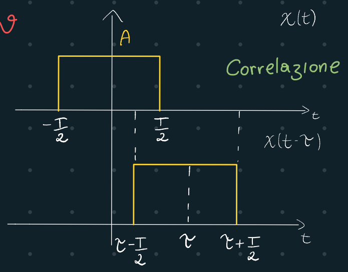
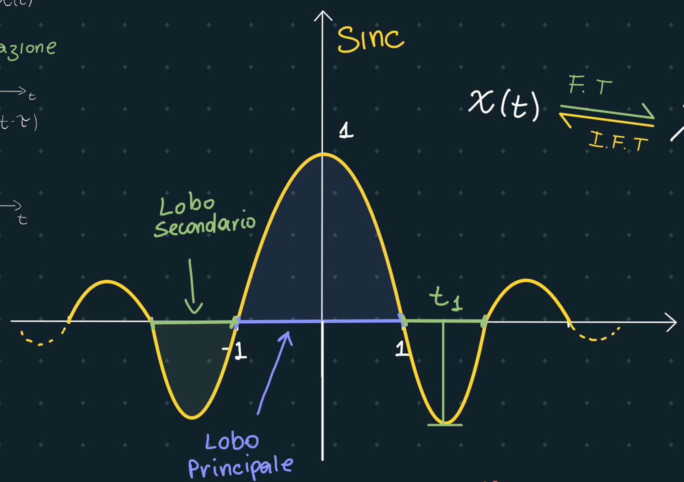
$$Q(x) = \int_{x}^{+\infty} \frac{1}{\sqrt{2\pi}} \, e^{-\frac{\vartheta}{2}} \, d\vartheta$$

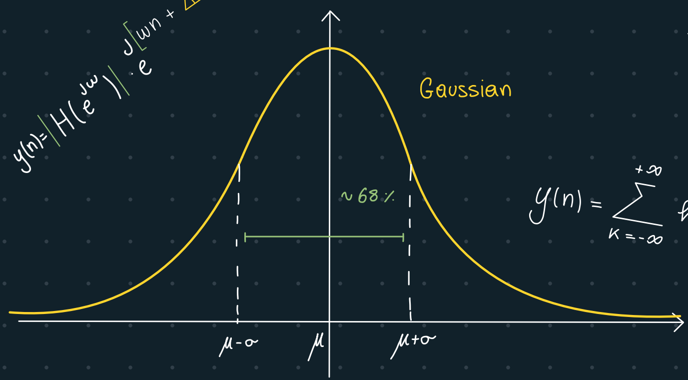


$$e^{j(\omega t+\varphi)}$$

$$y(n) = |H(e^{j\omega})| \cdot e^{j[\omega n + \angle H(e^{j\omega})]}$$



$$x(t) \underset{I.F.T}{\overset{F.T}{\rightleftarrows}} X(f)$$



$$y(n) = \sum_{k=-\infty}^{+\infty} h(k) \cdot x(n-k)$$

I/O I.R.

$$\mathcal{F}\left[x(t)\right] = \int_{-\infty}^{+\infty} x(t) \, e^{-j\omega t} \, dt$$

Modulo numero compl

- $z = a + ib \rightarrow |z| = \sqrt{a^2+b^2}$
- $z = r e^{i\theta} \rightarrow \underbrace{|z| = r}_{\text{Modulo}} \qquad \underbrace{\angle z = \theta}_{\text{fase / argomento}}$
- $z = r e^{i\theta} = r\left[\underbrace{\cos(\theta)}_{\text{Re}} + \underbrace{i \sin\theta}_{\text{Imm}}\right] \Rightarrow |z| = r \quad \angle z = \theta$

$$P_x = \lim_{T\to+\infty} \frac{1}{2T} \cdot \underbrace{\int_A^B dt}_{\text{Valore finito } N} = \lim_{T\to\infty} \frac{\overset{\text{Cost}}{N}}{\underbrace{2T}_{+\infty}} \rightarrow 0$$

$$\Delta = \begin{cases} 1-|t| & |t|< 1 \\ 0 & \end{cases}$$

$$\Rightarrow \int_{-\infty}^{+\infty} \left|1-|t|\right|^2 \underset{\text{Reale}}{=} \int_{-\infty}^{+\infty} (1-|t|)^2 dt = \int_{-\infty}^{+\infty} 1+t^2-2|t| \, dt = 2\int_0^1 1+t^2-2t \, dt$$

$$= 2\left[t+\frac{t^3}{3}-t^2\right]_0^1 = 2\left[1+\frac{1}{3}-1\right] = \frac{2}{3}$$

$$u(t) = \begin{cases} 1 & \text{per } t \geq 0 \\ 0 & \text{per } t < 0 \end{cases} \qquad P_u = \lim_{T \to \infty} \frac{1}{2T} \int_0^T dt = \lim_{T \to \infty} \frac{\cancel{T}}{2\cancel{T}} = \frac{1}{2}$$

$$y(n) = x(n) - x(n-1) \leftarrow$$ Sistema differenza prima

pongo $x(n) = a\,x(n) + b\,x(n)$ → Sostituisco in $y(n)$

$$\Rightarrow y(n) = a\,x(n) + b\,x(n) - \left[a\,x(n-1) + b\,x(n-1)\right]$$

$$= a\,x(n) + b\,x(n) - a\,x(n-1) - b\,x(n-1)$$

$$= a\underbrace{\left[x(n) - x(n-1)\right]}_{y_1} + b\underbrace{\left[x(n) - x(n-1)\right]}_{y_2}$$

$$\Rightarrow y(n) = a\,y_1(n) + b\,y_2(n) \leftarrow \text{Lineare}$$

1) pongo $x(n) = x(n-m)$ Ritardo $\qquad \Rightarrow y(n) = x(n-m) - x(n-m-1)$

2) $y(n-m) = x(n-m) - x(n-m-1)$

Pongo $n = n-m$

① = ② ⇒ Sys T.I

$h(n) = \delta(n) - \delta(n-1)$ $\qquad$ ES $x(n) = \delta(n)$

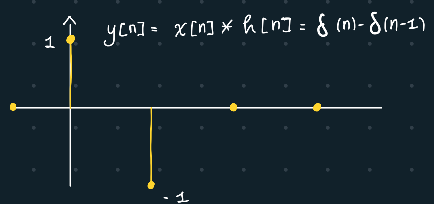
$$\Rightarrow y(n) = x(n) * h(n) = \sum_{k=-\infty}^{+\infty} x(k) \cdot h(n-k) = \sum_{k=-\infty}^{+\infty} \delta(n) \cdot \left[\delta(n-k) - \delta(n-k-1)\right]$$

$$= \sum_{k=-\infty}^{+\infty} \underbrace{\delta(n) \cdot \delta(n-k)}_{\delta(k)} - \sum_{k=-\infty}^{+\infty} \underbrace{\delta(n) \cdot \delta(n-(k-1))}_{\delta(k-1)} = \delta(k) - \delta(k-1)$$

Poniamo $n = \left[-1, -\frac{1}{2}, 0, \frac{1}{2}, 1\right]$ $\qquad$ $in = x(n) = u(n)$ → Sys → $out = y(n) = \delta(k) - \delta(k-1)$

$$\Rightarrow y[n] = \left[\underbrace{\delta(-1) - \delta(?)}_{0},\ \underbrace{\delta(-\tfrac{1}{2}) - \delta(-1)}_{0},\ \underset{1}{\delta(0)} - \underset{0}{\delta(-\tfrac{1}{2})},\ \underset{0}{\delta(\tfrac{1}{2})} - \underset{1}{\delta(0)},\ \underbrace{\delta(1) - \delta(\tfrac{1}{2})}_{0}\right]$$

$$= [0, 0, 1, -1, 0]$$



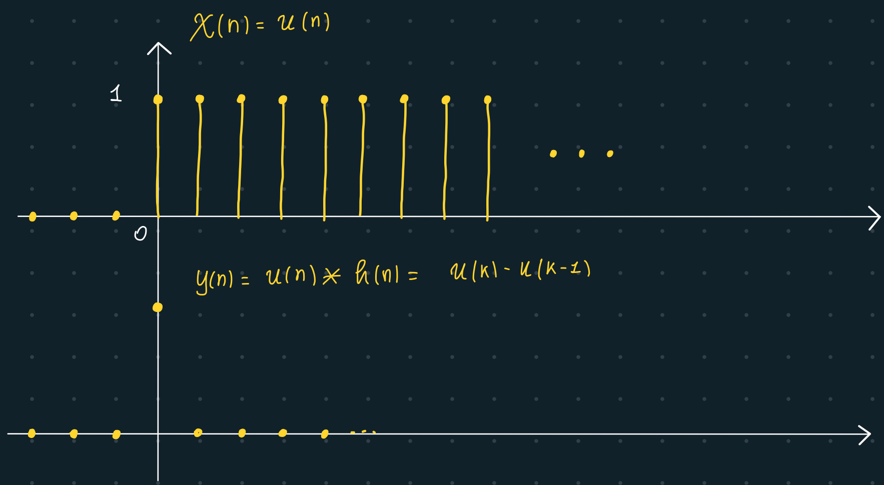
ES: $x[n] = u(n)$ $\quad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & otw. \end{cases}$

$$y[n] = u[n] * h[n] = \sum_{k=-\infty}^{+\infty} u(n) \cdot [\delta(n-k) - \delta(n-k-1)]$$

$$= \sum_{k=-\infty}^{+\infty} u(n) \cdot \delta(n-k) - \sum_{k=-\infty}^{+\infty} u(n)\, \delta(n-k-1) = \sum_{k=0}^{+\infty} u(k) - \sum_{k=0}^{+\infty} u(k-1)$$

Poniamo $n = [\underset{k=-1}{-\tfrac{1}{2}}, \underset{k=0}{0}, \underset{k=1}{\tfrac{1}{2}}, \underset{k=2}{1}, \underset{k=3}{\tfrac{3}{2}}]$

$$y[n] = u(k) - u(k-1) = [(u(0) - u(-\tfrac{1}{2})), (u(\tfrac{1}{2}) - u(0)), (u(1) - u(\tfrac{1}{2})) \dots]$$
$$= [1, 0, 0, \dots, 0]$$



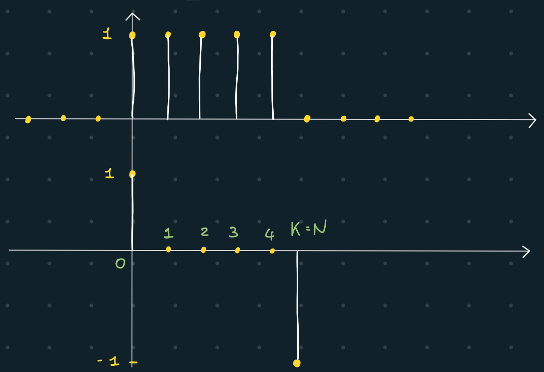
ES: $x(t) = R_N[n] = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & otw \end{cases}$

$$\Rightarrow h[n] = R_N[n] * h[n] = \sum_{k=-\infty}^{+\infty} R_N[n] \cdot [\delta(n-k) - \delta(n-k-1)]$$

$$= \sum_{k=-\infty}^{+\infty} R_N(n) \cdot \delta(n-k) - R_N(n) \cdot \delta(n-k-1) = \underbrace{\sum_{k=0}^{+\infty} R_N[k] - R_N[k-1]}_{y[n]}$$

Poniamo $n = [\underset{k=-1}{-\tfrac{1}{2}}, \underset{k=0}{0}, \underset{k=1}{\tfrac{1}{2}}, \underset{k=2}{1}, \underset{k=3}{\tfrac{3}{2}}]$
Asse del tempo

$$\Rightarrow y[n] = [(1-0), (1-1), (1-1), \dots, (0-1), (0-0), \dots] = [1, 0, 0, \dots, -1, 0, \dots]$$

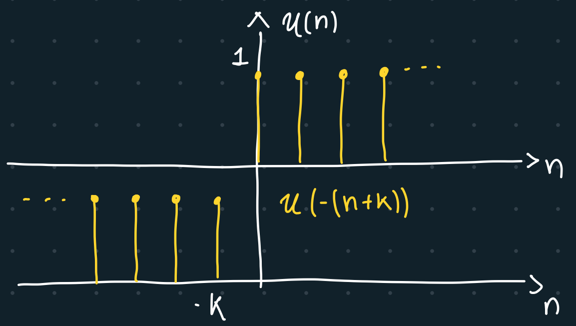


Convoluzione tra un gradino ed una seq exp mondatera.

$x(n) = u(n)$, $h(n) = a^{-n} \cdot u(n)$ con $0 < a < 1$

→ Calcolare l'uscita per un sistema che ha la risposta impulsiva $h(n)$ e per ingresso un gradino.

$$y(n) = u(n) * h(n) = \sum_{k=-\infty}^{+\infty} u(n)\, a^{n-k} \cdot u(n-k) = \sum_{k=-\infty}^{+\infty} u(n) \cdot u(n-k) \cdot \sum_{k=-\infty}^{+\infty} a^{n-k} \to \text{S. Geom.}$$



Caso 1: Per $-k < 0 \to k > 0 \Rightarrow y[n] = 0$

Caso 2: Per $-k > 0 \to k > 0$

$\Rightarrow u(n) \cdot u(n-k) = 1$ per $0 < n < k$

$\Rightarrow$ quindi per $0 \leq n \leq k \to y[n] = \sum_{k=0}^{+\infty} a^{n-k} = a^n \sum_{k=0}^{n} \left(\frac{1}{a}\right)^k \to$

$$\to a^n \cdot \frac{1 - \left(\frac{1}{a}\right)^{n+1}}{1 - \frac{1}{a}} = a^n \, \frac{1 - a^{-n-1}}{\frac{a-1}{a}} = a^n \, \frac{1 - a^{-(n+1)}}{\quad}$$

$$\to \frac{a^n - a^n a^{-(n+1)}}{\frac{a-1}{a}} = \frac{a^n \cdot a - a^n a^{-(n+1)}}{a - 1}$$

- $\int_{-\infty}^{+\infty} \delta(t)dt = 1$
- $x(t)\delta(t) = x(0)\delta(t)$
- $\int_{-\infty}^{+\infty} x(t)\delta(at)dt = \int_{-\infty}^{+\infty} \frac{1}{|a|} x\left(\frac{t}{a}\right) \delta(t)dt \rightarrow \delta(t a) = \frac{1}{|a|} \cdot \delta(t)$
- $x(t)\delta(t-\tau) = x(\tau)\delta(t-\tau)$
- $t \cdot \delta(t) = 0 \cdot \delta(t) = \emptyset$
- $\int_{t_1}^{t_2} x(t) \cdot \delta(t-t_0)\, dt = \begin{cases} x(t_0) & \text{se } t_1 < t_0 < t_2 \\ 0 & \text{Altrimenti} \end{cases}$
- $x(t) * \delta(t-t_0) = x(t-t_0)$

$$C_{XY}\begin{vmatrix}\sigma_X^2 & cov_{XY}\\ cov_{XY} & \sigma_Y^2\end{vmatrix}$$

- $x(t) * \delta(t-T_0) = x(t-T_0)$
- $x(t)\cdot\delta(t-\tau) = x(\tau)$

- $\tau_{xy}(\cdot) \Longleftrightarrow S_{xy}(\cdot)$

- $S_{y_1 y_2}(\cdot) = \left[H_1(\cdot)\cdot H_2^*(\cdot)\right]\cdot S_{x_1 x_2}(\cdot)$
- $S_y(\cdot) = |H_1(\cdot)|^2\cdot S_x(\cdot)$
- $S_{yx}(\cdot) = H(\cdot)\cdot S_x(\cdot)$
- $S_{xy}(\cdot) = H^*(\cdot)\, S_x(\cdot)$
- $\sum_{i\in A_X}\sum_{j\in A_Y} x_i\cdot y_j\cdot \underbrace{P(\{X=x\}\cap\{Y=y\})}_{\text{PMF congiunta}}$

- $P(\{X>Y\}) \rightarrow Z = X-Y>0 \rightarrow P(\{Z>0\})$

- $1\ GHz = 1\cdot 10^9\ Hz$
- $1\ MHz = 10^6\ Hz$
- $1\ KHz = 10^3\ Hz$

- $\angle = Tan^{-1}\left(\frac{C}{Re}\right) + 180 = \varphi$

  $|Z| = \sqrt{C^2 + Re^2}$

  $\rightarrow |Z|\,Cos(wt+\varphi)$