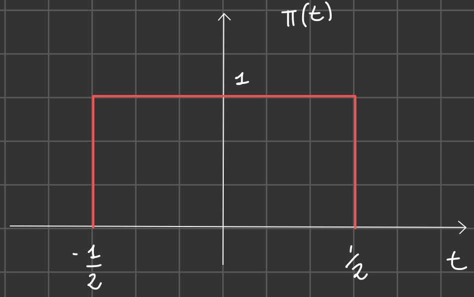
EX. 2

Sia $x(t)$ il treno di impulsi rettangolari

$$x(t) = \text{rep}_{1/50}\Pi\,(100t)\ .$$

Calcolarne la trasformata di Fourier. Il segnale $x(t)$ viene successivamente filtrato con un filtro passa-basso ideale avente frequenza di taglio pari a 60 Hz. Calcolare l'espressione nel dominio del tempo del segnale in uscita al filtro.

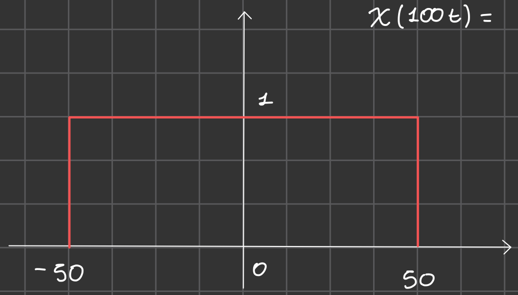
$x(t)$ e' un Treno di impulsi reTTangolari con periodo di rep $1/50 = T$



$$\begin{cases} 1 & -\frac{1}{2} < t < \frac{1}{2} \\ 0 & otw \end{cases}$$

$$x(100t) = \begin{cases} 1 & -\frac{1}{2}\frac{1}{100} < \cancel{100}t\frac{}{\cancel{100}} < \frac{1}{2}\frac{1}{100} \\ 0 \end{cases}$$

-50 50

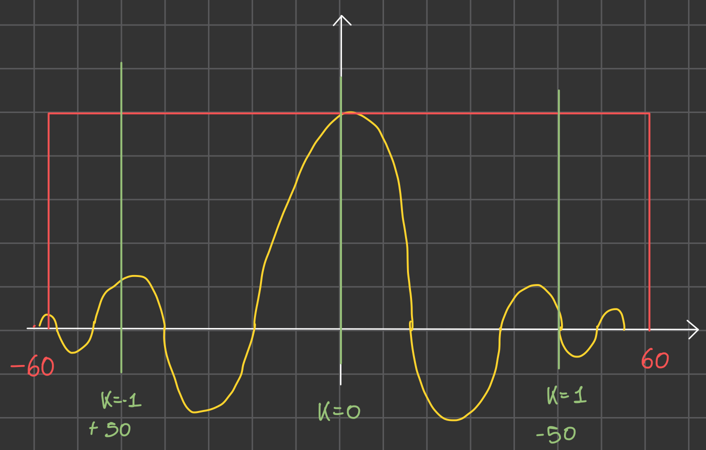


$$\Rightarrow rep_{\frac{1}{50}}\left[\pi(100t)\right] = \sum_{K=-\infty}^{+\infty} \pi(100t)\cdot\delta(t-KT) = \sum_{K=-\infty}^{+\infty} \pi\left(100t - \frac{K}{50}\right)$$

$$X(f) = \frac{1}{100}\cdot \text{Sinc}\left(\frac{f}{100}\right) \Rightarrow \tilde{X}_\delta(f) = \frac{1}{\frac{1}{50}}\cdot\frac{1}{100}\sum_{m=-\infty}^{+\infty} \text{Sinc}\left(\frac{f}{100}\right)\cdot\delta\left(f-\frac{K}{T}\right) = \frac{\cancel{50}}{\cancel{100}_{2}}\sum_{m=-\infty}^{+\infty}\text{Sinc}\left(\frac{f}{100}\right)\cdot\delta(f-50K)$$

$$= \frac{1}{2}\sum_{m=-\infty}^{+\infty}\text{Sinc}\left(\frac{\cancel{50}K}{\cancel{100}_2}\right)\cdot\delta(f-50K) = \frac{1}{2}\sum_{m=-\infty}^{+\infty}\text{Sinc}\left(\frac{1}{2}K\right)\cdot\delta(f-50K) \quad \tilde{X}_\delta(f)$$

$\Rightarrow$ Se viene filtrato con un filtro passa basso del tipo: $\pi\left(\frac{f}{120}\right)$



$\Rightarrow$ OTTENIAMO SOLO 3 CAMPIONAMENTI ovvero in: $K = \{-1, 0, 1\}$

$\Rightarrow$ Possiamo scriverci $\tilde{X}_\delta(f)\cdot\pi\left(\frac{f}{120}\right)$

$$\underbrace{\frac{1}{2}\left[\text{Sinc}\left(-\tfrac{1}{2}\right)\delta(f+50) + \text{Sinc}(0)\,\delta(f) + \text{Sinc}\left(\tfrac{1}{2}\right)\delta(f-50)\right]}_{\text{Spettro filtrato}}$$

$\Rightarrow$ Possiamo trovare il segnale $y(t)$ trasformando lo spettro filtrato:

$$y(t) = \frac{1}{2}\left[\text{Sinc}\left(-\tfrac{1}{2}\right)e^{J2\pi 50t} + 1 + \text{sinc}\left(\tfrac{1}{2}\right)e^{-J2\pi 50t}\right]$$

siccome la sinc e' pari: $\text{Sinc}(-x) = \text{Sinc}(x)$

$$\Rightarrow y(t) = \frac{1}{2}\,\text{Sinc}\left(\tfrac{1}{2}\right)\left[e^{J2\pi 50t} + e^{-J2\pi 50t}\right] + \frac{1}{2} = \text{Sinc}\left(\tfrac{1}{2}\right)\cos(100\pi t) + \frac{1}{2}$$