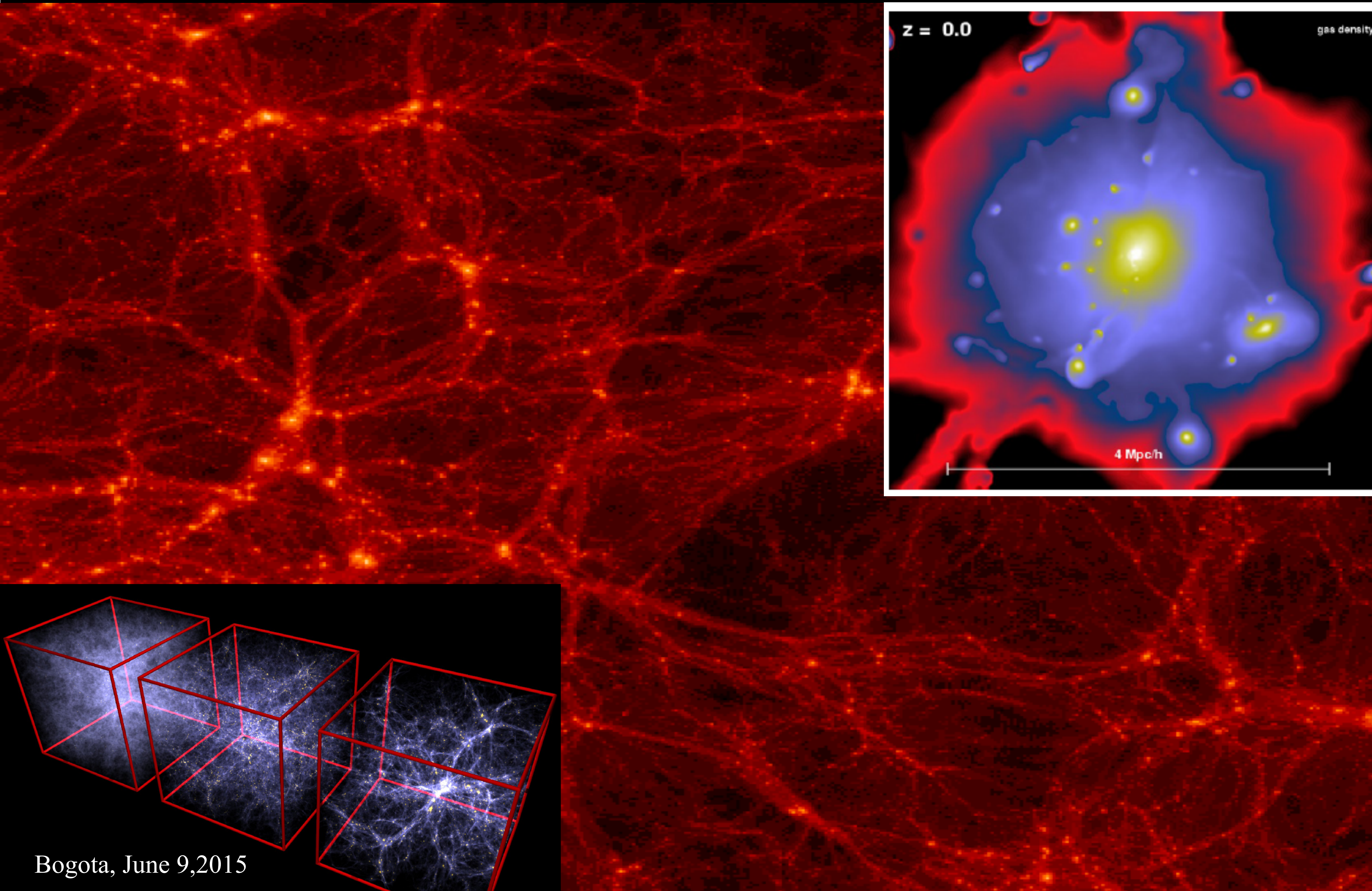
Numerical Cosmology



Bogota, June 9,2015

Overview

- Cosmological simulations
  - – - why do we need simulations
  - – - observational background
  - – - ingredients of numerical simulations
- Numerical techniques
  - – - initial conditions
  - – - evolving the initial conditions
  - – - analysing simulations
  - – - parallelization of codes
- State-of-the-art
  - – - Large volume simulations
  - – - Zoomed initial conditions: individual haloes, Local Group volumes

- Observational constraints and input for cosmological simulations

The need of numerical simulations

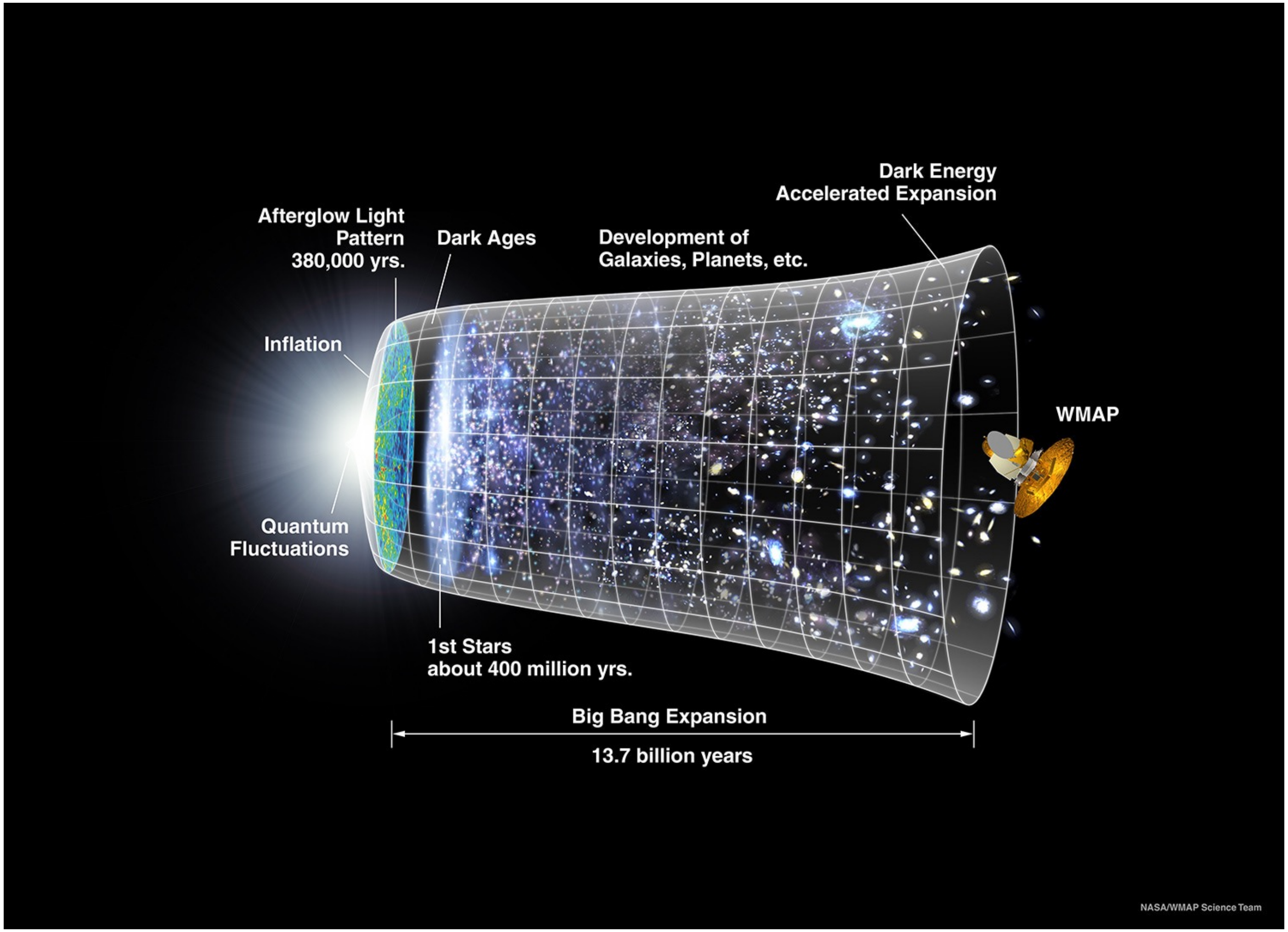
Numerical simulations are used to proof or disproof theoretical predictions because laboratory experiments

- are impossible (astrophysics)
- are too expensive
- are too time-consuming
- ...

- They are widely used in many research areas of physics, such as statistical physics, plasma physics, astrophysics, fluid dynamics.

Simulations of cosmic evolution

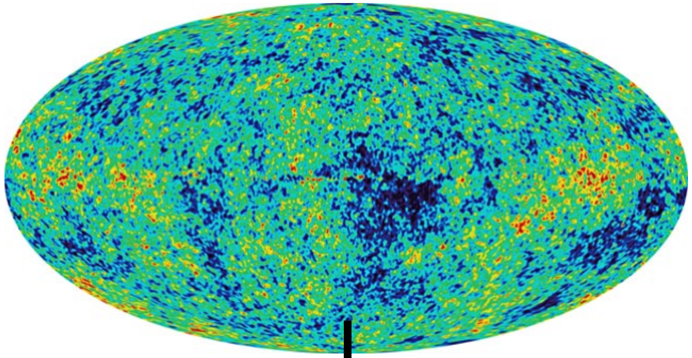
- Growth of structure in the universe is highly non-linear, multi-scale, free from any simplifying symmetries; not possible to follow analytically.

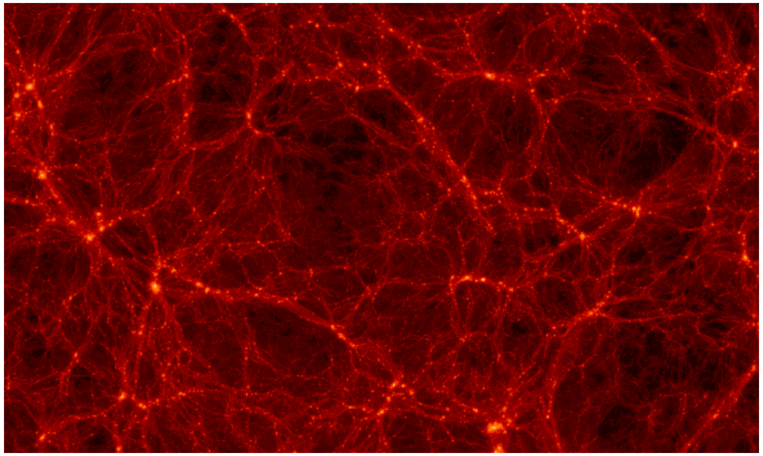


Ingredients of numerical simulations

- Initial conditions

(at some convenient time t_0)





Equations governing evolution

→ Gravity for collisionless component

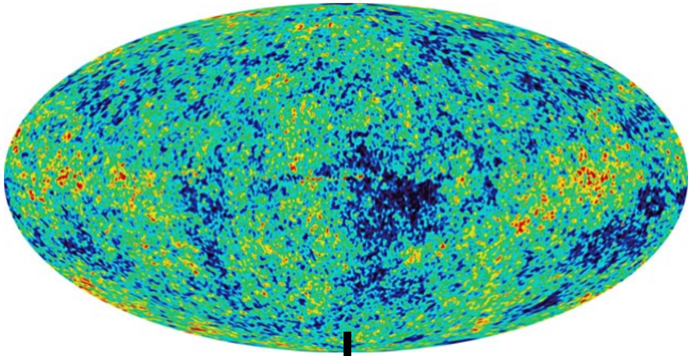
→ Fluid dynamics for collisional matter

→ Cosmological evolution of the universe

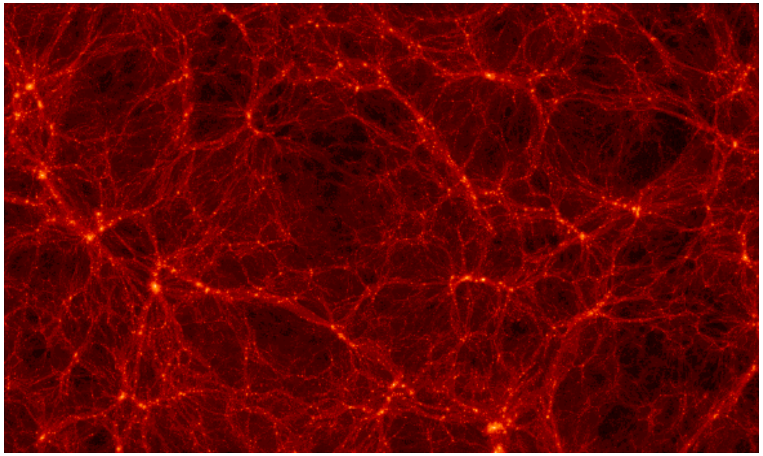
Ingredients of numerical simulations

- Initial conditions

(at some convenient time t_0)



The power spectrum of fluctuations infered from the observed CMB fluctuations



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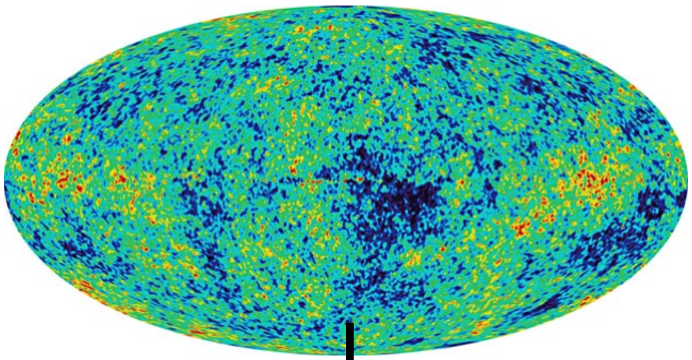
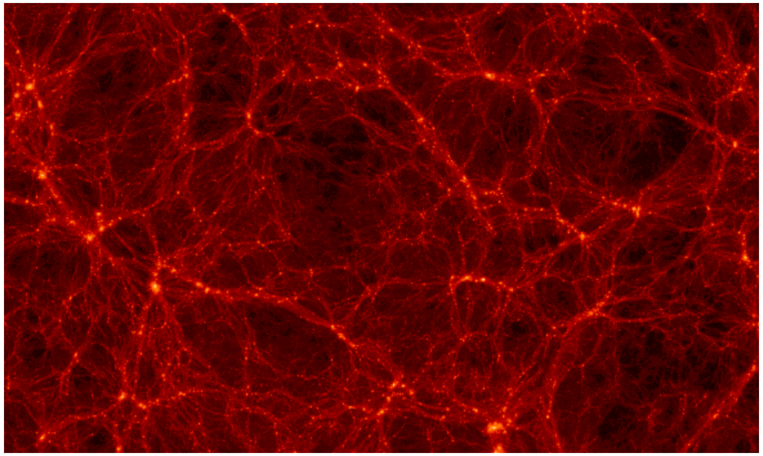




Equations governing evolution

Depend on cosmological model:

- matter content (baryons, dark matter)

- energy content (dark energy)

- Observations:
  - Spectrum of initial perturbations
  - Matter/energy content of the universe and cosmological parameters

The Cosmic Microwave Background

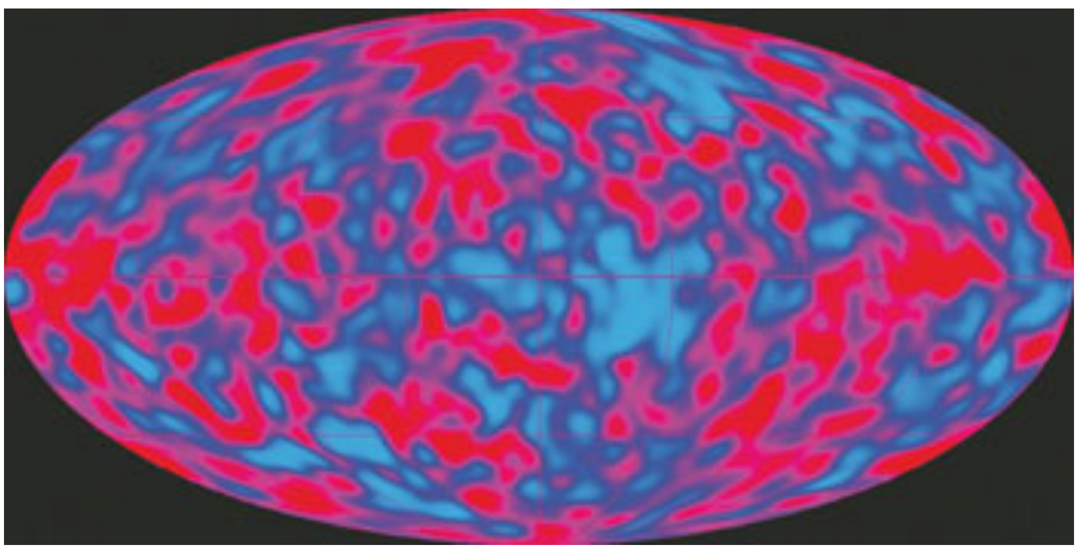
–CMB spectrum:

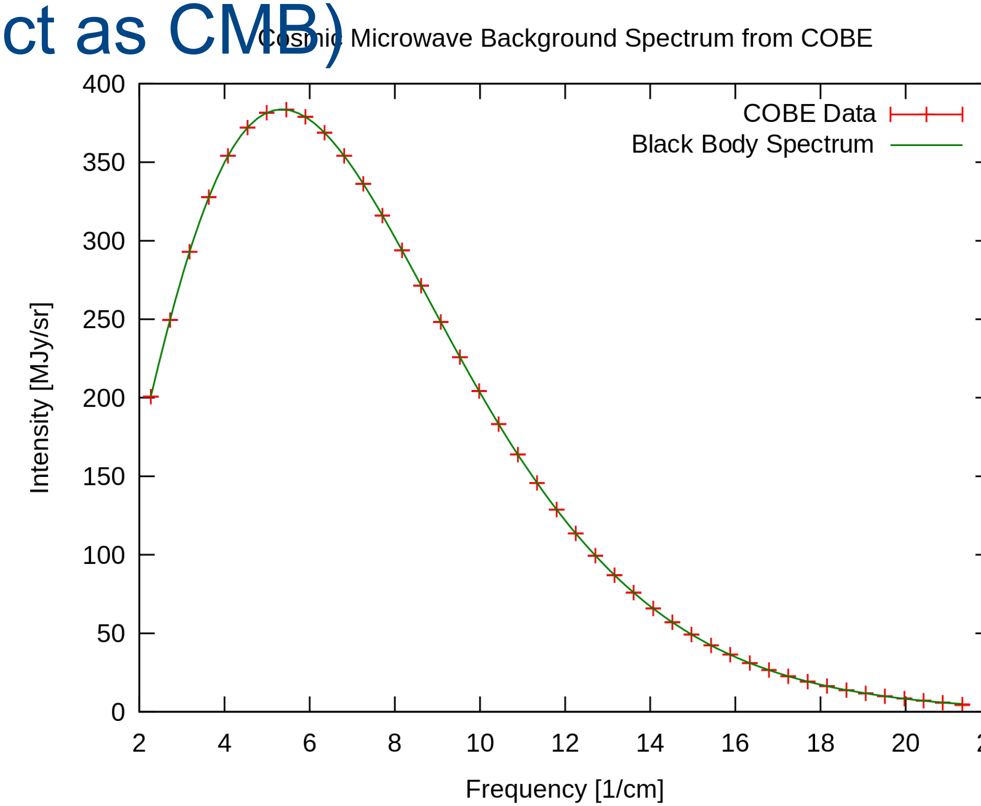
–→ isotropic to 1 part in 10000, fluctuations $\Delta T/T \sim 10^{-5}$

–→ has a perfect blackbody spectrum, T=2.73 K

–→ spectrum at recombination, 300000 years after Big Bang

(proof of Big Bang theory, no other blackbody spectrum in nature or laboratory as perfect as CMB)





COBE satellite, 1992

The Cosmic Microwave Background

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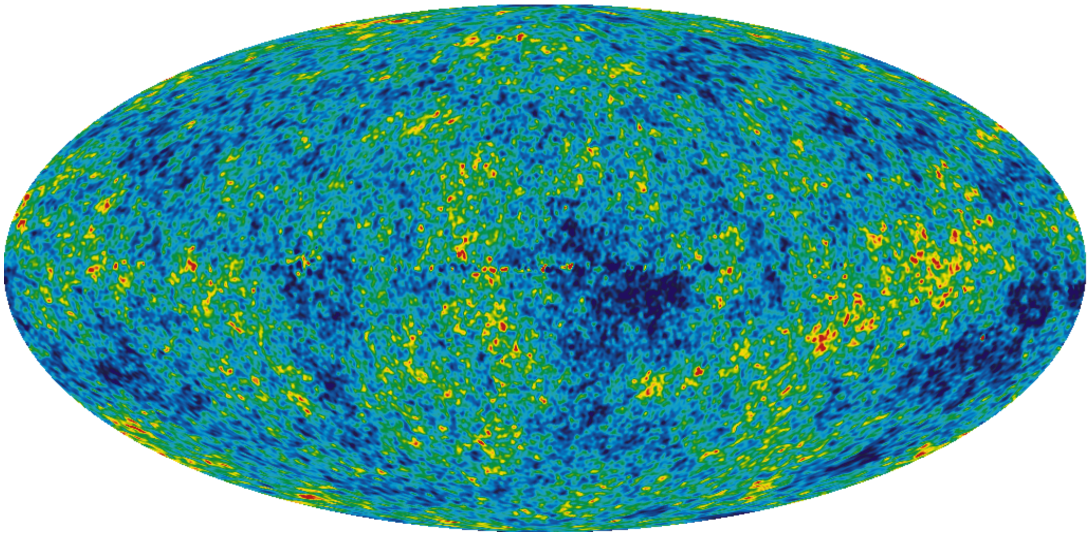
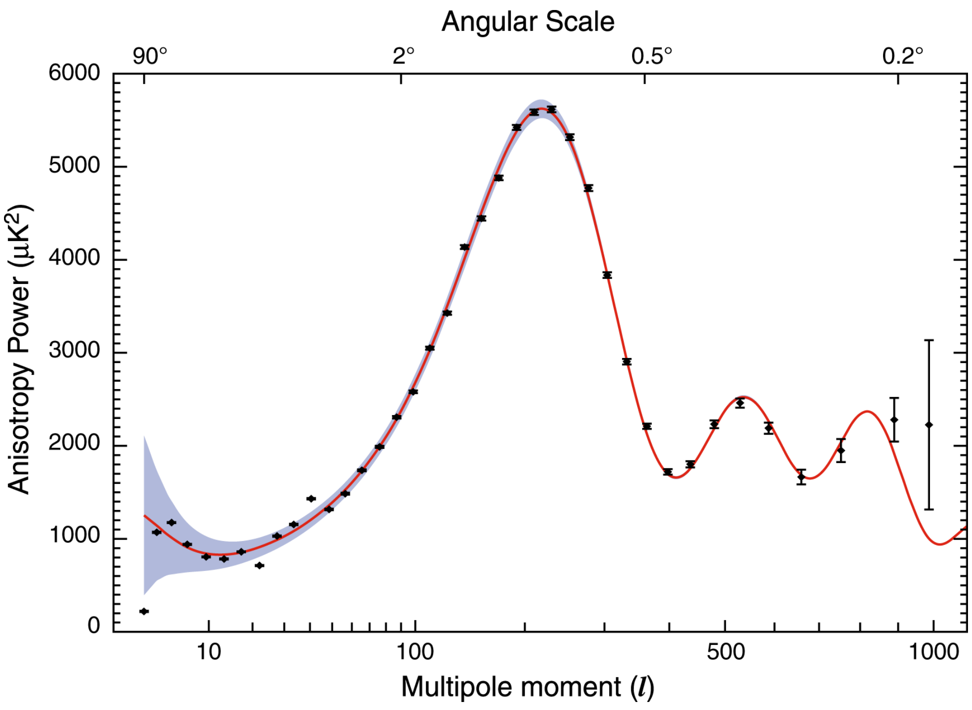


- WMAP, 2006



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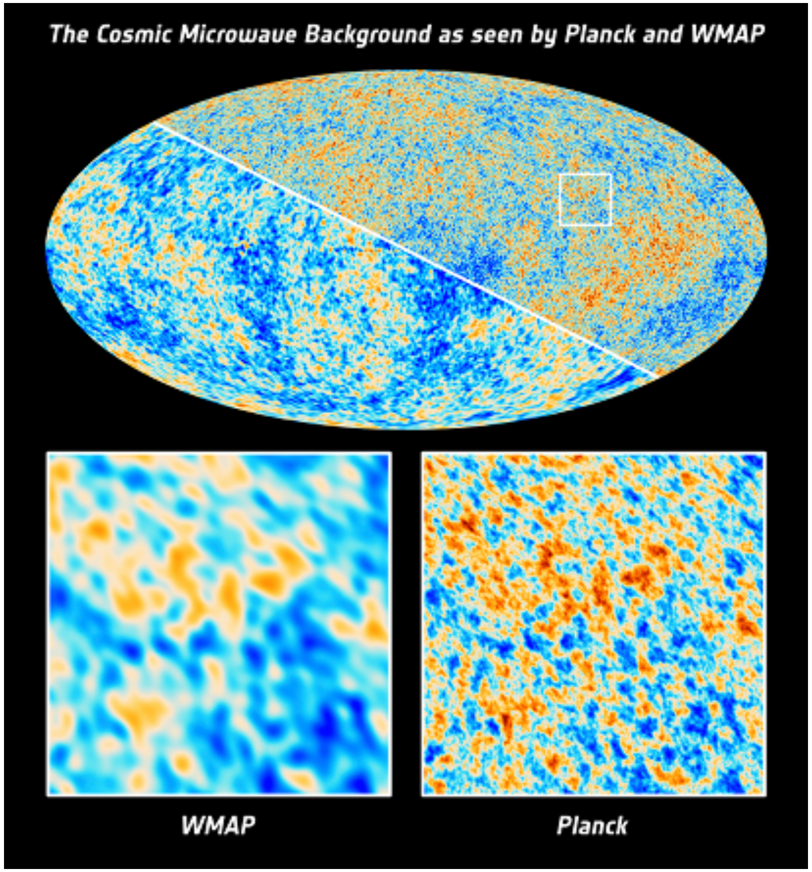
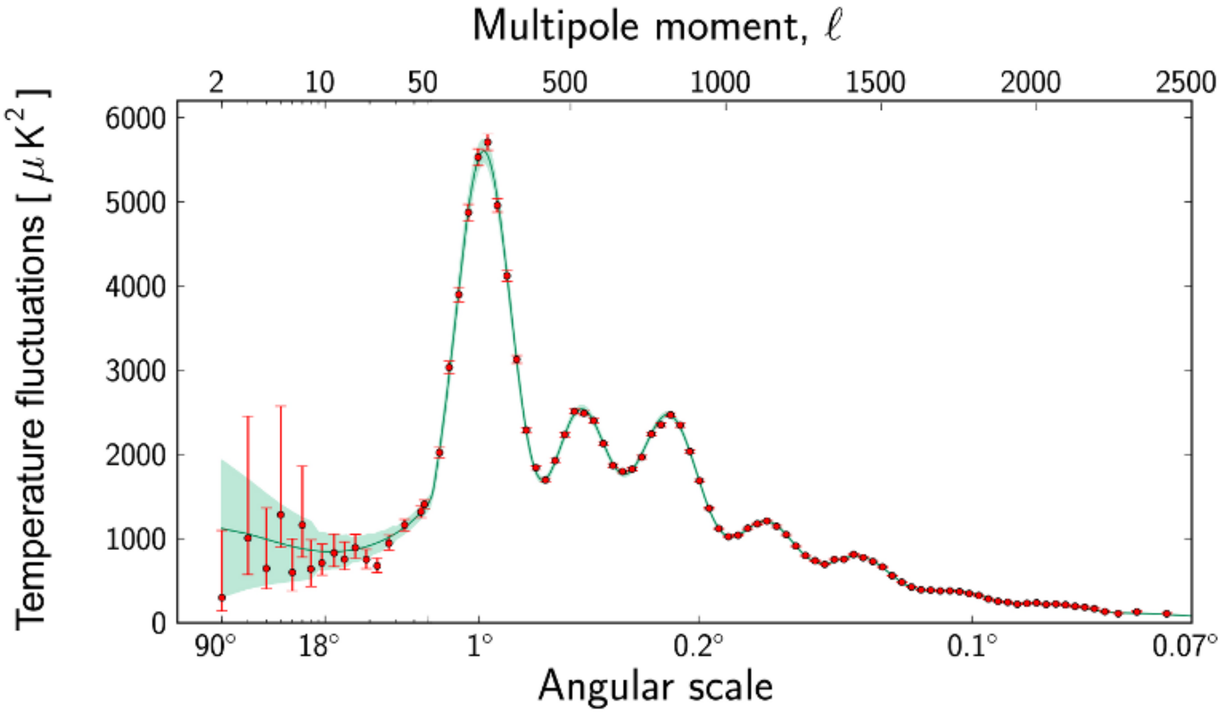
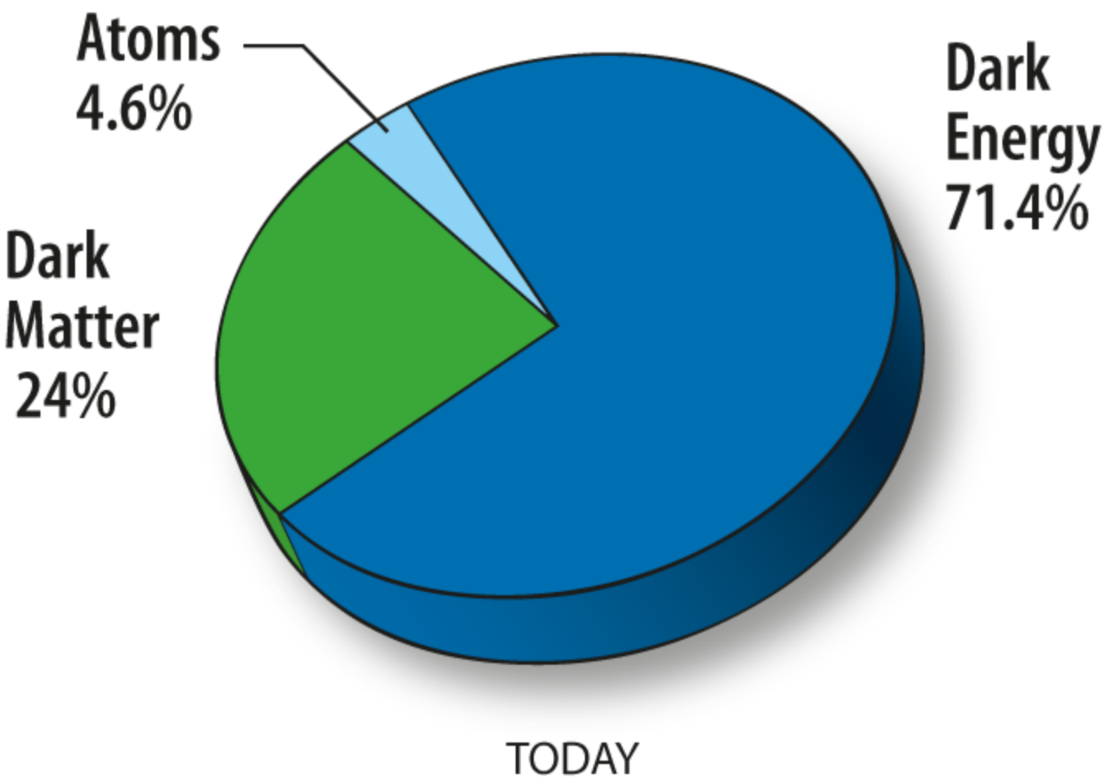




- Planck, 2013

Matter-Energy content of the Universe

Accurate measurements of the CMB allows to estimate the fundamental parameters of the Big Bang Model, including matter/energy content



→ About 70% of the matter-energy content of the universe is “dark energy” (TODAY)

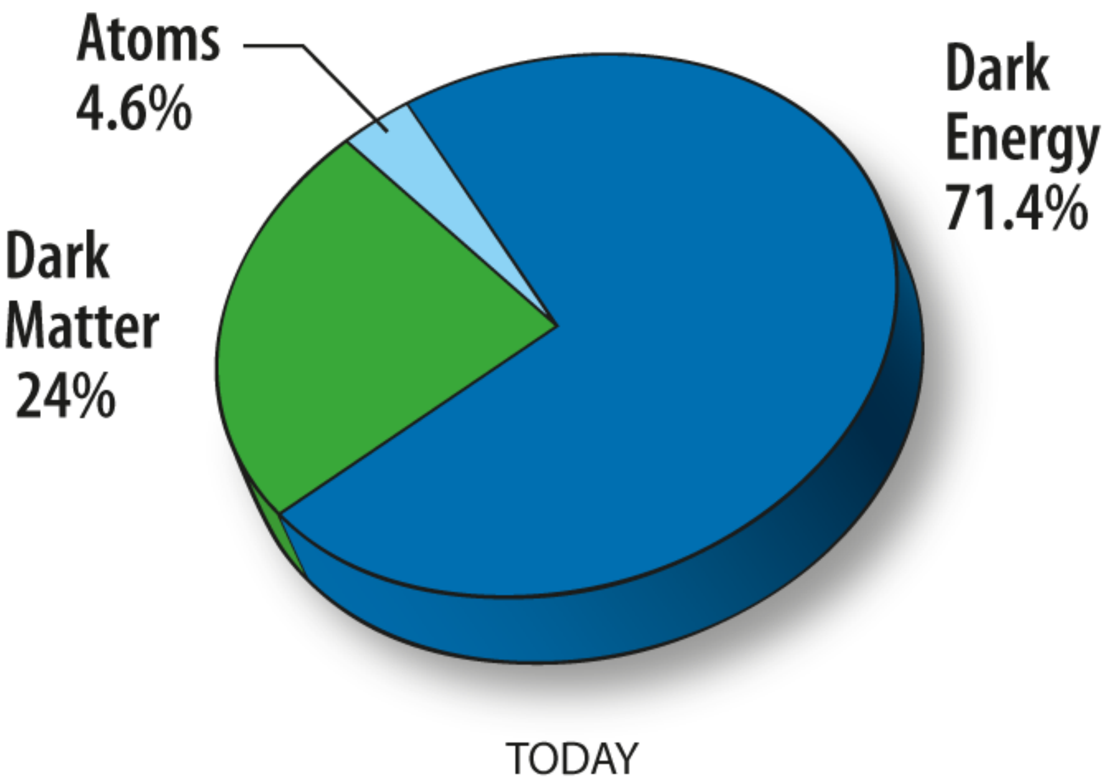
→ Only ~30% is matter (TODAY)

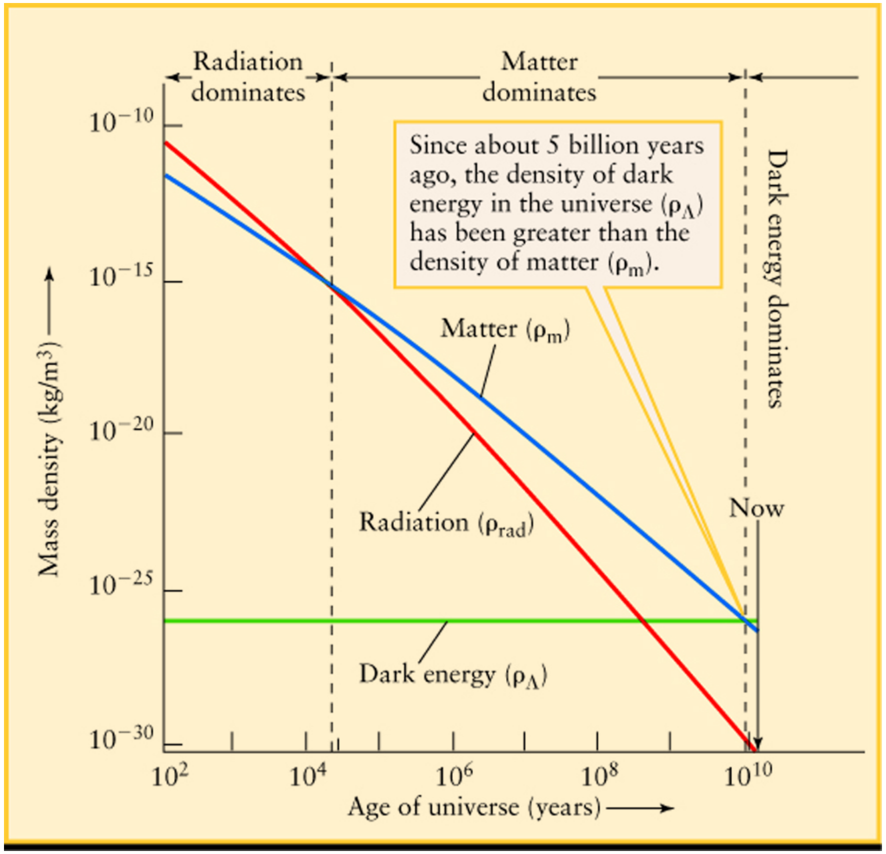
→ About 85% of all matter is “dark matter”

→ Only 15% of all matter, or equivalently ~5% of matter-energy, are baryons

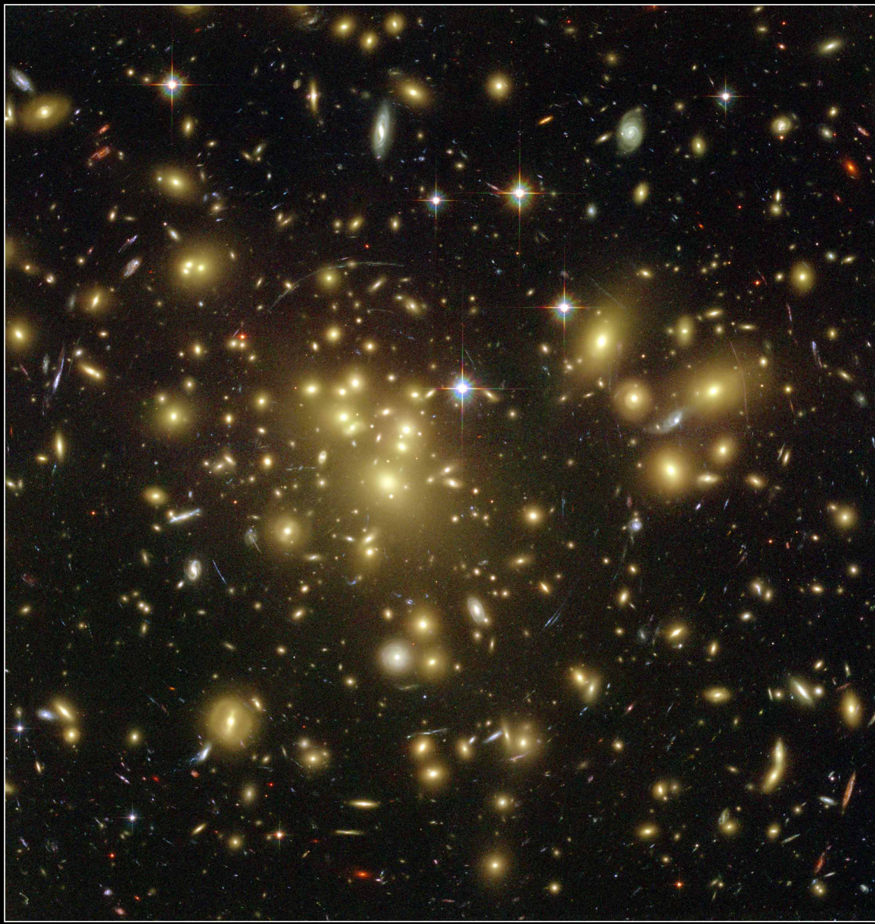
Matter-Energy content of the Universe

–Accurate measurements of the CMB allows to estimate the fundamental parameters of the Big Bang Model, including matter/energy content





Dark Matter: "observations"



Galaxy Cluster Abell 1689
Hubble Space Telescope • Advanced Camera for Surveys

NASA, N. Benitez (JHU), T. Broadhurst (The Hebrew University), H. Ford (JHU), M. Clampin(STScI), G. Hartig (STScI), G. Illingworth (UCO/Lick Observatory), the ACS Science Team and ESA
STScI-PRC03-01a

–→ Background galaxies get distorted and magnified by massive galaxy cluster → Gravitational lensing effect predicted by Einstein's General Relativity theory

–→ Predicted gravitative mass is much larger than the visible light

–→ evidence of the presence of a matter component that is "dark"

–→ Rotation curves of spiral galaxies are also an indication of the presence of dark matter (Lecture3)

Dark Matter: what do we know?

–→ About 85% of all matter must be dark

» ↓

–Dark matter is the main driver of gravitational evolution → responsible for large-scale structure

–→ Galaxies and galaxy clusters live within dark matter haloes (~10 times more massive and extended than galaxies themselves)

–→ Probably, not all dark matter haloes host galaxies (more about this on Thursday)

–→ Dark matter must be made of:

– → weakly interacting particles (WIMPs)

– → non-baryonic

– → more or less "cold"

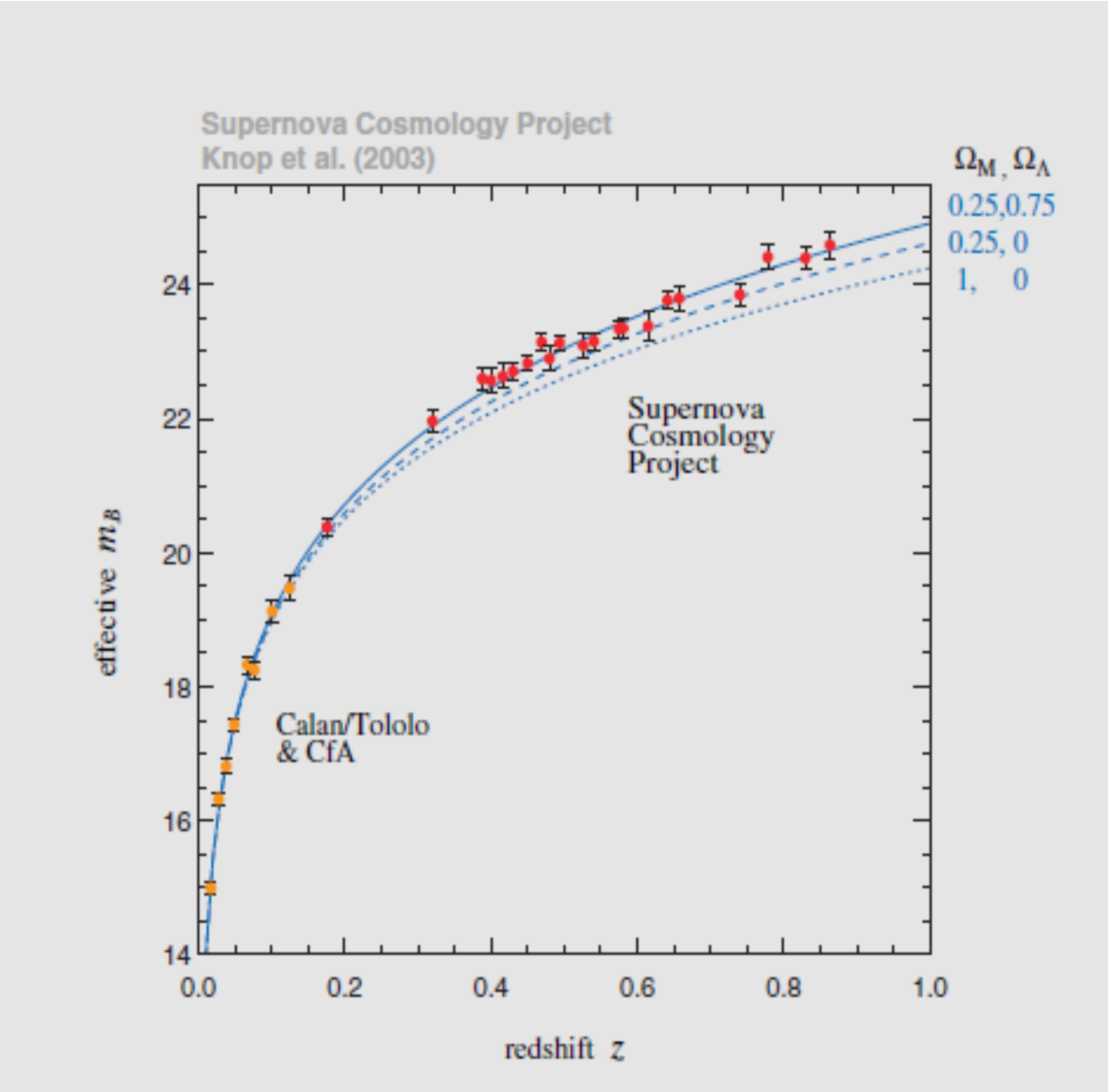
Dark Energy: “observations”

–Apparent magnitude vs redshift (measure of distance vs magnitude) of distant supernovae

–High redshift SNe are fainter than expected in standard (Λ=0) cosmology

–→ there must be an

– “accelerating force”: dark energy



Dark Energy: what do we know?

—→ About 70% of the universe is made of "dark energy", whose nature is unknown

—→ The dark energy is responsible for today's acceleration in the expansion of the universe

—→ Experiments are planned to measure the evolution of dark energy

– → simplest explanation: cosmological constant (= vacuum energy = additional constant term in Einstein's field equation)

– → some still unknown (scalar) field similar to the (still unknown) field which drives inflation

– → something else

Cosmological parameters (Planck, 2013)

- Dark matter density $\Omega_{DM} = 0.257$
- Dark energy density $\Omega_{\Lambda} = 0.693$
- Baryon density $\Omega_b = 0.048$
- Spatially flat universe $\Omega_{DM} + \Omega_{\Lambda} + \Omega_b = 1$
- Hubble parameter $h = 0.679$ (Thursday)
- Power spectrum:
- Shape $n = 0.96$
- Normalization $\sigma_8 = 0.823$

Cosmological parameters

Comparison of *Planck*-only and *WMAP*-only Six-Parameter ΛCDM Fits[a]

Parameter	*Planck* ("CMB+Lens")	*WMAP* (9-year)	Difference value	Difference *WMAP* σ
$\Omega_b h^2$	0.02217 ± 0.00033	0.02264 ± 0.00050	-0.00047	0.9
$\Omega_c h^2$	0.1186 ± 0.0031	0.1138 ± 0.0045	0.0048	1.1
Ω_Λ	0.693 ± 0.019	0.721 ± 0.025	-0.028	1.1
τ	0.089 ± 0.032	0.089 ± 0.014	0	0
t_0 (Gyr)	13.796 ± 0.058	13.74 ± 0.11	56 Myr	0.5
H_0 (km s^{-1}Mpc^{-1})	67.9 ± 1.5	70.0 ± 2.2	-2.1	1.0
σ_8	0.823 ± 0.018	0.821 ± 0.023	0.002	0.1
Ω_b	0.0481[b]	0.0463 ± 0.0024	0.0018	0.7
Ω_c	0.257[b]	0.233 ± 0.023	0.024	1.0

Baryon density

Cold Dark Matter density

Dark Energy or Cosmological Constant

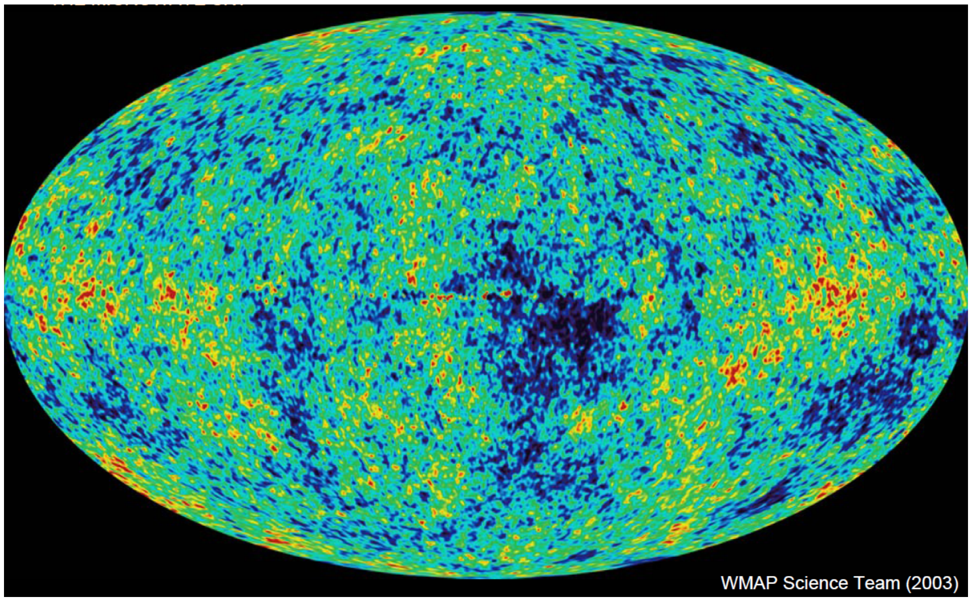
normalisation (fluctuations at 8 Mpc/h)

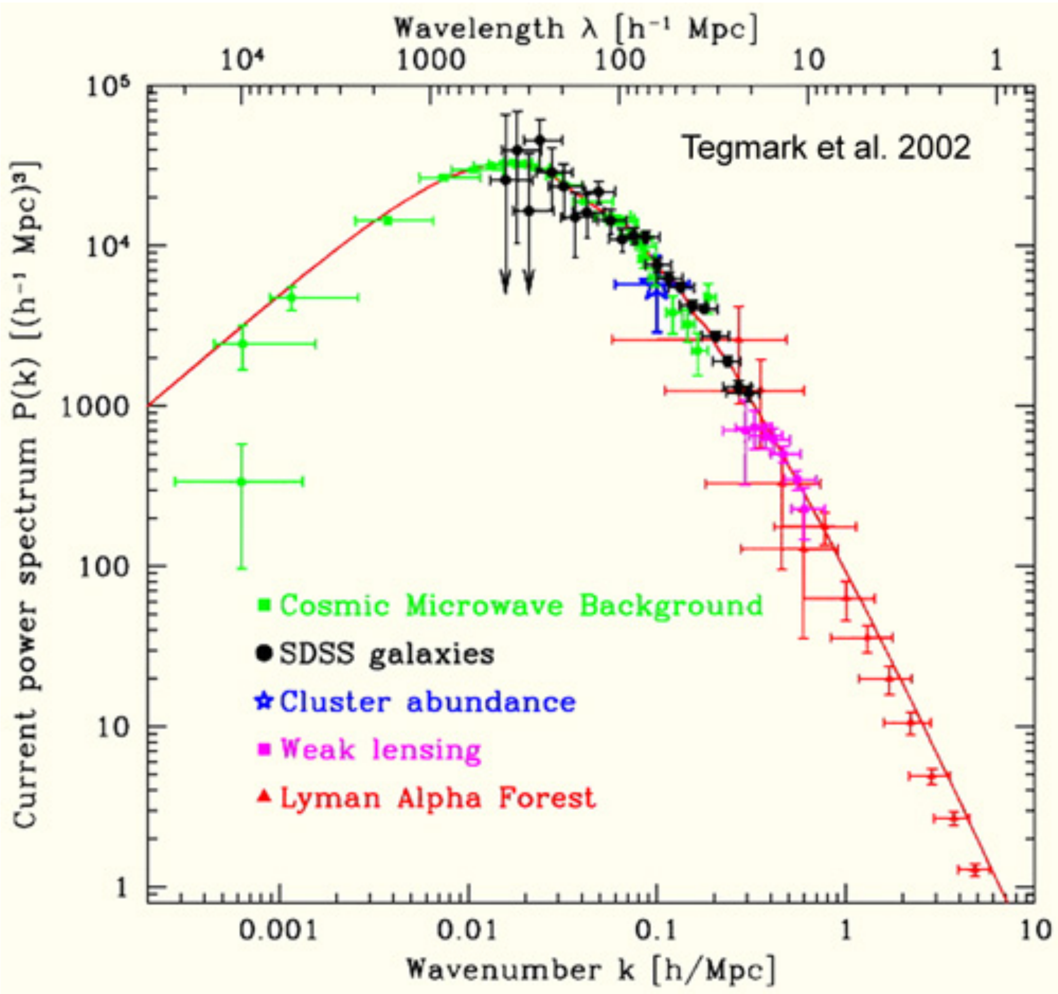
Hubble constant

- Ingredients of cosmological simulations:
  - summary

Initial conditions

- Initial conditions of cosmological structure formation are known (CMB)





Why is the universe spatially flat?

Why is the initial power spectrum almost flat?

Where do the initial fluctuations come from?

(a discussion at the white board)

Matter-energy content of the universe

–Dark Energy

- homogeneous
- negative pressure
- solution of the Friedmann equation for a given equation of state $p = p(\varepsilon)$ (simplest case $p = -\varepsilon = \text{const}$ (cosmological constant Λ))

Dark Matter

- collisionless
- Φ: potential
- $\rho_{\rm tot}$:matter density
- equation of motion: $\frac{d\mathbf{v}}{dt} = -\nabla\Phi$
- Poisson equation $\Delta\Phi = 4\pi G\rho_{\rm tot}$
- Newtonian dynamics, to be solved in the expanding cosmological coordinate system

Matter-energy content of the universe

–Baryons: gas

- collisional
- Euler equation: $\frac{d\mathbf{v}}{dt} = -\nabla\Phi - \frac{1}{\rho}\nabla P$
- equation of state $p = p(\rho)$
- "sub-grid" physics not (yet) well modelled
    - star formation
    - chemical evolution
    - formation of super-massive black holes and their influence on galaxies
    - magnetic fields
    - ...

--Baryons: stars

- collisionless

- A short and incomplete historical overview over the first simulations

1941: first N-body simulation by Erik Holmberg (Uppsala)

Flux at distance r of a point source:

$$F=L/4\pi\ r^2$$

N photo detectors
in a plane → measure F_n,
then displace detectors
Code complexity: O(N)!!

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME 94 NOVEMBER 1941 NUMBER 3

ON THE CLUSTERING TENDENCIES AMONG THE NEBULAE

II. A STUDY OF ENCOUNTERS BETWEEN LABORATORY MODELS OF STELLAR SYSTEMS BY A NEW INTEGRATION PROCEDURE

ERIK HOLMBERG

ABSTRACT

In a previous paper[1] the writer discussed the possibility of explaining the observed clustering effects among extragalactic nebulae as a result of captures. The present investigation deals with the important problem of whether the loss of energy resulting from the tidal disturbances at a close encounter between two nebulae is large enough to effect a capture. The tidal deformations of two models of stellar systems, passing each other at a small distance, are studied by reconstructing, piece by piece, the orbits described by the individual mass elements. The difficulty of integrating the total gravitational force acting upon a certain element at a certain point of time is solved by replacing gravitation by light. The mass elements are represented by light-bulbs, the candle power being proportional to mass, and the total light is measured by a photocell (Fig. 1). The nebulae are assumed to have a flattened shape, and each is represented by 37 light-bulbs. It is found that the tidal deformations cause an increase in the attraction between the two objects, the increase reaching its maximum value when the nebulae are separating, i.e., after the passage. The resulting loss of energy (Fig. 6) is comparatively large and may, in favorable cases, effect a capture. The spiral arms developing during the encounter (Figs. 4) represent an interesting by-product of the investigation. The direction of the arms depends on the direction of rotation of the nebulae with respect to the direction of their space motions.

I. THE EXPERIMENTAL ARRANGEMENTS

The present paper is a study of the tidal disturbances appearing in stellar systems which pass one another at small distances. These tidal disturbances are of some importance since they are accompanied by a loss of energy which may result in a capture between the two objects. In a previous paper[1] the writer discussed the clustering tendencies among extragalactic nebulae. A theory was put forth that the observed clustering effects are the result of captures between individual nebulae. The capture theory seems to be able to account not only for double and multiple nebulae but also for the large extragalactic clusters. The present investigation tries to give an answer to the important question of whether the loss of energy accompanying a close encounter between two nebulae is large enough to effect a capture.

A study of tidal disturbances is greatly facilitated if it can be restricted to only two dimensions, i.e., to nebulae of a flattened shape, the principal planes of which coincide with the plane of their hyperbolic orbits. In order to reconstruct the orbit described by

[1] *Mt. W. Contr.*, No. 633; *Ap. J.*, **92**, 200, 1940.

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1941: first N-body simulations by Holmberg

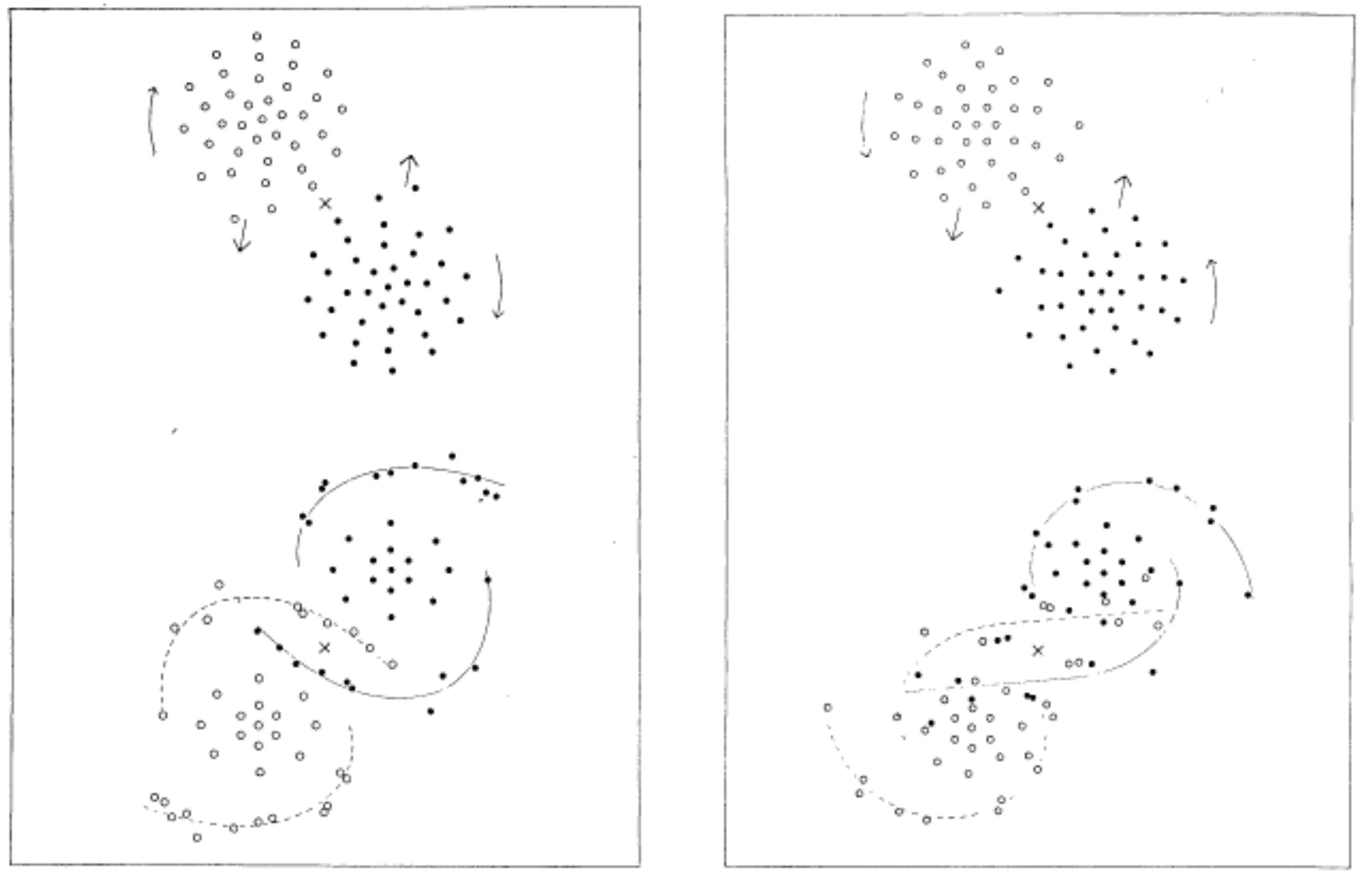
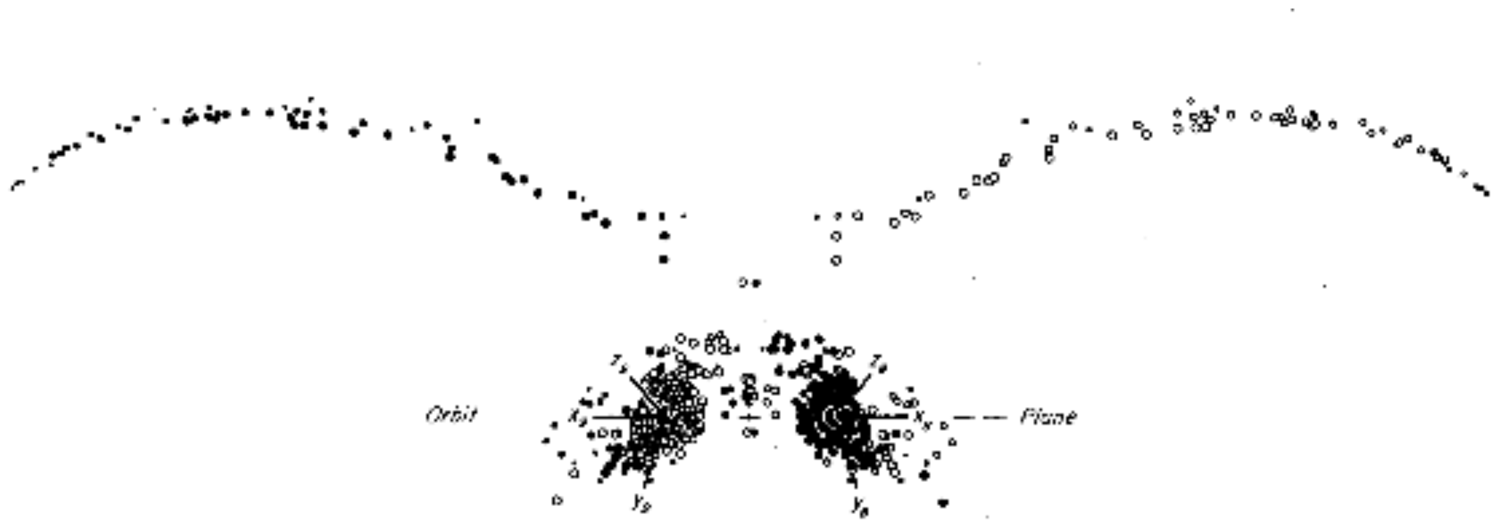


Fig. 4a Fig. 4b

Fig. 4a.—Tidal deformations corresponding to parabolic motions, clockwise rotations, and a distance of closest approach equal to the diameters of the nebulae. The spiral arms point in the direction of the rotation.

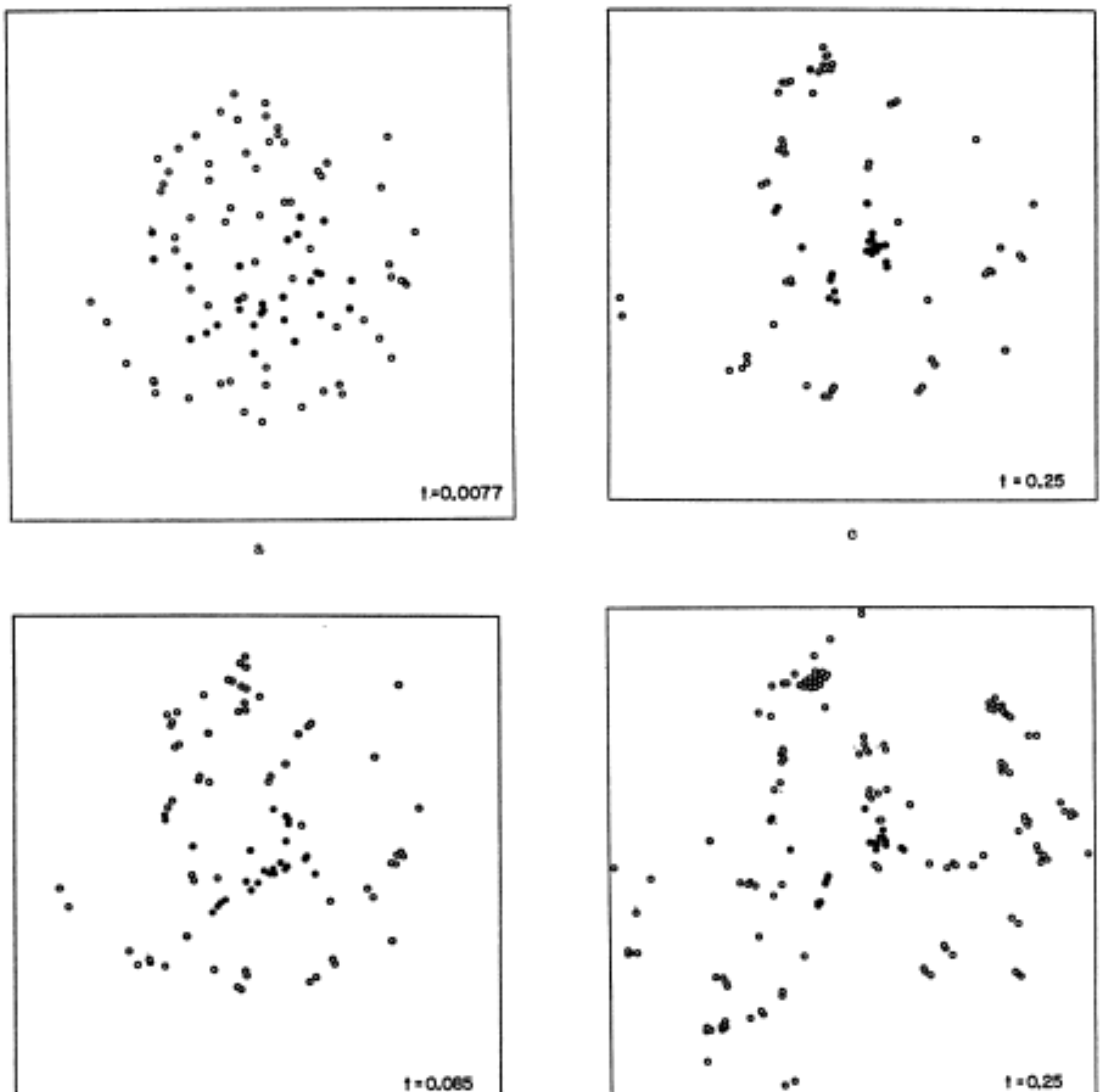
Fig. 4b.—Same as above, with the exception of counterclockwise rotations. The spiral arms point in the direction opposite to the rotation.

Toomre &Toomre 72: Antennae (NGC 4038/39)



Picture: Daniel Verschatse, APOD 2006-06-30

1st cosmological N-body simulation: Peebles 1971



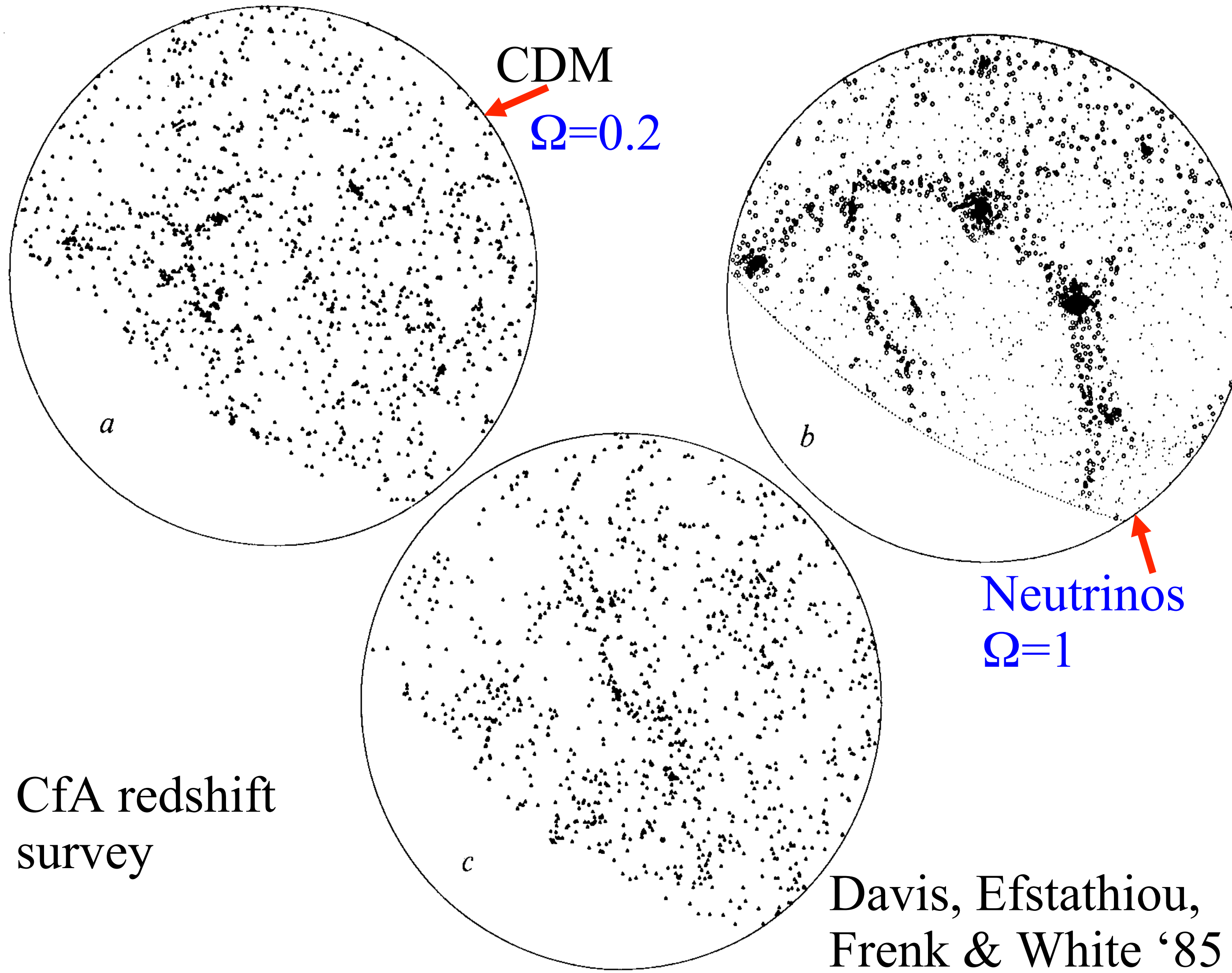
- Gravitational Clustering

- Clustered objects are slow rotators

$$\lambda \ll \lambda_{\rm disk}$$

CDM vs HDM vs CfA Survey

Neutrino dark matter produces unrealistic clustering



Davis, Efstathiou, Frenk & White '85

In CDM structure forms hierarchically

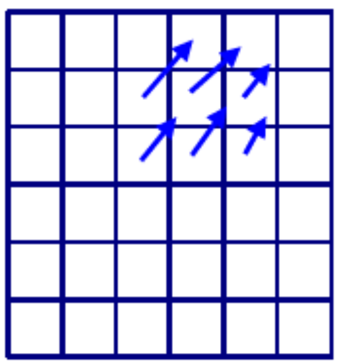
- Numerical simulations, dark matter-only: techniques
  - - Initial conditions
  - - Evolution of initial conditions – time integration
  - - Analysis

Numerical simulations: two approaches

– Two main methods to discretize a fluid

Eulerian

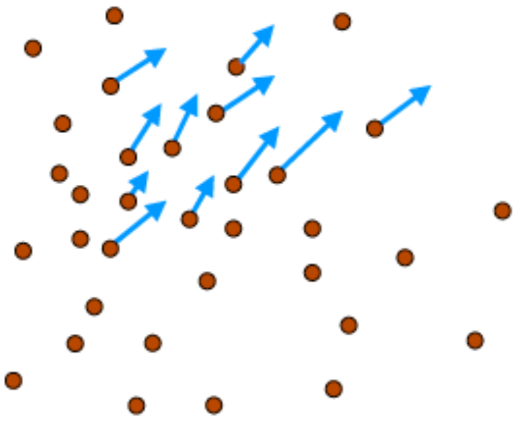
representation on a mesh
(volume elements)



Equations governing flow of physical quantities (mass, momentum, energy) through cell boundaries

Lagrangian

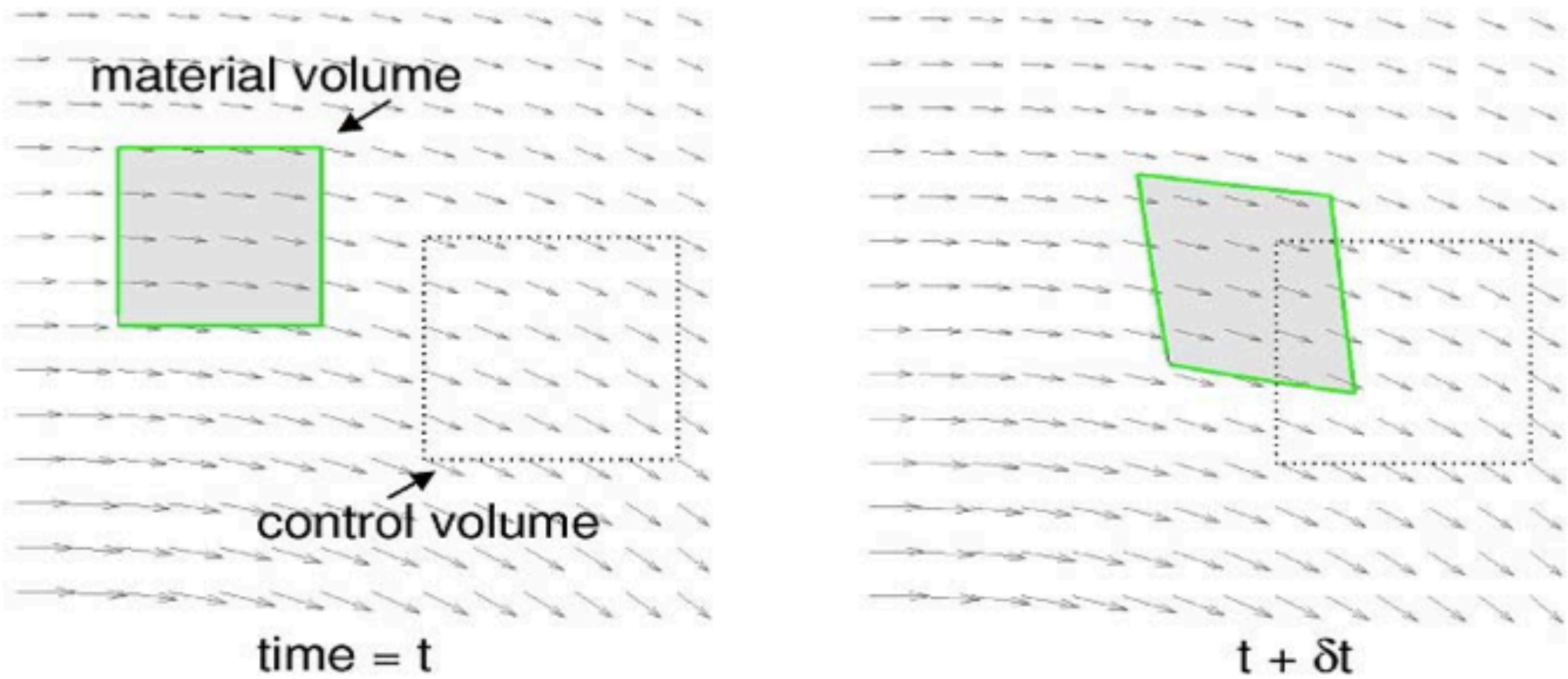
representation by fluid elements
(particles)



Equations governing evolution of physical properties (density, momentum, energy) of particles

Numerical simulations: two approaches

– Two main methods to discretize a fluid

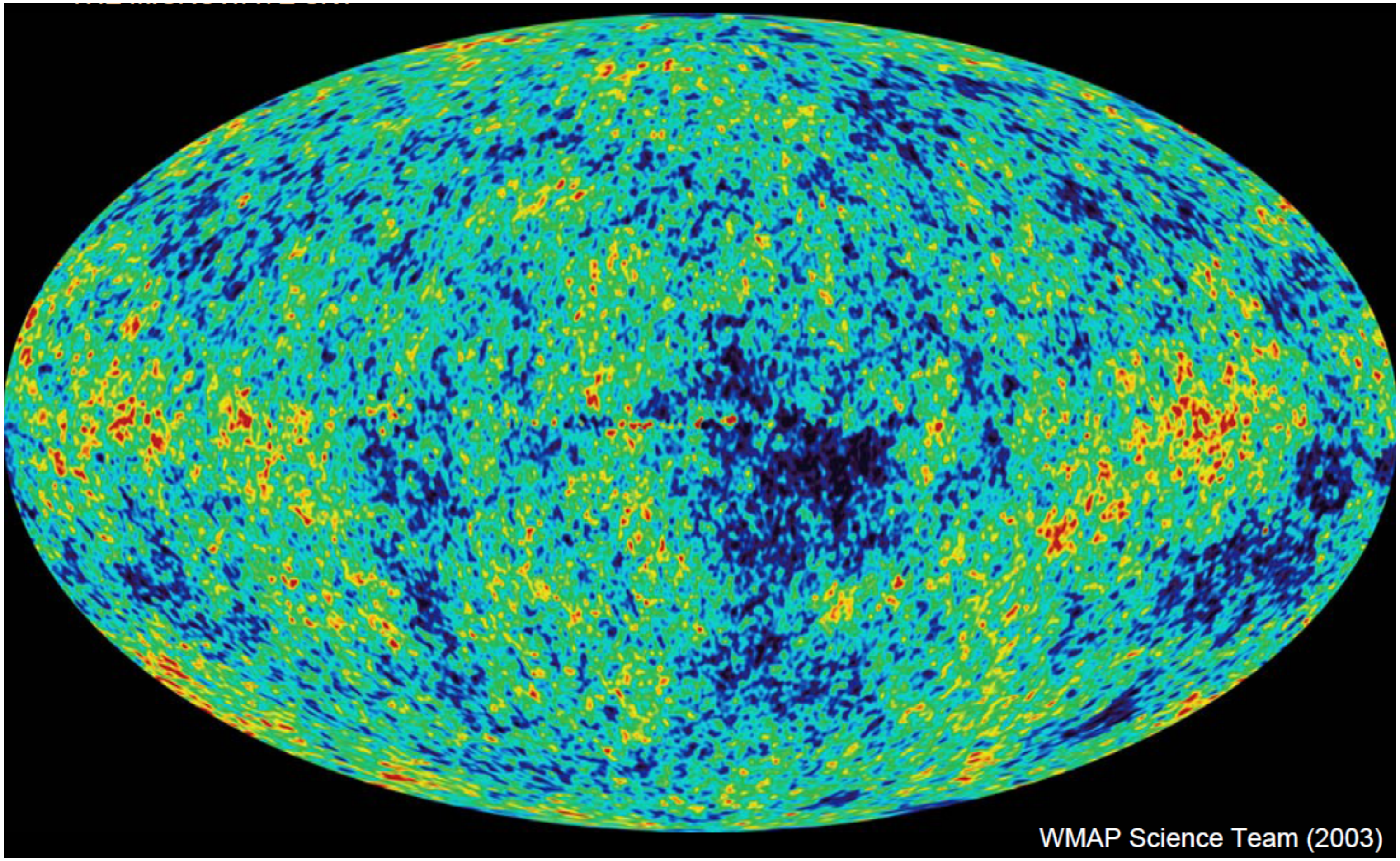


Lagrangian material volume (shaded area), position, pressure, and other properties of material volumes

Eulerian control volume (dotted boundary), fluid properties inside control volumes

Initial conditions

- Initial conditions of cosmological structure formation are known (CMB, z~1100)



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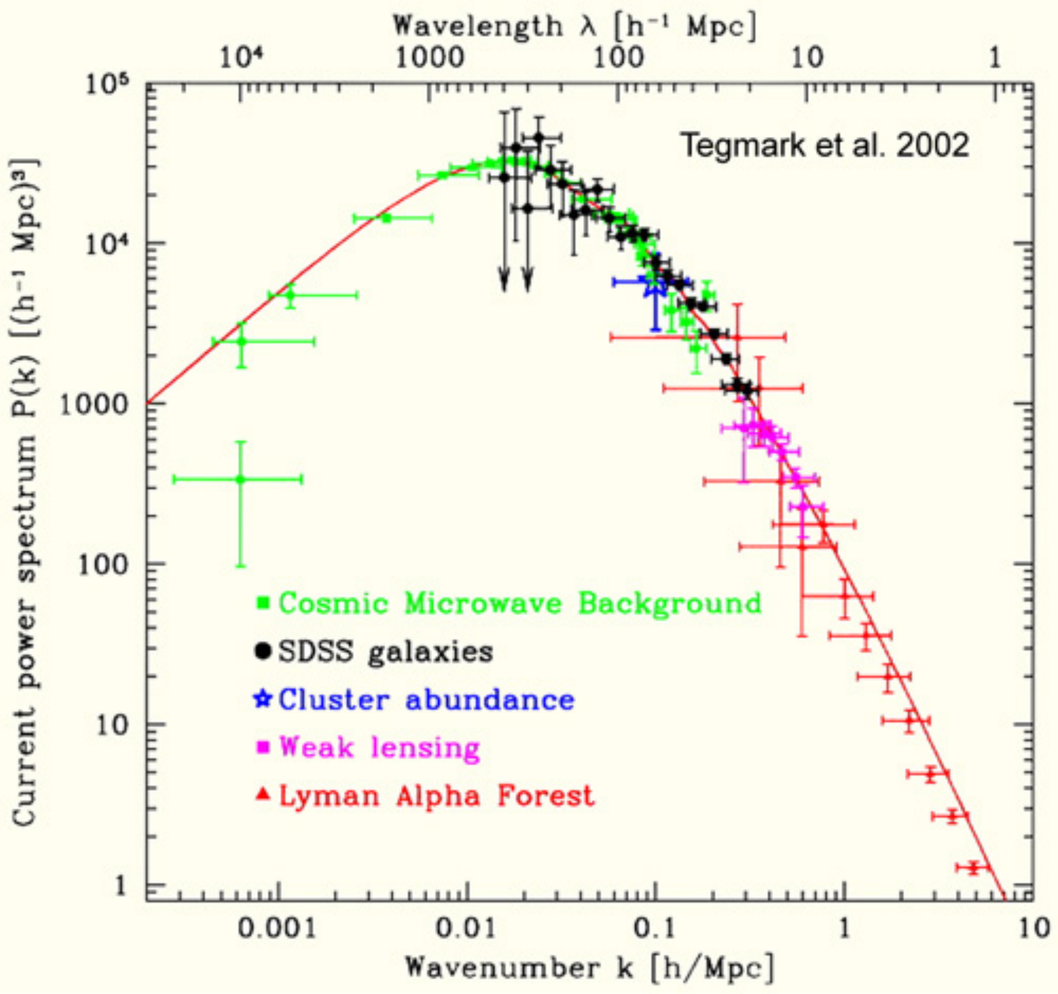

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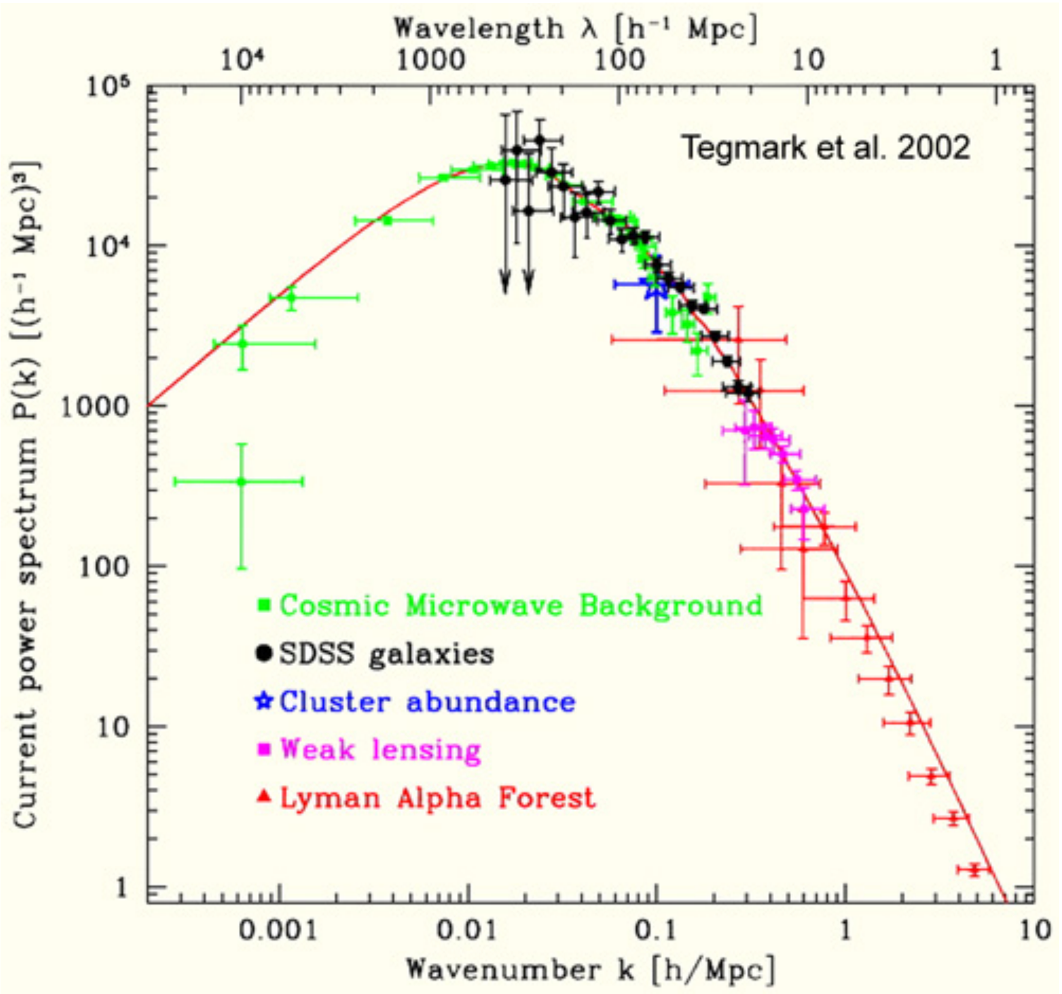
Initial conditions

$$P(k) = A \frac{k^n}{\{1 + [ak/\Gamma + (bk/\Gamma)^{3/2} + (ck/\Gamma)^2]^\nu\}^{1/\nu}}$$

Standard LCDM:

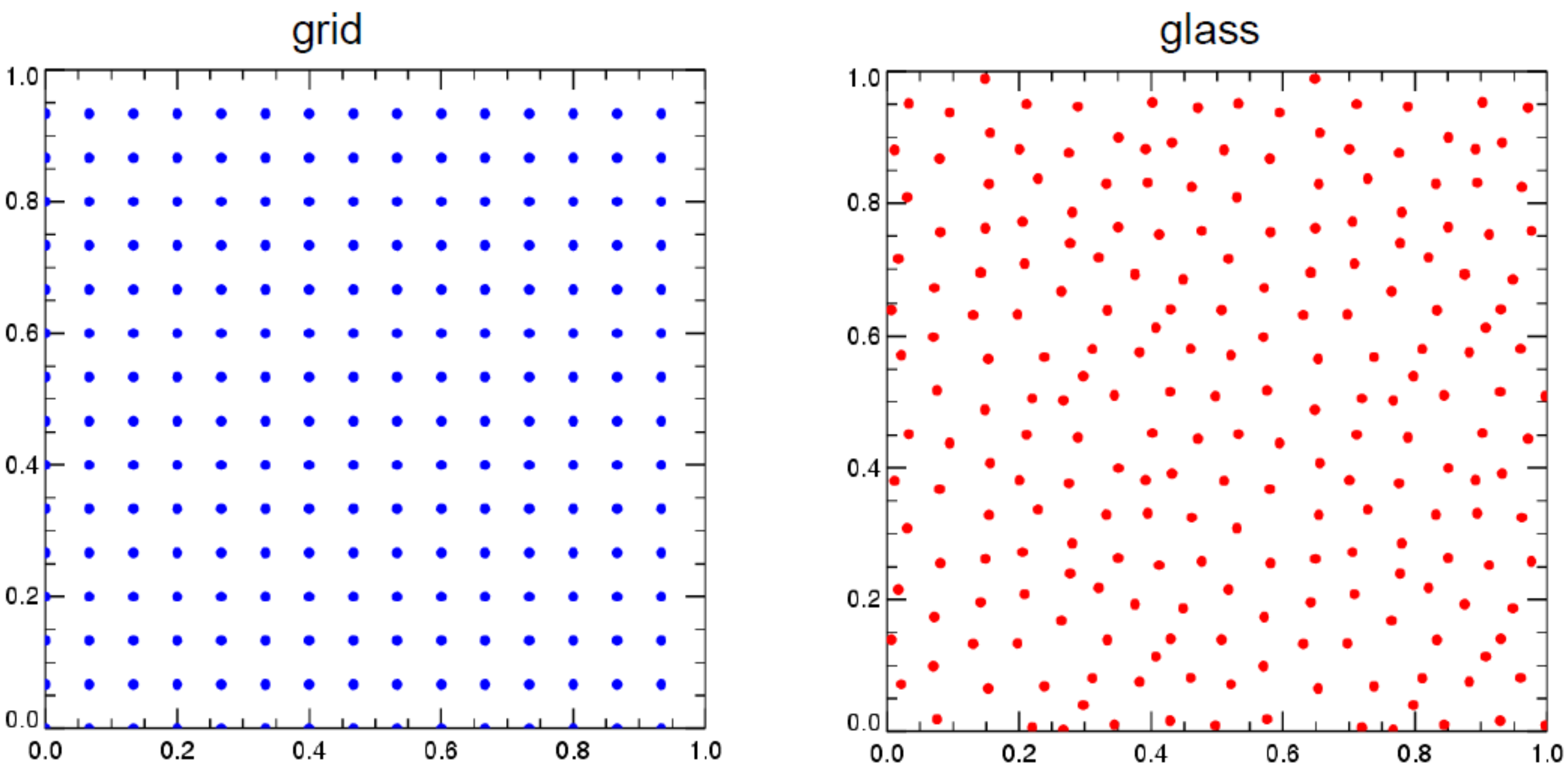
$$\begin{aligned} n &= 1.0 \\ \Gamma &= 0.21 \\ a &= 6.4\ h^{-1}\mathrm{Mpc} \\ b &= 3.0\ h^{-1}\mathrm{Mpc} \\ c &= 1.7\ h^{-1}\mathrm{Mpc} \\ \nu &= 1.13 \end{aligned}$$



We need to determine the normalization of the power spectrum.
For this we normalize the spectrum to observations of clusters

Initial conditions

- → Start with an unperturbed grid / glass-like distribution of N particles



Initial conditions

- → Start with an unperturbed grid / glass-like distribution of N particles

Dark matter particles

are not

dark matter particles

dark matter particles

- their nature is still unknown
- elementary particles (WIMPs)
- microscopic

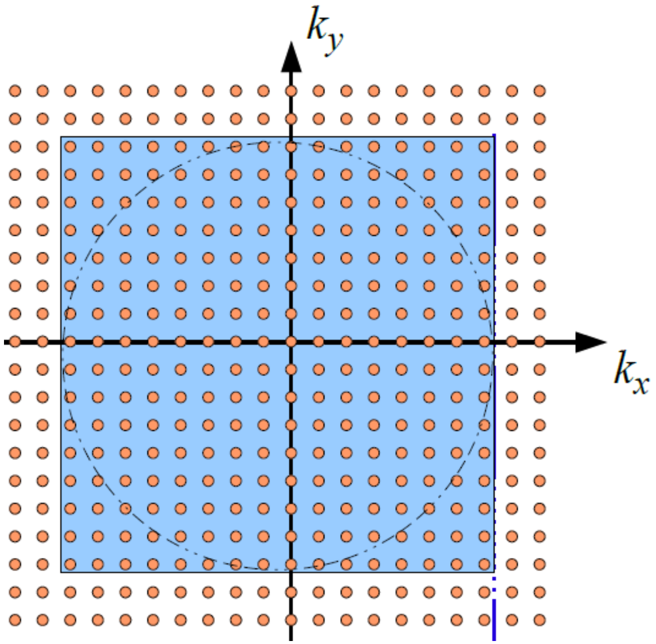
dark matter particles

- objects in DM simulations
- represent the total matter distribution
- typical masses in simulations: $10^5 h^{-1} \mathrm{M}_\odot$ — $10^{12} h^{-1} \mathrm{M}_\odot$

Initial conditions

- → Go to Fourier space *and generate fluctuations with power spectrum P(k).*
- *For each Fourier mode, draw a random phase, and an amplitude such that the variance is the square root of the power spectrum:*

$$\delta_{\mathbf{k}} = B_{\mathbf{k}} \exp^{\mathrm{i}\,\phi_{\mathbf{k}}} \qquad \left\langle \delta_{\mathbf{k}}^2 \right\rangle = P(k)$$



Yehuda introduced last week already the concept of the power spectrum and its normalisation to sigma_8.

Initial conditions

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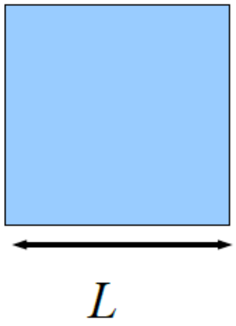
$$\delta_{\mathbf{k}} = B_{\mathbf{k}} \exp^{\mathrm{i}\,\phi_{\mathbf{k}}} \qquad \left\langle \delta_{\mathbf{k}}^2 \right\rangle = P(k)$$

For a simulation box of size L containing N^3 particles, the shortest mode that can be represented is given by the Nyquist frequency (twice the highest frequency)

- 

$$k_{\mathrm{Nyquist}} = \frac{2\pi}{L}\frac{N}{2}$$

Initial conditions

- → Compute displacement vector for each particle **S(q)**
- →FFT back to get displacement in real space $\mathbf{S}_x(\mathbf{q})$, $\mathbf{S}_y(\mathbf{q})$, $\mathbf{S}_z(\mathbf{q})$
- → Apply Zeldovich approximation to move particles from ***q*** to ***x***

$$\mathbf{x}(t) = \mathbf{q} + D_+(t)\mathbf{S}(\mathbf{q})$$

think about it as $\mathbf{x}(t) = \mathbf{x}_0 + t\mathbf{v}$

- $\mathbf{q}$ is the initial (Lagrangian) position of a DM particle
- $\mathbf{x}(t)$ is its position at time t (both $\mathbf{q}$ and $\mathbf{x}$ are *comoving*)
- $D_+(t)$ is the linear growth function which depends on the cosmological model
- $\mathbf{S}(\mathbf{q})$ is the displacement vector

Initial conditions

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- → Apply Zeldovich approximation to move particles from ***q*** to ***x***

- you need a good FFT and an excellent random number generator!
- check the power spectrum of your realisation
- check for artificial pattern (which you see if you have a bad random number generator)

http://ginnungagapgroup.github.io/ginnungagap

Ginnungagap

Inital conditions generator for cosmological simulations

Download .zip | Download .tar.gz | View on GitHub

Ginnungagap is a code for generating cosmological initial conditions. It is aimed at high scalability and ease of use. The main code will generate the velocity field at the requested starting redshift, which is then used by extra tools to generated a particle representation. Furthermore, tools are provided to generate realisations at different resolution levels while keeping the large scale structure fixed (by means of refining the underlying white noise field).

Ginnungagap is maintained by spilipenko
This page was generated by GitHub Pages. Tactile theme by Jason Long.

http://bitbucket.org/ohahn/music

MULTI
SCALE
INITIAL
CONDITIONS

Welcome to the homepage of MUSIC - a program to generate zoom initial conditions for cosmological simulations. © 2013 by Oliver Hahn

Info | Download | Contact

MUSIC - multi-scale cosmological initial conditions

MUSIC is a computer program to generate nested grid initial conditions for high-resolution "zoom" cosmological simulations. A detailed description of the algorithms can be found in Hahn & Abel (2011). You can download the user's guide here. Please consider joining the user mailing list.

Current MUSIC key features are:

- Supports output for RAMSES, ENZO, Arepo, Gadget-2/3, ART, Pkdgrav/Gasoline and NyX via plugins. New codes can be added.
- Support for first (1LPT) and second order (2LPT) Lagrangian perturbation theory, local Lagrangian approximation (LLA) for baryons with grid codes.
- Pluggable transfer functions, currently CAMB, Eisenstein&Hu, BBKS, Warm Dark Matter variants. Distinct baryon+CDM fields.
- Minimum bounding ellipsoid and convex hull shaped high-res regions supported with most codes, supports refinement mask generation for RAMSES.

Advancing the initial conditions

→ Discretize matter in N particles
→ assume that the only appreciable interaction of dark matter is gravity

$$\ddot{\mathbf{x}}_i = -\nabla_i \Phi(\mathbf{x}_i)$$

$$\Phi(\mathbf{x}) = -G \sum_{j=1}^{N} \frac{m_j}{[(\mathbf{x}-\mathbf{x}_j)^2 + \epsilon^2]}$$

<u>Gravitational softening</u>

→ prevents large-angle particle scattering
→ ensures two-body relaxation time is sufficiently large

ε sets the force resolution

Advancing the initial conditions

→ Discretize matter in N particles

→ assume that the only appreciable interaction of dark matter is gravity

$$\ddot{\mathbf{x}}_i = -\nabla_i \, \Phi(\mathbf{x}_i)$$

$$\Phi(\mathbf{x}) = -G \sum_{j=1}^{N} \frac{m_j}{[(\mathbf{x} - \mathbf{x}_j)^2 + \epsilon^2]}$$

System of 3N equations

→ coupled

→ non-linear

→ second-order

Advancing the initial conditions

→ How do we compute the gravitational forces accurately and efficiently?

→ How do we integrate the orbital equations in time?

The challenge of cosmological simulations

a) want small mass to resolve internal structure of objects

b) want large volume to get representative sample of the universe

→ Cosmological simulations require very large N!!

Numerical methods to solve gravitational force

Direct summation

$$m_i \frac{\mathbf{v}_i}{dt} = m_i \nabla \Phi_i = -\sum_{j=}^{N} N \frac{G m_i m_j}{|(\mathbf{r}_i - \mathbf{r}_j)^2 + \varepsilon^2|^{\frac{3}{2}}} (\mathbf{r}_i - \mathbf{r}_j)$$

CPU time scales as N^2 → too time-consuming if N is large

Numerical methods to solve gravitational force

PM method (Particle Mesh)

→ Poisson's equation can be solved in real space by a convolution of the density field with a Green's function:

$$\Phi(\mathbf{x}) = \int g(\mathbf{x} - \mathbf{x}')\, \rho(\mathbf{x})\, \mathrm{d}\mathbf{x}'$$

For example, for vacuum boundaries:

$$\boxed{\Phi(\mathbf{x}) = -G \int \frac{\rho(\mathbf{x})}{|\mathbf{x} - \mathbf{x}'|}\, \mathrm{d}\mathbf{x}' \qquad g(\mathbf{x}) = -\frac{G}{|\mathbf{x}|}}$$

PM method

→ Poisson's equation can be solved in real space by a convolution of the density field with a Green's function:

$$\Phi(\mathbf{x}) = \int g(\mathbf{x} - \mathbf{x}')\, \rho(\mathbf{x})\, \mathrm{d}\mathbf{x}'$$

In Fourier space, the convolution becomes a multiplication:

$$\hat{\Phi}(\mathbf{k}) = \hat{g}(\mathbf{k}) \cdot \hat{\rho}(\mathbf{k})$$

→ solve potential in three steps:

1) FFT the density field
2) Multiply with Green's function
3) FFT back to obtain potential

PM method

→ Poisson's equation can be solved in real space by a convolution of the density field with a Green's function:

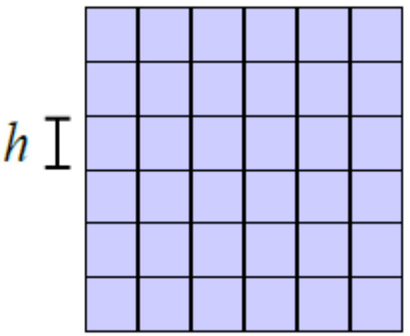
Need to calculate density field

→ solve potential in three steps:

1) FFT the density field
2) Multiply with Green's function
3) FFT back to obtain potential

PM method

Density assignment



$\{\mathbf{x}_m\}$ set of discrete mesh centres

Give particles a “shape” S(x). Then to each mesh cell, we assign the fraction of mass that falls into this cell. The overlap for a cell is given by:

$$W(\mathbf{x_m} - \mathbf{x}_i) = \int_{\mathbf{x_m} - \frac{h}{2}}^{\mathbf{x_m} + \frac{h}{2}} S(\mathbf{x}' - \mathbf{x}_i)\,\mathrm{d}\mathbf{x}'$$

The density on the mesh is then a sum over the contributions of each particle as given by the assignment function:

$$\rho(\mathbf{x_m}) = \frac{1}{h^3} \sum_{i=1}^{N} m_i\, W(\mathbf{x}_i - \mathbf{x_m})$$

PM method

Commoly used shape functions and assignment schemes

Name	Shape function S(x)	# of cells involved	Properties of force
NGP Nearest grid point	• $\frac{1}{\Delta x}\delta\left(\frac{x}{\Delta x}\right)$	$1^3 = 1$	piecewise constant in cells

Particles are point-like and all of particle's mass is assigned to the single grid cell that contains it

PM method

Commoly used shape functions and assignment schemes

Name	Shape function S(x)	# of cells involved	Properties of force
NGP Nearest grid point	$\frac{1}{\Delta x}\delta\left(\frac{x}{\Delta x}\right)$	$1^3 = 1$	piecewise constant in cells
CIC Clouds in cells	$\frac{1}{\Delta x}\begin{cases} 1, & \lvert x\rvert < \frac{1}{2}\Delta x \\ 0, & \text{otherwise} \end{cases}$	$2^3 = 8$	piecewise linear, continuous

Particles are cubes of uniform density and of one grid cell size

PM method

Commoly used shape functions and assignment schemes

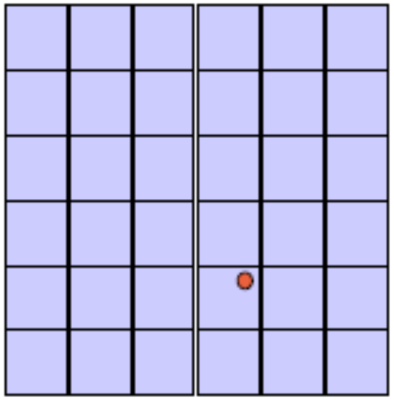
Name	Shape function S(x)	# of cells involved	Properties of force
NGP Nearest grid point	 $\frac{1}{\Delta x}\delta\left(\frac{x}{\Delta x}\right)$	$1^3 = 1$	piecewise constant in cells
CIC Clouds in cells	 $\frac{1}{\Delta x}\begin{cases} 1, & \lvert x\rvert < \frac{1}{2}\Delta x \\ 0, & \text{otherwise} \end{cases}$	$2^3 = 8$	piecewise linear, continuous
TSC Triangular shaped clouds	 $\frac{1}{\Delta x}\begin{cases} 1 - \lvert x\rvert/\Delta x, & \lvert x\rvert < \Delta x \\ 0, & \text{otherwise} \end{cases}$	$3^3 = 27$	continuous first derivative

Fraction of particle's mass assigned to a cell is the shape function averaged over this cell

PM method

Finite differencing of the potential to get the force field

Approximate the force field $\mathbf{f} = -\nabla\Phi$ with finite differencing



2nd order accurate scheme:

$$f^{(x)}_{i,j,k} = -\frac{\Phi_{i+1,j,k} - \Phi_{i-1,j,k}}{2h}$$

4th order accurate scheme:

$$f^{(x)}_{i,j,k} = -\frac{4}{3}\frac{\Phi_{i+1,j,k} - \Phi_{i-1,j,k}}{2h} + \frac{1}{3}\frac{\Phi_{i+2,j,k} - \Phi_{i-2,j,k}}{4h}$$

Interpolating the mesh-forces to the particle locations

$$F(\mathbf{x}_i) = \sum_{\mathbf{m}} W(\mathbf{x}_i - \mathbf{x}_{\mathbf{m}})\, f_{\mathbf{m}}$$

The interpolation kernel needs to be the same one used for mass-assignment to ensure force anti-symmetry.

PM method

The four steps of the PM algorithm

1) Density assignment

2) Computation of the potential

3) Determination of the force field

4) Assignment of forces to particles

PM method

Pros

→ Relative algorithmic simplicity

→ Speed: time scales as
$T \sim O(N_p) + O(N_c \log N_c)$
N_p=number of particles
N_c=number of cells

→ Natural incorporation of periodic boundary conditions

Cons

→ Force resolution limited to mesh size

→ Force errors somewhat anisotropic on the scale of the mesh size

→ This is a problem for cosmological simulations where matter is highly clustered

Variations of PM method

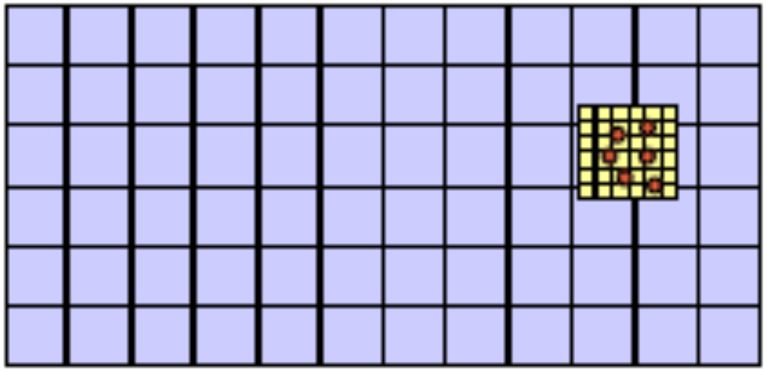
P^{3}M: particle-particle-particle mesh

Idea: Supplement the PM force with a direct summation short-range force at the scale of the mesh cells. The particles in cells are linked together by a chaining list.

Offers much higher dynamic range, but becomes slow when clustering sets in.

AP^{3}M: Adaptive particle-particle-particle mesh

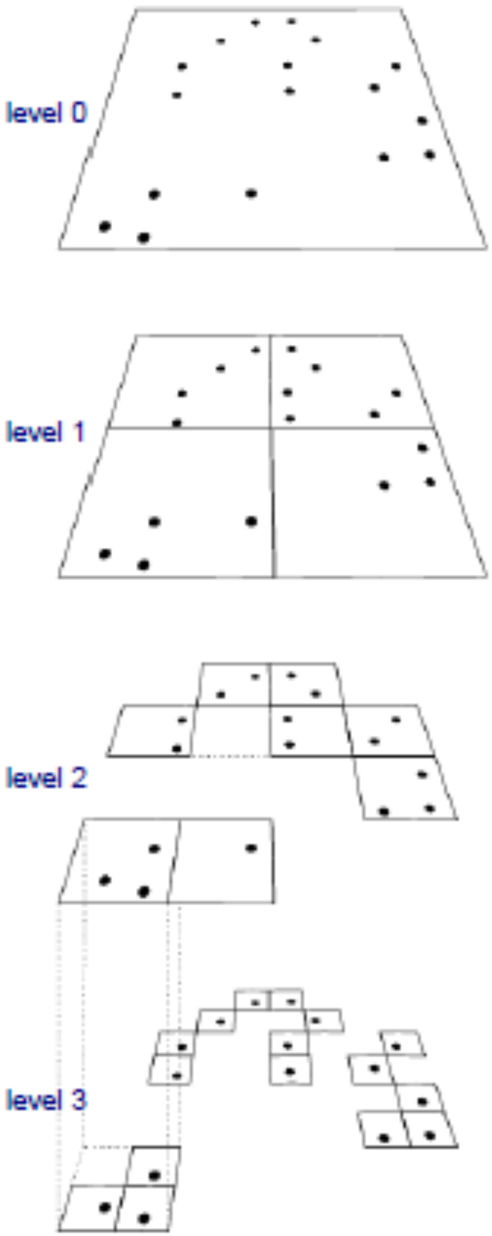
Mesh refinements are placed on clustered regions



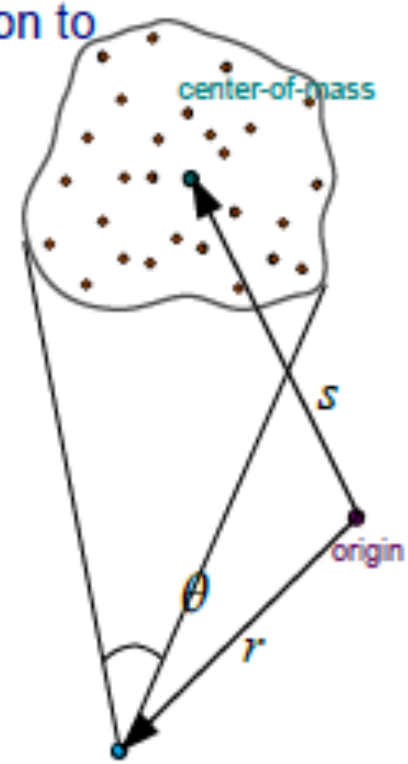
Can avoid clustering slow-down, but has higher complexity and ambiguities in mesh placement

Tree method

Oct-tree in two dimensions



Idea: Use hierarchical multipole expansion to account for distant particle groups



$$\Phi(\mathbf{r}) = -G \sum_i \frac{m_i}{|\mathbf{r} - \mathbf{x}_i|}$$

We expand:

$$\frac{1}{|\mathbf{r} - \mathbf{x}_i|} = \frac{1}{|(\mathbf{r} - \mathbf{s}) - (\mathbf{x}_i - \mathbf{s})|}$$

for $\quad |\mathbf{x}_i - \mathbf{s}| \ll |\mathbf{r} - \mathbf{s}| \qquad \mathbf{y} \equiv \mathbf{r} - \mathbf{s}$

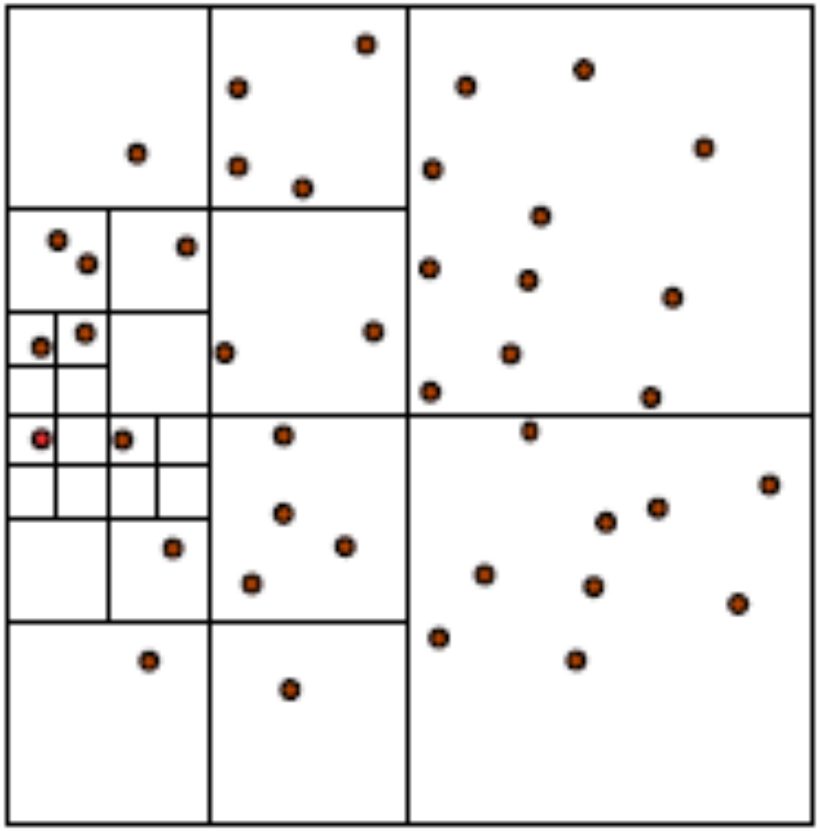
and obtain:

$$\frac{1}{|\mathbf{y} + \mathbf{s} - \mathbf{x}_i|} = \frac{1}{|\mathbf{y}|} - \frac{\mathbf{y} \cdot (\mathbf{s} - \mathbf{x}_i)}{|\mathbf{y}|^3} + \frac{1}{2} \frac{\mathbf{y}^T \left[3(\mathbf{s} - \mathbf{x}_i)(\mathbf{s} - \mathbf{x}_i)^T - \mathbf{I}(\mathbf{s} - \mathbf{x}_i)^2 \right] \mathbf{y}}{|\mathbf{y}|^5} + \ldots$$

the dipole term vanishes when summed over all particles in the group

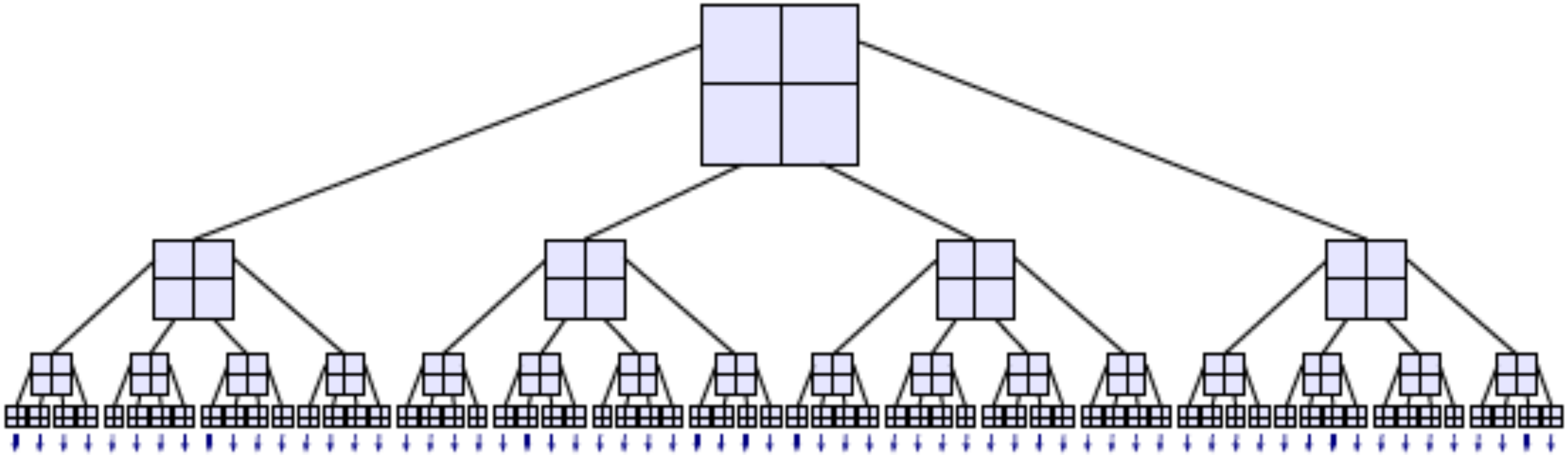
Numerical methods to solve gravitational force

Tree algorithms



Idea: group distant particles together, and use their multipole expansion

→ $\log(N_p)$ force terms per particle



Tree method

The multipole moments are computed for each node of the tree

Monpole moment:

$$M = \sum_i m_i$$

Quadrupole tensor:

$$Q_{ij} = \sum_k m_k \left[3(\mathbf{x}_k - \mathbf{s})_i (\mathbf{x}_k - \mathbf{s})_j - \delta_{ij} (\mathbf{x}_k - \mathbf{s})^2 \right]$$

Resulting potential/force approximation:

$$\Phi(\mathbf{r}) = -G \left[\frac{M}{|\mathbf{y}|} + \frac{1}{2} \frac{\mathbf{y}^T \mathbf{Q}\, \mathbf{y}}{|\mathbf{y}|^5} \right]$$

For a single force evaluation, not *N* single-particle forces need to be computed, but **only of order *log(N)* multipoles**, depending on opening angle.

Tree vs PM method

PM

→ Force resolution limited to mesh size

→ Force errors somewhat anisotropic on the scale of the mesh size

→ This is a problem for cosmological simulations where matter is highly clustered

Tree

→ No intrinsic restriction for dynamical range

→ Force accuracy can be conveniently adjusted to desired level

→ Speed depends only weakly on clustering level

→ geometrically flexible

TreePM method

Modern codes used the Tree-PM method:

→ decompose the potential (of a single particle) in Fourier space into long-range and short-range components

→ long-range potential solved with PM method

→ short-range potential solved with Tree (the Tree is expensive, particularly at high redshift)

Time integration

Once force at the position of each particle is determined, we need to evolve the system

Want to numerically integrate an **ordinary differential equation** (ODE)

$$\dot{\mathbf{y}} = f(\mathbf{y})$$

Note: $\mathbf{y}$ can be a vector

A numerical approximation to the ODE is a set of values $\{\mathbf{y}_0, \mathbf{y}_1, \mathbf{y}_2, \ldots\}$ at times $\{t_0, t_1, t_2, \ldots\}$

There are many different ways for obtaining this.

Time integration

Explicit Euler method

$$y_{n+1} = y_n + f(y_n)\Delta t$$

- Simplest of all
- Right hand-side depends only on things already known, **explicit method**
- The error in a single step is $O(\Delta t^2)$, but for the N steps needed for a finite time interval, the total error scales as $O(\Delta t)$!
- Never use this method, it's only **first order accurate**.

Implicit Euler method

$$y_{n+1} = y_n + f(y_{n+1})\Delta t$$

- **Excellent** stability properties
- Suitable for very stiff ODE
- Requires implicit solver for y_{n+1}

Time integration

Implicit mid-point rule

$$y_{n+1} = y_n + f\left(\frac{y_n + y_{n+1}}{2}\right)\Delta t$$

- **2nd order accurate**

- Time-symmetric
- But still implicit...

Runge-Kutta methods

whole class of integration methods

2nd order accurate

$$\begin{aligned} k_1 &= f(y_n) \\ k_2 &= f(y_n + k_1 \Delta t) \\ y_{n+1} &= y_n + \left(\frac{k_1 + k_2}{2}\right)\Delta t \end{aligned}$$

4th order accurate.

$$\begin{aligned} k_1 &= f(y_n, t_n) \\ k_2 &= f(y_n + k_1 \Delta t/2, t_n + \Delta t/2) \\ k_3 &= f(y_n + k_2 \Delta t/2, t_n + \Delta t/2) \\ k_4 &= f(y_n + k_3 \Delta t/2, t_n + \Delta t) \\ y_{n+1} &= y_n + \left(\frac{k_1}{6} + \frac{k_2}{3} + \frac{k_3}{3} + \frac{k_4}{6}\right)\Delta t \end{aligned}$$

Time integration

The Leapfrog

For a second order ODE: $\ddot{\mathbf{x}} = f(\mathbf{x})$

"Drift-Kick-Drift" version

$$
\begin{aligned}
x_{n+\frac{1}{2}} &= x_n + v_n \frac{\Delta t}{2} \\
v_{n+1} &= v_n + f(x_{n+\frac{1}{2}}) \Delta t \\
x_{n+1} &= x_{n+\frac{1}{2}} + v_{n+1} \frac{\Delta t}{2}
\end{aligned}
$$

"Kick-Drift-Kick" version

$$
\begin{aligned}
v_{n+\frac{1}{2}} &= v_n + f(x_n) \frac{\Delta t}{2} \\
x_{n+1} &= x_n + v_{n+\frac{1}{2}} \frac{\Delta t}{2} \\
v_{n+1} &= v_{n+\frac{1}{2}} + f(x_{n+1}) \frac{\Delta t}{2}
\end{aligned}
$$

- **2^{nd} order accurate**
- **symplectic**
- can be rewritten into time-centred formulation

http://www.mpa-garching.mpg.de/gadget/

GADGET - 2

A code for cosmological simulations of structure formation

General
- Description
- Features
- Authors and History
- Acknowledgments
- News

Software
- Download GADGET
- Download N-GenIC
- Requirements
- License
- Mailing List
- Change-Log
- Examples

Documentation
- Code Paper
- Users Guide
- Code Reference

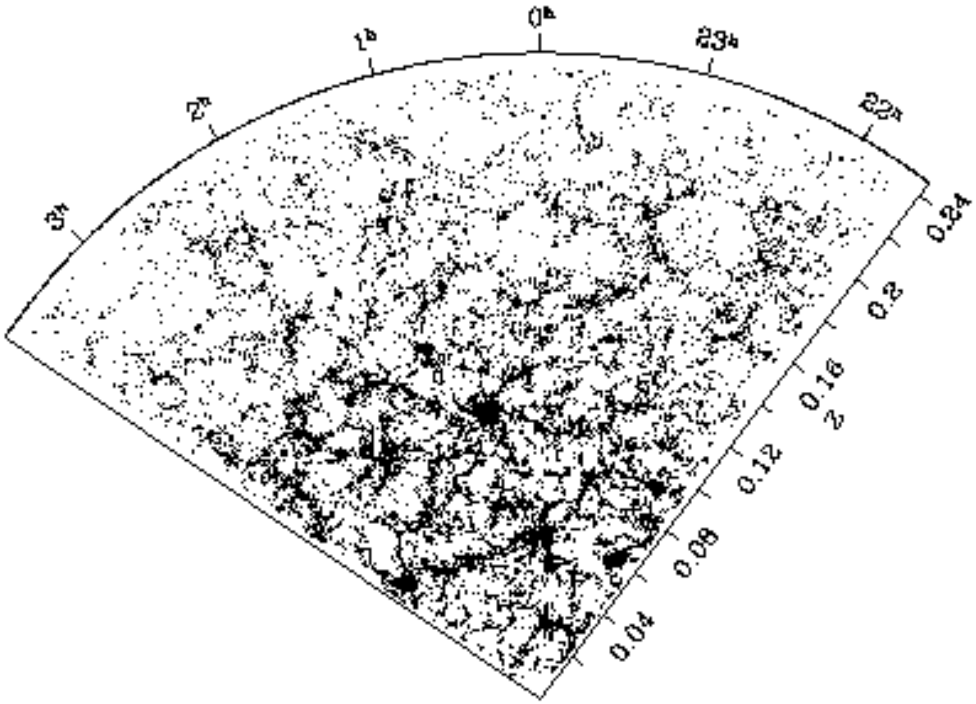
Publications

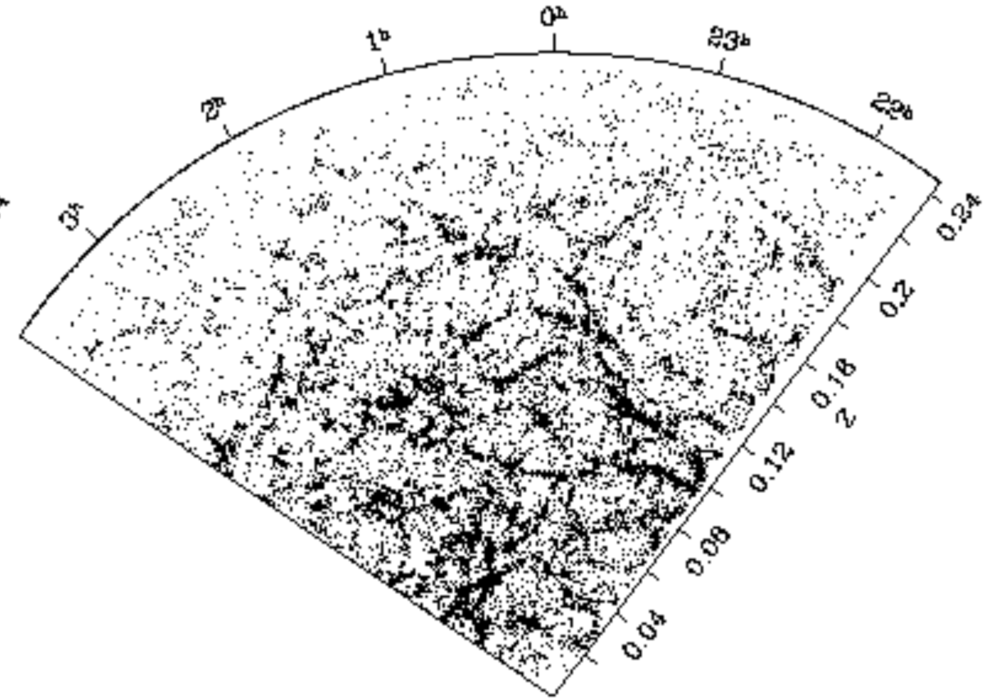
Description

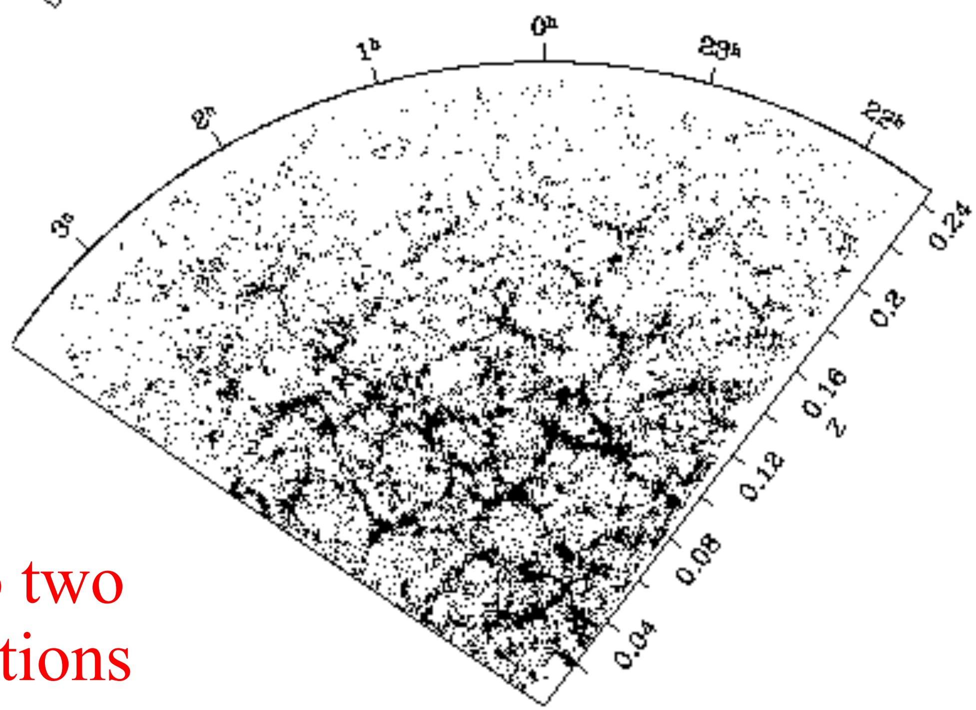
GADGET is a freely available code for cosmological N-body/SPH simulations on massively parallel computers with distributed memory. **GADGET** uses an explicit communication model that is implemented with the standardized MPI communication interface. The code can be run on essentially all supercomputer systems presently in use, including clusters of workstations or individual PCs.

GADGET computes gravitational forces with a hierarchical tree algorithm (optionally in combination with a particle-mesh scheme for long-range gravitational forces) and represents fluids by means of smoothed particle hydrodynamics (SPH). The code can be used for studies of isolated systems, or for simulations that include the cosmological expansion of space, both with or without periodic boundary conditions. In all these types of simulations, **GADGET** follows the evolution of a self-gravitating collisionless N-body system, and allows gas dynamics to be optionally included. Both the force computation and the time stepping of **GADGET** are fully adaptive, with a dynamic range which is, in principle, unlimited.

GADGET can therefore be used to address a wide array of astrophysically interesting problems, ranging from colliding and merging galaxies, to the formation of large-scale structure in the Universe. With the inclusion of additional physical processes such as radiative cooling and heating, **GADGET** can also be used to study the dynamics of the gaseous intergalactic medium, or to address star formation and its regulation by feedback processes.





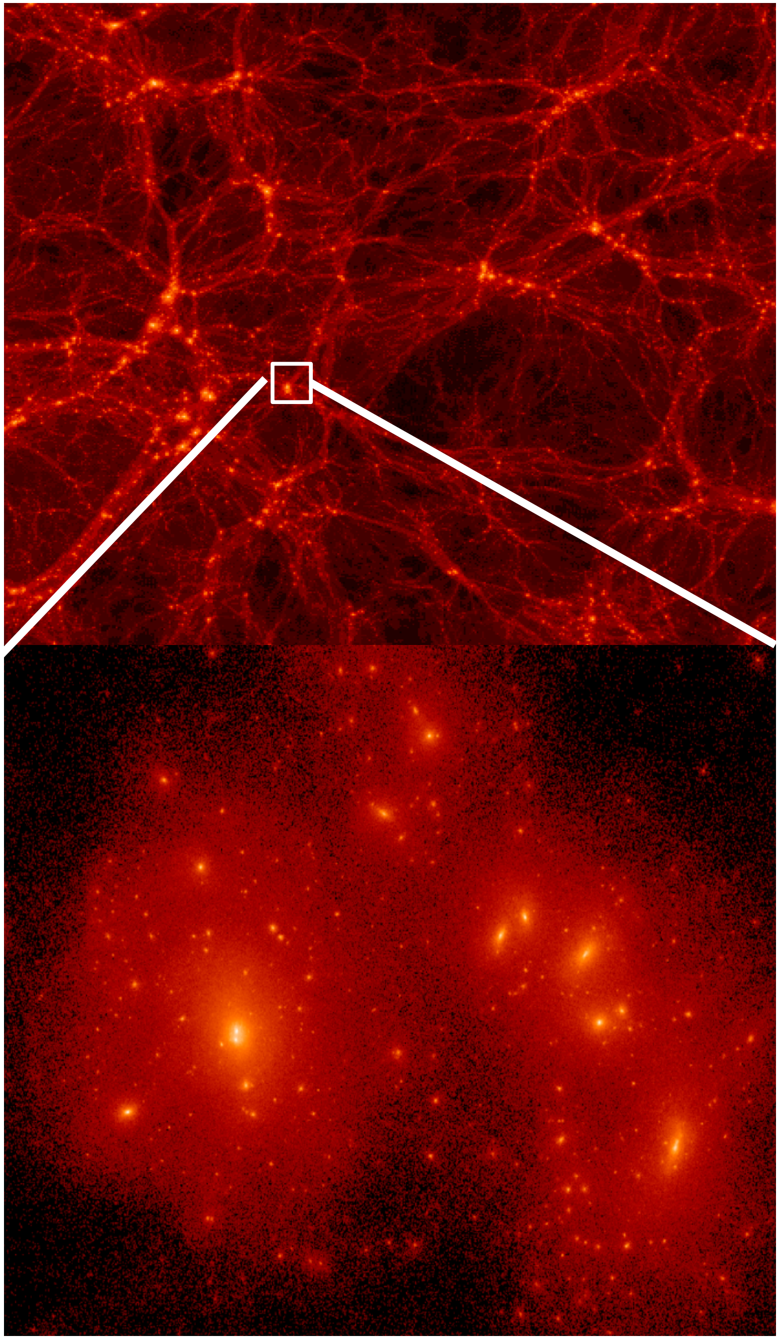


2dF compared to two computer simulations

Analysing simulations: Halo finding

FoF: friends of friends

→ Calculate inter-particle spacing (box volume divided by number of particles)
→ Pairs of particles closer together than the "linking length" – some fraction of the average inter-particle spacing (usually ~0.2) – are linked together. (Particles can be paired with more than one other particle.)
→ The final groups are formed by the networks of particles linked together by "friends".



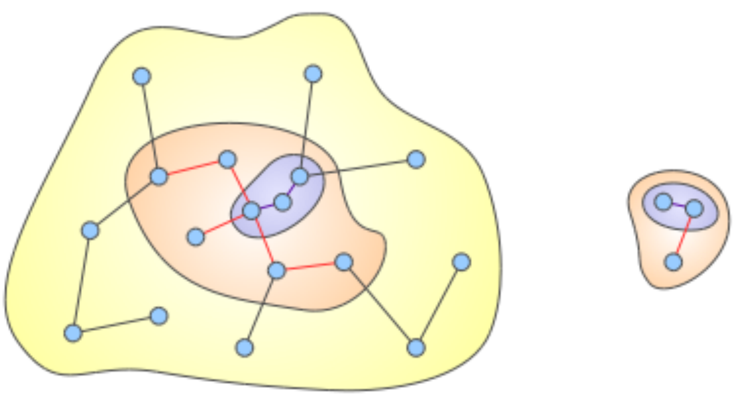
Analysing simulations: Halo finding

FoF: friends of friends

→ Linking length of 0.17 gives roughly virial overdensity (for ΛCDM at $z = 0$)

→ Linking 0.2 usually used

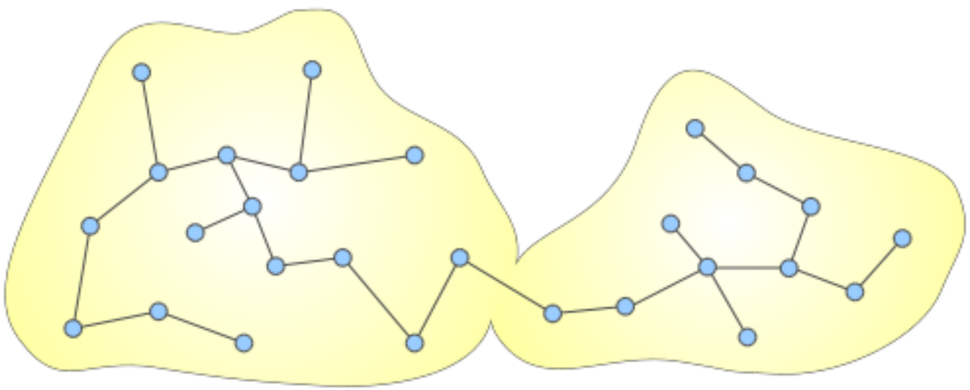
→ haloes have arbitrary shape



Different colors are groups of different linking-lengths

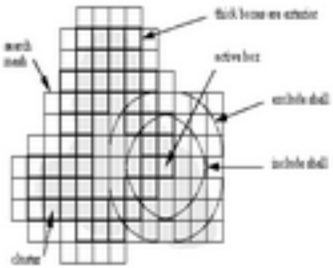
→ center of mass of the FoF group not necessarily highest density peak

→ subhalos can be found with a shorter linking length



FoF groups do not intersect

http://www-hpcc.astro.washington.edu/tools/afof.html

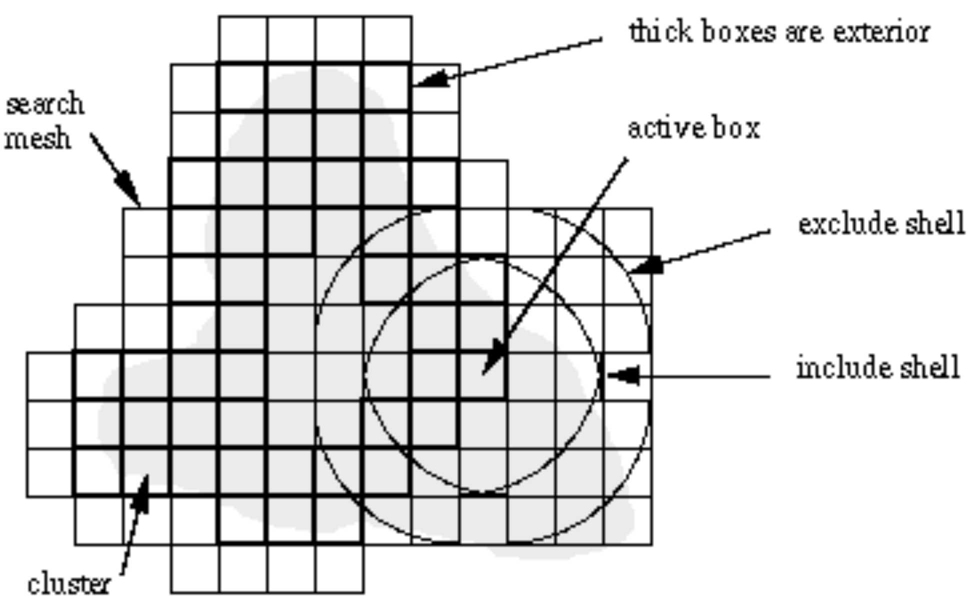


University Of Washington
N-Body Shop

AFOF

▷ N-Body Home

Overview of AFOF



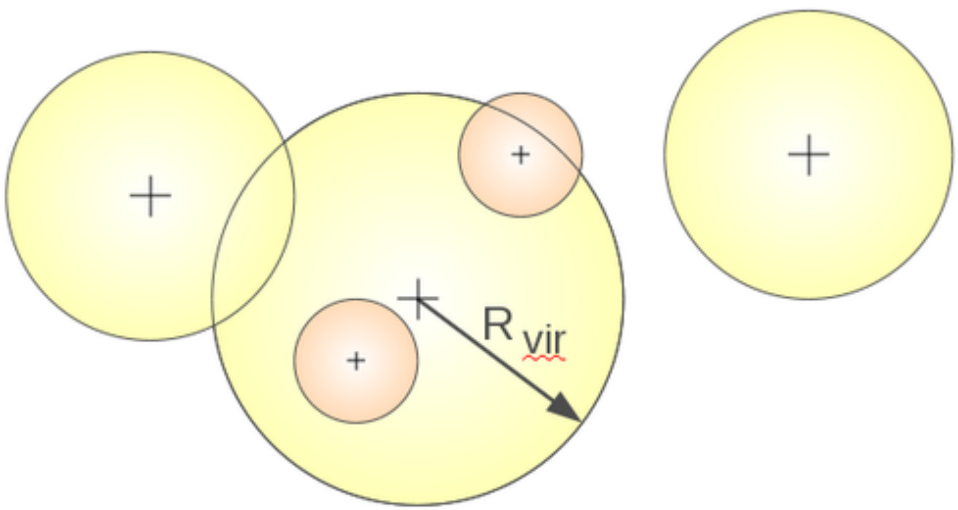
Gadget reading routines
by
Arman Khalatyan (AIP)

Analysing simulations: Halo finding

Spherical overdensity

→ Assign center of haloes to density peaks

→ Assign size of haloes, such that the radius is at virial overdensity. Remove unbound particles



→ Subhalos are found using the same procedure (local density peaks).

→ Halos are, by definition, spherical

→ Examples: BDM and AMIGA halo finders

→ Rockstar combines 6D FOF with SO

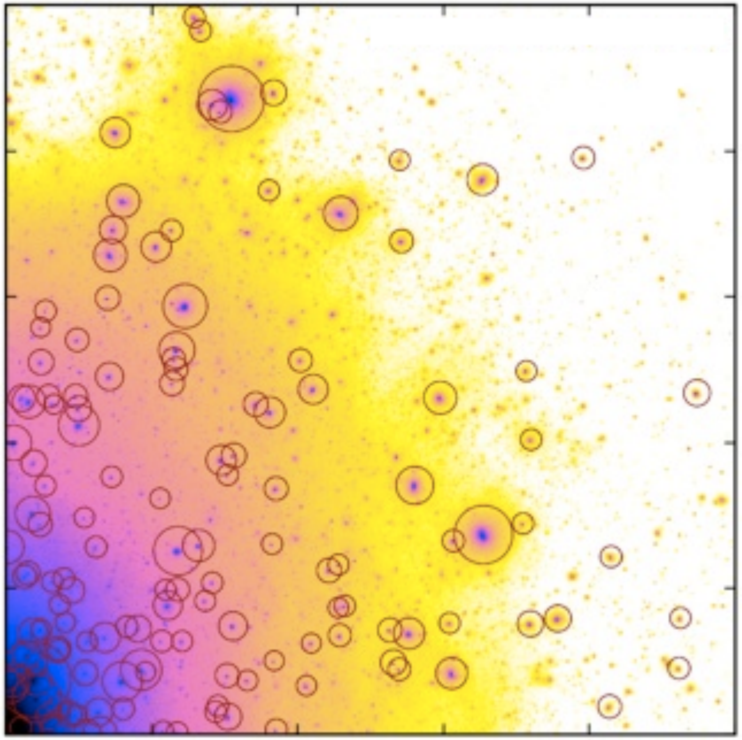
http://popia.ft.uam.es/AHF/

AMIGA HALO FINDER

DOWNLOAD DOCUMENTATION DISCUSSIONS

AHF is a code for finding gravitationally bound objects in cosmological simulations.

ahf-v1.0-084.tgz



https://bitbucket.org/gfcstanford/rockstar

The Rockstar Halo Finder

Most code: Copyright (C)2011-2014 Peter Behroozi

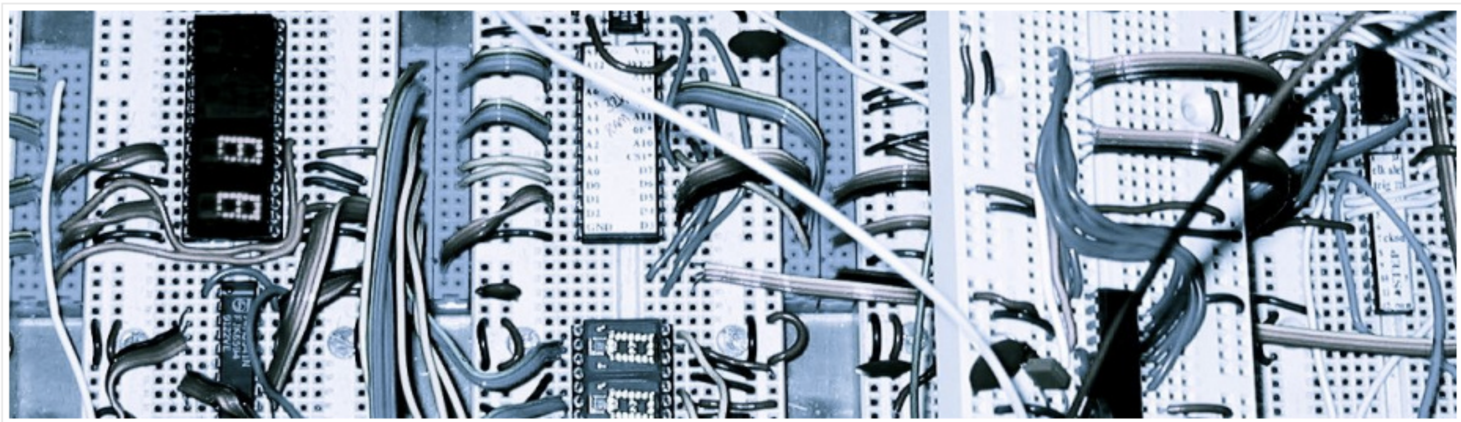
License: GNU GPLv3

Science/Documentation Paper: http://arxiv.org/abs/1110.4372

Contents

- Compiling
- Running

http://www.peterbehroozi.com/code.html



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The Rockstar Phase Space Halo Finder

Click here to access the BitBucket website (with source code and instructions).

Precision Halo Catalogs and Merger Trees

Click here to access the Google Code Repository (with source code and instructions).

Galaxy Number Density Evolution

Click here to access the Google Code Repository (with source code and instructions).

- A white board discussion

- FOF vs. SO

- Parallelization
  - –- Why do we need parallel codes?
  - –- Parallelization algorithms

Why do we need parallel codes?

Computational cost of large cosmological simulations is very high

examples:

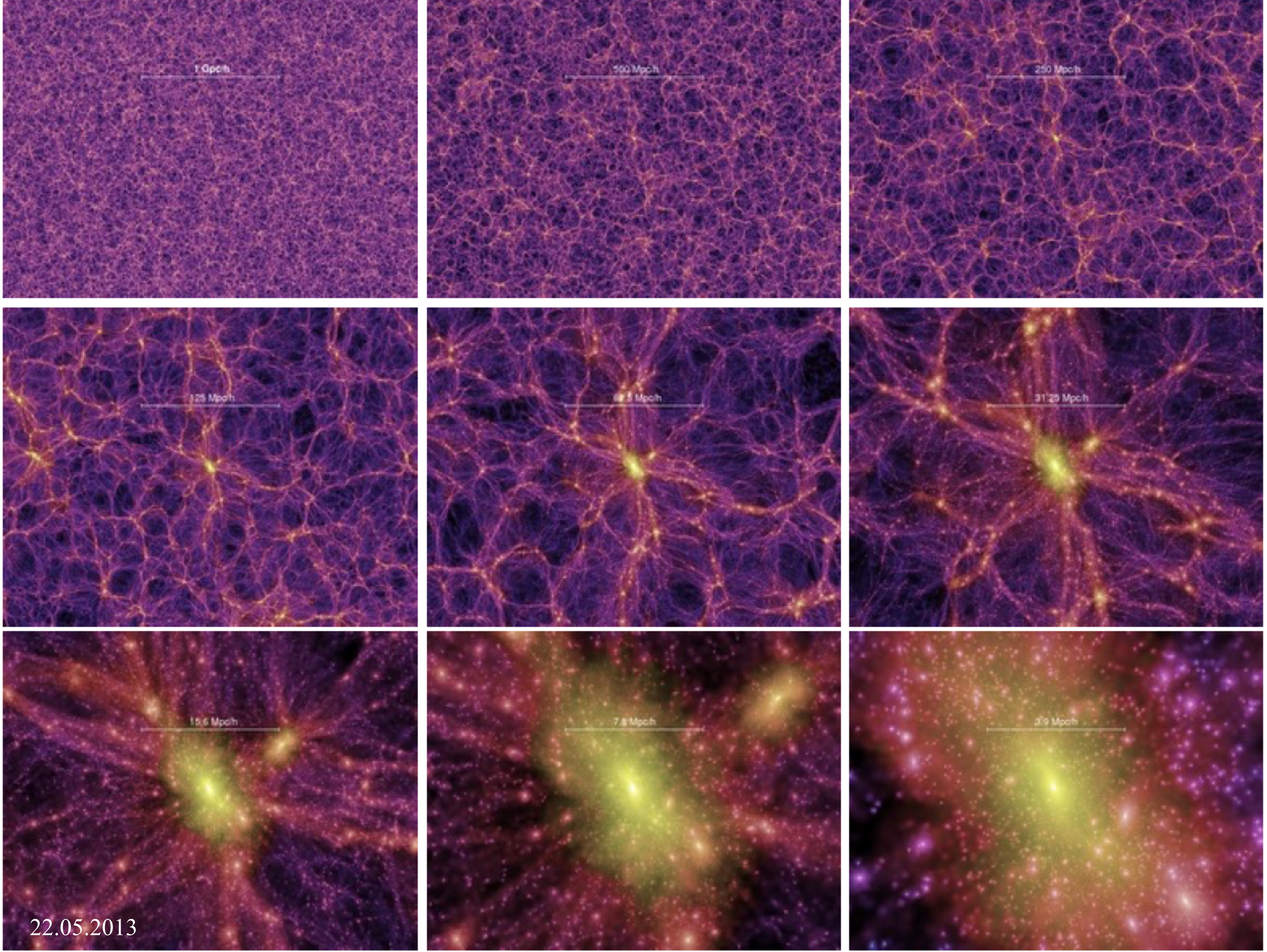
The Millenium simulation set

The Jubilee simulation (Thursday)

The Bolshoi and MultiDark simulation set (Thursday)

…

(HACC – Hardware/Hybrid Accelerated Cosmology Code)


1 Gpc/h
500 Mpc/h
250 Mpc/h
125 Mpc/h
62.5 Mpc/h
31.25 Mpc/h
15.6 Mpc/h
7.8 Mpc/h
3.9 Mpc/h
22.05.2013

Parallelization methods

OpenMp

- Open Multi-Processing
- Application Program Interface components
  - Compiler Directives
  - Runtime Library Routines
  - Environment Variables
- specified for C/C++ and Fortran
- `http://openmp.org`

MPI

- Message Passing Interface: libraries, designed to be a standard for parallel computing on distributed memory.
- Goal: to be practical, portable, efficient, and flexible
- `http://www.mpi-forum.org`

Parallelization methods

OpenMp

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OpenMP

- needs computer with shared memory

MPI

- Message Passing Interface: I
  standard for parallel comput
- Goal: to be practical, portat
- http://www.mpi-forum.org

MPI

- works on distributed memory

OpenMP vs MPI

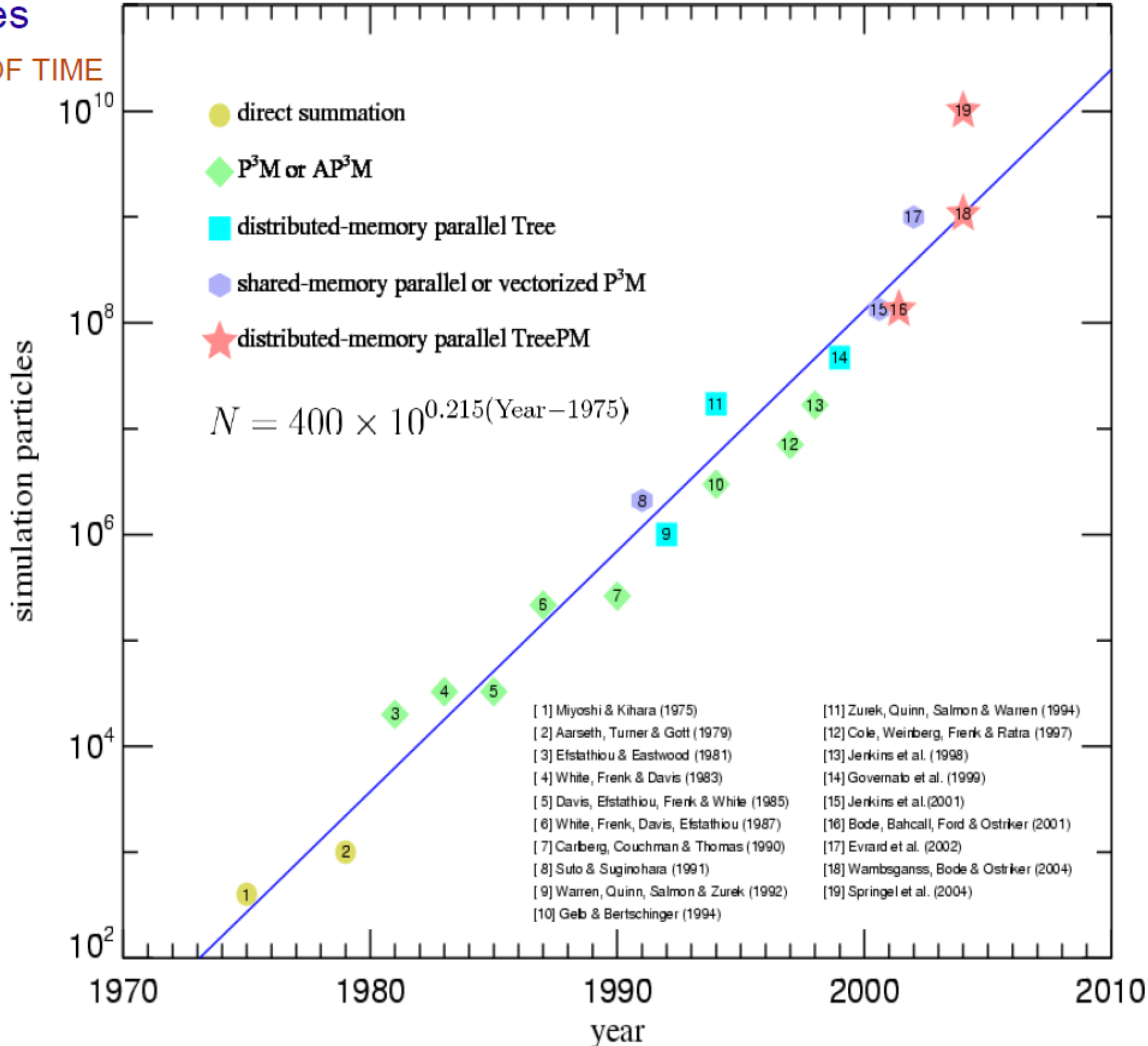
	openMP	MPI
HARDWARE	expensive SMP-machine	inexpensive cluster
PROGRAMMING	inexpensive (via pragmas)	expensive (message passing)
MIGRATION	easy	hard (start from scratch)
SCOPE / SCALABILTY	widely applicable / good	problem dependent

Better performance

Cosmological N-body simulations have grown rapidly in size over the last three decades

"**N**" AS A FUNCTION OF TIME

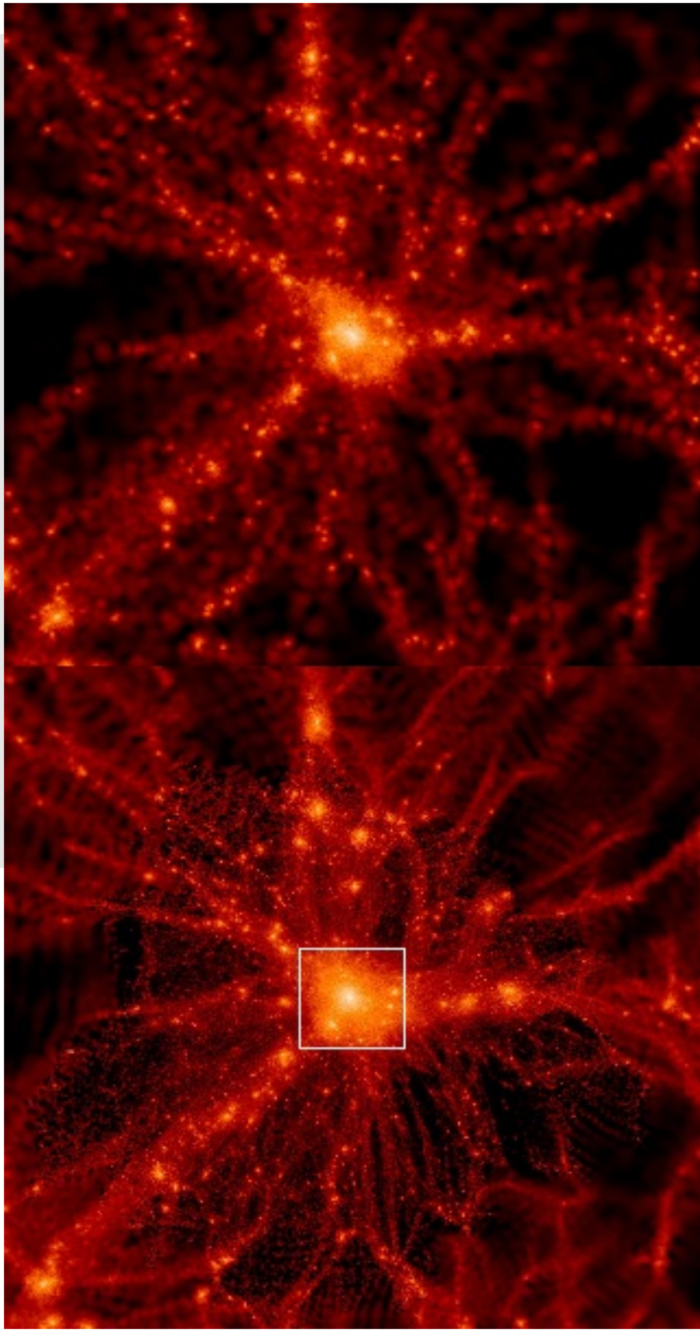
- Computers double their speed every 18 months (Moore's law)
- N-body simulations have doubled their size every 16-17 months
- Recently, growth has accelerated further.
  The Millennium Run should have become possible in 2010 – we have done it in 2004 !



- Zoomed initial conditions

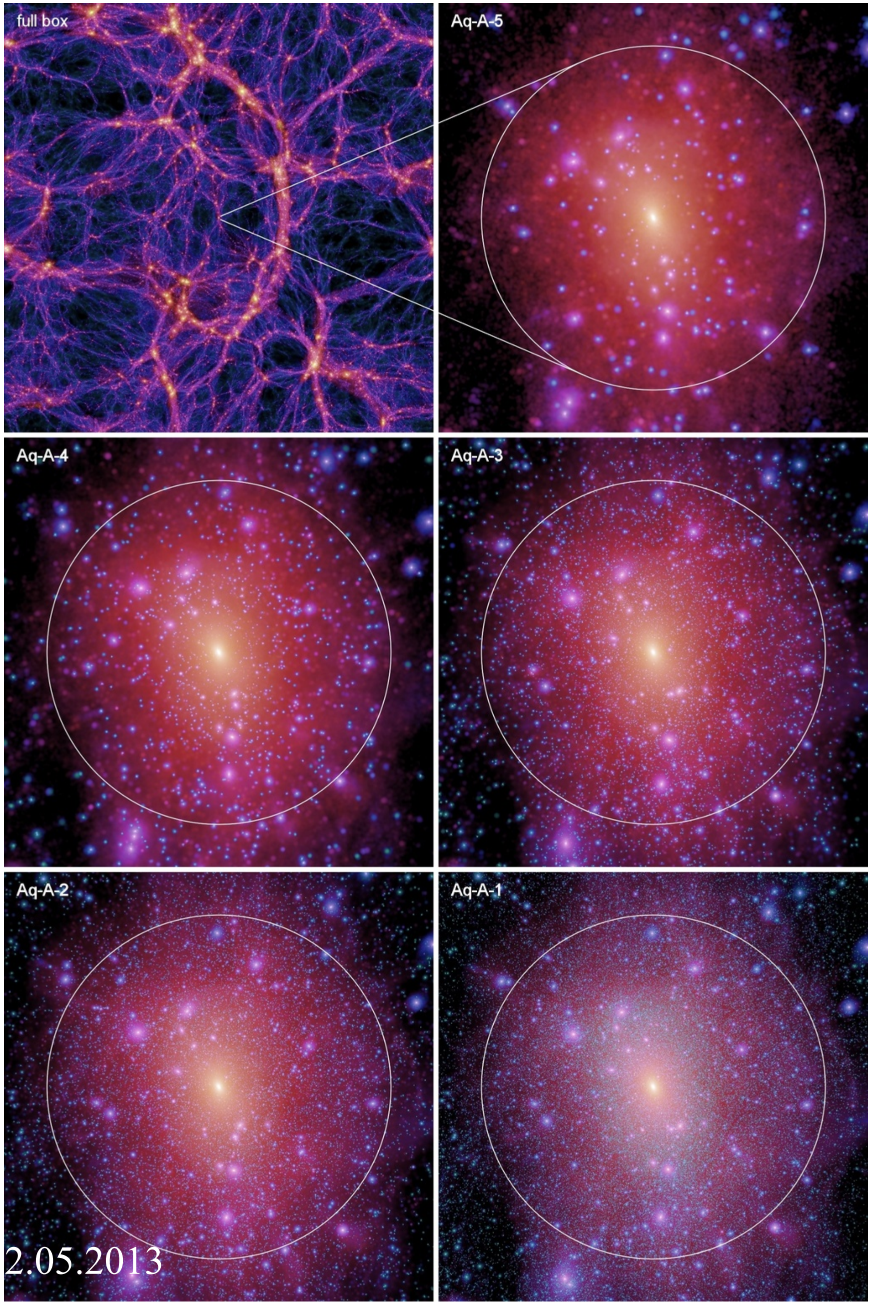
Zoomed initial conditions

- run a "low resolution" cosmological simulation
- find an object of interest
  - cluster
  - group of galaxies
  - isolated galaxy
  - low density region (void)
  - ....
- re-simulate this region in high resolution (mass and force) and keep the rest of the box in low resolution for the right cosmological environment

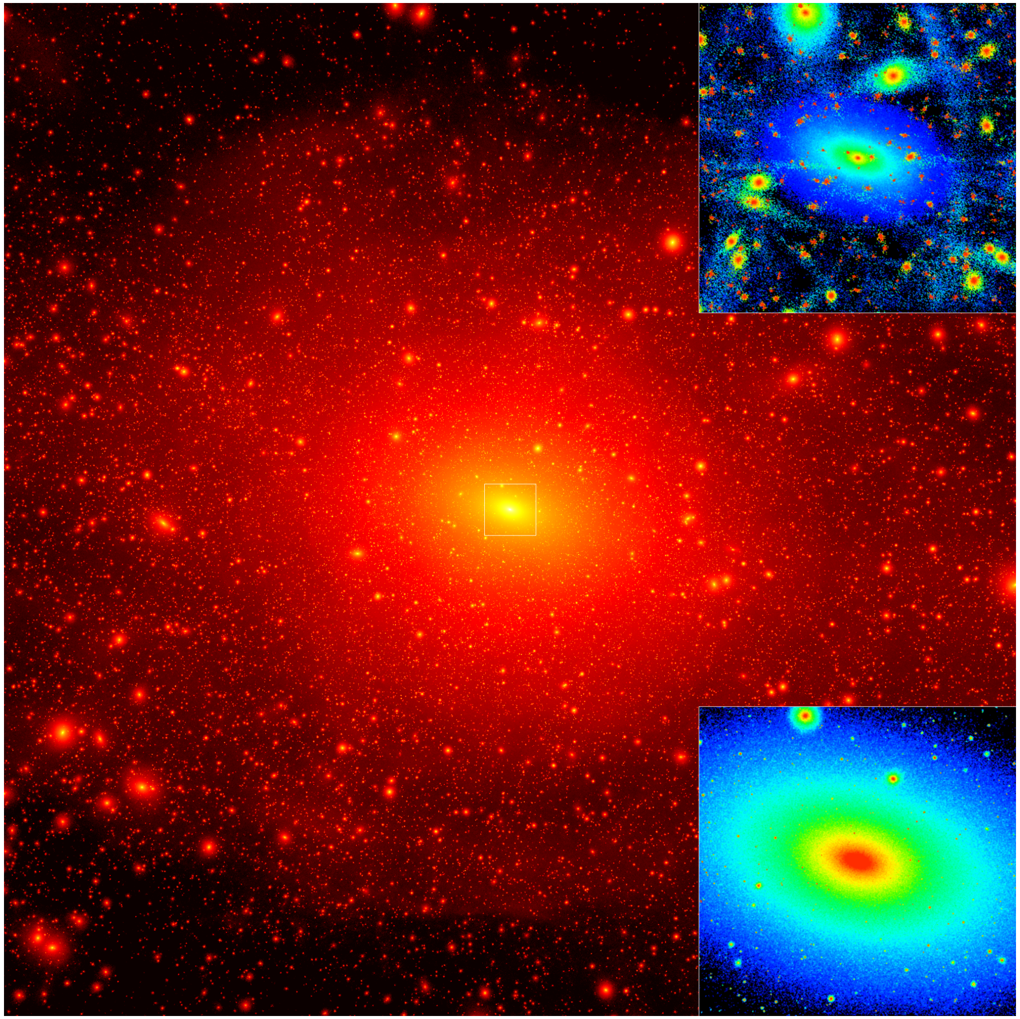


Zoomed initial conditions for individual halos

Aquarius Project

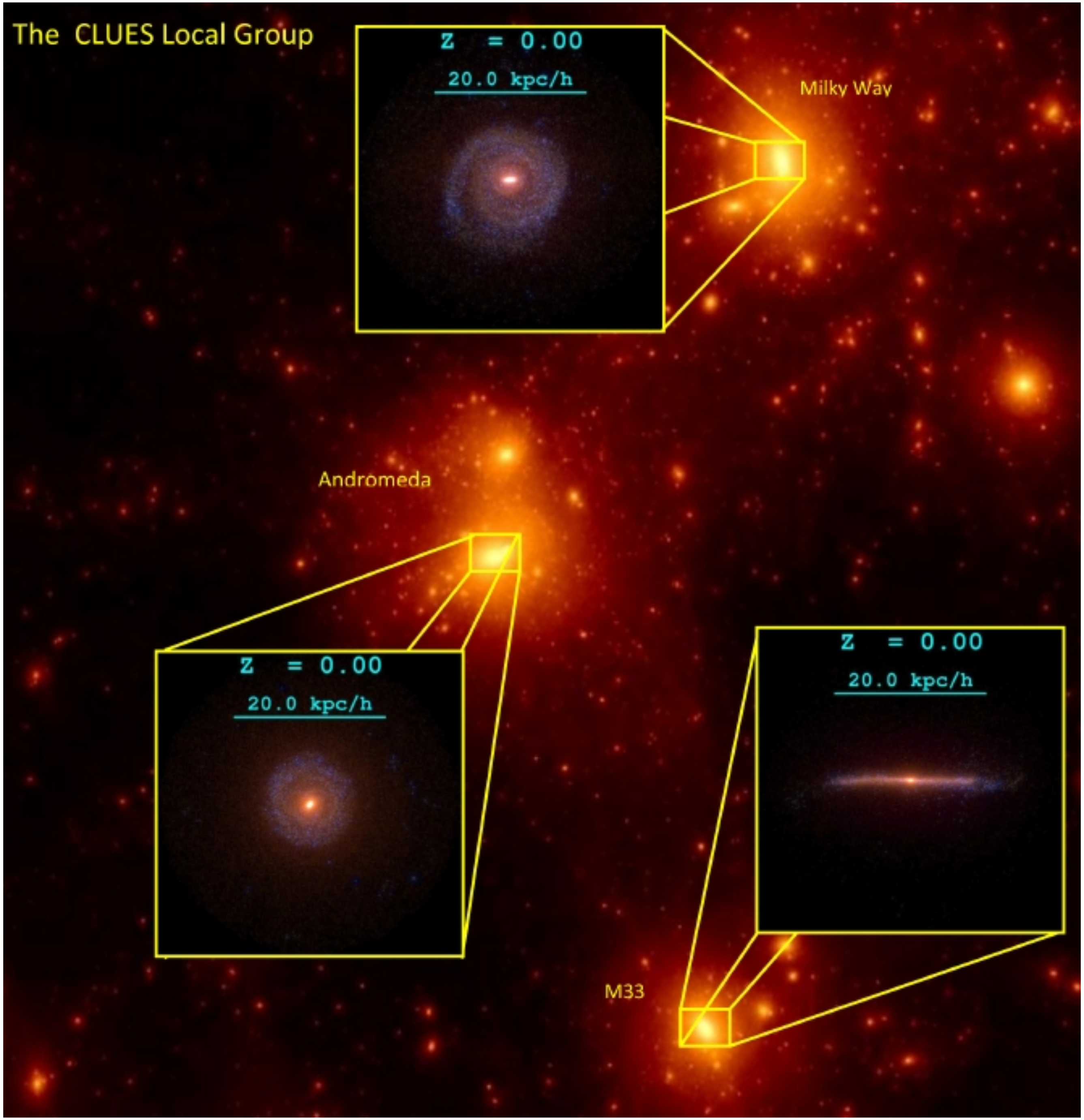


Via Lactea



2.05.2013

Zoomed initial conditions for Local Volume



CLUES Project

(Friday)