Advanced Models and Methods in Operations Research
Project: Nurse rostering

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For each problem considered, instances and a code skeleton containing an instance parser and a solution checker are provided in the `data/` and `python/` folders of the project.

The algorithms must be implemented in the provided files between the tags `TODO START` and `TODO END`.

They must be tested on all the provided instances with the command: `python3 problem.py -i instance.json -c certificate.json`

And each solution file must be validated by the provided checker: `python3 problem.py -a checker -i instance.json -c certificate.json`

The results must be reproducible.

The delivrable must contain:

- A *short* report describing and justifying the proposed algorithms
- The code implementing the algorithms
- The solution files obtained on the provided instances

Introduction

The nurse rostering problem is the problem of assigning nurses to shifts to create a roster satisfying some predetermined requirements.

A day is composed of three types of work shifts: the early shift, the late shift and the night shift. For each shift of each day of the scheduling horizon, a number of nurses is requested. The goal is to find a schedule for each nurse such that these requirements and the other constraints are satisfied, while minimizing the number of nurses.

1 Dynamic Programming

We consider the following shift selection problem:

- Input:
  - n days; for each day $j = 1, \ldots, n$, for each shift type $t = 1, 2, 3$, a profit $p_{j,t}$
  - a maximum work time c
- Problem: find a set of shifts such that
  - at most one shift is selected each day
  - the night shift of a day and the early shift of the next day are not both selected
  - at most c shifts are selected
- Objective: maximize the profit of the selected shifts

Propose and implement an algorithm based on Dynamic Programming for this problem.

2 Heuristic Tree Search

We consider the following shift selection problem including the constraint "Maximum numbers of shifts of each type":

- Input:
  - n days; for each day $j = 1, \ldots, n$, for each shift type $t = 1, 2, 3$, a profit $p_{j,t}$
  - a maximum work time c
  - **for each shift type $t = 1, 2, 3$, a maximum number of shifts of this type m_t**
- Problem: find a set of shifts such that

- at most one shift is selected each day
- the night shift of a day and the early shift of the next day are not both selected
- at most c shifts are selected
- **at most m_t shifts of a type t are selected**

- Objective: maximize the profit of the selected shifts

Propose and implement an algorithm based on Heuristic Tree Search with Dynamic Programming for this problem.

3 Column Generation + Dynamic Programming

We consider the following nurse rostering problem including the constraint:

- Input:
  - n days; for each day $j = 1, \dots, n$, for each shift type $t = 1, 2, 3$, a requested number of nurses $s_{j,t}$
  - a maximum work time c
- Problem: for each nurse, find an assignment of shifts such that
  - $s_{j,t}$ nurses are assigned to shift t of day j
  - a nurse is assigned at most one shift each day
  - a nurse is not assigned the night shift of a day and the early shift of the next day
  - a nurse is not assigned more than c shifts
- Objective: minimize the number of nurses

Propose an exponential formulation and implement an algorithm based on a Column Generation heuristic for this problem.

4 Column Generation + Heuristic Tree Search

We consider the following nurse rostering problem including the constraint "Maximum numbers of shifts of each type that can be assigned to nurses":

- Input:
  - n days; for each day $j = 1, \dots, n$, for each shift type $t = 1, 2, 3$, a requested number of nurses $s_{j,t}$
  - a maximum work time c
  - **for each shift type $t = 1, 2, 3$, a maximum number of shifts of this type m_t**
- Problem: for each nurse, find an assignment of shifts such that
  - $s_{j,t}$ nurses are assigned to shift t of day j
  - a nurse is assigned at most one shift each day
  - a nurse is not assigned the night shift of a day and the early shift of the next day
  - a nurse is not assigned more than c shifts
  - **a nurse is not assigned more than m_t shifts of a type t**
- Objective: minimize the number of nurses

Propose an exponential formulation and implement an algorithm based on a Column Generation heuristic for this problem.