

# Mass and occupation fraction of dark matter halos hosting Lyman- $\alpha$ emitters at $z \sim 3$

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## ABSTRACT

We derive constraints on the mass and occupation fraction of dark matter halos hosting Ly $\alpha$  Emitting galaxies (LAEs) at a redshift of  $z = 3.1$  by matching the number density and the angular correlation function between mock and observed fields. We explicitly take into account the cosmic variance on the typical observed field size by constructing mock fields from a large cosmological N-body simulation matching the geometries of observed fields. To populate the halos in the simulation we use a model where a dark matter halo with mass in the range  $M_{\min} < M_h < M_{\max}$  can only host one detectable LAE with a probability  $f_{\text{occ}}$ . The main motivation to build such a model is to avoid the large uncertainties in the estimation of Ly $\alpha$  luminosities. Our main result is that the clustering and number density information are insufficient to impose a tight constraint on the **occupation** fraction. On the other hand the minimum mass and maximum mass can be tightly constrained to the range  $M_{\max} < 10^{12} h^{-1} M_{\odot}$  and  $10^{10} h^{-1} M_{\odot} \leq M_{\min} \leq 10^{11.5} h^{-1} M_{\odot}$ . We also find that all models have a narrow mass range  $\Delta M \equiv \log_{10} M_{\max} - \log_{10} M_{\min}$  smaller than 1.0 dex, with the vast majority in the range  $\Delta M < 0.5$  dex. Already published constraints on the mass range and occupation fraction are a sub-set of the models we report in this paper. Our results present an inconclusive response as to which is the occupation fraction for LAEs at  $z = 3.1$ . A precise answer requires observations on larger fields and robust physical models for the connection between dark matter, star formation and the escape of Ly $\alpha$  radiation. That kind of modelling should also be able to provide a plausible explanation for the narrow range in mass  $\Delta M < 1.0$  dex derived here for the halos hosting LAEs at  $z = 3.1$ .

**Key words:** cosmology: theory cosmology: large-scale structure of universe galaxies: formation galaxies: high-redshift galaxies: statistics galaxy: haloes

## 1 INTRODUCTION

Lyman- $\alpha$  emitting galaxies (LAEs) are helpful in a diverse range of subjects in extragalactic astronomy. LAEs can be used as probes of reionization (Dijkstra et al. 2011), tracers of large scale structure (Koehler et al. 2007), signposts for low metallicity stellar populations, markers of the galaxy formation process at high redshift (Dayal et al. 2009; Forero-Romero et al. 2012) and tracers of active star formation (Guaita et al. 2013).

Capitalizing LAEs observations requires an understanding of their place in the context of a given structure formation model in a cosmological context. Under the current paradigm the dominant matter content of the Universe is to be found in dark matter (DM) and each galaxy is thought to be hosted by larger dark matter structure known as a halo. (Peebles 1980; Springel et al. 2005).

Galaxy formation models find that halo mass predicts with high accuracy galaxy properties such as stellar mass and star formation rate (Behroozi et al. 2013b). This suggests that the physical processes that regulate the star formation cycle are dependent on halo mass. For that reason, finding the typical dark matter halo mass hosting LAEs represents an advance to understand the nature of this galaxy population in the context of Lambda Cold Dark Matter ( $\Lambda$ CDM) paradigm.

Some theoretical attempts to solve this problem using an ab-initio approach. They start from the DM halo population to infer the intrinsic star formation rates and Ly $\alpha$  luminosities. From these values they estimate the amount of Ly $\alpha$  photons that escape each galaxy and compute the observed luminosity for each galaxy. These models can predict different observables including: the luminosity function, correlation function and the equivalent width distributions.

Such modelling has been implemented in semi-analytic models (Garel et al. 2012; Orsi et al. 2012; ?) and full N-body hydrodynamical simulations (Laursen & Sommer-Larsen 2007; Dayal et al. 2009; Forero-Romero et al. 2011; Yajima et al. 2012).

However, there are many uncertain steps in these calculations. The usual ones are the astrophysical processes describing star formation in galactic populations. But the dominant uncertainty is estimating the fraction of Ly $\alpha$  photons that escape the galaxy, a number known as the escape fraction. Given the resonant nature of the Ly $\alpha$  line, the escape fraction is sensitive to the density, temperature, topology and kinematics of the neutral Hydrogen in the interstellar medium (ISM), besides the amount of dust that absorb the Ly $\alpha$  photons. The complex radiative transfer of Ly $\alpha$  photons in the ISM requires Monte Carlo simulations to be solved. Until now it has been difficult to achieve a consensus on the expected value for the Ly $\alpha$  escape fraction in high redshift galaxies (Neufeld 1991; Verhamme et al. 2006; Forero-Romero et al. 2011; Dijkstra & Kramer 2012; Laursen et al. 2013; Orsi et al. 2012).

A different approach to infer the typical mass of halos hosting LAEs is based on clustering information. This method uses the fact that in CDM cosmologies the spatial clustering of galaxies on large scales is entirely dictated by the halo distribution (Colberg et al. 2000), which in turn has a strong dependence on halo mass. Using measurements of the angular correlation function of LAEs, observers have put constraints on the typical mass and occupation fraction of the putative halos hosting these galaxies (Hayashino et al. 2004; Gawiser et al. 2007; Nilsson et al. 2007; Ouchi et al. 2010). In these studies the observations are done on fields of  $\sim 1 \text{ deg}^2$  and the conclusions derived on the halo host mass do not delve too deeply into the possible impact of cosmic variance.

Recently Yamada et al. (2012) observed a wide area of  $2.4 \text{ deg}^2$  under homogeneous instrumental and data reduction conditions. This data set is constructed from 12 different sub-fields. This large and homogeneous data-set allows us to use clustering statistics and cosmic variance to constrain the mass and occupation fraction of halos hosting LAEs.

Our method first populates DM halos in cosmological simulations with LAEs. We only consider whether a DM can host a detectable LAE or not without predicting a Ly $\alpha$  luminosity. This is constructed to bypass all the physical uncertainties involved in the estimation of observed Ly $\alpha$  luminosities. Once we have the mock LAE catalogs, we compare them against the statistics on the number density distribution derived from the observations over the 12 fields of Yamada et al. (2012). Additionally, we use measurements of the angular correlation function to improve these constraints. This method allows us to find the preferred mass range of DM halos hosting these galaxies and its occupation fraction.

This paper is structured as follows. In the next section we present the simulation and the model used to produce the mock catalogs. We also list the criteria used to compare the mocks against observations. In §3 we present the main results for the halo mass and occupation fraction. We continue with a discussion of these results under the light

of other observational and theoretical results. Finally, we present our conclusions in §5.

Throughout this paper we assume a  $\Lambda$ CDM cosmology with the following values for the cosmological parameters,  $\Omega_m = 0.27$ ,  $\Omega_\Lambda = 0.73$  and  $h = 0.70$ , corresponding to the matter density, vacuum density and the Hubble constant in units of  $100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

## 2 METHODOLOGY

Our method is based on the comparison of observations and mock catalogs. We use two different kinds of statistics to perform the comparison: (i) the distribution of the surface number density across fields and (ii) the angular correlation function measured in some fields.

In the next subsections we describe in detail the four key elements of this work-flow. First, we present the observations we take as a benchmark. Second, we describe the main characteristics of the N-body simulation and the halo catalogs we use. Third, we recount the important parameters of the simplified model that we use to populate the halo catalogs with LAEs. Finally we describe some of the statistical tests we adopt to compare observations and mocks.

### 2.1 Observational constraints

The first observational constraint we use in this paper is the LAE number density information at  $z = 3.1$  obtained by the panoramic narrow-band survey presented by Yamada et al. (2012) from a survey conducted with the Subaru 8.2m telescope and the Subaru Prime Focus Camera, which has a field of view covering  $34 \times 27 \text{ arc-min}$ , corresponding to a comoving scale of  $46 \times 35 \text{ Mpc } h^{-1}$  at  $z = 3.09$ . The narrow band filter used in the survey is centered at  $4977 \text{ \AA}$  with  $77 \text{ \AA}$  width, corresponding to the redshift range  $z = 3.062\text{--}3.125$  and  $41 \text{ } h^{-1} \text{ Mpc}$  comoving scale for the detection of the Lyman- $\alpha$  line centered at  $z = 3.09$ . The authors reported a total 2161 LAEs with an observed equivalent width, in the observer frame, larger than  $190 \text{ \AA}$  over a total survey area of  $2.42 \text{ deg}^2$  that includes 12 sub-fields, this corresponds to average surface number density of  $0.20 \pm 0.01 \text{ arcmin}^{-2}$ .

The survey covered four independent fields:

(i) The first is the SSA22 field of  $1.38 \text{ deg}^2$  with 1394 detected LAEs (7 sub-fields), this field has been known to harbor a region with a large density excess of galaxies. **The first sub-field of is taken around the same region observed in Hayashino et al. (2004)** as it is reported by as by Yamada et al. (2012) in section 2.1.

(ii) The second observed region is composed by the fields Subaru/*XMM-Newton* Deep Survey (SXDS)-North, -Center and -South, with a total of  $0.58 \text{ deg}^2$  and 386 LAEs (3 sub-fields).

(iii) The third field is the Subaru Deep Field (SDF) with  $0.22 \text{ deg}^2$  and 196 LAEs (1 sub-field), and

(iv) The fourth one is the field around the Great Observatory Optical Deep Survey (GOODS-N) with  $0.24 \text{ deg}^2$  and 185 LAEs (1 sub-field).

There is abundant observational work done on LAEs at redshift  $z = 3.1$  (Kudritzki et al. 2000; Matsuda et al. 2005; Gawiser et al. 2007; Nilsson et al. 2007; Ouchi et al. 2008).

However, we decide to focus on the data from Yamada et al. (2012) because it has the largest covered area with homogeneous instrumentation conditions (telescope, narrow band filter), data reduction pipeline and conditions to construct the LAE catalog. This ensures that the number density variations among fields are *not* due to different observational conditions or criteria to construct the catalogs.

A secondary benchmark is the angular correlation function (ACF). Yamada et al. (2012) does not report an ACF measurement for the entire field. Instead we use the results by Hayashino et al. (2004) who has reported the ACF in the SSA22 region, including its densest subfield. In addition to Hayashino's ACF we use as a matter of comparison the ACF reported by Ouchi et al. (2008). This ACF is computed over an observed field of  $1 \text{ deg}^2$  over the SXDS field.

There are some differences between Ouchi et al. (2008) observations and those by Yamada et al. (2012). The details in the color selection, corresponding limiting luminosities and EW thresholds are different in these references. The average LAEs number density inferred by Hayashino et al. (2004) is  $0.37 \text{ arcmin}^{-2}$ , which is 15% lower than the new measurement by Yamada et al. (2012). In the case of Ouchi et al. (2008) the number density is  $0.10 \text{ arcmin}^{-2}$ , which is  $\sim 50\%$  higher than the value inferred by Yamada et al. (2012) for a portion of the same region.

## 2.2 The adequacy of including SSA22 as an observational benchmark

The field SSA22 has been known to harbor a significant galaxy overdensity. Yamada et al. (2012) estimates that the densest subfield is likely to be a rare density peak with  $3 - 4\sigma$  significance on the scale of  $\sim 60h^{-1} \text{ Mpc}$ .

Excluding that high density sub-field in SSA22, all the fields are average with a similar number density as the blank fields rare. The left panel in Figure 2 shows the integrated distribution of LAEs number density in all the fields reported in Yamada et al. (2012). It is evident that there is only one sub-field that stands out as an outlier in the distribution (it corresponds to the densest field in SSA22). All the other fields form a continuous distribution in number density.

Therefore, using the whole SSA22 region as a benchmark in the number density distribution does not impose any bias. However, one has to keep in mind that it is not plausible that the full number density distribution from a simulation will include a very dense field. The statistical test we perform to compare mock distributions against observations does not require such a perfect match.

We will not require the presence of such field to judge a mock distribution as successful. In terms of the ACF the results that include the densest field in SSA22 (Hayashino et al. 2004) are consistent, within the uncertainties, with the results of other fields (Ouchi et al. 2008).

Finally, two very different number densities in LBGs versus LAEs evidences that the two populations have different biases. In a  $\Lambda$  CDM cosmology this implies that LAEs are, on average, hosted by halos less massive than those hosting LBGs. The interpretation of how rare is this subfield in terms of the number densities of dark matter halos has to assume a defined model for galaxy formation; i.e. defining the link between halo mass, star formation rate,  $\text{Ly}\alpha$  luminosity and  $\text{Ly}\alpha$  escape fraction.

The purpose of our work is to avoid any loose of generality by assuming a defined halo-LAE connection and instead explore all the possible halo populations to host LAEs.

## 2.3 Simulation and halo catalogs

The Bolshoi simulation (Klypin et al. 2011) we use in this paper was performed in a cubic volume of  $250 h^{-1} \text{ Mpc}$  on a side. The dark matter distribution is sampled using  $2048^3$  particles, which translates into a particle mass of  $m_p = 1.35 \times 10^8 h^{-1} M_\odot$ . The cosmological parameters are consistent with a WMAP5 and WMAP7 data with a matter density  $\Omega_m = 0.27$ , cosmological constant  $\Omega_\Lambda = 0.73$ , dimensionless Hubble constant  $h = 0.70$ , slope of the power spectrum  $n = 0.95$  and normalization of the power spectrum  $\sigma_8 = 0.82$  (Komatsu et al. 2009; Jarosik et al. 2011).

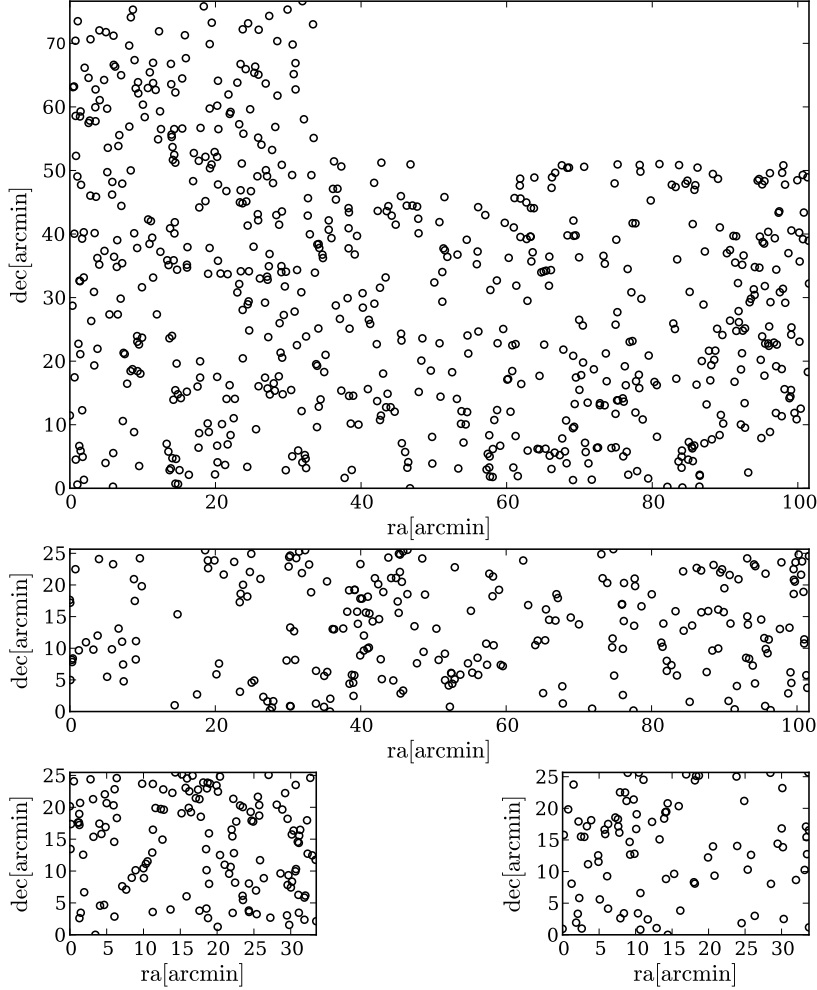
We use halo catalogs constructed with a Friend-of-Friends (FOF) algorithm with a linking length of 0.17 times the inter-particle distance. The catalogs were obtained from the publicly available Multidark database<sup>1</sup> (Riebe et al. 2013). For each halo in the box we store its comoving position in the box (3-D coordinates) and FOF mass. We focus our work on halos more massive than  $1 \times 10^{10} h^{-1} M_\odot$  that are resolved with at least 70 particles, the reasons for this choice are explained in the next sub-section.

## 2.4 A model to populate halos with LAEs

We assume that a dark matter halo can only host one detectable LAE at most. There are three parameters that decide whether a halo host a LAE: the lower and upper bounds for the mass range,  $M_{\min} < M_h < M_{\max}$ , where LAEs reside and the fraction  $f_{\text{occ}}$  of such halos that host a detectable LAE. The reader must keep in mind that the physical interpretation of the occupation fraction  $f_{\text{occ}}$  convolves two phenomena: the actual presence of a star forming galaxy in a halo and its detectability as a LAE. We do not perform an explicit modeling for LAE detectability. Our model does not assign a luminosity or escape fraction to each LAE. We are only interested in constraining the halo mass range hosting detectable LAEs under the conditions defined by Yamada et al. (2012).

In what follows we note by the letter  $\mathcal{M}$  a model defined by a particular choice of the three scalar parameters  $M_{\min}$ ,  $M_{\max}$  and  $f_{\text{occ}}$ . For each model  $\mathcal{M}$  we create a set of mock fields from disjoint volumes in the simulation. Each volume has the same geometry probed by Suprime-CAM

<sup>1</sup> <http://www.multidark.org/MultiDark/>



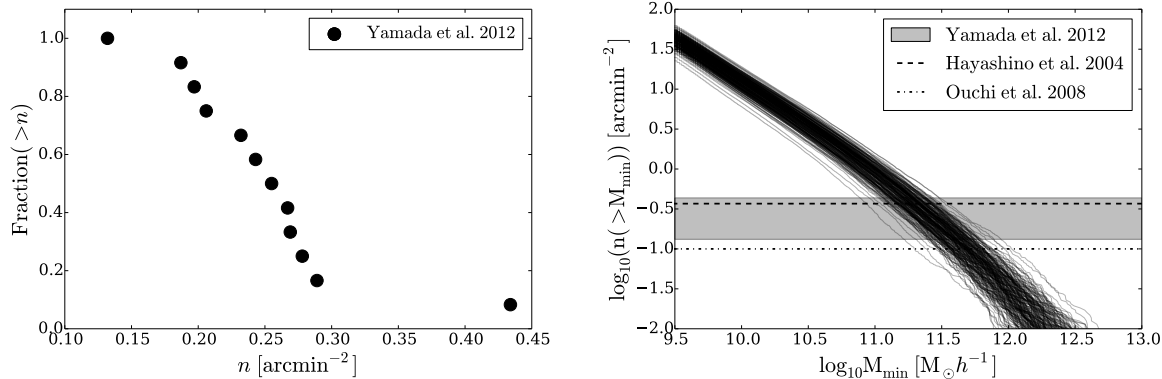
**Figure 1.** Spatial distribution of a LAEs mock survey for a model with parameters  $\log_{10} M_{\min} = 10.4$ ,  $\log_{10} M_{\max} = 10.5$  and  $f_{\text{occ}} = 0.1$  for the **match** method. The larger panel shows 7 mock fields together mimicking the SSA22 region. The intermediate panel shows 3 mock fields corresponding to the SXDS region. The lower panels represent the SDF and GOODS-North fields. The same model has another 14 associated mock surveys build from different regions in the simulation, but constructed following the same pattern in the **match** method.

and the narrow band filter, namely rectangular cuboids of dimensions  $46 \times 35 \times 41 h^{-3} \text{ Mpc}^3$  where the last dimension goes in the redshift direction. This corresponds to a total area of  $880 \text{ arcmin}^2$  in each mock field. We construct a total  $5 \times 7 \times 6 = 210$  of such volumes from a snapshot in the Bolshoi simulation. In each mock field a LAE is assigned to the position of a dark matter halo if the halo mass is in the range allowed by the model  $M_{\min} < M_h < M_{\max}$  and a random variable taken from an homogeneous distribution  $0 \leq \xi < 1$  is smaller than the occupation fraction  $\xi < f_{\text{occ}}$ .

Next we construct mock surveys by making groups of 12 mock fields out of the 210 available volumes. In total 15 mock surveys are constructed for each model  $\mathcal{M}$ . The grouping of the 12 mock fields into a mock catalog is done in two different ways. The first is called **match**, it follows the clustering of the observational fields. From the 12 mock fields, 7 are constructed from contiguous fields in the simulation to mimic the SSA22 region, 3 are also contiguous between them but not to the first 7 fields to mimic the SXDS fields and finally 2 non-contiguous fields to imitate the SDF and GOODS-North field. The second way to group the mock

fields is called **random**, whereby all the 12 fields are selected in such a way as to avoid that any two volumes are contiguous. In this paper we only report the results obtained by the **match** method and mention explicitly differences observed with the **random** selection.

Figure 1 shows the spatial distribution for one mock survey constructed using the **match** method. Each field corresponds to one of the observational fields. The model parameters to build it are  $M_{\min} = 10^{10.4} h^{-1} \text{ M}_{\odot}$ ,  $M_{\max} = 10^{10.5} h^{-1} \text{ M}_{\odot}$  and  $f_{\text{occ}} = 0.1$ . It is only one out of the 15 different mock surveys that are constructed for each model. We note that we use  $15 \times 12 = 180$  mock fields out of the total of 210 available sub-volumes. The reason is that the **match** method imposes constraints on the way the 7 fields mimicking the SSA22 can be distributed. This restriction makes unable some of the sub-volumes in the box. We decide to keep the number of mock surveys fixed to 15 also for the **random** method in order to allow a fair comparison between the two methods.



**Figure 2.** Left panel. Distribution of LAE number densities in all the fields observed by Yamada et al. (2012). The point with the highest surface density corresponds to the densest sub-field in the SSA22 field. Right panel. Surface density of dark matter halos as a function of a minimum halo mass to count the total number of elements in a volume. Each line represents one of the 210 volumes of dimensions  $46 \times 35 \times 41 h^{-3} \text{ Mpc}^3$  in the Bolshoi simulation. The horizontal gray band represents the range of surface densities observed for LAEs at  $z = 3.1$  as reported by (Yamada et al. 2012).

## 2.5 Exploring and selecting good models

We make a thorough exploration of the parameter space for the models  $\mathcal{M}$  where  $\log_{10} M_{\min}$  takes 30 values from 10.0 up to 12.9 with an even spacing of 0.1 dex.  $\log_{10} M_{\max}$  takes values in the same range as  $\log_{10} M_{\min}$  only with a displacement of 0.1 dex in the whole range. The occupation fraction  $f_{\text{occ}}$  takes 10 different values from 0.1 to 1 regularly spaced by 0.1. In total the number of different models  $\mathcal{M}$  that are explored is  $30 \times 30 \times 10 = 9000$ .

The lower limit for the parameter  $M_{\min}$  is set by the minimum occupation fraction we are able to consider. At  $M_{\min} = 10^{10} h^{-1} M_{\odot}$  the halo number density is already  $\sim 10$  times higher than the observational constraints for LAEs. This means that models in that mass range and an occupation fraction  $f_{\text{occ}} = 0.1$  have the possibility to be compatible with observations. Lower values for  $M_{\min}$  require  $f_{\text{occ}} < 0.1$ , which are not considered in this paper.

For each mock survey generated in a given model  $\mathcal{M}$  we compute the surface density in the 12 mock fields. We perform a Kolmogorov-Smirnov (KS) to compare this mock data against the 12 observational values. From this test we obtain a value  $0 < P < 1$  to reject the null hypothesis, namely that two data sets come from the same distribution. In this paper we consider that for values  $P > 0.05$  the two distributions can be thought as coming from the same distribution.

We begin by considering that a model  $\mathcal{M}$  that has at least one mock survey (out of 15) consistent with the observed distribution of LAE number densities has viable parameters that deserve to be considered for further analysis. Later on we impose harder constraints to reduce the number of models by asking that all the 15 mocks to be consistent with observations.

The second constraint comes from the angular correlation function (ACF) for all the models having the 15 mock surveys consistent with observations.

**The ACF is computed using the Landy & Szalay estimator (Landy & Szalay 1993) on two different fields in the mock survey. The first field is the densest to be compared against the results of Hayashino**

**et al. (2004). The second field has an average number density and is compared against Ouchi et al. (2010).**

The observed and mock ACF are fit to a power-law function:

$$\omega(\theta) = \left( \frac{\theta}{\theta_0} \right)^{-\beta}, \quad (1)$$

where  $\theta_0$  and  $\beta$  are free parameters. The fit is done using a least square minimization procedure. For each mock field we obtain a covariance matrix that gives us the uncertainty in the parameters  $\beta$  and  $\theta_0$ . Using the 15 mock fields in a mock survey, we compute a mean value for the parameters and its associated uncertainty as the quadratic average of the uncertainties in the fitting procedure. We consider that a model is consistent with observations if the two parameters  $\beta$  and  $\theta_0$  are equal within a  $1-\sigma$  range.

## 3 RESULTS

The main purpose of this section is to show how different observational constraints narrow down the parameters space of allowed models. Each sub-section presents the effect of adding new pieces of observational or statistical evidence.

### 3.1 Dark Matter Halo Number Density

In Figure 2 we present the results for the integrated dark matter halo surface density as a function of the minimum halo mass  $M_{\min}$ . Each line corresponds to one of the 210 sub-volumes in the Bolshoi simulation. The gray band indicates the surface density values for LAEs allowed by observations (Yamada et al. 2012).

This result allows us to understand why only a specific range of models  $\mathcal{M}$  can be expected to be consistent with observations. From Figure 2 we can read that models with a minimum mass  $M_{\min} > 10^{11.5} h^{-1} M_{\odot}$  will always have a surface number density lower than the observational constrain. This makes impossible that models in that range can be compatible with observations, there are simply too few halos

compared to observed LAEs. The opposite is true in models with  $M_{\min} < 10^{10.5} h^{-1} M_{\odot}$  that have a surface number density larger observations. In those cases the occupation fraction  $f_{\text{occ}} < 1.0$  can be tuned in order to lower the halo number density to match observations.

Figure 2 also illustrates the impact of cosmic variance. At fixed mass there is a scatter of  $0.3 - 0.6$  dex in the number density abundance. A consequence of this variation is that models with the same mass range and occupation fraction can have mock fields with number densities varying up to a factor of  $\sim 2 - 5$ . This scatter is comparable to the  $0.3$  dex scatter coming from observations. In the same Figure 2 we see that at fixed number density, around the range allowed by observations, there is a scatter in the masses of  $0.4 - 0.5$  dex.

This scatter induced by cosmic variance is naturally included in the mock construction process. This will allow us to have a wider range of models compatible with observations, as compared with a model that neglects cosmic variance.

### 3.2 Models consistent with the surface density distributions

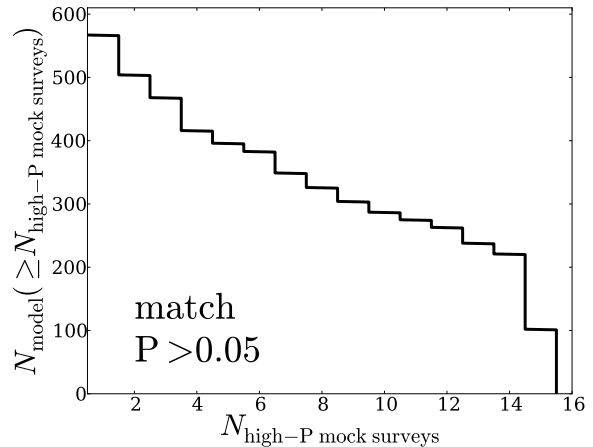
Figure 3 presents regions in parameter space  $M_{\min} - M_{\max}$ ,  $M_{\min} - f_{\text{occ}}$  where the KS test yields values of  $P > 0.05$  at least for one mock survey, which is the most conservative assumption for agreement between a model and observations. For those models it is not possible to reject the hypothesis that the simulated and observed data for the surface number density come from the same parent distribution. In total, there are between 550 to 600 models out of the original 9000 models that have at least one mock survey consistent with observations. By inspection of Figure 3 we see that the halo number abundance is able to constraint the minimum mass  $M_{\min}$  to a narrow range, while  $M_{\max}$  and  $f_{\text{occ}}$  remain largely unconstrained.

In Figure 3 there are three regions of parameter space that can be clearly distinguished. The first corresponds to models where the minimum mass is high  $M_{\min} > 10^{11.5} h^{-1} M_{\odot}$ . None of these models is compatible with observations as expected from the arguments presented in the previous subsection. For these models the number density of LAEs is too low compared to observations.

The second region corresponds to an intermediate range for the minimum mass  $10^{10.5} h^{-1} M_{\odot} < M_{\min} < 10^{11.5} h^{-1} M_{\odot}$  where, regardless of the value of the maximum mass  $M_{\max}$ , it is possible to tune the occupation fraction  $f_{\text{occ}}$  to bring some of the mock surveys into good agreement with observations. In this region of parameter space one can find two extreme kinds of models. One kind where the mass interval  $\Delta M \equiv \log_{10} M_{\max} - \log_{10} M_{\min}$  is narrow with  $\Delta M < 1.0$  dex, others where the mass interval is extended  $\Delta M > 1.0$  dex going up to the maximum halo mass present in the simulation at that redshift, with  $\Delta M = 2.5$  dex in those cases.

The third region in parameter space corresponds to  $M_{\min} < 10^{10.5} h^{-1} M_{\odot}$ . In this case only models with a very narrow mass interval of at most  $0.5$  dex ( $M_{\max} < 10^{11.0} h^{-1} M_{\odot}$ ) and low occupation fractions  $f_{\text{occ}} \leq 0.3$  are allowed.

Without any additional information our method allows



**Figure 4.** Number of models with at least  $N_{\text{high-P}}$  mock surveys consistent with the observed surface number density distribution in terms of the KS-test values  $P > 0.05$ . Only  $\sim 100$  models have all their mocks consistent with observations.

us to infer that most of the successful models are found in the second and third regions of parameter space. This result was already expected from halo abundance calculations shown in Figure 2 and discussed in the previous subsection.

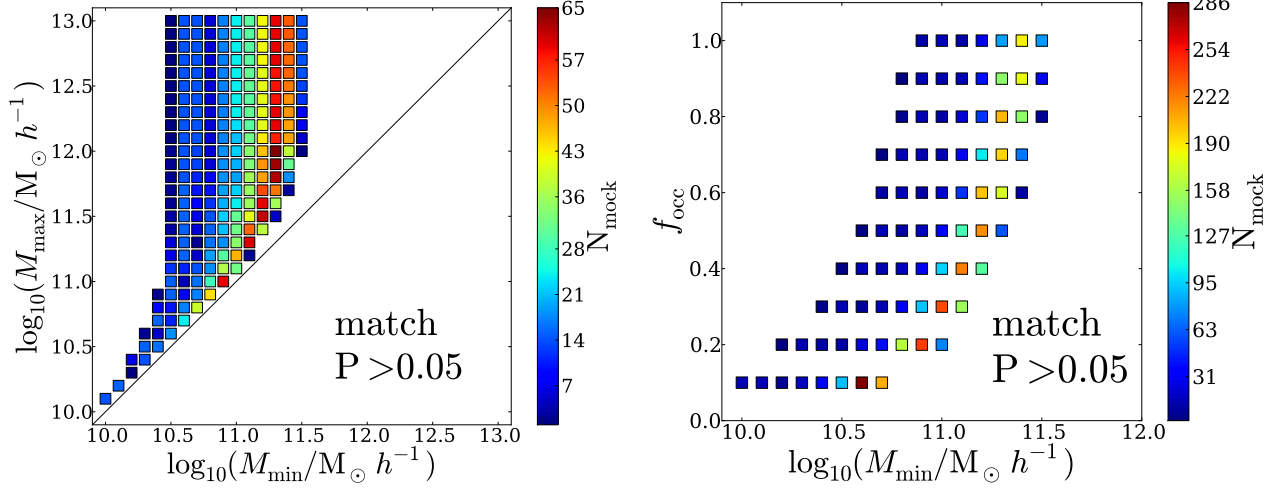
However, the additional information we gain with this test is the relative abundance of models in the parameter space. Not all models in the second region have an equal abundance. By inspection of Figure 3 it seems that models with  $\log_{10} M_{\min} \sim 10^{11.3} h^{-1} M_{\odot}$  and low occupation fraction  $f \leq 0.3$  are preferred. In the next sub-sections we explore in detail the models in this region, imposing tighter constraints on the KS test results and exploring the mocks' consistency with the angular correlation function.

### 3.3 Models with the largest number of consistent mock surveys

In the previous sub-section we presented a conservative criterion of agreement by selecting the models that had at least one mock survey with  $P > 0.05$  in the KS test. Now we turn to a more strict selection by requiring all constructed mock surveys to be consistent with observations.

Figure 4 shows the number of models that have at least  $N_{\text{high-P}}$  mocks with  $P > 0.05$ . For each model  $\mathcal{M}$  there are 15 different mock surveys. We discard the models that have 14 mocks or less consistent with observations. With this cut we find  $\sim 100$  models with all the 15 mock survey realizations with  $P > 0.05$ . This cut represents a reduction by a factor of  $\sim 6$  with respect to the total number of models with at least one consistent mock.

Figure 5 presents the locii of these models in the parameter space  $M_{\min} - M_{\max}$  and  $M_{\min} - f_{\text{occ}}$ . With this constraints the number of consistent models with  $10.5 < \log_{10} M_{\min} < 11.0$  are greatly reduced. This corresponds to the regions in the parameter space in Figure 3 that already had a low number of consistent mock surveys. On the other hand, from the right panel in Figure 5 one can see that the reduction of favored values for the occupation fraction  $f_{\text{occ}}$  is not strong.



**Figure 3.**  $M_{\min}$ - $M_{\max}$  (left) and  $M_{\min}$ - $f_{\text{occ}}$  (right) planes for all models with KS test values  $P > 0.05$ . The color code corresponds to the number of mock surveys compatible with observations. Only regions of parameter space with at least one consistent mock survey are included.

We conclude that conditions on the number density statistics, even if they are very strict, only put strong constraints on the minimum mass  $M_{\min}$  but not on the other two parameters of the model  $M_{\max}$  and  $f_{\text{occ}}$ . In the next section we use additional observational information to constrain these parameters.

### 3.4 Consistency with the Angular Correlation Function

Figure 6 shows the main results in a  $\theta_0 - \beta$  plane where the average and standard deviation over the mocks is shown in comparison with the result derived from observations. In the left panel Figure 6 we see how the observational ACF measured by Hayashino et al. (2004) can be successful in reducing the total number of possible models. The right panel shows the results for fields in the mock catalog with an average number density compared against the fit of the data reported by Ouchi et al. (2010). In this case we find that there are large uncertainties coming from the observational data, and only a reduced fraction of the models can be discarded.

In Figure 7 we present the preferred models in the planes  $M_{\min}$ - $M_{\max}$  and  $M_{\min}$ - $f_{\text{occ}}$  both for the comparisons against Hayashino et al. (2004) and Ouchi et al. (2010). To make this selection we consider that a model is consistent with observations if there is a  $1-\sigma$  overlap between both the correlation length  $\theta_0$  and the power  $\beta$ . We clearly see from this Figure that the conditions imposed by Hayashino et al. (2004) are the tightest. Comparing this Figure with the results shown in Figure 5 we see that practically all models with  $M_{\max} > 10^{12}$  are removed. On the other hand the comparison against Ouchi et al. (2010) does not substantially reduce the number of models.

At this point we take the most strict decision to keep models consistent with both observational constraints. This reduces the selection to the models already restricted by the

data in Hayashino et al. (2004) as presented in the lower panels of Figure 7. Our main conclusions after this comparison is that clustering information is able to put meaningful constraints on the values of  $M_{\max}$ . However, the possible values for the occupation fraction, despite having a strong correlation with  $M_{\min}$ , still are in the initial range  $0.1 \leq f_{\text{occ}} \leq 1$ .

## 4 DISCUSSION

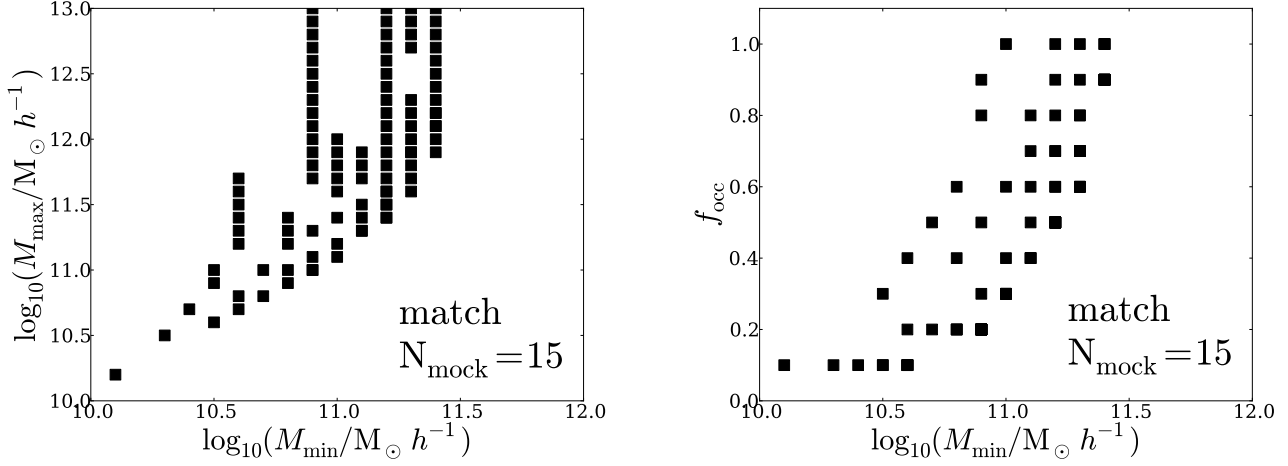
In the previous Section we started by matching the rough numbers for the galaxy surface number density. This sets the median mass of all successful models to the broad range  $(10^{10}-10^{12}) h^{-1} M_{\odot}$  for  $M_{\min}$  and  $M_{\max}$ . Then we included additional criteria on the number of mock surveys that must be consistent observations and the information from the angular correlation function, reducing the number of feasible models. We ended up with 23 models out of the initial 9000 possible combination of parameters.

Now we define the halo mass range  $\Delta M = \log_{10} M_{\max} - \log_{10} M_{\min}$ , to use it together with the occupation fraction,  $f_{\text{occ}}$ , and the minimum mass  $M_{\min}$  to classify all the successful models into two families:

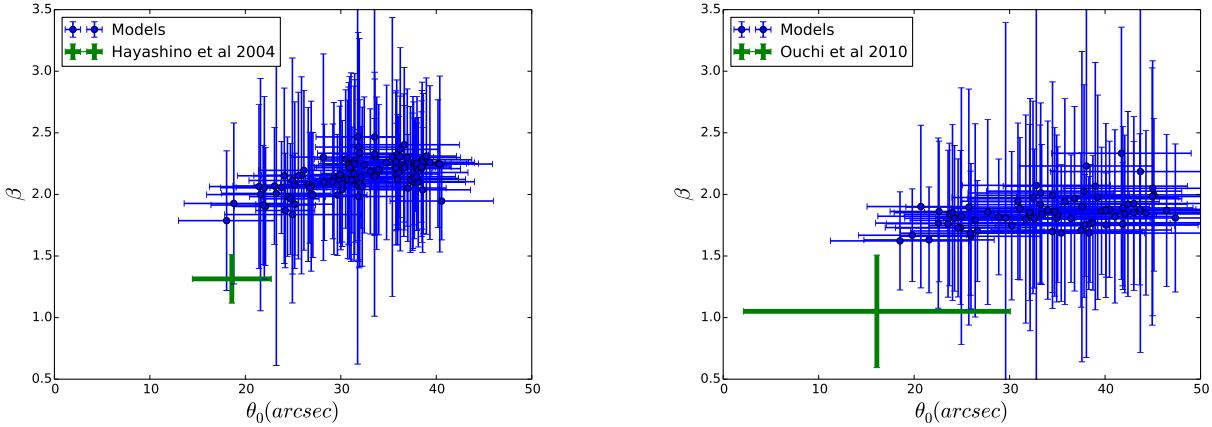
- (1) Low  $f_{\text{occ}} \leq 0.3$ ,  $M_{\min} \leq 10^{10.8} h^{-1} M_{\odot}$  and low  $\Delta M \leq 1.0$  dex: 12 models.
- (2) High  $f_{\text{occ}} > 0.3$ ,  $M_{\min} \geq 10^{10.8} h^{-1} M_{\odot}$  and low  $\Delta M \leq 1.0$ : 11 models

The full list of parameters in the two families is summarized in Table 1 and Table 2. We do not have a clear majority of any family. This leads us to the following two conclusions. First, the occupation fraction cannot be constrained based on clustering analysis. Second, the halos hosting LAEs have a narrow mass range.

The existence of a narrow mass range for halo masses has different possible explanations. A cut at the low mass end,  $M_{\min}$ , can be interpreted in terms of the minimal halo



**Figure 5.** Favored regions in parameter space when the constraints on the maximal number of consistent mocks is imposed. Only the results for the *match* method are shown.



**Figure 6.** Values for the free parameters  $\theta_0$  and  $\beta$  in the fitting formula (Eq. 1) for the angular correlation function. Blue dots correspond to simulations and the green cross to observations by Hayashino et al. (2004) (left) and Ouchi et al. (2010) (right). The error bars in the theoretical data correspond to the quadratic average of the fitting errors for each mock survey

star formation rate needed to produce the Ly $\alpha$  luminosity needed to be above a given detection threshold. However, under the reasonable assumption of star formation rate increasing with halo mass, the cut at higher halo masses  $M_{\text{max}}$  requires a different justification. There are two complementary physical scenarios that could provide it.

One scenario can be presented in terms of a decreasing escape fraction of Ly $\alpha$  radiation in massive systems. Detailed galaxy formation models support the idea that massive galaxies with higher metallicities have larger dust contents and less concentrated ISM than lower mass systems. Due to the resonant nature of the Ly $\alpha$  line the probability of absorption of Ly $\alpha$  photons increases in massive systems, producing high absorption of the Ly $\alpha$  line but not of UV continuum or other non-resonant lines (Laursen et al. 2009; Forero-Romero et al. 2011). In a second scenario larger systems have more extended gaseous envelopes which due to resonance effects of the Ly $\alpha$  line, induces a low surface brightness and a broader line, making these systems less ob-

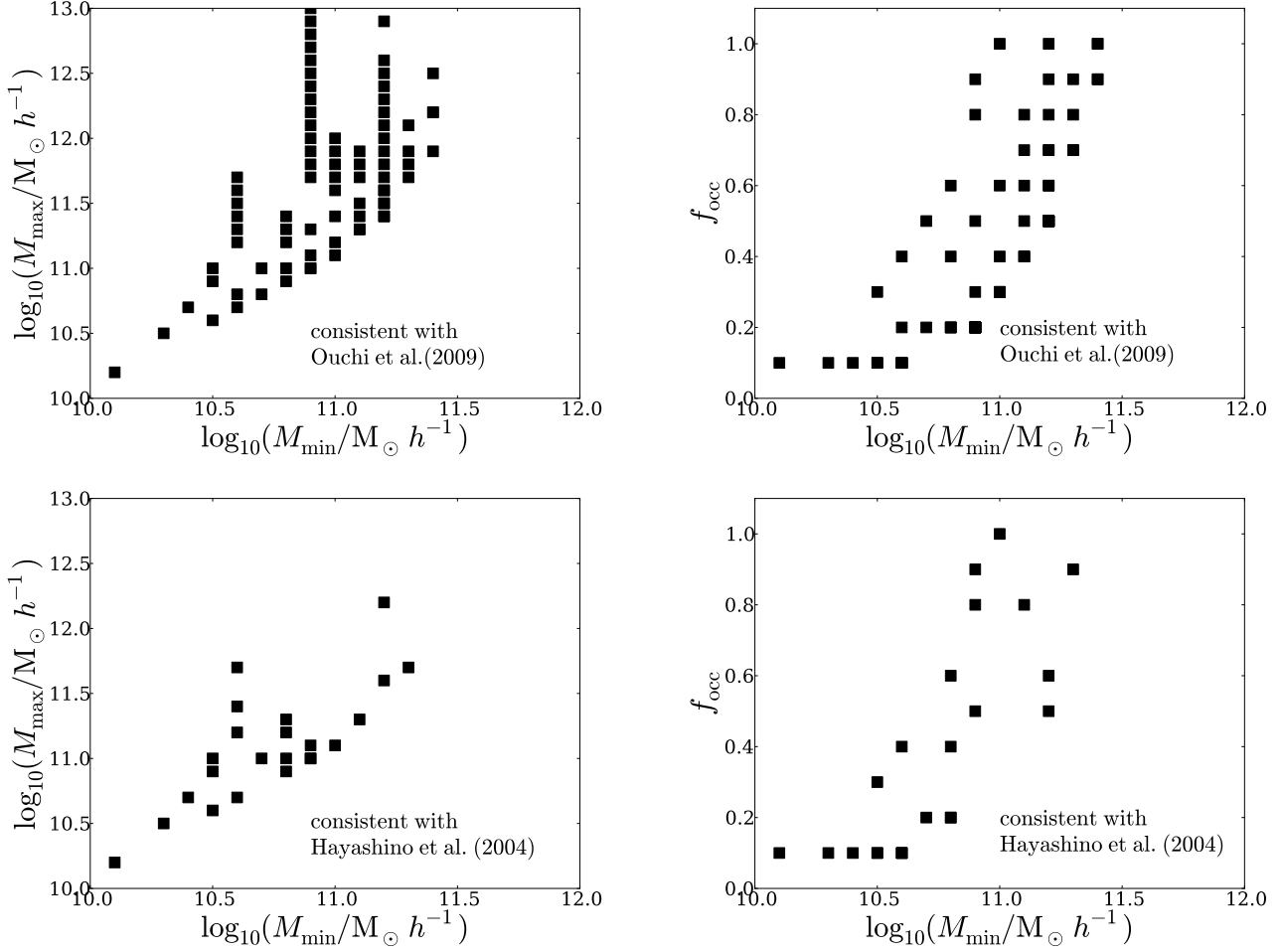
servable in narrow band filter surveys (Laursen et al. 2009; Zheng et al. 2010).

#### 4.1 Comparison against results from galaxy surveys

Theoretical interpretations of blind galaxy surveys at  $z = 2.2$  find a volume weighted escape fraction  $f_{\text{esc}} \sim 0.1 - 0.2$  Hayes et al. (2010), similar results are obtained in a recent theoretical study of observational data in a wide redshift range  $0 < z < 6$  (Dijkstra & Jeeson-Daniel 2013).

Hayes et al. (2010) showed that the luminosity function for LAEs at  $z = 2.2$  is consistent with the escape fraction being constant for every galaxy regardless of its luminosity. From this results they derive that almost 90% of the star forming galaxies emit insufficient Lyman  $\alpha$  to be detected, effectively setting the occupation fraction to be  $f_{\text{occ}} = 0.10$ . Dijkstra & Jeeson-Daniel (2013) use similar methods to find an average Ly $\alpha$  escape fraction in high- $z$  galaxies, they con-





**Figure 7.** Planes  $M_{\min}$ - $M_{\max}$  (left) and  $M_{\min}$ - $f_{\text{occ}}$  (right). with the models fulfilling both constraints on the maximal number of consistent mocks and the angular correlation function. Upper panels correspond checks the consistency with the observational data of Ouchi et al. (2010). Lower panels show results after comparison against Hayashino et al. (2004)

clude that there is a range in Ly $\alpha$  luminosity where the escape fraction they derive at  $z \sim 3.0$ ,  $f_{\text{esc}} = (17 \pm 5)\%$ , could be interpreted as an occupation fraction  $f_{\text{occ}} \sim 0.2$ . We consider a success that our model indeed includes such low occupation fraction models. The observational evidence from galaxy surveys seem to disfavor the second family of models with high occupation fractions. Nevertheless, it remains an open task to explain the narrow mass range  $\Delta M < 0.5$  in most of the models.

#### 4.2 Comparison to other clustering estimates

Observational evidence based on the ACF inferred from photometric measurements in the Extended Chandra Deep Field South has shown that the median dark matter masses of  $h$  los hosting LAEs is  $\log_{10} M_{\text{med}} = 10.9^{+0.5}_{-0.9} M_{\odot}$ , with a corresponding occupation fraction of 1–10% (Gawiser et al. 2007). Ouchi et al. (2010) presents analysis of LAE observations in the redshift interval  $3.1 < z < 7.0$  and at  $z = 3.1$  They quote an average mass for the host dark matter halos of  $M_h = 2.9^{+24.0}_{-2.9} \times 10^{10} h^{-1} M_{\odot}$  with a corresponding duty cycle of  $0.008 \pm 0.03$ .

Our results are in a general good agreement with those estimates for the host mass. This is not completely unexpected given that we have also required consistency with ACF measurements. These expectations are matched by the first family of models, also summarized in Table 1. These models, which favor only the very low occupations fractions, are also consistent in that regards with the observational expectations.

The novelty in our results is that we have a detailed estimate for host halo mass range together with the escape fraction. This allows us to demonstrate that the halo mass range can in some cases be narrow  $\Delta M < 0.3$  dex, something that cannot be inferred from ACF analysis alone. Furthermore, In contrast to (Gawiser et al. 2007) and (Ouchi et al. 2010) we find that an ACF analysis on a single data-set is not enough to rule out models with a high occupation fraction  $f_{\text{occ}} > 0.3$ , which represent half of our best models.

#### 4.3 In the context of abundance matching models

The abundance matching methods are based on observational results for Lyman Break Galaxies (LBGs) (Behroozi

$\log_{10} M_{\min}$	$\log_{10} M_{\max}$	$f_{\text{occ}}$	$\Delta M$
10.1	10.2	0.1	0.1
10.3	10.5	0.1	0.2
10.4	10.7	0.1	0.3
10.5	10.6	0.3	0.1
10.5	10.9	0.1	0.4
10.5	11.0	0.1	0.5
10.6	11.2	0.1	0.6
10.6	11.4	0.1	0.8
10.6	11.7	0.1	1.1
10.7	11.0	0.2	0.3
10.8	11.2	0.2	0.4
10.8	11.3	0.2	0.5

**Table 1.** List of parameters for the first family of models. Narrow mass range  $\Delta M \leq 1.0$  dex and low occupation fraction  $f_{\text{occ}} \leq 0.3$ .

$\log_{10} M_{\min}$	$\log_{10} M_{\max}$	$f_{\text{occ}}$	$\Delta M$
10.6	10.7	0.4	0.1
10.8	10.9	0.6	0.1
10.8	11.0	0.4	0.2
10.9	11.0	0.8	0.1
10.9	11.0	0.9	0.1
10.9	11.1	0.5	0.2
11.0	11.1	1.0	0.1
11.1	11.3	0.8	0.2
11.2	11.6	0.6	0.4
11.2	12.2	0.5	1.0
11.3	11.7	0.9	0.4

**Table 2.** List of parameters for the second family of models. Narrow mass range  $\Delta M \leq 1.0$  dex and high occupation fraction  $f_{\text{occ}} > 0.3$ .

et al. 2013b,a). In the case of Behroozi et al. (2013b) the minimum halo mass considered to be relevant in their analysis is  $10^{11.4} h^{-1} M_{\odot}$ . They report stellar mass around  $(1.0 \pm 0.3) \times 10^{9.0} h^{-1} M_{\odot}$ , while their star formation rate is in the range  $0.6 \pm 0.2 M_{\odot} \text{yr}^{-1}$ , which nevertheless are close to the lower bound of values inferred for LAEs at high redshift (Gawiser et al. 2007; Nilsson et al. 2009; Pentericci et al. 2009).

In our results, all the preferred models have a halo mass range lower than the minimum of  $M_{\min} < 10^{11.4} h^{-1} M_{\odot}$  considered in abundance matching at  $z = 3$ . Our results confirm the expectations that most of LAEs are to be found in less massive halos LBG hosts. A detailed analysis of the spectral and photometric properties of LAEs coupled to the kind of analysis performed in this paper can be a guide in the study of the properties of low mass dark matter halos at  $z = 3.1$ , extending the capabilities of abundance matching methods.

#### 4.4 Caveats of our method

There are two important caveats for the work presented here. The first is the assumption of a single LAE per dark matter halo. This contradicts the general expectation of dark matter sub-halos in the simulation to host satellite galaxies.

However, it has been found in analysis based on the shape of the correlation function (Jose et al. 2013) that satellite galaxies are not a dominant population, making our initial approximation a reasonable one. Therefore we conclude that including the effect of satellite galaxies won't change the main results reported in this paper.

The second caveat are the precise values for the mass intervals. These values are defined from the halos defined in using the FOF halo finder. Different halo finders and definitions for the detection density threshold can yield different masses up to a factor. For instance a Friends-of-Friends algorithm with linking length  $l = 0.20$  times the average inter-particle distance finds halos on average 1.4 times less massive than halos defined with an spherical overdensity algorithm halos (Klypin et al. 2011). Therefore, the mass values for  $M_{\min}$  and  $M_{\max}$  should not be considered exact within less than  $\sim 0.2$  dex.

#### 4.5 On the reproducibility of our results

All the software, raw and processed data to produce the results and plots in this paper are publicly available in a github repository <sup>2</sup>. Most of the code to produce the plots can be found as an Ipython notebook (Pérez & Granger 2007) in the same repository.

### 5 CONCLUSIONS

In this paper we look for constraints on the preferred mass and occupation fraction of dark matter halos hosting Lyman Alpha Emitters at redshift  $z = 3.1$  in a  $\Lambda$ CDM cosmology. We perform this study paying special attention to the impact of cosmic variance on these results. To this end we build a large number of mock catalogs matching observational geometries. The mocks are constructed from a N-body simulation following a simple recipe to assign a single LAE to each halo. Only a fraction  $f_{\text{occ}}$  of halos with a mass range  $M_{\min} < M_h < M_{\max}$  can host a LAE. We proceed with a thorough exploration of the space of free parameters  $M_{\min}$ ,  $M_{\max}$  and  $f_{\text{occ}}$  to find mocks that are consistent with two observational constraints: the surface number density and the angular correlation function. Out of the initial 9000 combinations of parameters in the model we find 23 arrangements consistent with observations.

We find that the observational information we use is insufficient to impose a tight constraint on the escape fraction. On the other hand the minimum mass and maximum mass are tightly constrained as follows  $M_{\max} < 10^{12} h^{-1} M_{\odot}$  and  $10^{10} h^{-1} M_{\odot} \leq M_{\min} \leq 10^{11.5} h^{-1} M_{\odot}$ . Furthermore, we find that the mass range defined as  $\Delta M = \log_{10} M_{\max} - \log_{10} M_{\min}$  is always  $\Delta M \leq 1.0$ .

The wide range in solutions is facilitated by the large dispersion in the statistics derived from the mocks. Nevertheless, all the halo mass ranges and occupation fractions deduced previous analysis (i.e. Gawiser et al. 2007; Ouchi et al. 2010) are a subset of the models we find in this paper. All of our models support the notion that the most massive halos at  $z = 3.1$  do not host the brightest LAEs.

<sup>2</sup> <https://github.com/forero/CosmicVarianceLAES>

The existence of a narrow mass range for halos hosting LAEs,  $\Delta M \leq 1.0$  dex, has two different explanations at least. The first is having LAEs with a decreasing Ly $\alpha$  escape fraction with increasing mass, the second is having larger screening effects by neutral Hydrogen around the most massive systems (Laursen et al. 2009; Forero-Romero et al. 2011).

However, there are extreme cases with a very narrow  $\Delta M < 0.3$ , meaning that there is barely a factor of  $\sim 2$  between the minimum and halo mass. If these models turn out to be confirmed, this would be a great challenge for galaxy formation models to explain how is that LAEs can be hosted in such a narrow range of halo mass.

In summary, we show how spatial information of LAEs cannot put a tight constraint on their host halos and occupation fraction. We foresee that the new observations with new instruments (such as MUSE, Hyper SuprimeCam and HETDEX) covering larger fields and a wider range of luminosities will be key in imposing tighter constraints on the properties of dark matter halos hosting LAEs. At the same time, additional modeling for Ly $\alpha$  radiation transfer, is needed to put a tighter constrain on the Ly $\alpha$  escape fraction in high redshift galaxies, also paying attention to newly framed physical phenomena, such as the stochasticity (Forero-Romero & Dijkstra 2013) in the star formation process, which might play a role in inducing detection biases in high redshift LAEs.

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