P2) Choose h and k such that has given properties

$x_1 + 3x_2 = 2$

$3x_1 + hx_2 = k$

a) no solution

- this will occurs when eq's are $\parallel$
- we are going to set the coefficients to be equal ratios of each other but not the result

like this

$x_1 + 3x_2 = 2$

$3x_1 + 3 \cdot 3x_2 = 0$

again w/ the elimination method

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & 9 & 0 \end{array}\right] \Rightarrow \begin{array}{l} r_1 = r_1 \\ r_3 = r_3 - 3r_1 \end{array} \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 0 & -6 \end{array}\right]$$

$0 \neq -6$ thus no solution

thus $h = 9$ and $k = 0$ works!

P2) continued ...

b) a unique solution

- this occurs when eq's cross
- let's pick a couple easy numbers to calculate.

$x_1 + 3x_2 = 2$

$3x_1 + 3x_2 = 0$

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & 3 & 0 \end{array}\right] \Rightarrow \begin{array}{l} r_1 = r_1 \\ r_3 = r_3 - 3r_1 \end{array} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -6 & -6 \end{bmatrix}$$

$$\Rightarrow \begin{array}{l} r_1 = r_1 \\ r_3 = \frac{r_3}{-6} \end{array} \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow r_1 = r_1 - 3r_3 \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$(x_1, x_2) = (-1, 1)$

is indeed a unique solution

P2 continued ...

c) many solution

- this will happen when eq's are on top of each other
- we can cook up some values that work by making the coeff and result have equal ratios, like so...

$x_1 + 3x_2 = 2$

$3x_1 + 3 \cdot 3x_2 = 2 \cdot 3$

again w/ augmented matrix reduction

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & 9 & 6 \end{array}\right] \Rightarrow \begin{array}{l} r_1 = r_1 \\ r_2 = r_2 - 3r_1 \end{array} \left[\begin{array}{ccc} 1 & 3 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$\boxed{x_1 + 3x_2 = 2 \text{ is are the solutions}}$ ✓