

Exercises for Seminar Classes

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Exercise 1: Generate and plot data from the following models.

(a) Gaussian AR(1) model:

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t, \quad \{\varepsilon_t\}_{t \in \mathbb{Z}} \sim NID(0, \sigma_\varepsilon^2).$$

(b) Gaussian random coefficient autoregressive (RCAR) model:

$$y_t = \alpha + \beta_t y_{t-1} + \varepsilon_t, \quad \beta_t \sim NID(b, \sigma_\beta^2), \quad \{\varepsilon_t\}_{t \in \mathbb{Z}} \sim NID(0, \sigma_\varepsilon^2).$$

(c) Linear observation driven local-level model:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t, & \{\varepsilon_t\}_{t \in \mathbb{Z}} &\sim NID(0, \sigma_\varepsilon^2), \\ \mu_{t+1} &= \omega + \beta \mu_t + \alpha(y_t - \mu_t). \end{aligned}$$

(d) Linear parameter driven local-level model:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t, & \{\varepsilon_t\}_{t \in \mathbb{Z}} &\sim NID(0, \sigma_\varepsilon^2), \\ \mu_{t+1} &= \alpha + \beta \mu_t + \eta_t, & \{\eta_t\}_{t \in \mathbb{Z}} &\sim NID(0, \sigma_\eta^2). \end{aligned}$$

(e) Generalized autoregressive conditional heteroskedasticity (GARCH) model:

$$\begin{aligned} y_t &= \sigma_t \varepsilon_t, & \{\varepsilon_t\}_{t \in \mathbb{Z}} &\sim NID(0, 1), \\ \sigma_{t+1}^2 &= \omega + \beta \sigma_t^2 + \alpha y_t^2. \end{aligned}$$

(f) Stochastic volatility (SV) model:

$$\begin{aligned} y_t &= \exp(\sigma_t) \varepsilon_t, & \{\varepsilon_t\}_{t \in \mathbb{Z}} &\sim NID(0, \sigma_\varepsilon^2), \\ \sigma_{t+1} &= \alpha + \beta \sigma_t + \eta_t, & \{\eta_t\}_{t \in \mathbb{Z}} &\sim NID(0, \sigma_\eta^2). \end{aligned}$$