Notes on projected two-mode field states for atoms detected in states $|0\rangle$ and $|2\rangle$. For the moment we will assume that the state $|1\rangle$ is essentially unpopulated as it would be out of resonance. So for

I. The atom detected in atomic state $|0\rangle$: The fields are projected into the state

$$|\psi(t);0\rangle = \sum_{n_a=0}^{\infty}\sum_{n_b=0}^{\infty} B_{n_a n_b}(t)|n_a, n_b\rangle$$

where

$$B_{n_a n_b}(t) = \frac{1}{N_0(t)}\tilde{Q}_{n_a n_b} C_0(n_a, n_b, t)$$

and where $\tilde{Q}_{n_a n_b}$ and $C_0(n_a, n_b, t)$ are given in the 1991 paper by Lai, Buzek, and Knight (PRA 44, 6043 (1991)). Only consider the case where the initial field states are coherent states. The factor $N_0(t)$ is given by

$$N_0(t) = \left[\sum_{m_a=0}^{\infty}\sum_{m_b=0}^{\infty} \left|\tilde{Q}_{m_a m_b}\right|^2 \left|C_0(m_a, m_b, t)\right|^2\right]^{1/2}.$$

The time *t* in the above equations is the time the atom and field interact and unitarily evolve and at which time the atom is detected in the state $|0\rangle$. There's a subtle point about this that I will explain later.

The first thing to do is to check the normalization which means checking to see if

$$\sum_{n_a=0}^{\infty}\sum_{n_b=0}^{\infty} \left|B_{n_a n_b}(t)\right|^2 = 1$$

holds. Then we'll need to look at the joint photon number probabilities

$$P_{n_a n_b}(t) = \left|B_{n_a n_b}(t)\right|^2$$

at different times *t*. We'll need 3-D bar charts of this versus n_a and n_b at times *t*.

II. For the atom detected in state $|2\rangle$: The fields are projected into the state

$$|\psi(t);2\rangle = \sum_{n_a=1}^{\infty}\sum_{n_b=0}^{\infty} D_{n_a n_b}(t)|n_a - 1, n_b + 1\rangle$$

where

$$D_{n_a n_b}(t) = \frac{1}{N_2(t)}\tilde{Q}_{n_a n_b} C_2(n_a, n_b, t)$$

and where

$$N_2(t) = \left[\sum_{m_a=1}^{\infty}\sum_{m_b=0}^{\infty}\left|\tilde{Q}_{m_a m_b}\right|^2 \left|C_2(m_a, m_b, t)\right|^2\right]^{1/2}.$$

Again we'll want to check that the state is normalized according to

$$\sum_{n_a=0}^{\infty}\sum_{n_b=0}^{\infty}\left|D_{n_a n_b}(t)\right|^2 = 1,$$

and we'll need to look at the joint photon number probabilities

$$P_{n_a n_b}(t) = \left|D_{n_a n_b}(t)\right|^2.$$

~~More to be added.~~

Notes on linear entropy calculations.

1. The density operator for the $|0\rangle$ state is:

$$\hat{\rho}_{ab} = |\psi(t);0\rangle\langle\psi(t);0| = \sum_{m_a}\sum_{m_b}\sum_{n_a}\sum_{n_b}|n_a n_b\rangle B_{n_a n_b}(t) B^*_{m_a m_b}(t)\langle m_a m_b|$$

2. The reduced density operator for field *a* is

$$\begin{aligned}
\hat{\rho}_a &= Tr_b(\rho_{ab}) = \sum_{l_b=0}^{\infty} {}_b\langle l|\hat{\rho}_{ab}|l\rangle_b \\
&= \sum_{l}\sum_{m_a}\sum_{m_b}\sum_{n_a}\sum_{n_b}\langle l|n_b\rangle|n_a\rangle B_{n_a n_b}(t) B^*_{m_a m_b}(t)\langle m_a|\langle m_b|l\rangle \\
&= \sum_{l}\sum_{n_a}\sum_{m_a}|n_a\rangle B_{n_a l}(t) B^*_{m_a l}(t)\langle m_a| \\
&= \sum_{n_a}\sum_{m_a}|n_a\rangle\left[\sum_{l}^{\infty} B_{n_a l}(t) B^*_{m_a l}(t)\right]\langle m_a| \\
&= \sum_{n_a}\sum_{m_a}|n_a\rangle F^{l}_{n_a m_a}(t)\langle m_a|
\end{aligned}$$

3. Squaring this quantity:

$$\begin{aligned}
\hat{\rho}_a^2 &= \sum_{n'_a}\sum_{m'_a}\sum_{n_a}\sum_{m_a}\times|n_a\rangle F^{l}_{n_a m_a}(t)\langle m_a|n'_a\rangle F^{l'}_{n'_a m'_a}(t)\langle m'_a| \\
&= \sum_{m'_a}\sum_{n_a}\sum_{m_a}\times|n_a\rangle F^{l}_{n_a m_a}(t) F^{l'}_{n'_a m'_a}(t)\langle m'_a|
\end{aligned}$$

4. And taking the trace

$$\begin{aligned}
Tr(\hat{\rho}_a^2) &= \sum_{q}\sum_{m'_a}\sum_{n_a}\sum_{m_a}\times\langle q|n_a\rangle F^{l}_{n_a m_a}(t) F^{l'}_{n'_a m'_a}(t)\langle m'_a|q\rangle \\
&= \sum_{q}\sum_{m_a} F^{l}_{q m_a}(t) F^{l'}_{q m_a}(t)
\end{aligned}$$

5. Linear Entropy is $S = 1 - Tr(\hat{\rho}^2)$

   For a pure state, the trace of Ro squared is 1, and for a mixed state the trace of Ro squared is less than 1. Meaning, that if S (entropy) is 0, the states separable aka not entangled.