Dark matter at the electroweak scale with neutrino masses (based on arxiv 1704.01162)

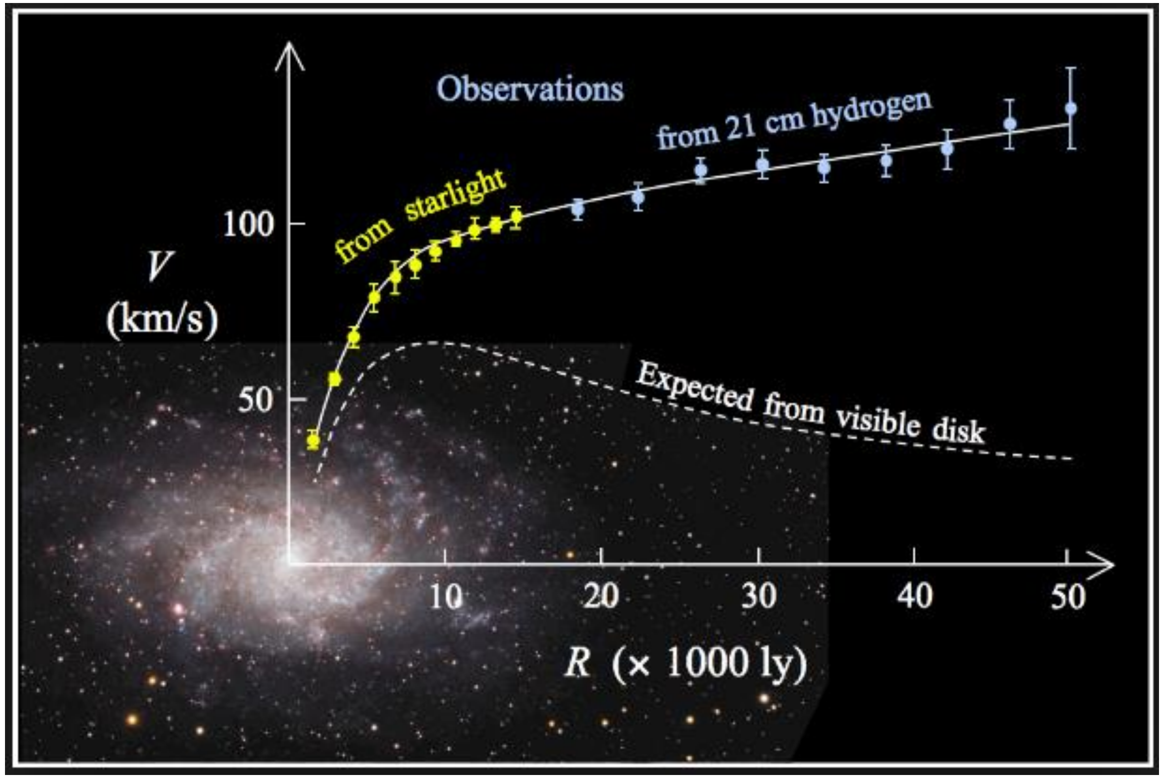
MOCa
June 27, 2017

By Amalia Betancur. In collaboration with
Robinson Longas and Óscar Zapata

Outline:

- Motivation
- The doublet-triplet model
- Direct Detection
- Indirect Detection
- Higgs diphoton decay rate
- Neutrino masses
- Conclusions

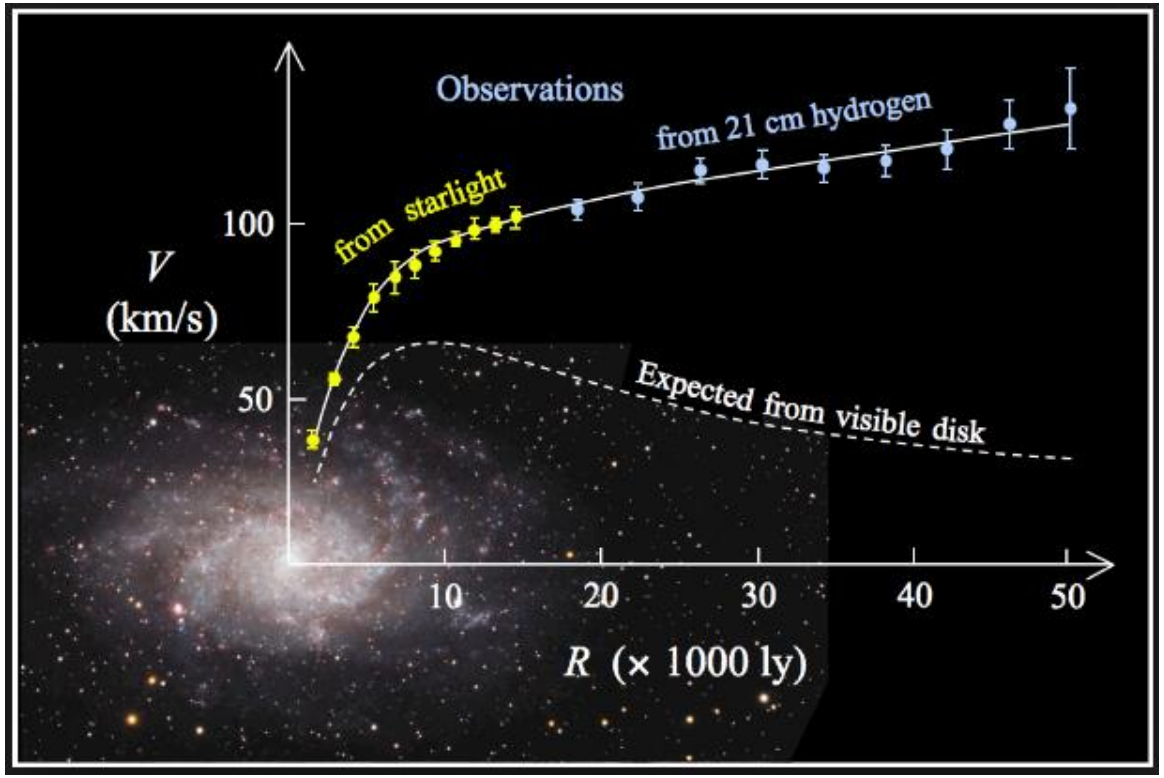
Motivation: Dark Matter



- Dark matter accounts for 25% of matter-energy content of the universe.
- Evidence comes from a myriad of observations, ranging from small scales to large scales, galaxy rotation curves, astrophysical simulations, CMB, etc.

Image credit: Wikipedia

Motivation: Dark Matter



Dark matter candidate:

- Neutral
- Cold
- Stable

Image credit: Wikipedia

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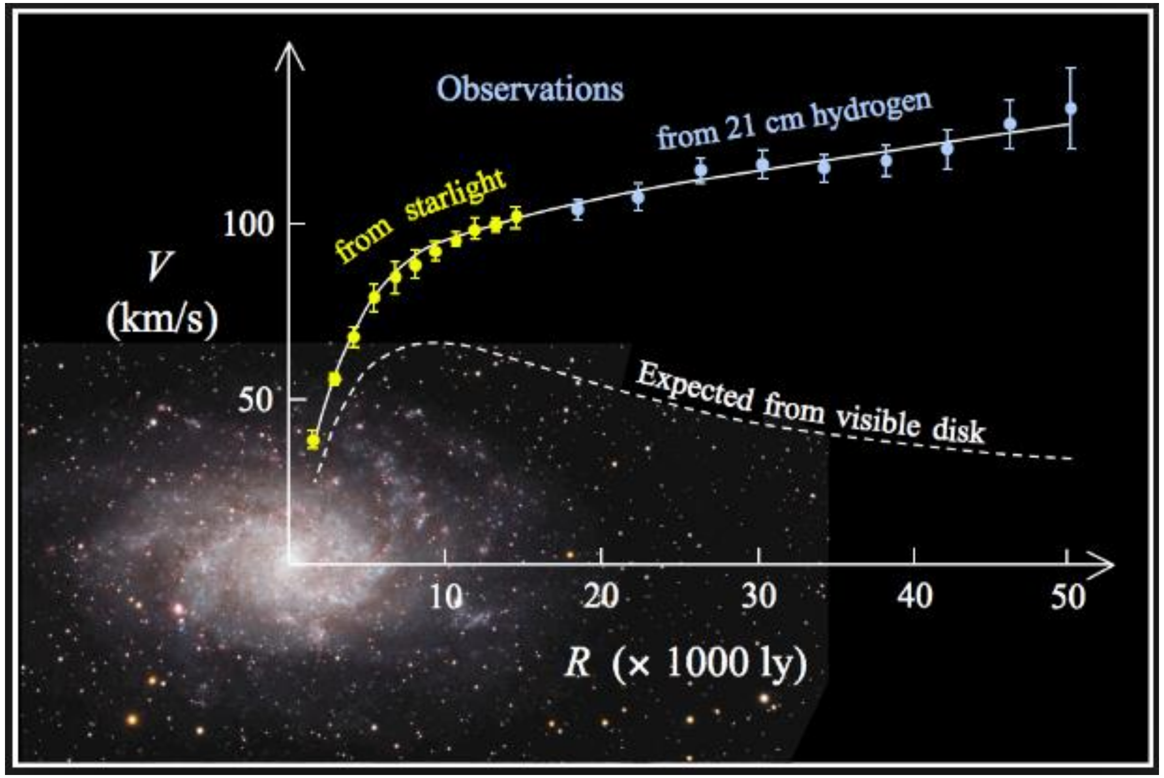

Dark matter candidate:

- Neutral
- Cold
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No particle within the SM fulfills these criteria!

Image credit: Wikipedia

Motivation: Neutrino Masses

- Neutrino oscillation data shows that neutrinos must have masses
- It is not possible to accommodate this fact within the Standard Model.

Image credit:
www.nobelprize.org

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We need BSM physics!

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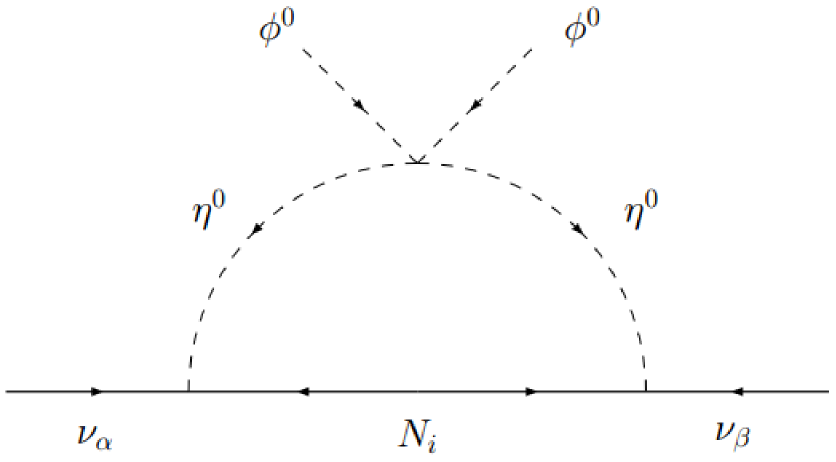
No neutrino masses at tree-level due to the imposed Z_2 symmetry



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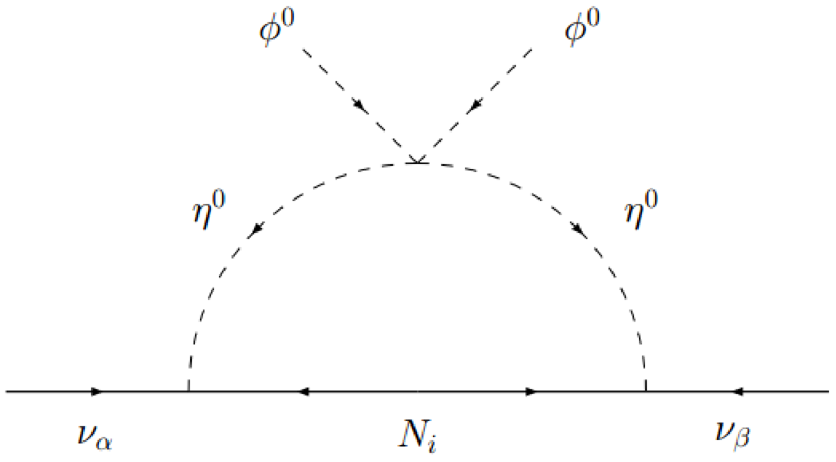

Small neutrino masses due to the loop suppression

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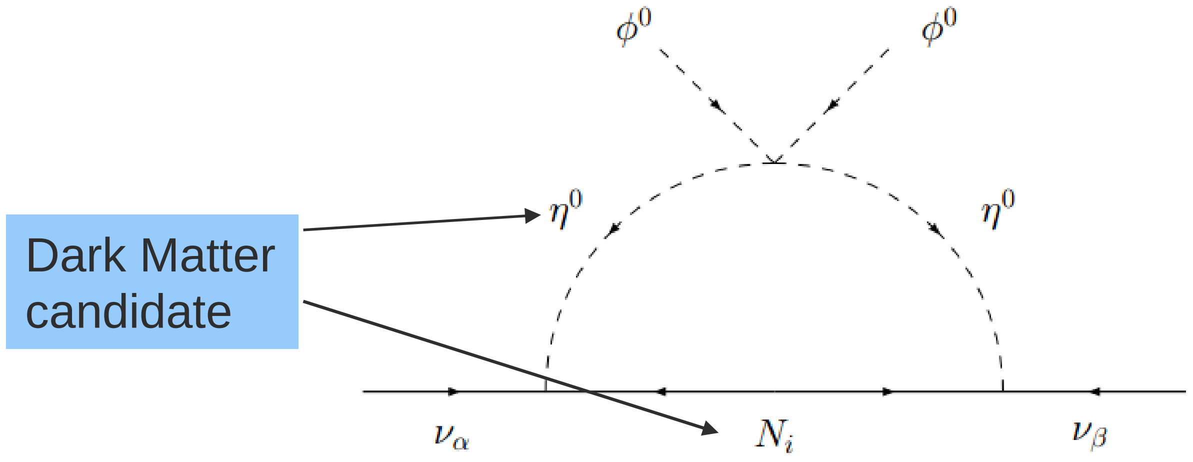

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Small neutrino masses due to the loop suppression

Doublet-Triplet model: Scalar and fermions

$$\psi_L = \begin{pmatrix} \psi_L^0 \\ \psi_L^- \end{pmatrix} \qquad \psi_R = \begin{pmatrix} \psi_R^0 \\ \psi_R^- \end{pmatrix} \quad \Sigma_L \equiv \sqrt{2}\Sigma_L^i \tau^i = \begin{pmatrix} \Sigma_L^0/\sqrt{2} & \Sigma_L^+ \\ \Sigma_L^- & -\Sigma_L^0/\sqrt{2} \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H^+ \\ \frac{H^0 + iA^0}{\sqrt{2}} \end{pmatrix} \qquad \Delta = \frac{1}{2}\begin{pmatrix} \Delta_0 & \sqrt{2}\,\Delta^+ \\ \sqrt{2}\,\Delta^- & -\Delta_0 \end{pmatrix}$$

All odd under the Z_2 symmetry.The SM fields are even

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Dark matter candidates!

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Either the IDM or
Masses at the TeV scale

$$H_2 = \begin{pmatrix} H^+ \\ \frac{H^0 + iA^0}{\sqrt{2}} \end{pmatrix} \qquad \Delta = \frac{1}{2}\begin{pmatrix} \Delta_0 & \sqrt{2}\,\Delta^+ \\ \sqrt{2}\,\Delta^- & -\Delta_0 \end{pmatrix}$$

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Doublet-Triplet: Fermion sector

$$\mathcal{L}_\psi = \bar{\psi} i\gamma^\mu D_\mu \psi - M_\psi(\bar{\psi}_R \psi_L + \text{h.c.})$$

$$\mathcal{L}_{\Sigma_L} = \text{Tr}[\bar{\Sigma}_L i\gamma^\mu D_\mu \Sigma_L] - \frac{1}{2}\text{Tr}(\bar{\Sigma}_L^c M_\Sigma \Sigma_L + \text{h.c.})$$

$$\mathcal{L}_1 = -y_1 H_1^\dagger \overline{\Sigma_L^c} \epsilon \psi_R^c + y_2 \overline{\psi_L^c} \epsilon \Sigma_L H_1 + \text{h.c.}$$

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Mixing among fermions !

Only allowed new interaction terms due to the Z_2 symmetry

Doublet-Triplet fermion: The model

$$\mathbf{M}_{\Xi^0} = \begin{pmatrix} M_\Sigma & \frac{1}{\sqrt{2}} yv\cos\beta & \frac{1}{\sqrt{2}} yv\sin\beta \\ \frac{1}{\sqrt{2}} yv\cos\beta & 0 & M_\psi \\ \frac{1}{\sqrt{2}} yv\sin\beta & M_\psi & 0 \end{pmatrix}$$

$$\mathbf{M}_{\Xi^\pm} = \begin{pmatrix} M_\Sigma & yv\cos\beta \\ yv\sin\beta & M_\psi \end{pmatrix}$$

Majorana fermions

χ_1^0 χ_2^0 χ_3^0

$\chi_1^\pm$ $\chi_2^\pm$

Doublet-Triplet fermion: Blind spot

$$\mathcal{L}_1 = -\boxed{y_1} H_1^\dagger \overline{\Sigma_L^c} \epsilon \psi_R^c + \boxed{y_2} \overline{\psi_L^c} \epsilon \Sigma_L H_1 + \text{h.c.}$$

$$\boldsymbol{y_1 = y_2 = y}$$

$$\mathbf{M}'_{\Xi^0} = \begin{pmatrix} M_\Sigma & yv & 0 \\ yv & M_\psi & 0 \\ 0 & 0 & -M_\psi \end{pmatrix}$$

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Decoupled eigenvalue!

$$\chi_1^0 = 1/\sqrt{2}(\psi_L^0 + \psi_R^{0c})$$

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Decoupled eigenvalue!

$$m_{\chi_1^0} = -M_\psi$$

$$m_{\chi_2^0} = m_{\chi_1^\pm}$$

$$m_{\chi_3^0} = m_{\chi_2^\pm}$$

$$\chi_1^0 = 1/\sqrt{2}(\psi_L^0 + \psi_R^{0c})$$

$$m_{\chi_1^\pm, \chi_2^\pm} = \frac{1}{2}\left[M_\psi + M_\Sigma \mp \sqrt{(M_\psi - M_\Sigma)^2 + 2y^2v^2}\right]$$

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Decoupled eigenvalue!

$$m_{\chi_1^0} = -M_\psi$$

DM candidate

$$m_{\chi_2^0} = m_{\chi_1^\pm}$$

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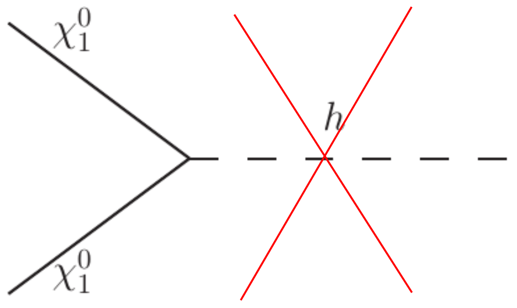
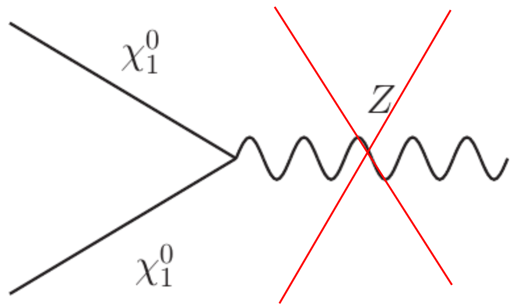
Doublet-Triplet fermion: Phenomenology

Dark matter χ_1^0





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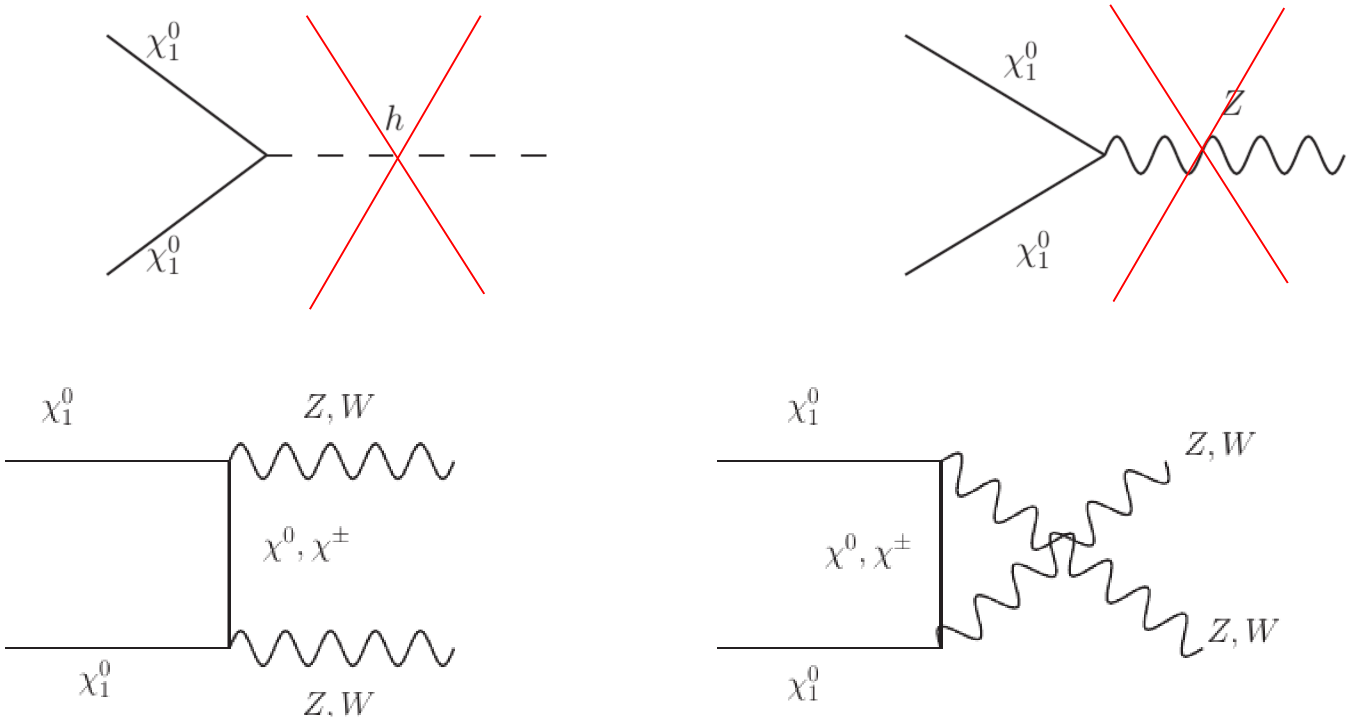
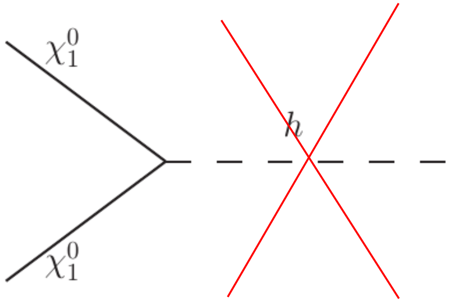


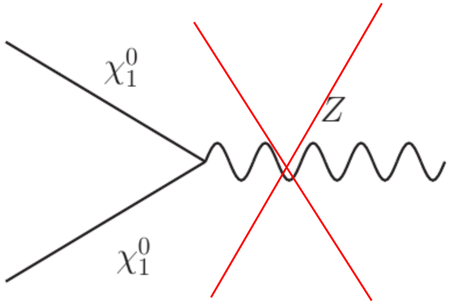
Annihilation through t and u-channels Suppressed!! When no coannihilations are considered

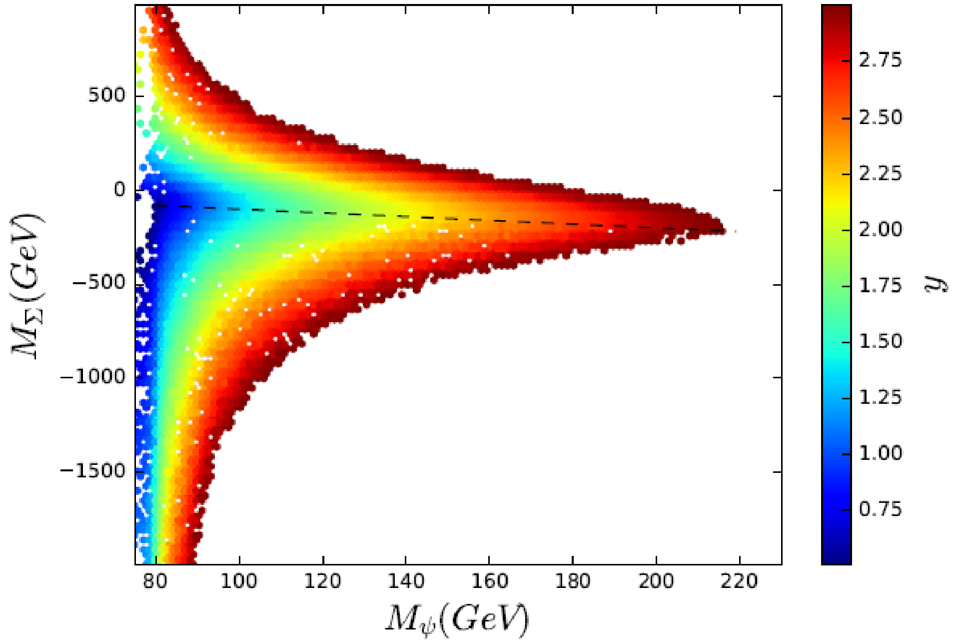
Doublet-Triplet fermion: Phenomenology

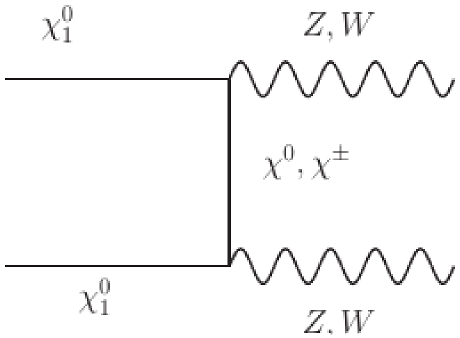
Freytas, JHEP 2015
Dedes, PRD 2014
Abe, PRD 2014

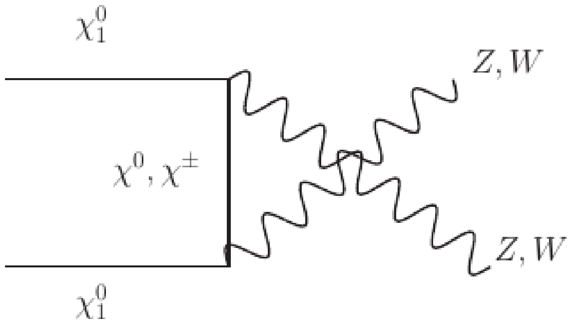
Dark matter χ_1^0







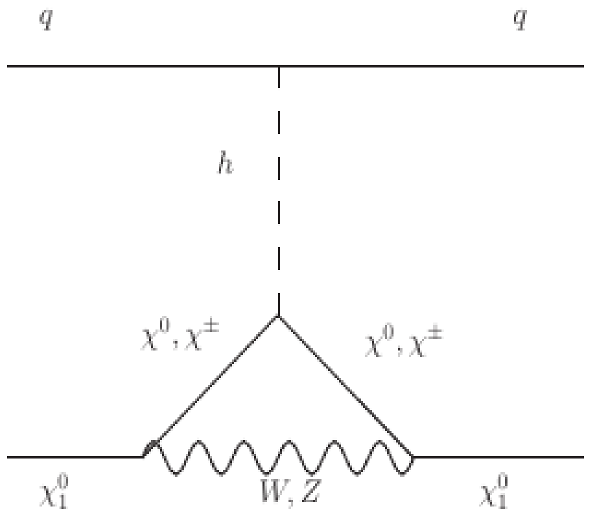


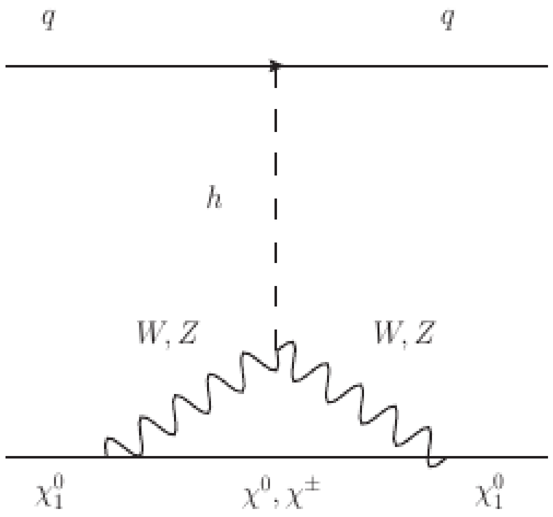


Annihilation through t and u-channels Suppressed!

Doublet-Triplet fermion: Direct Detection

Direct Detection: Main channels



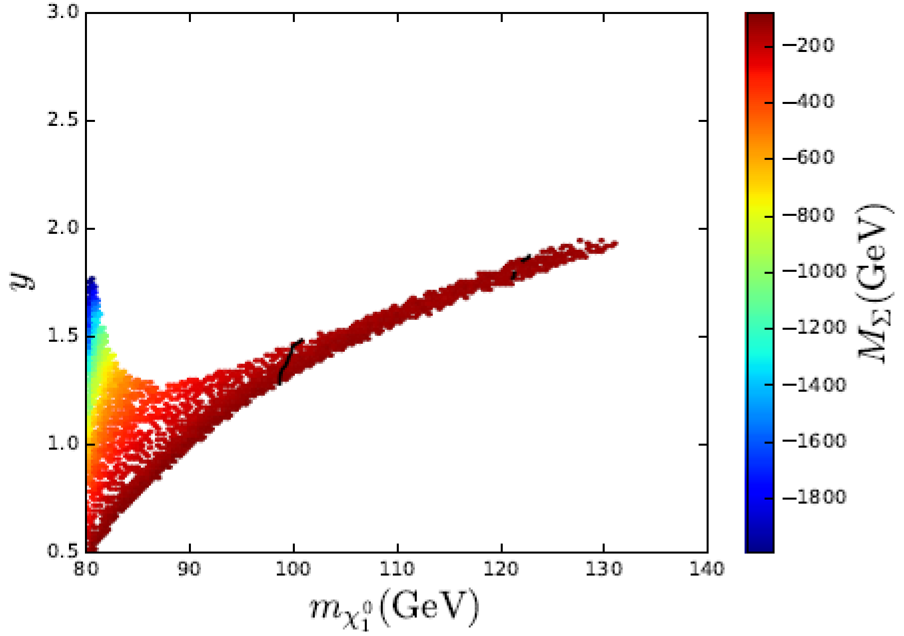


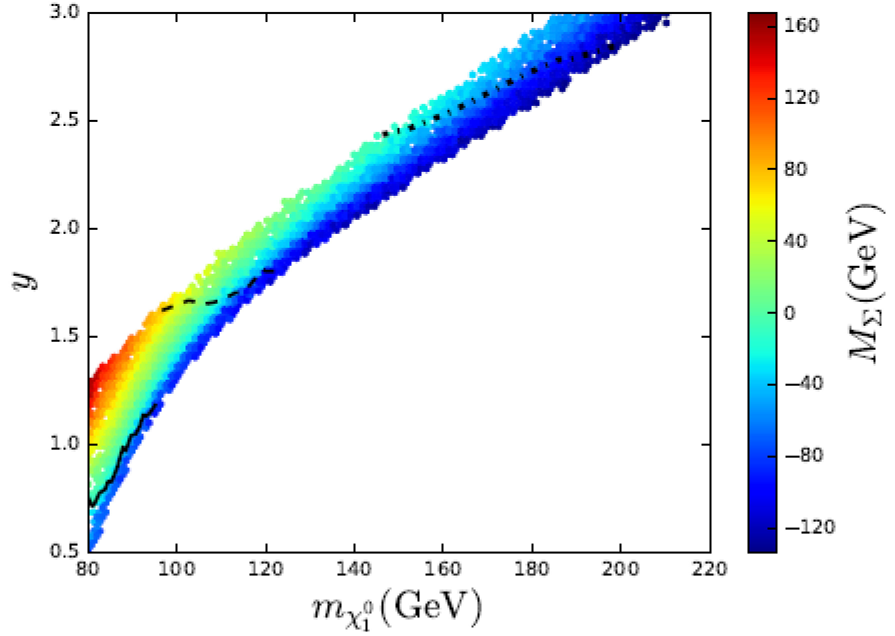
$$\boxed{\frac{\delta y}{2}\chi_1^0\chi_1^0 h}$$

Freitas, JHEP, 2015

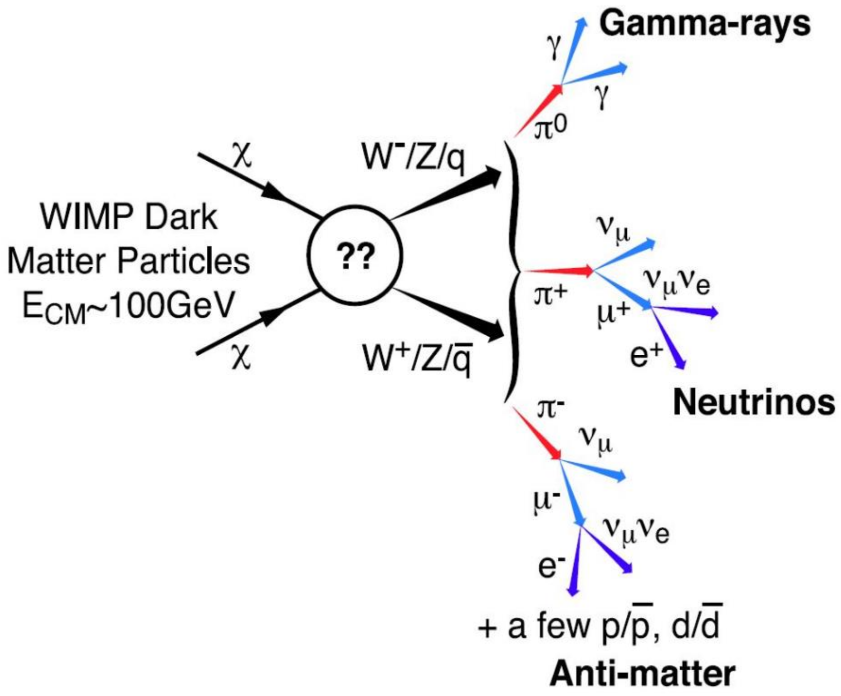
Doublet-Triplet fermion: Direct Detection

Direct detection including LUX 2016 results





Doublet-Triplet fermion: Indirect Detection



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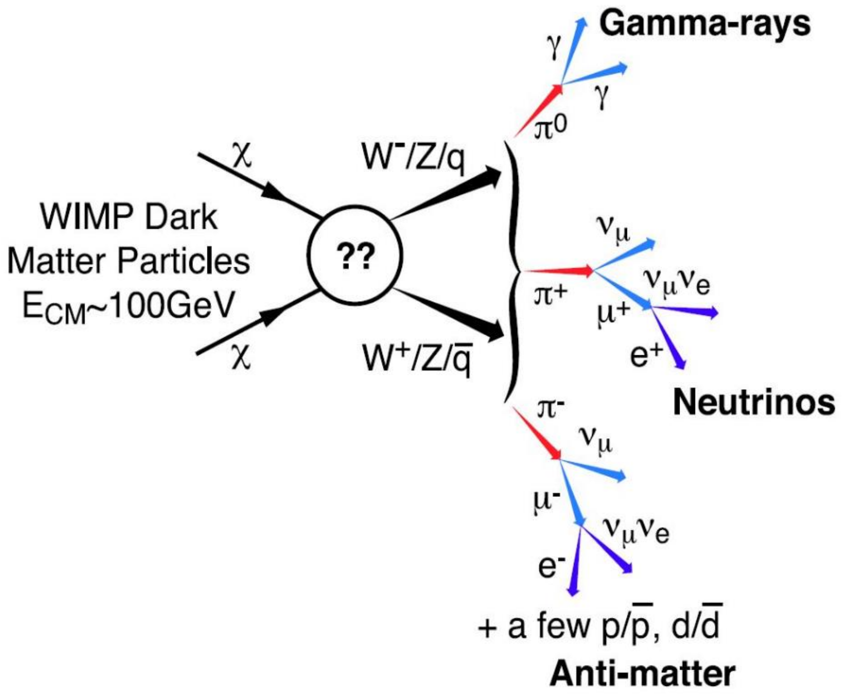

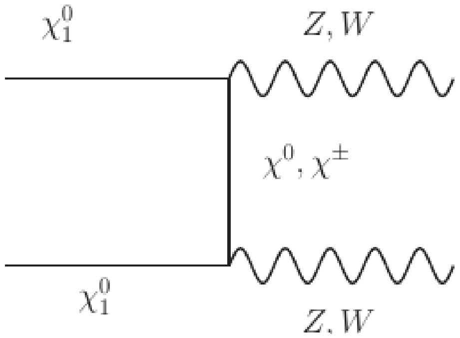

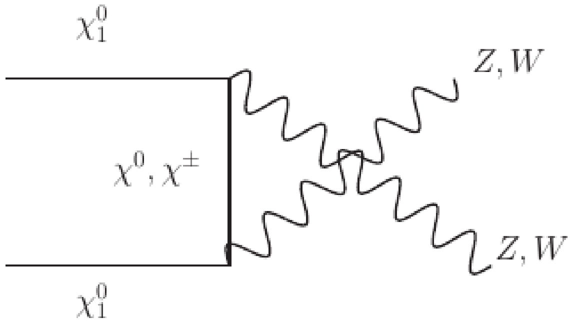

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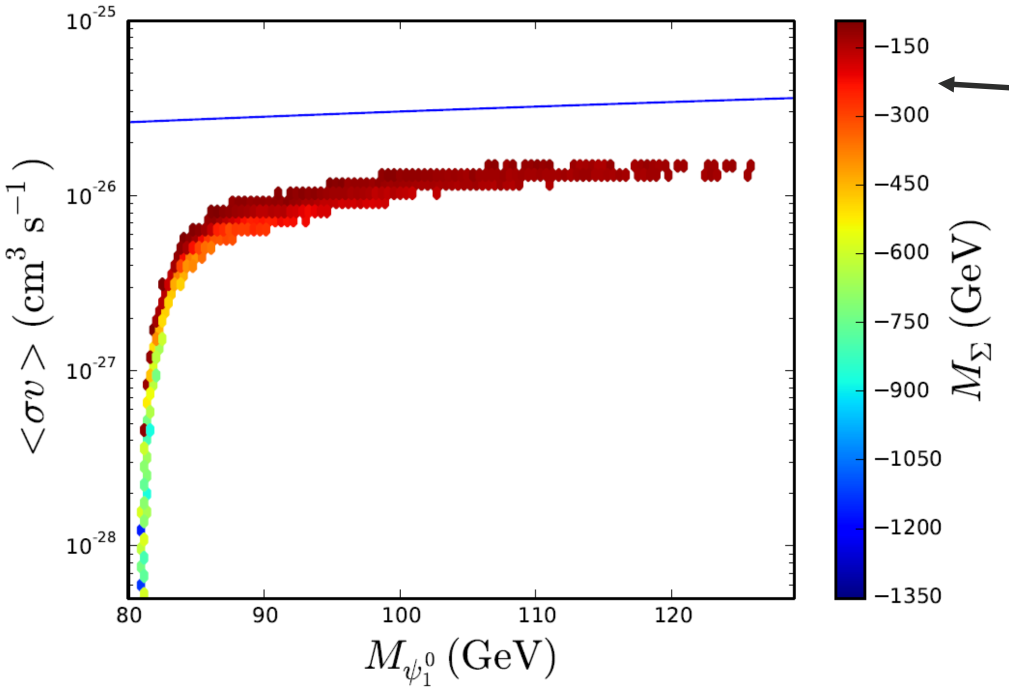
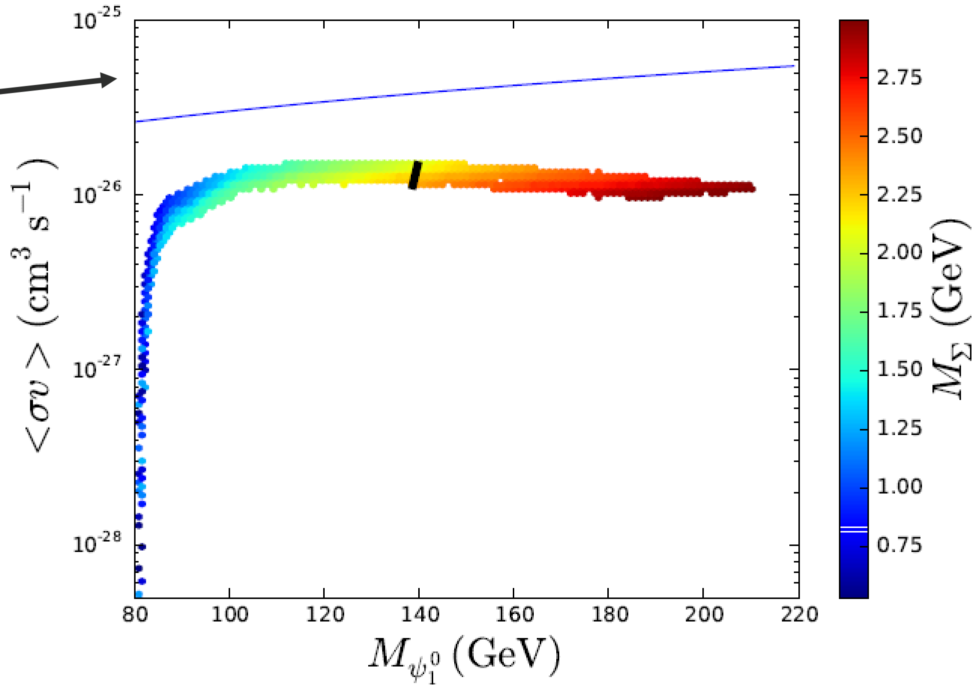
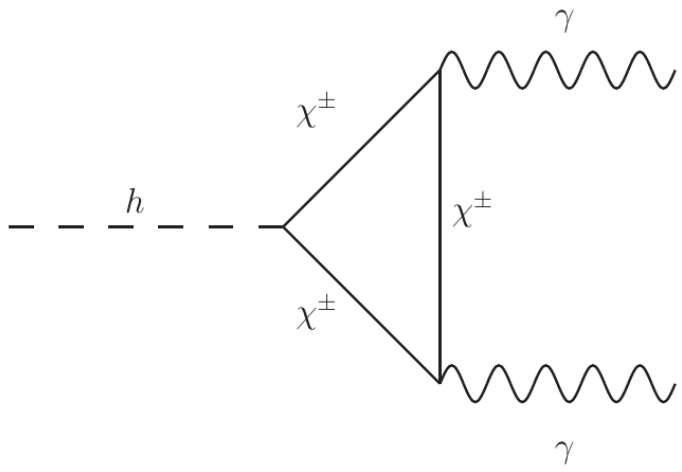


Current limits from dSph galaxies from Fermi-LAT 6 years



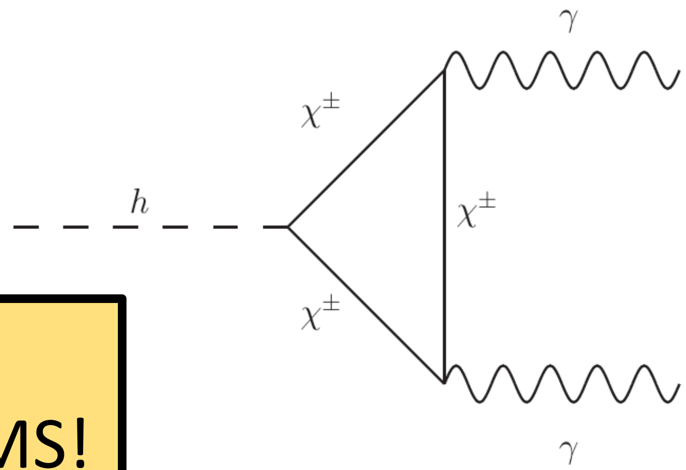
Images retrieved from: https://astrobites.org

Doublet-Triplet fermion: Higgs diphoton decay rate



$$R_{\gamma\gamma} = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{\text{SM}}} = \left| 1 + \frac{1}{\underbrace{A_{\text{SM}}}_{<0}} \frac{y^2 v^2}{m_{\chi_2^{\pm}} - m_{\chi_1^{\pm}}} \left[\overbrace{\frac{A_F(\tau_2)}{m_{\chi_2^{\pm}}} - \frac{A_F(\tau_1)}{m_{\chi_1^{\pm}}}}^{>0} \right] \right|^2$$

Doublet-Triplet fermion: Higgs diphoton decay rate



Diphotn decay rate suppressed:
Mostly Excluded by ATLAS and CMS!

$$R_{\gamma\gamma} = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{\rm SM}} = \left| 1 + \frac{1}{\underbrace{A_{\rm SM}}_{<0}} \frac{y^2 v^2}{m_{\chi_2^\pm} - m_{\chi_1^\pm}} \left[\frac{\overbrace{A_F(\tau_2)}^{>0}}{m_{\chi_2^\pm}} - \frac{A_F(\tau_1)}{m_{\chi_1^\pm}} \right] \right|^2$$

Doublet-Triplet : Scalar sector

$$H_2 = \begin{pmatrix} H^+ \\ \frac{H^0 + iA^0}{\sqrt{2}} \end{pmatrix} \qquad \Delta = \frac{1}{2}\begin{pmatrix} \Delta_0 & \sqrt{2}\,\Delta^+ \\ \sqrt{2}\,\Delta^- & -\Delta_0 \end{pmatrix}$$

Both odd under Z_2 symmetry

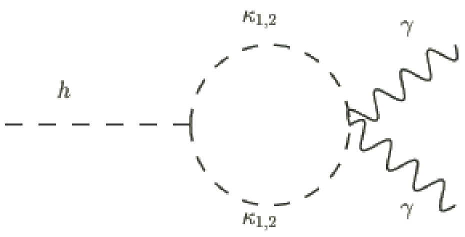
$$\mathcal{V}_2 = -\mathcal{L}_2 = \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^\dagger H_2|^2 + \frac{\lambda_5}{2}\left[(H_1^\dagger H_2)^2 + \text{h.c.}\right]$$
$$+ \lambda_3' (H_1^\dagger H_1)\text{Tr}[\Delta^2] + \lambda_{\Delta H_2}(H_2^\dagger H_2)\text{Tr}[\Delta^2] + \mu\left[\text{H}_1^\dagger \Delta \text{H}_2 + \text{h.c.}\right]$$

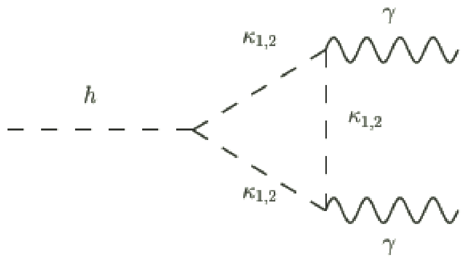
$$\lambda_3 + \sqrt{\lambda_1 \lambda_2} > 0;\ \lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} > 0;\ \lambda_3' + \sqrt{\lambda_1 \lambda_\Delta} > 0$$
$$\lambda_{H_2\Delta} + \sqrt{\lambda_2 \lambda_\Delta} > 0;\ \lambda_3, \lambda_3', \lambda_{\Delta H_2} < 4\pi;\ \lambda_2, \lambda_\Delta < \frac{4\pi}{3}$$

Vacuum stability and perturbativity conditions

Doublet-Triplet fermion-scalar:Higgs diphoton decay rate

$$R = \left| 1 + \frac{1}{A_{SM}} \frac{\lambda_3}{4m_{\kappa_1}^2} A_0\left(\frac{m_h^2}{4m_{\kappa_1}^2}\right) + \frac{1}{A_{SM}} \frac{\lambda_3'}{4m_{\kappa_2}^2} A_0\left(\frac{m_h^2}{4m_{\kappa_2}^2}\right) + \frac{1}{A_{\mathrm{SM}}} \frac{y^2 v^2}{m_{\chi_>^\pm} - m_{\chi_<^\pm}} \left[\frac{A_F(\tau_>)}{m_{\chi_>^\pm}} - \frac{A_F(\tau_<)}{m_{\chi_<^\pm}} \right] \right|^2$$





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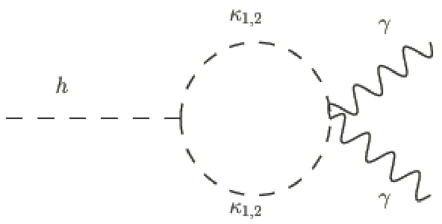
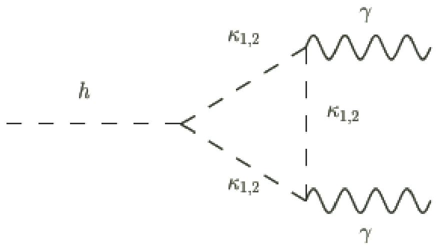




The Scan

Perturbativity

$$0 < \lambda_2, \lambda_\Delta < \frac{4\pi}{3} \,, -1.0 < \lambda_3, \lambda_3' < 4\pi$$

No-coannihilations

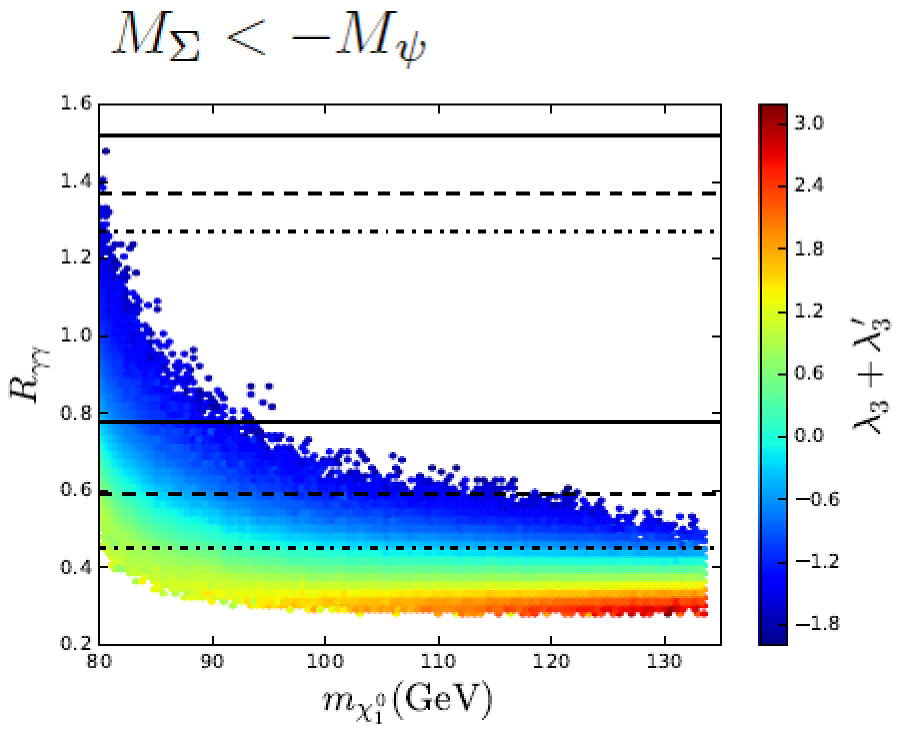
$$1.2 < m_{\kappa_1,\kappa_2}/m_{\chi_1^0} < 3.0 \,, 1.2 < m_{\eta_1,\eta_2}/m_{\chi_1^0} < 3.0$$

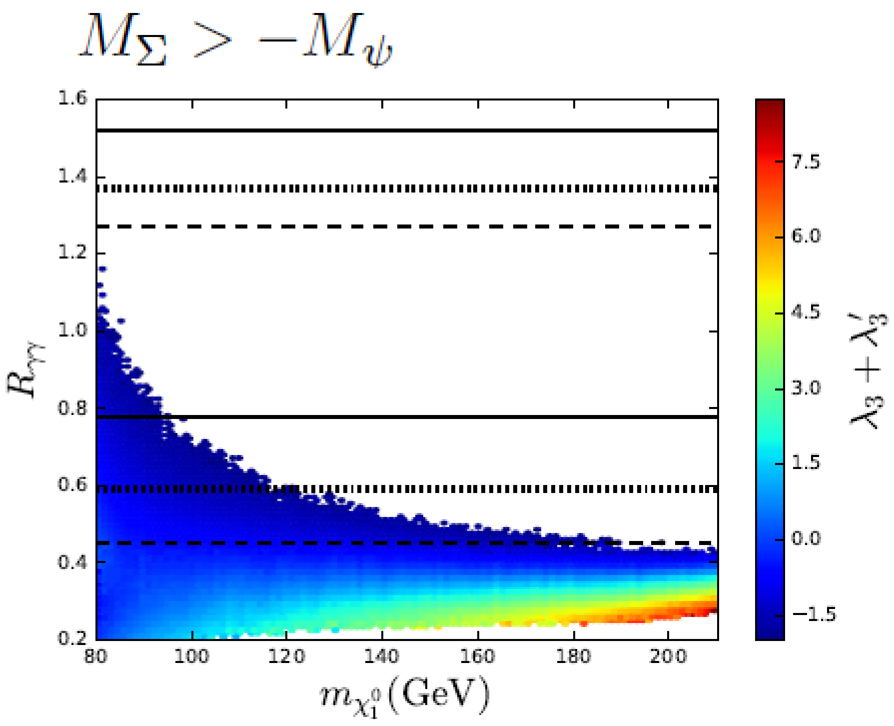
No STU corrections

$$m_{A^0} = m_{\eta_1} \,, \mu = 0$$

Doublet-Triplet fermion-scalar:Higgs diphoton decay rate

Scan Results: All points satisfy relic abundance, direct detection and Higgs diphoton decay rates





Doublet-Triplet fermion-scalar:Higgs diphoton decay rate

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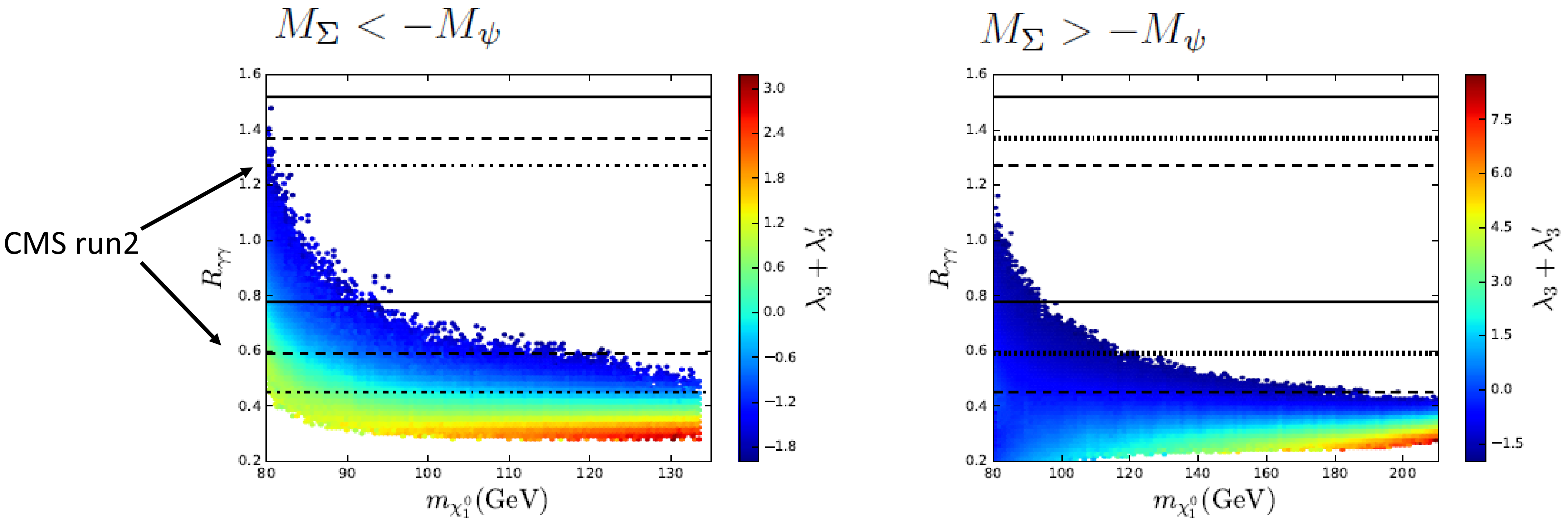

Doublet-Triplet fermion-scalar:Higgs diphoton decay rate

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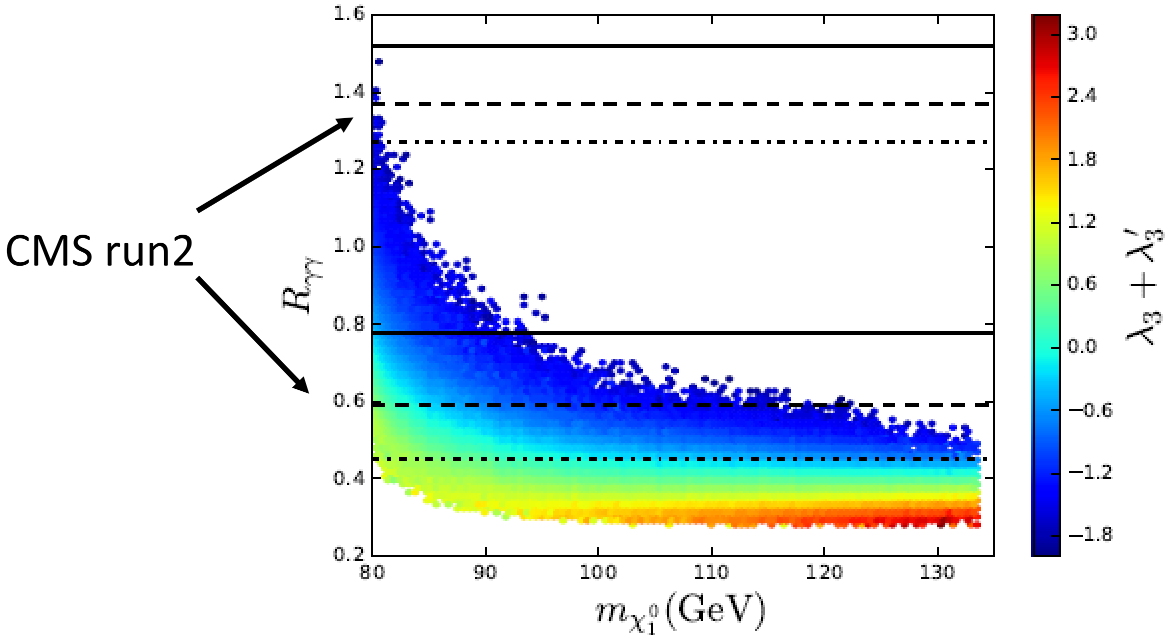
$M_\Sigma < -M_\psi$



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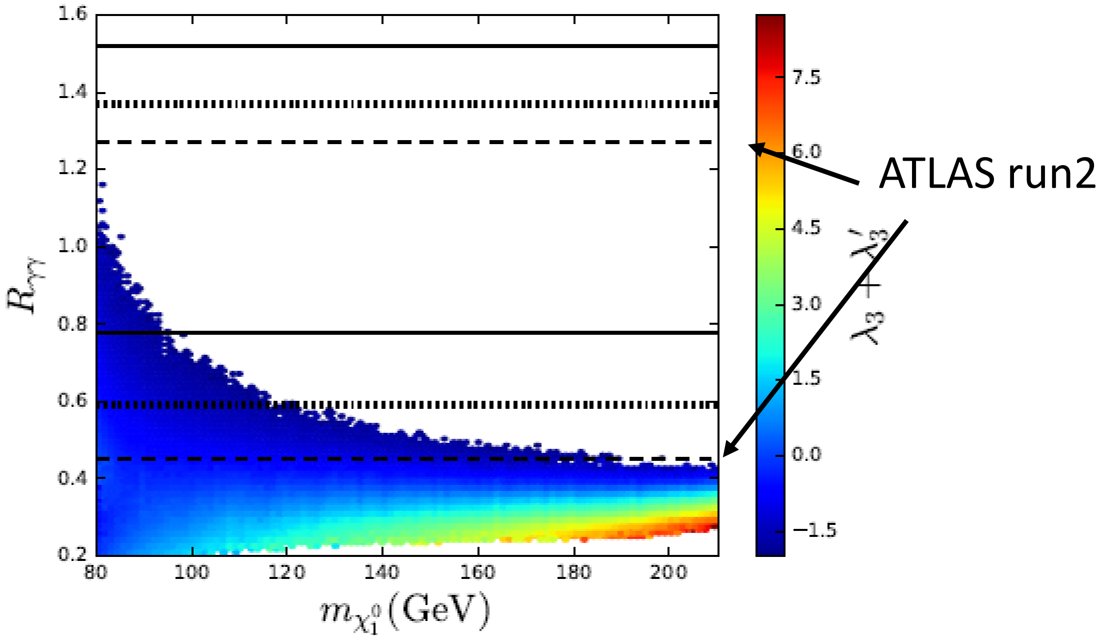

Doublet-Triplet scalar-fermion:Neutrino masses

$$\mathcal{L}_3 = -\zeta_i \bar{L}_{Li} \Sigma_L^c \tilde{H}_2 - \rho_i \bar{\psi}_L H_2 e_{Ri} - f_i \overline{L_{Li}} \Delta \psi_R + \text{h.c.}$$

No neutrino masses at tree level

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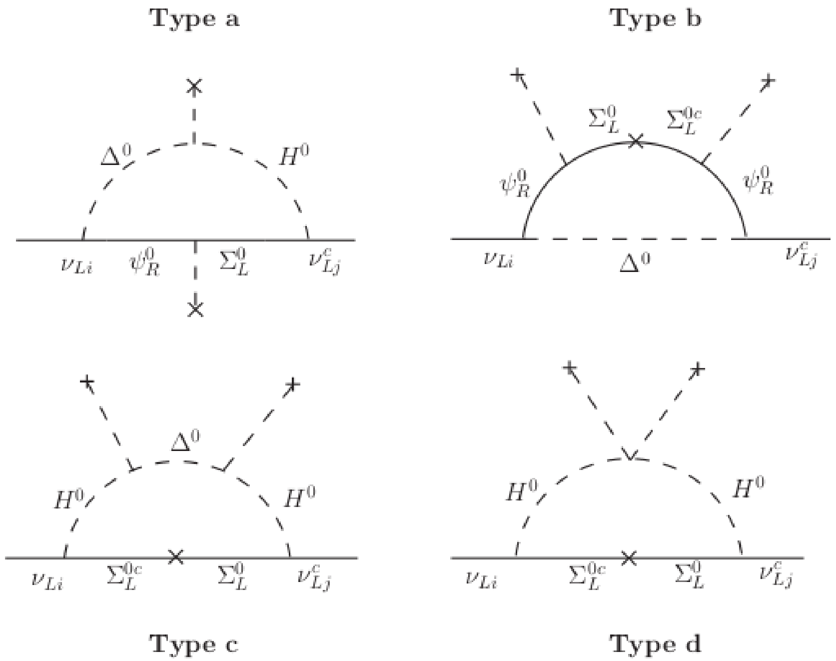

Radiative neutrino masses in four different topologies.

All new fields participate in the mass generation

Doublet-Triplet scalar-fermion:Neutrino masses

Neutrino mass-matrix

$$\boxed{M^{\nu} = \Lambda_{\zeta}\zeta_i\zeta_j + \Lambda_f f_i f_j + \Lambda_{f\zeta}(\zeta_i f_j + f_i\zeta_j)}$$

$$\widetilde{M}_{\nu} = U^{\mathrm{T}}_{\mathrm{PMNS}}\, M_{\nu}\, U_{\mathrm{PMNS}}$$

The matrix has zero determinant, only two massive majorana neutrinos

Doublet-Triplet scalar-fermion:Neutrino masses

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$$\boxed{M^{\nu} = \Lambda_{\zeta}\zeta_i\zeta_j + \Lambda_f f_i f_j + \Lambda_{f\zeta}(\zeta_i f_j + f_i\zeta_j)}$$

It is possible to parametrize all the relevant new Yukawas in terms of neutrino observables and one free Yukawa

ζ_1

Free Yukawa

$$f_i = \frac{1}{\Lambda_f}\left[\pm\sqrt{\Lambda_f\alpha_{ii} - \tilde{\Lambda}\zeta_i^2} - \Lambda_{\zeta f}\zeta_i\right], \qquad i = 1, 2, 3,$$

$$\zeta_j = \frac{\pm 1}{\Lambda_f\tilde{\Lambda}\alpha_{11}}\sqrt{\lambda^2 m_2 m_3 \Lambda_f^2 \tilde{\Lambda}(V_{13}^* V_{j2}^* - V_{12}^* V_{j3}^*)^2(\Lambda_f\alpha_{ii} - \tilde{\Lambda}\zeta_i^2)} + \frac{\alpha_{1j}\zeta_1}{\alpha_{11}}$$

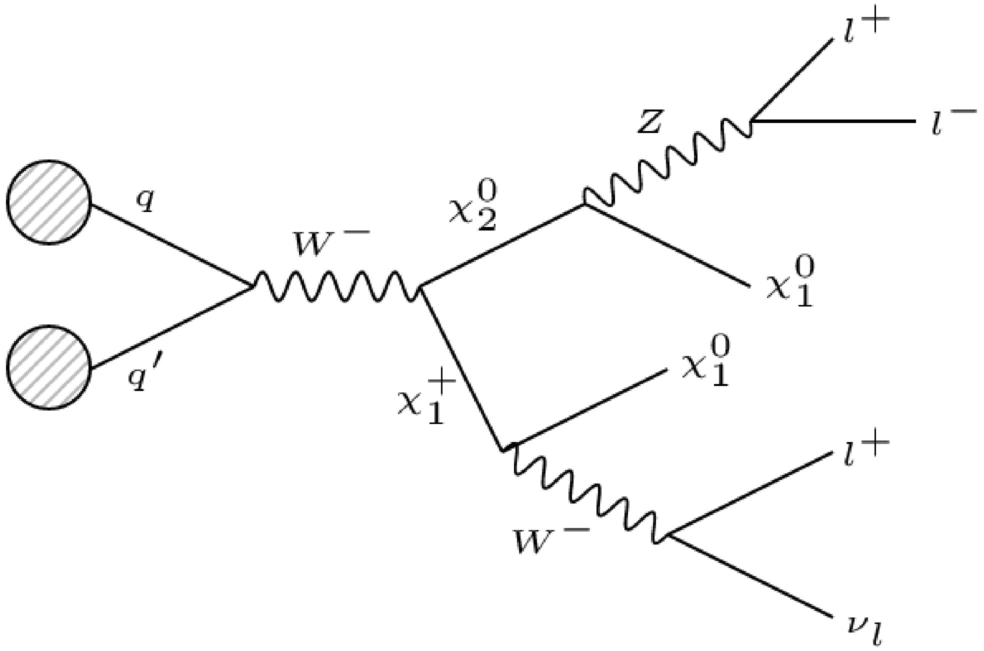
Conclusions:

- The doublet-triplet fermion model may account for the DM of the universe at the electroweak scale, when the Yukawa couplings to the Higgs y_1 and y_2 are the same.
- The fermión sector of the model is mostly excluded because it generates a too low Higgs diphoton decay rate. When the contribution of the doublet and a triplet scalars are included the correct decay rate is obtained.
- Neutrino masses are not generated at tree level, but it is possible to generate them at one-loop with only two massive neutrinos.

Backup: Collider searches

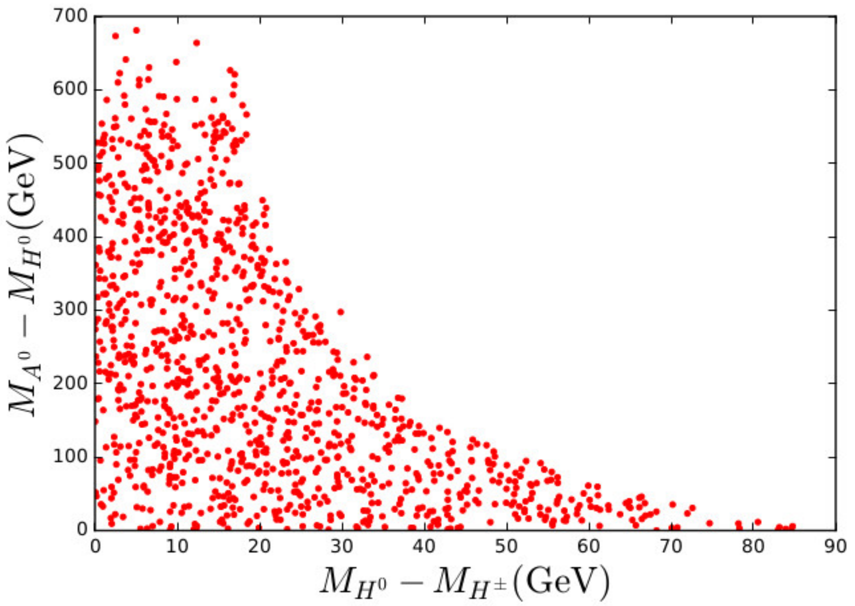
The most relevant search is in trileptons
Explored by CMS

Part of the parameter space of the doublet-triplet fermion will be explored 13 TeV and 30 fb^{-1} the whole region will be probed at 300 fb^{-1} **Freytas, JHEP(2015).** Due to the presence of the scalars we expect modifications in the trilepton plus missing energy search.



Backup: STU

Oblique parameter S and T in the doublet scalar



For the doublet-triplet scalar no additional corrections since $\mu = 0$

For the doublet triplet fermions no additional corrections since y_1=y_2 hence the T parameter is protected and it was found in Dedes PRD 2014 that the S parameter contribution is small.

Backup:Neutrino masses

$$
\begin{aligned}
\Lambda_\zeta &= \frac{1}{32\pi^2}\frac{1}{2}\sum_{k=1}^{3} m_{\chi_k^0}\,(U_{1k})^2\left[c_\alpha^2 F_1(m_{\eta_1}^2, m_{\chi_k^0}^2) + s_\alpha^2 F_1(m_{\eta_2}^2, m_{\chi_k^0}^2) - F_1(m_{A^0}^2, m_{\chi_k^0}^2)\right] \\
\Lambda_f &= \frac{1}{16\pi^2}\frac{1}{4}\sum_{k=1}^{3} m_{\chi_k^0}\,(U_{3k})^2\left[s_\alpha^2 F_2(m_{\eta_1}^2, m_{\chi_k^0}^2) + c_\alpha^2 F_2(m_{\eta_2}^2, m_{\chi_k^0}^2)\right] \\
\Lambda_{\zeta f} &= \frac{1}{32\pi^2}\left[\frac{1}{2}s_\alpha c_\alpha \sum_{k=1}^{3} m_{\chi_k^0}\, U_{1k}U_{3k}\left[F_1(m_{\eta_2}^2, m_{\chi_k^0}^2) - F_1(m_{\eta_1}^2, m_{\chi_k^0}^2)\right]\right. \\
&\left.\quad + s_\beta c_\beta \sum_{k=1}^{2} m_{\chi_k^\pm}\, V_{1k}^L V_{2k}^{R*}\left[F_1(m_{\kappa_1}^2, m_{\chi_k^\pm}^2) - F_1(m_{\kappa_2}^2, m_{\chi_k^\pm}^2)\right]\right]
\end{aligned}
$$

Backup:Neutrino masses

Parametrization:
For the case of normal hierarchy

ζ_1 ← $\boxed{\text{Free Yukawa}}$

$$f_i = \frac{\pm}{\Lambda_f}\left(\sqrt{-\Lambda_f\Lambda_\zeta\zeta_i^2 + \Lambda_f m_2 e^{i\alpha/2} V_{i2}^{*2} + \Lambda_f m_3 V_{i3}^{*2} + \Lambda_{\zeta f}^2\zeta_i^2}\right) - \frac{\Lambda_{\zeta f}\zeta_i}{\Lambda_f}, \qquad i = 1, 2, 3.$$

$$\zeta_j = \pm\left(\frac{\sqrt{\Lambda_f^2 e^{i\alpha/2} m_2 m_3(\Lambda_f\Lambda_\zeta - \Lambda_{\zeta f}^2)(V_{13}^* V_{j2}^* - V_{12}^* V_{j3}^*)^2(-\Lambda_f\Lambda_\zeta\zeta_1^2 + m_2 V_{12}^{*2} e^{i\alpha/2}\Lambda_f + m_3 V_{13}^{*2}\Lambda_f + \Lambda_{\zeta f}^2\zeta_1^2)}}{(\Lambda_f^2\Lambda_\zeta - \Lambda_f\Lambda_{\zeta f}^2)(e^{i\alpha/2} m_2 V_{12}^{*2} + m_3 V_{13}^{*2})}\right.$$

$$\left. \pm \frac{\left(e^{i\alpha/2} m_2\, V_{12}^* V_{j2}^* + m_3 V_{13}^* V_{j3}^*\right)(\Lambda_f^2\Lambda_\zeta\zeta_1 - \Lambda_f\Lambda_{\zeta f}^2\zeta_1)}{(\Lambda_f^2\Lambda_\zeta - \Lambda_f\Lambda_{\zeta f}^2)(e^{i\alpha/2} m_2 V_{12}^{*2} + m_3 V_{13}^{*2})}\right)$$