- 1. Find the singular values of the matrix  $\begin{bmatrix} -5 & 0 \\ 0 & 0 \end{bmatrix}$ .
- 2. Suppose the factorization below is an SVD of a matrix A, with the entries in U and V rounded to two decimal places.

$$A = \begin{bmatrix} -0.86 & -0.11 & -0.50 \\ 0.31 & 0.68 & -0.67 \\ 0.41 & -0.73 & -0.55 \end{bmatrix} \begin{bmatrix} 12.48 & 0 & 0 & 0 \\ 0 & 6.34 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.66 & -0.03 & -0.35 & 0.66 \\ -0.13 & -0.90 & -0.39 & -0.13 \\ 0.65 & 0.08 & -0.16 & -0.73 \\ -0.34 & 0.42 & -0.84 & -0.08 \end{bmatrix}$$

- (a) What is the rank of A?
- (b) Use this decomposition of A, with no calculations, to write a basis for Col A and a basis for Nul A.
- 3. Suppose A is square and invertible. Find a singular value decomposition of  $A^{-1}$ .
- 4. Show that if A is square, then  $|\det A|$  is the product of the singular values of A.
- 5. Find the minimal length least-squares solution of the equation  $A\mathbf{x} = \mathbf{b}$ , where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}.$$