Math 425 Applied and Computational Linear Algebra Spring 2022

hw5 , **due on iLearn by 12:30pm on Tuesday, April 26**

1. Find an orthogonal basis for the column space of the matrix $A = \begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$.

2. Find an orthonormal basis for the column space of the matrix $A = \begin{bmatrix} 3 & -3 & 0 \\ -4 & 14 & 10 \\ 5 & -7 & -2 \end{bmatrix}$.

3. Let $\mathbf{u}_1, \ldots, \mathbf{u}_p$ be an orthogonal basis for the subspace W of $\mathbb{R}^n$, and let $T : \mathbb{R}^n \to \mathbb{R}^n$ be defined by $T(\mathbf{x}) = \text{proj}_W \mathbf{x}$. Show that T is a linear transformation.

4. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$. Find (a) the orthogonal projection of $\mathbf{b}$ onto Col A and (b) a least-squares solution of $A\mathbf{x} = \mathbf{b}$.

5. Let $A = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 2 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -5 \\ 8 \\ 1 \end{bmatrix}$. Find the least-squares solution of $A\,\mathbf{x} = \mathbf{b}$.

6. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad \text{and } \mathbf{b} = \begin{bmatrix} 7 \\ 2 \\ 3 \\ 6 \\ 5 \\ 4 \end{bmatrix}.$$

Describe all least-squares solutions of the equation $A\mathbf{x} = \mathbf{b}$.

7. Let

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 4 \\ 1 & -1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \\ 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix},$$

be the factorization $A = QR$ and let $\mathbf{b} = \begin{bmatrix} -1 \\ 6 \\ 5 \\ 7 \end{bmatrix}$. Use the QR factorization to find the least-squares solution of $A\mathbf{x} = \mathbf{b}$.

8. A healthy child's systolic blood pressure p (in millimeters of mercury) and weight w (in pounds) are approximately related by the equation

$$\beta_0 + \beta_1 \ln w = p.$$

Use the following experimental data to estimate the systolic blood pressure of a healthy child weighing 100 pounds.

w	44	61	81	113	131
$\ln w$	3.78	4.11	4.41	4.73	4.88
p	91	98	103	110	112

9. To measure the takeoff performance of an airplane, the horizontal position of the plane was measured every second, from $t = 0$ to $t = 12$. The positions (in feet) were: 0, 8.8, 29.9, 62.0, 104.7, 159.1, 222.0, 294.5, 380.4, 471.1, 571.7, , 686.8, 809.2 .

(a) Find the least-squares cubic curve $y = \beta_0 + \beta_1 t + \beta_2 t^2 + \beta_3 t^3$ for these data.

(b) Use the result of (a) to estimate the velocity of the plane when $t = 4.5$ seconds.

10. Find the singular values of the matrix $\begin{bmatrix} -5 & 0 \\ 0 & 0 \end{bmatrix}$.

11. Suppose the factorization below is an SVD of a matrix A, with the entries in U and V rounded to two decimal places.

$$A = \begin{bmatrix} -0.86 & -0.11 & -0.50 \\ 0.31 & 0.68 & -0.67 \\ 0.41 & -0.73 & -0.55 \end{bmatrix} \begin{bmatrix} 12.48 & 0 & 0 & 0 \\ 0 & 6.34 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.66 & -0.03 & -0.35 & 0.66 \\ -0.13 & -0.90 & -0.39 & -0.13 \\ 0.65 & 0.08 & -0.16 & -0.73 \\ -0.34 & 0.42 & -0.84 & -0.08 \end{bmatrix}$$

(a) What is the rank of A?

(b) Use this decomposition of A, with no calculations, to write a basis for Col A and a basis for Nul A.

12. Suppose A is square and invertible. Find a singular value decomposition of A^{-1}.

13. Show that if A is square, then $|\det A|$ is the product of the singular values of A.

14. Find the *minimal length* least-squares solution of the equation $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}.$$