Consideriamo il sistema caratterizzato dalla rel in-out $y(t) = 2x(t) + x(t-t_0)$

Q_1: Modulo della risposta in frequenza

Per prima cosa dobbiamo calcolare $Y(f)$ Tenendo a mente $A \cdot x(t) \rightleftharpoons AX(f)$, $x(t-T_0) \rightleftharpoons X(f) \cdot e^{-j2\pi f T_0}$

$$\to Y(f) = 2X(f) + X(f) \cdot e^{-j2\pi f t_0}$$

La risposta in freq è $H(f)$ $\Rightarrow$ sappiamo che: $Y(f) = X(f) \cdot H(f) \Rightarrow H(f) = \frac{Y(f)}{X(f)}$

$$\Rightarrow H(f) = \frac{2\cancel{X(f)}}{\cancel{X(f)}} + \frac{\cancel{X(f)}}{\cancel{X(f)}} \cdot e^{-j2\pi f t_0} \to H(f) = 2 + e^{-j2\pi f t_0}$$

complesso!

Il modulo di un numero complesso dipende dalla sua forma, ma in generale $|a+ib| = \sqrt{a^2+b^2}$

Siccome $e^{-j2\pi f t_0} = \cos(2\pi f t_0) - j\sin(2\pi f t_0) \to H(f) = 2 + \cos(2\pi f t_0) - j\sin(2\pi f t_0)$

otteniamo $|H(f)| = \sqrt{\underbrace{[2+\cos(2\pi f t_0)]^2}_{a} + \underbrace{\sin^2(2\pi f t_0)}_{b}} = \sqrt{4 + \cos^2(2\pi f t_0) + 4\cos(2\pi f t_0) + \sin^2(2\pi f t_0)}$

Sappiamo che $\cos^2(w) + \sin^2(w) = 1$

$$\to \sqrt{5 + 4\cos(2\pi f t_0)}$$

Q_2A: Consideriamo in input $x(t) = \Pi\left(\frac{t}{t_0}\right)$ calcolare $y(t)$

$\to$ Siccome abbiamo già $H(f)$ Trasformiamo $x(t)$ $\quad A\Pi\left(\frac{t}{t_0}\right) \rightleftharpoons At_0 \operatorname{Sinc}(ft_0)$

$$\to \Pi\left(\frac{t}{t_0}\right) \rightleftharpoons t_0 \operatorname{sinc}(ft_0)$$

$$\Rightarrow \text{Sappiamo che } Y(f) = X(f) \cdot H(f) = t_0 \operatorname{Sinc}(ft_0) \cdot \left[2 + e^{-j2\pi f t_0}\right] = \underbrace{2t_0 \operatorname{Sinc}(ft_0) + t_0 \operatorname{Sinc}(ft_0)\, e^{-j2\pi f t_0}}_{Y(f)}$$

Per trovare $y(t)$ dobbiamo Trasformare tenendo a mente le proprietà:

$$\begin{cases} X(f) \cdot e^{-j2\pi f T_0} \rightleftharpoons x(t-t_0) \\ A \operatorname{Sinc}(2Bf) \rightleftharpoons \frac{A}{2B}\Pi\left(\frac{t}{2B}\right) \end{cases} \qquad \text{Siccome } At_0 \operatorname{Sinc}(ft) \rightleftharpoons A\,\Pi\left(\frac{t}{t_0}\right)$$

- $2t_0 \operatorname{Sinc}(ft_0) \rightleftharpoons 2\Pi\left(\frac{t}{t_0}\right)$
- $t_0 \operatorname{sinc}(ft_0) \cdot e^{-j2\pi f t_0} \rightleftharpoons \Pi\left(\frac{t-t_0}{t_0}\right)$

$$\Big\} \; y(t) = 2\Pi\left(\frac{t}{t_0}\right) + \Pi\left(\frac{t-t_0}{t_0}\right)$$

Time 24 (Q_1+Q_2A)

Q_2B: S_y ?

Sappiamo che $S_y = |Y(f)|^2$, siccome abbiamo già $Y(f)$ ci basta solo calcolare il mod quadro

$$|Y(f)|^2 = Y(f) \cdot Y^*(f) = \left[2t_0\operatorname{Sinc}(ft_0) + t_0\operatorname{Sinc}(ft_0)e^{-j2\pi f t_0}\right] \cdot \left[2t_0\operatorname{Sinc}(ft_0) + t_0\operatorname{Sinc}(ft_0)e^{j2\pi f t_0}\right]$$

$$= 4t_0^2\operatorname{Sinc}^2(ft_0) + 2t_0^2\operatorname{Sinc}(ft_0)e^{j2\pi f t_0} + 2t_0^2\operatorname{Sinc}(ft_0)e^{-j2\pi f t_0} + t_0^2\operatorname{Sinc}^2(ft_0) \to$$

Mettiamo insieme i due fasori:

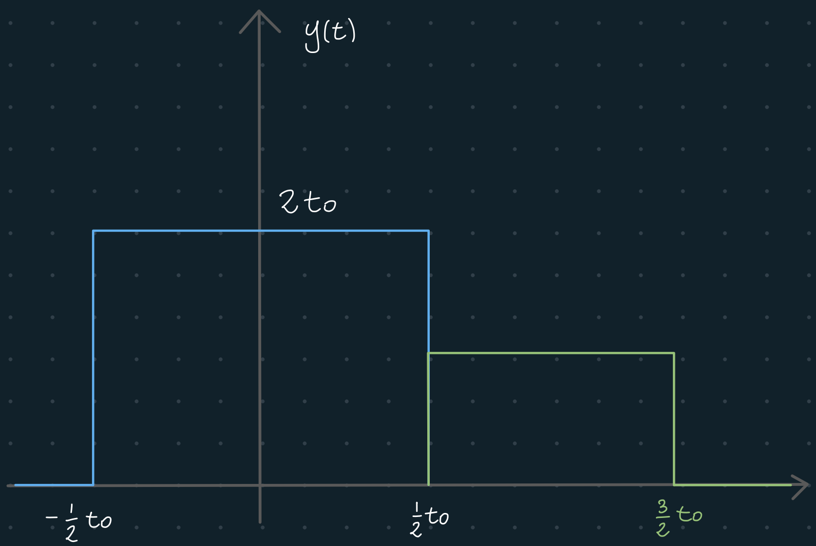
$$= 4t_0^2\operatorname{Sinc}^2(ft_0) + t_0^2\operatorname{Sinc}^2(ft_0) + 2t_0^2\operatorname{Sinc}(ft_0)\left[e^{j2\pi f t_0} + e^{-j2\pi f t_0}\right] \to 2\cos(2\pi f t_0)$$

$$= 5t_0^2\operatorname{Sinc}^2(ft_0) + 2t_0^2\operatorname{Sinc}(ft_0)\cos(2\pi f t_0)$$

$$y(t) = 2\Pi\left(\frac{t}{t_0}\right) + \Pi\left(\frac{t-t_0}{t_0}\right)$$

Per trovare gli estremi di $\Pi\left(\frac{t-t_0}{t_0}\right)$

$$t_0 - \frac{1}{2}t_0 = \frac{1}{2}t_0 \quad , \quad t_0 + \frac{1}{2}t_0 = \frac{3}{2}t_0$$



Procedura alternativa

Abbiamo $H(f)$, $x(t)$ e dobbiamo trovare $S_y(f)$

$$\Rightarrow S_y(f) = |Y(f)|^2 \quad \text{ma} \quad Y(f) = X(f)\cdot H(f) \Rightarrow S_y(f) = |X(f)\cdot H(f)|^2$$

Troviamo $X(f) = x(t) = \Pi\left(\frac{t}{t_0}\right) \rightleftharpoons t_0 \,\text{Sinc}\left(\frac{f}{t_0}\right)$

$$\Rightarrow S_y(f) = \left| t_0 \,\text{Sinc}\left(\frac{f}{t_0}\right) \cdot \left[2 + e^{-j2\pi f t_0}\right] \right|^2 = \left| 2t_0 \,\text{Sinc}\left(\frac{f}{t}\right) + t_0 e^{-j2\pi f t_0} \right|^2$$

$$\Rightarrow S_y(f) = 4t_0^2 \,\text{Sinc}^2\left(\frac{f}{t}\right) + 2t_0^2 \,\text{Sinc}\left(\frac{f}{t_0}\right) e^{j2\pi f t_0} + 2t_0^2 \,\text{Sinc}\left(\frac{f}{t_0}\right) e^{-j2\pi f t_0} + t_0^2$$

$$\Rightarrow S_y(f) = 2t_0^2 \,\text{Sinc}\left(\frac{f}{t_0}\right)\left[e^{j2\pi f t_0} + e^{-j2\pi f t_0}\right] + 6t_0^2 \,\text{Sinc}^2\left(\frac{f}{t}\right) + t_0^2$$

$$= 4t_0^2 \,\text{Sinc}\left(\frac{f}{t_0}\right)\cos(2\pi f t_0) + 6t_0^2 \,\text{Sinc}^2\left(\frac{f}{t_0}\right) + t_0$$

$\leftarrow$ escono 2 risultati leggermente diversi (?)