Advanced Models and Methods in Operations Research
Project: Vehicle routing

Florian Fontan

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For each problem considered, instances and a code skeleton containing an instance parser and a solution checker are provided in the `data/` and `python/` folders of the project.

The algorithms must be implemented in the provided files between the tags `TODO START` and `TODO END`.

The algorithms must be tested on all the given instances with the command: `./problem.py -i instance.txt -c certificate.txt`

Each solution must be verified with the provided checker: `./problem.py -a checker -i instance.txt -c certificate.txt`

The delivrable must contain:

- A *short* report describing and justifying the proposed algorithms
- The code implementing the algorithms
- The solution files obtained on the provided instances

1 Dynamic Programming

We consider the Elementary shortest path problem with resource constraint and a single slot:

- Input:
  - 1 depot
  - $n-1$ customers; for each customer $j = 2, \dots, n$, a visit interval $[s_j, e_j[$, $s_j \in \mathbf{N}_+$, $e_j \in \mathbf{N}_+$, $s_j < e_j$
  - an $n \times n$ symmetric matrix t specifying the times in $\mathbf{R}_+$ to travel between each pair of locations
  - an $n \times n$ matrix c specifying the costs in $\mathbf{R}$ to travel between each pair of locations
- Problem: find a sub-tour starting and ending at the depot such that
  - each customer is visited at most once

  - the arrival at a customer j is before s_j
  - the departure from a customer is at e_j (even if the arrival was before s_j)

- Objective: minimize the cost of the sub-tour

Note that the costs might be negative.

Propose and implement an algorithm based on Dynamic Programming for this problem.

2 Heuristic Tree Search

We consider the Elementary shortest path problem with resource constraint and two slots:

- Input:
  - 1 depot
  - $n-1$ customers; for each customer $j = 2, \ldots, n$, two visit intervals (which might overlap)
    - $[s_j^1, e_j^1[$, $s_j^1 \in \mathbf{N}_+$, $e_j^1 \in \mathbf{N}_+$, $s_j^1 < e_j^2$
    - $[s_j^2, e_j^2[$, $s_j^2 \in \mathbf{N}_+$, $e_j^2 \in \mathbf{N}_+$, $s_j^2 < e_j^2$
  - an $n \times n$ symmetric matrix t specifying the times in $\mathbf{R}_+$ to travel between each pair of locations
  - an $n \times n$ matrix c specifying the costs in $\mathbf{R}$ to travel between each pair of locations
- Problem: find a sub-tour starting and ending at the depot such that
  - each customer is visited at most once
  - the arrival and the departure from a customer include one of its two visit intervals
- Objective: minimize the cost of the sub-tour

Propose and implement an algorithm based on Heuristic Tree Search with Dynamic Programming for this problem.

3 Column Generation Heuristic + Dynamic Programming

We consider the Vehicle routing problem with a single slot:

- Input:
  - 1 depot
  - $n-1$ customers; for each customer $j = 2, \ldots, n$, a visit interval $[s_j, e_j[$, $s_j \in \mathbf{N}_+$, $e_j \in \mathbf{N}_+$, $s_j < e_j$

  - an $n \times n$ symmetric matrix t specifying the times in $\mathbf{R}_+$ to travel between each pair of locations

- Problem: find a set of routes starting and ending at the depot such that
  - each customer is visited exactly once
  - the arrival at a customer j is before s_j
  - the departure from a customer is at e_j (even if the arrival was before s_j)
- Objective: minimize the total traveled distance

Propose an exponential formulation and implement an algorithm based on a Column Generation heuristic for this problem.

4 Column Generation Heuristic + Heuristic Tree Search

We consider the Vehicle routing problem with two slots:

- Input:
  - 1 depot
  - $n-1$ customers; for each customer $j = 2, \ldots, n$, two visit intervals (which might overlap)
    - $[s_j^1, e_j^1[$, $s_j^1 \in \mathbf{N}_+$, $e_j^1 \in \mathbf{N}_+$, $s_j^1 < e_j^2$
    - $[s_j^2, e_j^2[$, $s_j^2 \in \mathbf{N}_+$, $e_j^2 \in \mathbf{N}_+$, $s_j^2 < e_j^2$
  - an $n \times n$ symmetric matrix t specifying the times in $\mathbf{R}_+$ to travel between each pair of locations
- Problem: find a set of routes starting and ending at the depot such that
  - each customer is visited exactly once
  - the arrival and the departure from a customer include one of its two visit intervals
- Objective: minimize the total traveled distance

Propose an exponential formulation and implement an algorithm based on a Column Generation heuristic for this problem.