Advanced Models and Methods in Operations Research Project: Nurse rostering

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For each problem considered, instances and a code skeleton containing an instance parser and a solution checker are provided in the `data/` and `python/` folders of the project.

The algorithms must be implemented in the provided files between the tags `TODO START` and `TODO END`.

They must be tested on all the provided instances with the command: `python3 problem.py -i instance.json -c certificate.json`

And each solution file must be validated by the provided checker: `python3 problem.py -a checker -i instance.json -c certificate.json`

The results must be reproducible.

The delivrable must contain:

- A *short* report describing and justifying the proposed algorithms

- The code implementing the algorithms

- The solution files obtained on the provided instances

Introduction

The nurse rostering problem is the problem of assigning nurses to shifts to create a roster satisfying some predetermined requirements.

A day is composed of three types of work shifts: the early shift, the late shift and the night shift. For each shift of each day of the scheduling horizon, a number of nurses is requested. The goal is to find a schedule for each nurse such that these requirements and the other constraints are satisfied, while minimizing the number of nurses.

1 Dynamic Programming

We consider the following shift selection problem:

- Input:

    - n days; for each day $j = 1, \ldots, n$, for each shift type $t = 1, 2, 3$, a profit $p_{j,t}$
    - a maximum work time c

- Problem: find a set of shifts such that

    - at most one shift is selected each day
    - the night shift of a day and the early shift of the next day are not both selected
    - at most c shifts are selected

- Objective: maximize the profit of the selected shifts

Propose and implement an algorithm based on Dynamic Programming for this problem.

2 Heuristic Tree Search

We consider the following shift selection problem including the constraint "Maximum numbers of shifts of each type":

- Input:

    - n days; for each day $j = 1, \ldots, n$, for each shift type $t = 1, 2, 3$, a profit $p_{j,t}$
    - a maximum work time c
    - **for each shift type $t = 1, 2, 3$, a maximum number of shifts of this type m_t**

- Problem: find a set of shifts such that

– at most one shift is selected each day

– the night shift of a day and the early shift of the next day are not both selected

– at most c shifts are selected

– **at most m_t shifts of a type t are selected**

• Objective: maximize the profit of the selected shifts

Propose and implement an algorithm based on Heuristic Tree Search with Dynamic Programming for this problem.

3 Column Generation + Dynamic Programming

We consider the following nurse rostering problem including the constraint:

• Input:

– n days; for each day $j = 1, \ldots, n$, for each shift type $t = 1, 2, 3$, a requested number of nurses $s_{j,t}$

– a maximum work time c

• Problem: for each nurse, find an assignment of shifts such that

– $s_{j,t}$ nurses are assigned to shift t of day j

– a nurse is assigned at most one shift each day

– a nurse is not assigned the night shift of a day and the early shift of the next day

– a nurse is not assigned more than c shifts

• Objective: minimize the number of nurses

Propose an exponential formulation and implement an algorithm based on a Column Generation heuristic for this problem.

4 Column Generation + Heuristic Tree Search

We consider the following nurse rostering problem including the constraint "Maximum numbers of shifts of each type that can be assigned to nurses":

• Input:

– n days; for each day $j = 1, \ldots, n$, for each shift type $t = 1, 2, 3$, a requested number of nurses $s_{j,t}$

– a maximum work time c

– **for each shift type $t = 1, 2, 3$, a maximum number of shifts of this type m_t**

Problem: for each nurse, find an assignment of shifts such that

• $s_{j,t}$ nurses are assigned to shift t of day j

• a nurse is assigned at most one shift each day

• a nurse is not assigned the night shift of a day and the early shift of the next day

• a nurse is not assigned more than c shifts

• **a nurse is not assigned more than m_t shifts of a type t**

Objective: minimize the number of nurses

Propose an exponential formulation and implement an algorithm based on a Column Generation heuristic for this problem.