

Sum of uncorrelated variables with random sample size

There are cases when a sample is taken without knowing, in advance, how many observations will be acceptable according to some criterion. In such cases, the sample size N is a random variable whose variation adds to the variation of X , such that,

$$\text{Var}\left(\sum_{i=1}^N X_i\right) = \text{E}[N] \text{Var}(X) + \text{Var}(N)(\text{E}[X])^2 \quad [1]$$

which follows from the law of total variance.

If N has a Poisson distribution, then $\text{E}[N] = \text{Var}(N)$ with estimator $n = N$. So, the estimator of $\text{Var}\left(\sum_{i=1}^n X_i\right)$ becomes $nS_x^2 + n\bar{X}^2$, giving $\text{SE}(\bar{X}) = \sqrt{\frac{S_x^2 + \bar{X}^2}{n}}$ (see standard error of the sample mean).

References

1. Cornell, J R, and Benjamin, C A, *Probability, Statistics, and Decisions for Civil Engineers*, McGraw-Hill, NY, 1970, pp.178-9.