The GAP functions contained here are intended to be used for analyzing several properties of groups E defined by ordinary group presentations of the form  $\langle a, x : a^n, W(a, x) \rangle$  where:

- W(a, x) is a word in the free group F(a, x);
- $a \in E$  has order n; and
- E is finite.

To this end, it will be assumed that, in GAP, the group e is defined via the presentation with a = e.1 and M is the coset table resulting from CosetTable(e,Subgroup(e,[e.1]));;.

## MakeUnified (M, n[, f])

This function creates the list

where  $\mathbf{n}=n$  is the order of  $a\in E$ . Optionally, an integer  $\mathbf{f}$  may be input when there exists a retraction  $\nu^{\mathbf{f}}:E\to\langle a\rangle$  satisfying  $\nu^{\mathbf{f}}(a)=a$  and  $\nu^{\mathbf{f}}(x)=a^{\mathbf{f}}$ . In such a case, the kernel ker  $\nu^{\mathbf{f}}$  is cyclically presented. See citation. If  $\mathbf{f}$  is input, then it becomes the fifth entry in the list. Otherwise, there is no default value for  $\mathbf{f}$ . For the functions below, the result of MakeUnified is assumed to be defined to U.

### ShiftOrder (M)

Used by MakeUnified. Intended to return the value of the shift of the cyclically presented kernel  $\ker \nu^{\mathbf{f}}$  when a retraction  $\nu^{\mathbf{f}}: E \to \langle a \rangle$  exists.

### MakeTree (M)

Used by MakeUnified. Creates a list containing data used to determine a coset representative for each coset in  $\langle a \rangle \backslash E$ . Each entry in the list is a list itself containing two values. The first value is either 0 or 1 corresponding to x or a respectively. The second value is an index corresponding to the coset obtained by multiplying the chosen coset on the right by either  $x^{-1}$  or  $a^{-1}$  (as determined by the first value).

ModifyRetraction (U[, f]) Used to modify the U[5] data about a retraction  $\nu^{f}: E \to \langle a \rangle$  if one exists. If f is not input, the retraction data is removed from U.

# MakeOrbit (U, pos[, row])

This function creates a list of the column indices in U[1] that correspond to the cosets in  $\langle a \rangle \backslash E$  in the orbit of the coset with index pos under successive right multiplication by the element  $a^{\pm 1}, x^{\pm 1} \in E$  that corresponds to the row index of U[1] given by row. By default, row has a value of one, corresponding to multiplication by a.

## OrbitSizes (U[, row])

This function creates a list containing two lists of equal length. The entries in the first list are the distinct sizes of the orbits made by the MakeOrbit function taken over every column index of U[1]. The second list contains the number of distinct orbits of the size given in the first list in the same index. If row is input, it is used as an optional parameter when calling MakeOrbit. By default, row has a value of one.

# FixedPoints (U[, pow, prim])

MakeWordList
TraceWordlist
MakePowers
Orderlist
MakeCenter
MakeGroupFromList
DuplicateFree
MakeWord
CentralizingIndices