$$

$$= \underbrace{-\frac{1}{4}e^{-16}}_{<<} + \frac{1}{4} \sim \frac{1}{4} \quad b_1$$

$\Rightarrow \mathcal{E} \neq 0 \neq +\infty \Rightarrow$ S. di energia $\Uparrow$

$$\mathcal{P}_s = \langle \|s(t)\|^2 \rangle = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{+T} \|s(t)\|^2 dt = \frac{1}{2T}\underbrace{\int_0^4 e^{-4t} dt}_{\text{numero}} = \emptyset \quad b_2$$

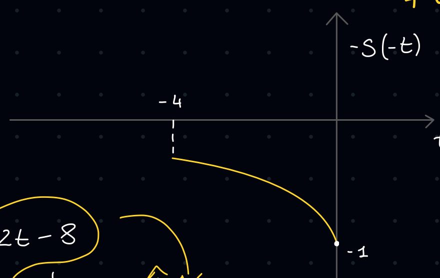
b Bonus Consideriamo

$$s(t) = e^{-2t}$$

$$\Rightarrow \mathcal{E}_s = \int_{-\infty}^{+\infty} e^{-2t} dt = +\infty$$ ovviamente

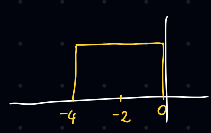
$$\Rightarrow \mathcal{P}_s = \lim_{T\to+\infty} \frac{1}{2T}\int_{-T}^{T} e^{-4t} dt = \frac{1}{2T}\cdot\frac{1}{4}\left[-e^{-4t}\right]_{-T}^{T} = \frac{1}{2}\lim \frac{\overbrace{-e^{-4T}}^{\to \emptyset} + e^{4T}}{4T} \sim \underbrace{\frac{e^{4T}}{4T}}_{+\infty}$$

$\Rightarrow$ Sia $\mathcal{E}$ che $\mathcal{P}$ sono Infinite!

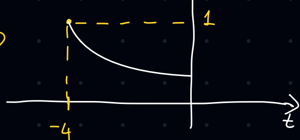


c: $z(t) = -s(-t) = -\left[rect\left(\frac{-t-2}{4}\right) e^{2t}\right]$

$$v(t) = s(t+4) = rect\left(\frac{t+2}{4}\right)\cdot e^{-2(t+4)} = rect\left(\frac{t+2}{4}\right) e^{-2t-8}$$



$\Rightarrow$



Esercizio 2

Si consideri il segnale:

$$s(t) = sgn\left(a \cdot cos\left(\frac{2\pi}{T_0}t\right)\right)$$

e si risponda alle seguenti domande:

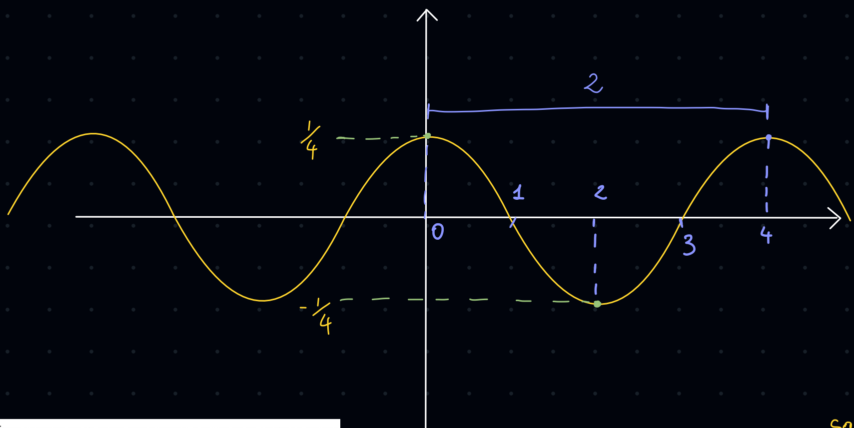
a) rappresentare graficamente il segnale;

b) calcolare l'energia e la potenza del segnale e discutere se è un segnale a energia finita o a potenza finita.

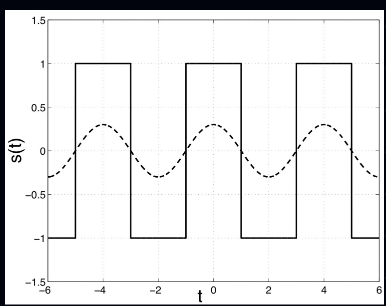
$$S(t) = Sgn\left(a \cdot cos\left(\frac{2\pi}{T_0}t\right)\right)$$

$$\begin{cases} -1 & t < 0 \\ 0 & t = 0 \\ 1 & t > 0 \end{cases}$$

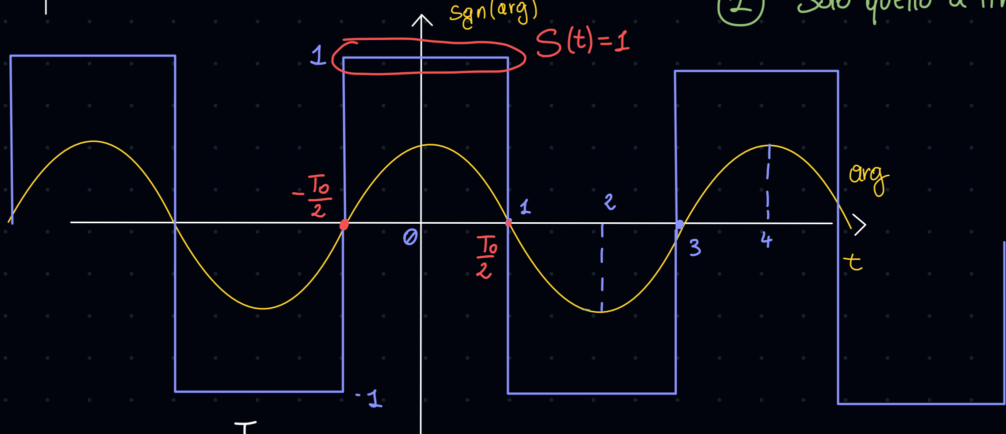
$a \cdot cos\left(\frac{2\pi}{T_0}t\right)$ Segnale periodico di ampiezza a e periodo T_0 : pongo $a = \frac{1}{2}$ e $T_0 = 4$



⇒ Siccome Sgn(t) vale 0 quando $t > 0$, 1 per $t > 0$ e -1 per $t < 0$ otteniamo



① N.B. Il segnale è solo quello a linea viola



b_1: I S. periodici sono TUTTI di potenza:

$$P_s = \langle \| S^2(t) \| \rangle = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} S^2(t)\,dt =$$ Siccome S(t) è periodico possiamo considerarlo nell'intervallo $(-T_0, +T_0)$ moltiplicato per n volte

$$= \lim_{n \to +\infty} \frac{n}{2nT_0} \int_{-T_0}^{+T_0} S^2(t)\,dt =$$ Siccome nell'intervallo (ciclo/periodo) $-T_0$ T_0 il segnale vale 1, scriviamo:

$$= \lim_{n \to \infty} \frac{\cancel{n}}{2\cancel{n}T_0} \int_{-T_0}^{+T_0} 1\,dt = \frac{1}{2T_0} \cdot t\Big|_{-T_0}^{+T_0} = \frac{1}{2T_0} \cdot 2T_0 = 1$$

$P_s \neq 0 \neq +\infty$

$$\mathcal{E}_s = \int_{-\infty}^{+\infty} S^2(t) = \lim_{n \to +\infty} n \int_{-T_0+\varepsilon}^{+T_0+\varepsilon} S^2(t) = c \lim_{n \to \infty} n = +\infty$$

→ Valore finito

$\mathcal{E}_s = +\infty$

⇒ Segnale di potenza

Esercizio 3

Si consideri il segnale:

$$s(t) = 2tr\left(\frac{t}{4}\right)$$

e si risponda alle seguenti domande:

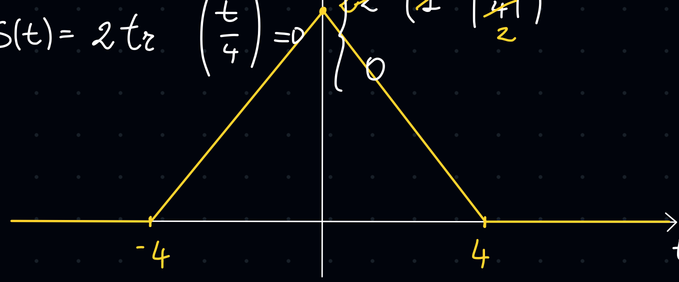
a) rappresentare graficamente il segnale;

b) calcolare l'energia e la potenza del segnale e discutere se è un segnale a energia finita o a potenza finita;

c) scrivere l'espressione analitica e rappresentare graficamente il segnale:

$$v(t) = s(2t)$$

$$S(t) = 2tr\left(\frac{t}{4}\right) = \begin{cases} \cancel{2}2\cdot\left(\cancel{1}^{\,2} - \left|\frac{t}{\cancel{4}_{\,2}}\right|\right) \\ 0 \end{cases}$$

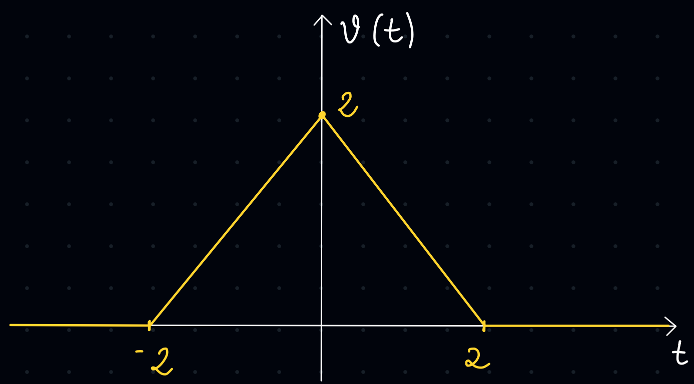


$$\mathcal{E}_s = \int_{-\infty}^{+\infty} s^2(t)\,dt = \int_{-4}^{4}\left(2-\left|\frac{t}{2}\right|\right)^2 dt =$$

$$= 2\int_0^4 4+\frac{t^2}{4}-2t = 2\left(4t+\frac{1}{4}\frac{t^3}{3}-t^2\right)\Big|_0^4 = 2\left(16+\frac{64}{12}-16\right) = 2\cdot\frac{16}{3} = \frac{32}{3}$$ $\mathcal{E}_s \neq 0 \neq \infty$

$$P_s = \langle \|s(t)\|^2 \rangle = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} s^2(t)\,dt = \lim_{T\to\infty}\frac{1}{2T}\int 4tr^2\left(\frac{t}{4}\right)dt$$

c : $v(t) = s(2t) = 2tr\left(\frac{2t}{4}\right) = 2tr\left(\frac{t}{2}\right)$



Il segnale viene compresso

Esercizio 4

Studiare le proprietà di simmetria del segnale s(t) e scomporlo nella sua parte pari e parte dispari:

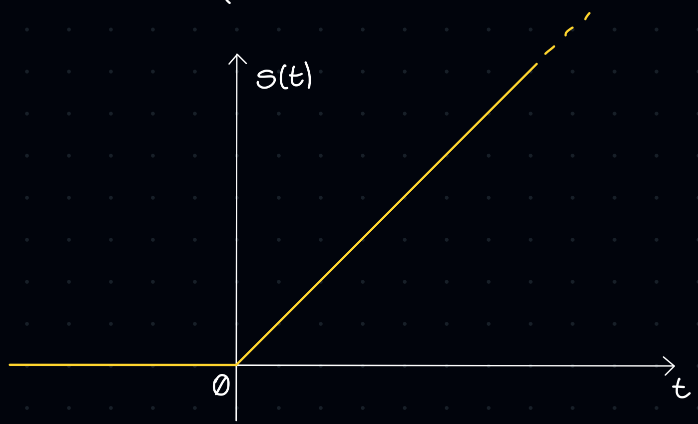
$$s(t) = t \cdot \mathrm{u}(t)$$

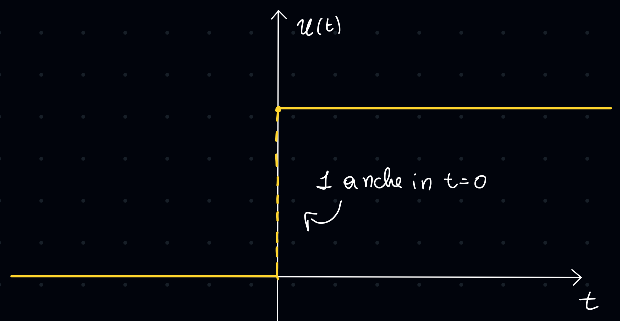
$S(t) = t \cdot u(t)$

$u(t)$ gradino unitario

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$\Rightarrow \quad t \cdot u(t) \begin{cases} 0 & \text{per } t < 0 \\ t & \text{per } t \geq 0 \end{cases}$$





Un segnale reale è definito pari o dispari se soddisfa le seguenti relazioni:

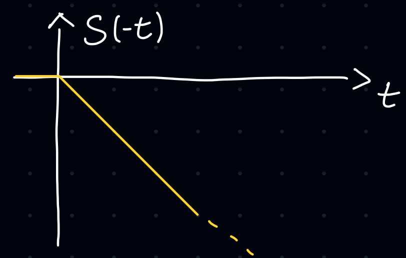
$$s(t) \quad \text{pari} \quad \Longleftrightarrow \quad s(t) = s(-t) \quad \forall t$$

$$s(t) \quad \text{dispari} \quad \Longleftrightarrow \quad \begin{cases} s(t) = -s(-t) & \forall t, t \neq 0 \\ s(t) = 0 & t = 0 \end{cases}$$

- $S(t) = S(-t) \ \forall t$? guardando il grafico: NO $\Rightarrow$ Dispari

Proof

$$S(t) = \begin{cases} 0 & \text{per } t < 0 \\ t & \text{per } t \geq 0 \end{cases} \Rightarrow S(-t) = \begin{cases} 0 & \text{per } t < 0 \\ -t & \text{per } t \geq 0 \end{cases}$$



- $S(t) = \begin{cases} S(t) = -S(-t) & \forall t, t \neq 0 \\ S(t) = 0 & t = 0 \end{cases}$

$$S(t) = -S(-t) \ ? \quad \Rightarrow \quad -S(-t) = \begin{cases} 0 & t = 0 \\ \underbrace{-(-t)}_{t} & t \geq 0 \end{cases} \Rightarrow S(t) = -S(-t) \ \forall t \ \checkmark$$

$$S(0) = 0 \ \checkmark$$

Per scomporre un segnale nella sua parte pari e parti dispari si procede nel seguente modo:

$$s_p(t) = \frac{s(t) + s(-t)}{2}$$
$$s_d(t) = \frac{s(t) - s(-t)}{2}$$

$$\begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \qquad \begin{cases} 0 & -t < 0 \\ 1 & -t \geq 0 \end{cases} \rightarrow \begin{cases} 0 & t > 0 \\ 1 & t \leq 0 \end{cases}$$

$$\Rightarrow S_p(t) = \frac{t \cdot u(t) + (-t \cdot u(-t))}{2} = \frac{t u(t) - t u(-t)}{2} = \frac{t\,\overbrace{(u(t) - u(-t))}^{1}}{2} = \frac{|t|}{2}$$

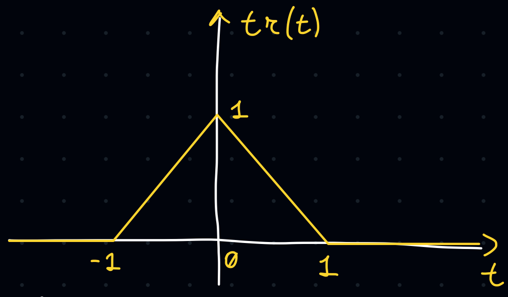
$$S_d = \frac{t u(t) - (-t\, u(-t))}{2} = \frac{t u(t) + t u(-t)}{2} = \frac{t\,\overbrace{(u(t) + u(-t))}^{1}}{2} = \frac{t}{2}$$

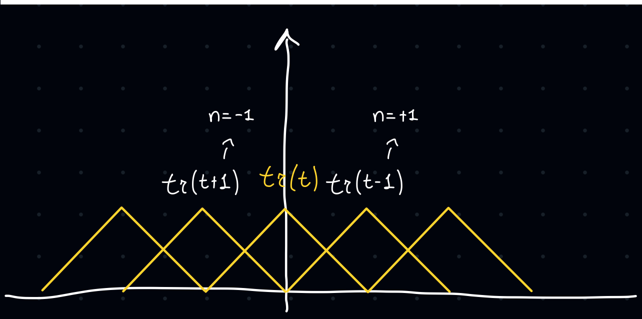
Esercizio 5

Disegnare il grafico del seguente segnale:

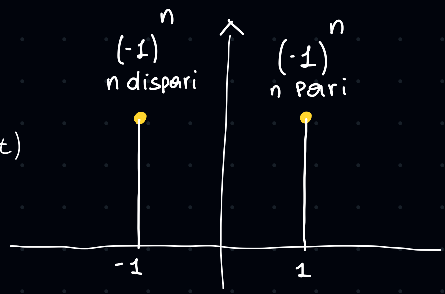
$$s(t) = \sum_{n=-\infty}^{\infty} (-1)^n \mathrm{tr}(t-n)$$

$$S(t) = \sum_{n=-\infty}^{\infty} (-1)^n \, tr(t-n)$$

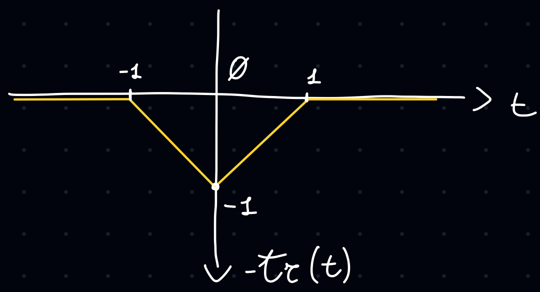


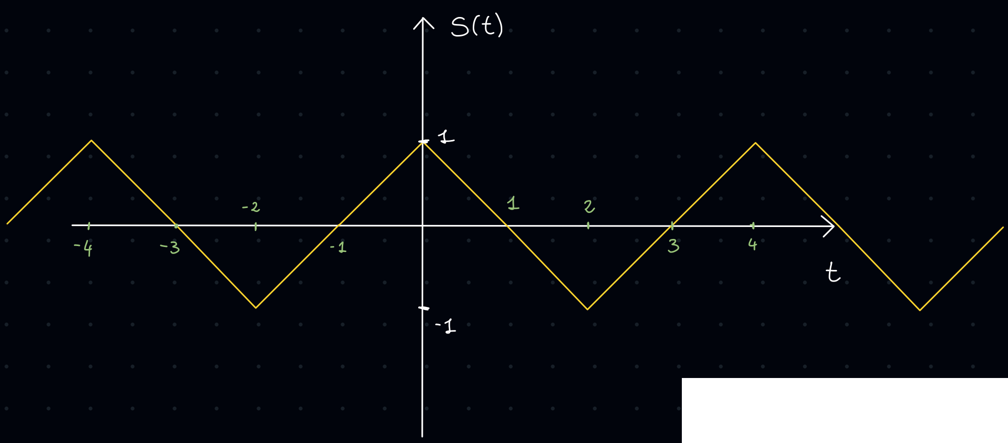


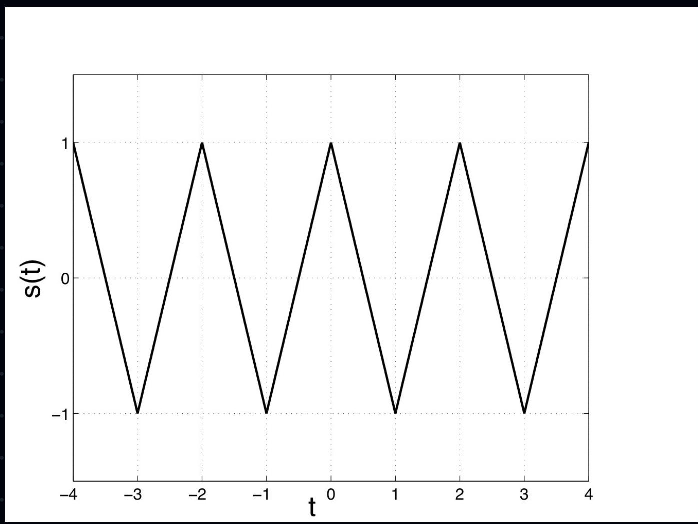
$$\Rightarrow \sum_{n=-\infty}^{+\infty} tr(t-n) = \text{ripetizione infinita di } tr(t)$$



$$\Rightarrow \sum_{n=-\infty}^{+\infty} (-1)^n \, tr(t-1) = \begin{cases} tr(t-1) & n \text{ pari} \\ -tr(t-1) & n \text{ dispari} \end{cases}$$







Altri esercizi

1. Disegnare il grafico dei seguenti segnali:

   a) $\mathrm{rect}(t) - \mathrm{rect}(t-1)$

   b) $\mathrm{tr}(t) \cdot \mathrm{rect}(t)$

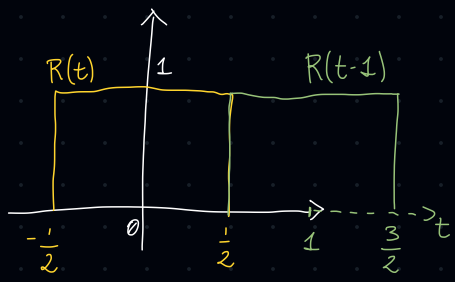
   c) $\sum_{n=-\infty}^{\infty} (-1)^n \mathrm{rect}\left(t - \frac{3}{4}n\right)$

   d) $1 + \mathrm{sgn}(1-t)$

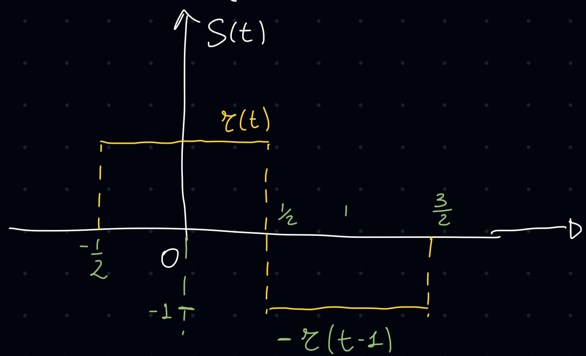
   e) $\mathrm{sinc}(t) \cdot \mathrm{sgn}(t)$

   f) $\sum_{n=1}^{\infty} \frac{1}{2^n} \mathrm{rect}\left(\frac{t}{n}\right)$

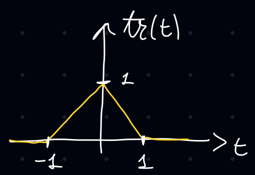
a) $\mathrm{rect}(t) - \mathrm{rect}(t-1)$

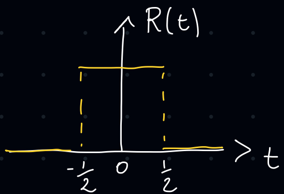


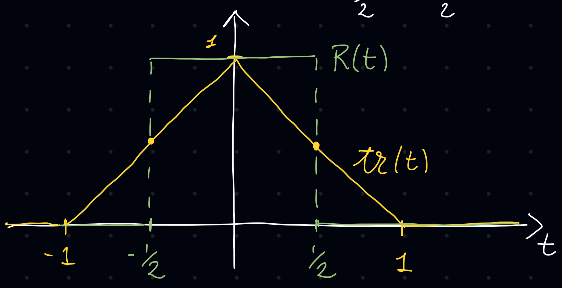
$$\begin{cases} 1 & -\frac{1}{2} < t < \frac{1}{2} \\ 0 & \text{Altrove} \end{cases} \qquad \begin{cases} 1 & \frac{1}{2} < t < \frac{3}{2} \\ 0 & \text{Altrove} \end{cases}$$

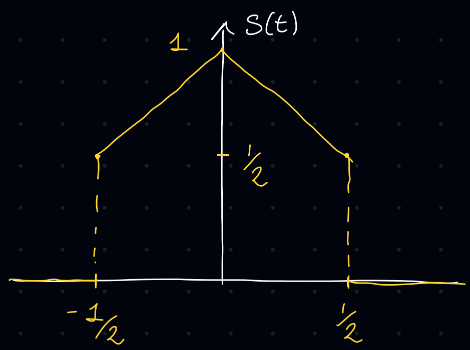


b) $\mathrm{tr}(t) \cdot \mathrm{rect}(t) = S(t)$

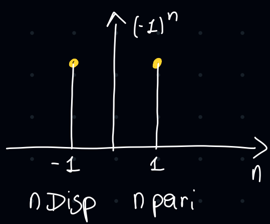


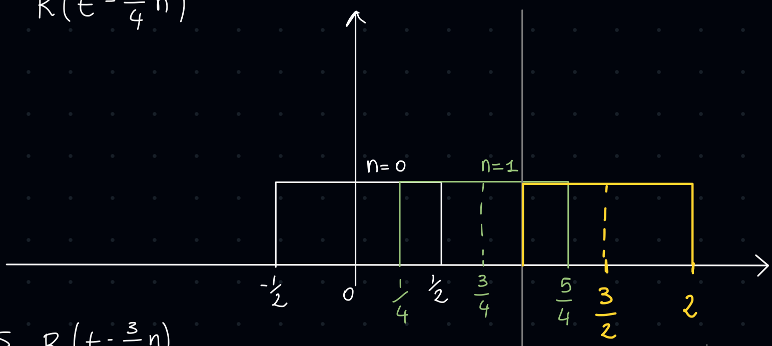




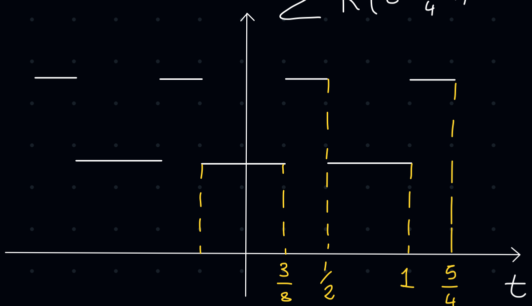


c) $s(t) = \sum_{n=-\infty}^{\infty} (-1)^n R\left(t - \frac{3}{4}n\right)$

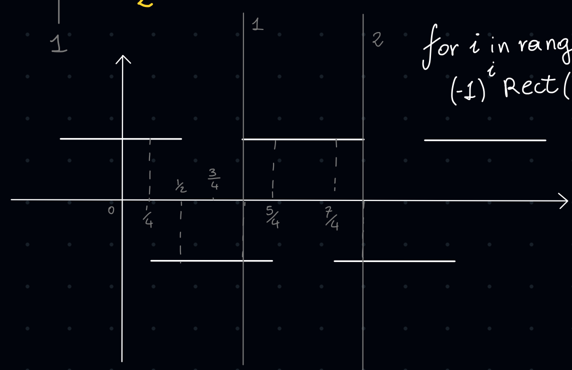


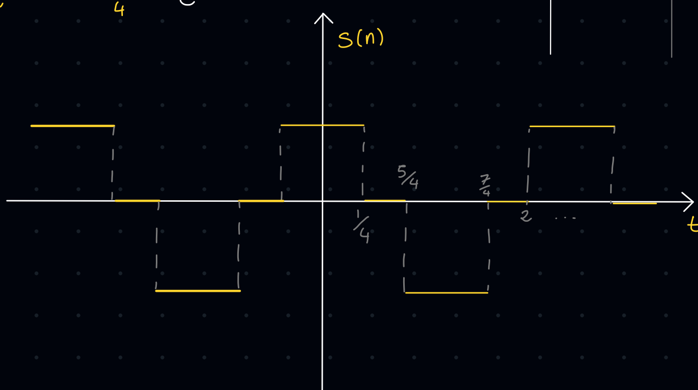


$\sum R\left(t - \frac{3}{4}n\right)$

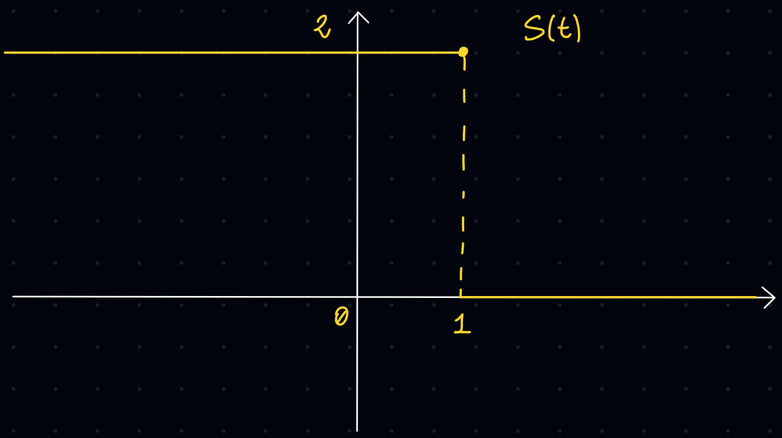


for i in range $(-2,2)$
$(-1)^i \mathrm{Rect}\left(t - \frac{3}{4}i\right)$

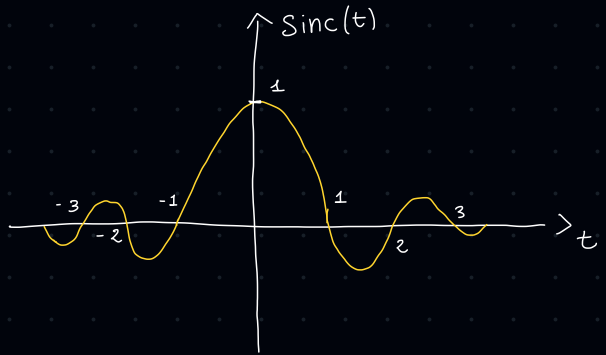


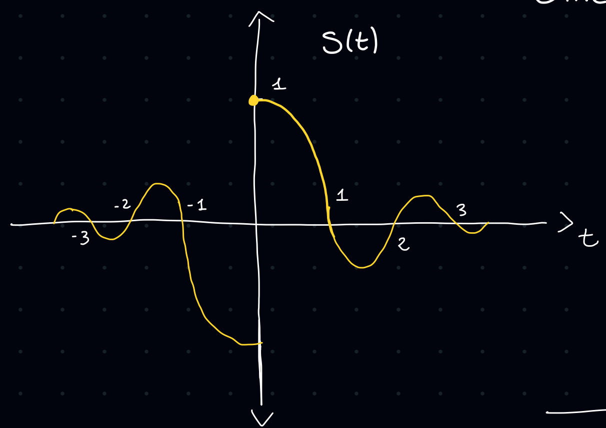


(d) : $1 + \mathrm{Sgn}(1-t)$ $\quad -\mathrm{Sgn}(t-1) \equiv \mathrm{Sgn}(1-t)$

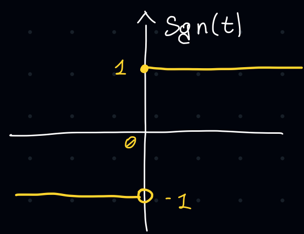


(e) : $\mathrm{Sinc}(t) \cdot \mathrm{Sgn}(t)$

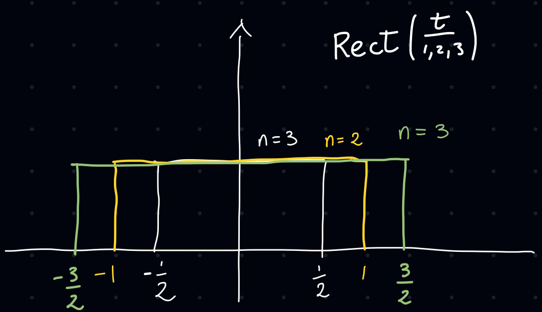


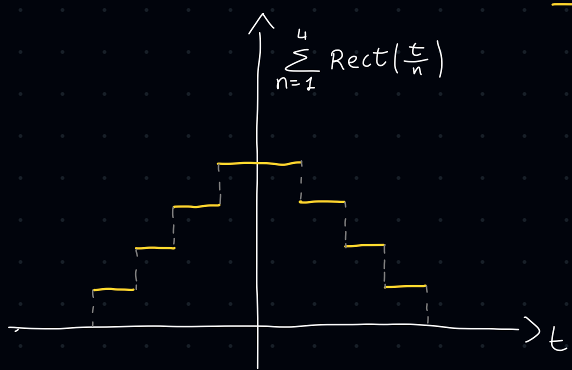


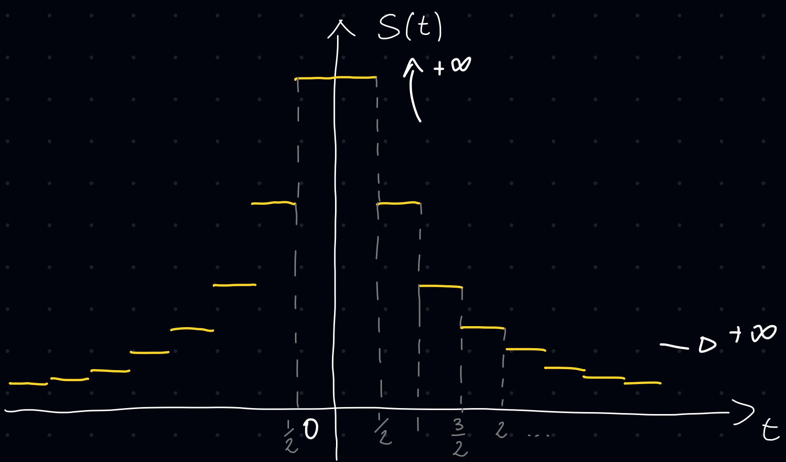
$$\mathrm{Sinc} = \begin{cases} \dfrac{\sin(\pi t)}{\pi t} & t \neq 0 \\ 1 & t = 0 \end{cases}$$



(f) : $\sum_{n=1}^{\infty} \frac{1}{2^n} \mathrm{Rect}\left(\frac{t}{n}\right)$







2. Calcolare energia e potenza dei seguenti segnali:

a) $A\cos(2\pi f_0 t + \phi_0) + B\cos(2\pi f_1 t + \phi_1)$

b) $e^{-t}\cos(t)\mathrm{u}(t)$

c) $e^{-t}\cos(t)$

d) $e^{-t}\cdot rect\left(\frac{t-1}{2}\right) - e^{-t}\cdot rect\left(\frac{t-3}{2}\right)$

a: $S(t) = A\cos(2\pi f_0 t + \varphi_1) + B\cos(2\pi f_1 t + \varphi_2)$

$S_1(t) = A\cos(2\pi f_0 t + \varphi_1)$

$$\Rightarrow P_{S_1} = \langle \| S_1 \|^2 \rangle = \langle A^2 \cos^2(2\pi f_0 t + \varphi_1) \rangle = \langle A^2 \left[\frac{1}{2}\cos(4\pi f_0 t + 2\varphi_1) + \frac{1}{2}\right] \rangle$$

$$= \langle \frac{A^2}{2}\cos(4\pi f_0 t + 2\varphi_1) + \frac{A^2}{2} \rangle$$

C

$\cos(2t) = 2\cos^2(t) - 1 \Rightarrow \cos^2(t) = \frac{1}{2}\cos(2t) + \frac{1}{2}$

$\Rightarrow \cos^2(2\pi f_0 t + \varphi_1) = \frac{1}{2}\cos(4\pi f_0 t + 2\varphi_1) + \frac{1}{2}$

$$= \frac{A^2}{2} + \langle \cos(4\pi f_0 t + 2\varphi_1 \rangle = \frac{A^2}{2} \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \cos(4\pi f_0 t + 2\varphi_1\, dt$$

funct

$$= \frac{A^2}{2} + \lim_{n\to\infty} \frac{1}{2nT_0} \int_{nT_0}^{nT_0} \cos(z)\, dt = \frac{A^2}{2} + \lim_{n\to\infty} \frac{\cancel{n}}{2\cancel{n}T_0} \int_{-T_0}^{T_0} \cos(z)\, dt$$

$$= \frac{A^2}{2} + \underbrace{\int_{-T_0}^{T_0} \cos(z)\, dz}_{\emptyset} = \frac{A^2}{2} \quad P_{S_1}$$

$$\Rightarrow P_{S_2} = \frac{B^2}{2}$$

$$\Rightarrow \mathcal{E}_{S_1} = \int_{-\infty}^{+\infty} S_1^2(t)\, dt = \int_{-\infty}^{+\infty} A^2\cos^2(2\pi f_0 t + \varphi_1)\, dt = A^2 \underbrace{\int_{-\infty}^{+\infty} \cos^2(2\pi f_0 t + \varphi_1)\, dt}_{\emptyset}$$

$$= \emptyset \quad \mathcal{E}_{S_1} = \mathcal{E}_{S_2}$$

$$P_{S_1S_2} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} S_1 \cdot S_2^* \, dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} AB\cos(2\pi f_0 t + \varphi_1)\cos(2\pi f_0 t + \varphi_2)\, dt$$

$$= \lim_{T\to\infty} \frac{AB}{2T} \int_{-T}^{T} \cos(2\pi f_0 t + \varphi_1) \cdot \cos(2\pi f_0 t + \varphi_2)\, dt$$

- Diversi casi possibili:
  - $f_0 = f_1 = f$ e S_1 ha amp A e S_2 ha amp B $\Rightarrow$ c é l'ampiezza di $S_1 + S_2$

    $\Rightarrow c^2 = a^2 + b^2 + 2ab\cos((\varphi_2 - \varphi_2))$ fase

    $\Rightarrow$ Siccome abbiamo dimostrato che $P_{S_1} = \frac{A^2}{2}$ e $P_{S_2} = \frac{B^2}{2}$ $\Rightarrow A_{S_1+S_2} = \frac{C^2}{2}$

    $$\Rightarrow P_{S_1+S_2} = \frac{a^2 + b^2 + 2ab\cos(\varphi_2 - \varphi_1)}{2} = \frac{1}{2}A^2 + \frac{1}{2}B^2 + AB\cos(\varphi_2 - \varphi_1)$$

    $$= P_{S_1} + P_{S_2} + AB\cos(\varphi_2 - \varphi_1)$$

    $f_0 = f_1 = f$ $\quad \varphi_1 \neq \varphi_2$

    $$= P_{S_1} + P_{S_2} + AB$$

    $f_0 = f_1 = f$

    $\varphi_1 = \varphi_2$