Ex 1: Consideriamo le variabili Aleatorie

$Y_1 = 3X_1 + 2$ e $Y_2 = 2X_2$ le V.A X_1 e X_2 sono gaussiane Standard indip.

$\Rightarrow X_1, X_2 \sim \mathcal{N}(0,1)$ Indip $\Rightarrow Y_1, Y_2 \sim \mathcal{N}(\mu_{Y_1Y_2}, \sigma^2_{Y_{1,2}})$

Q_1: Matrice di covarianza Tra Y_1 e Y_2

$$\begin{bmatrix} \sigma^2_{Y_1} & C_{Y_1Y_2} \\ C_{Y_1Y_2} & \sigma^2_{Y_2} \end{bmatrix}$$

Dobbiamo calcolare

$E[X_1] = E[X_2] = \emptyset$

$E[Y_1] = E[3X_1 + 2] = 3\mu_{X_1} + 2 \Rightarrow \mu_{Y_1} = 2$

$E[Y_2] = 2\mu_{X_2} \Rightarrow \mu_{Y_2} = 0$

$\sigma^2_{Y_1} = E[(Y_1 - \mu_{Y_1})^2] = E[\bar{Y}^2] - \mu^2_{Y_1} \Rightarrow \bar{Y}^2 = E[(3X_1+2)^2] = 9E[X_1^2] + 12\cancel{\mu_{X_1}}^{0} + 4$

$\Rightarrow \bar{X}_1^2 = \mu^2_{X_1} + \sigma^2_{X_1} = 1 \Rightarrow \sigma^2_{Y_1} = 9 + 4 - 4 = 9$ $\sigma^2_{Y_1}$

$\mu^2_{Y_1}$

$\sigma^2_{Y_2} = E[(2X_2)^2] = 4\bar{X}_2^2 \Rightarrow \bar{X}_2 = \mu^2_{X_2} + \sigma^2_{X_2} = 1 \Rightarrow \sigma^2_{Y_2} = 4$ $\sigma^2_{Y_2}$

$\Rightarrow Y_1 \sim \mathcal{N}(2,9) \quad Y_2 \sim \mathcal{N}(0,4)$

$Cov(Y_1, Y_2) = E[(Y_1 - \mu_{Y_1})(Y_2 - \mu_{Y_2})] = E[Y_1Y_2] - \overset{\emptyset}{\mu_1\mu_2} - \cancel{\mu_1\mu_2} + \cancel{\mu_1\mu_2}$

$E[Y_1Y_2] = E[(3X_1+2)(2X_2)] = 6\underbrace{E[X_1X_2]}_{\substack{X_1 \text{ e } X_2 \text{ indip} \\ \Rightarrow \mu_{X_1}\mu_{X_2}}} + 4\mu_{X_2} = 0 + 0 = \emptyset$

$\Rightarrow cov_{Y_1Y_2} = \emptyset$

$$\Rightarrow C = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}$$