Math 425 Computation Linear Algebra

HW3, Part A

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Uniqueness, linear transformations, range and domain.

In [1]:
```
# environment setup, try to make it clear which library I'm using for what
import numpy as np  # nice arrays and other stuff
import sympy as sym # symbollic maths
from sympy.matrices import Matrix # pretty matrices
from sympy import Eq # pretty equations
from sympy.physics.quantum.dagger import Dagger # we'll want this later...
from math import e, pi, sqrt # Mathy math math
from mpl_toolkits.mplot3d import Axes3D # we like 3d quivers for tutorials
import matplotlib.pyplot as plt # old standby for plotting like a villian
from IPython.display import display, Math, Latex # used to display formatted re
sults in the console
sym.init_printing()  # initialize pretty printing
```

1. Let $T(\mathbf{x}) = A\mathbf{x}$. If $A = \begin{bmatrix} 1 & -3 & 2 \\ 0 & 1 & -4 \\ 3 & -5 & -9 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix}$, find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$, and determine whether $\mathbf{x}$ is unique.

In [2]:

```
# "We are Legion. We do not forgive. We do not forget. Expect us." -Anonymous
A = Matrix([[1,-3,2],[0,1,-4],[3,-5,-9]])
b = Matrix([6,-7,-9])
x1,x2,x3 = sym.symbols('x1 x2 x3')
X = Matrix([x1,x2,x3])
x, = sym.linsolve((A,b), (x1,x2,x3)) # ',' represents sequence unpacking in pyt
hon, somewhat abstract usage here

eq = Eq(A * X, b)  # flex on sympy
display(Latex(f'$Ax=b = {sym.latex(eq)}$'))
display(Latex(f'$x={sym.latex(Matrix(x))}$ *$x$ is a solution, but is it uniqu
e?'))

Au = A.col_insert(3, b)  # Ought to make this a function, but I bet there's a r
eason this isn't builtin already...
display(Latex(f'$Au={sym.latex(Au)}$, $Au.rref()={sym.latex(Au.rref(pivots=Fals
e))}$ **'))
display(Latex('**The full rank indicates that there are no free variables and \
being a consistant system, this is indeed a unique solution.'))
display(Latex(f"\
$Au.nullspace()={sym.latex(Au.nullspace())}$ \
***note homogeneous nature of the nullspace.")) # become a Nullspace Cadet
```

$$Ax = b = \begin{bmatrix} x_1 - 3x_2 + 2x_3 \\ x_2 - 4x_3 \\ 3x_1 - 5x_2 - 9x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -7 \\ -9 \end{bmatrix}$$

$x = \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix}$ *x is a solution, but is it unique?

$Au = \begin{bmatrix} 1 & -3 & 2 & 6 \\ 0 & 1 & -4 & -7 \\ 3 & -5 & -9 & -9 \end{bmatrix}$, $Au.rref() = \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ **

**The full rank indicates that there are no free variables and being a consistant system, this is indeed a unique solution.

$Au.nullspace() = \left[\begin{bmatrix} 5 \\ 3 \\ -1 \\ 1 \end{bmatrix}\right]$ ***note homogeneous nature of the nullspace.

2. Let $A = \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 4 \end{bmatrix}$. Is $\mathbf{b}$ in the range of the linear transformation $T(\mathbf{x}) = A\mathbf{x}$?

In [3]:

```
x1,x2,x3 = sym.symbols('x1 x2 x3')
x = Matrix([x1,x2,x3])
A = Matrix([[1,3,9,2],[1,0,3,-4],[0,1,2,3],[-2,3,0,5]])
b = Matrix([-1,3,-1,4])
Au = A.col_insert(4, b)
display(Latex(f'$A={sym.latex(A)}$, \
$A.rref()={sym.latex(A.rref(pivots=False))}$,\
$A.rank()={sym.latex(A.rank())}$, \
*not a fully ranked system'))
display(Latex(f'$Au={sym.latex(Au)}$, \
$Au.rref()={sym.latex(Au.rref(pivots=False))}$, \
$Au.rank()={sym.latex(Au.rank())}$, \
**'))

display(Latex('**The augmented rank indicates that there are is a free variable
which is fine, \
just means the solution is not unique, \
however being an iconsistant system we can clearly see $b$ is NOT in the $span$
of $A$.  \
I suppose we might consider $\overrightarrow{\mathbb{0}}$ to be a unique, albei
t trival solution, but that is just being weird.'))

display(Latex(f"\
$Ax=b$, ${sym.latex(A)}{sym.latex(x)}={sym.latex(b)}$, \
$x={sym.latex(sym.linsolve((A,b), (x1,x2,x3)))}$ \
*indeed, sympy confirms that only the trivial solution."))

display(Latex(f"Let's check the nullspace of our augmented matrix: \
$Au.nullspace()={sym.latex(Au.nullspace())}$ \
*nonhomogeneous nullspace further confirms $b$ in not in the range of $Ax$")) #
Nonhomogeneous Nullspace Cowboy
```

$$A = \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix}, A.rref() = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A.rank() = 3,$$ *not a fully ranked system

$$Au = \begin{bmatrix} 1 & 3 & 9 & 2 & -1 \\ 1 & 0 & 3 & -4 & 3 \\ 0 & 1 & 2 & 3 & -1 \\ -2 & 3 & 0 & 5 & 4 \end{bmatrix}, Au.rref() = \begin{bmatrix} 1 & 0 & 3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, Au.rank() = 4,$$ **

**The augmented rank indicates that there are is a free variable which is fine, just means the solution is not unique, however being an iconsistant system we can clearly see b is NOT in the $span$ of A. I suppose we might consider $\vec{0}$ to be a unique, albeit trival solution, but that is just being weird.

$$Ax = b, \begin{bmatrix} 1 & 3 & 9 & 2 \\ 1 & 0 & 3 & -4 \\ 0 & 1 & 2 & 3 \\ -2 & 3 & 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ -1 \\ 4 \end{bmatrix}, x = \varnothing$$ *indeed, sympy confirms that only the trivial solution.

Let's check the nullspace of our augmented matrix: $Au.nullspace() = \begin{bmatrix} \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}$ *nonhomogeneous nullspace

further confirms b in not in the range of Ax

3. Let $T: \mathbf{R}^n \to \mathbf{R}^m$ be a linear transformation, and let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set in $\mathbf{R}^n$. Explain why the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent.

THEOREM 12

Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let A be the standard matrix for T. Then:

a. T maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if the columns of A span $\mathbb{R}^m$;
b. T is one-to-one if and only if the columns of A are linearly independent.

Statement (a) in Theorem 12 is equivalent to the statement “T maps R^n onto R^m if and only if every vector in R^m is a linear combination of the columns of A.” *Theorem12 and Text, excerpt from Lay's "Linear Algebra and It's Applicaitons"

The point being that linear combinations in R^{n} map onto R^{m}, iff the columns of A $span$ R^{m}, however the columns don't even $span$ R^{n} so mapping, onto or not, R^{m} is also dependent. There's propably a more mathy way to say this but let's call this a 'point and grunt' proof.

4. Consider a linear transformation from $T: \mathbf{R}^3 \to \mathbf{R}^2$, where

$$T\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}7\\11\end{bmatrix}, T\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}6\\9\end{bmatrix}, \text{ and } T\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-13\\17\end{bmatrix}.$$

Find the standard matrix A of the transformation T.

In [4]:
```
e1 = Matrix([1,0,0])
e2 = Matrix([0,1,0])
e3 = Matrix([0,0,1])
e = Matrix([e1.T,e2.T,e3.T]).T # our basis for R**3
A = Matrix([[7,6,-13],[11,9,17]])  # just write it out for R**2

display(Latex(f'$A=[T(e1),T(e2),T(e3)]$'))
display(Latex(f'$A=[T({sym.latex(e1)},T({sym.latex(e2)},T({sym.latex(e3)}]$'))
display(Latex(f'Thus our standard matrix of $A$ is ${sym.latex(A)}$'))

display(Latex(f' \
    $A{sym.latex(e1)}={sym.latex(A*e1)}$, \
    $A{sym.latex(e2)}={sym.latex(A*e2)}$, \
    $A{sym.latex(e3)}={sym.latex(A*e3)}$ *Checks out!!'\
)) #A*e1, A*e2, A*e3, # looks good
```

$A = [T(e1), T(e2), T(e3)]$

$A = [T(\begin{bmatrix}1\\0\\0\end{bmatrix}, T(\begin{bmatrix}0\\1\\0\end{bmatrix}, T(\begin{bmatrix}0\\0\\1\end{bmatrix}]$

Thus our standard matrix of A is $\begin{bmatrix}7 & 6 & -13\\11 & 9 & 17\end{bmatrix}$

$A\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}7\\11\end{bmatrix}$, $A\begin{bmatrix}0\\1\\0\end{bmatrix} = \begin{bmatrix}6\\9\end{bmatrix}$, $A\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-13\\17\end{bmatrix}$ *Checks out!!

5. Let $T: \mathbf{R}^2 \to \mathbf{R}^3$ be a linear transformation such that

$$T(x_1, x_2) = (x_1 - 2x_2, \ -x_1 + 3x_2, 3x_1 - 2x_2).$$

Find $\mathbf{x}$ such that $T(\mathbf{x}) = (-1, 4, 9)$.

In [5]:
```
x1,x2 = sym.symbols('x1 x2')
x = Matrix([x1,x2])
A = Matrix([[1,-2],[-1,3],[3,-2]])
b = Matrix([-1,4,9])
display(Latex(f'$A=[T(x1),T(x2)]$, *$A$ not $Ax$, right? think about this...'))
# let's be rather verbose here and show unity of the trinity
display(Latex(f' \
    $Ax={sym.latex(A)}*{sym.latex(x)}={sym.latex(A*x)}$ \
    * yes, that looks about right '))

X, = sym.linsolve((A,b), (x1,x2)) # ',' represents sequence unpacking in python
x = Matrix(X)

#display(Latex(f'$A={sym.latex(A)}$, $x={sym.latex(x)}$'))
display(Latex(f'\
    example: $Ax={sym.latex(A)}{sym.latex(x)}$ = ${sym.latex(A*x)}$ \
    *indeed, $x={sym.latex(x)}$, checks out ')) # develop usage of line continu
aton for aesthetic purposes
```

$A = [T(x1), T(x2)]$, *A not Ax, right? think about this...

$Ax = \begin{bmatrix} 1 & -2 \\ -1 & 3 \\ 3 & -2 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 2x_2 \\ -x_1 + 3x_2 \\ 3x_1 - 2x_2 \end{bmatrix}$ * yes, that looks about right

example: $Ax = \begin{bmatrix} 1 & -2 \\ -1 & 3 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 5 \\ 3 \end{bmatrix}$ = $\begin{bmatrix} -1 \\ 4 \\ 9 \end{bmatrix}$ *indeed, $x = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$, checks out

6. Find the standard matrix for the linear transformation $T: \mathbf{R}^2 \to \mathbf{R}^2$

which is a horizontal shear transformation that leaves $\mathbf{e}_1$ unchanged and maps

$\mathbf{e}_2$ into $\mathbf{e}_2 + 3\mathbf{e}_1$.

In [6]:
```
e1 = Matrix([1,0])
e2 = Matrix([0,1])
A = Matrix([[1,0],[3,1]]) # nope, that's not what the problems is asking... I have a mind like a steel Hermition Matrix; transposed and conjugated.
A= Matrix([[1,3],[0,1]])  # really? okay sure
x1,x2=sym.symbols('x1 x2')
x=Matrix([x1,x2])
display(Latex(f'$A={sym.latex(A)}$'))
display(Latex(f'example: $Ax={sym.latex(A)}{sym.latex(x)}$ = ${sym.latex(A*x)}$ *okay, not what I expected'))
display(Latex(f'$A{sym.latex(e1)}={sym.latex(A*e1)}$, $A{sym.latex(e2)}={sym.latex(A*e2)}$ *Checks out!!'))
display(Latex('Indeed, I have learned something today.'))
```

$A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

example: $Ax = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ = $\begin{bmatrix} x_1 + 3x_2 \\ x_2 \end{bmatrix}$ *okay, not what I expected

$A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ *Checks out!!

Indeed, I have learned something today.

7. The color of light can be represented in a vector $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$ where R = amount of red, G = amount of green, and B = amount of blue.

The human eye and the brain transform the incoming signal into the signal $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$,

where

$$\begin{array}{lcll} \text{intensity} & I & = & \dfrac{R+G+B}{3} \\ \text{long-wave signal} & L & = & R-G \\ \text{short-wave signal} & S & = & B-\dfrac{R+G}{2}. \end{array}$$

(a) Find the matrix P representing the transformation from $\begin{bmatrix} R \\ G \\ B \end{bmatrix}$ to $\begin{bmatrix} I \\ L \\ S \end{bmatrix}$

This simply a matter of reading the transforamtion in terms of the original basis (RGB).

The basis is $e_{rgb} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Thus, standard form is $P = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1 & -1 & 0 \\ -1/2 & -1/2 & 1 \end{bmatrix}$

(b) Consider a pair of yellow sunglasses for water sports which cuts out all blue light and passes all red and green light. Find the matrix A which represents the transformation incoming light undergoes as it passes through the sunglasses.

No humans involved so this is a this a simple matter of filtering out the blue basis...

The new basis is $e'_{rgb} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

thus $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) Find the matrix for the composite transformation which light undergoes as it first passes through the sunglasses and then the eye.

Again simply a matter of writting our P but this time, filtering out the blue base too.

The basis is $e'_{rgb} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Thus, standard form is $P = \begin{bmatrix} 1/3 & 1/3 & 0 \\ 1 & -1 & 0 \\ -1/2 & -1/2 & 0 \end{bmatrix}$

8. Let $\mathbf{v}$ be a fixed vector in $\mathbf{R}^n$ and let $T: \mathbf{R}^n \rightarrow \mathbf{R}$ be the mapping defined by $T(\mathbf{x}) = \mathbf{v}^T\mathbf{x}$ (i.e. the standard inner product).

(a) Is T a linear operator?

(b) Is T a linear transformation?

a) The question is asking if the inner product is a linear operator. The inner product has both additivity $f(u+v) = f(u) + f(v)$ and homogeneity $f(\alpha u) = \alpha f(u)$ so yes it's a linear transformation, but since the domain and co-domain do NOT match, the inner product is NOT a linear operator. *see question to myself below

b) Is the inner product a linear transformation? Below I've included Theorem 1 (more pointing and grunting) showing the properties of the inner product and the definition of a linear transformation. The properties (b) and (c) match the transformation definitions (i) and (ii), thus by definition, the inner product is a linear transformation. Which we knew anyway because inner product has both additivity $f(u+v) = f(u) + f(v)$ and homogeneity $f(\alpha u) = \alpha f(u)$.

THEOREM 1

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in $\mathbb{R}^n$, and let c be a scalar. Then

a. $\mathbf{u}\cdot\mathbf{v} = \mathbf{v}\cdot\mathbf{u}$

b. $(\mathbf{u}+\mathbf{v})\cdot\mathbf{w} = \mathbf{u}\cdot\mathbf{w} + \mathbf{v}\cdot\mathbf{w}$

c. $(c\mathbf{u})\cdot\mathbf{v} = c(\mathbf{u}\cdot\mathbf{v}) = \mathbf{u}\cdot(c\mathbf{v})$

d. $\mathbf{u}\cdot\mathbf{u} \geq 0$, and $\mathbf{u}\cdot\mathbf{u} = 0$ if and only if $\mathbf{u} = \mathbf{0}$

Properties (b) and (c) can be combined several times to produce the following useful rule:

$$(c_1\mathbf{u}_1 + \cdots + c_p\mathbf{u}_p)\cdot\mathbf{w} = c_1(\mathbf{u}_1\cdot\mathbf{w}) + \cdots + c_p(\mathbf{u}_p\cdot\mathbf{w})$$

DEFINITION

A transformation (or mapping) T is **linear** if:

(i) $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ for all $\mathbf{u}, \mathbf{v}$ in the domain of T;

(ii) $T(c\mathbf{u}) = cT(\mathbf{u})$ for all scalars c and all $\mathbf{u}$ in the domain of T.

*Theorem 1, from "Linear Algebra and It's Applications" by Lay, et alia

*My question is what is the difference between an operator and a transformation. How does this relate to function space?

Think about how inner product collaspes a dimension... Alright, now we can see that for the linear operator, the domain and co-domain match, however for a linear transformation, the domain and co-domain need not match. Thus, every linear operator is a linear transformation, however not every linear transformation is a linear operator. The second part of my question is a bit more convoluted * . *Okay, don't get me started with the puns!

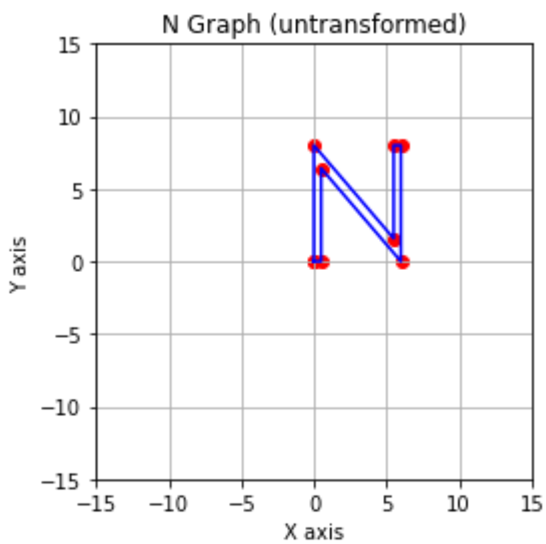
9. Find the 3×3 matrices that produce the described composite 2D transformations, using homogeneous coordinates. Apply the transformations to the 'letter N' data, ``letterN.pny'' and submit the corresponding plots as well.

In [7]:
```python
N = np.load('letterN.npy')

def myplot(title, N, lim=15):
    plt.title(f"{title}"); plt.xlabel("X axis"); plt.ylabel("Y axis")
    plt.scatter(N[0], N[1], color ="red")
    plt.plot(N[0], N[1], color ="blue")
    plt.xlim(-lim,lim); plt.ylim(-lim,lim)
    plt.grid(); plt.gca().set_aspect("equal") # square grids are pretty
    plt.show()

display(Latex("Show some information about the data we've just loaded."))
display(Latex(f'$N.shape= {N.shape}$, type(N)={type(N)}'))
display(Latex(f'$N= {sym.latex(Matrix(N))}$'))

myplot('N Graph (untransformed)', N)
```

Show some information about the data we've just loaded.

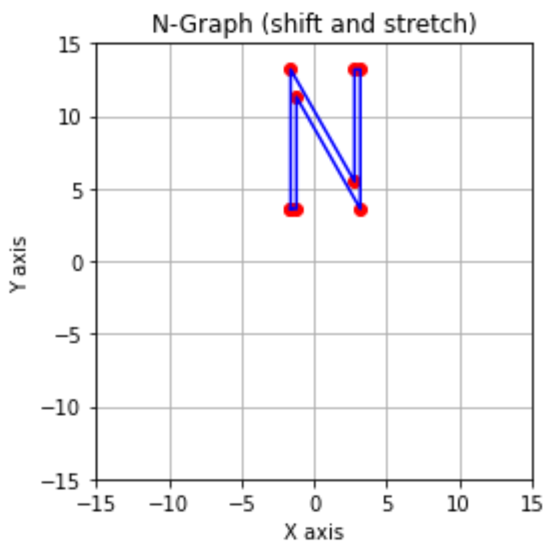
$N.shape = (2, 9)$, type(N)=

$$N = \begin{bmatrix} 0.0 & 0.5 & 0.5 & 6.0 & 6.0 & 5.5 & 5.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 6.42 & 0.0 & 8.0 & 8.0 & 1.58 & 8.0 & 0.0 \end{bmatrix}$$



(a) Translate by $(-2, 3)$, and then scale the x-coordinate by 0.8 and the y-coordinate by 1.2

In [8]:
```
NN = np.asmatrix(N)  # note: N was imported above
r = np.ones(N.shape[1]) # form a new homogeneous row
NN = np.vstack([NN, r]) # append our row

# see also: Ch2.7 P4E in Lay text
T1 = np.matrix([[1,0,-2],[0,1,3],[0,0,1]])
T2 = np.matrix([[0.8,0,0],[0,1.2,0],[0,0,1]])
T = np.dot(T2,T1)    # inner product of <T2|T1>, break out some Dirac notation
Q = np.dot(T,NN)    # inner product of <T|N>
myplot('N-Graph (shift and stretch)',np.array(Q),15)

#print(f'T = {T}')  # numpy print is ugly
display(Latex(f'$N={sym.latex(Matrix(NN))}$ *show our homogeneous data'))
display(Latex(f'$T={sym.latex(Matrix(T2))}{sym.latex(Matrix(T1))}={sym.latex(Ma
trix(T))}$'))
display(Latex(f'$TN={sym.latex(Matrix(T))}{sym.latex(Matrix(NN))}$')) # so pret
ty
display(Latex(f'$TN={sym.latex(Matrix(Q))}$'))
```



$N = \begin{bmatrix} 0.0 & 0.5 & 0.5 & 6.0 & 6.0 & 5.5 & 5.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 6.42 & 0.0 & 8.0 & 8.0 & 1.58 & 8.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$ *show our homogeneous data

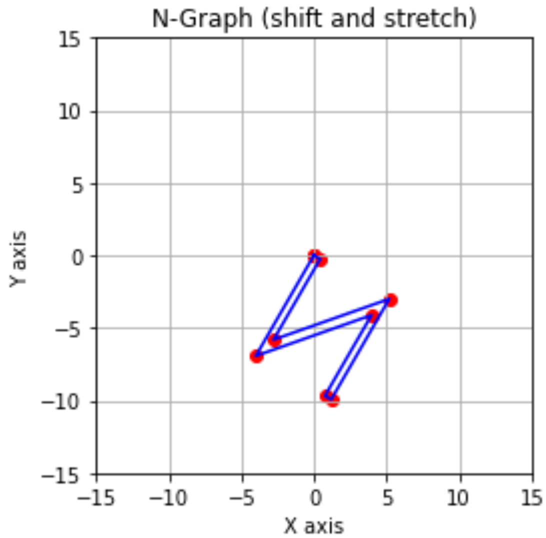
$$T = \begin{bmatrix} 0.8 & 0.0 & 0.0 \\ 0.0 & 1.2 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.8 & 0.0 & -1.6 \\ 0.0 & 1.2 & 3.6 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$TN = \begin{bmatrix} 0.8 & 0.0 & -1.6 \\ 0.0 & 1.2 & 3.6 \\ 0.0 & 0.0 & 1.0 \end{bmatrix} \begin{bmatrix} 0.0 & 0.5 & 0.5 & 6.0 & 6.0 & 5.5 & 5.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 6.42 & 0.0 & 8.0 & 8.0 & 1.58 & 8.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$$TN = \begin{bmatrix} -1.6 & -1.2 & -1.2 & 3.2 & 3.2 & 2.8 & 2.8 & -1.6 & -1.6 \\ 3.6 & 3.6 & 11.304 & 3.6 & 13.2 & 13.2 & 5.496 & 13.2 & 3.6 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

(b) Rotate points $\frac{\pi}{6}$, and then reflect through the x-axis.

In [9]:
```
# use syms to make pretty
theta = sym.pi/6
T1 = Matrix([[sym.cos(theta), -sym.sin(theta),0], [sym.sin(theta), sym.cos(thet
a),0],[0,0,1]]) #rot
T2 = Matrix([[1,0,0],[0,-1,0],[0,0,1]]) # flip y
T= T2 * T1
Q = T*Matrix(NN) # note: NN was defined in the previous section
myplot('N-Graph (shift and stretch)',np.array(Q),15)
display(Latex(f'$T={sym.latex(Matrix(T2))}{sym.latex(Matrix(T1))}={sym.latex(Ma
trix(T))}$'))
display(Latex(f'$TN={sym.latex(Matrix(T))}{sym.latex(Matrix(NN))}$'))
display(Latex(f'$TN={sym.latex(Q.n(2))}$ *rounded to two decimal points')) # so
pretty
```



$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$TN = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.0 & 0.5 & 0.5 & 6.0 & 6.0 & 5.5 & 5.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 6.42 & 0.0 & 8.0 & 8.0 & 1.58 & 8.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$

$$TN = \begin{bmatrix} 0 & 0.43 & -2.8 & 5.2 & 1.2 & 0.76 & 4.0 & -4.0 & 0 \\ 0 & -0.25 & -5.8 & -3.0 & -9.9 & -9.7 & -4.1 & -6.9 & 0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \end{bmatrix}$$ *rounded to two decimal points

Appendix 0. The Matrix Alphabet

sym	matrix	sym	matrix
A	Any Matrix	P	Permutation Matrix
B	Basis Matrix	P	Projection Matrix
C	Cofactor Matrix	Q	Orthogonal Matrix
D	Diagonal Matrix	R	Upper Triangular Matrix
E	Elimination Matrix	R	Reduced Echelon Matrix
F	Fourier Matrix	S	Symmetric Matrix
H	Hadamard Matrix	T	Linear Transformation
I	Identity Matrix	U	Upper Triangular Matrix
J	Jordan Matrix	U	Left Singular Vectors
K	Stiffness Matrix	V	Right Singular Vectors
L	Lower Triangular Matrix	X	Eigenvector Matrix
M	Markov Matrix	Λ	Eigenvalue Matrix
N	Nullspace Matrix	Σ	Singular Value Matrix

Linear Algebra by Gilbert Strang

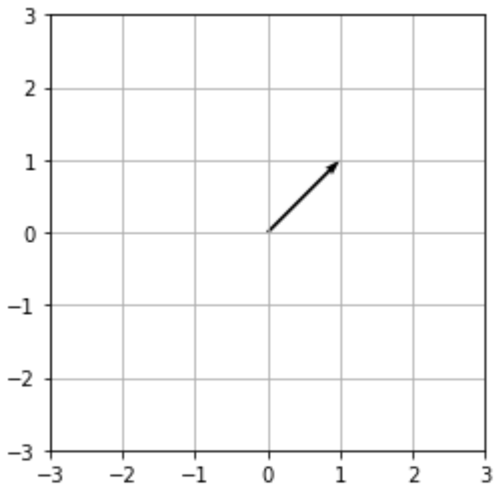
Appendix 1Damage

"I Remember Damage. And Escape. Then Adrift In A Stranger's Galaxy For A Long Time. But I'm Safe Now. I Found It Again. My Home."

****Station Eleven by Emily St. John Mandel***

Appendix 2D. quiver

```
In [10]: # 2d quiver
plt.quiver(0,0,1,1, angles='xy', scale_units='xy', scale=1)
lim = 3
plt.xlim(-lim,lim)
plt.ylim(-lim,lim)
plt.grid()
plt.gca().set_aspect("equal") # square grids are pretty
plt.show()
```



Appendix 3D. quiver

```
In [11]: fig = plt.figure()
ax = fig.gca(projection='3d')
ax.set_xlim3d(0, 1)
ax.set_ylim3d(0, 1)
ax.set_zlim3d(0, 1)
ax.quiver(0, 0, 0, 1, 1, 1, length = sqrt(2), normalize = True)
plt.show()
```