The GAP [2] functions contained in `cpgtools.gap` are intended to be used for analyzing several properties of groups E defined by ordinary group presentations of the form $\langle a, x : a^n, W(a,x)\rangle$ where:

- $W(a,x)$ is a word in the free group $F(a,x)$;
- $a \in E$ has order n; and
- E is finite.

To this end, it will be assumed that, in GAP, the group `e` is defined via the presentation with $a =$ `e.1`, $x =$ `e.2`, and `M` is the coset table resulting from `CosetTable(e,Subgroup(e,[e.1]));;`.

`MakeUnified (M, n[, f])`
This function creates the list

`[M, MakeTree(M), ShiftOrder(M), n]`

where $\mathtt{n} = n$ is the order of $a \in E$. Optionally, an integer `f` may be input when there exists a retraction $\nu^{\mathtt{f}} : E \to \langle a\rangle$ satisfying $\nu^{\mathtt{f}}(a) = a$ and $\nu^{\mathtt{f}}(x) = a^{\mathtt{f}}$. In such a case, the kernel $\ker \nu^{\mathtt{f}}$ is cyclically presented. See [1]. If `f` is input, then it becomes the fifth entry in the list. Otherwise, there is no default value for `f`. For the functions below, the result of `MakeUnified` is assumed to be defined to `U`.

`ShiftOrder (M)`
Used by `MakeUnified`. Intended to return the value of the shift of the cyclically presented kernel $\ker \nu^{\mathtt{f}}$ when a retraction $\nu^{\mathtt{f}} : E \to \langle a\rangle$ exists.

`MakeTree (M)`
Used by `MakeUnified`. Creates a list containing data used to determine a coset representative for each coset in $\langle a\rangle\backslash E$. Each entry in the list is a list itself containing two values. The first value is either `0` or `1` corresponding to x or a respectively. The second value is an index corresponding to the coset obtained by multiplying the chosen coset on the right by either x^{-1} or a^{-1} (as determined by the first value).

`ModifyRetraction (U[, f])` Used to modify the `U[5]` data about a retraction $\nu^{\mathtt{f}} : E \to \langle a\rangle$ if one exists. If `f` is not input, the retraction data is removed from `U`.
`MakeOrbit (U, pos[, row])`
This function creates a list of the column indices in `U[1]` that correspond to the cosets in $\langle a\rangle\backslash E$ in the orbit of the coset with index `pos` under successive right multiplication by the element $a^{\pm 1}, x^{\pm 1} \in E$ that corresponds to the row index of `U[1]` given by `row`. By default, `row` has a value of one, corresponding to multiplication by a.

`OrbitSizes (U[, row])`
This function creates a list containing two lists of equal length. The entries in the first list are the distinct sizes of the orbits made by the `MakeOrbit` function taken over every column index of `U[1]`. The second list contains the number of distinct orbits of the size given in the first list in the same index. If `row` is input, it is used as an optional parameter when calling `MakeOrbit`. By default, `row` has a value of one.

`FixedPoints (U[, pow, prim])`
This function creates the list of column indices of `U[1]` corresponding to the cosets in $\langle a\rangle\backslash E$ where

$\langle a\rangle\, wa^{\texttt{pow}} = \langle a\rangle\, w$ for the optional positive integer input `pow` (by utilizing the data in row one of `U[1]`). By default, `pow` is equal to one. If anything is input for `prim`, the resulting list will contain only those indices corresponding to the cosets for which `pow` is minimal.

`CentralizingIndices (U[, pow])`
To eventually replace `FixedPoints`. Removes an optional argument.

`MakeWordList (U, pos[, f])`
This function creates a list of zeros and ones which correspond to x, a to determine a coset representative in E for the coset corresponding to the column index of `U[1]` of value `pos`. If $U[5]$ is defined or if the optional parameter `f` is input, the chosen representative will be an element of the cyclically presented kernel $\nu^{\texttt{f}} : E \to \langle a\rangle$. Entering a value for `f` overrides any retraction data at `U[5]`.

`MakeWord (wordlist, a, x)`
This function creates a string from the word encoded in `wordlist` from the output of `MakeWordlist` . The resulting string uses the characters given as strings `a` and `x` in place of the respective ones and zeros in `wordlist`. The `EvalString` function may be used to have GAP evaluate the resulting string as a word in the chosen characters `a` and `x` as variables.
`TraceWordlist (U, wordlist[, pos])`
Similar to `TracedCosetFpGroup`, this function returns the column index of $U[1]$ corresponding to the coset in $\langle a\rangle\backslash E$ obtained from right multiplication of the coset with column index value `pos` by the word encoded in `wordlist` (as an output from `MakeWordlist`). By default, `pos` has a value of one.

`MakePowers (U, wordlist)`
`MakePowers (U, pos[, f])`
This function creates a list of the column indices of `U[1]` corresponding to the cosets in $\langle a\rangle\backslash E$ containing (1) the powers of the word $w \in E$ encoded by `wordlist` or (2) the powers of the word $w \in E$ encoded by the wordlist obtained by passing the parameters into `MakeWordlist`. In case (2), the optional parameter `f` may be entered to override any retraction data present in `U[5]`.

`Orderlist (U[, f])`
This function creates a list containing the order of each coset in $\langle a\rangle\backslash E$ in the group structure described in [1]. By default, the function uses the retraction data in `U[5]`, but his may be overridden by including a value for the optional parameter `f`.
`MakeCenter (U[, f])`
This function creates a list of column indices of `U[1]` corresponding to the cosets in $\langle a\rangle\backslash E$ contained in the center of the kernel $\ker \nu^{\texttt{f}}$ of the retraction $\nu^{\texttt{f}} : E \to \langle a\rangle$. By default, the function will use the retraction data in `U[5]`, but this my be overridden by including a value for the optional parameter `f`. The group structure of $\langle a\rangle\backslash E$ is described in [1].

`MakeGroupFromList (U, indexlist[, f])`
This function creates the subgroup of the kernel $\ker \nu^{\texttt{f}}$ of a retraction $nu^{\texttt{f}} : E \to \langle a\rangle$ containing the cosets in $\langle a\rangle\backslash E$ corresponding to the column indices of `U[1}` given in. the list `indexlist`. The resulting group is given as a finitely presented group. By default, the function will use the retraction data in `U[5]`, but this may be overridden by including a value for the optional parameter `f`. The group structure of $\langle a\rangle\backslash E$ is described in [1].

`DuplicateFree (list)`
This function takes a list `list` of integers, then orders the list and removes all duplicates. It is more efficient than `AsDuplicateFreeList` as this function will only work with lists of integers.

References

[1] W. A. Bogley. On shift dynamics for cyclically presented groups. *J. Algebra*, 418:154–173, 2014.

[2] *GAP – Groups, Algorithms, and Programming, Version 4.8.7*, 2017.