Frans' Savings Plan

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My idea is it would be wise to save for pension or large investment such as an apartment. My interest in this, beyond the end result, is limited: I have no intension to spend time on fundamental analysis or follow developments on a regular basis. However, I do write this document, and I intend to do a modern portfolio theory (MPT) analysis of the portfolio. I have in interest to sleep well, without worrying about this.

Individual stocks are, as far as I can tell, hence irrelevant because I won't manage it, rebalance, and so forth. Therefore I think it makes sense to look at passive indices and active management.

The computations contains no strategic decisions, they are a procedural, technical matter. The interesting questions is what assets to choose, and that is the hard part, not calculating the portfolio weights.

1 Details

This is the broad picture:

- "Long" horizon. Easily 5-10 years
- Generally offensive/"high" risk investment
- Bank of choice is Avanza in Sweden
- Other banks of mine are Nordea in Sweden and Norway. A student's economy, only savings accounts
- Account type for the investments: Investeringssparkonto (ISK)
- Start amount: 10 000 SEK
- Monthly investment: 500 SEK.

2 Investment Objects

The following portfolio has been selected automatically by Avanza's platform for a time horizon of ten years or more, where highest Morningstar rating is key.

Name
SPP Global Plus A SEK
AMF Aktiefond Global
AMF Aktiefond Småbolag
Handelsbanken Sverige Tema (A1 SEK)
Öhman Etisk Emerging Markets A
Nordea 1 - Emerging Stars Equity BP SEK
SEB Europe Equity Fund C EUR
SPP Aktiefond USA A SEK

Table 1: Selected assets for the portfolio.

In short it achieves diversification across USA, Europe and Sweden, and is high return/risk. Therefore it matches my objectives.

2.1 Mean-variance Efficient Frontier

Mathematically, the problem is:

$$\max_{w} w'\mu$$

w is a $N \times 1$ vector of asset weights, where N is the number of assets. μ is a $N \times 1$ vector of expected returns. The solution can be found in the mean-variance efficient frontier, which is:

$$\bar{u}_p = \frac{A}{C} + \frac{1}{C}\sqrt{(C\sigma_p^2 - 1)(BC - A^2)},$$

where

$$A = \mu'\Sigma^{-1}\iota$$
$$B = \mu'\Sigma^{-1}\mu$$
$$C = \iota\Sigma^{-1}\iota$$

Σ is an $N \times N$ variance-covariance matrix of returns, ι an $N \times 1$ vector of ones.

The mean-variance frontier for the portfolio is plotted in figure 1.

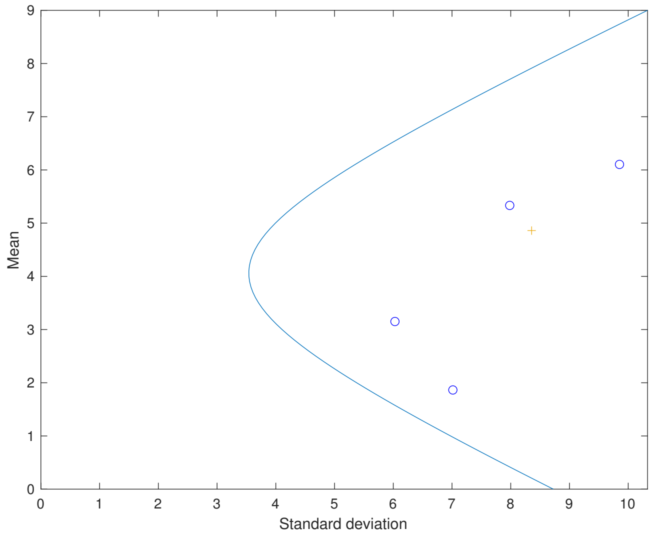

Figure 1: Mean-variance frontier of portfolio.

3 Optimal Weights

The definition of optimal weights is those that give maximum return. Another definition could have been that which leads to minimum variance, that is, catering to risk aversion.

The theoretically optimal weights may involve shorting, but due to the bank not allowing this and that simplicity is preferred, the optimisation has been constrained to not include shorting. No position is taken in the risk-free asset.

The optimal weights of the assets in the portfolio are found in table 2.

Asset Name	Weighting	Weighting (%)
Asset 1	0.238727	24
Asset 2	0.213544	21
Asset 3	0.328114	33
Asset 4	0.130888	13
Asset 5	0.088728	9

Table 2: Optimal weights.

The Sharpe ratio of the portfolio is ≈1.68.

However, DeMiguel, Garlappi, and Uppal (2007) concludes that naive diversification outperforms the mean-variance based determination of weights, in terms of Sharpe ratio. Hence this paper is mostly for me an educational exercise – though appreciated. I will go for a naive weighting, the bank's default.

4 Remaining Questions

References

DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal (2007). "Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy?" In: *The Review of Financial Studies* 22.5, pp. 1915–1953. ISSN: 0893-9454. DOI: 10.1093/rfs/hhm075.

A Code

Listing 1: Matlab code

```

clear all;
close all;

%% The data.
% 5 assets are made up for now until we have real data from
    Bloomberg.
nassets = 5; % = size(mu, 1);

% Expected returns.
ers = [2, 5, 3, 6, 7];

asset_names = string(1:nassets);

for i = 1:nassets
    asset_names(i) = strcat("Asset ", asset_names(i));
end

% 3 years of daily observations.
n_obs = 30 * 12 * 3;

g_sd = [7, 8, 6, 10, 12];

rng('default');

% Array of returns, columns are assets.
mktret = [];

ret = [];

% Randomly generate the returns.
for i = 1:nassets
    new = normrnd(ers(i), g_sd(i), [n_obs, 1]);
    ret = cat(2, ret, new);
end

%% Basic paramaters.
mu = mean(ret)';
sd = std(ret)';
correl = corr(ret);

% Covariance of returns.
Covar = diag(sd) * correl * diag(sd);

%% Compute the optimal weights through maximisation.
f = @(x) sqrt(x' * Covar * x) * 100;
```

```
% Our starting values for fmincon(). It is the assets equally weighted.
w_0 = repmat(1 / nassets, nassets, 1);

% Linear equality constraints.
Aeq = ones(1, nassets);
beq = 1;

% We do this because we cannot short.
lb = zeros(nassets, 1);

% Compute and write out our optimal portfolio weights.
% Aeq * x = beq
w_opt = fmincon(f, w_0, [], [], Aeq, beq, lb, []);

fid = fopen('generated_weights.tex', 'w');

for i = 1:size(w_opt, 1)
    fprintf(fid,                    ...
            '%s & %f & %g \\\\\n',  ...
            asset_names(i),         ...
            w_opt(i),               ...
            round(w_opt(i) * 100, 0));
end

fclose(fid);

%% Draw the MV frontier.

mu_bar = (0 : 0.1 : round(max(mu) * 1.2))'; % TODO right?

% Pre-define a matrix for the weights.
w_MV = zeros(size(mu_bar, 1), nassets);

% Pre-define a matrix for standard deviations.
sigma_MV = zeros(size(mu_bar, 1), 1);

Aeq = [ones(1, nassets); mu'];

for i = 1 : size(mu_bar, 1)
    w_opt_cand = fmincon(f, w_0, [], [], Aeq, [1; mu_bar(i)]);

    % Saving the optimal weights.
    w_MV(i, :) = w_opt_cand';

    % Saving the corresponding S.D.
    sigma_MV(i) = sqrt(w_opt_cand' * Covar * w_opt_cand);
end

figure;

% The MV frontier.
plot(sigma_MV, mu_bar);

xlim([0, max(sigma_MV)]);
ylim([0, max(mu_bar)]);

hold all;

% The original portfolios.
plot(sd, mu, 'ob');
```

```
xlabel('Standard deviation');
ylabel('Mean');

%% Add the optimal portfolio to the plot.
% Note, risk-free is irrelevant in our case, so the equations are adapted
% accordingly.

w_tilde = inv(Covar) * mu;
w_opt = w_tilde / (w_tilde' * ones(nassets, 1));

opt_mu = w_opt' * mu;
opt_sigma = w_opt' * sd;

plot(opt_sigma, opt_mu, '+');

saveas(gcf, "generated_mv_frontier.eps", 'epsc');

%% Compute & write out SR.
opt_SR = opt_mu / sqrt(opt_sigma);

fid = fopen('generated_constants.tex', 'w');
fprintf(fid, '\\def\\optSR{%g}\n', round(opt_SR, 2));
fclose(fid);
```

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