Weak Gravitational Lensing

Andean Cosmology School,
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Weak Lensing: basics

- Fermat Principle->Euler-Lagrange equations.
- Integrating the EL equations along the light path*, we obtain the **deflection angle***:

$$\boxed{\hat{\vec{\alpha}} = \frac{2}{c^2}\int \nabla_\perp \Phi \, dl}$$

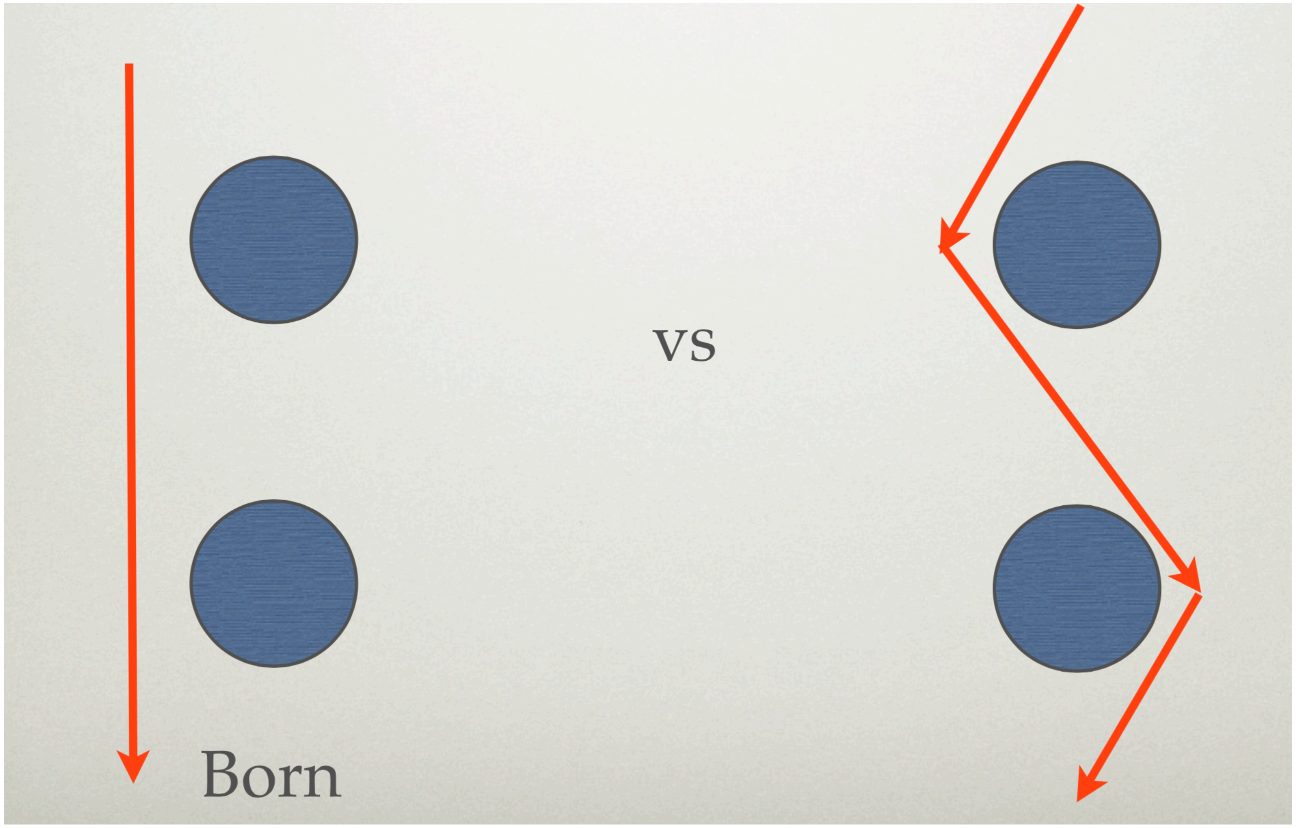
If you measure the deflection angle, we can obtain information about the gravitational potential of the lens and /or the metric! -**> Dark matter, modified gravity!**

* In WL we usually use the so-called Born approximation: the integrals can be taken in the radial direction

**The deflection angle can also be derived by looking at the geodesic equation in GR

Weak Lensing: basics

As it stands, the equation for $\hat{\vec{\alpha}}$ is not useful, as we would have to integrate over the actual light path. However, since $\Phi/c^2 \ll 1$, we expect the deflection angle to be small. Then, we can adopt the Born approximation familiar from scattering theory and integrate over the unperturbed light path.



Weak Lensing: basics

- Modified gravity: consider two types of scalar perturbations to the metric

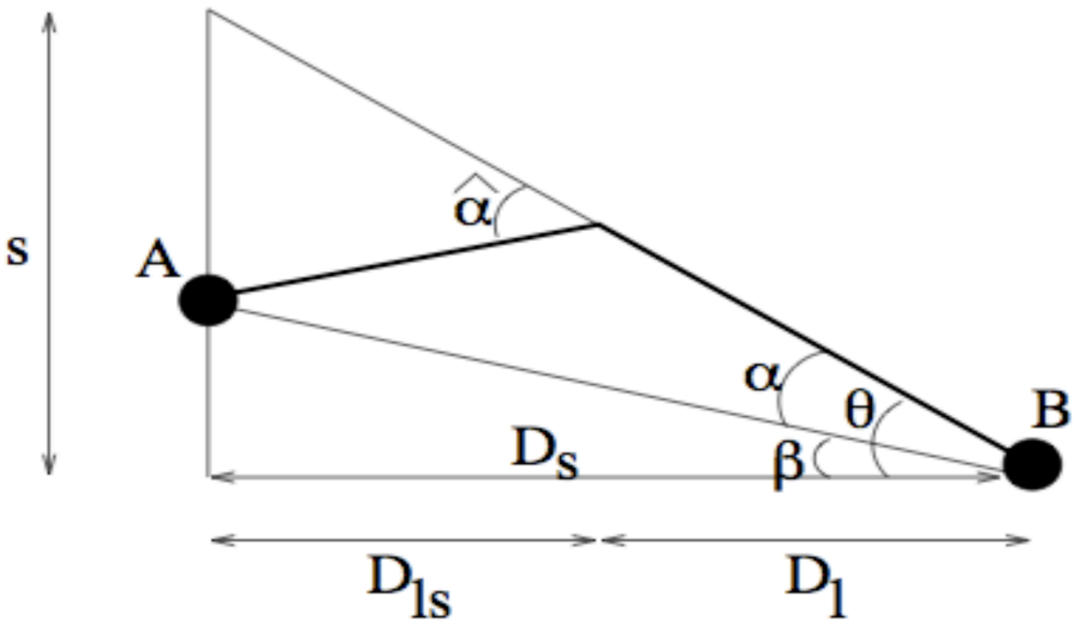
$$ds^2 = \left(1 + \frac{2\Psi}{c^2}\right) dt^2 - \left(1 - \frac{2\Phi}{c^2}\right) dl^2$$

In GR, they are **both the same.**

For this metric, the deflection angle responds to the combination of the two perturbations:

$$\hat{\vec{\alpha}} = \frac{1}{c^2} \int \nabla_\perp (\Phi + \Psi) dl$$

Weak Lensing: geometry



All angular diameter distances

$$\hat{\vec{\alpha}} = \frac{2}{c^2} \int \nabla_\perp \Phi \, dl$$

$$\boxed{\boxed{\vec{\theta} = \vec{\beta} + \vec{\alpha}}}$$

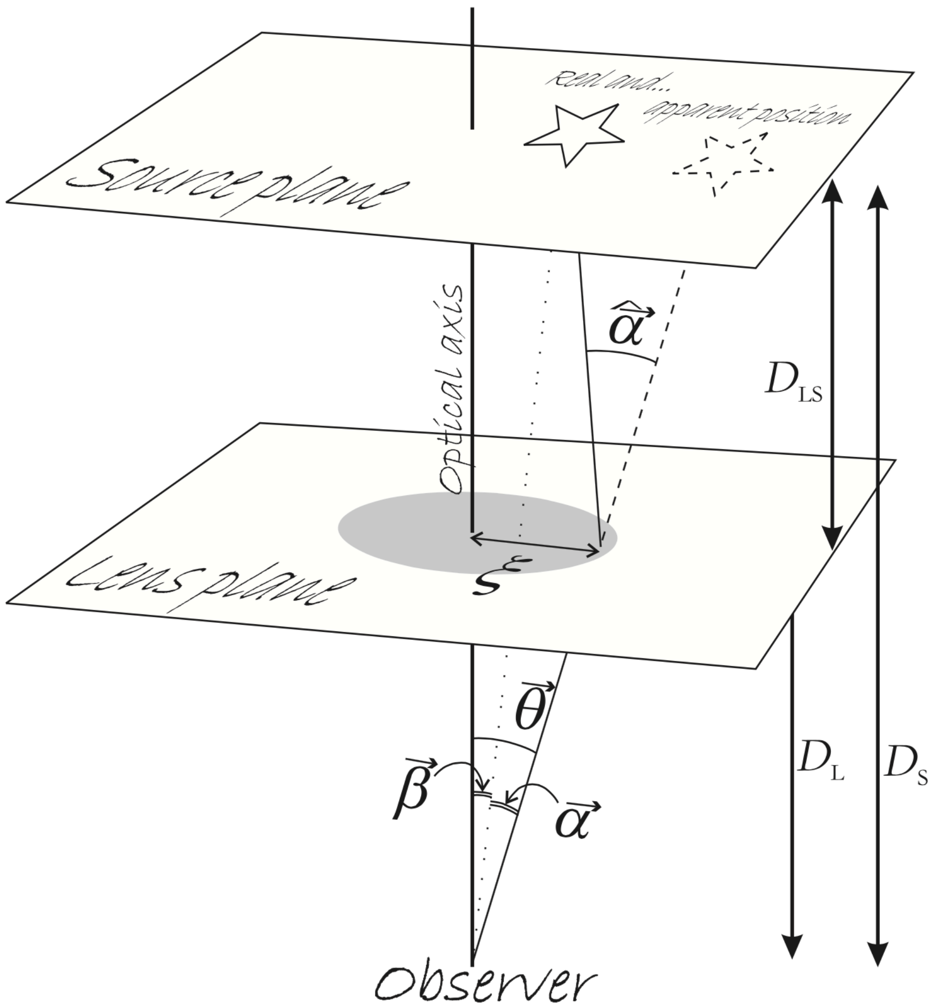
The Lens Equation

$$\vec{\alpha} D_s = \hat{\vec{\alpha}} D_{ls}$$

$$\vec{\alpha} = \frac{D_{ls}}{D_s} \hat{\vec{\alpha}}$$

Reduced deflection angle

Weak Lensing: geometry



Massey

We have also used the thin lens approximation: the mass distribution of the lens can be confined to a plane.

Reasonable approximation given the distances involved.

$$\boxed{\hat{\vec{\alpha}} = \frac{2}{c^2}\int \nabla_\perp \Phi \, dl}$$

$$\boxed{\vec{\theta} = \vec{\beta} + \vec{\alpha}}$$

Weak Lensing: basics

- The lens equation:

$$\boxed{\vec{\theta} = \vec{\beta} + \vec{\alpha}}$$

It is non-linear in general and can have multiple solutions for **θ**, potentially creating multiple images in the strong-lensing regime.

For weak gravitational lensing, we will assume that this mapping between the source and the lens plane is linear, and be characterized by its Jacobian.

Weak Lensing: the lensing potential

- Note that the reduced deflection angle can be written as:

$$\vec{\alpha} = \nabla_{\perp}\left(\frac{2D_{ds}}{D_s c^2}\int \Phi ds\right)$$

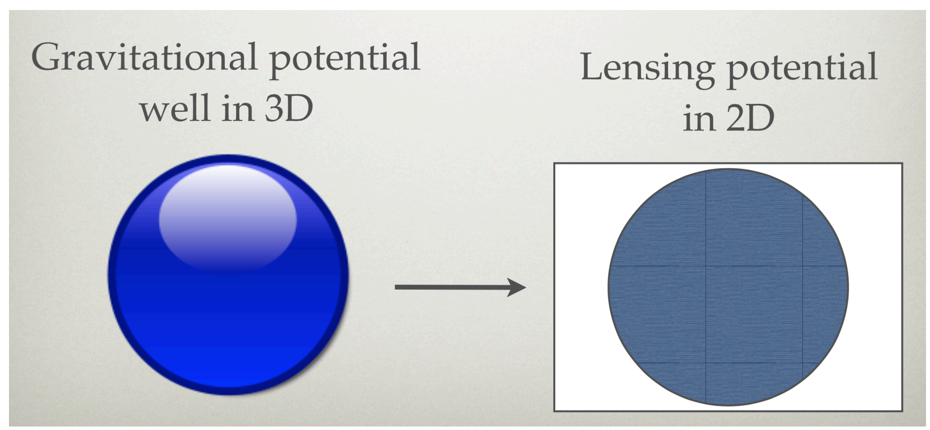
-So it can be thought as the angular gradient of a potential:

$$\vec{\nabla}_{\theta}\psi$$

-It is called the **Lensing Potential,** a scaled projection of the 3D Newtonian potential.

$$\vec{\nabla}_{\theta} = D_d\vec{\nabla}_{\perp}$$

$$\psi(\vec{\theta}) = \frac{2D_{ds}}{D_d D_s}\int \Phi(D_d\vec{\theta}, s)ds$$



Weak Lensing: observables

- In WL, the deflection angle itself is not observable: we are not at liberty to remove the foreground lens structures to observe the unlensed position.

- So we look at the gradient of the deflection angle, which is the Laplacian of the lensing potential. A 2D Poisson Equation! (s=line of sight)

$$
\begin{aligned}
\nabla_\theta^2 \psi &= \frac{2}{c^2}\frac{D_{ds}D_d}{D_s}\int\left(\nabla^2 - \frac{\partial^2}{\partial s^2}\right)\Phi\ ds \\
&= \frac{D_{ds}D_d}{D_s}\frac{8\pi G}{c^2}\int \rho ds \\
&= \frac{D_{ds}D_d}{D_s}\frac{8\pi G\Sigma(D_d\vec{\theta})}{c^2} \equiv \frac{2\Sigma(D_d\vec{\theta})}{\Sigma_c} \equiv 2\kappa
\end{aligned}
$$

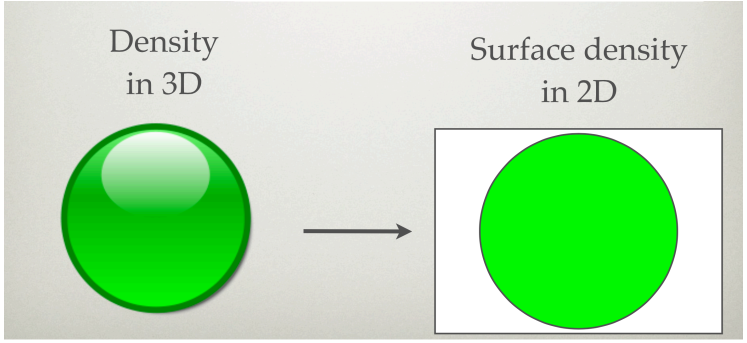
- Assume: derivative of the potential vanishes far away from the lens.

-Use: $\nabla^2\Phi = 4\pi G\rho$

-Define: $\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, s)ds$

-Define the critical surface mass density as (contains constants and distance ratios):

$$\Sigma_c \equiv \frac{c^2 D_s}{4\pi G D_{ds} D_d}$$



Weak Lensing observables: convergence

$$\nabla_\theta^2 \psi = \frac{2\Sigma}{\Sigma_c} \qquad \kappa = (1/2)(\partial_1\partial_1\psi + \partial_2\partial_2\psi) \qquad \kappa(\vec{\theta}) = \frac{\Sigma(D_d\vec{\theta})}{\Sigma_{cr}}$$

$$\psi(\vec{\theta}) = \frac{1}{\pi}\int d^2\theta' \ln|\vec{\theta} - \vec{\theta}'|\,\kappa(\vec{\theta}')$$

Solution to 2D Poisson Equation

It can be viewed as a measure of the integrated mass density of the lens, and it describes the magnification of the image due to the focusing of light rays caused by the lens

The convergence is proportional to the projected density (the surface mass density)

Weak Lensing basics

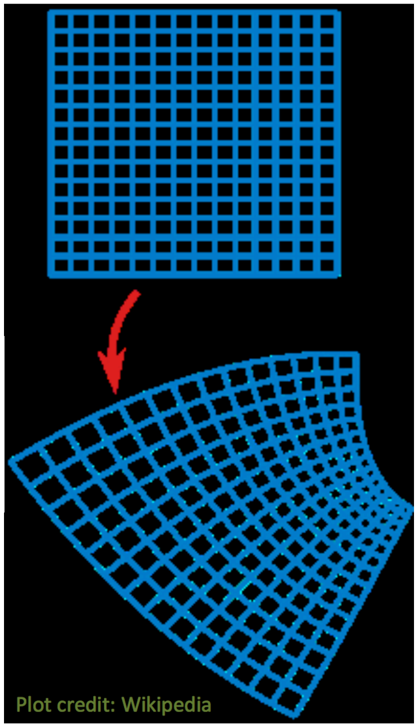
- The effect of **weak** gravitational lensing can be expressed locally as a linear map of the surface brightness distribution of the source

$$I^{\mathrm{obs}}(\theta_i) = I^{\mathrm{s}}(A_{ij}\theta_j)$$

- **I** is the intensity or surface brightness distribution measured in units of luminosity per unit area at position (x,y)

- This coordinate transformation is specified by the **lens equation** introduced above, and can be characterized by its Jacobian matrix A

$$\boxed{\vec{\theta} = \vec{\beta} + \vec{\alpha}}$$



$$A_{ij} = \frac{\partial \beta_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} = \delta_{ij} - \psi_{,ij}$$

$$= \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j}$$

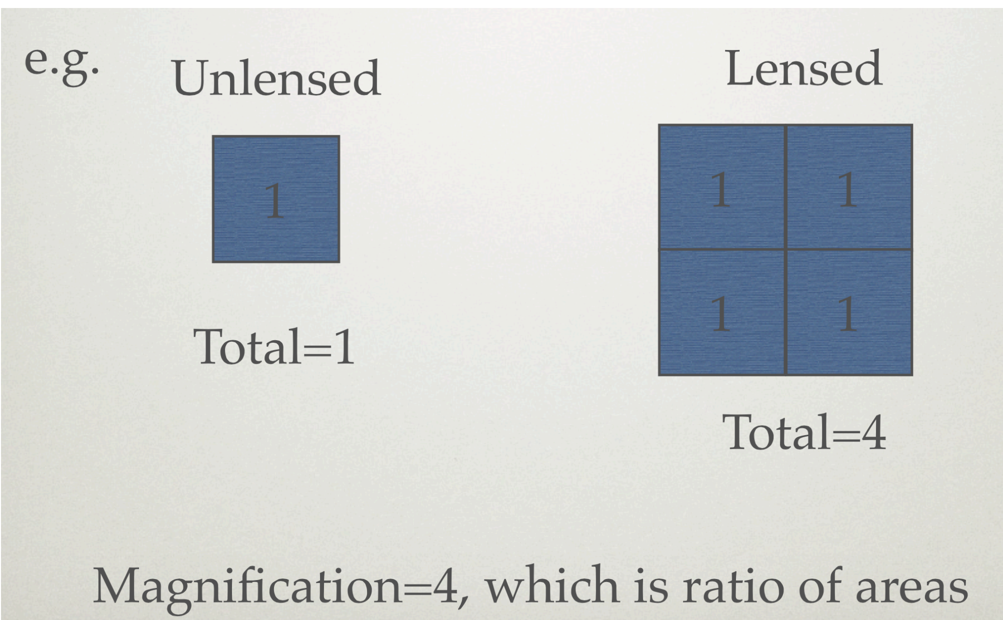
Matrix A: how a position in the image plane maps to a position in the source plane.

Weak Lensing basics

The inverse of A is called the magnification tensor $M \equiv A^{-1}$

$$\det M \equiv \mu$$

Magnification: ratio of lensed to unlensed luminosity



But by Liouville's theorem* **lensing conserves surface brightness** (L/A), so magnification becomes ratio of areas.

Magnification: describes how the original area element changes under the lensing map

*The number of particles per volume in phase space is constant in time

Weak Lensing: shear

Define the complex shear: $\gamma = \gamma_1 + i\gamma_2$

$$\gamma_1 \equiv \frac{1}{2}(\partial_1\partial_1\psi - \partial_2\partial_2\psi)$$

$$\gamma_2 \equiv \partial_1\partial_2\psi,$$

Can be written as derivatives of the lensing potential, like the convergence

$$\kappa = (1/2)(\partial_1\partial_1\psi + \partial_2\partial_2\psi)$$

So the distortion matrix can be written as:

$$A = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} + \begin{pmatrix} -\gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 \end{pmatrix} = (1-\kappa)\begin{pmatrix} 1-g_1 & -g_2 \\ -g_2 & 1+g_1 \end{pmatrix}$$

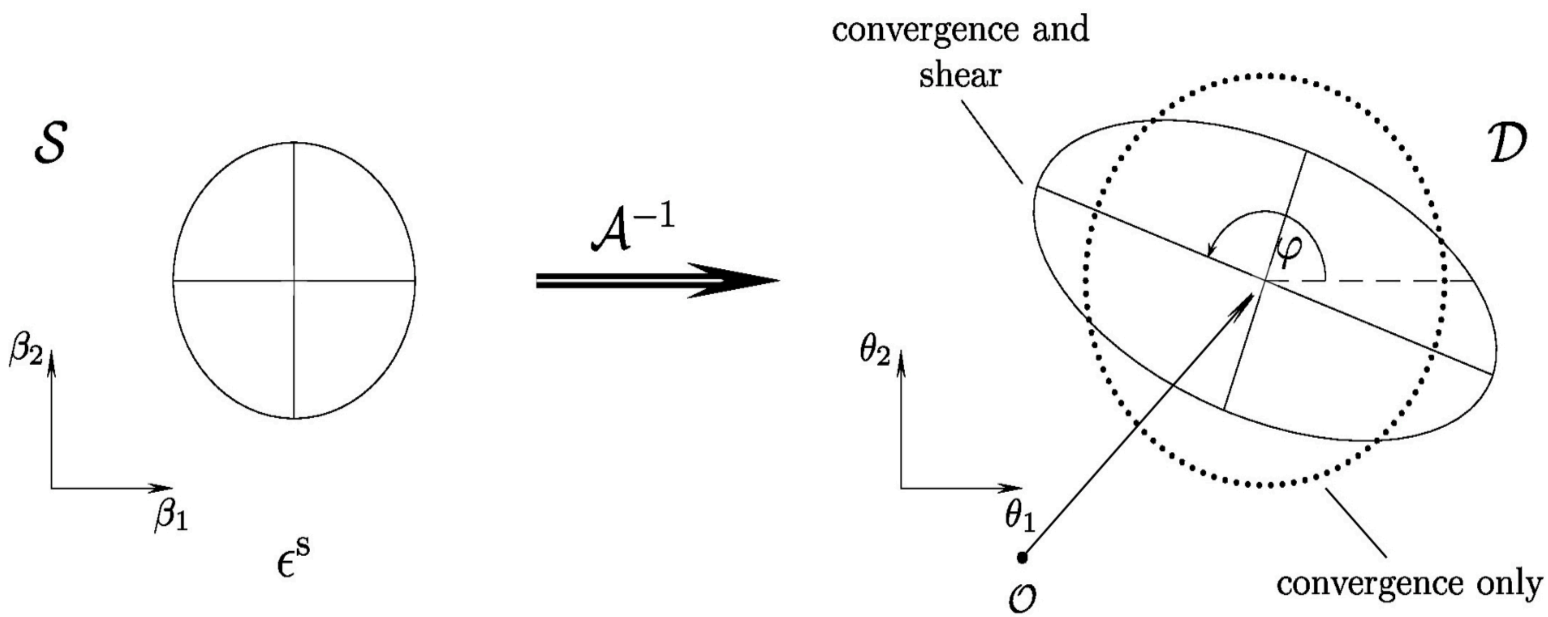
From above, we see that the change in magnification is given by the term (1 –κ), and the distortion by the reduced shear:

$$g_i \equiv \gamma_i/(1-\kappa).$$

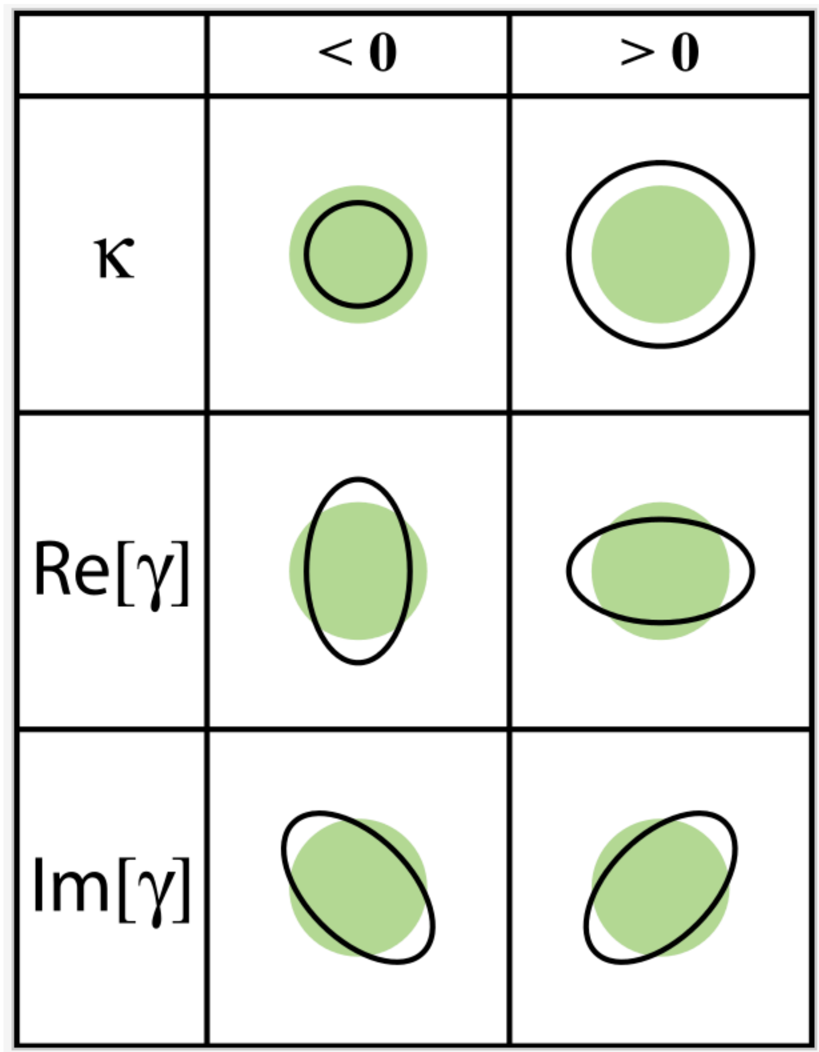
Weak Lensing basics

- Distortion is **elliptical**: a circular source of unit radius will turn into a magnified ellipse

$$a = (1 - \kappa - \gamma)^{-1} \qquad b = (1 - \kappa + \gamma)^{-1}$$



Weak Lensing basics



Weak lensing basics

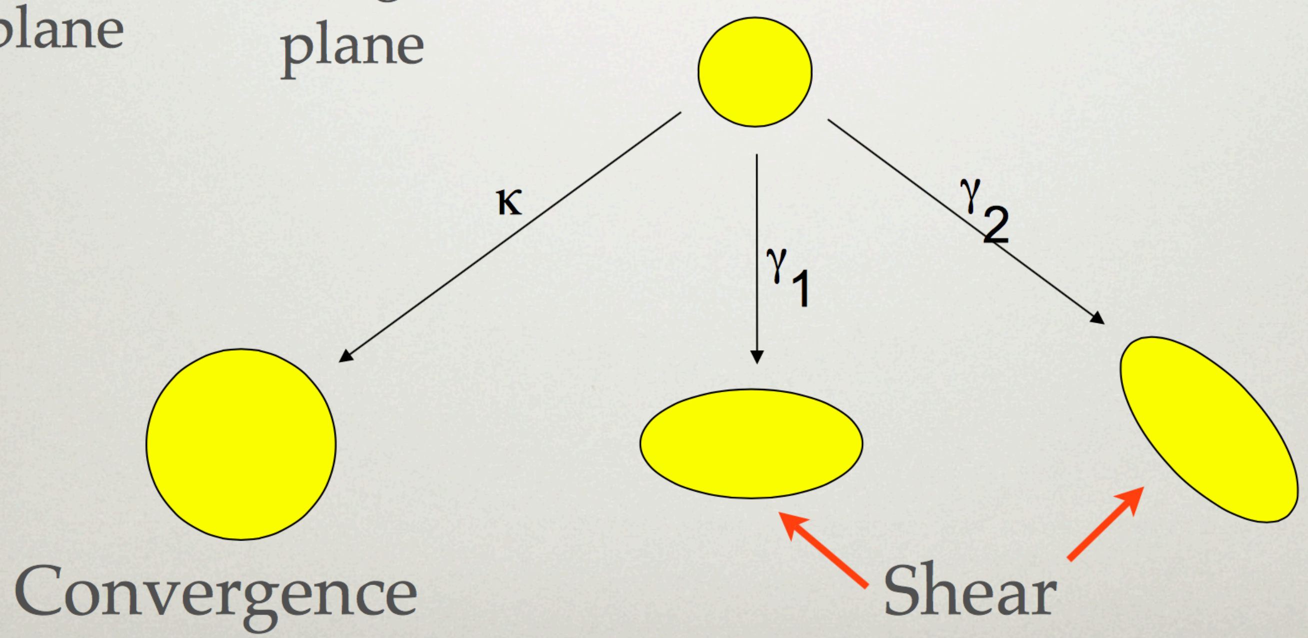
To recap:

$$\beta_i = A_{ij}\theta_j$$

Source plane

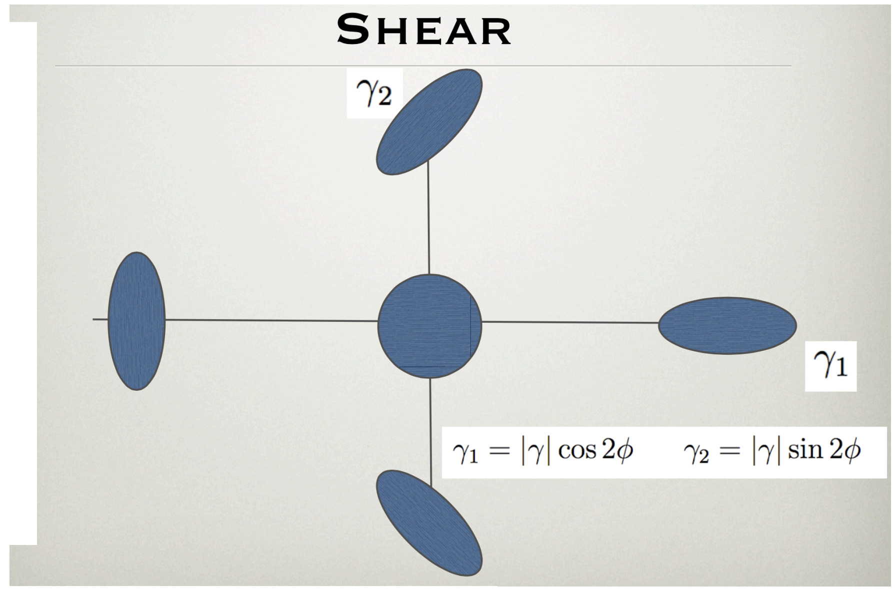
Image plane

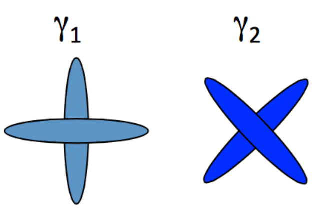
$$A = \begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix} + \begin{pmatrix} -\gamma_1 & -\gamma_2 \\ -\gamma_2 & \gamma_1 \end{pmatrix}$$

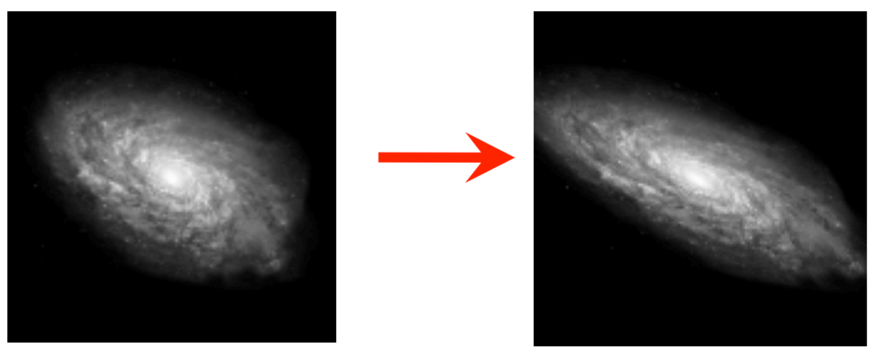


Weak lensing basics

- Shear is a *spin-weight 2* field
  - Symmetric under rotations of 180 deg.
  - *Polarisation also an example of spin-2*
  - *Convergence is a spin-wight 0 field (symmetric under any rotation)*
  - *Spin 0 = scalar*
  - *Spin 1 = vector*





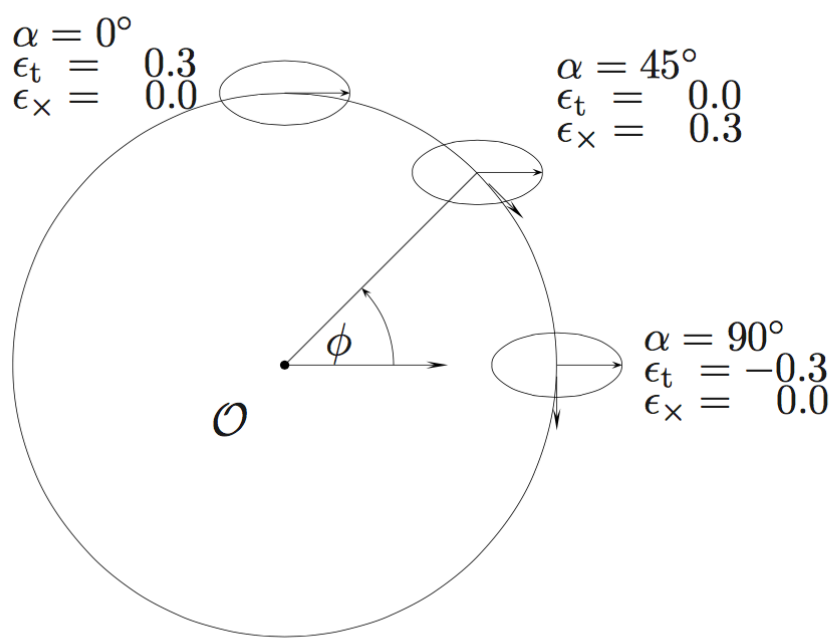


The shear is the additional ellipticity imprinted on an object

Weak lensing basics

- It is often useful to consider the shear components in a rotated reference frame: tangential and cross-component shear

$$\gamma_{\rm t} = -\mathcal{R}{\rm e}\left[\gamma\, {\rm e}^{-2{\rm i}\phi}\right] \quad , \quad \gamma_{\times} = -\mathcal{I}{\rm m}\left[\gamma\, {\rm e}^{-2{\rm i}\phi}\right]$$

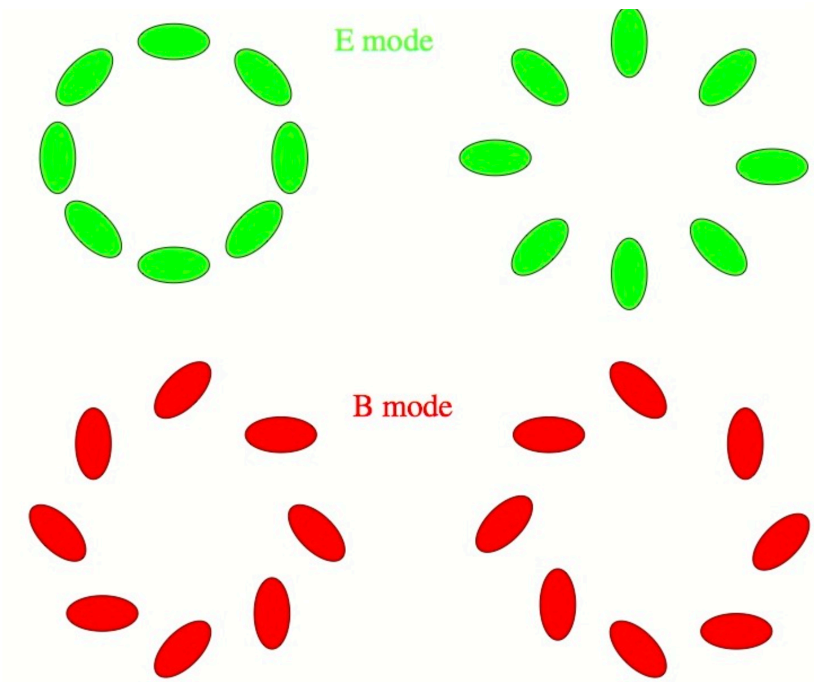


Shear: E and B modes

- The shear signal produced around mass concentrations is tangent to their centers: E mode
- The signal produced about voids is radial: E mode

- A signal rotated 45 degrees from the two above is unphysical: B mode

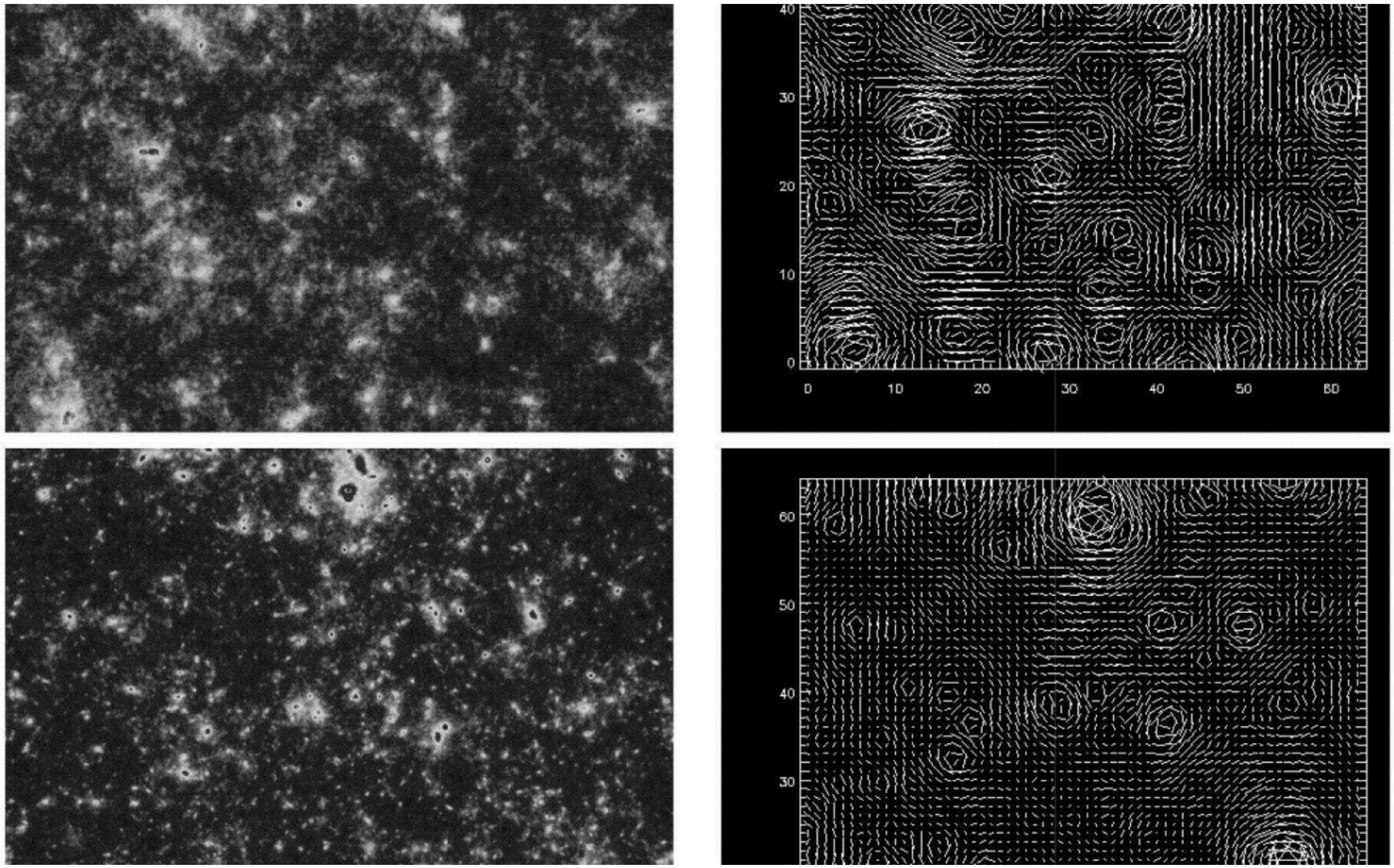
$$
\begin{aligned}
\tilde{\kappa}^{\mathrm{E}} &= \cos(2\phi_\ell)\,\tilde{\gamma}_1 + \sin(2\phi_\ell)\,\tilde{\gamma}_2\ , \\
\tilde{\kappa}^{\mathrm{B}} &= -\sin(2\phi_\ell)\,\tilde{\gamma}_1 + \cos(2\phi_\ell)\,\tilde{\gamma}_2
\end{aligned}
$$



van Waerbeke & Mellier 2003

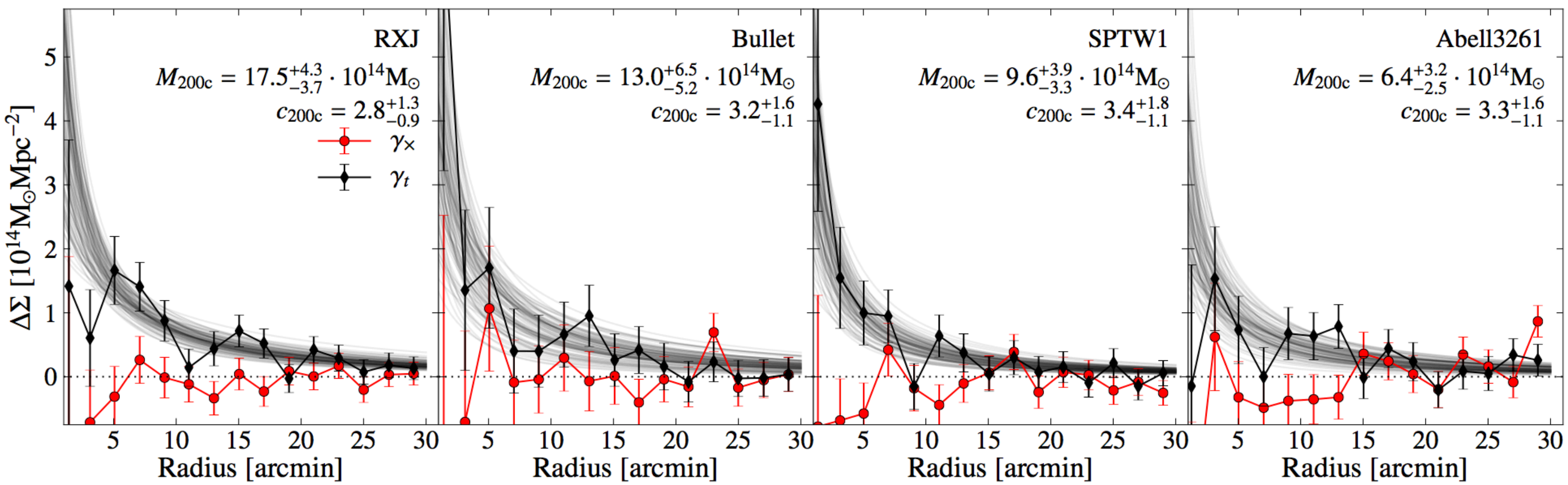
If you see a B-mode in your analysis, something is wrong! -> systematic errors! Sadly, the converse is not true.

Shear: E and B modes



Jain et al. 2000

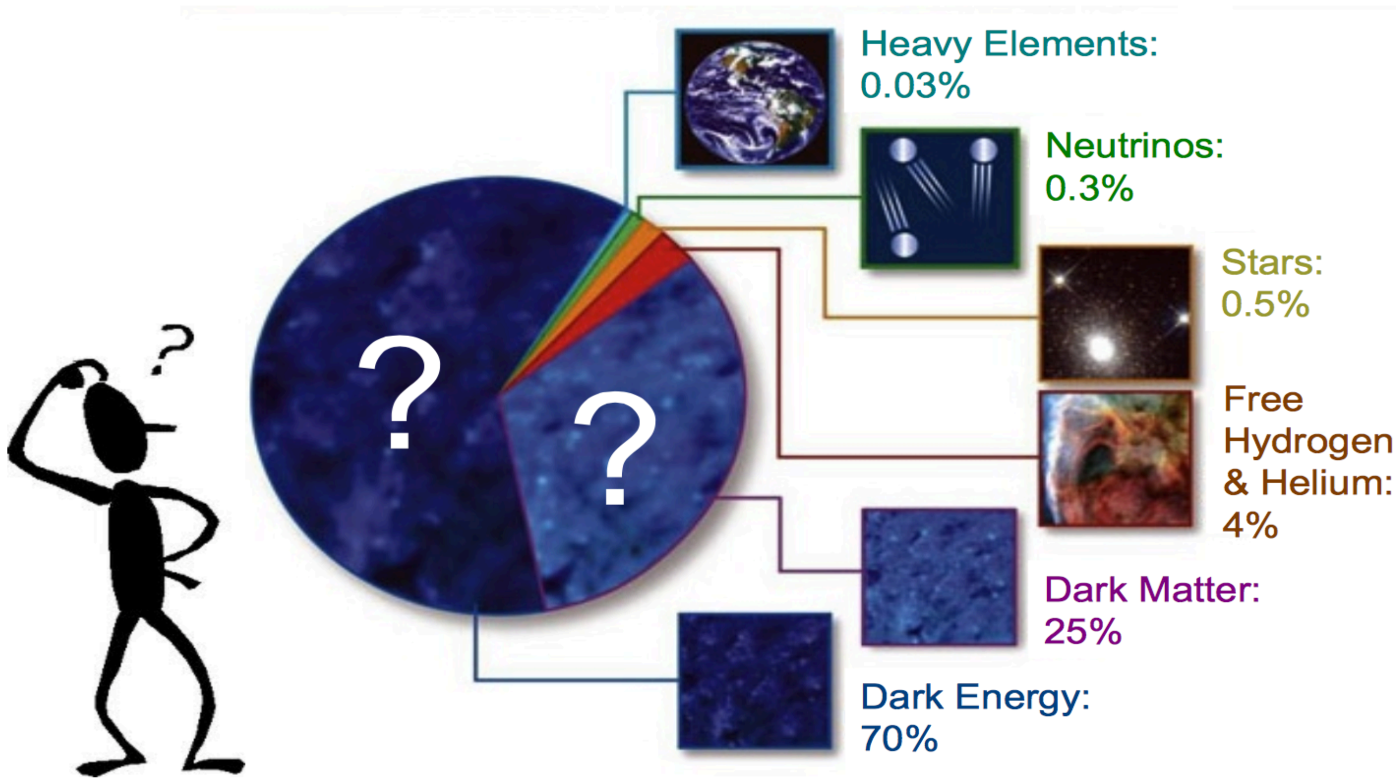
Shear: tangential shear about clusters



Melchior et al. 2014

Weak lensing applications

- Through weak lensing, we can know more about dark energy and dark matter. **Two of the biggest mysteries in modern Physics.**



Credit: LSST

Weak lensing applications: dark energy


Photo: Lawrence Berkeley National Lab

Saul Perlmutter


Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

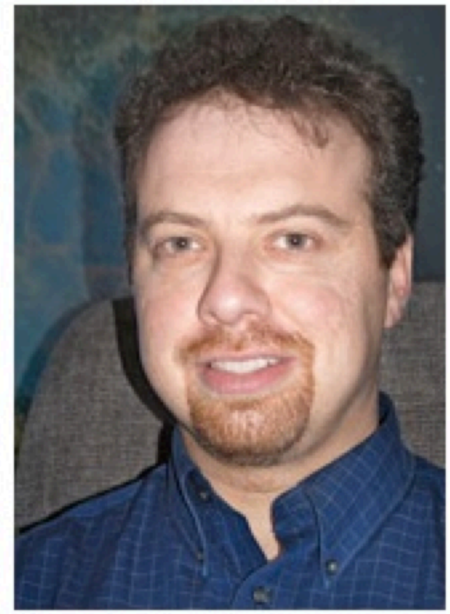


Photo: Scanpix/AFP

Adam G. Riess

The Nobel Prize in Physics 2011 was awarded *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"* with one half to Saul Perlmutter and the other half jointly to Brian P. Schmidt and Adam G. Riess.

Weak lensing applications: Dark Energy

- 1) Do we need to modify/expand GR at cosmic scales?

- 2) If GR is right, we need a new form of energy that exerts repulsive gravity: **Dark Energy**

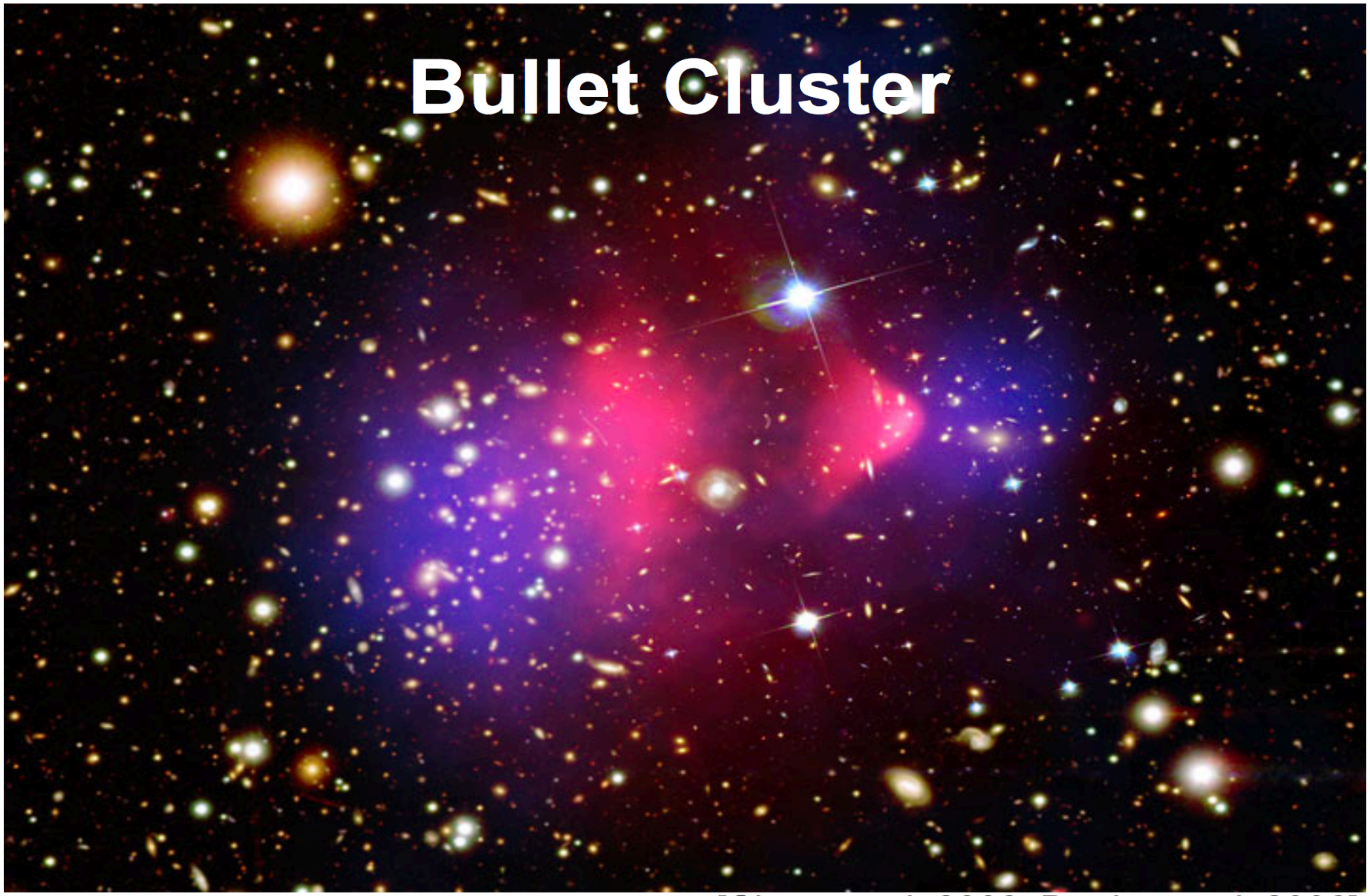
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\sum_i \left(\rho_i + \frac{3p_i}{c^2}\right) \longrightarrow$$

Acceleration equation:
Time evolution of the Universe
In GR

Dark energy: equation of state parameter $w < -1/3$

- Weak lensing is sensitive to the expansion rate (geometry) and growth of structures in the Universe. These depend on dark energy

Weak lensing applications: dark matter



[Clowe et al. 2006; Bradac et al. 2006]

Weak lensing: applications

- Once you have a shear catalog…
- You can (amongst other things):

  - Create mass maps (Kaiser-Squires inversion)

  - Measure correlation functions of the shear field and their evolution with redshift: **dark energy**

  -Tangential shear around clusters:

Weak lensing applications: mass maps

- Create **mass maps** (Kaiser-Squires inversion)

Assume that you have a catalogue of shear and convergence values binned on a grid

Recall: $\gamma_1 = \frac{1}{2}(\partial_1^2 - \partial_2^2)\psi \quad \gamma_2 = \partial_1 \partial_2 \psi \quad \kappa = \frac{1}{2}(\partial_1^2 + \partial_2^2)\psi$

FFT: $\tilde{\gamma}_1 = -\frac{1}{2}(k_1^2 - k_2^2)\tilde{\psi} \quad \tilde{\gamma}_2 = -k_1 k_2 \tilde{\psi} \quad \tilde{\kappa} = -\frac{1}{2}k^2\tilde{\psi}$

Weak lensing applications: mass maps

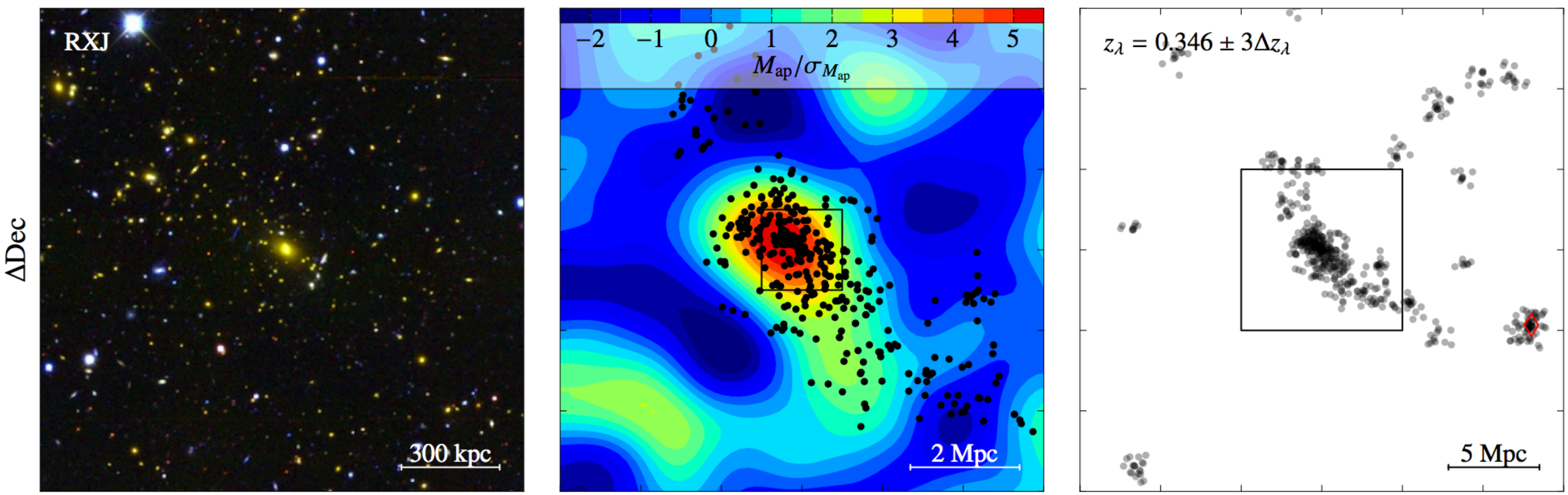
$$\tilde{\gamma}_1 = -\frac{1}{2}(k_1^2 - k_2^2)\tilde{\psi} \qquad \tilde{\gamma}_2 = -k_1 k_2 \tilde{\psi} \qquad \tilde{\kappa} = -\frac{1}{2}k^2\tilde{\psi}$$

Eliminating the potential: $$\tilde{\gamma}(\vec{k}) = \left(\frac{k_1^2 - k_2^2}{k^2} + 2i\frac{k_1 k_2}{k^2}\right)\tilde{\kappa}(\vec{k}) = e^{2i\beta}\tilde{\kappa}(\vec{k})$$

Inverting: $$\boxed{\tilde{\kappa} = \frac{1}{k^2}[(k_1^2 - k_2^2)\tilde{\gamma}_1 + 2k_1 k_2 \tilde{\gamma}_2]}$$

We then take the inverse Fourier transform, and we have a map of the derivative of the lensing potential: mass

Weak lensing applications: mass maps



Melchior et al. 2014

Weak lensing applications: mass maps

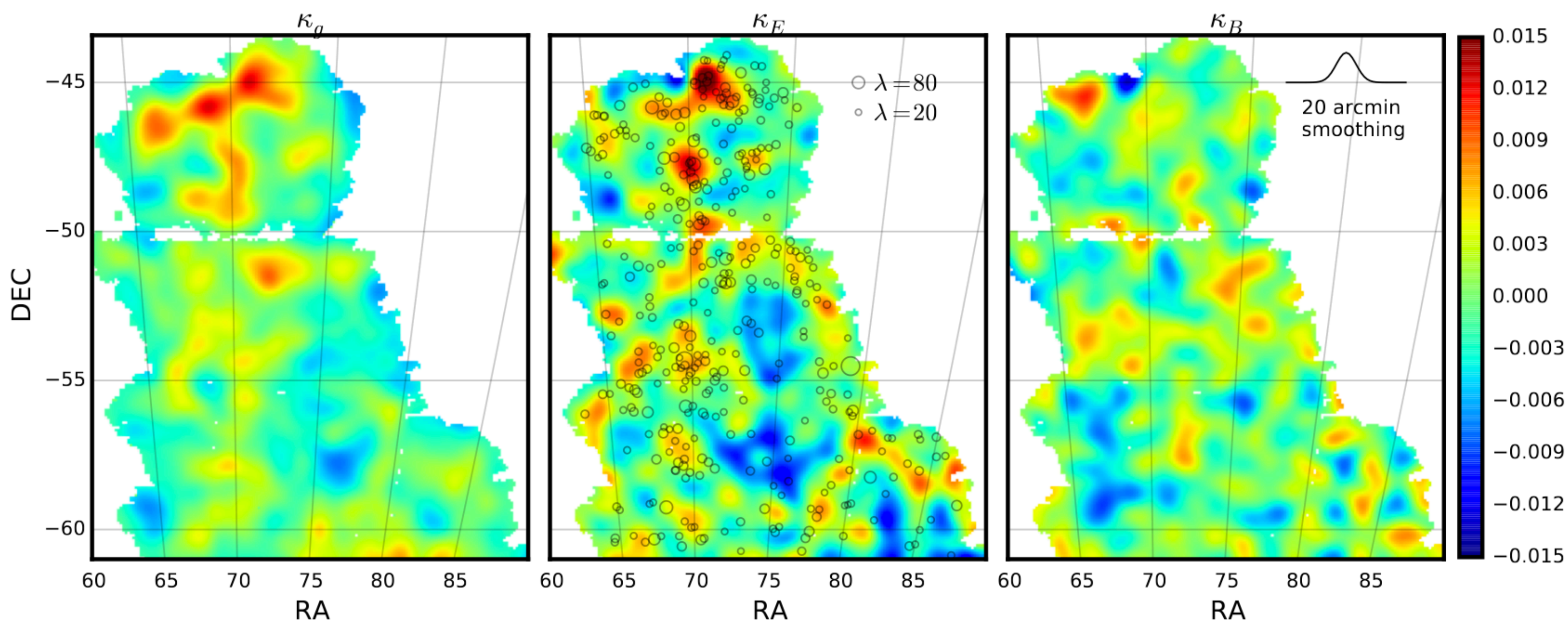


FIG. 1: The DES SV main foreground galaxy maps $\kappa_{g,main}$ (left), E-mode convergence map κ_E (middle) and B-mode convergence map κ_B (right) are shown in these panels. All maps are generated with 5×5 arcmin^2 pixels and 20 arcmin RMS Gaussian smoothing. In the κ_g and κ_E maps, red areas corresponds to overdensities and blue areas to underdensities. White regions correspond to the survey mask. The scale of the Gaussian smoothing kernel is indicated by the Gaussian profile on the upper right corner of the right panel. The κ_E map is overlaid by Redmapper galaxy clusters with optical richness $\lambda > 20$. The radius of the circles scale with λ.

Chang et al. 2015, Vikram et al. 2015

Weak lensing applications: correlation functions

- When averaged over sufficient area the shear field has a mean of zero
- Use 2-point correlation function or power spectra which contains cosmological information

$$\xi_+(\theta) = \langle\gamma\gamma^*\rangle(\theta) = \langle\gamma_t\gamma_t\rangle(\theta) + \langle\gamma_\times\gamma_\times\rangle(\theta);$$
$$\xi_-(\theta) = \langle\gamma\gamma\rangle(\theta) = \langle\gamma_t\gamma_t\rangle(\theta) - \langle\gamma_\times\gamma_\times\rangle(\theta).$$

$$\xi_+(\theta) = \frac{1}{2\pi}\int \mathrm{d}\ell\,\ell \mathrm{J}_0(\ell\theta)[P_\kappa^{\mathrm{E}}(\ell) + P_\kappa^{\mathrm{B}}(\ell)];$$
$$\xi_-(\theta) = \frac{1}{2\pi}\int \mathrm{d}\ell\,\ell \mathrm{J}_4(\ell\theta)[P_\kappa^{\mathrm{E}}(\ell) - P_\kappa^{\mathrm{B}}(\ell)].$$

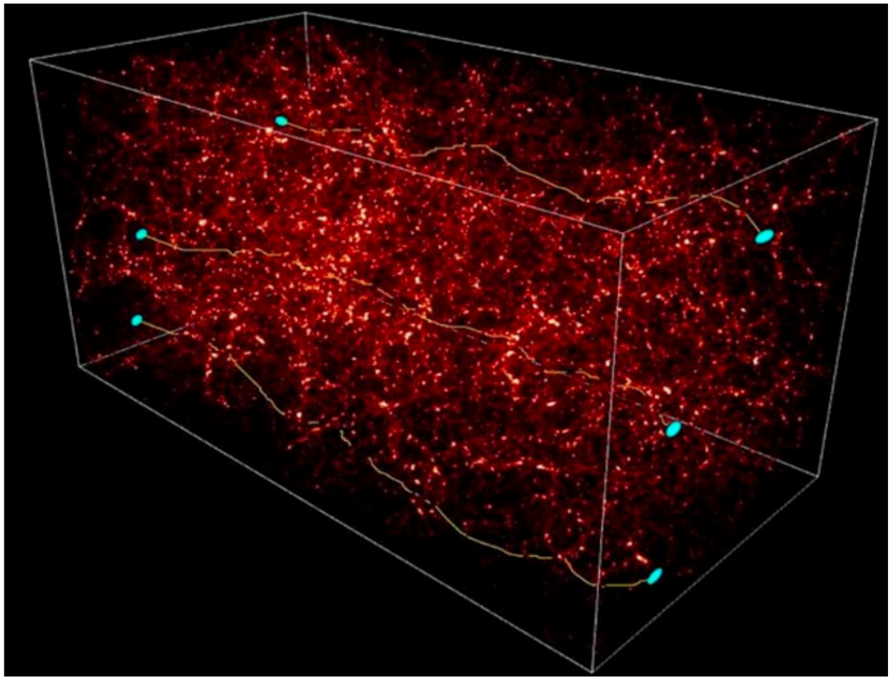
$$\xi_\pm = X_+ \pm X_\times$$

Estimator:

$$X_{+/\times} = \frac{\sum_{i,j} w_i w_j (e^i)_{+/\times}(e^j)_{+/\times}}{\sum_{i,j} w_i w_j}$$

Weak lensing applications: correlation functions

- **Cosmic shear:** distortions that lensing induces as photons encounter inhomogeneities due to the **large-scale structure**



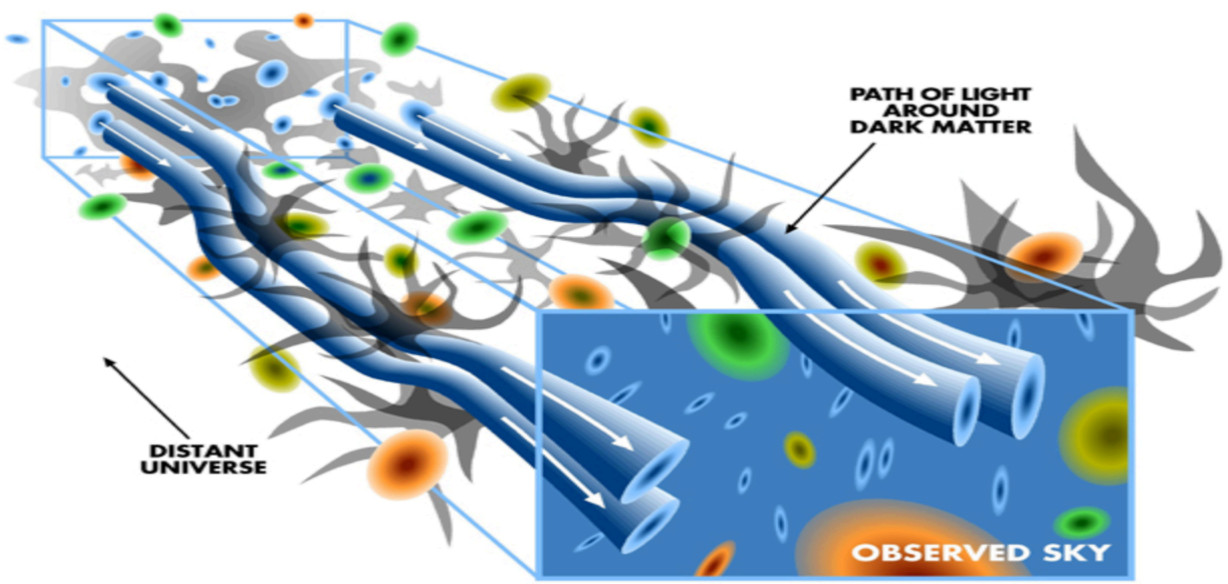
$$\hat{\xi}^{ij}_{\pm}(\theta) = \frac{\sum \mathrm{w}_a \mathrm{w}_b \left[\epsilon^i_t(\boldsymbol{x_a})\epsilon^j_t(\boldsymbol{x_b}) \pm \epsilon^i_\times(\boldsymbol{x_a})\epsilon^j_\times(\boldsymbol{x_b})\right]}{\sum \mathrm{w}_a \mathrm{w}_b}$$

Correlation function at different redshift bins: tomography

HU 200..

Weak lensing applications: correlation functions

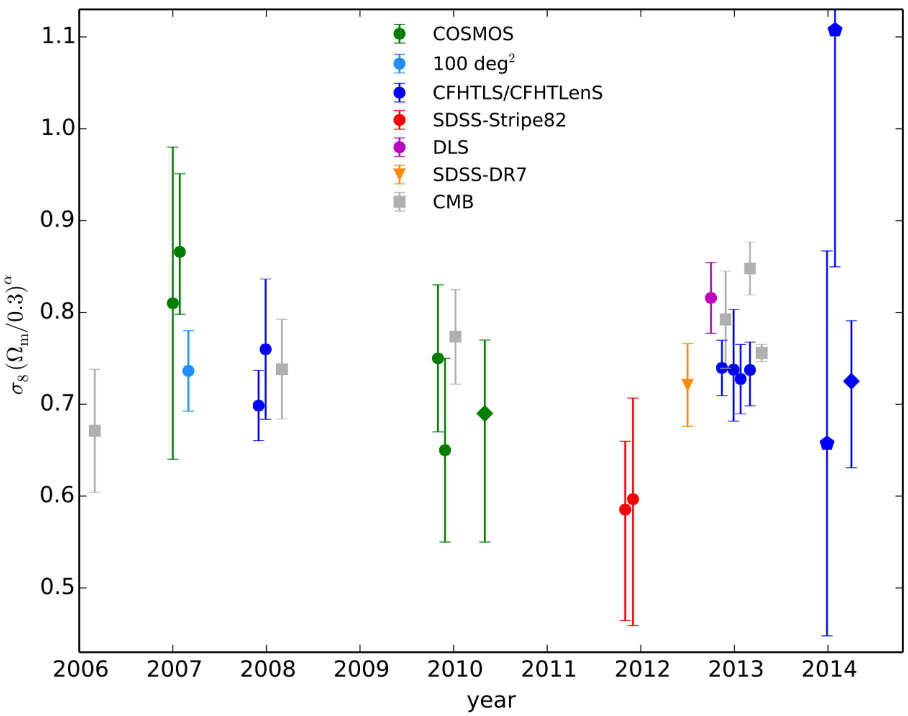
- Cosmic shear was first detected in 2000: large volumes of deep digital imaging are necessary. Also, the signal is very subtle and must be carefully distinguished from image distortions caused by the atmosphere and telescope optics

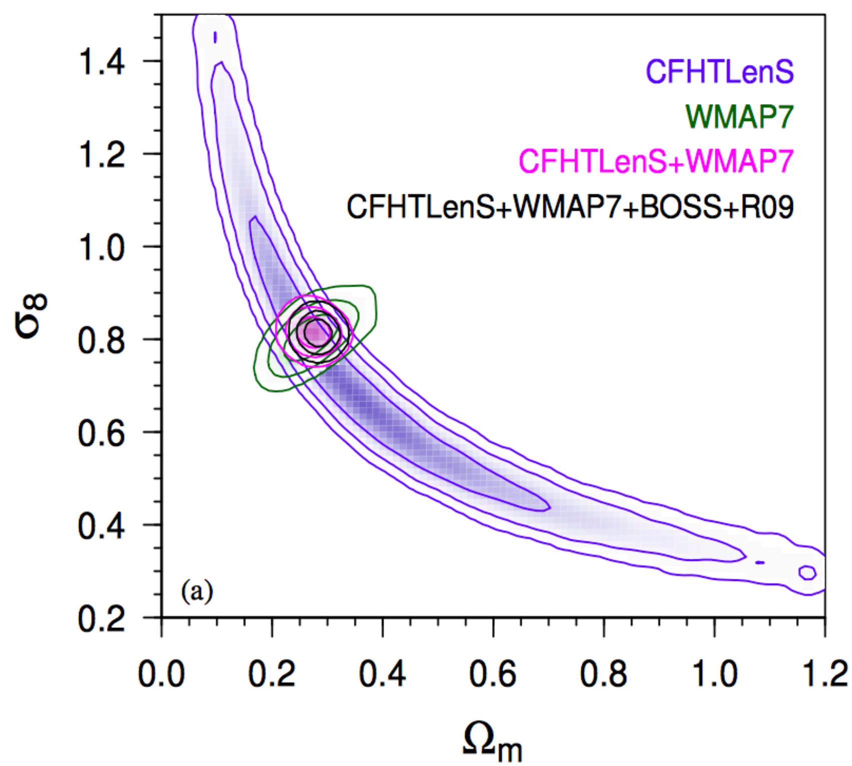


Weak lensing applications: correlation functions

Cosmic shear is most sensitive to the combination:

$$\sigma_8 \Omega_m^\alpha \qquad \alpha \approx 0.5 - 0.7.$$





Weak lensing applications: correlation functions

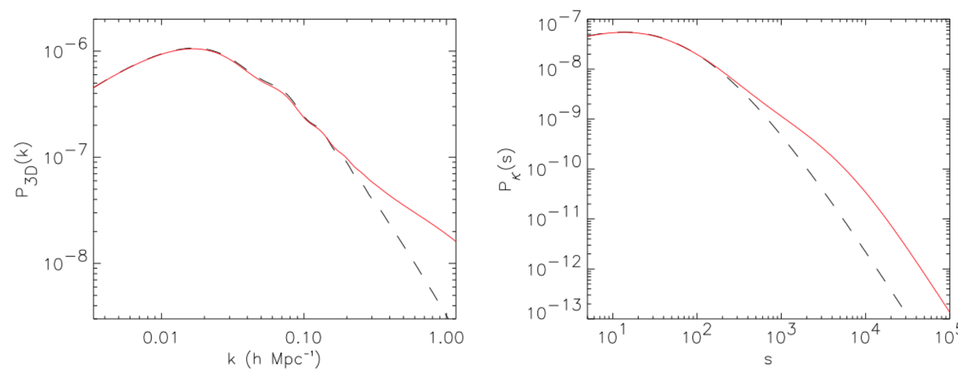
- The convergence and shear spectra are the same. In Fourier space, they only differ by a phase (Kaiser-Squires):

$$\tilde{\gamma}(\boldsymbol{\ell}) = \left(\frac{\ell_\ell^2 - \ell_2^2}{|\boldsymbol{\ell}|^2} + 2\mathrm{i}\frac{\ell_1 \ell_2}{|\boldsymbol{\ell}|^2} \right) \tilde{\kappa}(\boldsymbol{\ell}) = \mathrm{e}^{2\mathrm{i}\phi_\ell} \tilde{\kappa}(\boldsymbol{\ell})$$

The lensing spectra can be written in terms of the density field power spectrum (Kaiser 1992) :

$$P^{\kappa\kappa}(\ell) = P^{\gamma\gamma}(\ell) = \frac{9H_0^2\Omega_m^2}{4c^2} \int_0^{\chi_h} d\chi \frac{g^2(\chi)}{a^2(\chi)} P_\delta \left(\frac{\ell}{d_A(\chi)}, \chi(z) \right)$$

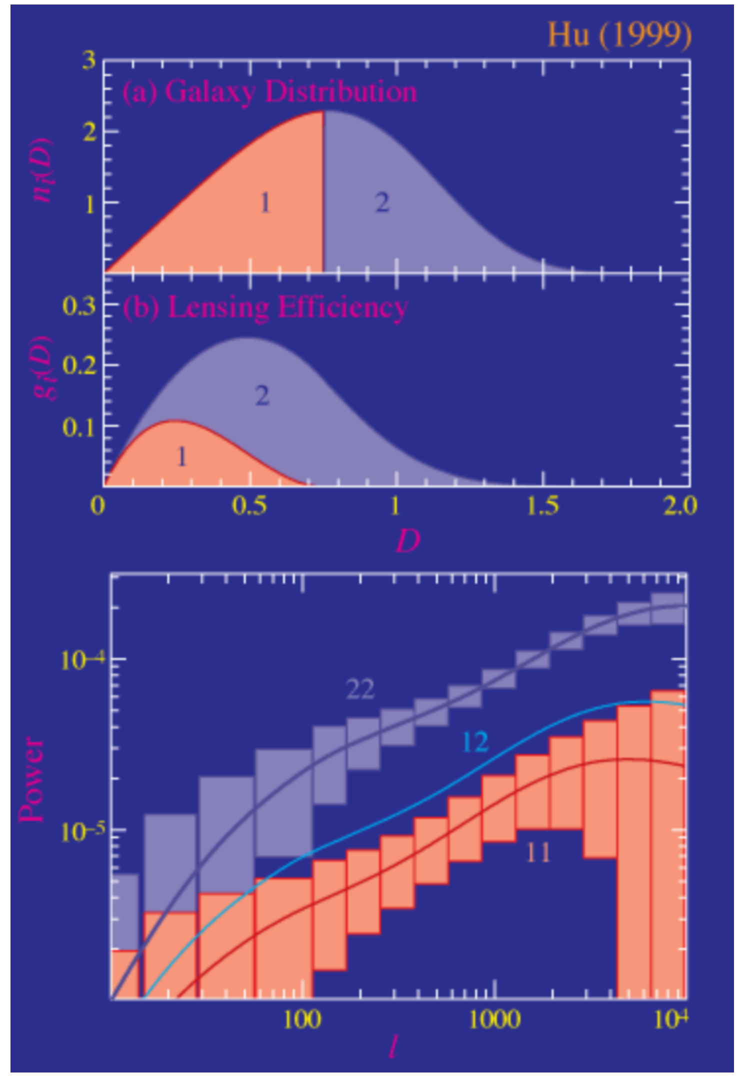
g function: “lensing efficiency”



Weak lensing applications: correlation functions

Photometric surveys such as DES and LSST will measure redshifts of hundreds of millions and billions (10^9) of galaxies

-Divide the survey into multiple redshift bins.
-Low-redshift bins: lensed by structures very near to us: non-linear matter P(k)
-High-redshift bins will be lensed by structures over a wide range of redshift.
-We can map out 3D distributions of dark matter: **tomography!**
-The third dimension involves distance and cosmic time
-Sensitive to matter power spectrum today and also to its evolution over the history of the universe, and the expansion history of the universe during that time. This tells us about dark energy!!



Weak lensing applications: correlation functions

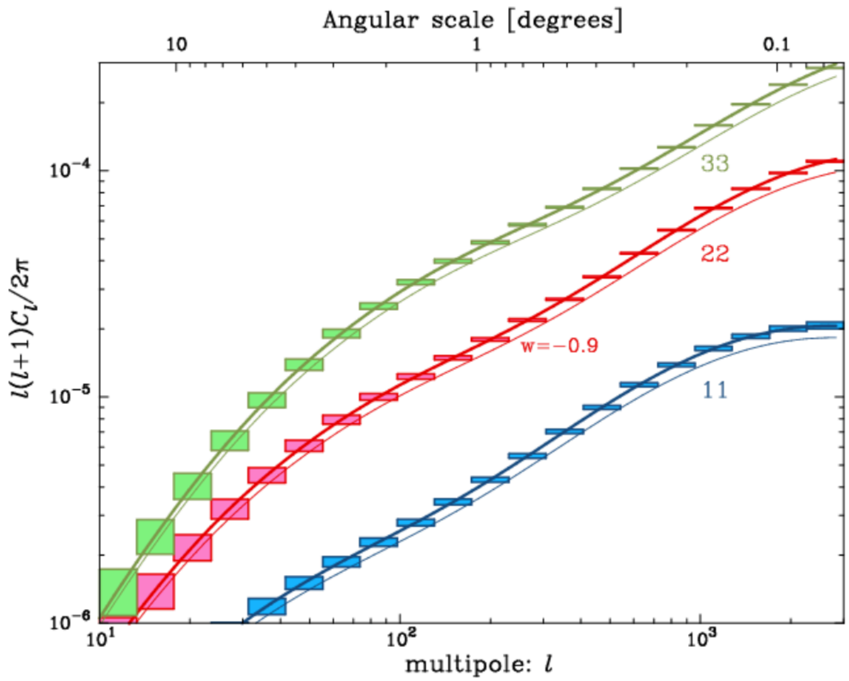
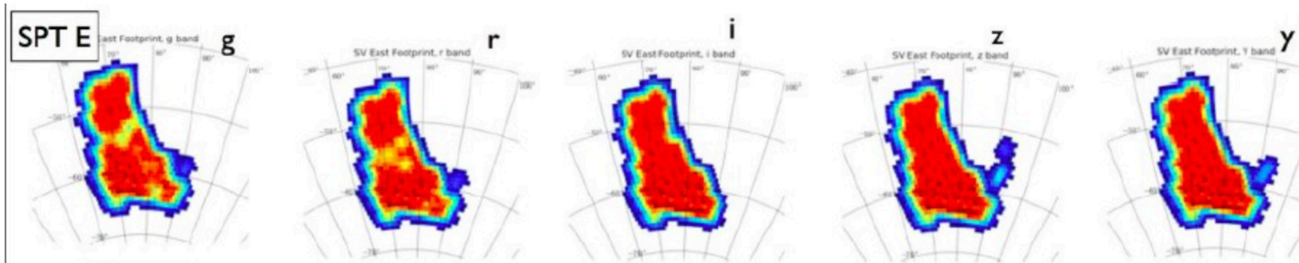


Figure 14.3: The lensing power spectra constructed from galaxies split into three broad redshift bins: $z < 0.7, 0.7 < z < 1.2$, and $1.2 < z < 3$. The solid curves are predictions for the fiducial ΛCDM model and include nonlinear evolution. The boxes show the expected measurement error due to the sample variance and intrinsic ellipticity errors (see text for details). The thin curves are the predictions for a dark energy model with $w = -0.9$. Clearly such a model can be distinguished at very high significance using information from all bins in ℓ and z. Note that many more redshift bins are expected from LSST than shown here, leading to over a hundred measured auto- and cross-power spectra.

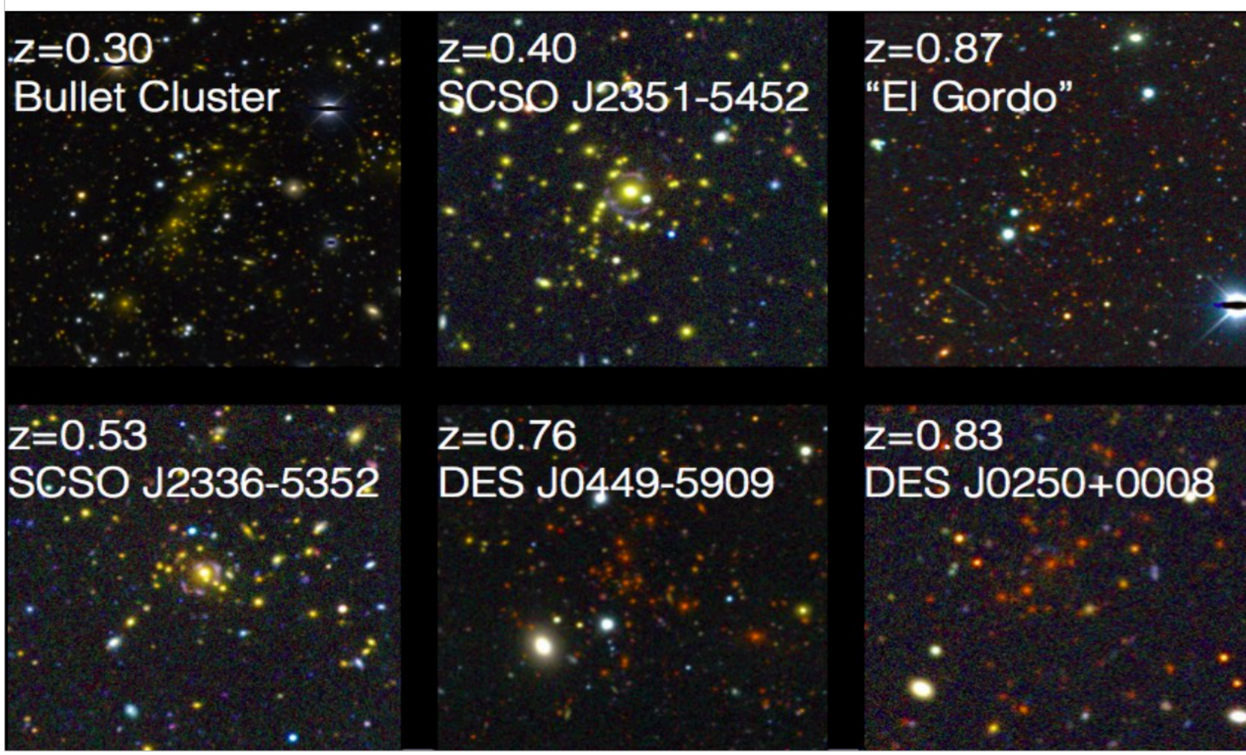
LSST Science Book

Weak lensing applications: clusters

Clusters from SV region



Approximately 125 sq-deg after cuts



RedMaPPer algorithm (Rykoff et al., 2014): find over densities in color and real space.

Weak lensing applications: clusters

Cluster Cosmology in 3 Easy Steps

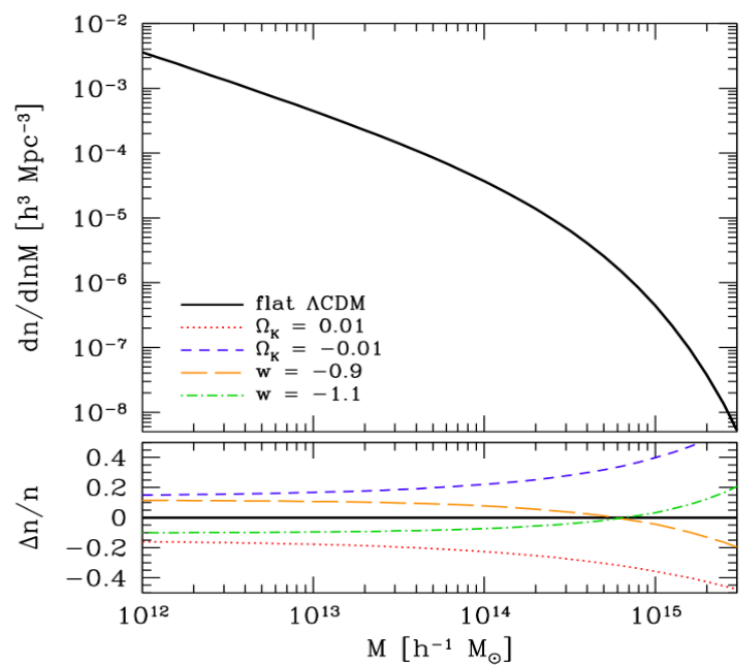
1. Find massive halos (i.e. clusters) using an observable signature, e.g. X-rays, or galaxy counts.
2. Determine the relation between the observable signature X and the mass M, i.e. P(X|M).
3. Use P(X|M) and N_{theory}(M) to predict the observed abundance of clusters N_{obs}(X).

E. Rozo

Weak lensing applications: clusters

- Clusters: largest structures to have undergone gravitational collapse
- We can predict (analytically and numerically) the number of clusters per unit volume per unit mass:

$$dN/(dM\,dV) \longrightarrow \text{Mass function}$$



Credit: Weinberg et al. 2012

- At high M, sensitive to amplitude of matter perturbations

- Detection: x-rays, SZ effect
  - Problem: do not measure mass directly

- Weak lensing allows the direct determination of masses, providing a calibration for the mass-observable relation

Main takeaway: Measurements of the halo abundance constrain growth of matter fluctuations.

Weak lensing applications: clusters

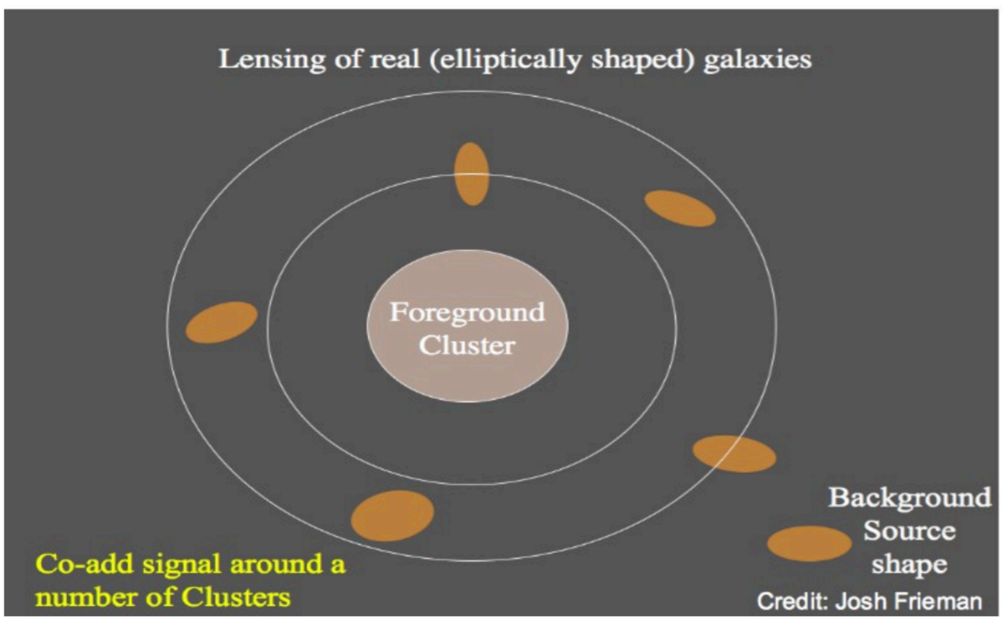
- Cluster-galaxy or galaxy lensing (cross-correlation lensing)

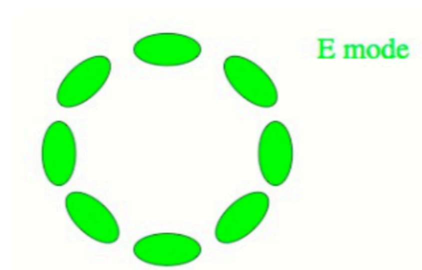
Cross-correlation of lens position and source galaxy shapes

Mean tangential shear: $\gamma_T(R) \times \Sigma_{\rm crit} = \bar{\Sigma}(< R) - \bar{\Sigma}(R) \equiv \Delta\Sigma$

Geometric factor:

$$\Sigma_{\rm crit}^{-1} = \frac{4\pi G D_{LS} D_L}{c^2 D_S}$$





Stack the signal from many clusters.

Weak lensing applications: clusters

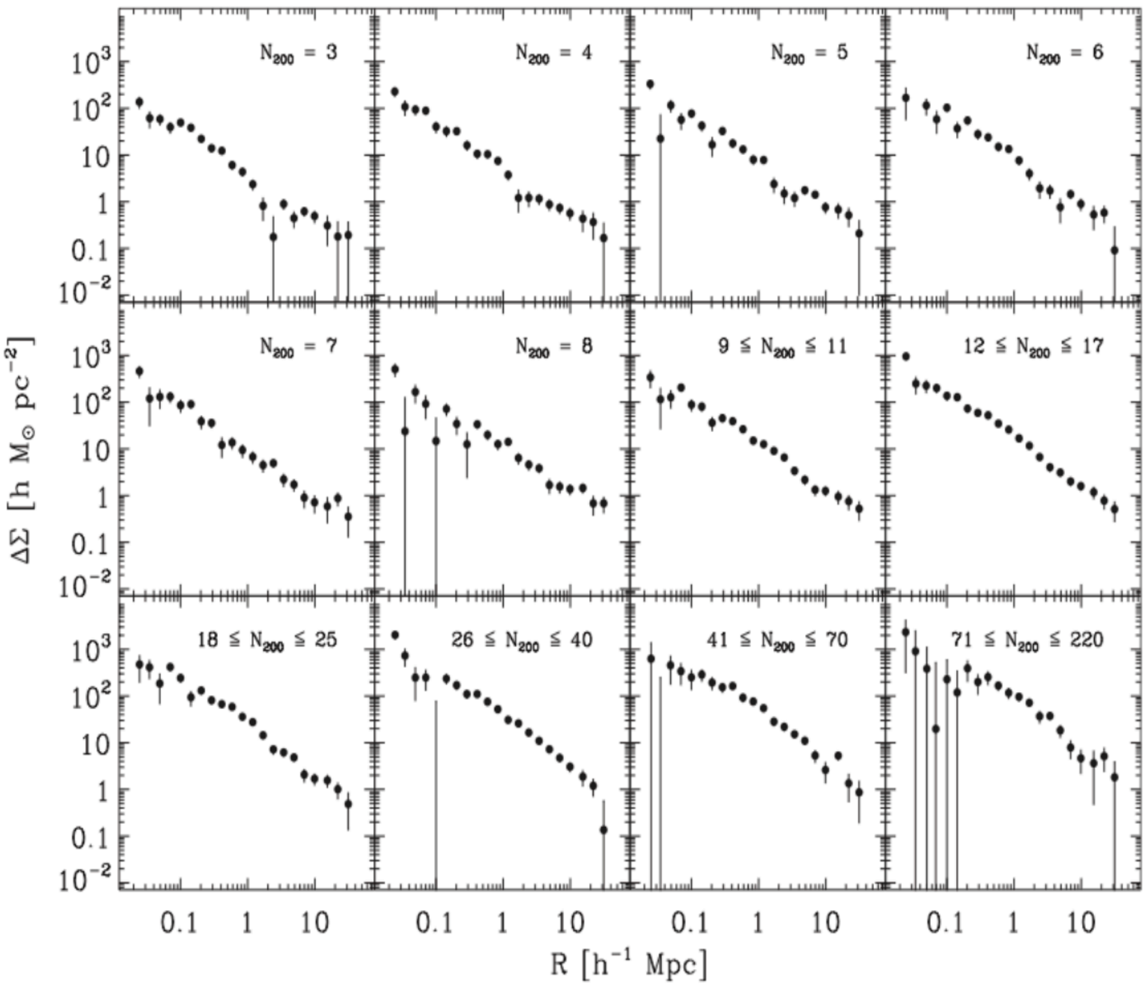


Figure 8. $\Delta\Sigma$ from 25 to 30 h^{-1} Mpc in 12 bins of N_{200}, the number of galaxies (> 0.4 L_*) within r_{200}^{gals}. The signal measured around random points is subtracted from these profiles (see Figure 6). The correction for clustering of sources with the lenses is also applied (see Figure 7). The errors are from jackknife re-sampling.

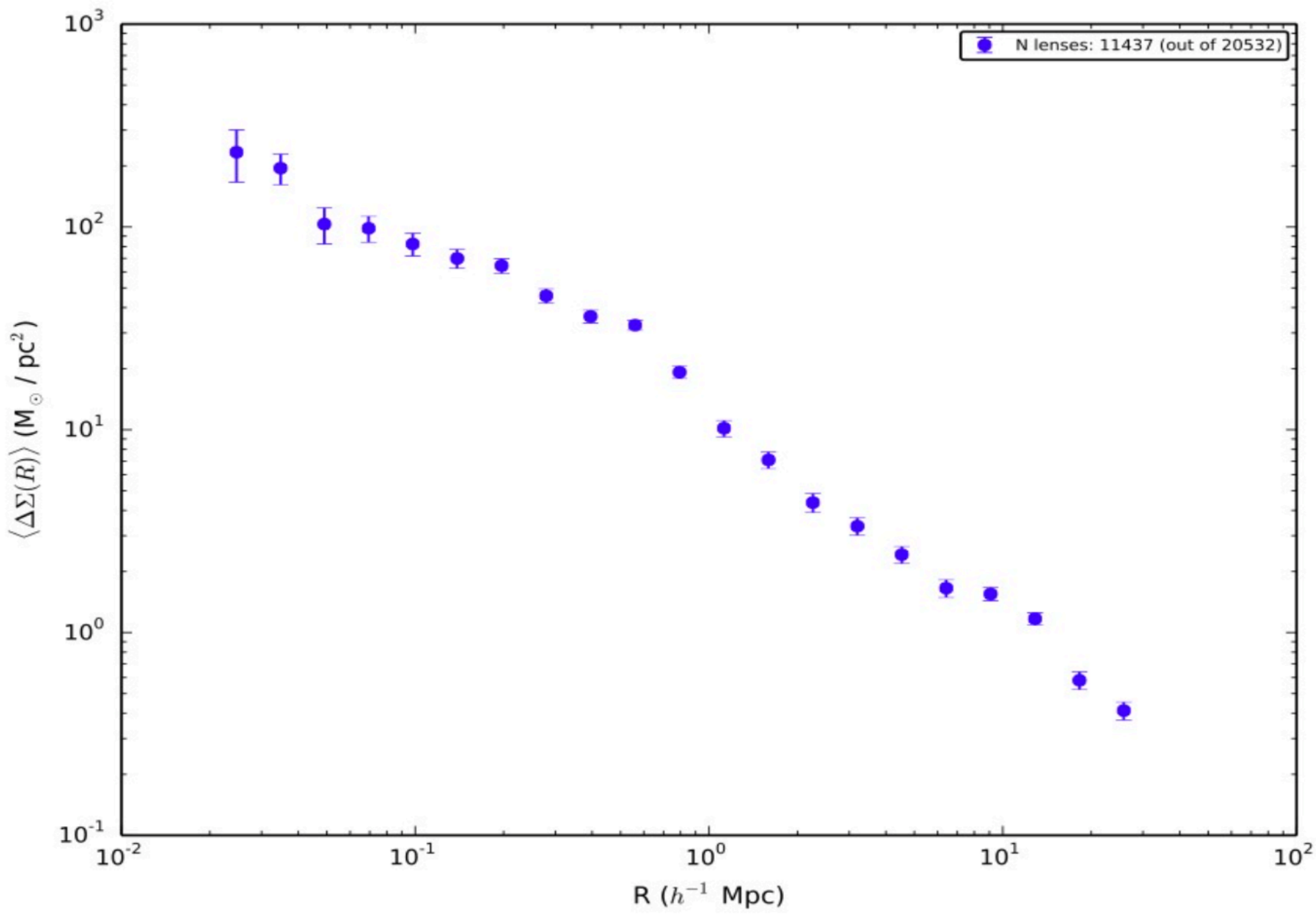
For SDSS:

Sheldon et al., 2009

DES: will allow the study of redshift evolution

Weak lensing applications: clusters

Preliminary result using early DES data!



Sources:
ngmix (E. Sheldon)

Lenses: RedMapper

Weak Lensing Systematics

-Weak Lensing *"...is likely to be the most powerful individual technique, and also the most powerful component in a multi-technique program..."* to learn about Dark Energy **if systematics** can be controlled (DETF 06, *Albrecht et al.*)

- **PSF measurement, modeling, and interpolation**
- **Multiplicative (calibration) errors in the signal (shear)**
- **Shear Additive errors**
- **Instrumental signatures**
- Photo-z biases
- Intrinsic Alignments: physically close galaxies
- Errors in theory (non-linear power spectrum predictions)

References

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