Advanced Models and Methods in Operations Research
Heuristic tree search

Florian Fontan

November 14, 2023

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Overview

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- Branching scheme: representing the search space as an implicit decision tree.
- Tree search algorithm: exploring this search tree in a smart way to visit the most promising regions in priority

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Definition

Branching scheme: representing the search space as an implicit decision tree.

A branching scheme is defined by:

- Its root node
- How to generate the children of a node

Example: travelling salesman problem

- Input:
  - n locations
  - an $n \times n$ symmetric matrix containing the distances between each pair of locations
- Problem: find a tour such that each location is visited exactly once
- Objective: minimize the total length of the tour

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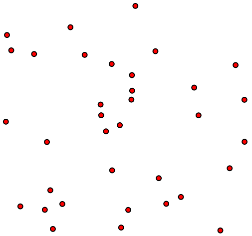

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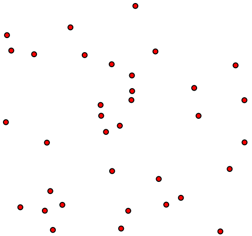
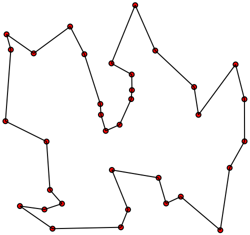


Solution



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Branching scheme

- A node corresponds to a partial tour
- Root node: contains only location 1
- Children of a node: append to the partial tour the next location to visit; generate one child for each remaining location to visit

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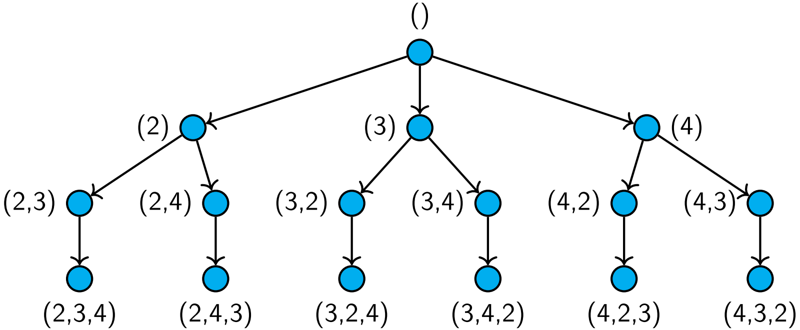
Example with 4 nodes:



Example: sequential ordering problem

- Input:
  - n locations
  - an $n \times n$ matrix containing the distances between each pair of locations (not necessarily symmetric)
  - a directed acyclic graph G such that each vertex corresponds to a location
- Problem: find a route from location 1 such that:
  - each location is visited exactly once
  - if there exists an arc from vertex j_1 to vertex j_2 in G, then location j_1 is visited before location j_2
- Objective: minimize the total length of the route

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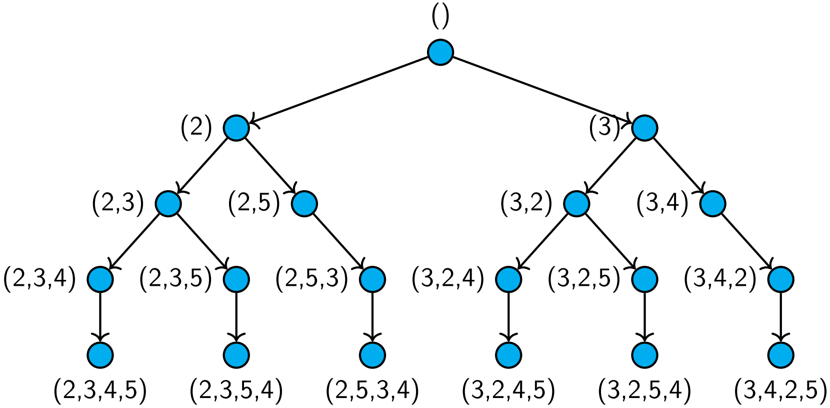
Example with 5 nodes, precedences: $2 \rightarrow 5$, $3 \rightarrow 4$:



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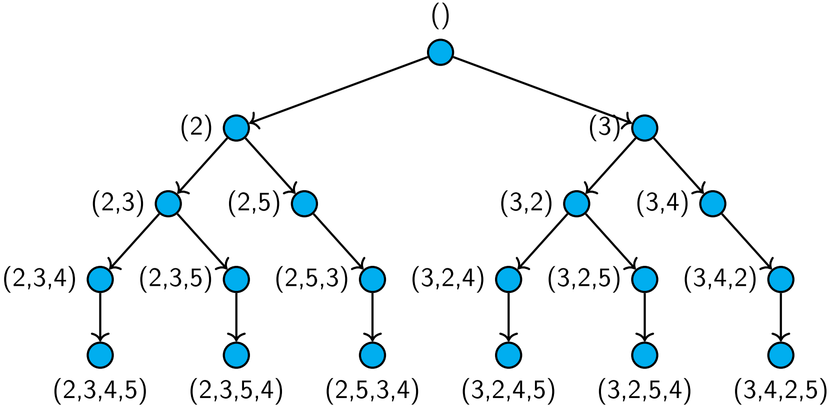

- Usually, more constraints $\implies$ less nodes.

Transition

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- These search trees usually become very large when the depth increases
- It is not possible to explore them exhaustively
- We need to find smart ways to explore the most promising nodes

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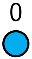
Greedy algorithm

Select the best child until reaching a leaf.
The value next to the nodes is the criteria used to compare them. The lesser the value, the better the node.



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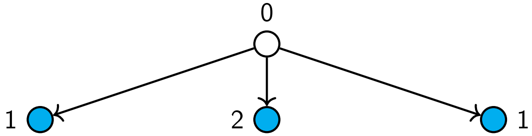

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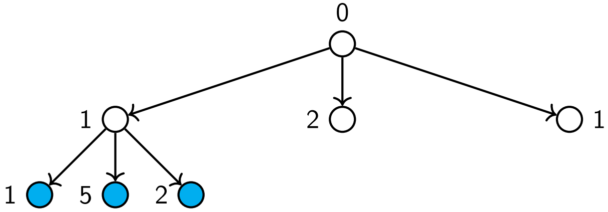

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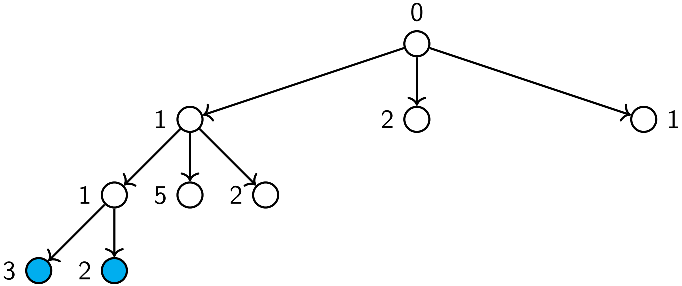

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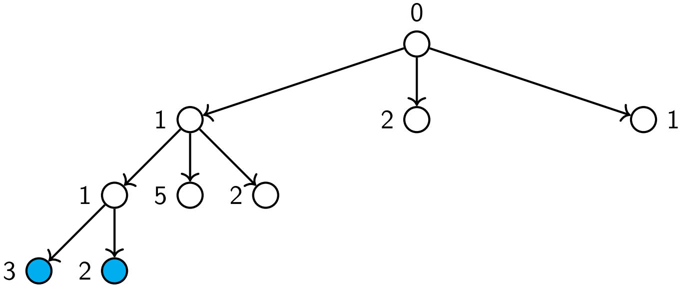

function Greedy(branching_scheme)
    node ← branching_scheme.root()
    while branching_scheme.children(node) is not empty **do**
        node ← "best" node from branching_scheme.children(node)

Greedy algorithm

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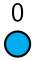
At each iteration, we expand the "best" node.



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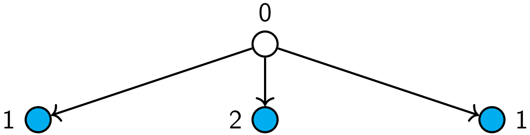

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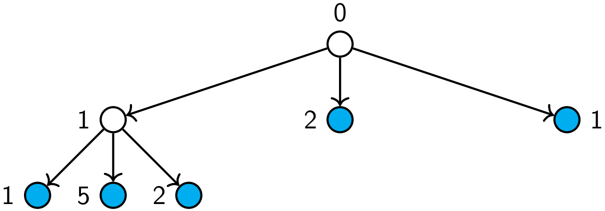

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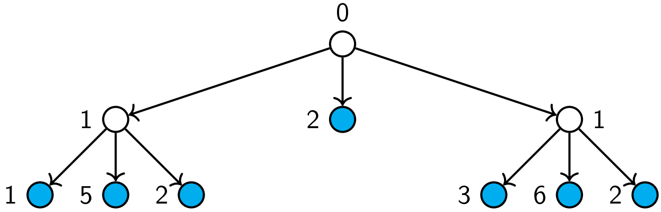

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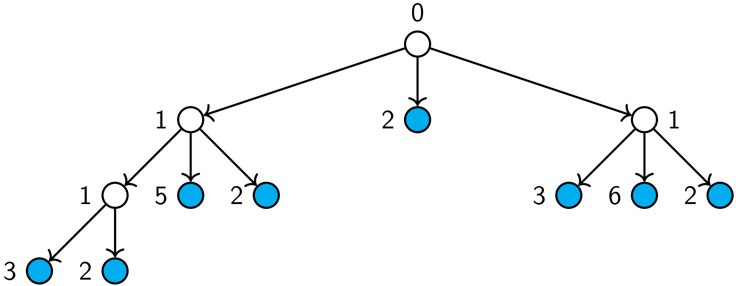

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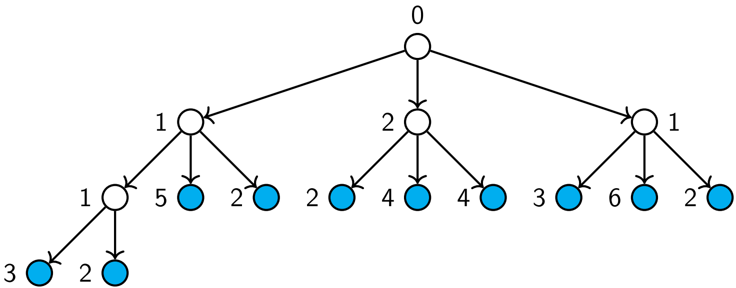

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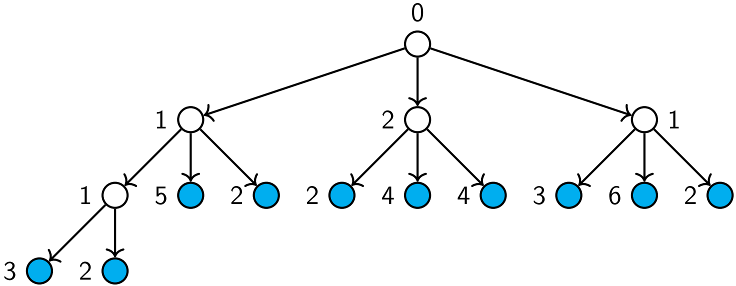

function A(branching_scheme)
  queue ← {branching_scheme.root()}
  while queue is not empty **do**
    node ← extract “best” node from queue
    queue ← queue ∪ branching_scheme.children(node)

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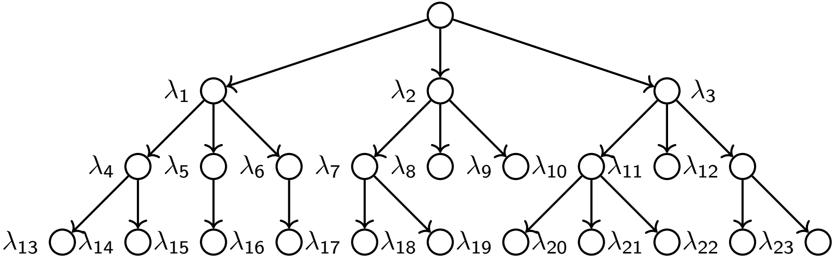
- The node queue quickly becomes too large

Limited discrepancy search

Nodes are explored by increasing value of their discrepancy.



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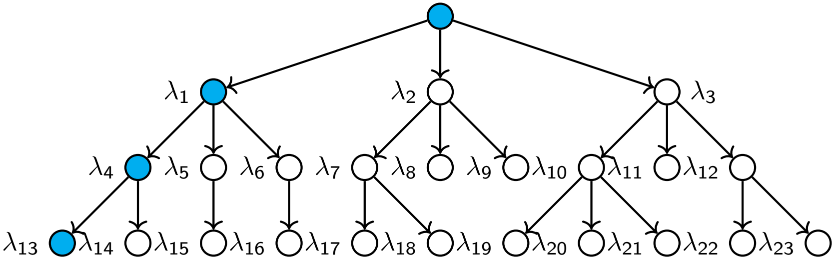

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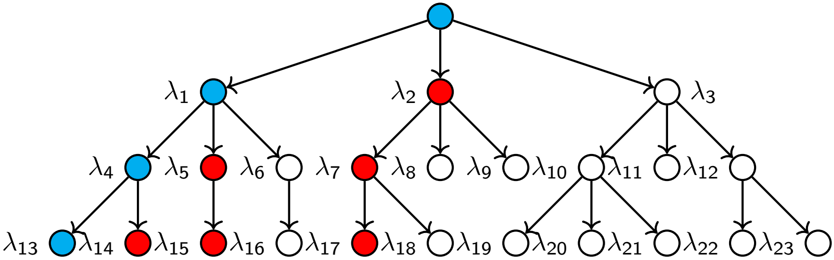

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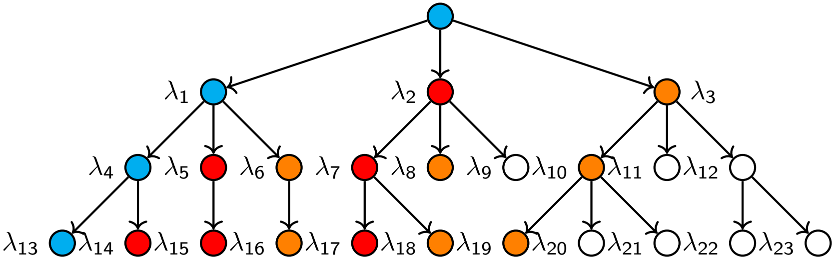

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- Quickly reaches leaves
- Works well with unbalanced trees
- A node is only compared with its brothers:
  $\Longrightarrow$ very easy to design a guide

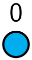
Drawbacks:

- Does not work as well with more balanced trees
  A node is only compared with its brothers:
  $\Longrightarrow$ succession of decisions are never challenged

Beam search

Breadth first search with a maximum width (called “beam width”).
At each stage, the “worst” nodes are discarded.

Example with a beam width of 5



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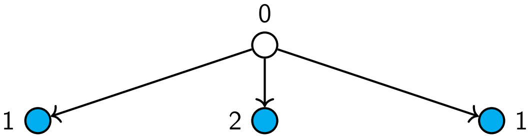

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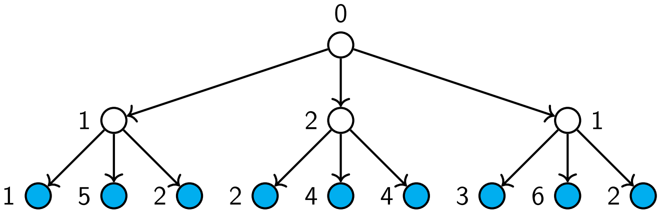

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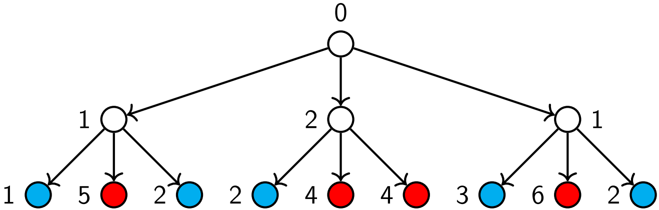

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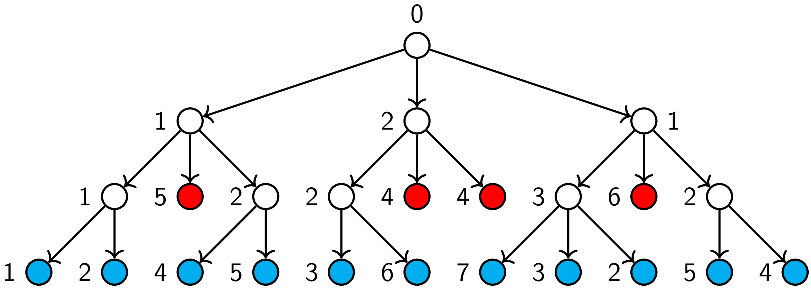

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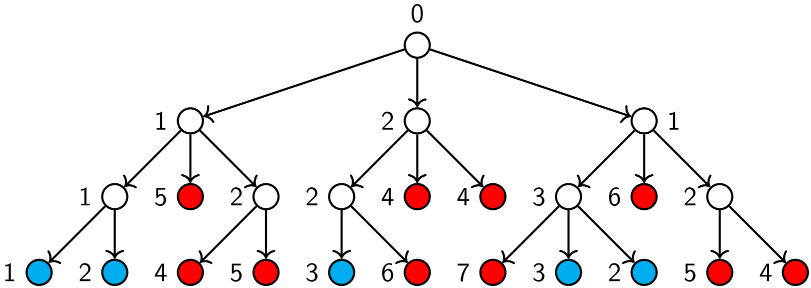

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Drawbacks:

- How to choose the beam width?

Iterative beam search

How to choose the beam width:

- small: close to greedy
- large: close to breadth first search

Iterative beam search:

- Succesive executions of a beam search while increasing the beam width: 1, 2, 4, 8, 16...
- Growth rate: between 1.25 and 2, small influence on the algorithm performances
- Anytime

Dominances

Travelling salesman problem example:

- Node N_1: $1 \to 2 \to 3 \to 4$, length 10
- Node N_2: $1 \to 3 \to 2 \to 4$, length 11

We can safely prune Node N_2.

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More generally, let

- $\text{visited}(N)$ be the list of visited locations of node N.
- $\text{last}(N)$ be the last visited location of node N.
- $\text{length}(N)$ be the length of the partial tour of node N.

Consider two nodes N_1 and N_2. If

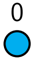
- $\text{visited}(N_1) \supseteq \text{visited}(N_2)$
- $\text{last}(N_1) = \text{last}(N_2)$
- $\text{length}(N_1) \leq \text{length}(N_2)$

then node N_1 dominates node N_2 and therefore node N_2 can be safely pruned.

A / best first search + dynamic programming

Each time a node is added to the queue:

- if it is dominated by another node which is in the queue, it is not added
- the nodes from the queue that it dominates are removed from the queue



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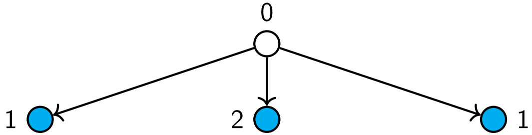

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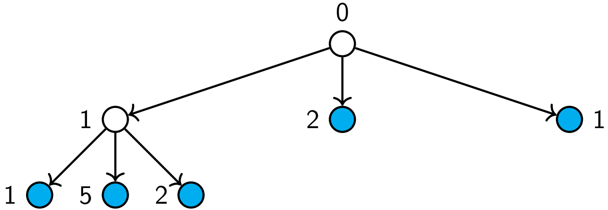

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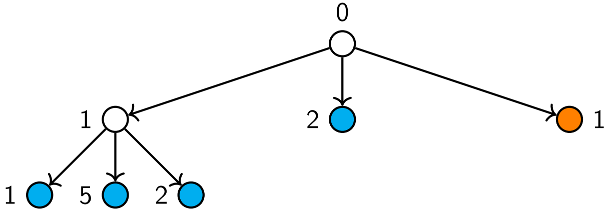

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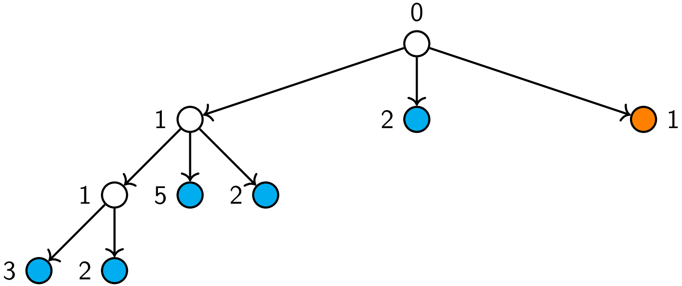

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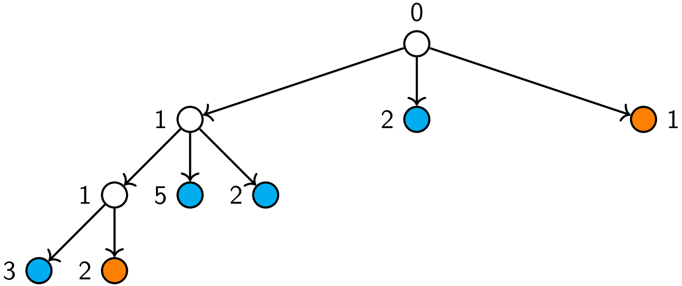

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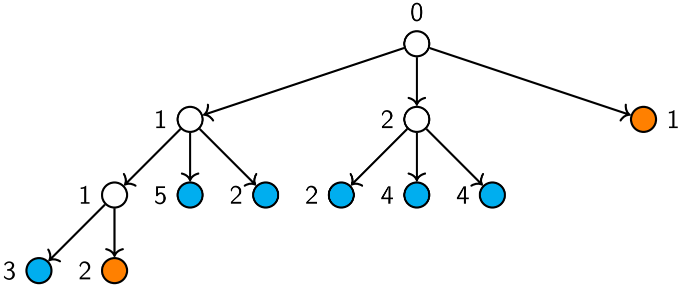

Beam search + dynamic programming

Before discarding the “worst” nodes, the dominated ones are first removed.

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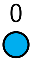

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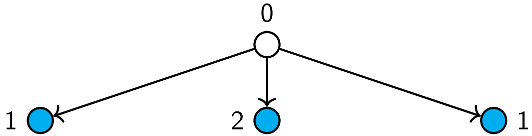

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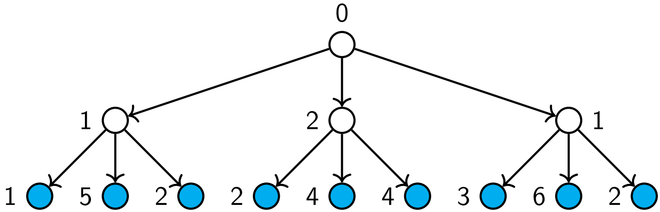

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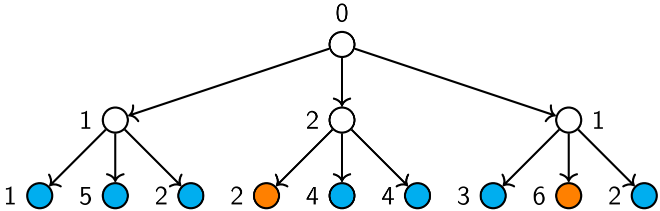

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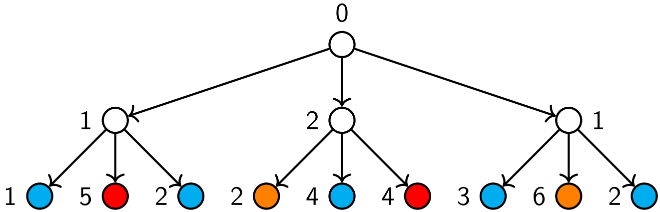

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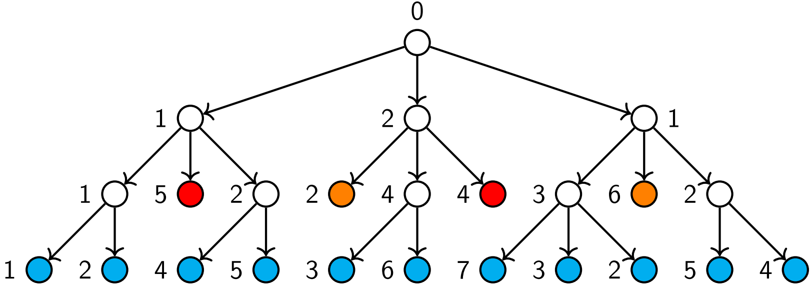

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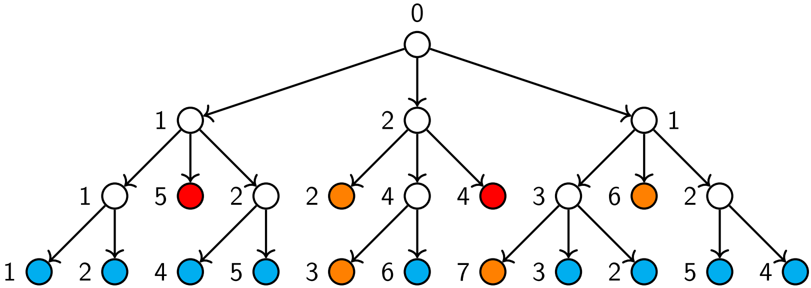

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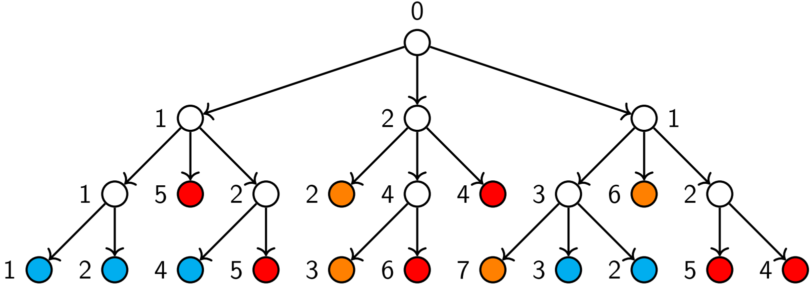
Beam width: 5



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Guides

All the previously presented algorithms require a criteria to compare nodes:

- First idea: define and use the objective of the partial solutions:
  - Examples:
    - Travelling salesman problem: length of the partial tour
  - Advantage: simple, might be good as a first approach
  - Drawbacks: does not take into account the rest of the solution
    - Travelling salesman problem: a forgotten location near the first ones

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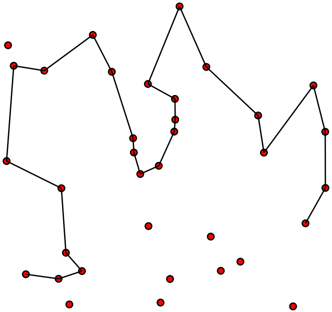
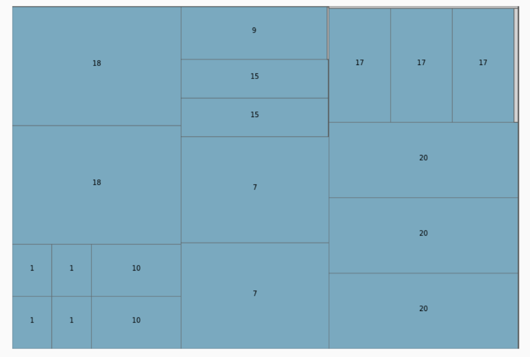


Guides

- The criteria needs to take into account what is and what is not in the partial solution
- Other ideas: bound, expected value of the best reachable leaf...
- The complexity of computing the score needs to be taken into account. A better score with a greater complexity might not be worth
- In practice, heuristic tree search does not work well for the traveling salesman problem, but it provides state-of-the-art results on instances of the sequential ordering problem with a dense precedence graph

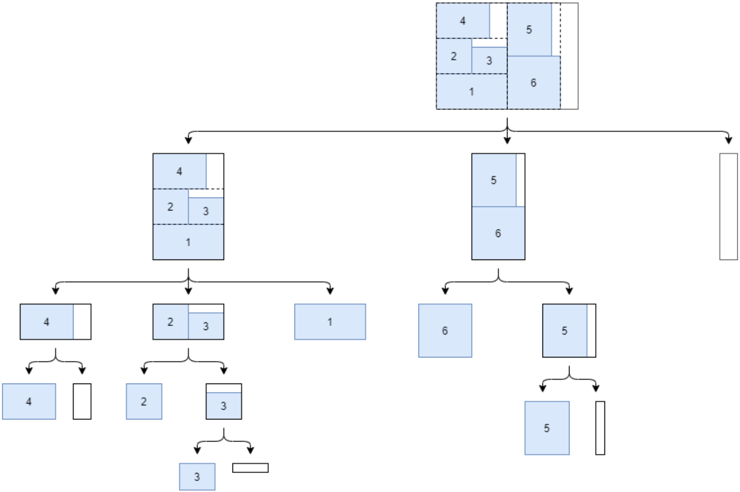
Example: two-dimensional guillotine knapsack problem

- Input
  - A bin with width W and height H
  - n items; for each item $j = 1, \ldots, n$, a width w_j, a height h_j and a profit p_j
- Problem: find a 3-staged guillotine cutting plan such that:
  - each item is cut at most once
- Objective: maximize the total profit of the item cut

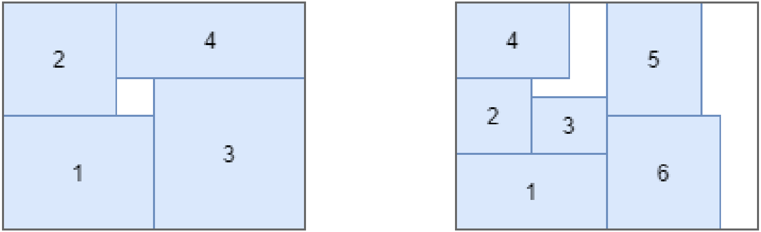


Example: two-dimensional guillotine knapsack problem

Guillotine cutting plan: items can be extracted with only edge-to-edge cuts:



Example: two-dimensional guillotine knapsack mroblem

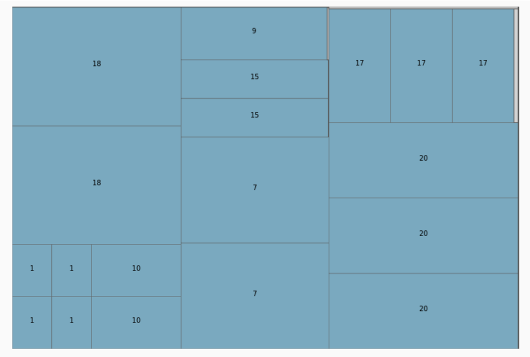


Non-guillotine pattern

Guillotine pattern

Example: two-dimensional guillotine knapsack problem

Order of the items in a cutting plan:



Example: two-dimensional guillotine knapsack problem

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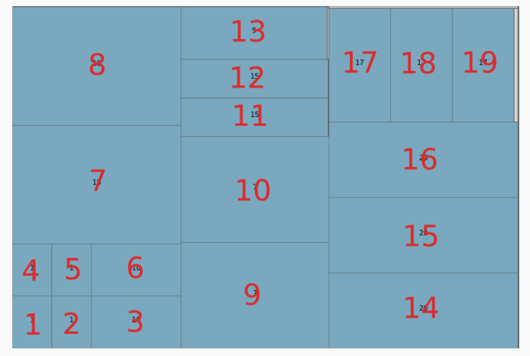
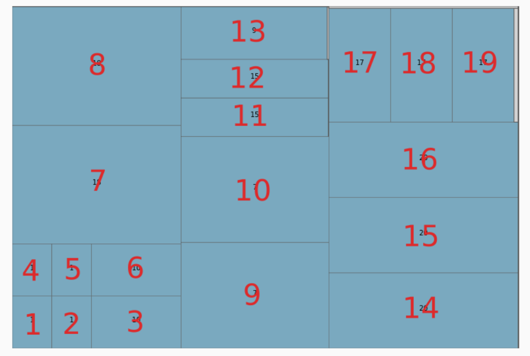


Example: two-dimensional guillotine knapsack problem

Order of the items in a cutting plan:



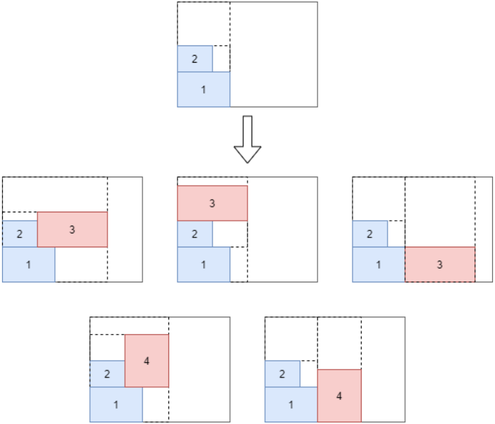
Branching scheme:

- Root node: empty solution, no item
- Add the next item (following the order defined above) to the partial solution; generate one child for each remaining item at each possible position

Example: two-dimensional guillotine knapsack mroblem

There are at most 3 valid positions to pack a next item:

- In the current second-level subplate, in a new third-level subplate
- In the current first-level subplate, a new second-level subplate
- In a new first-level subplate



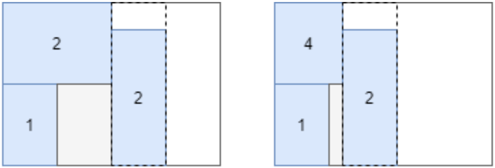
Example: two-dimensional guillotine knapsack problem

Guiding the search:

- First idea, guide with the profit of the partial solution:

$$\frac{1}{\text{profit}(S)}$$

- This is not as bad as for the traveling salesman problem, since a wrong choice may less likely lead to a very bad final solution.
- Still, with this guide, we favor solutions containing high profit items, even if these packed items are not tightly packed



Partial solution A

Partial solution B

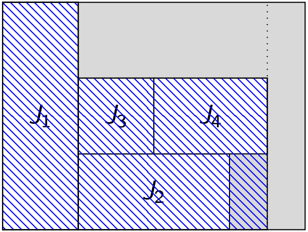
If the profit of an item is equal to its area, then partial solution A is more profitable than partial solution B. Still, partial solution B seems more favorable since it contains less waste

Example: two-dimensional guillotine knapsack problem

One way to overcome this issue is to guide with:

$$\frac{\mathrm{area}(S)}{\mathrm{profit}(S)}$$

where $\mathrm{area}(S)$ is the used area of a partial solution S as illustrated below:



Example: two-dimensional guillotine knapsack problem

- In some cases, this guide favors partial solutions with many small items since it is easier to generate partial solutions with less waste using small items
- Then, at the end, only large items remains and packing them generates a lot of waste
- One way to overcome this issue is to integrate the mean area of the packed items in the guide:

$$\frac{\text{area}(S)}{\text{profit(S)}} \frac{1}{\text{mean\_item\_area}(S)}$$

- In practice, we use the last two guide functions

Example: ROADEF/EURO 2022/2023 challenge

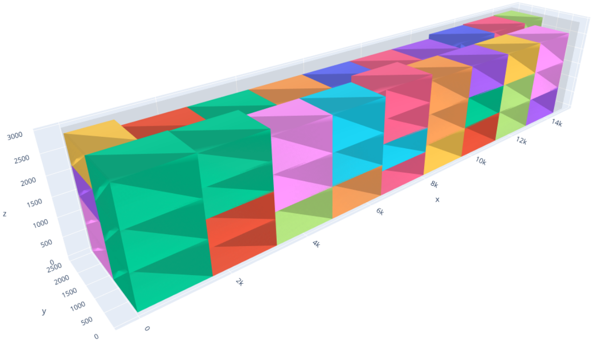
3D knapsack subproblem:

- Input
  - A bin with length L, width W and height H
  - n items; for each item $j = 1, \dots, n$, a length l_j, a width w_j, a height h_j, a quantity d_j, a profit $p_j \dots$
- Problem: find a packing plan such that each constraint is satisfied
- Objective: maximize the total profit of the items packed

Packing constraints: stacks

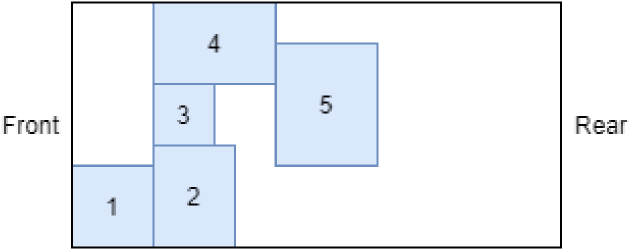
The third dimensions is only possible through stacks

- A stack necessarily contains items with the same length and width



Packing constraints: stacks

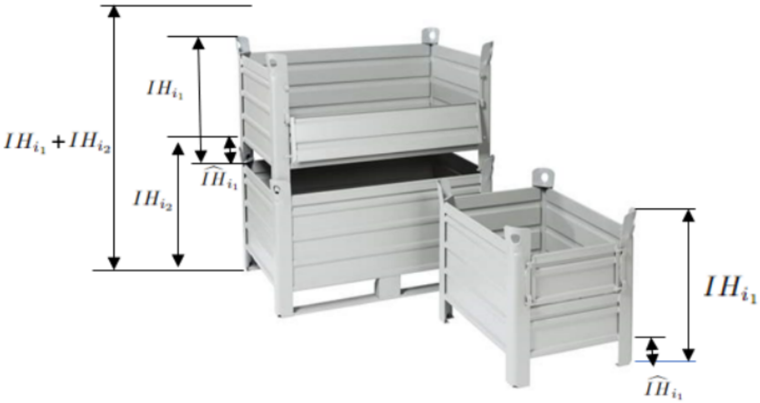
Any stack must be adjacent to another stack on its left on the X-axis, or to the front of the truck (adjacent to the truck driver)



View from the top.
The placement of stacks 3 and 4 is not allowed.

Packing constraints: nesting height

For some items, their height is reduced when they are packed above another item.



Packing constraints: loading order

Loading order:

- Stacks must be placed in an increasing fashion from the front to the rear of the truck according to the suppliers' pickup order

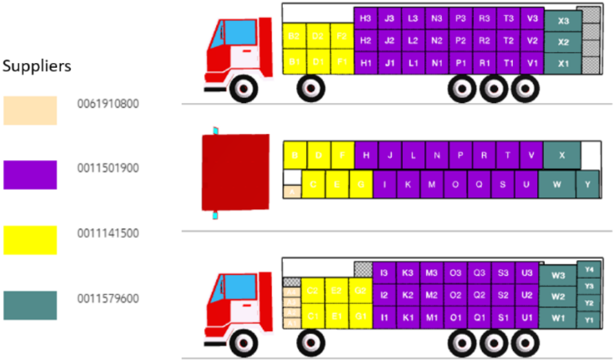


Figure 5: A truck with 4 picked-up suppliers

Packing constraints: axle weights

Maximum weight on the middle and rear axles of the truck

- Formulas to compute axle weights:

$$ej^e = \frac{\sum_{s \in \widetilde{TS}_t} (sx_s^o + \frac{(sx_s^e - sx_s^o)}{2}) \times sm_s}{tm_t}$$

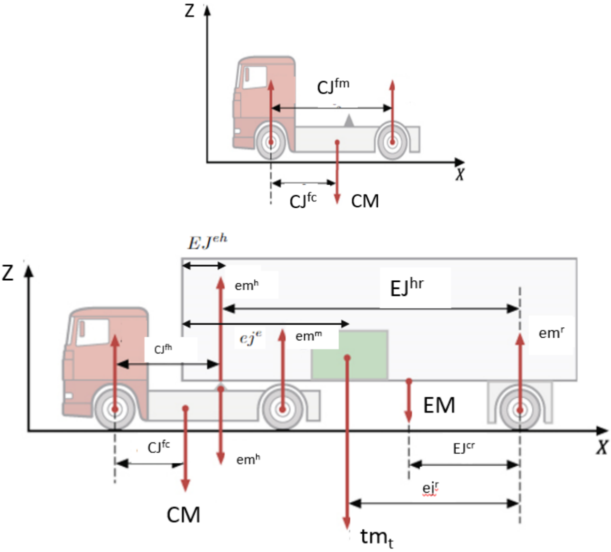
$$ej^r = EJ^{eh} + EJ^{hr} - ej^e$$

$$em^h = \frac{tm_t \times ej^r + EM \times EJ^{cr}}{EJ^{hr}}$$

$$em^r = tm_t + EM - em^h$$

$$em^m = \frac{CM \times CJ^{fc} + em^h \times CJ^{fh}}{CJ^{fm}}$$

Packing constraints: axle weights

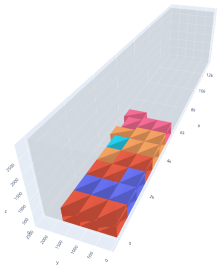


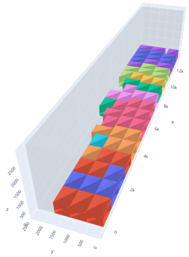
Packing constraints: axle weights

The maximum axle weight constraints has some counter-intuitive properties:

- The solution on the left is infeasible
  - The weight on the middle axle weight is too high
- The solution on the right is feasible

Adding some items to a solution without removing or moving the current items might decrease the axle weights!





Packing constraints: other constraints

Remaining constraints:

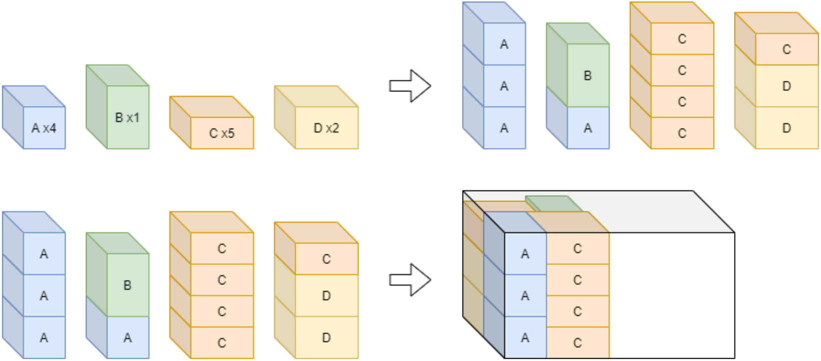
- Maximum weight allowed in the truck
- For each item, maximum weight allowed above
- For each item, maximum number of items in a stack containing this item
- Maximum density of a stack

Solution method

To solve the 3D knapsack subproblems, we design two algorithms:

- An algorithm that decomposes the 3D problem into
  - A 1D problem that builds stacks
    - Build the least number of stacks with all the items
    - This is a 1D bin packing problem
  - A 2D packing problem that packs these stacks.
    - This is a 2D knapsack problem
  - This algorithm works very well when axle weight constraints are not critical
- An algorithm that directly builds 3D packings
  - This algorithm works very well when weight constraints are critical and therefore, sparse packings are needed

3D knapsack subproblem, sequential 1D 2D



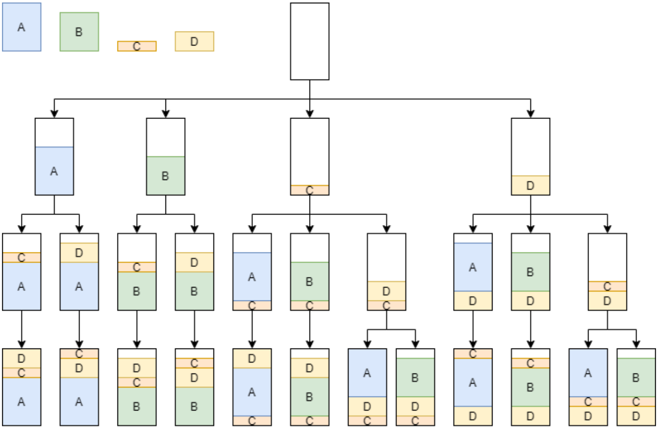
1D bin packing subproblem

- For a given set of items, we want to pack them into the least number of stacks
- Remaining constraints:
  - Maximum height of a stack (height of the truck)
  - Nesting height
  - Maximum number of items in a stack containing an item of a given type
  - Maximum weight of a stack (stack density)
  - Maximum weight allowed above an item of a given type
- We decompose the problem into a sequence of (1D) knapsack subproblems
- The 1D knapsack subproblems are solved with a tree search algorithm

1D knapsack subproblem, branching scheme

Branching scheme:

- Root node: empty stack
- Children of a node: we generate a child node for each item which is valid to add *on top* of the stack

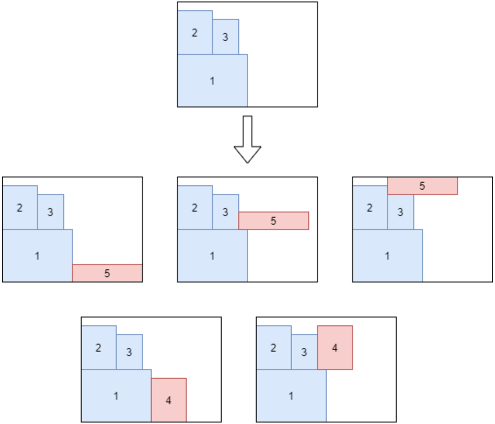


2D knapsack problem

- For a set of given stacks, we want to pack as many as possible of them inside a truck
- Remaining constraints:
  - Geometrical constraints
  - Maximum weight of the truck
  - Loading order
  - We ignore axle weight constraints

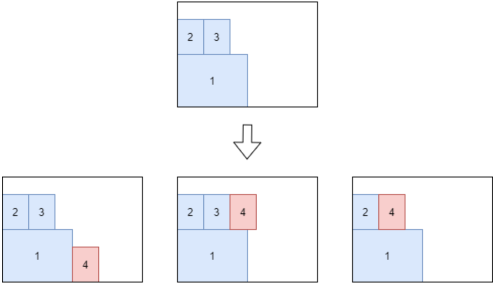
2D knapsack problem, branching scheme

We solve this 2D knapsack subproblem with a tree search algorithm based on a skyline



3D knapsack problem, direct approach

- Our second approach to solve the 3D knapsack problem is a tree search algorithm that directly builds 3D solutions
- The branching scheme is similar to the 2D case, except that we allow adding an item above one of the "last" stack



3D knapsack problem, direct approach

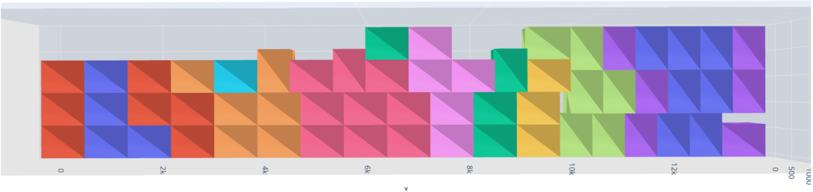
- We allow partial solutions that do not satisfy the weight distribution constraints, since the path to feasible full solutions often contains infeasible partial solutions
- But we guide the search towards full feasible solutions

Guides for 3D packing

- The SOR algorithm fails to find a good enough solution when the middle axle weight constraint is so critical, that a sparse packing is required to get a good feasible solution
- The goal of the 3D packing algorithm is to generate such a sparse solution
- The solution must be sparse enough to be feasible, but dense enough to pack as many items as possible
- In particular, the sparsity is mostly important at the front of the truck, since the items packed at the front contribute more to the middle axle weight

Guides for 3D packing

Here is an example of a sparse solution (from top):



Guides for 3D packing

- To generate dense solutions with the SOR algorithm, we guide the search with:

$$\frac{\text{space of the items packed}}{\text{space used}}$$

- To generate sparse solutions with the 3D tree search algorithm, we estimate in advance, depending on the current number of items packed
  - The maximum length that the partial solution should not exceed
  - The maximum middle axle weight that the partial solution should not exceed

  and the guide looks like

$$\text{length excess} + \text{middle axle weight excess}$$

  How do we generate these estimates?

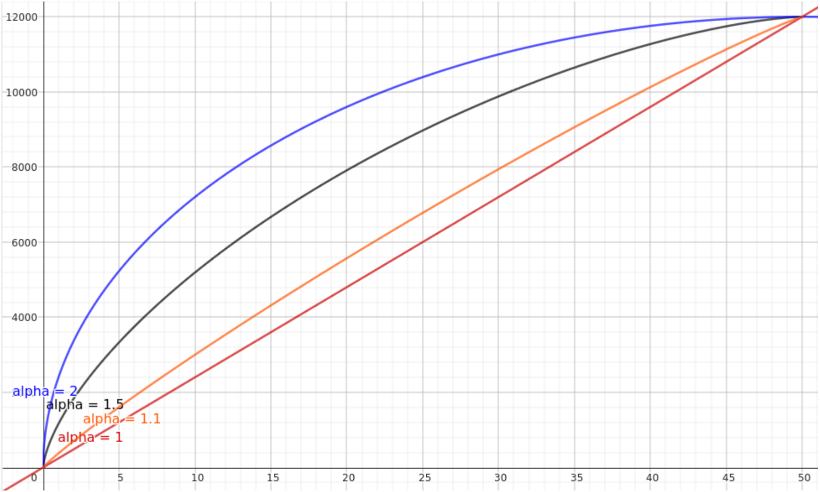
Guides for 3D packing

- We consider a one-dimensional problem with only the x-component
- We ignore item overlap
- Let n be the number of items that we plan to pack
- Let L be the length of the truck
- We consider that the position of the jth item is given by

$$x_j = L \sqrt[\alpha]{1 - \left(1 - \frac{j}{n}\right)^{\alpha}}$$

Guides for 3D packing

Here is an example for $L = 12000$, $n = 50$ and different values of α:



Guides for 3D packing

- When $\alpha = 1$, items are uniformely distributed inside the truck
- When α increases the front of the truck becomes sparser of the rear of the truck becomes denser
- We look for the smallest α for which the solution satisfies the middle axle weight constraints
- We find this value with a dichotomic search
- Then we deduce the maximum length and maximum middle axle weight for each number of items

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Heuristic tree search vs LP-based branch-and-bound

Heuristic tree search vs LP-based branch-and-bound

- An LP-based branch-and-bound is also a tree search:
  - Root node: no variable bounds have been tightened
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- Works better than a heuristic tree search approach if the bounds are strong
  - Example: arc-flow formulation of a bin packing problem

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- Works better than a heuristic tree search approach if the bounds are strong
  - Example: arc-flow formulation of a bin packing problem
- Performs poorly if the bounds are weak (a lot of time is spent in the nodes, but the number of nodes remains too high)
  - Example: two-dimensional bin packing

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`treesearchsolverpy`

- A package that simplifies the implementation of tree search based algorithms
- Written in `Python3` (original version in `C++`)
- `https://github.com/fontanf/treesearchsolverpy`
- Install with: `pip3 install treesearchsolverpy`
- It includes an iterative beam search + dynamic programming
- To solve a problem, one needs to create a `BranchingScheme` class that implements the requried methods (about 100–200 lines of code). Then:
  `iterative_beam_search(branching_scheme)`

- For the branching scheme:
  - `Node` class with `__lt__(self, other)` (guide)
  - `root()` method
  - `next_child(father)` method
  - `infertile(node)` method
  - `leaf(node)` method
  - `bound(node_1, node_2)` method
- For the solution pool:
  - `better(node_1, node_2)` method (main objective, not guide)
  - `equals(node_1, node_2)` method (same solution, not same objective value)
- For the dominances:
  - `comparable(node)` method
  - `Bucket` class with `__init__(self, node)`, `__hash__(self)` and `__eq__(self, other)`
  - `dominates(node_1, node_2)` method (called only if both nodes are in the same bucket)
- `display(node)` method

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Conclusion

- Heuristic tree search: branching scheme + tree search algorithm (+ guides, dominances)
- New optimization method to add to your toolbox
- As for all other methods, does not work well for all problems
- Works well for medium-sized problems
  - depth ≤ 1000
- Works well for problems with many constraints
- Rather robust to the addition of new constraints
- Less robust to changes in the objective function

Advanced Models and Methods in Operations Research
Heuristic tree search

Florian Fontan

November 14, 2023