# Math 425 Computation Linear Algebra

## Final Project, Problem 1.

***Topics in Matrix Transformation, Least-squares, Linear Modelling and Singular Vaule Decomposition.***

### Group 3

- Anneke Moeller; math validation, code review and testing
- Shem Cheng; visualization enhancements, code review and testing
- Rai'd Muhammad; code review and user testing
- Brent Thorne; scheduling, software and reporting

In [1]:
```
# environment setup...
# we will try to make it clear which library we're using for what
import sympy as sym # symbolic maths
from sympy.matrices import Matrix # pretty matrices
from mpl_toolkits.mplot3d import Axes3D # spatial visuals, like 3d quivers
import matplotlib.pyplot as plt # old standby for plotting like a villian
from IPython.display import display, Math, Latex # used to display formatted resul
sym.init_printing()  # pretty printing
```

### Problem 1. Escher in the Matrix - Tiling the plane

***Create an Escher like artwork by tiling the model bird.***

In [2]:
```
class bird:  # free bird
    """
    import point data and return bird object
    provides functions transform and plot
    assumes sympy Matrix is 2-rows (2xn)
    recall: A m×n matrix has m rows and n columns.
    """
    def __init__(self, D):
        assert isinstance(D, Matrix) # eat 2xm matrix of points and make homogenou
        self.T = sym.eye(3)  # consider feature to set transform on creation
        self.D = D
        self.D = self.D.col_join(sym.ones(1,self.D.cols)) # make homogenous
        self.colors = ["green", "red",  "blue", "yellow"]
        self.colorIndex = 0

    def eye(self):
        self.T = sym.eye(3) # we see you
        self.colorIndex = 0;

    def plot(self, title = 'Fly like an Eagle.'): # hold figure: see bird.show()
     #   lim=1.5 # consider feature to set limits based on origin and average poin
        DD = self.T * self.D  # do inner product at plotting
        plt.title(f"{title}"); plt.xlabel("X axis"); plt.ylabel("Y axis")
        plt.scatter(list(DD.row(0)), list(DD.row(1)), color ="red", s=10)
        plt.plot(list(DD.row(0)), list(DD.row(1)), color ="blue", linewidth=1, marl
```

```
        plt.fill(list(DD.row(0)), list(DD.row(1)), color =self.colors[self.colorInd
        self.colorIndex+=1

     #   plt.xlim(-lim,lim); plt.ylim(-lim,lim)
        plt.grid(); plt.gca().set_aspect("equal") # square grids are pretty
        plt.axhline(0, color='black', linestyle='--')
        plt.axvline(0, color='black', linestyle='--')

    def show(self): # this stops the figure hold: see bird.plot()
        self.colorIndex = 0;
        plt.show()

    def __mul__(self, other): # dot the transform
        if isinstance(other, Matrix):
            self.T = other * self.T
        else:
            return NotImplemented

    def dot(self, other): # dot yourself
        return self.__mul__(other)

    def translate(self, a, b):
        self.T = Matrix([[1,0,a], [0,1,b], [0,0,1]]) * self.T

    def mirror(self, a,b,c): # mirror on line formed by f(a,b,c)= ax+by-c=0, dot yo
        self.T = 1/(a**2+b**2)*Matrix([\
        [b**2-a**2,    -2*a*b, -2*a*c],\
        [   -2*a*b, a**2-b**2, -2*b*c],\
        [        0,         0, a**2+b**2]]) * self.T

    def rotate(self, theta): # rot and dot yourself
        self.T = Matrix([[sym.cos(theta),-sym.sin(theta),0],\
        [sym.sin(theta),sym.cos(theta),0],\
        [0,0,1]]) * self.T

    def report(self): # so pretty
        display(Latex(f'$TD={sym.latex(self.T)}\
        {sym.latex(self.D.n(2))}$'))
        display(Latex(f'$TD={sym.latex(Matrix(self.T*self.D).n(2))}$ * rounded to 
```

**(a) Take four tiles of the model bird and fit them together according to the instructions**

In [3]:
```
# The ten endpoints of the line segments are
P = [(0.3036, 0.1960), (0.6168, 0.2977), (0.7128,0.4169), (0.7120, 0.1960), (0.937
     (0.7120, 0.5680), (0.3989, 0.6697), (0.3028,0.7889), (0.3036, 0.5680), (0.529

# format our data as a matrix
for i in range(len(P)+1):
    if i == 0:
        V = Matrix(P[0]) # create model bird shape
    elif i >= len(P):
        V = V.row_join(Matrix(P[0])) # add an extra endpoint vector to close our l
    else:
        V = V.row_join(Matrix(P[i]))
```

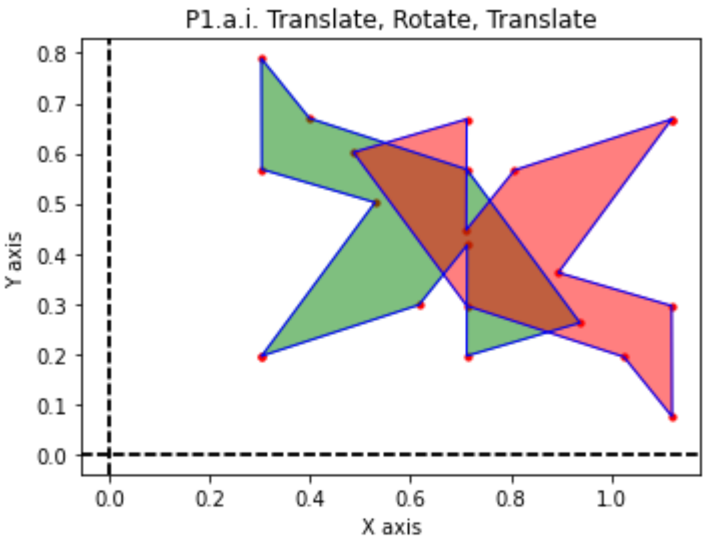
***i. Create the first tile by rotating the model bird through π radians about the point (0.7120, 0.4320). Provide the matrix for the transformation in homogeneous coordinates.***

In [4]:
```
# P1.a.i
```

```
B = bird(V) # fly the model bird shape
#B.report()
B.plot("P1.a Untransformed")
#B.show()

# Create first tile
#B.dot(Matrix([[1,0,-0.7120],[0,1,-0.4320],[0,0,1]])) # shift point to origin, see
B.translate(-0.7120,-0.4320) # use translate to shift instead of matrix
B.rotate(sym.pi) # rotate by pi
B.translate(0.7120,0.4320) # shift back
B.plot("P1.a.i. Translate, Rotate, Translate")
B.report()
T1 = B.T # save transpose
```

$$TD = \begin{bmatrix} -1 & 0 & 1.424 \\ 0 & -1 & 0.864 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.3 & 0.62 & 0.71 & 0.71 & 0.94 & 0.71 & 0.4 & 0.3 & 0.3 & 0.53 & 0.3 \\ 0.2 & 0.3 & 0.42 & 0.2 & 0.26 & 0.57 & 0.67 & 0.79 & 0.57 & 0.5 & 0.2 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$$TD = \begin{bmatrix} 1.1 & 0.81 & 0.71 & 0.71 & 0.49 & 0.71 & 1.0 & 1.1 & 1.1 & 0.89 & 1.1 \\ 0.67 & 0.57 & 0.45 & 0.67 & 0.6 & 0.3 & 0.19 & 0.075 & 0.3 & 0.36 & 0.67 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$ * rounded to two decimal points



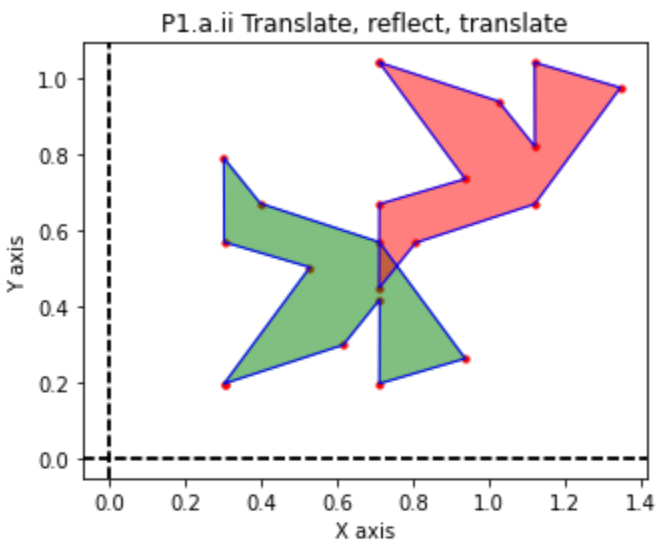

***ii. Form the second tile by reflecting the model bird through the horizontal line y = 0.6180 and then translating this image by 0.4084 units along the x-axis. Provide the matrix for the transformation in homogeneous coordinates.***

In [5]:

```
# P1.a.ii
B.eye() # restore model bird
B.plot()
# B.mirror(0,-0.6180,0) # reflect on line, not working as expected TESTME!!!
# Actually, above works however there is an extra step to format the line equation
# Much easiler to take dot products to shift, reflect, shift

B.translate(0,-0.6180) # shift y
B.dot(Matrix([[1,0,0],[0,-1,0],[0,0,1]])) # mirror y-axis
B.translate(0,0.6180) # shift y
B.translate(0.4084,0) # shift x
B.plot("P1.a.ii Translate, reflect, translate")
B.report()
T2 = B.T
```

$$TD = \begin{bmatrix} 1 & 0 & 0.4084 \\ 0 & -1 & 1.236 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.3 & 0.62 & 0.71 & 0.71 & 0.94 & 0.71 & 0.4 & 0.3 & 0.3 & 0.53 & 0.3 \\ 0.2 & 0.3 & 0.42 & 0.2 & 0.26 & 0.57 & 0.67 & 0.79 & 0.57 & 0.5 & 0.2 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$$TD = \begin{bmatrix} 0.71 & 1.0 & 1.1 & 1.1 & 1.3 & 1.1 & 0.81 & 0.71 & 0.71 & 0.94 & 0.71 \\ 1.0 & 0.94 & 0.82 & 1.0 & 0.97 & 0.67 & 0.57 & 0.45 & 0.67 & 0.73 & 1.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$ * rounded to two decimal points

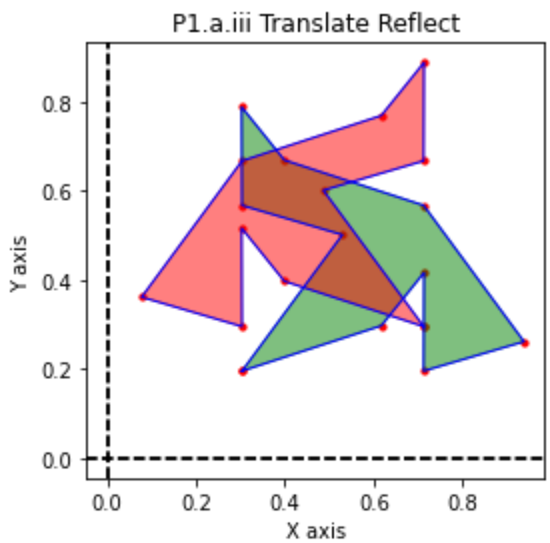



***iii. To create the third tile, reflect the model bird through the vertical line x = 0.5078 and translate the image by 0.1000 along the y-axis. Provide the matrix for the transformation in homogeneous coordinates.***

In [6]:
```
# P1.a.iii
B.eye()
B.plot()

B.translate(-0.5078,0) # shift x
B.dot(Matrix([[-1,0,0],[0,1,0],[0,0,1]])) # mirror on x-axis
B.translate(0.5078,0) # shift x back
B.translate(0,0.1) # translate y

B.plot("P1.a.iii Translate Reflect")
B.report()

T3 = B.T
```

$$TD = \begin{bmatrix} -1 & 0 & 1.0156 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.3 & 0.62 & 0.71 & 0.71 & 0.94 & 0.71 & 0.4 & 0.3 & 0.3 & 0.53 & 0.3 \\ 0.2 & 0.3 & 0.42 & 0.2 & 0.26 & 0.57 & 0.67 & 0.79 & 0.57 & 0.5 & 0.2 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$$TD = \begin{bmatrix} 0.71 & 0.4 & 0.3 & 0.3 & 0.078 & 0.3 & 0.62 & 0.71 & 0.71 & 0.49 & 0.71 \\ 0.3 & 0.4 & 0.52 & 0.3 & 0.36 & 0.67 & 0.77 & 0.89 & 0.67 & 0.6 & 0.3 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$ * rounded to two decimal points

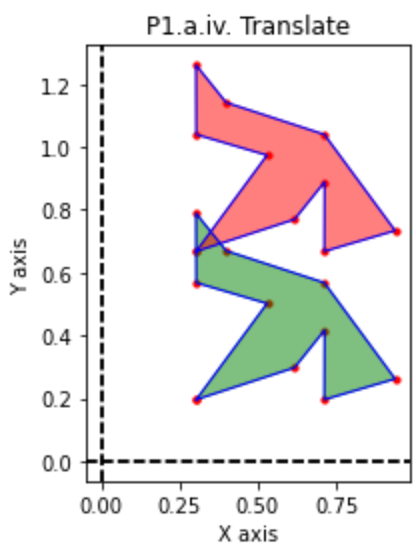



***iv. Create the fourth tile by translating the model bird along the y-axis by 0.4720. Provide the matrix for the transformation in homogeneous coordinates.***

In [7]:
```
# P1.a.iv
B.eye()
B.plot()
B.translate(0,0.4720) # shift y
B.plot("P1.a.iv. Translate")
B.report()

T4 = B.T
```

$$TD = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.472 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.3 & 0.62 & 0.71 & 0.71 & 0.94 & 0.71 & 0.4 & 0.3 & 0.3 & 0.53 & 0.3 \\ 0.2 & 0.3 & 0.42 & 0.2 & 0.26 & 0.57 & 0.67 & 0.79 & 0.57 & 0.5 & 0.2 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$$TD = \begin{bmatrix} 0.3 & 0.62 & 0.71 & 0.71 & 0.94 & 0.71 & 0.4 & 0.3 & 0.3 & 0.53 & 0.3 \\ 0.67 & 0.77 & 0.89 & 0.67 & 0.73 & 1.0 & 1.1 & 1.3 & 1.0 & 0.97 & 0.67 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$ * rounded to two decimal points




***v. Graph all four tiles together to produce the base pattern.***

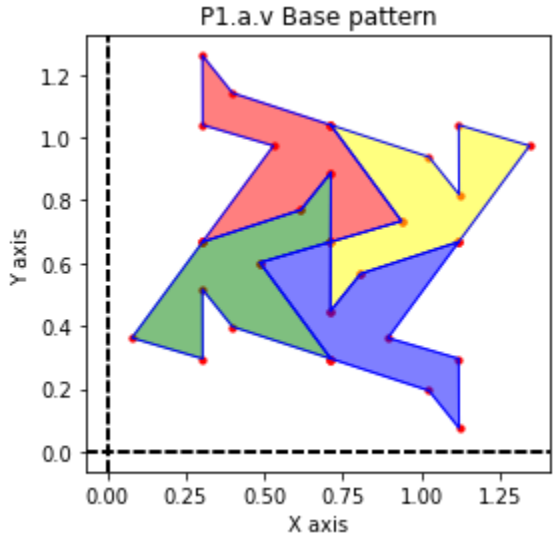
In [8]:
```
TT = [T1, T2, T3, T4]
for T in TT:
    B.T = T; B.plot("P1.a.v Base pattern")
    display(Latex(f'${sym.latex(B.T)}$'))
```

$$\begin{bmatrix} -1 & 0 & 1.424 \\ 0 & -1 & 0.864 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0.4084 \\ 0 & -1 & 1.236 \\ 0 & 0 & 1 \end{bmatrix}$$

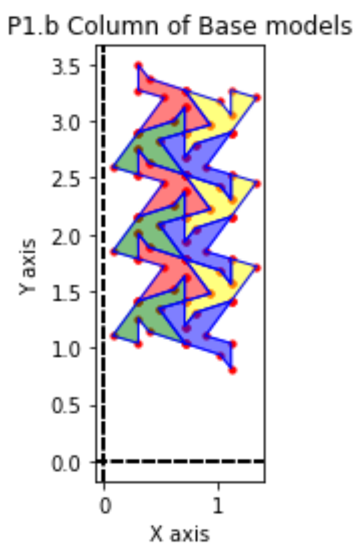
$$\begin{bmatrix} -1 & 0 & 1.0156 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0.472 \\ 0 & 0 & 1 \end{bmatrix}$$




**(b) Repeat the pattern of four model birds in part (a) but translate the entire pattern by 0.7441n, n = 1, 2, 3 along the y-axis to produce a column of the tilings.**

In [9]:
```
TT = [T1, T2, T3, T4]
for m in range(1,3+1):
    for T in TT:
        B.T = T
        B.translate(0,0.7441*m)
        B.plot("P1.b Column of Base models")
```




**(c) Repeat parts (b) and translate the column of tilings by –0.8168n, n = 1, 2, 3, 4, 5 (along the x-axis) to produce the final pattern.**

In [10]:
```
TT = [T1, T2, T3, T4]
for n in range(1,5+1):
    for m in range(1,3+1):
        for T in TT:
            B.T = T
            B.translate(-0.8168*n,0.7441*m)
            B.plot()
```