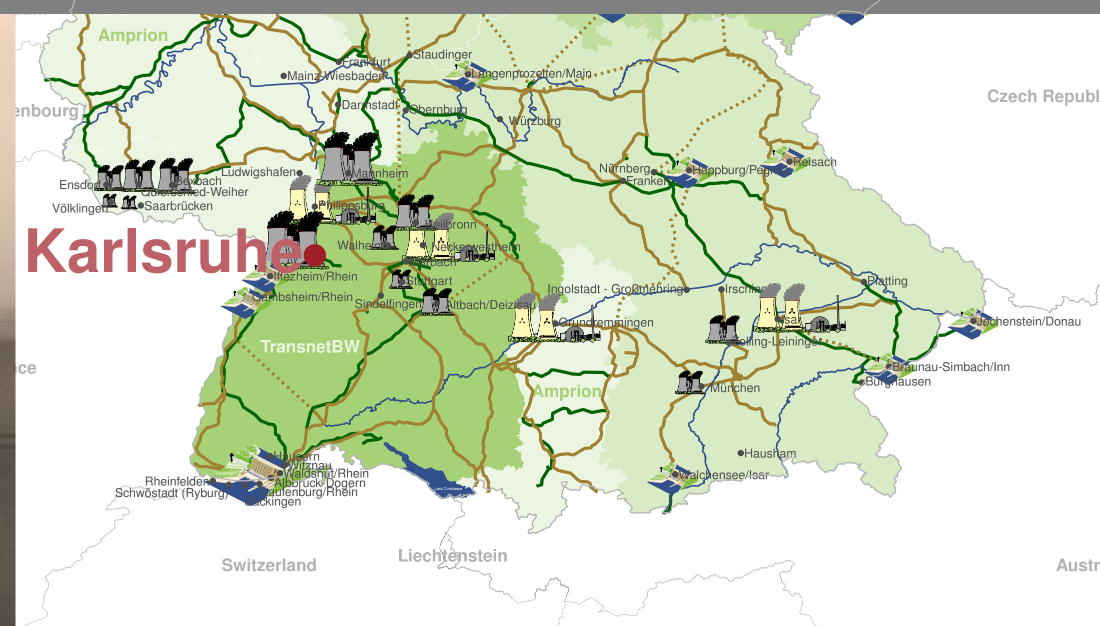
Karlsruhe Institute of Technology

Placement and Planning Problems in Power Grids

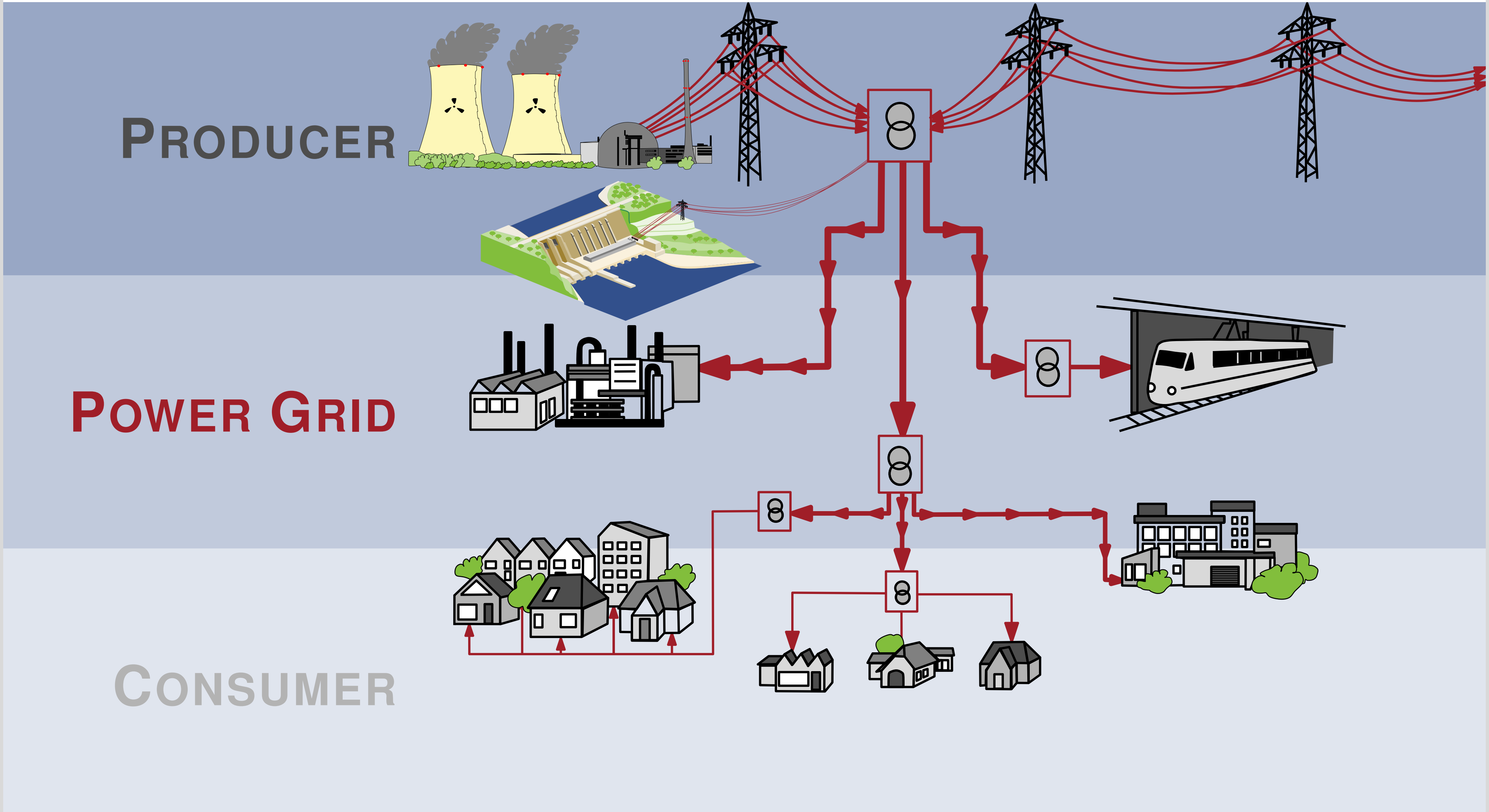
FRICO · 16. August 2017

Franziska Wegner

INSTITUTE OF THEORETICAL INFORMATICS · ALGORITHMICS GROUP

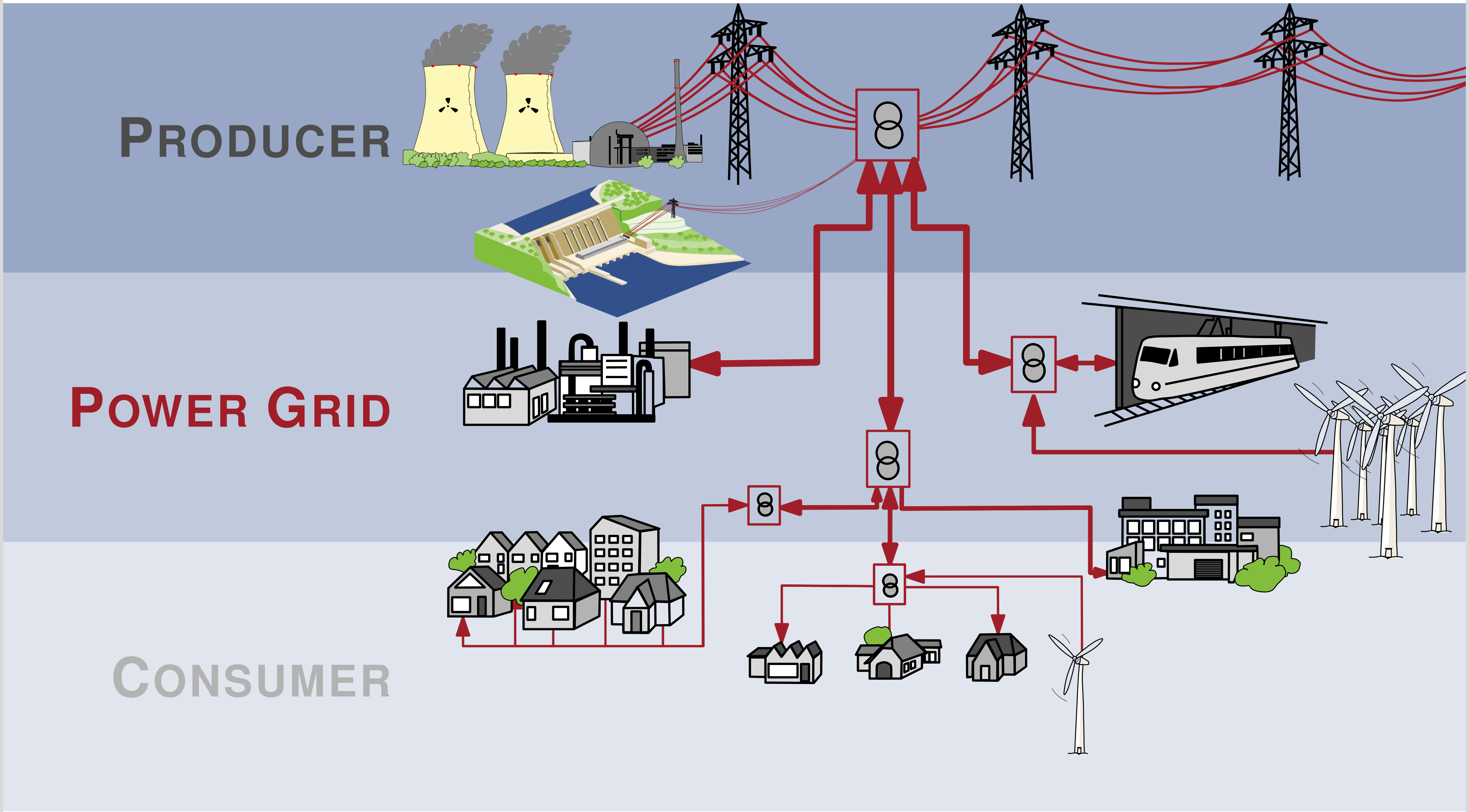


Recent Development in Power Grids



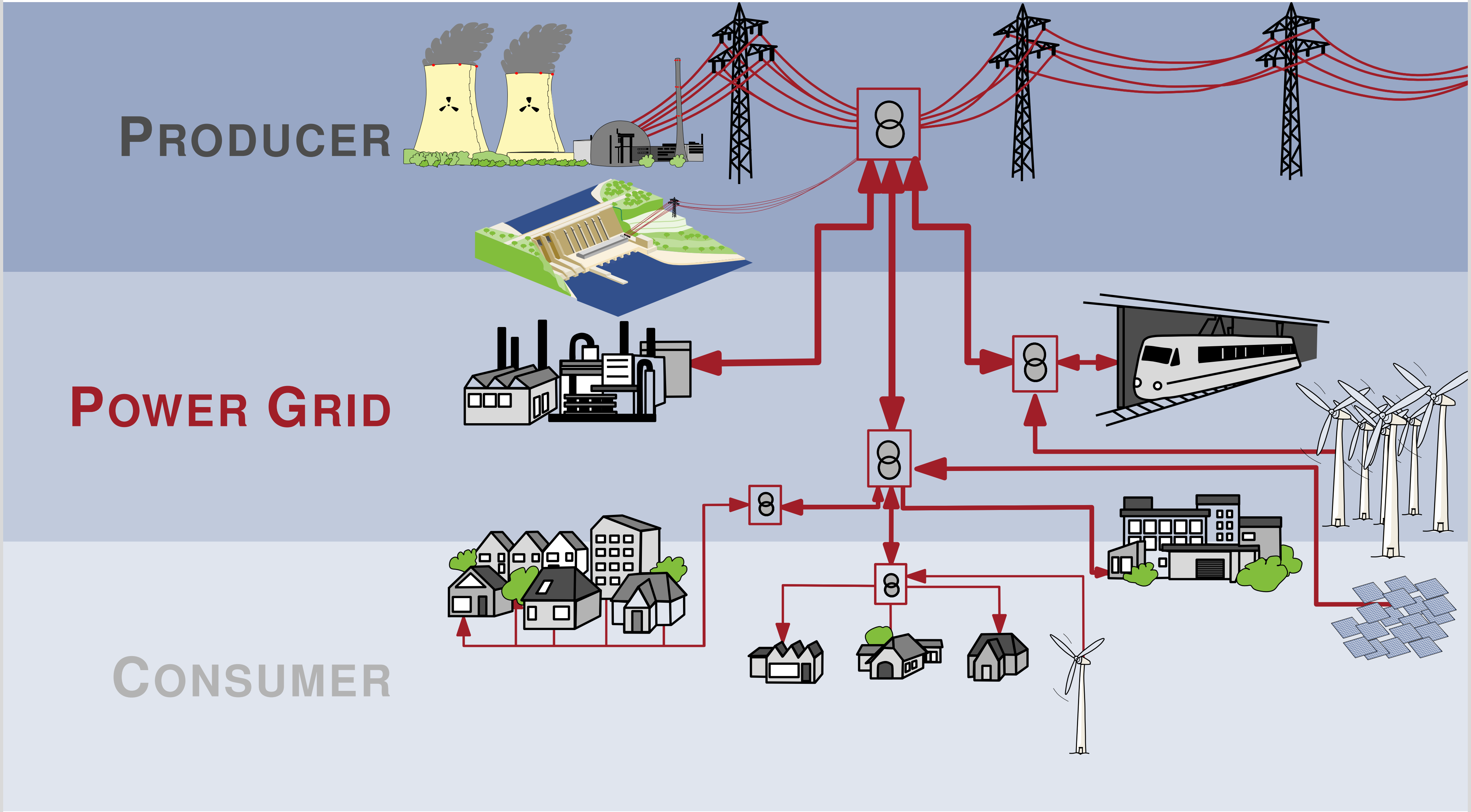
Recent Development in Power Grids

Karlsruhe Institute of Technology



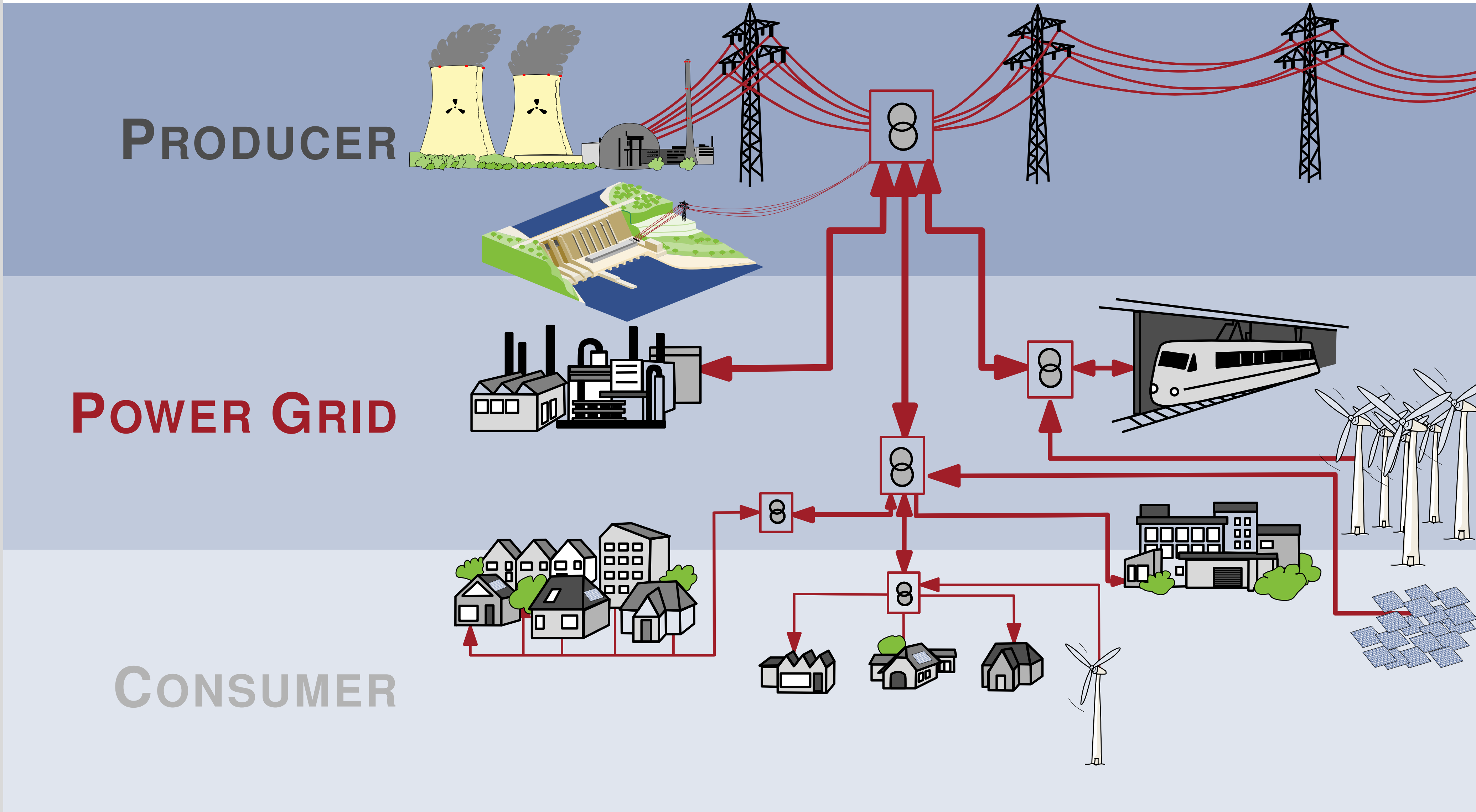
Institute of Theoretical Informatics
Algorithmics Group

Recent Development in Power Grids

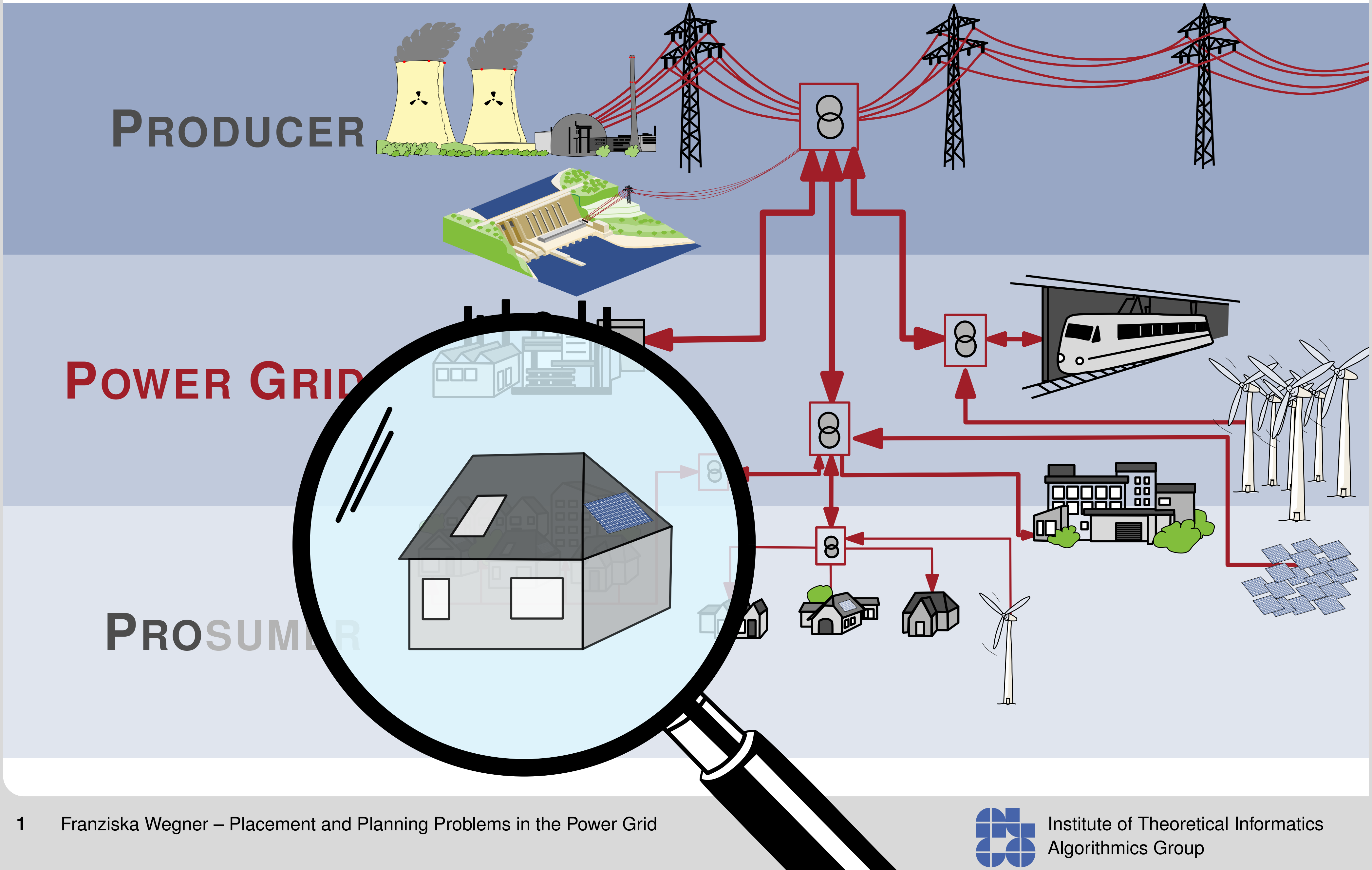


Institute of Theoretical Informatics
Algorithmics Group

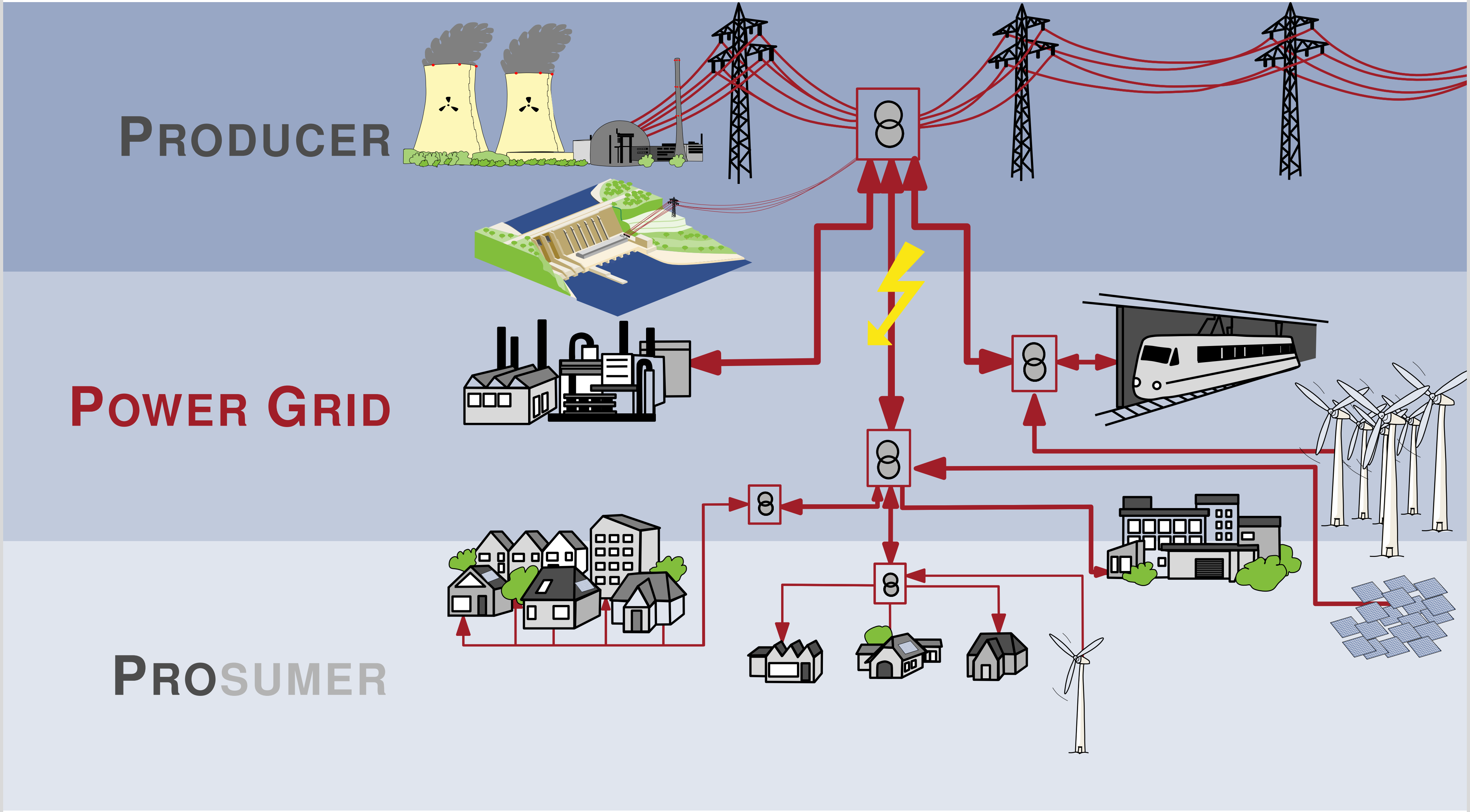
Recent Development in Power Grids



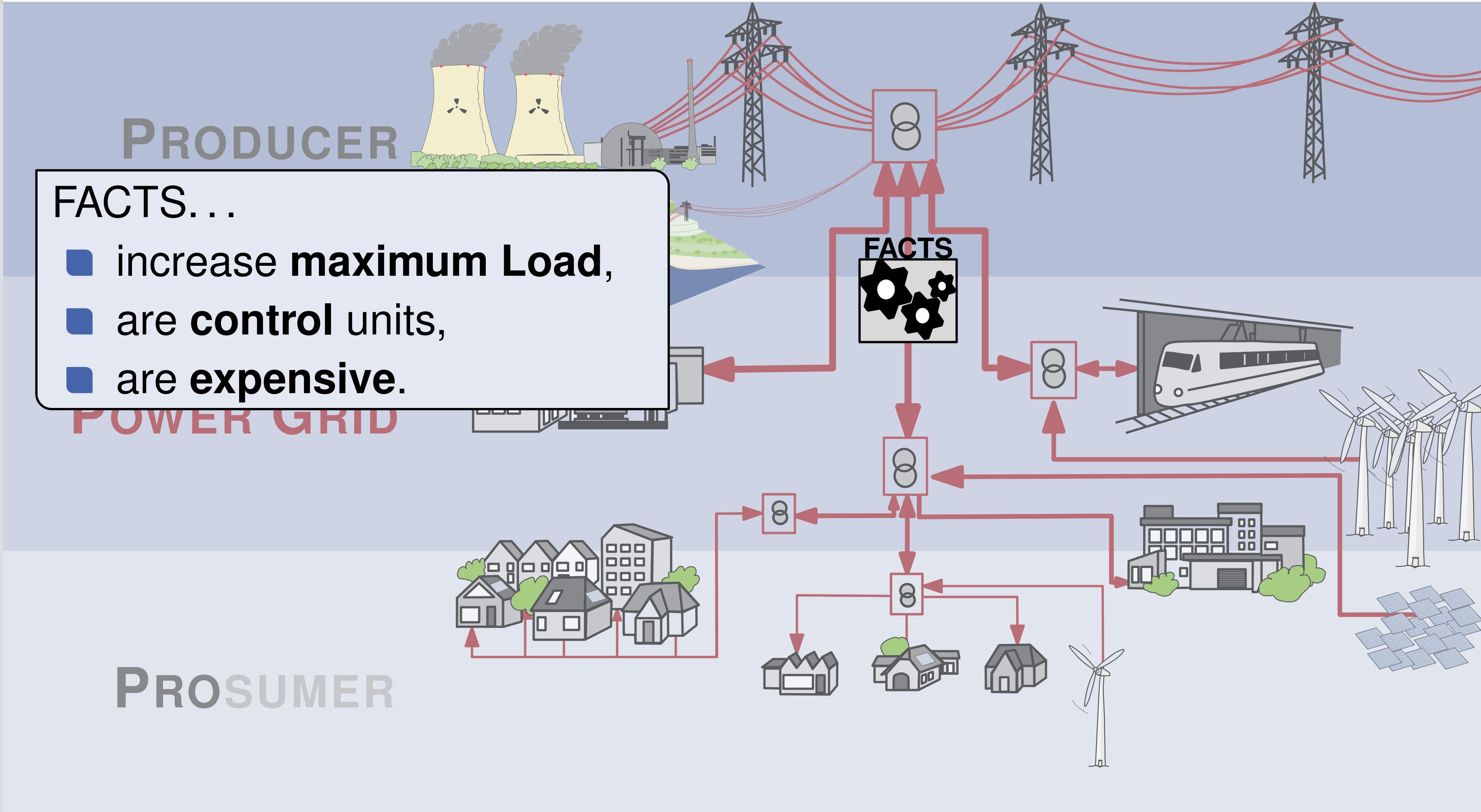
Recent Development in Power Grids



Recent Development in Power Grids



Recent Development in Power Grids

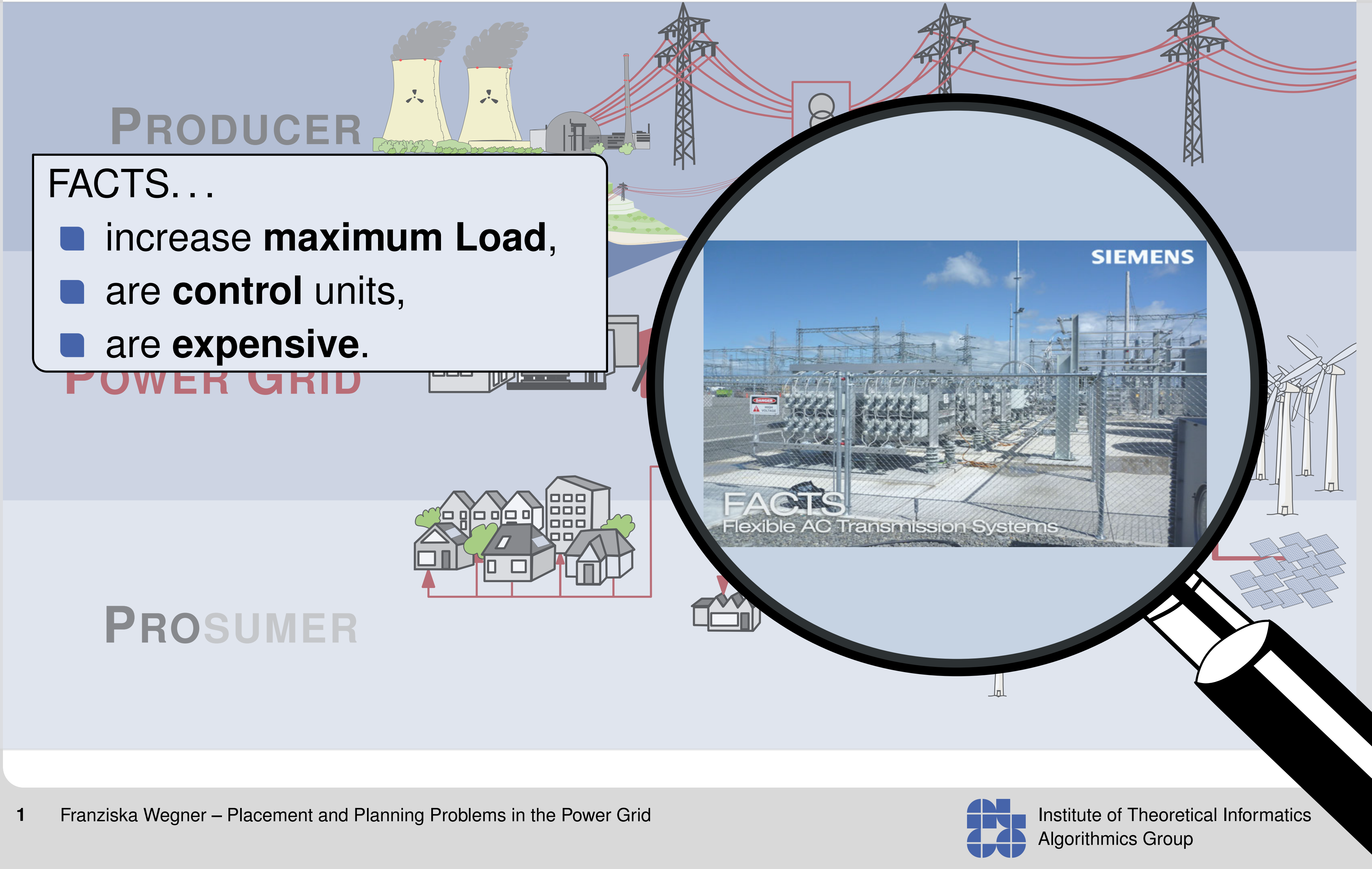


FACTS...

- increase **maximum Load**,
- are **control** units,
- are **expensive**.

Institute of Theoretical Informatics
Algorithmics Group

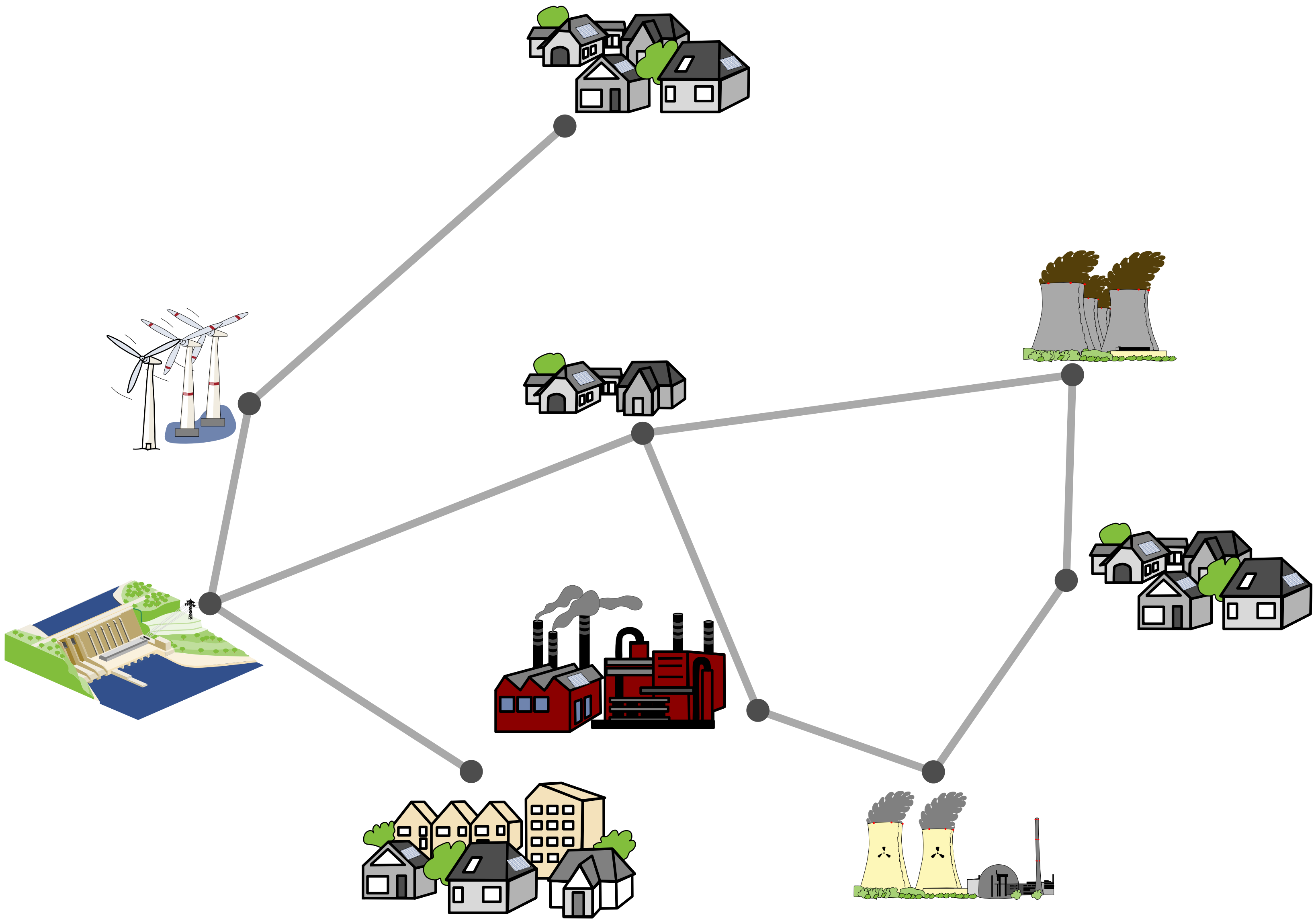
Recent Development in Power Grids



FACTS. . .

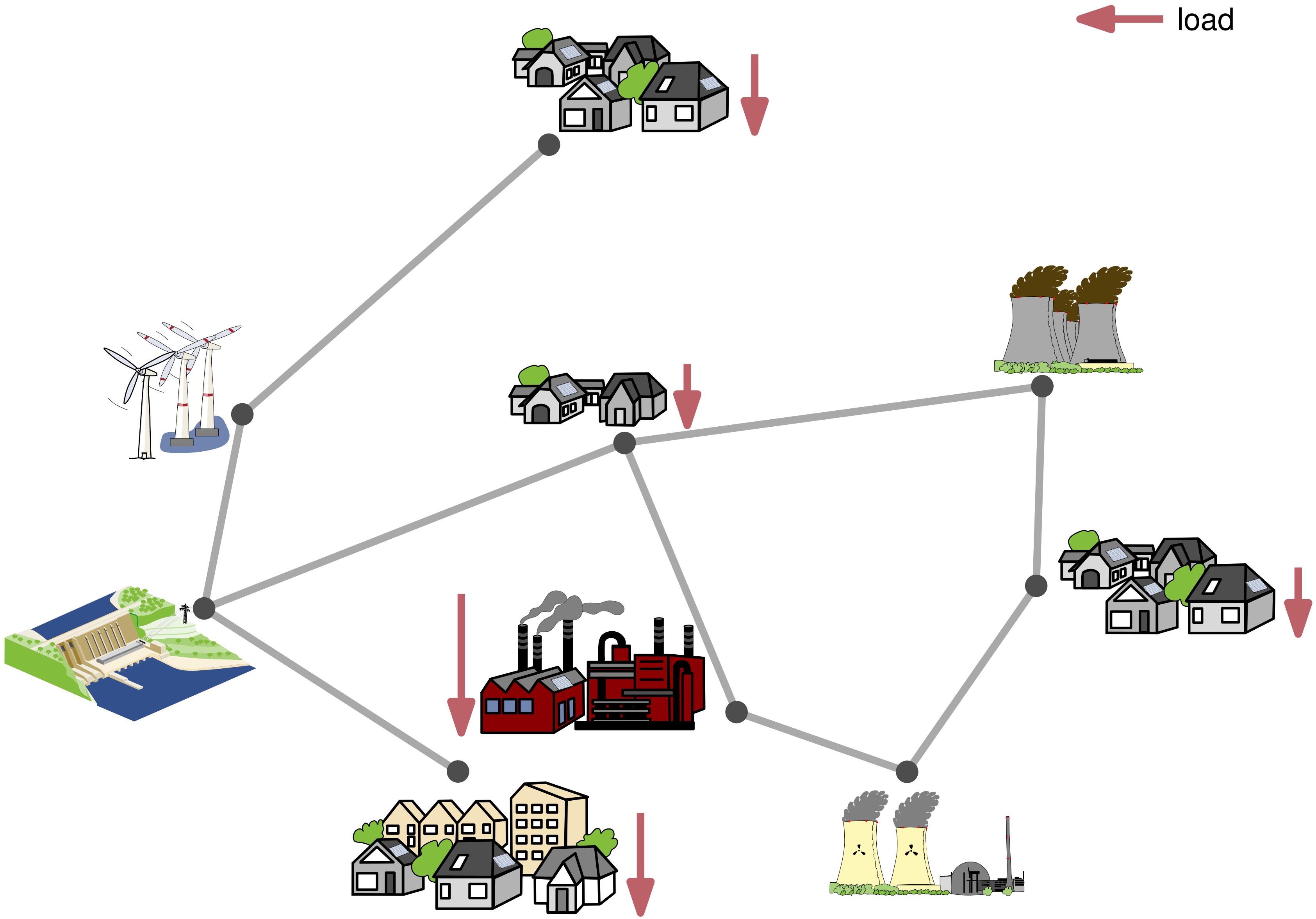
- increase **maximum Load**,
- are **control** units,
- are **expensive**.

AC Conservation of Flow in Power Grids

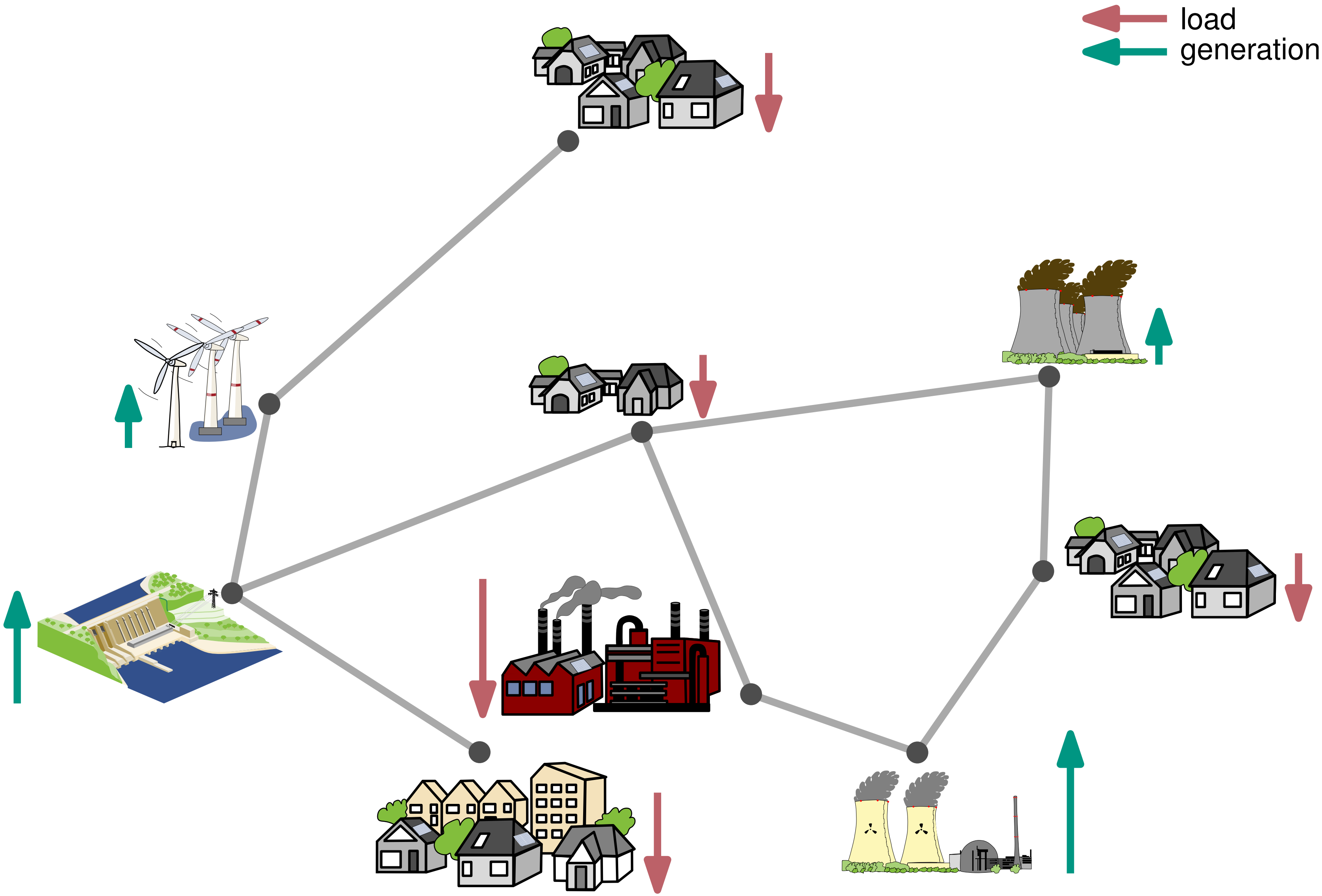


Institute of Theoretical Informatics
Algorithmics Group

AC Conservation of Flow in Power Grids

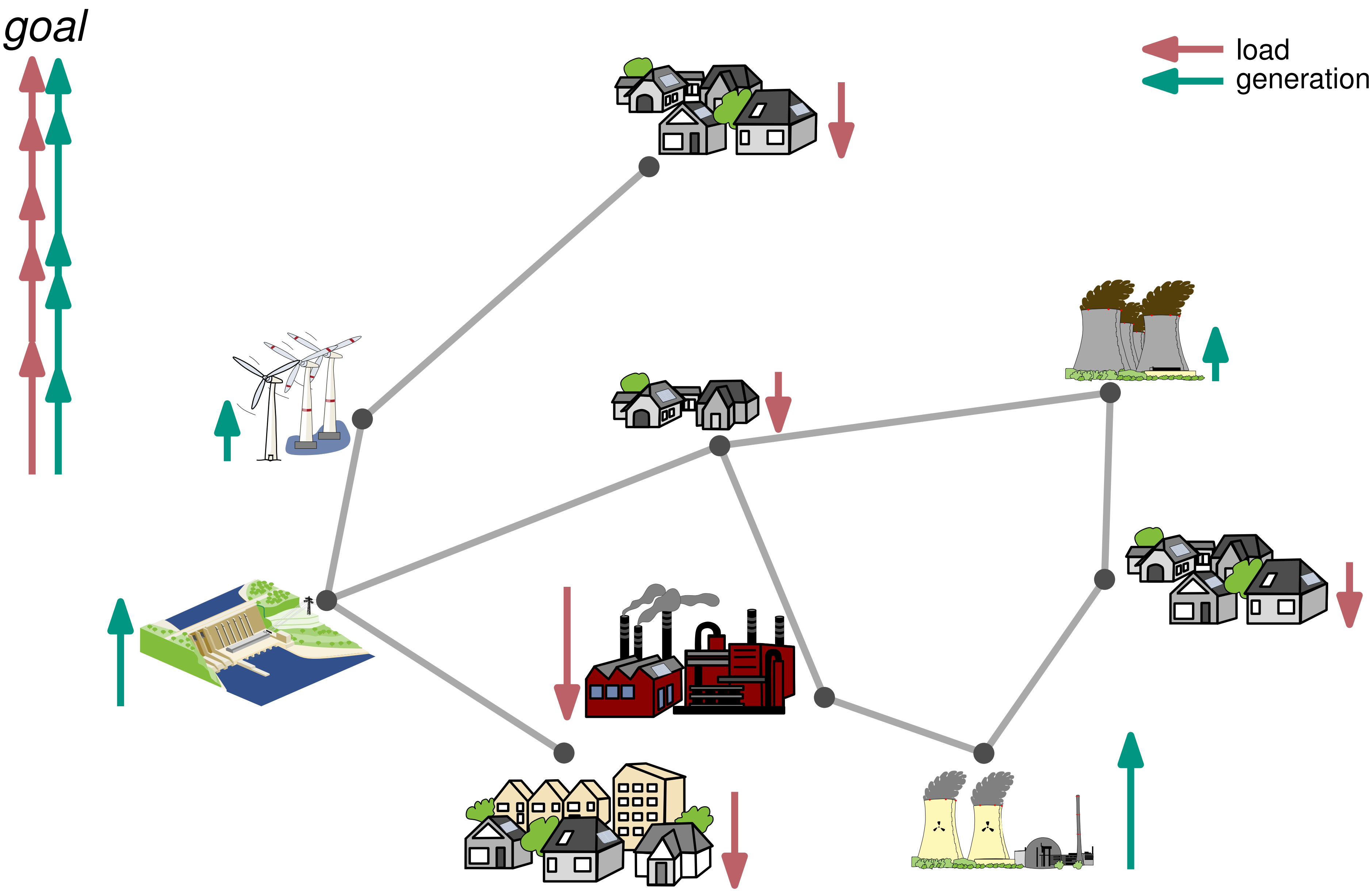


AC Conservation of Flow in Power Grids



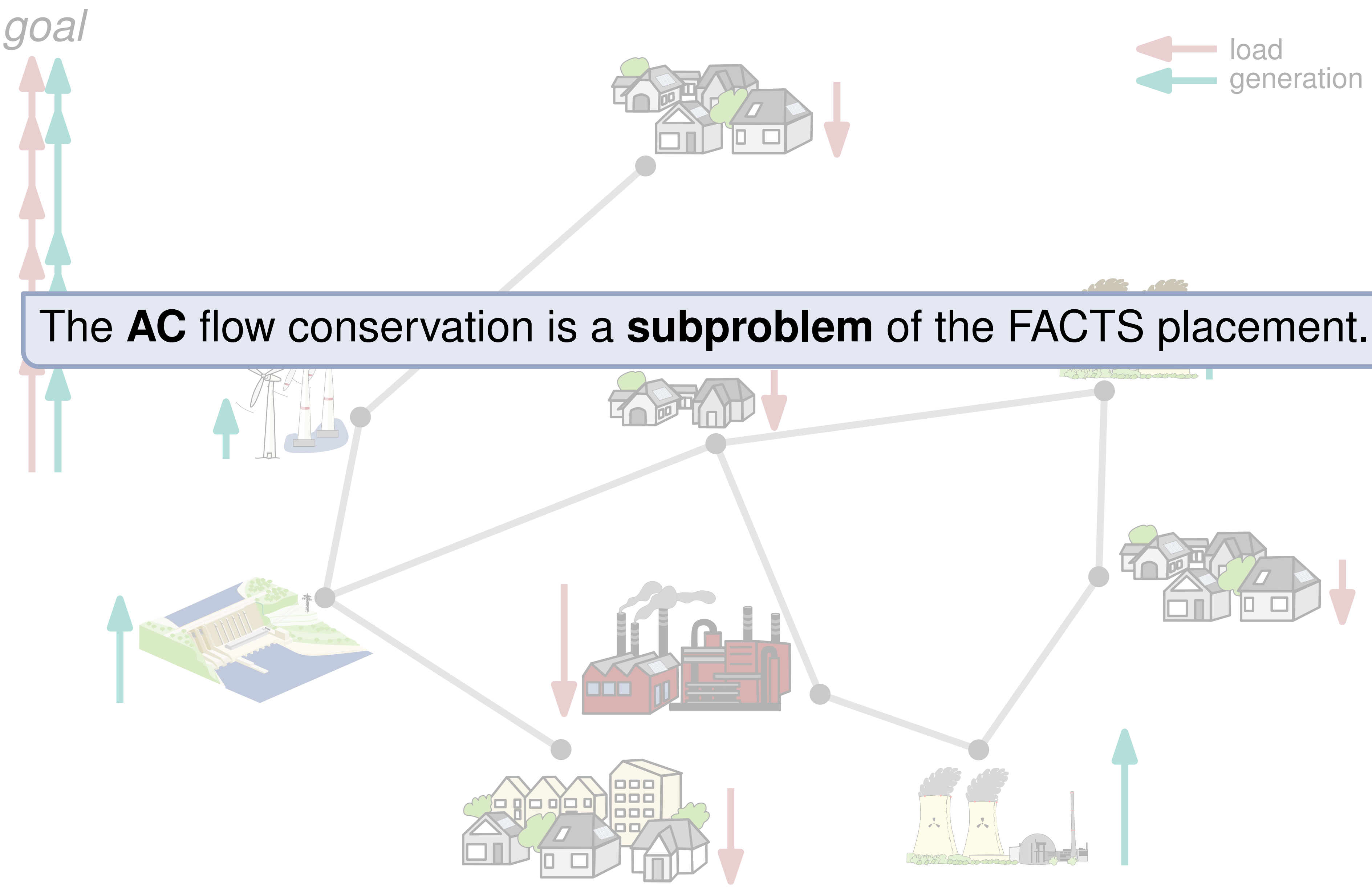
Institute of Theoretical Informatics
Algorithmics Group

AC Conservation of Flow in Power Grids



Institute of Theoretical Informatics
Algorithmics Group

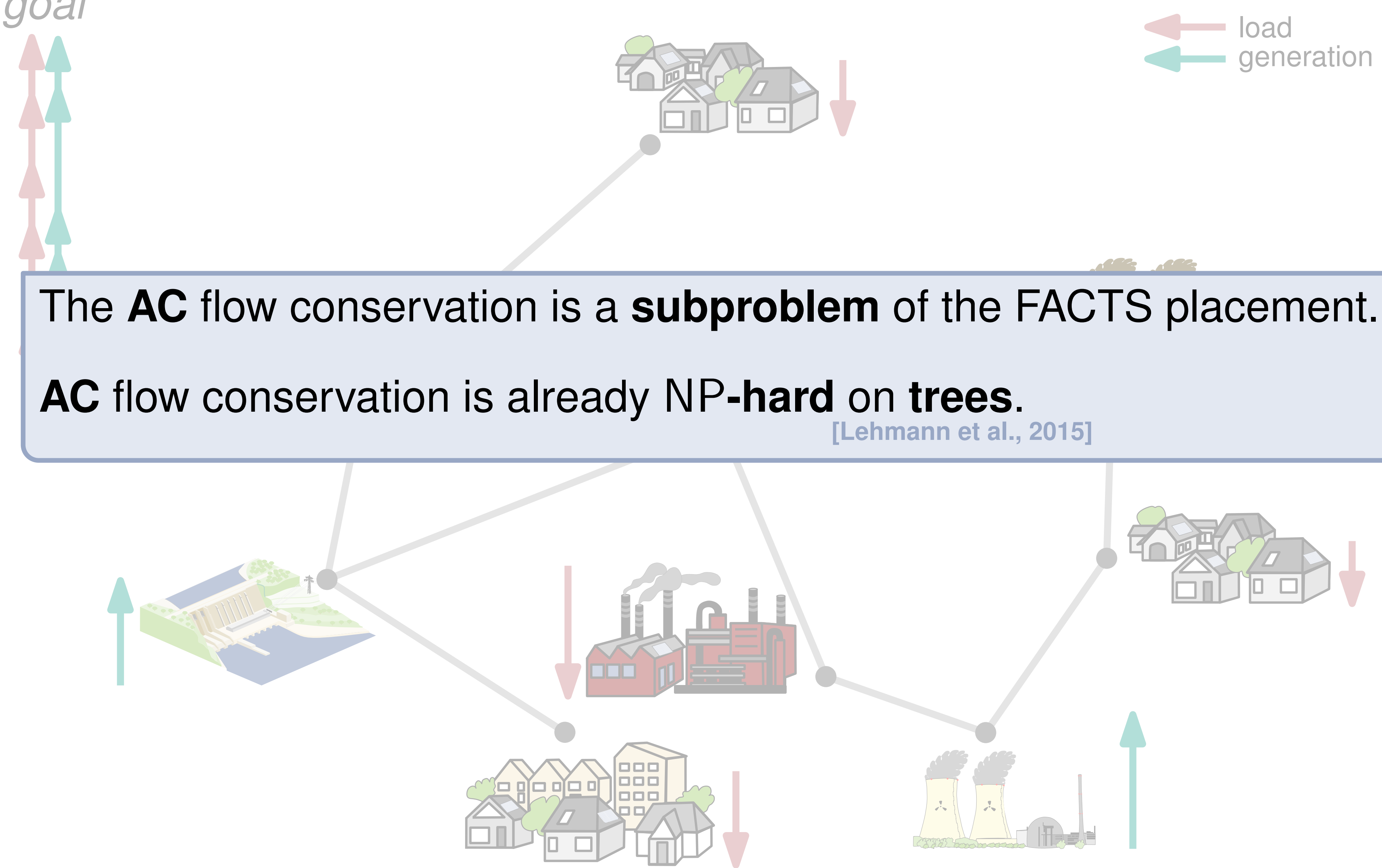
AC Conservation of Flow in Power Grids



The **AC** flow conservation is a **subproblem** of the FACTS placement.

AC Conservation of Flow in Power Grids

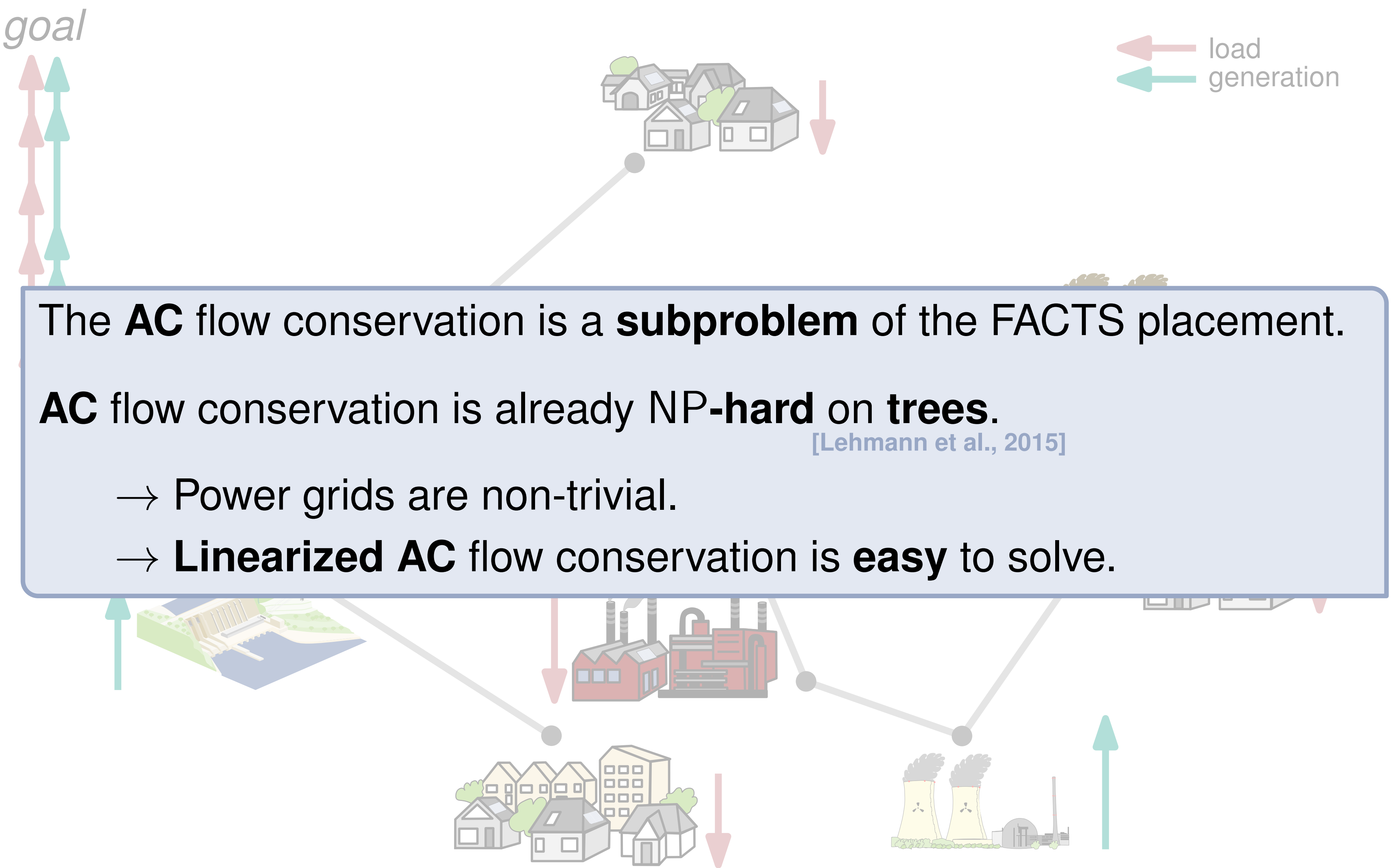
goal



The **AC** flow conservation is a **subproblem** of the FACTS placement.

AC flow conservation is already NP-**hard** on **trees**.
[Lehmann et al., 2015]

AC Conservation of Flow in Power Grids



The **AC** flow conservation is a **subproblem** of the FACTS placement.

AC flow conservation is already NP-**hard** on **trees**. [Lehmann et al., 2015]

→ Power grids are non-trivial.

→ **Linearized AC** flow conservation is **easy** to solve.

Institute of Theoretical Informatics
Algorithmics Group



FACTS placement in the power grid

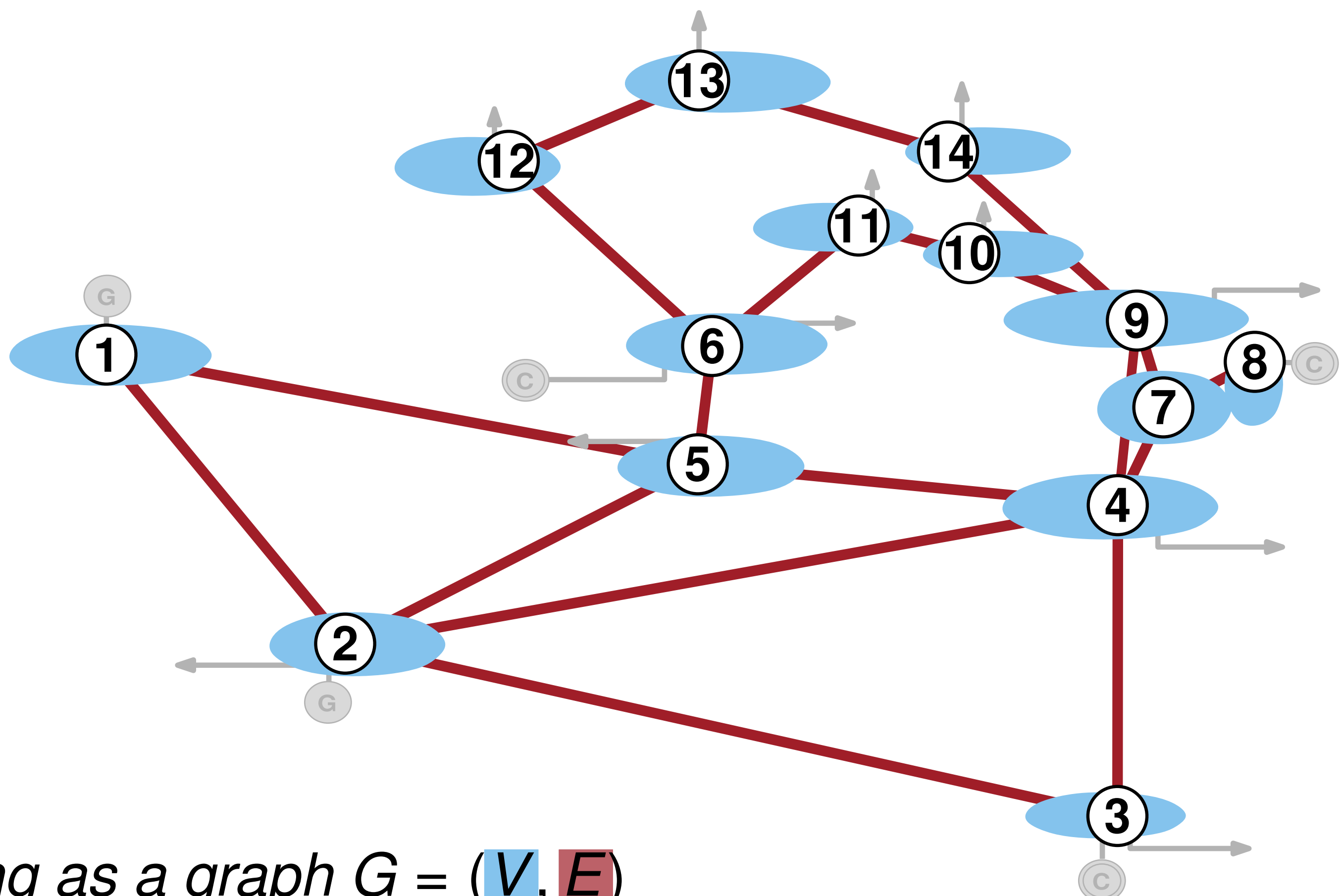
Left Figure:
[2] http://www.lichtenwald-mentaltraining.de/files/bild_licht_im_wald.jpg

Institute of Theoretical Informatics
Algorithmics Group

Optimal ideal FACTS Flow (OiFF)

given *V* set of grid vertices

E set of branches



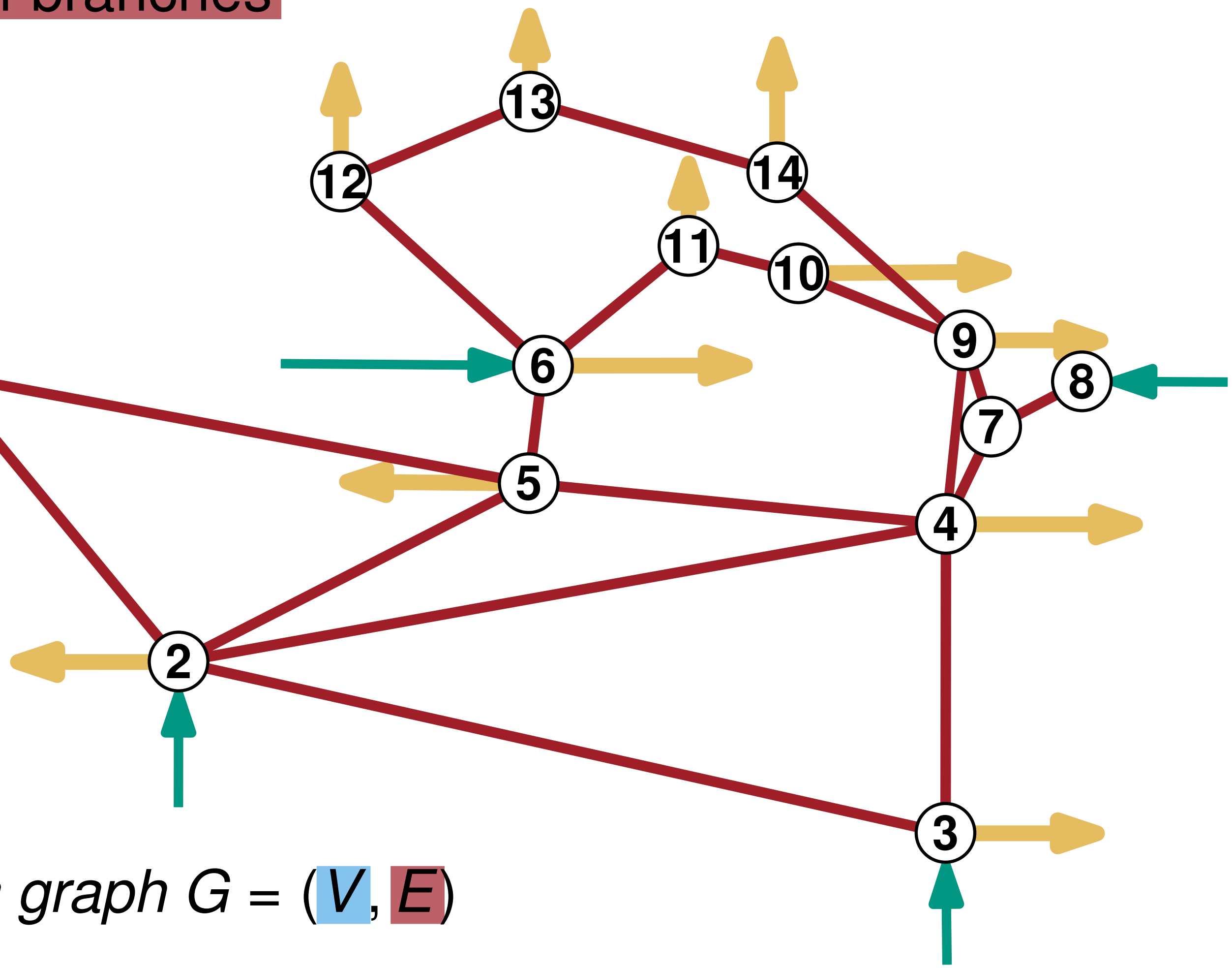
Modeling as a graph $G = (V, E)$

Optimal ideal FACTS Flow (OiFF)

given V set of grid vertices, $V_L \subseteq V$ set of consumers

$V_G \subseteq V$ set of generators (with **capacities** and **costs**)

E set of branches



Modeling as a graph $G = (V, E)$

Optimal ideal FACTS Flow (OiFF)

given V set of grid vertices, $V_L \subseteq V$ set of consumers

$V_G \subseteq V$ set of generators (with **capacities** and **costs**)

E set of branches

Input

Institute of Theoretical Informatics
Algorithmics Group

Optimal ideal FACTS Flow (OiFF)

given V set of grid vertices, $V_L \subseteq V$ set of consumers
$V_G \subseteq V$ set of generators (with **capacities** and **costs**)
E set of branches

find for every generator: the **power generation**
for every branch: **FACTS placement**

variables

Institute of Theoretical Informatics
Algorithmics Group

Optimal ideal FACTS Flow (OiFF)

given V set of grid vertices, $V_L \subseteq V$ set of consumers
$V_G \subseteq V$ set of generators (with **capacities** and **costs**)
E set of branches

find for every generator: the **power generation**
for every branch: **FACTS placement**

minimize **costs**

Institute of Theoretical Informatics
Algorithmics Group

Optimal ideal FACTS Flow (OiFF)

given V set of grid vertices, $V_L \subseteq V$ set of consumers

$V_G \subseteq V$ set of generators (with **capacities** and **costs**)

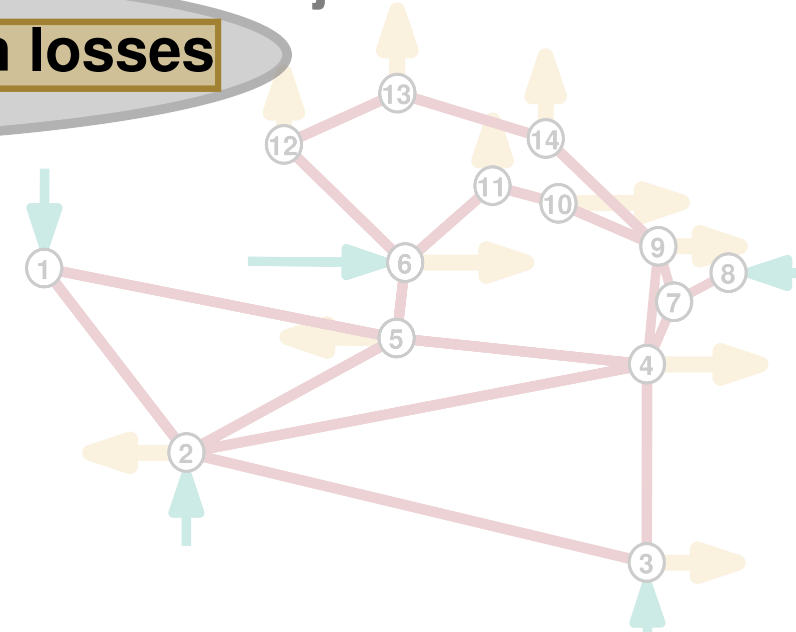
E set of branches (with **impedances**, **susceptance**, **capacities)**

find for every generator: the **power generation**

for every branch: **FACTS placement**

objective

minimize **total generation costs** and **branch losses**



Institute of Theoretical Informatics
Algorithmics Group

Optimal ideal FACTS Flow (OiFF)

Karlsruhe Institute of Technology

given V set of grid vertices, $V_L \subseteq V$ set of consumers

$V_G \subseteq V$ set of generators (with **capacities** and **costs**)

E set of branches (with **impedances**, **susceptance**, **capacities)**

find for every generator: the **power generation**

for every branch: **FACTS placement**

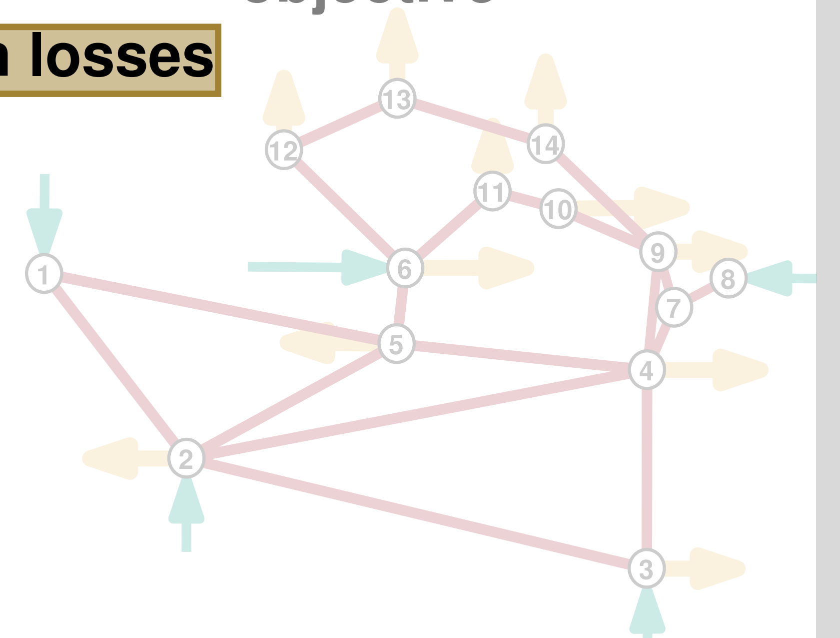
minimize **total generation costs** and **branch losses**

subject to branch capacities

generator capacities

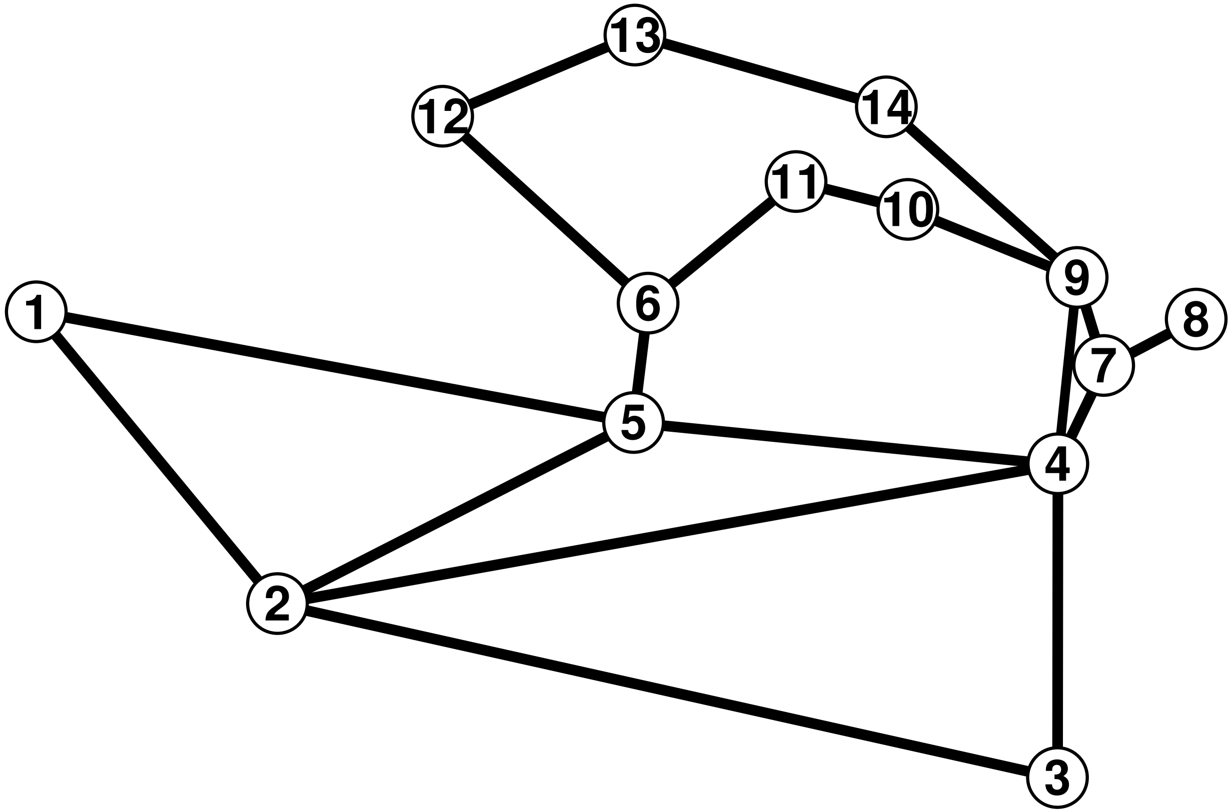
power flow constraints

objective



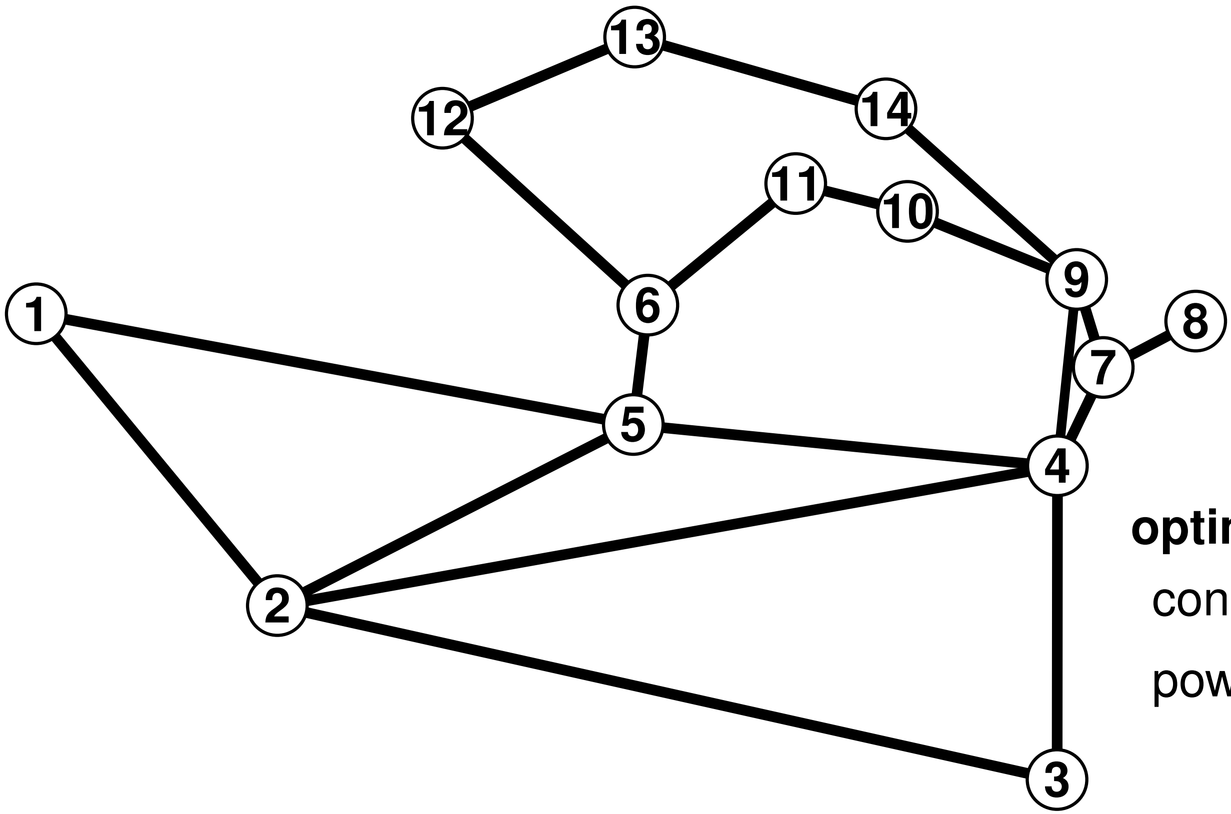
Institute of Theoretical Informatics
Algorithmics Group

Modeling the Power Flow Constraint



Institute of Theoretical Informatics
Algorithmics Group

Modeling the Power Flow Constraint



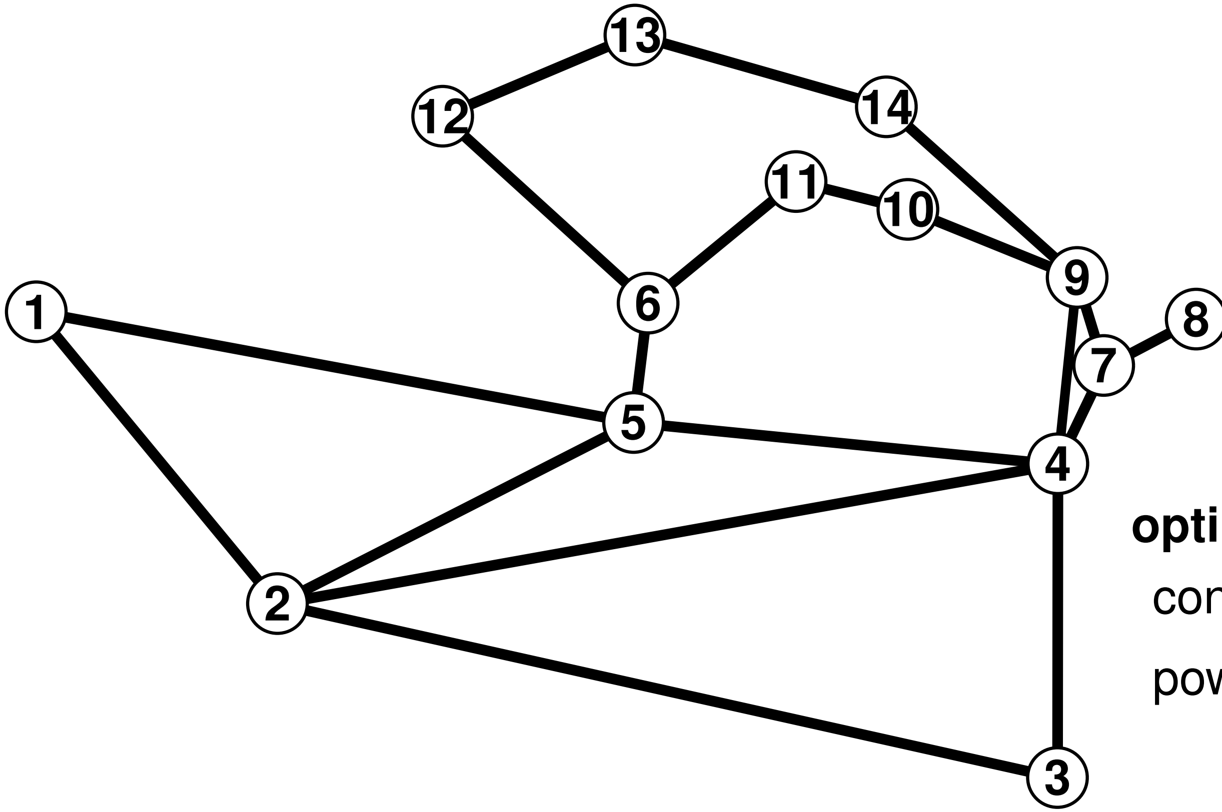
optimize subject to:

conservation of flow

power flow constraints

Institute of Theoretical Informatics
Algorithmics Group

Modeling the Power Flow Constraint



optimize subject to:

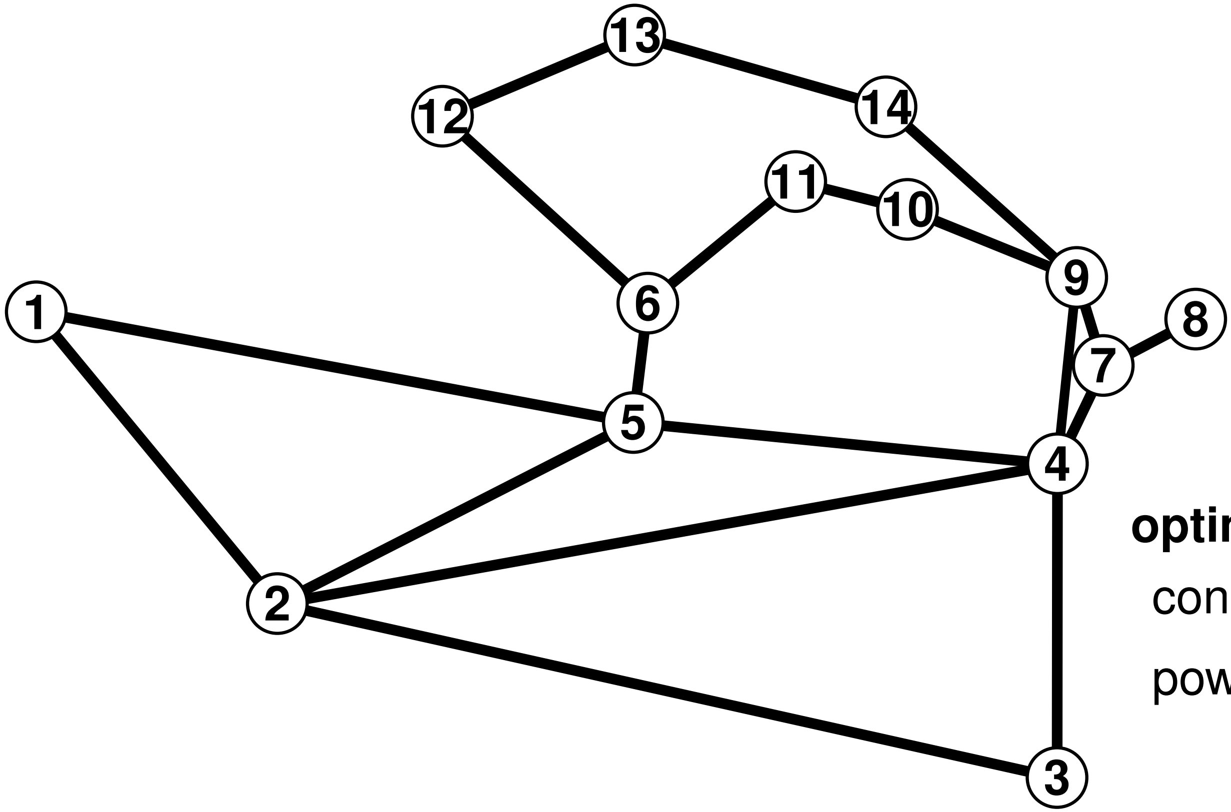
conservation of flow

power flow constraints

minimize costs

Institute of Theoretical Informatics
Algorithmics Group

Modeling the Power Flow Constraint



optimize subject to:

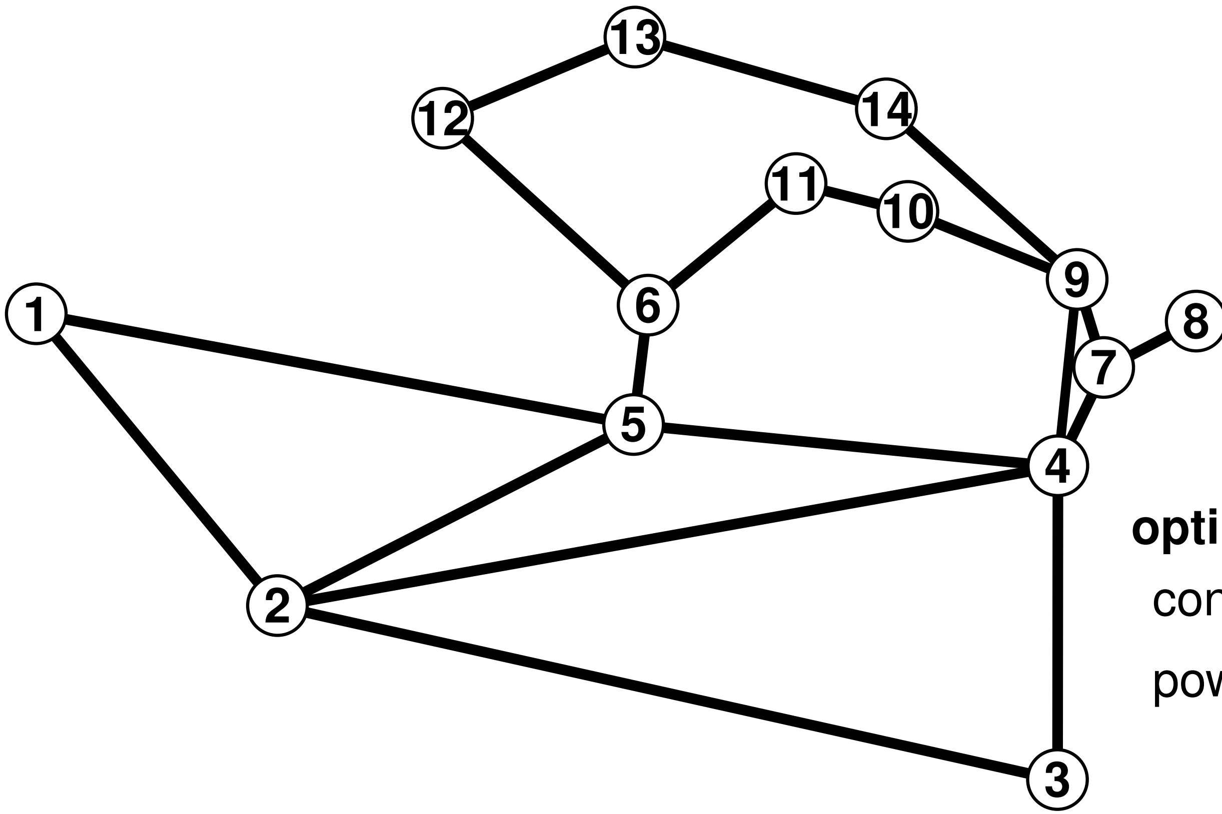
conservation of flow ✓

power flow constraints ✓

minimize costs

physical model

Modeling the Power Flow Constraint



optimize subject to:

conservation of flow ✓ ✓

power flow constraints ✓ ✗

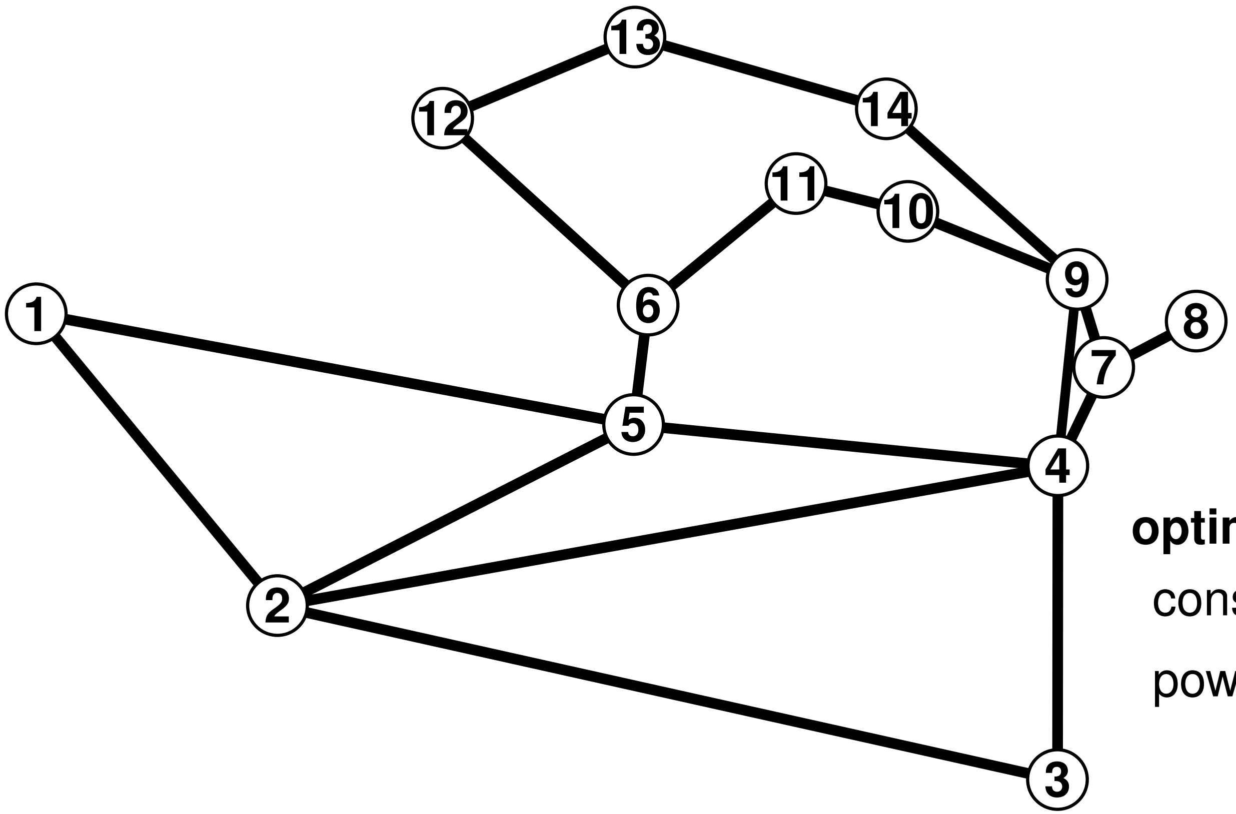
minimize costs

physical model

flow model

Institute of Theoretical Informatics
Algorithmics Group

Modeling the Power Flow Constraint



optimize subject to:

conservation of flow ✓ ✓

power flow constraints ✓ ✗

minimize costs

flow model

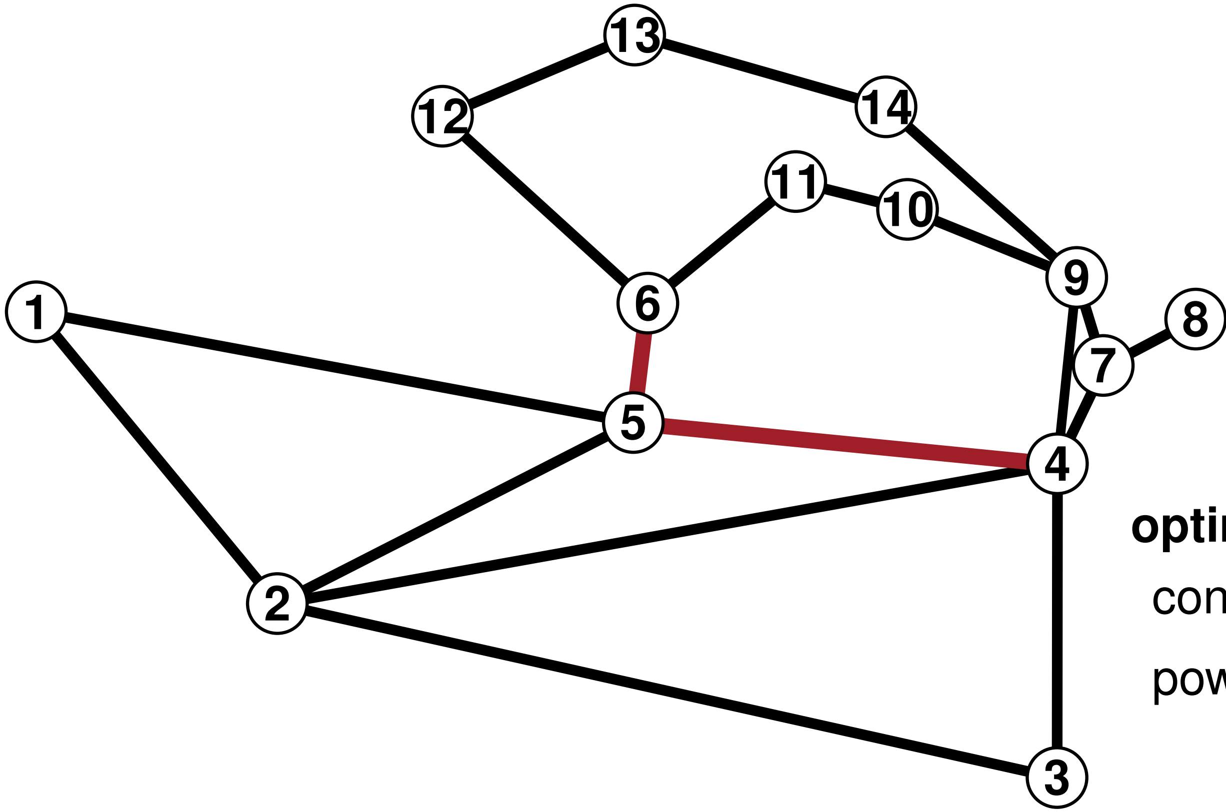
lower bound

physical model

upper bound

Institute of Theoretical Informatics
Algorithmics Group

Modeling the Power Flow Constraint



ideal FACTS

optimize subject to:

conservation of flow	✓	✓	✓
power flow constraints	✓	✗	✓

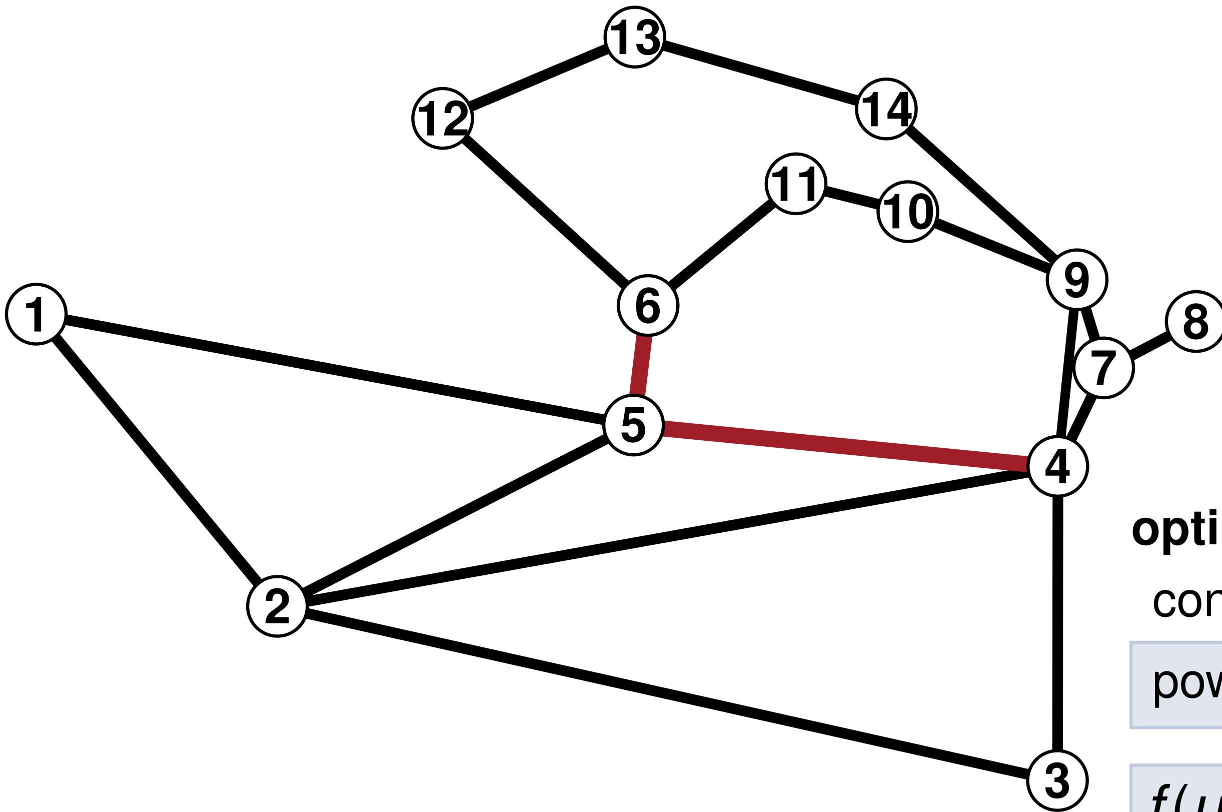
flow model
lower bound

minimize costs
objective

physical model
upper bound

Institute of Theoretical Informatics
Algorithmics Group

Modeling the Power Flow Constraint



ideal FACTS

optimize subject to:

conservation of flow	✓	✓	✓
power flow constraints	✓	✗	✓

$$f(u,v) = B(u,v) \cdot (\theta(u) - \theta(v))$$

flow model

lower bound

minimize costs

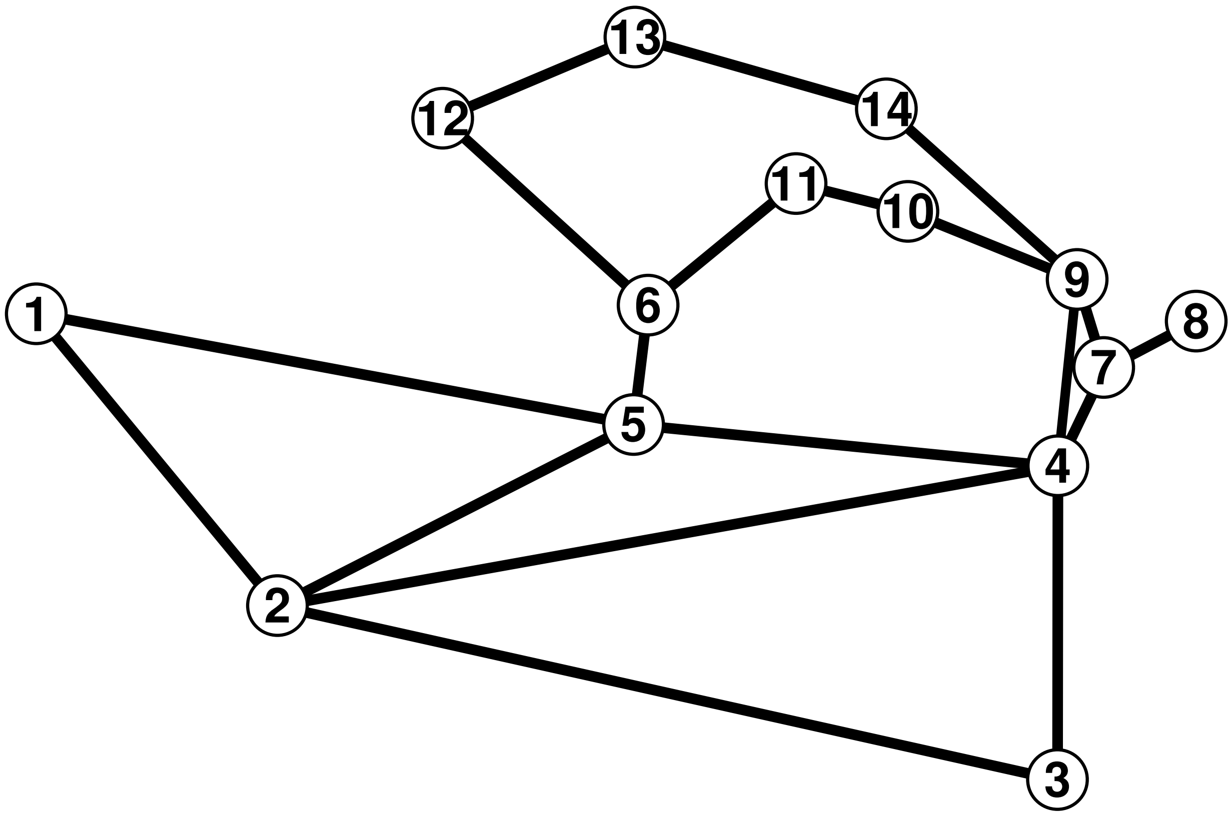
objective

physical model

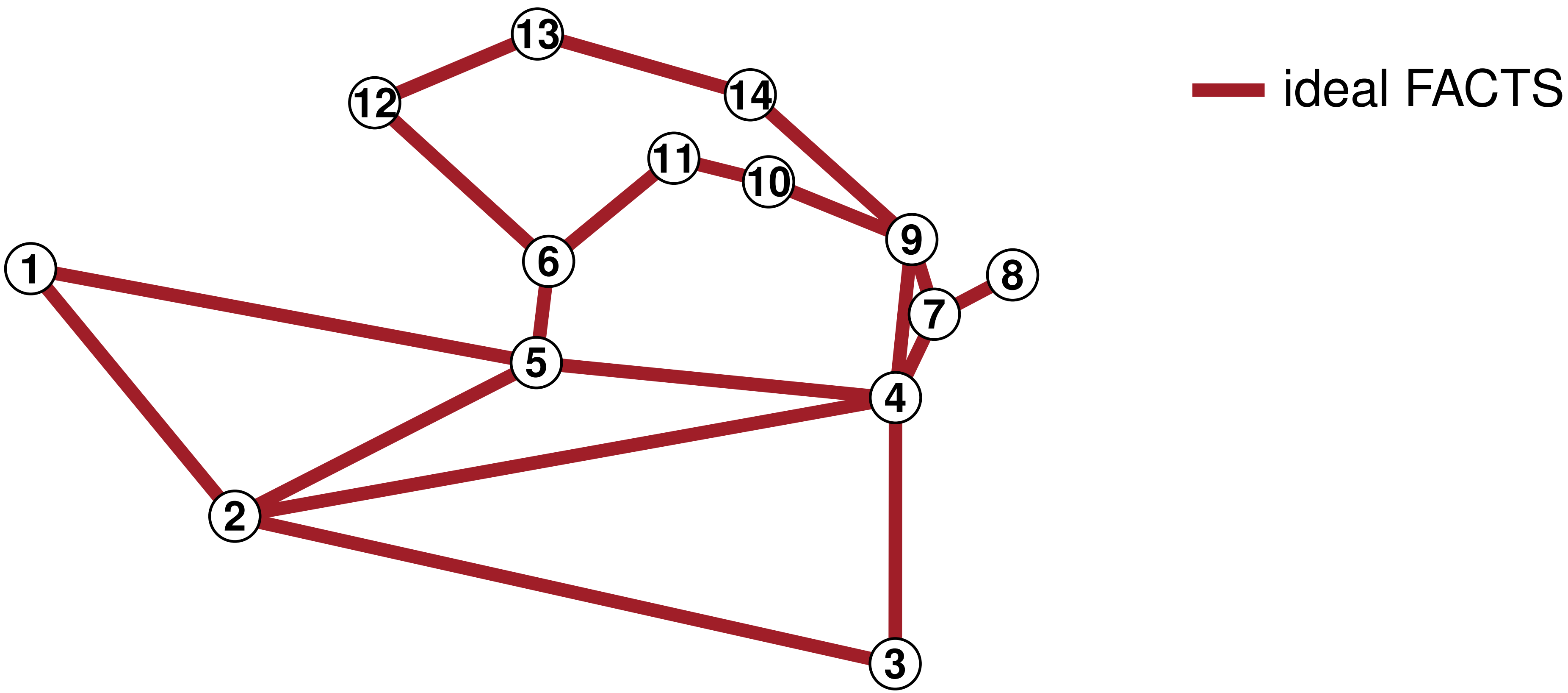
upper bound

Institute of Theoretical Informatics
Algorithmics Group

Global Optimal Power Flow



Global Optimal Power Flow

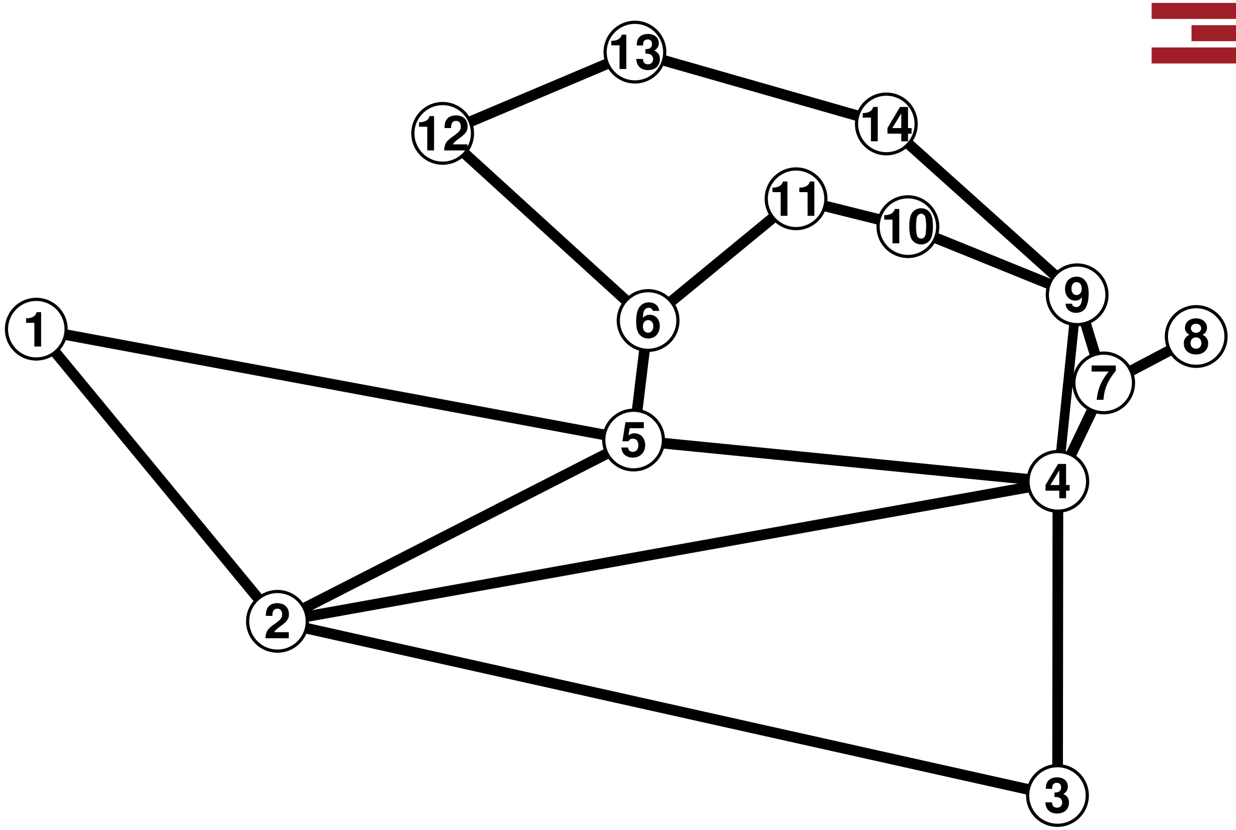


Can we become as good as the flow model
with fewer FACTS?

Institute of Theoretical Informatics
Algorithmics Group

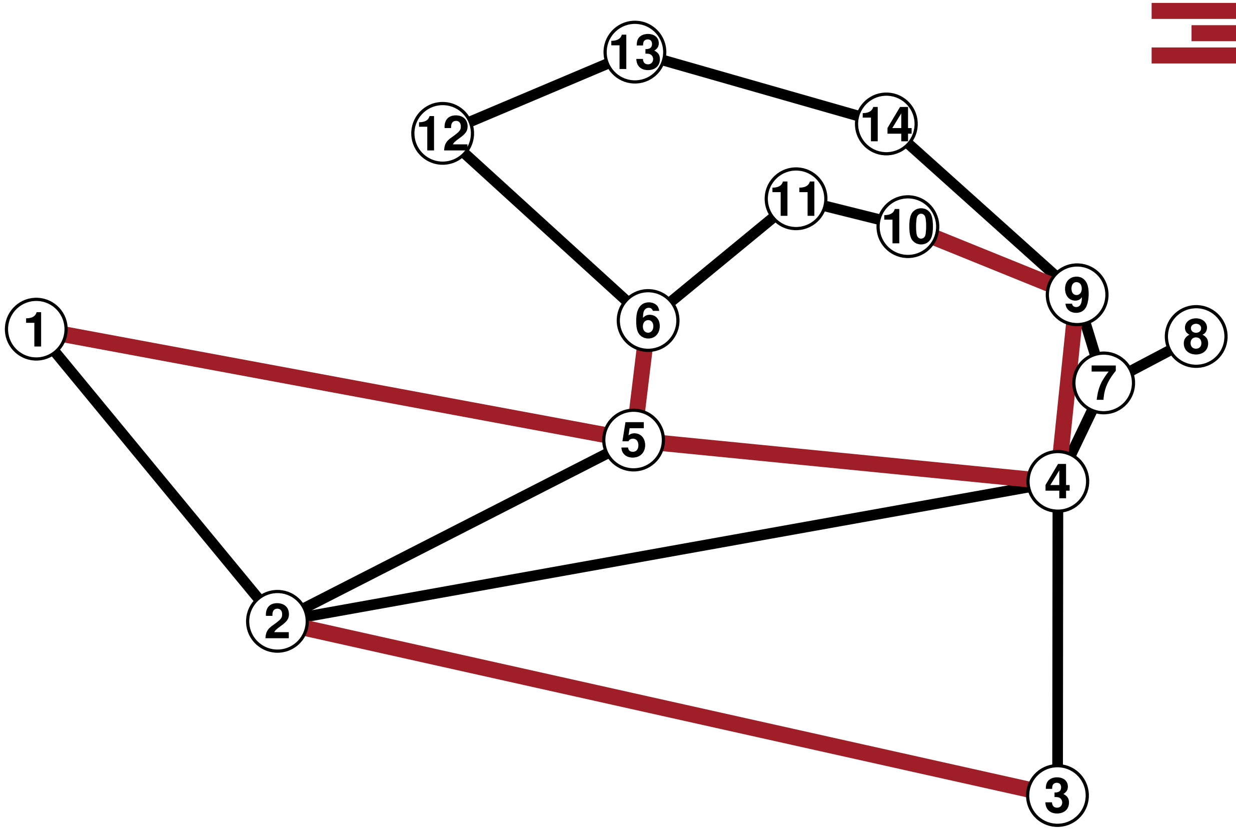
Feedback Forest Set

Feedback Forest Set



Institute of Theoretical Informatics
Algorithmics Group

Feedback Forest Set



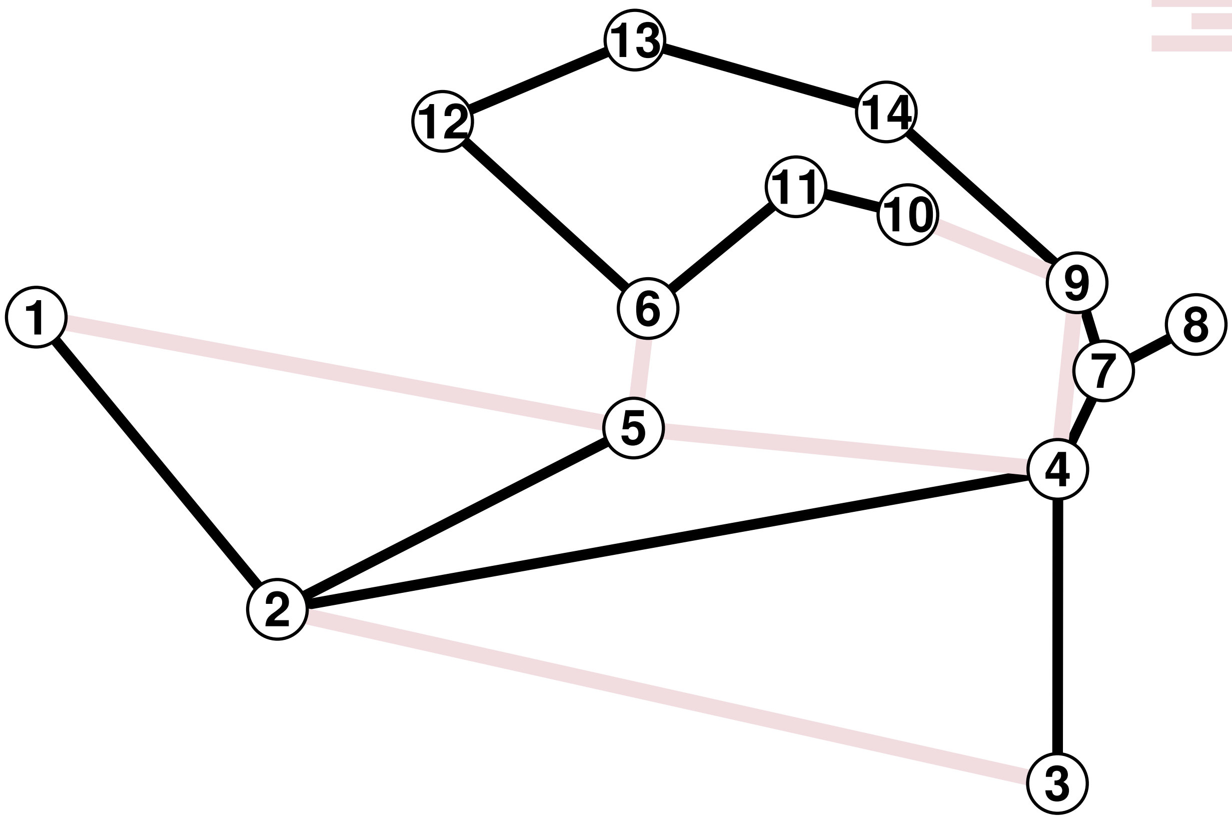
Feedback Forest Set

⇓

A set of Trees (*Forests*) remains!

Institute of Theoretical Informatics
Algorithmics Group

Feedback Forest Set



Feedback Forest Set

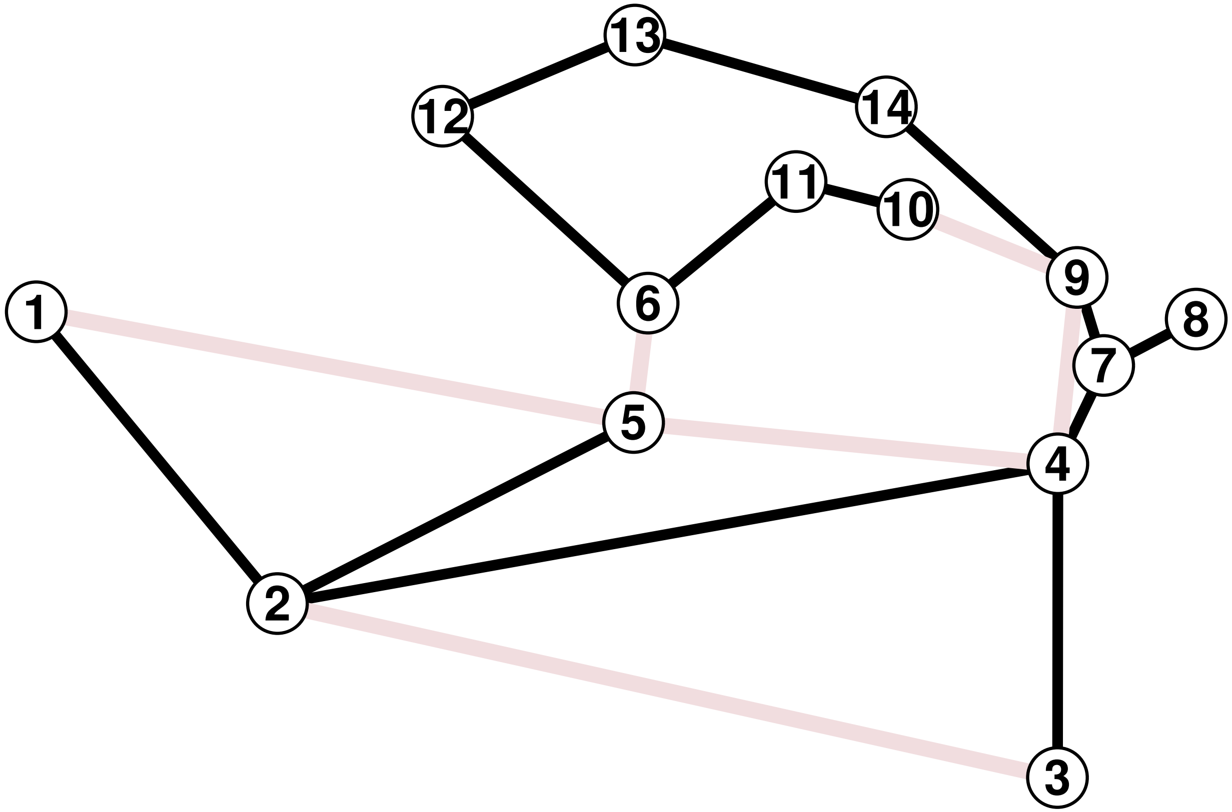
⇓

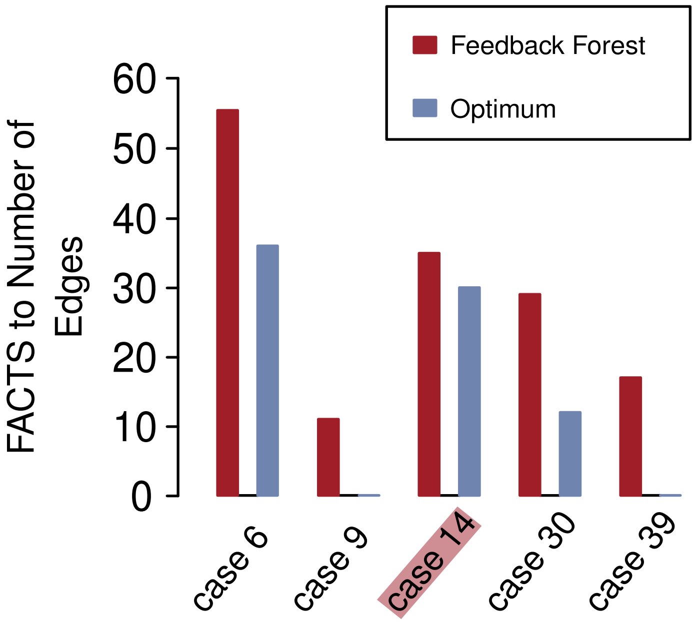
A set of Trees (*Forests*) remains!

Theorem 1

Physical subgrid forest ⇒ All **graph theoretical flows** are **electrical flows**

Feedback Forest Set

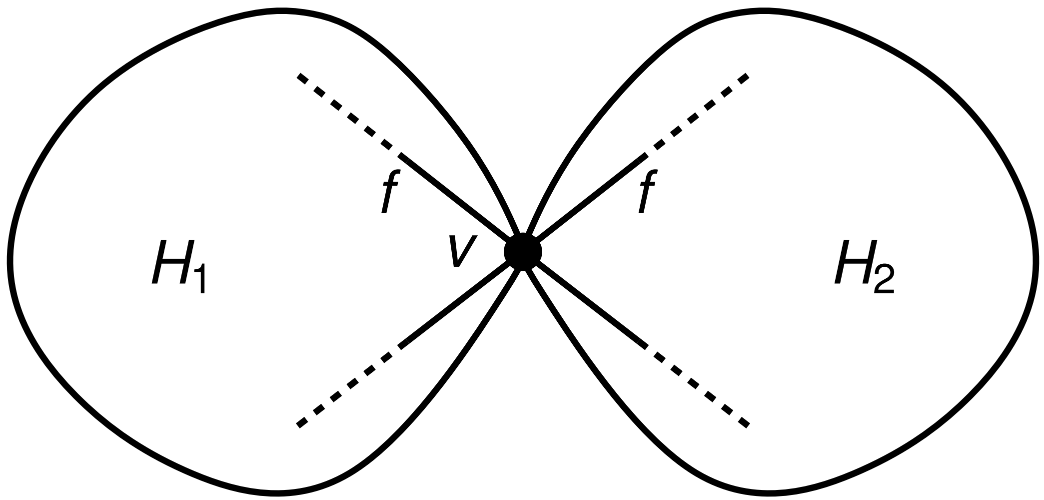




Are less than 6 FACTS possible?

Institute of Theoretical Informatics
Algorithmics Group

Idea of Proof for the Feedback Forest Set

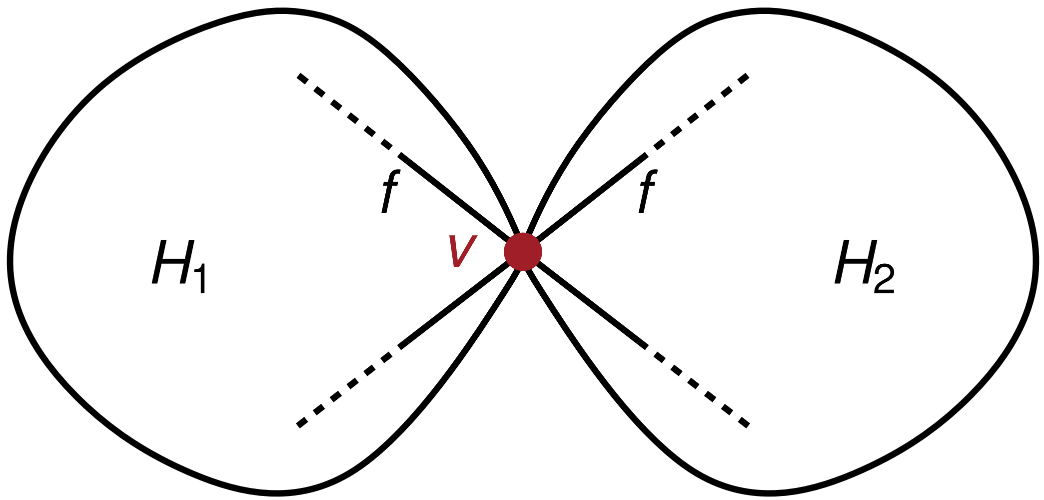


Assumption

- network $N(G)$ has an electrically feasible flow f
- network $N(G)$ has a cut vertex v

Institute of Theoretical Informatics
Algorithmics Group

Idea of Proof for the Feedback Forest Set

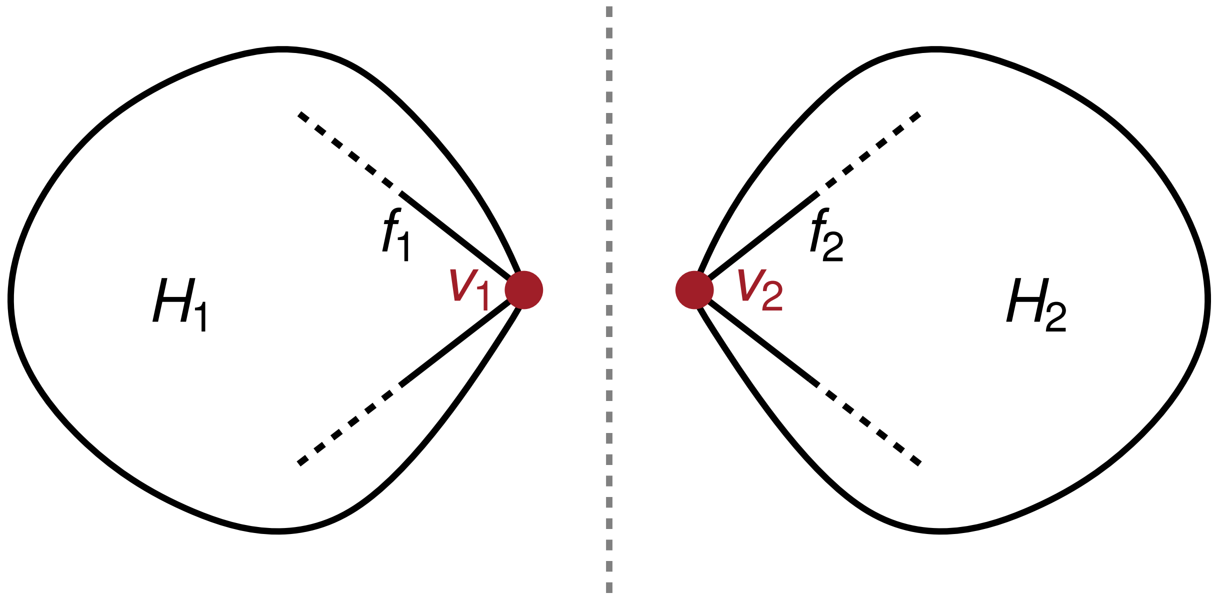


Assumption

- network $N(G)$ has an electrically feasible flow f
- network $N(G)$ has a cut vertex v

Institute of Theoretical Informatics
Algorithmics Group

Idea of Proof for the Feedback Forest Set



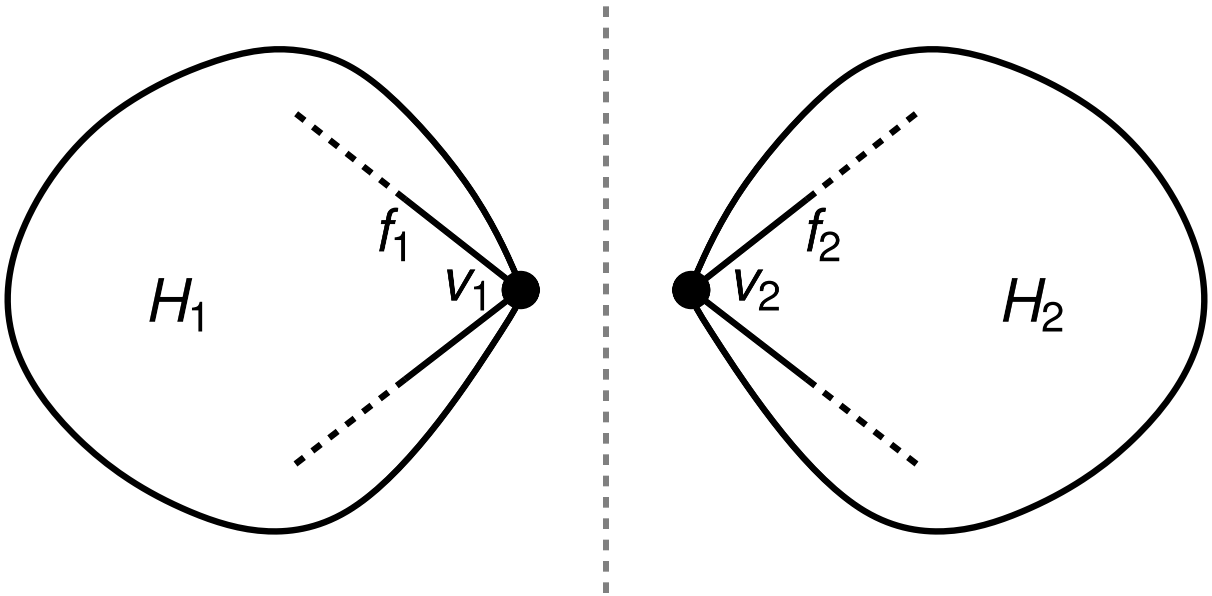
Idea

- cut $N(G)$ at the cut vertex v with $v = v_1 = v_2$
- function f_1 in H_1 und f_2 in H_2 remain unchanged

$\Rightarrow$ electrically feasible

Institute of Theoretical Informatics
Algorithmics Group

Idea of Proof for the Feedback Forest Set

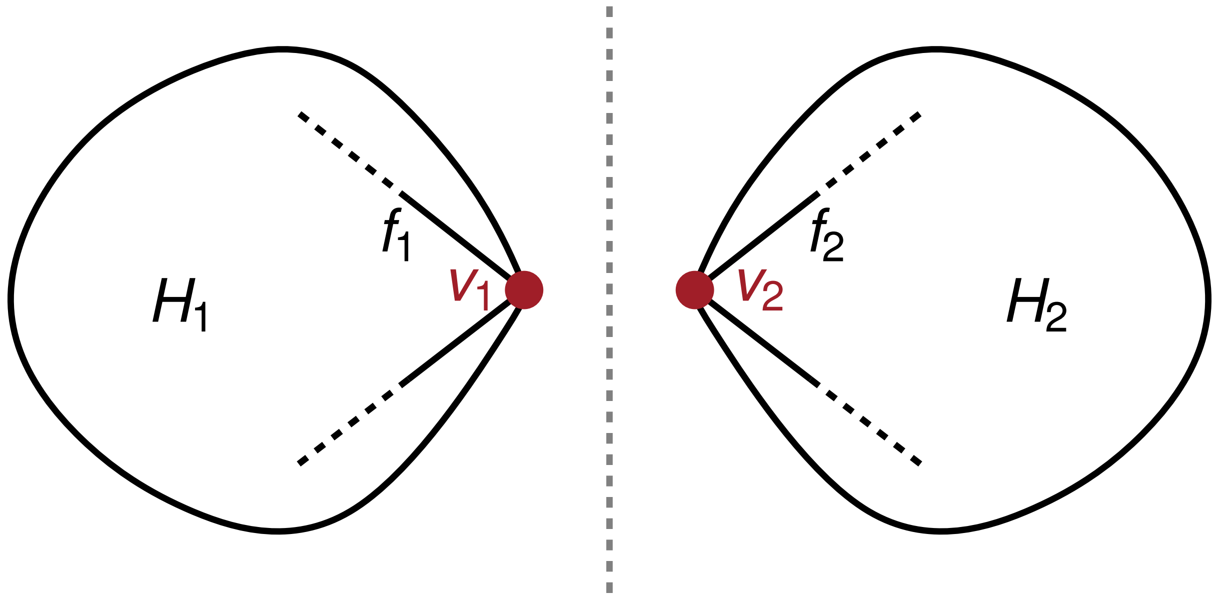


Assumption

- function f_1 in H_1 and f_2 in H_2 are electrically feasible
- find all copies of $v = \{v_1, v_2\}$

Institute of Theoretical Informatics
Algorithmics Group

Idea of Proof for the Feedback Forest Set



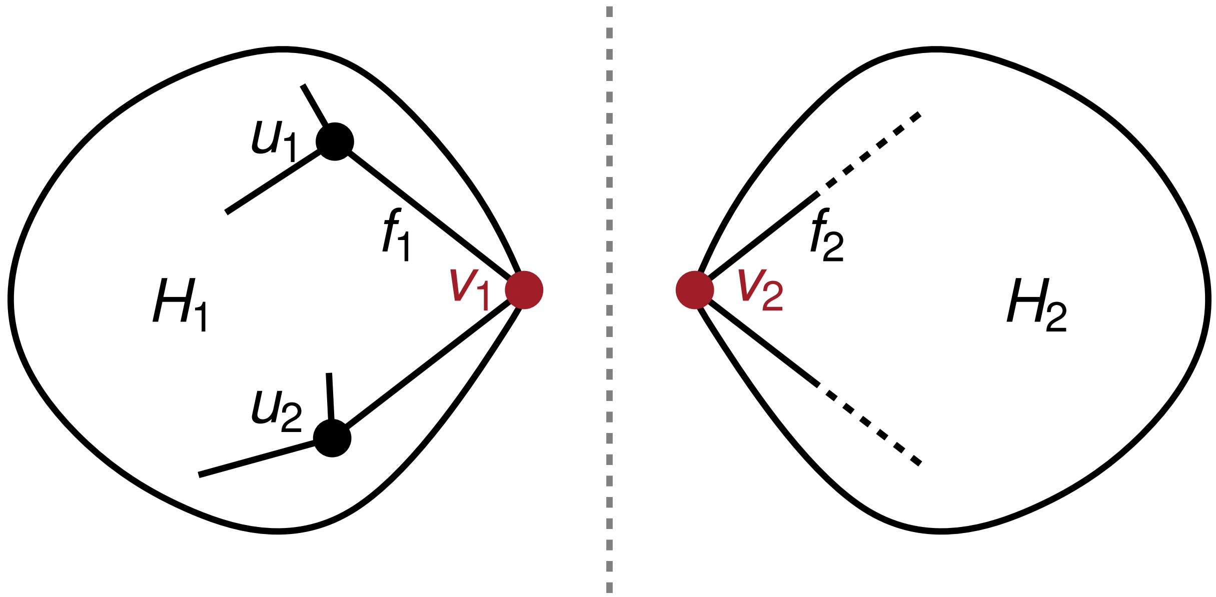
Idea

$$\Theta'(v_1) = \Theta'(v_2) = c$$

$$\Delta_1 = \Theta(v_1) - c \qquad \Delta_2 = \Theta(v_2) - c$$

Institute of Theoretical Informatics
Algorithmics Group

Idea of Proof for the Feedback Forest Set



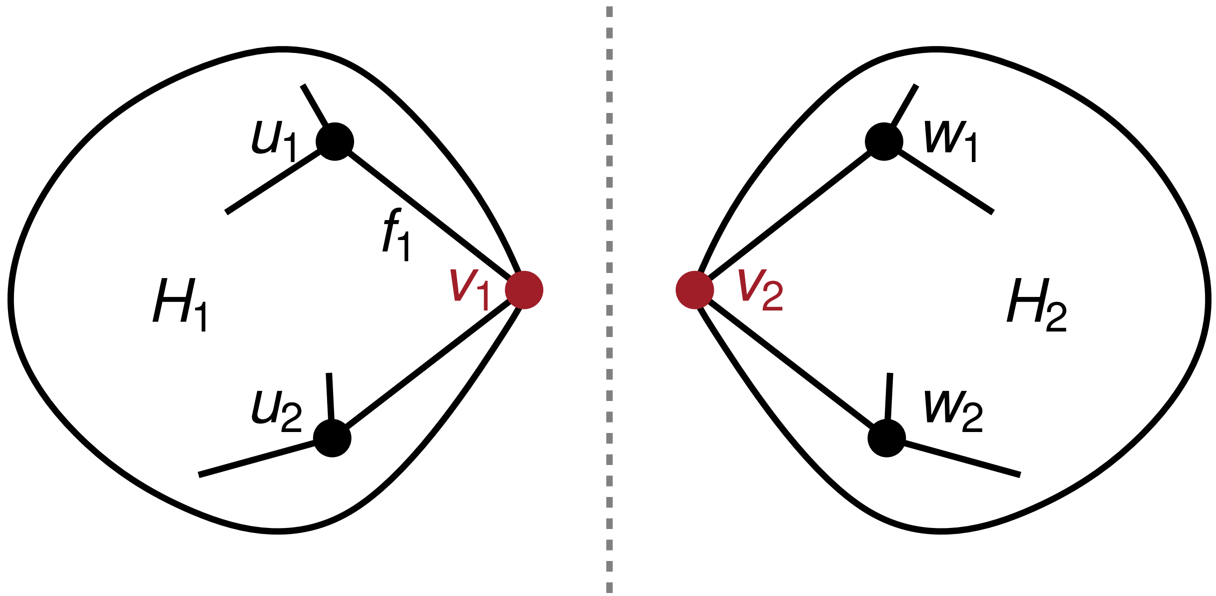
Idea

$$\Theta'(u_1) = \Theta(u_1) - \Delta_1$$

$$\Theta'(u_2) = \Theta(u_2) - \Delta_1$$

Institute of Theoretical Informatics
Algorithmics Group

Idea of Proof for the Feedback Forest Set

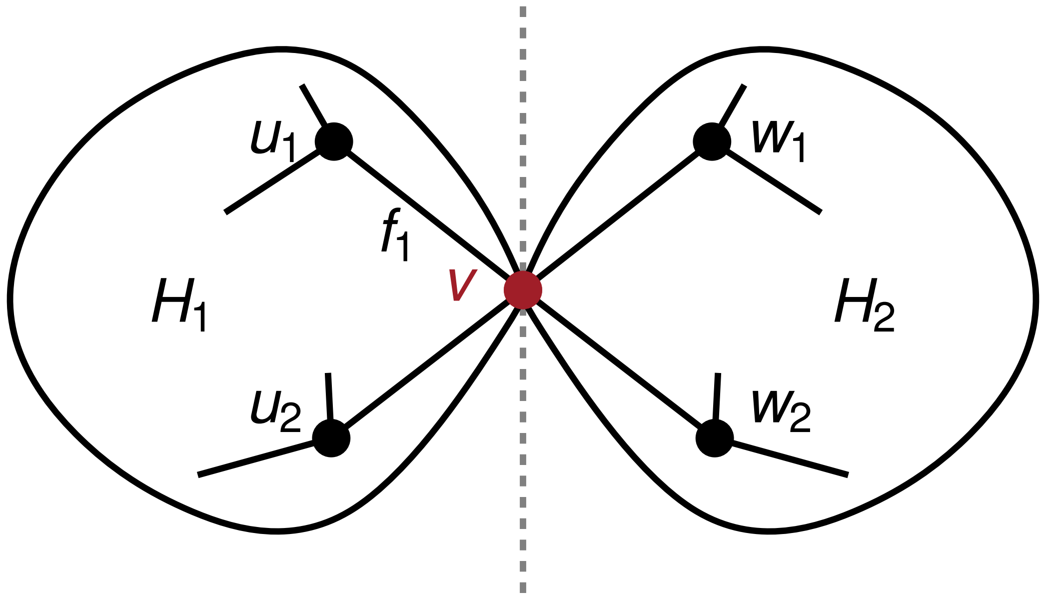


Idea

$$\Theta'(w_1) = \Theta(w_1) - \Delta_2$$

$$\Theta'(w_2) = \Theta(w_2) - \Delta_2$$

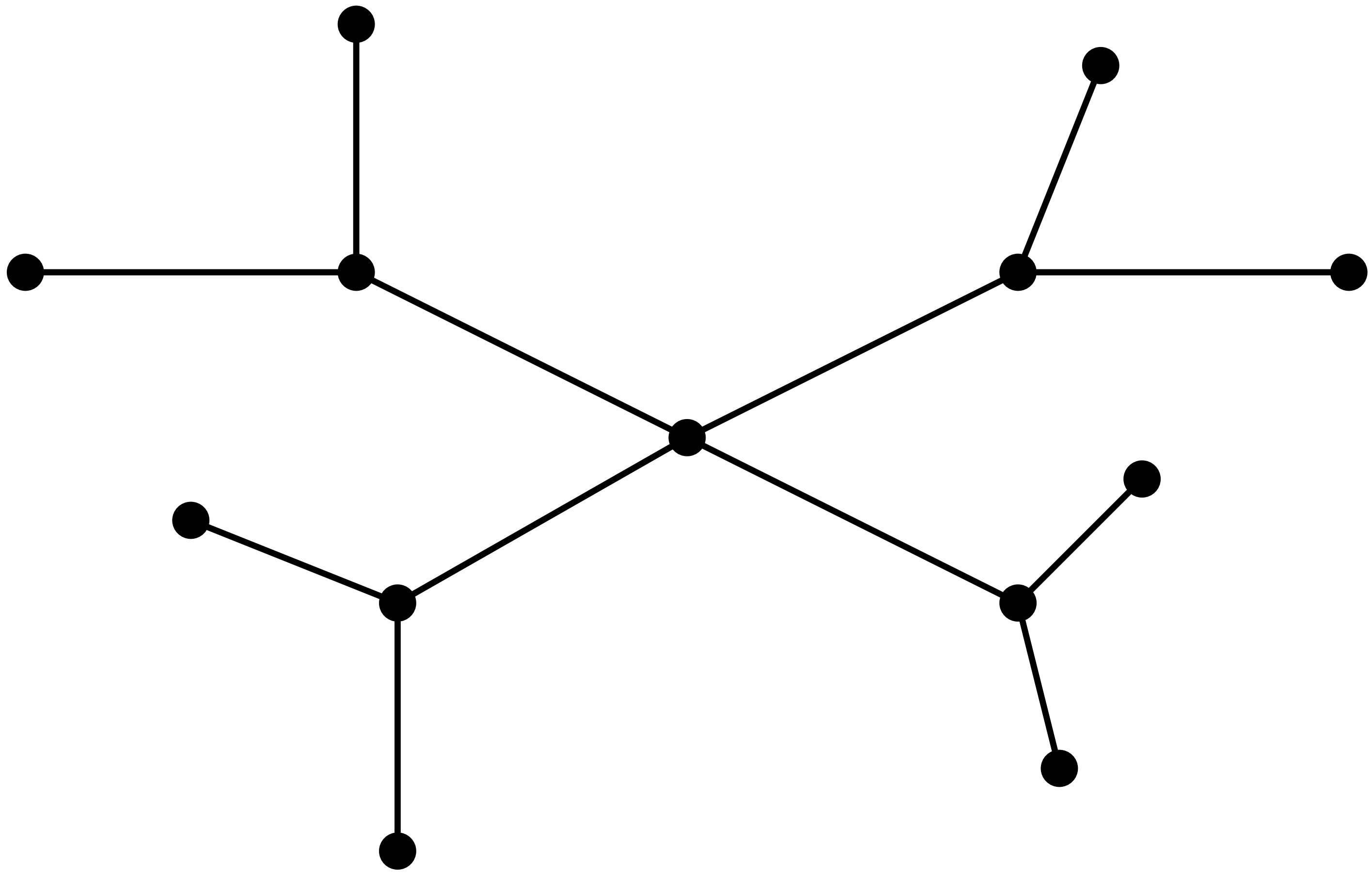
Idea of Proof for the Feedback Forest Set



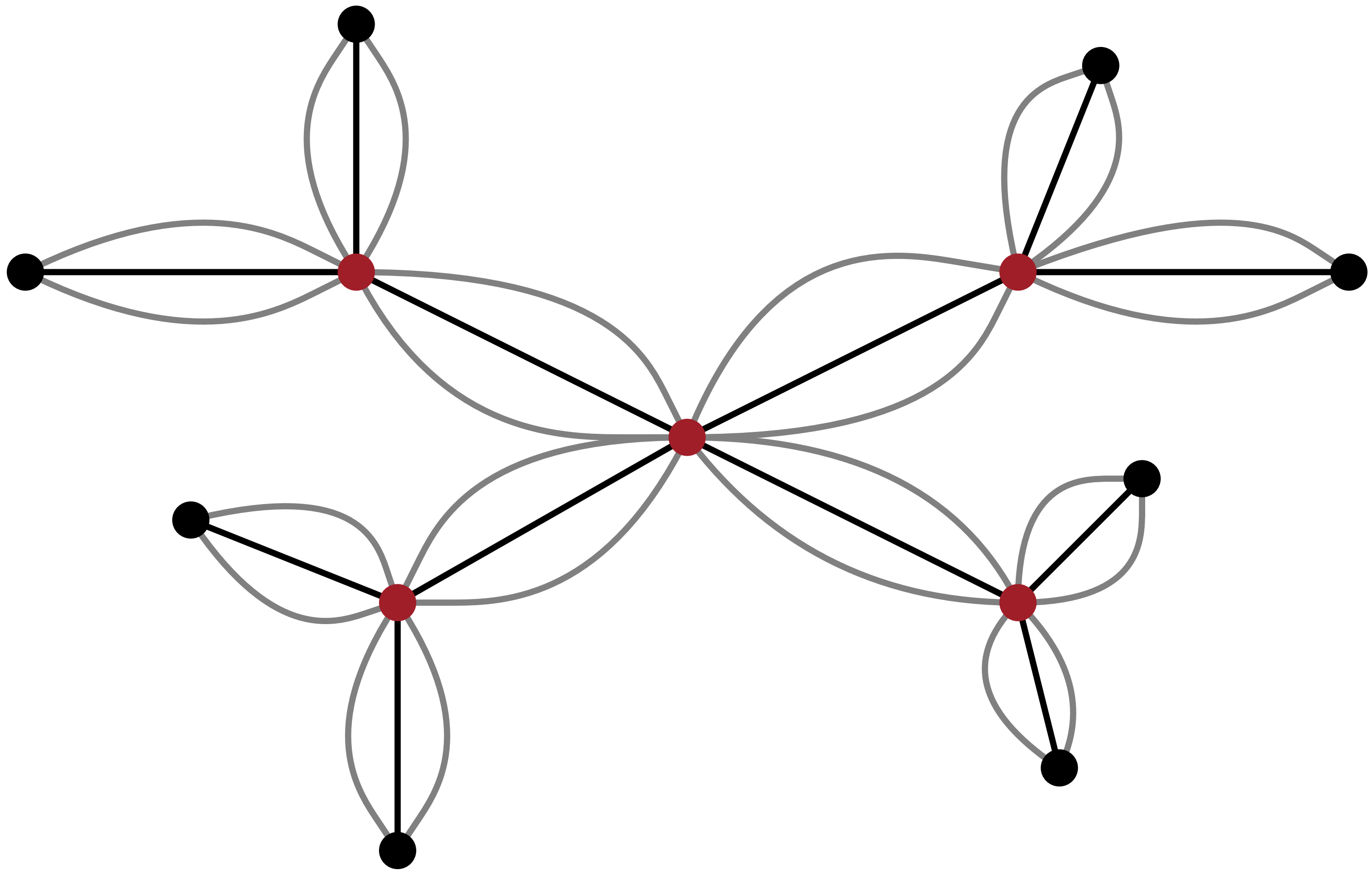
Idea

$$\Theta(v) = \Theta'(v_1) = \Theta'(v_2)$$

Idea of Proof for the Feedback Forest Set



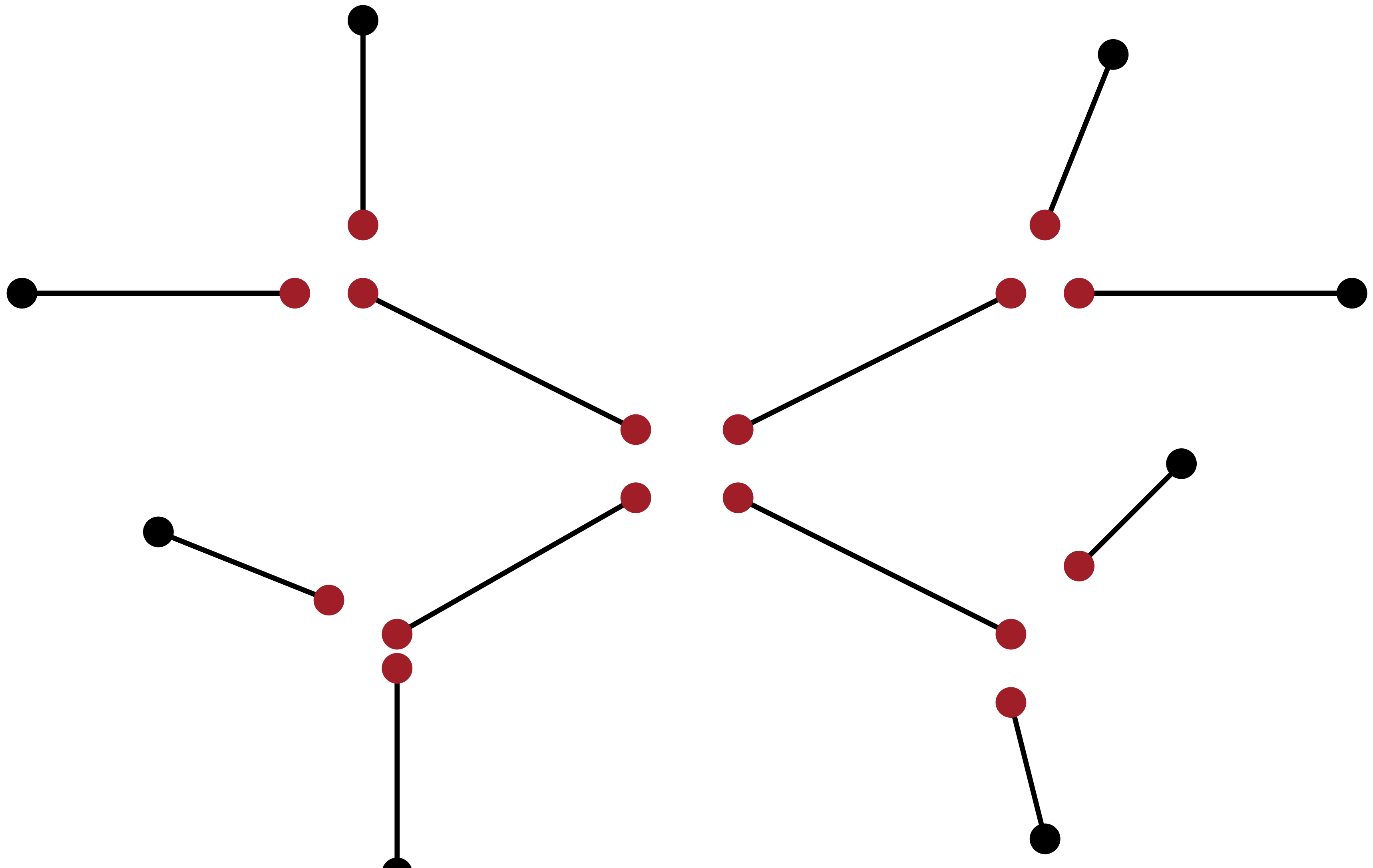
Idea of Proof for the Feedback Forest Set



⇒ Trees can be cut into *edges*

Institute of Theoretical Informatics
Algorithmics Group

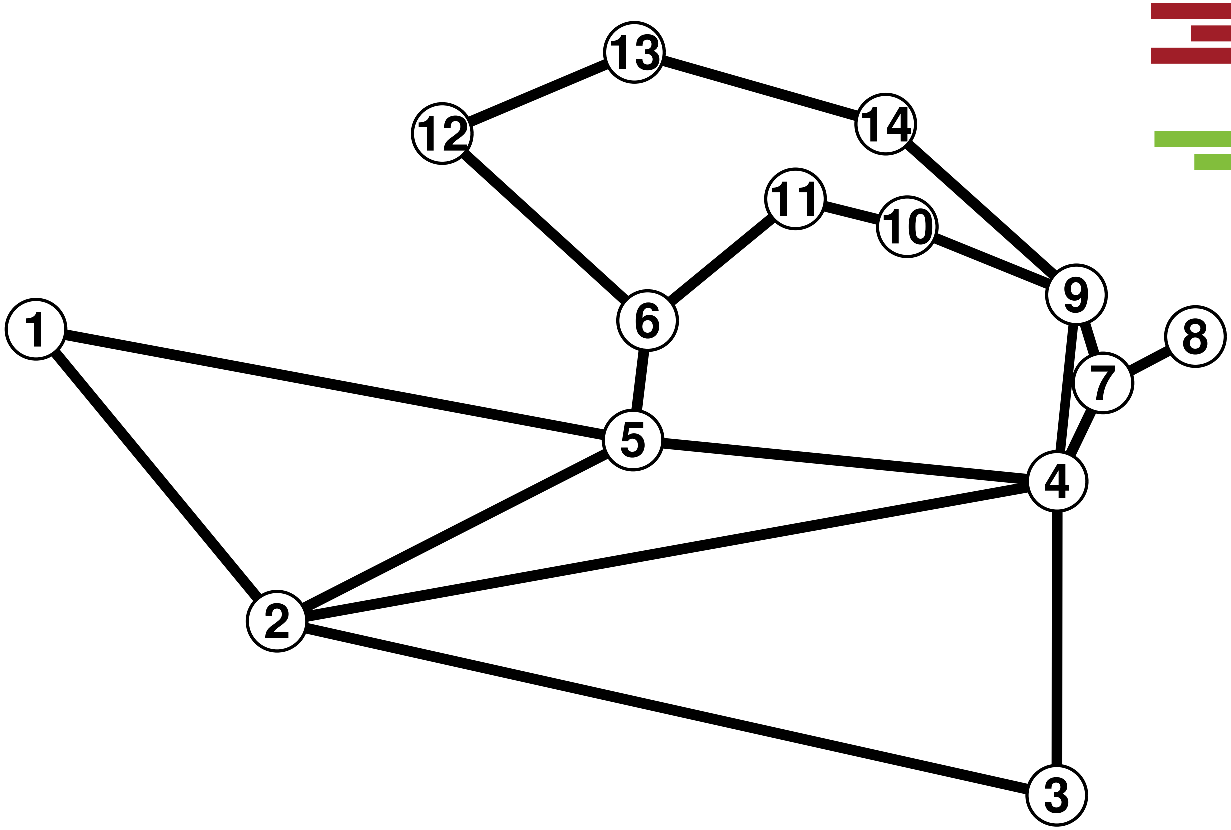
Idea of Proof for the Feedback Forest Set



$\Rightarrow$ For every *edge* there exists a solution!

Institute of Theoretical Informatics
Algorithmics Group

Feedback Cactus Set

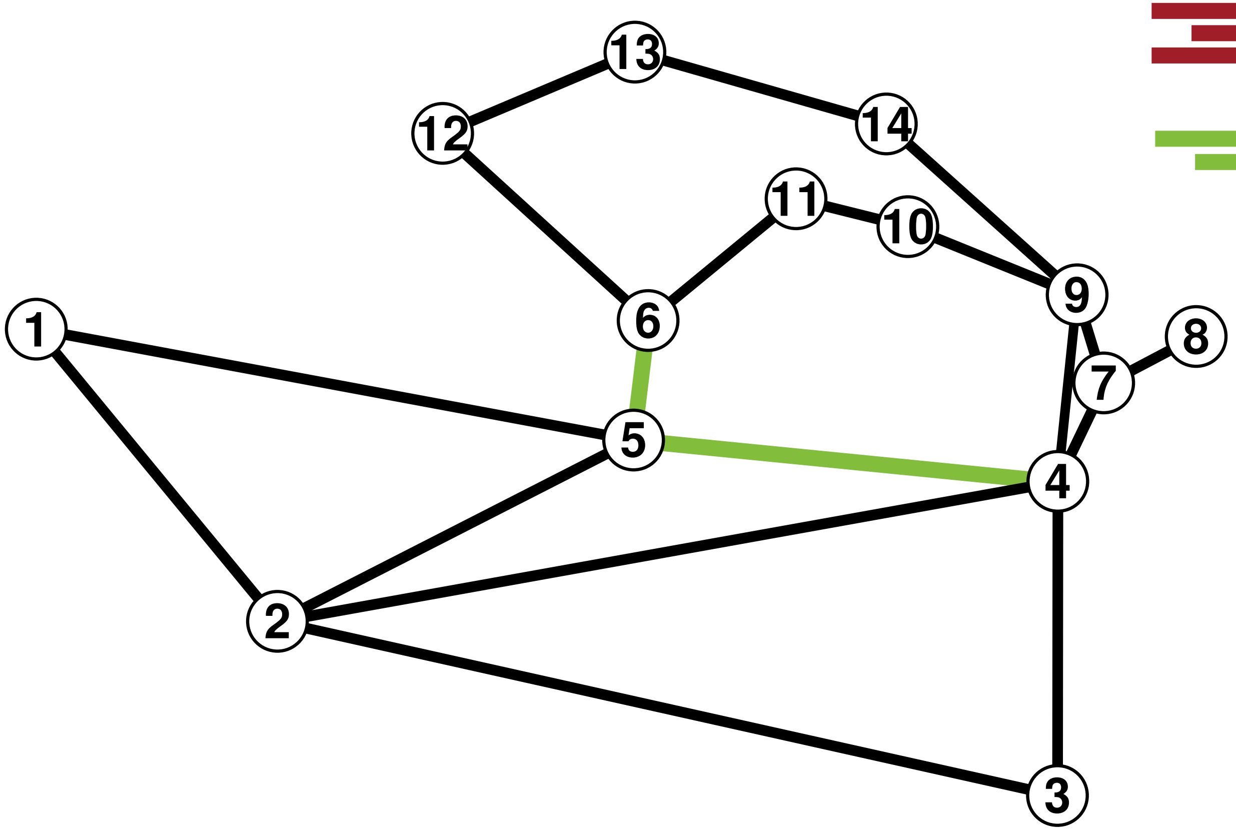


Feedback Forest Set

Feedback Cactus Set

Institute of Theoretical Informatics
Algorithmics Group

Feedback Cactus Set



Feedback Forest Set

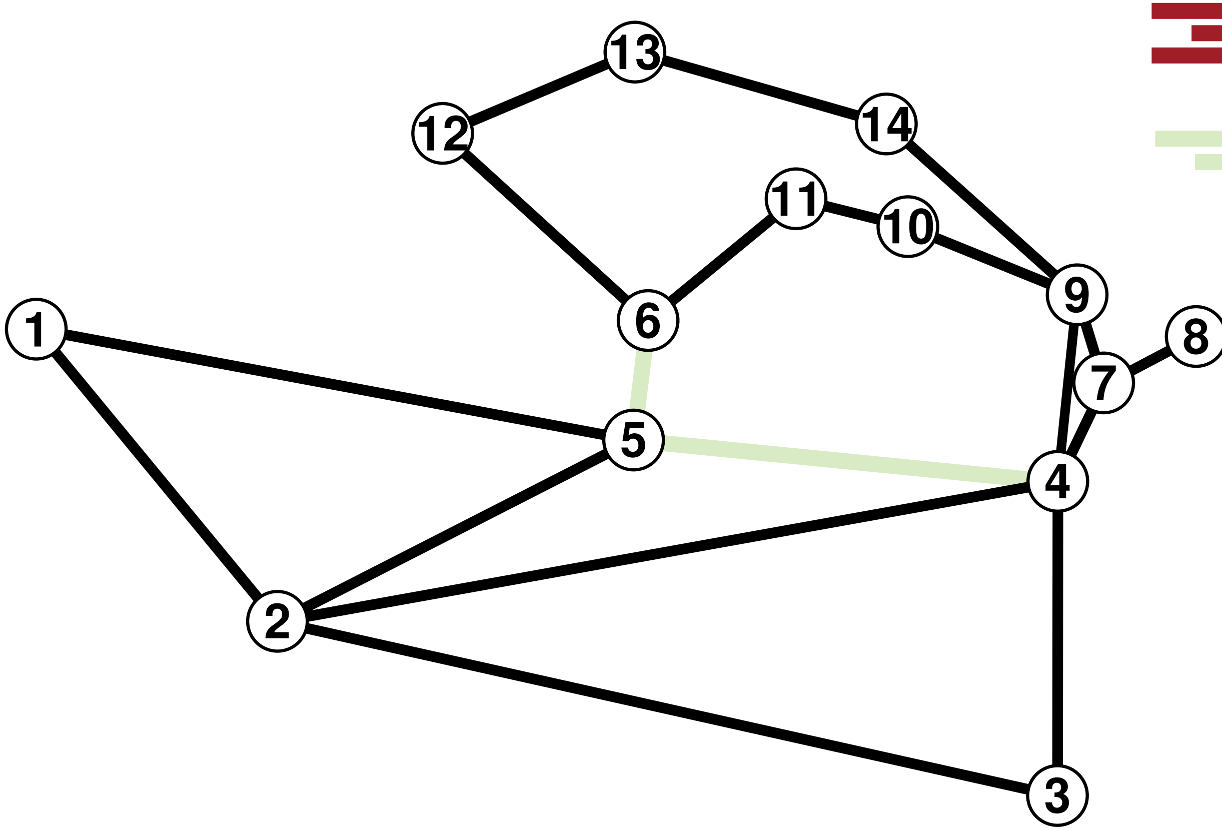
Feedback Cactus Set

⇓

A set of *cacti* remains!

Institute of Theoretical Informatics
Algorithmics Group

Feedback **Cactus** Set



Feedback Forest Set

Feedback Cactus Set

⇓

A set of *cacti*
remains!

Theorem 2

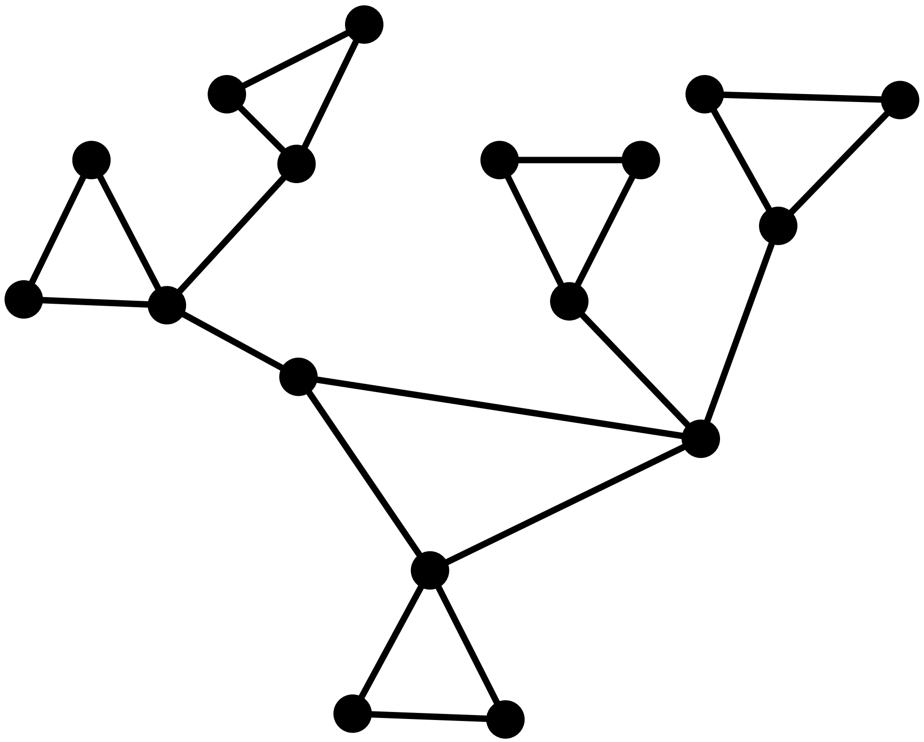
Physical subgrid **cactus**, line limits on cactus suitably bounded.

$\Rightarrow$

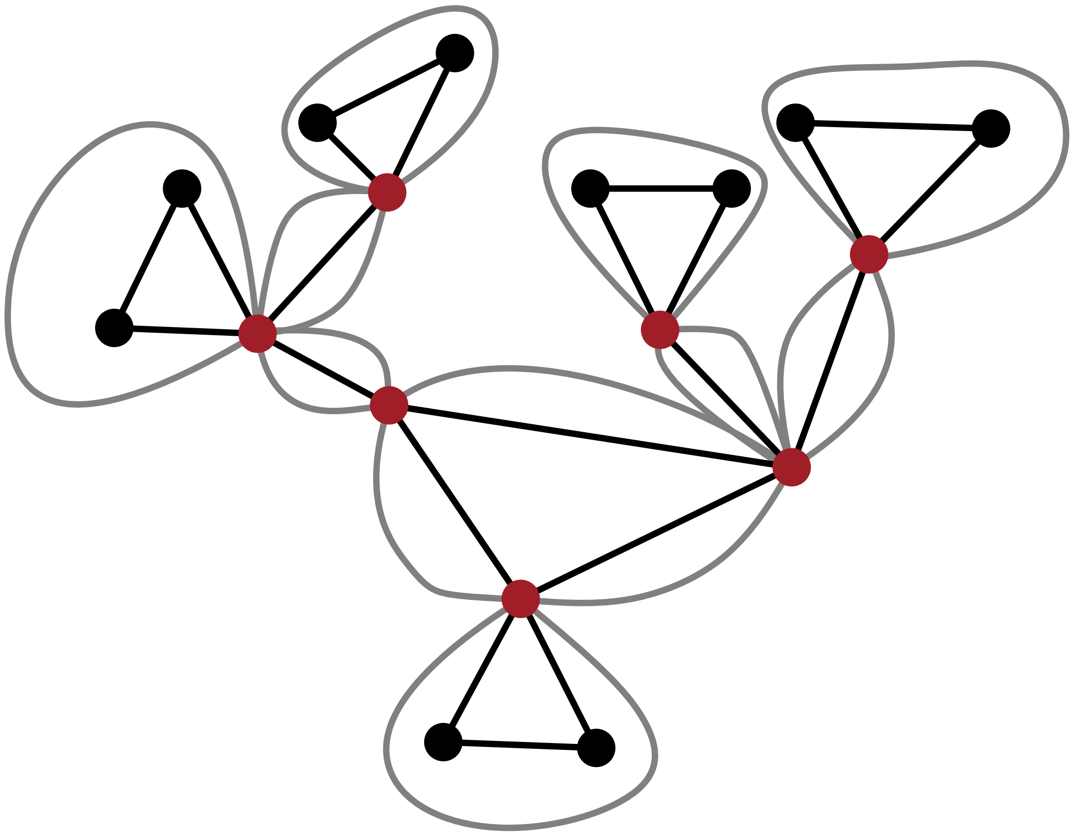
For every **graph theoretical flow** there is a **cost-equivalent flow** obeying voltage laws.

Institute of Theoretical Informatics
Algorithmics Group

Idea of Proof for the Feedback Cactus Set



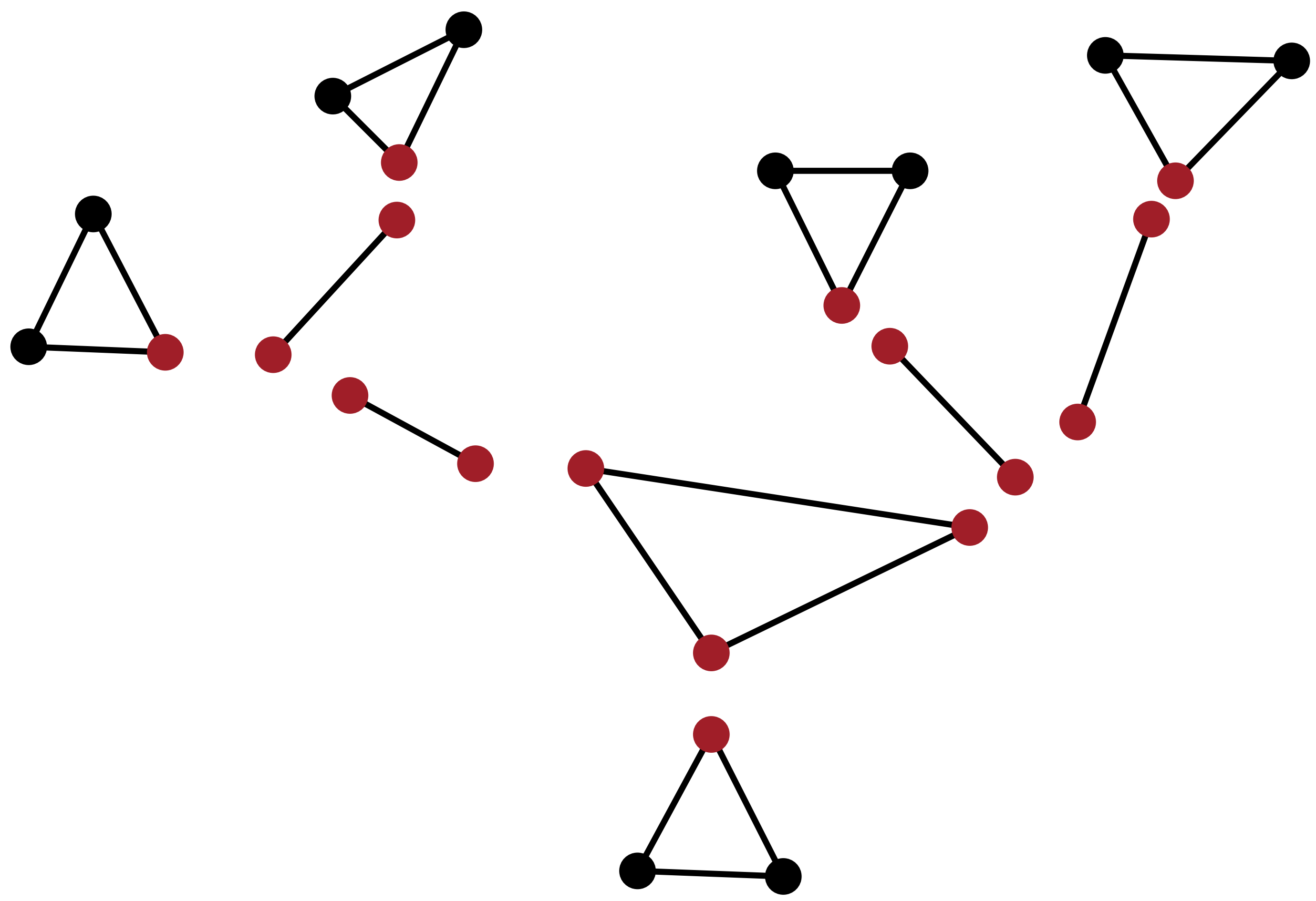
Idea of Proof for the Feedback Cactus Set



$\Rightarrow$ Cacti can be cut into single *edges* and single *cycles*

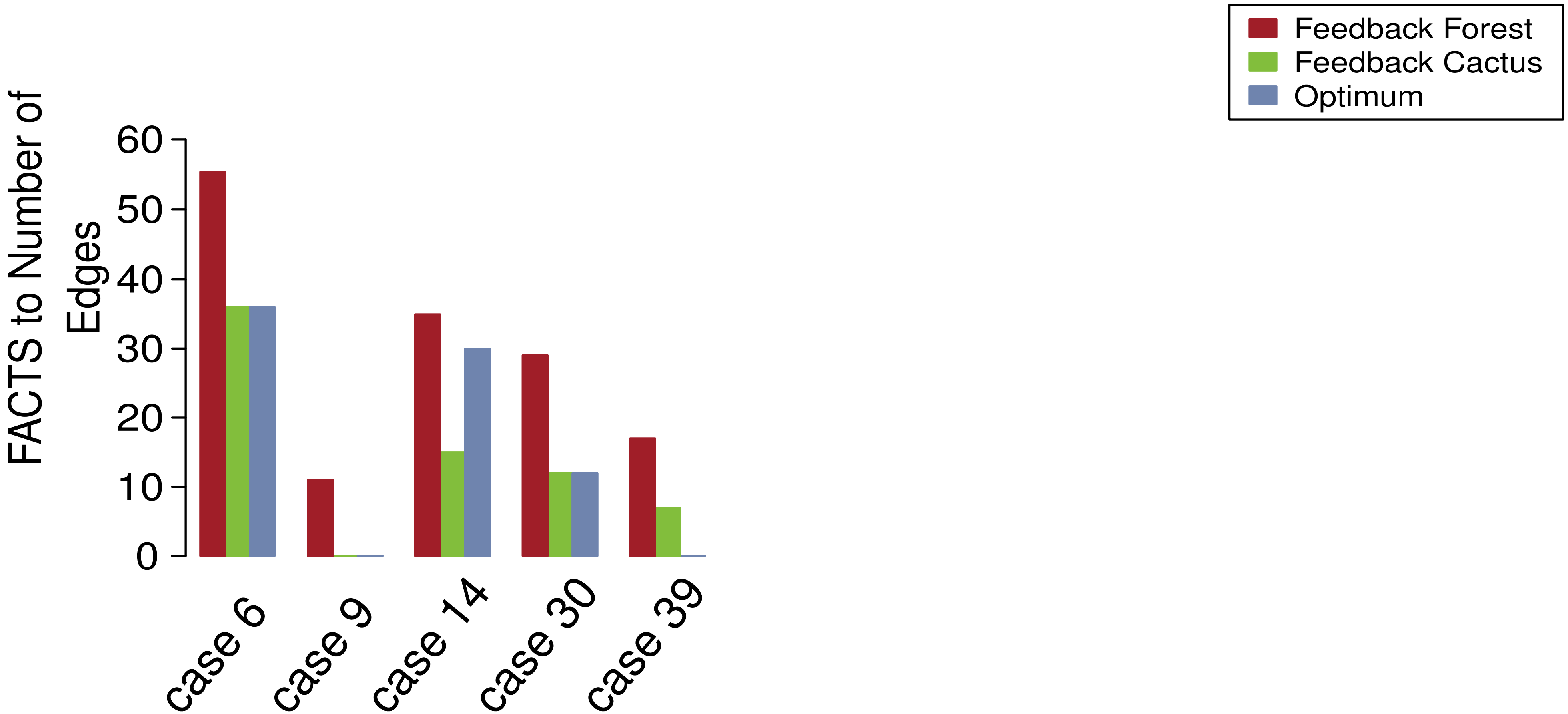
Institute of Theoretical Informatics
Algorithmics Group

Idea of Proof for the Feedback Cactus Set

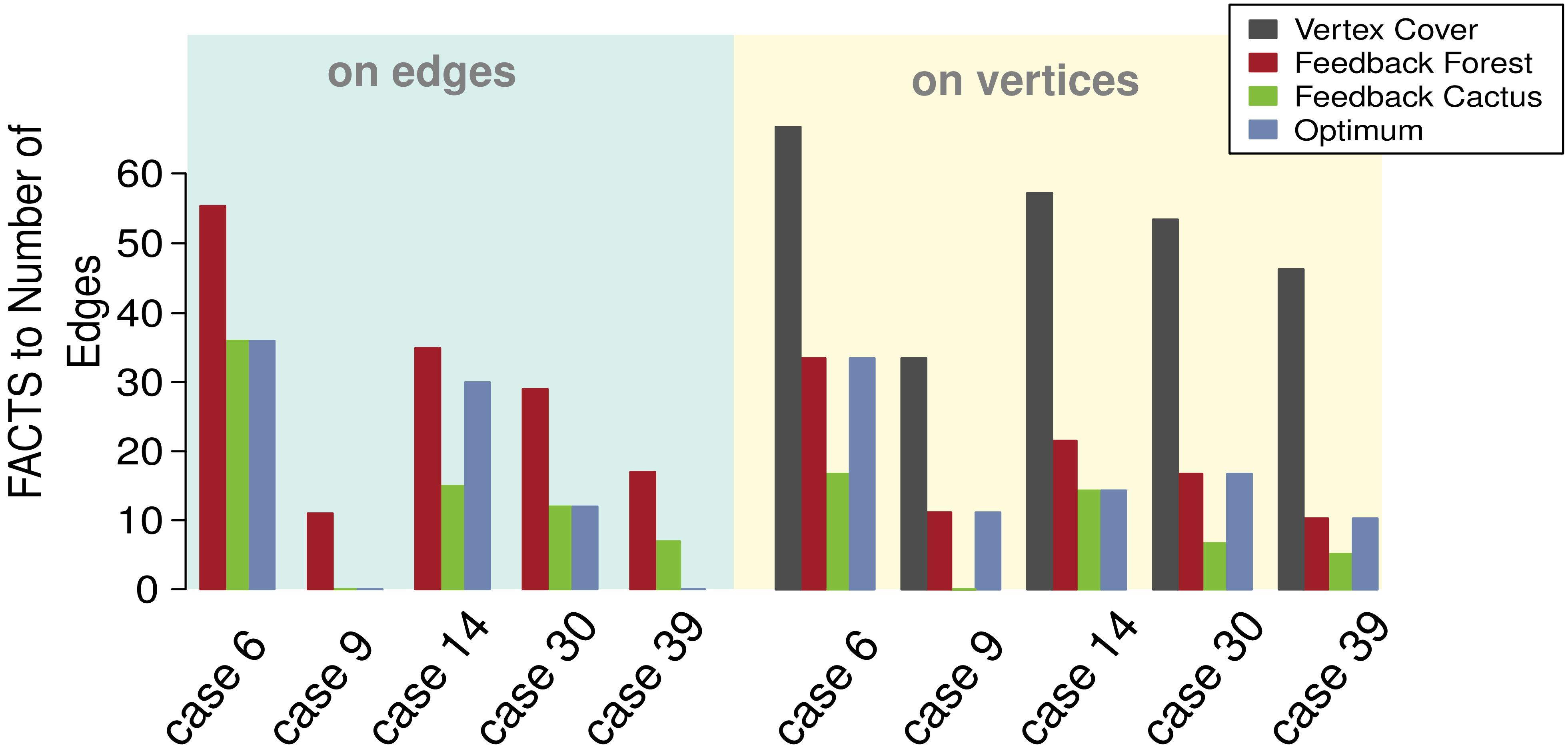


$\Rightarrow$ For every *edge* and every *cycle* always exists a solution!

Number FACTS $\leftrightarrow$ structural Results



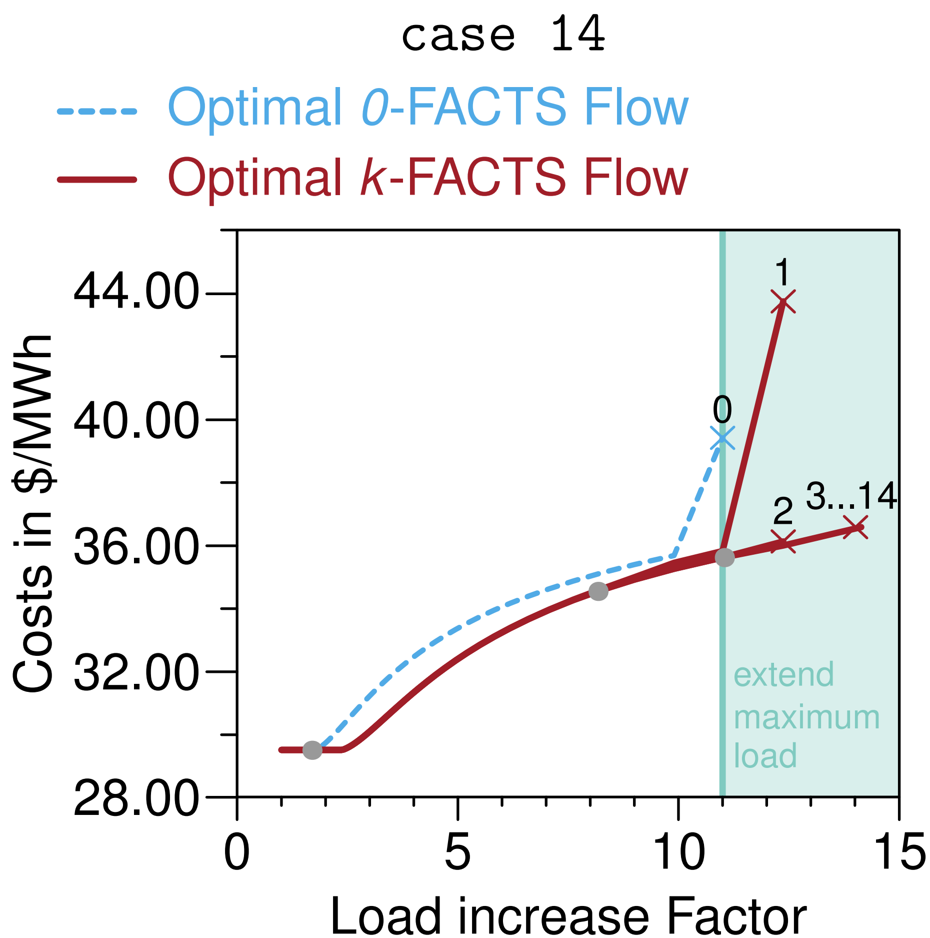
Number FACTS ↔ structural Results

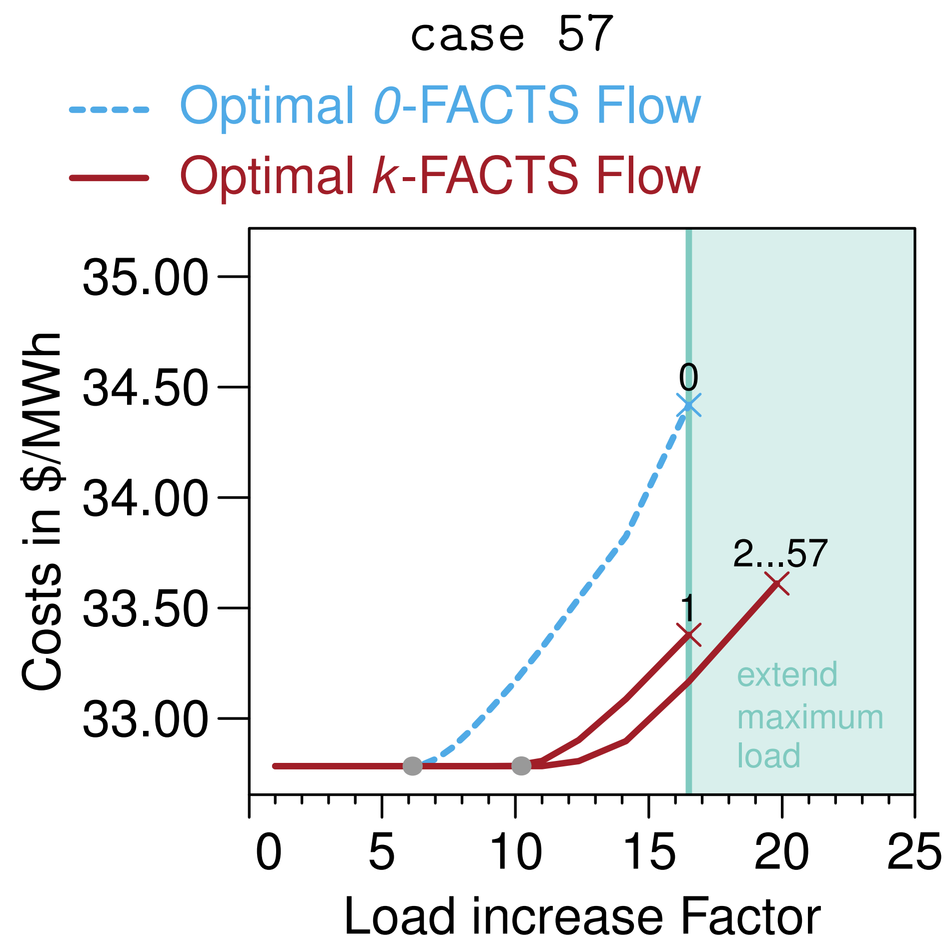


Often a **small number** of ideal FACTS suffices for **matching cost** of the flow model.

Institute of Theoretical Informatics
Algorithmics Group

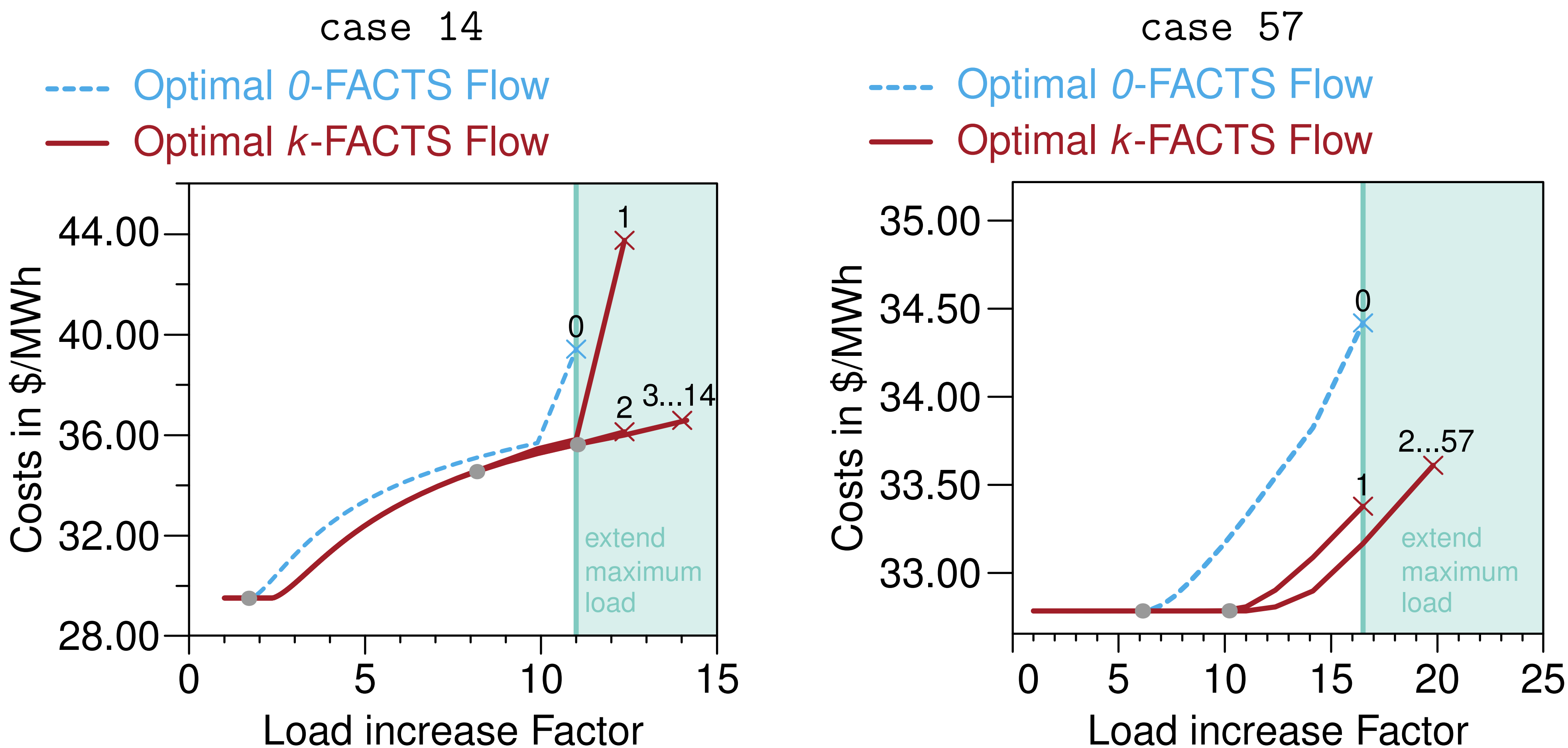
Operation under Increasing Loads





Institute of Theoretical Informatics
Algorithmics Group

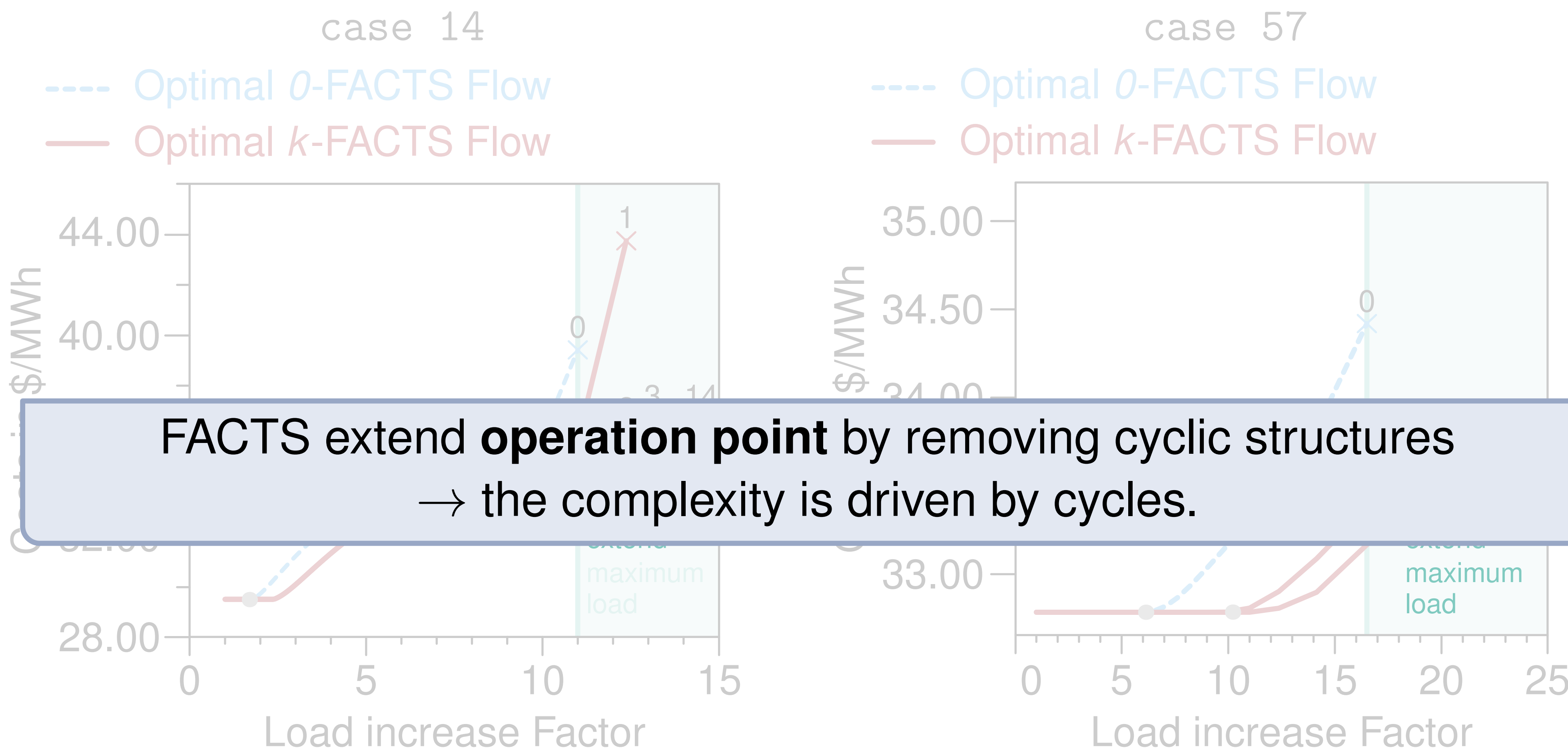
Operation under Increasing Loads



Very few flow control branches extend **maximum load** at **lower cost**.

Institute of Theoretical Informatics
Algorithmics Group

Operation under Increasing Loads



FACTS extend **operation point** by removing cyclic structures
$\rightarrow$ the complexity is driven by cycles.

Very few flow control branches extend **maximum load** at **lower cost**.

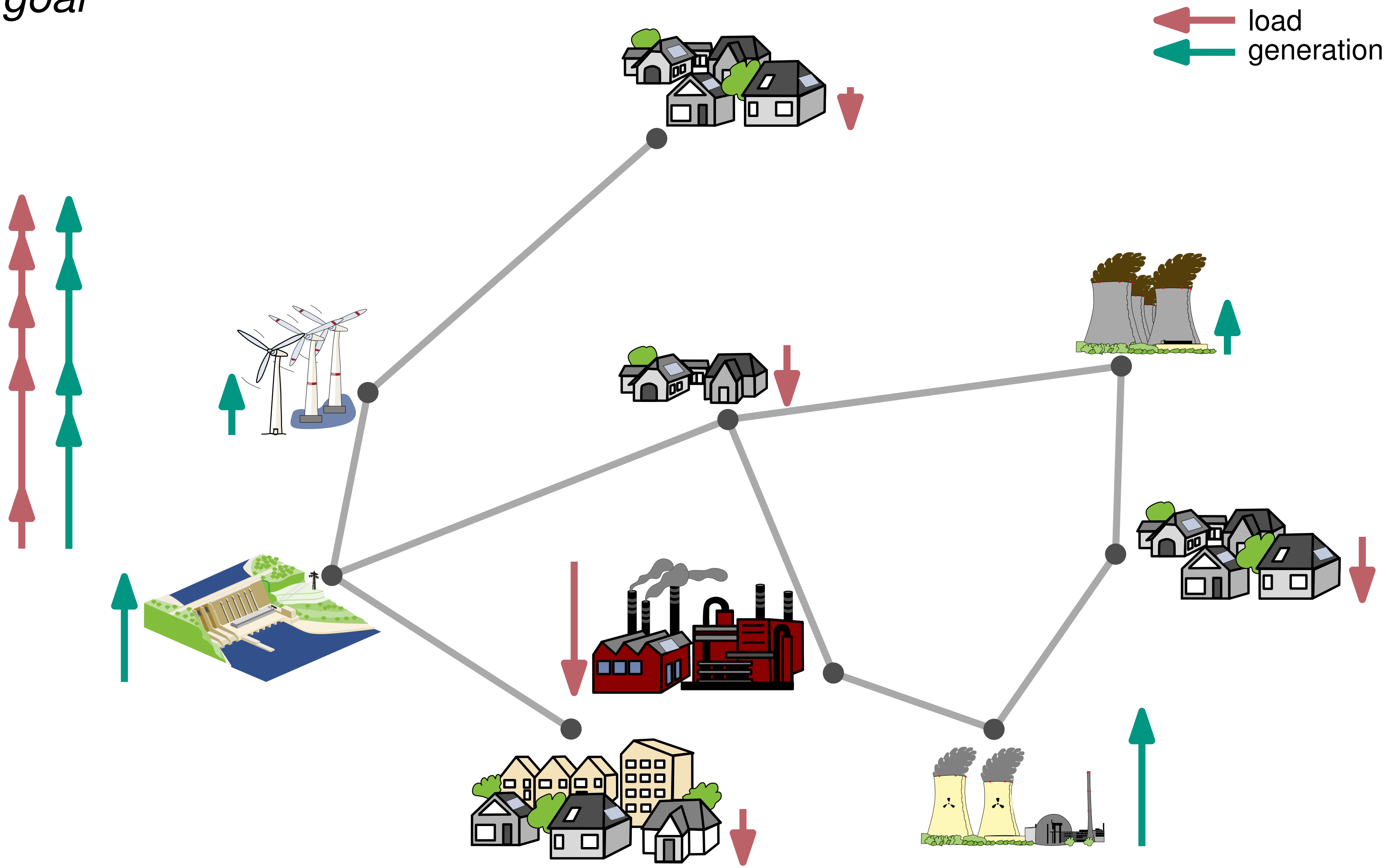
Institute of Theoretical Informatics
Algorithmics Group

From ideal FACTS to real FACTS

Figures FLTR:
[3] http://www.abb.com/cawp/seitp202/c36f4e62da52ab46c1257670003690d3.aspx
[4] http://electrical-engineering-portal.com/facts-flexible-ac-transmission-systems

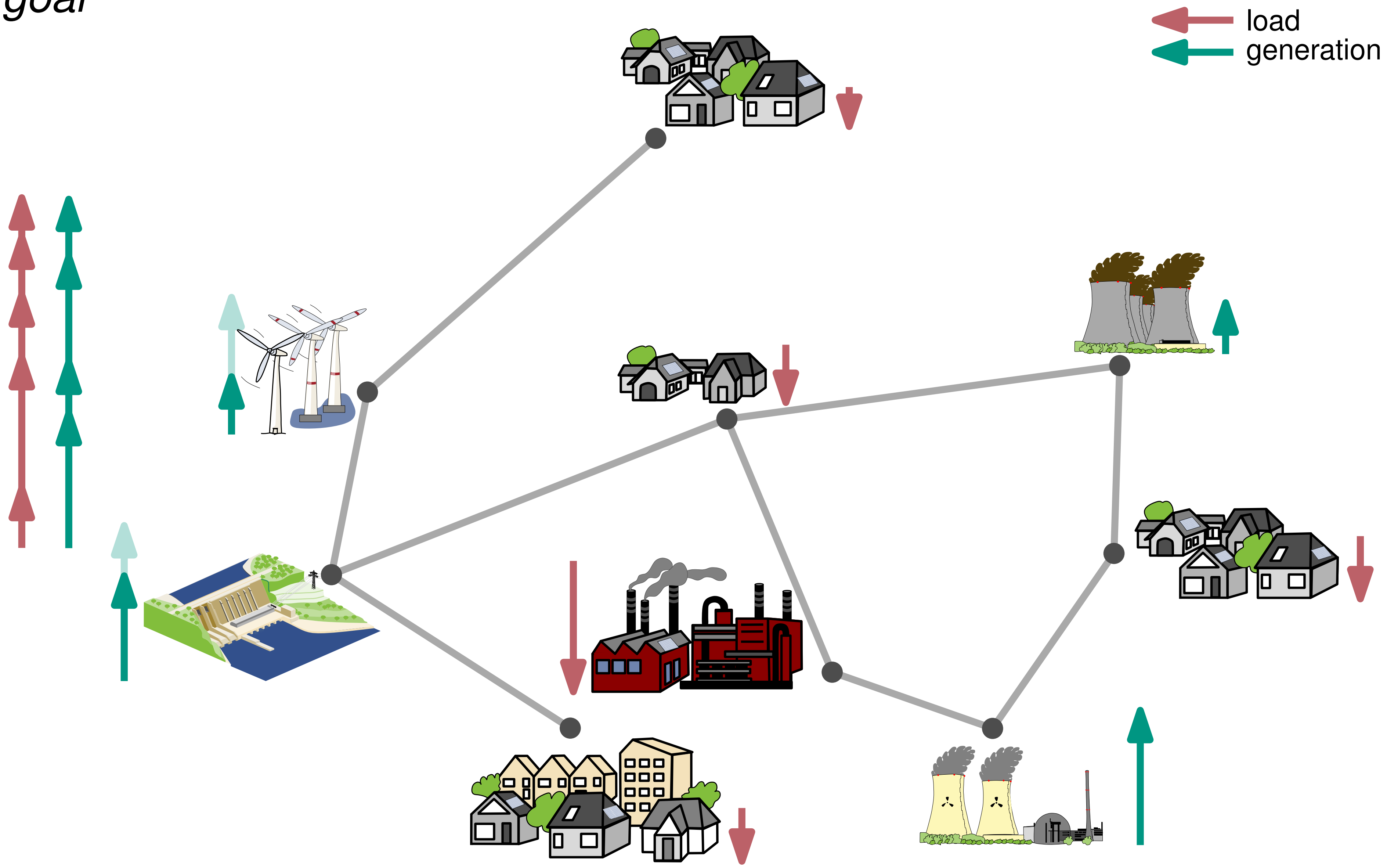
AC Conservation of Flow in Power Grids

goal



AC Conservation of Flow in Power Grids

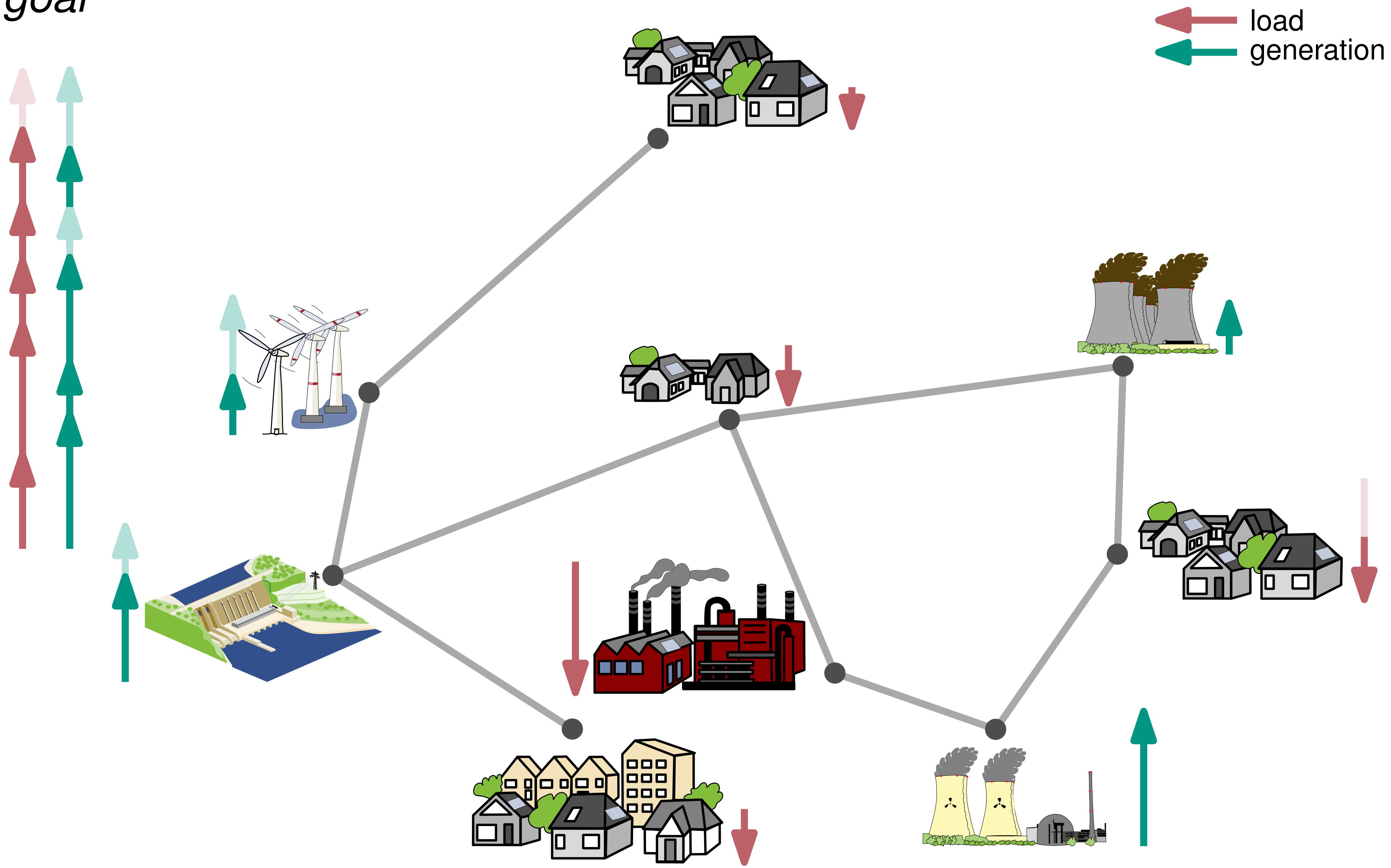
goal



Institute of Theoretical Informatics
Algorithmics Group

AC Conservation of Flow in Power Grids

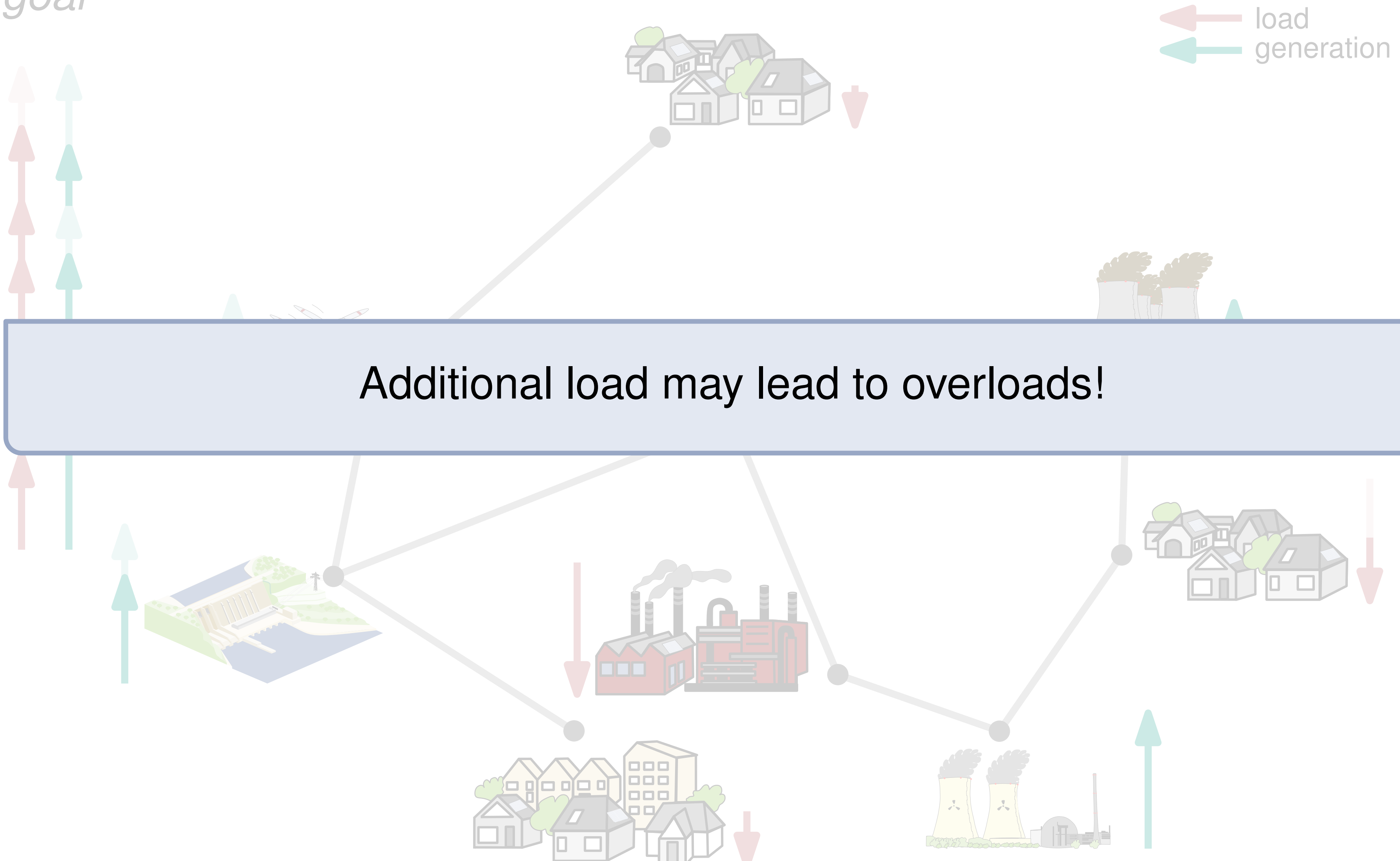
goal



Institute of Theoretical Informatics
Algorithmics Group

AC Conservation of Flow in Power Grids

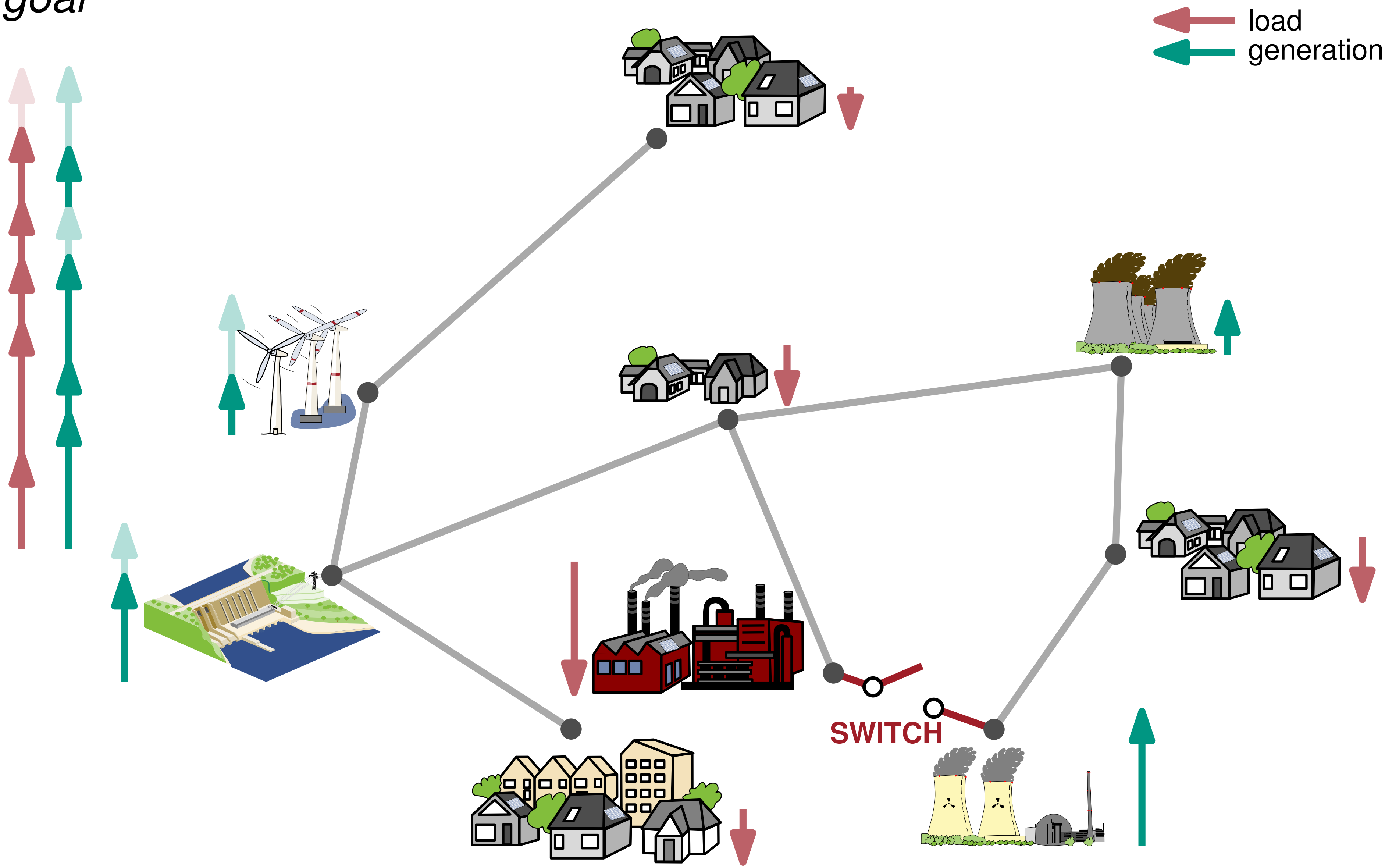
goal



Additional load may lead to overloads!

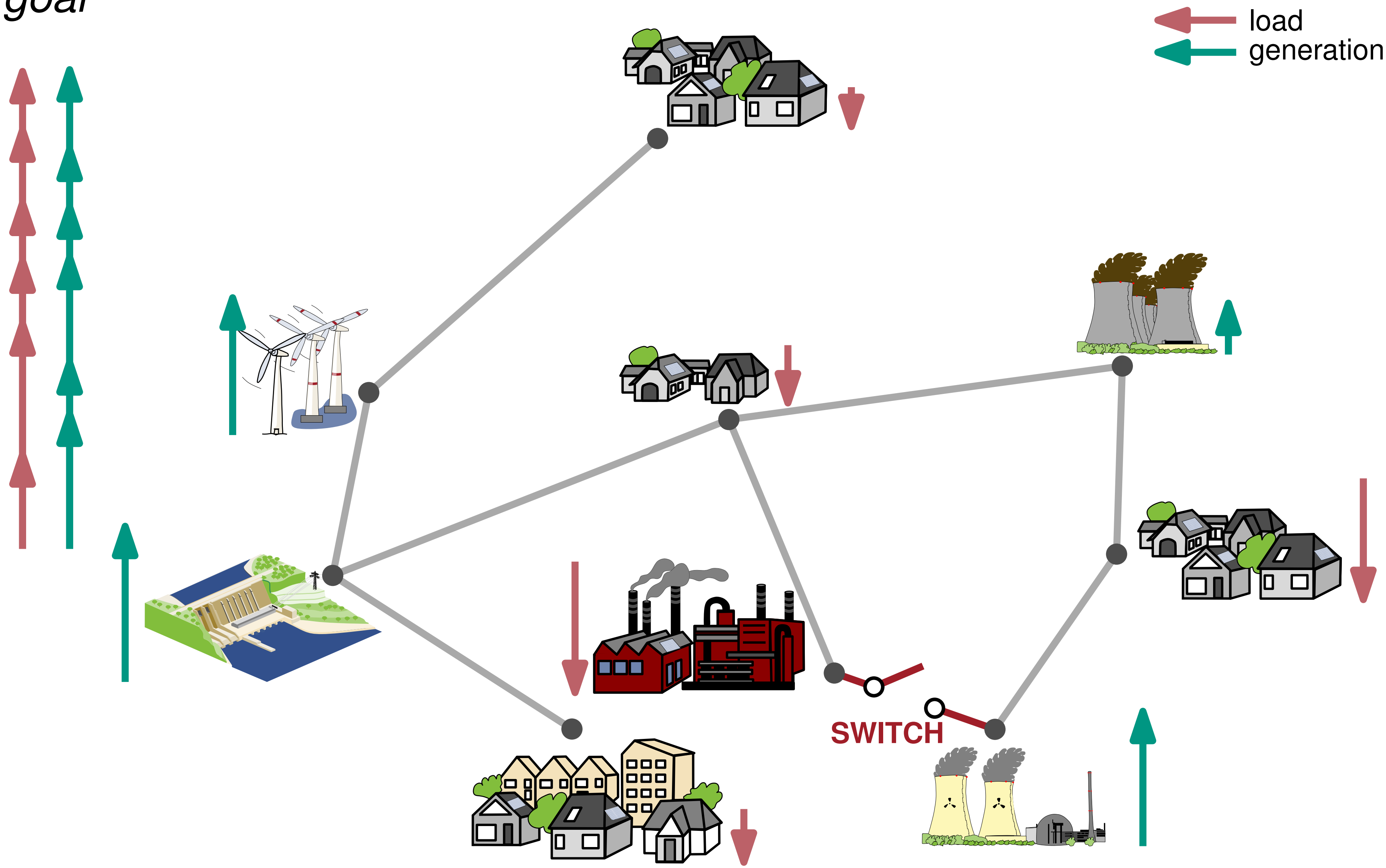
Switches

goal



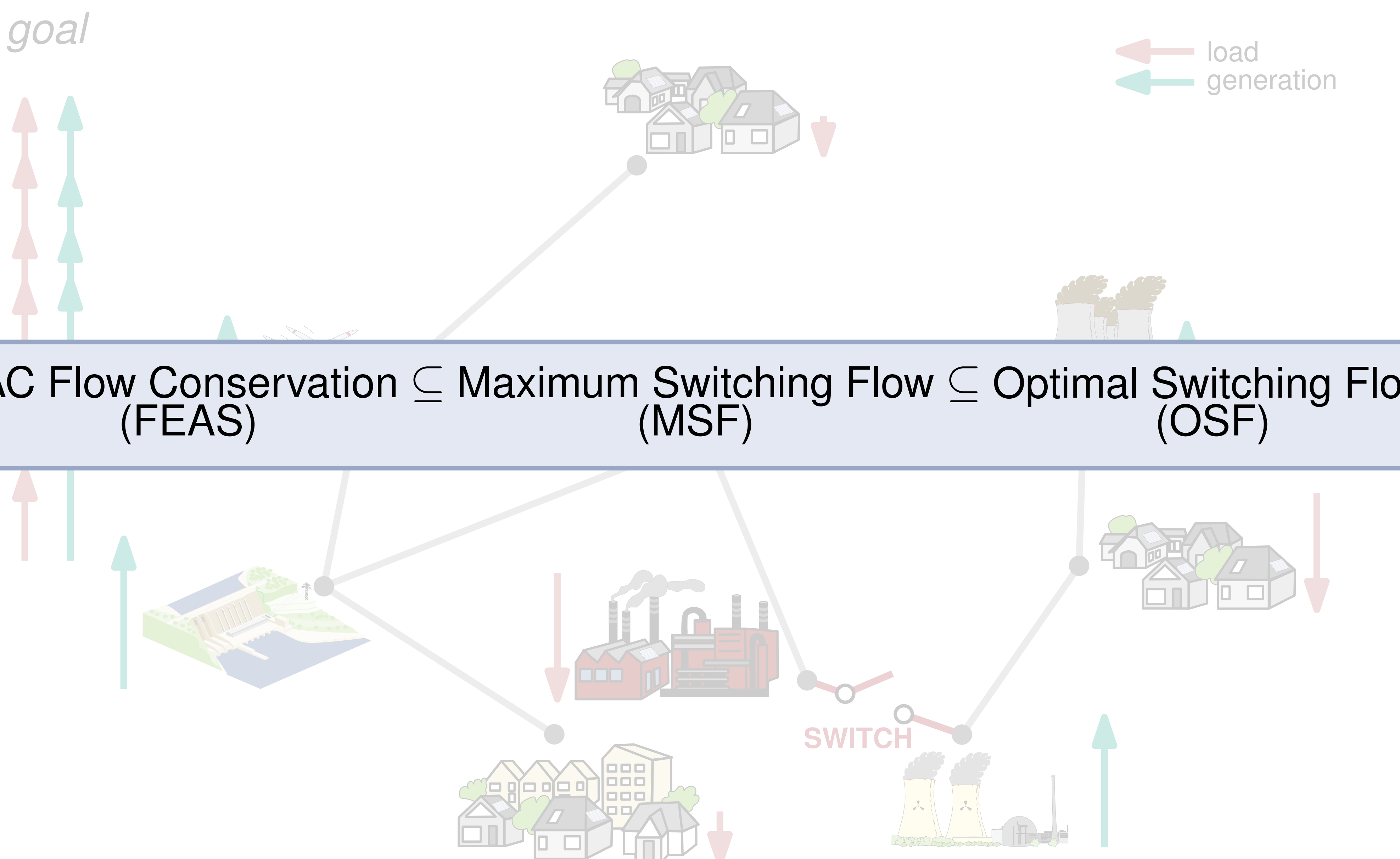
Switches

goal



Institute of Theoretical Informatics
Algorithmics Group

Switches

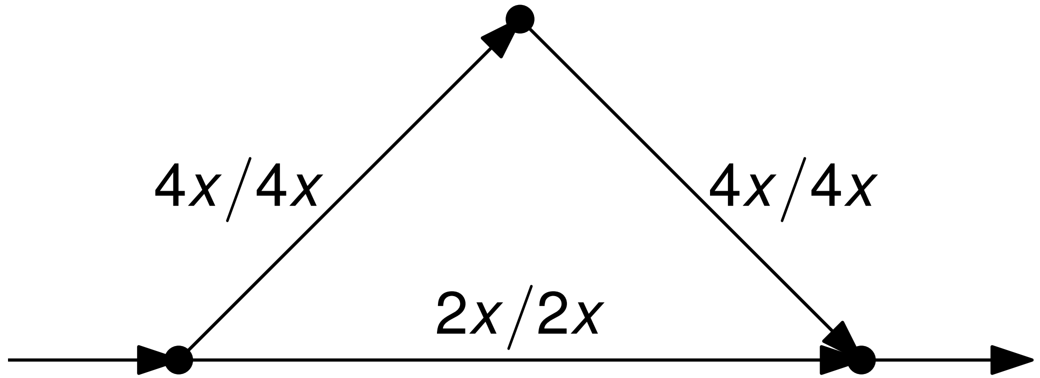


AC Flow Conservation $\subseteq$ Maximum Switching Flow $\subseteq$ Optimal Switching Flow
(FEAS) (MSF) (OSF)

Institute of Theoretical Informatics
Algorithmics Group

Maximum Switching Flow (MSF)

Maximum Flow (MF) of $6x$

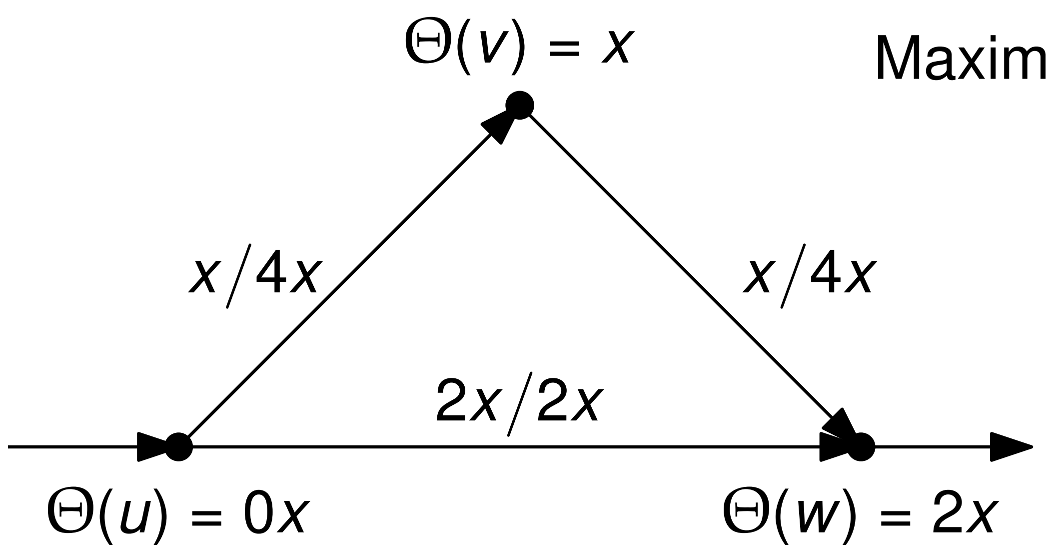


Institute of Theoretical Informatics
Algorithmics Group

Maximum Switching Flow (MSF)

Maximum Flow (MF) of $6x$

Maximum Power Flow (MPF) of $3x$



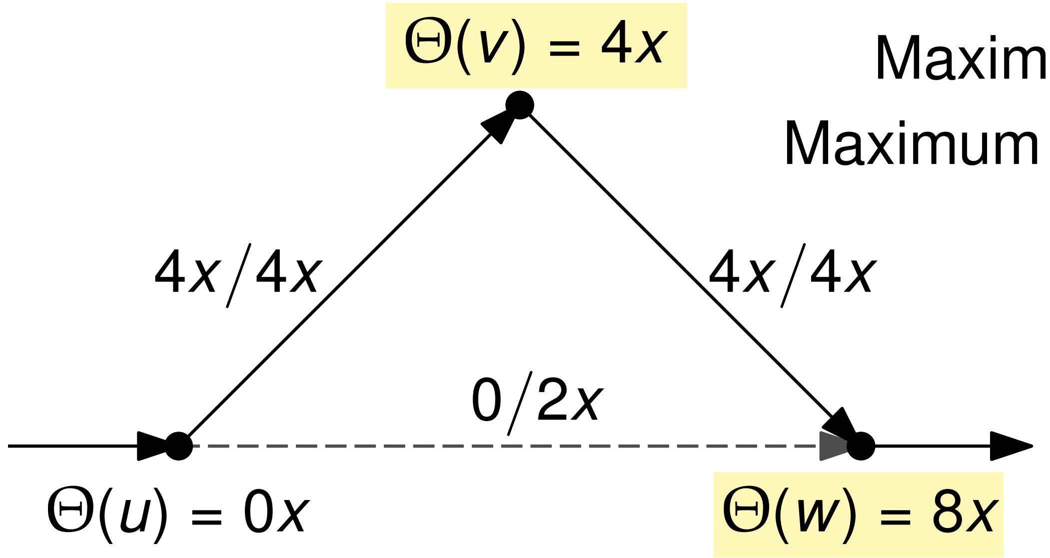
$$P(u, v) = B(u, v)(\Theta(u) - \Theta(w))$$

Maximum Switching Flow (MSF)

Maximum Flow (MF) of $6x$

Maximum Power Flow (MPF) of $3x$

Maximum Switching Flow (MSF) of $4x$



$$P(u, v) = B(u, v)(\Theta(u) - \Theta(w))$$

Institute of Theoretical Informatics
Algorithmics Group

Complexity of the Maximum Switching Flow (MSF)

Graph Structure	Complexity
cacti with max degree 3	NP-hard
2-level trees	NP-hard
planar graphs with max degree of 3	strong NP-hard
arbitrary graphs	Non-APX

Institute of Theoretical Informatics
Algorithmics Group

Complexity of the Maximum Switching Flow (MSF)

Graph Structure	Complexity
cacti with max degree 3	NP-hard
2-level trees	NP-hard
planar graphs with max degree of 3	strong NP-hard
arbitrary graphs	Non-APX

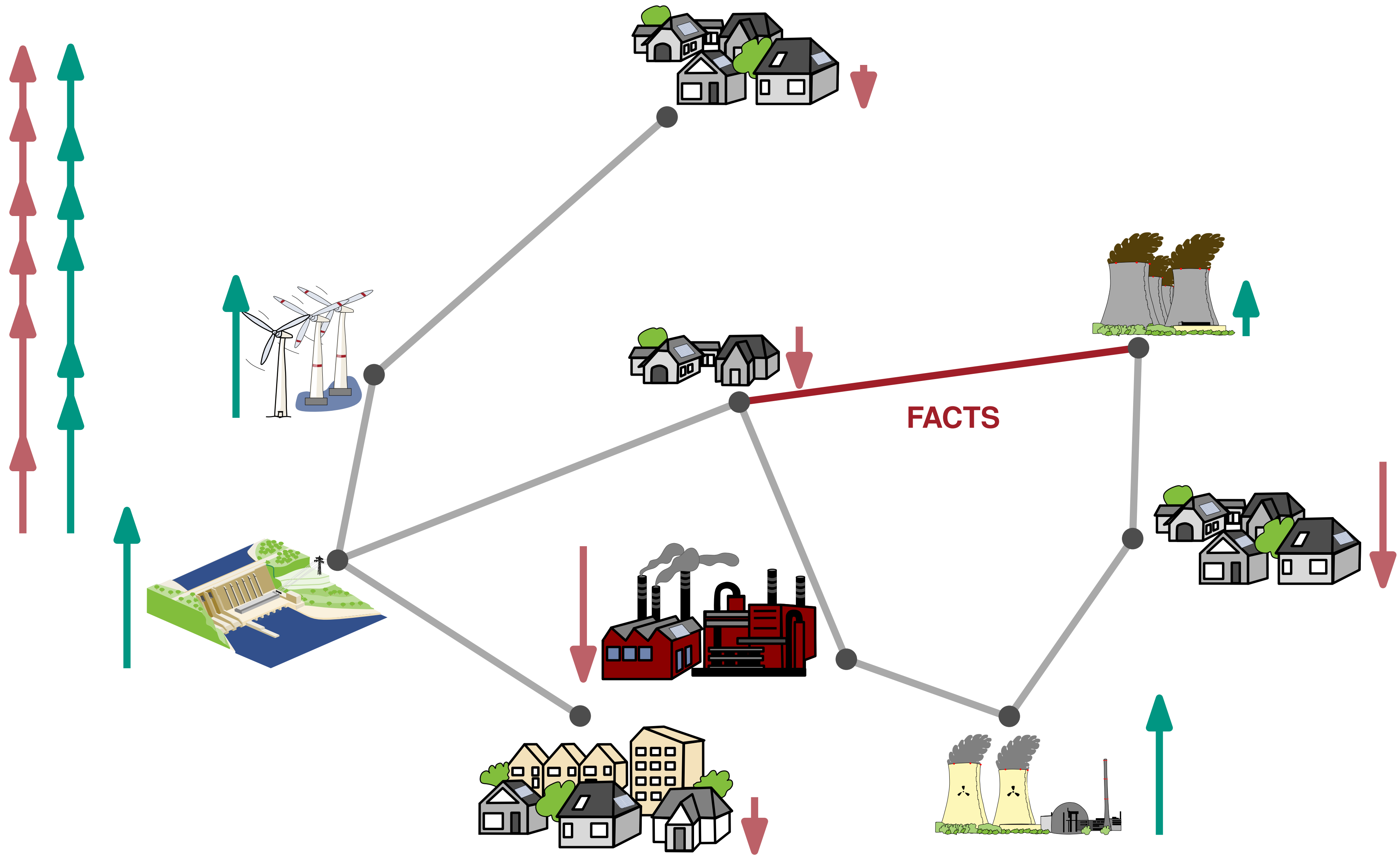
Theorem 3

The maximum switching flow (MSF) is 2-approximatable on cacti.

Institute of Theoretical Informatics
Algorithmics Group

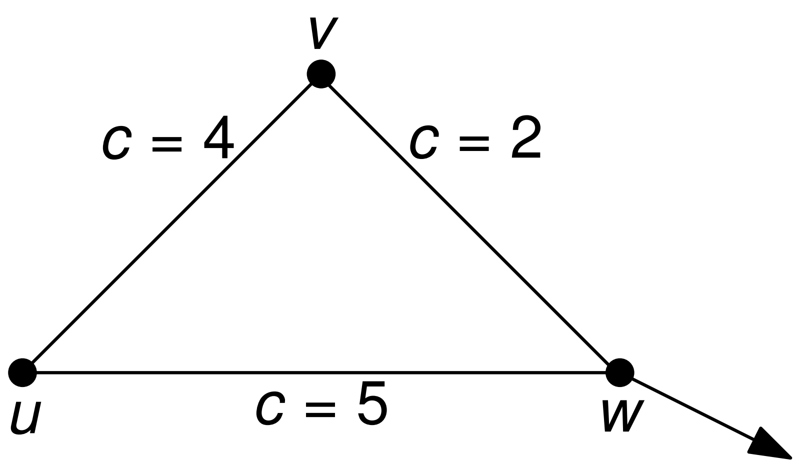
FACTS with Susceptance Interval

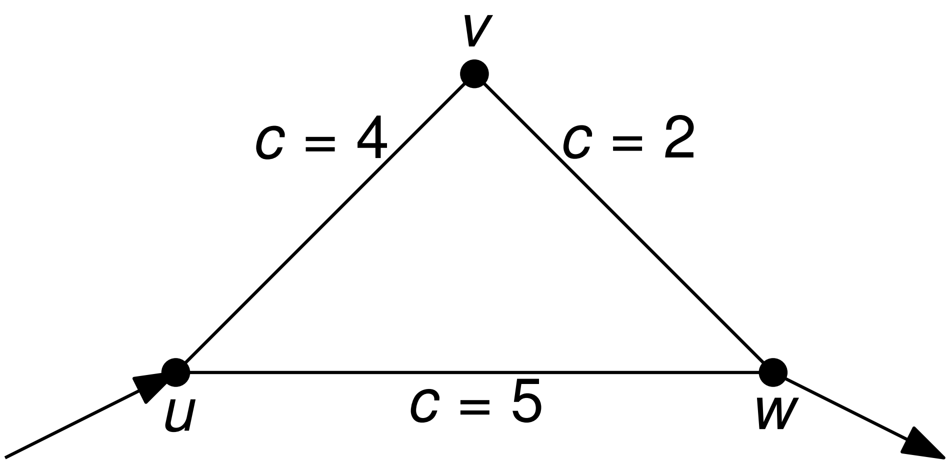
goal



Institute of Theoretical Informatics
Algorithmics Group

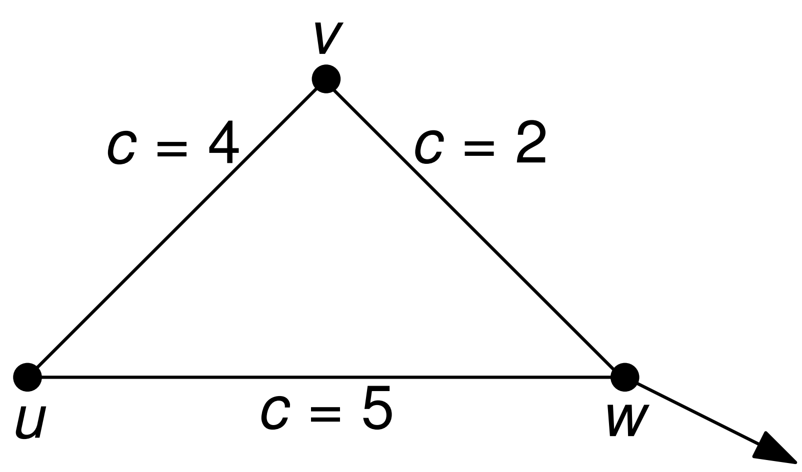
Maximum FACTS Flow

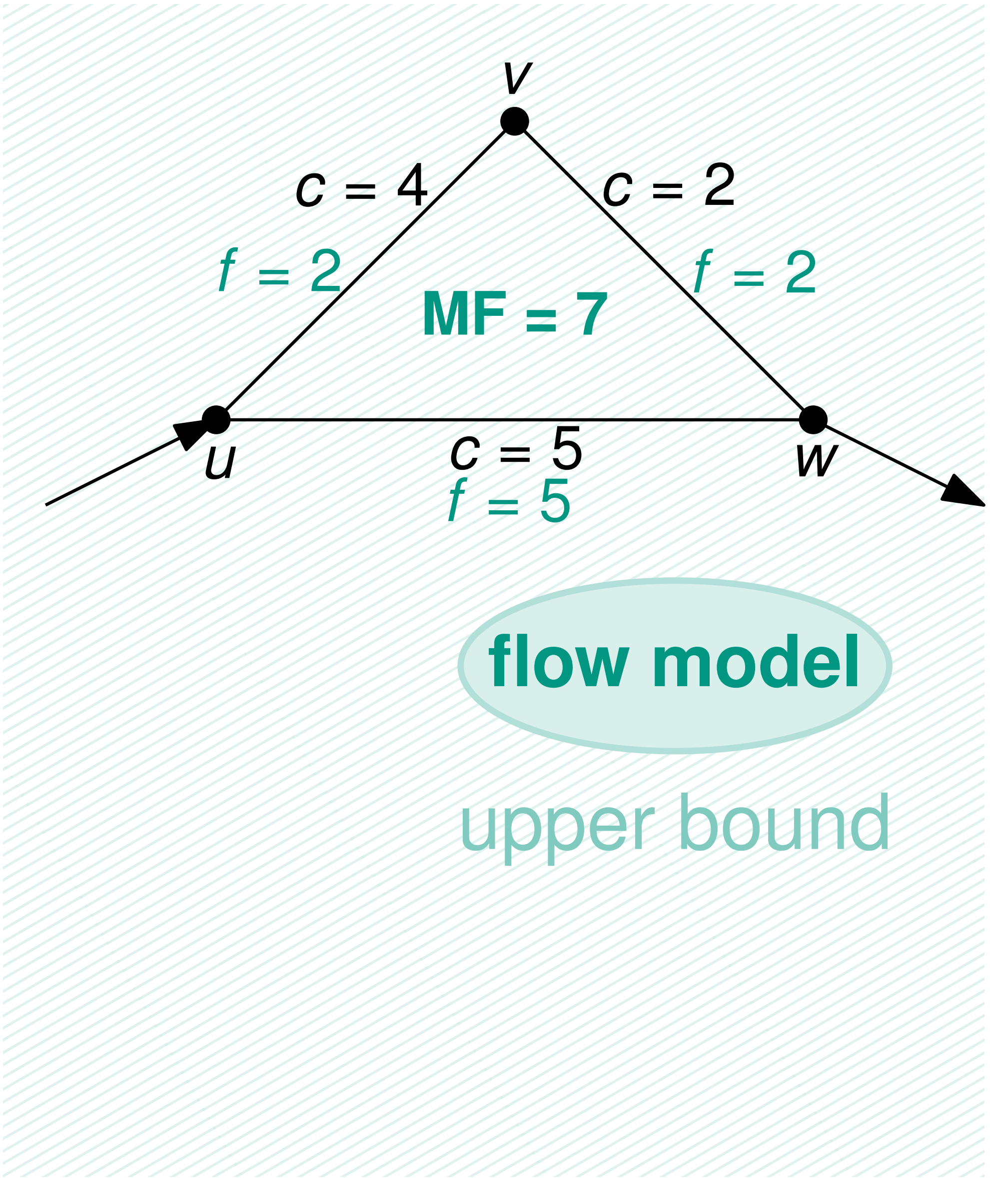




Institute of Theoretical Informatics
Algorithmics Group

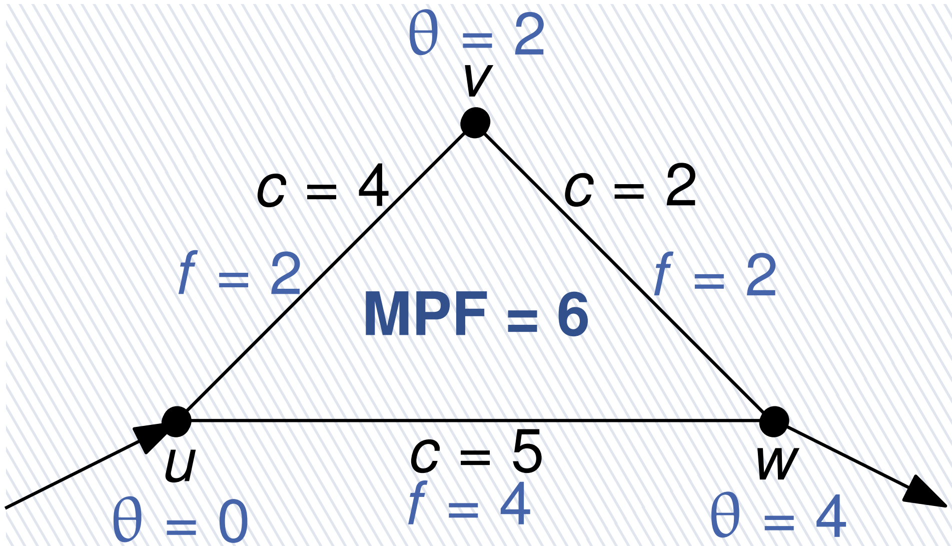
Maximum FACTS Flow





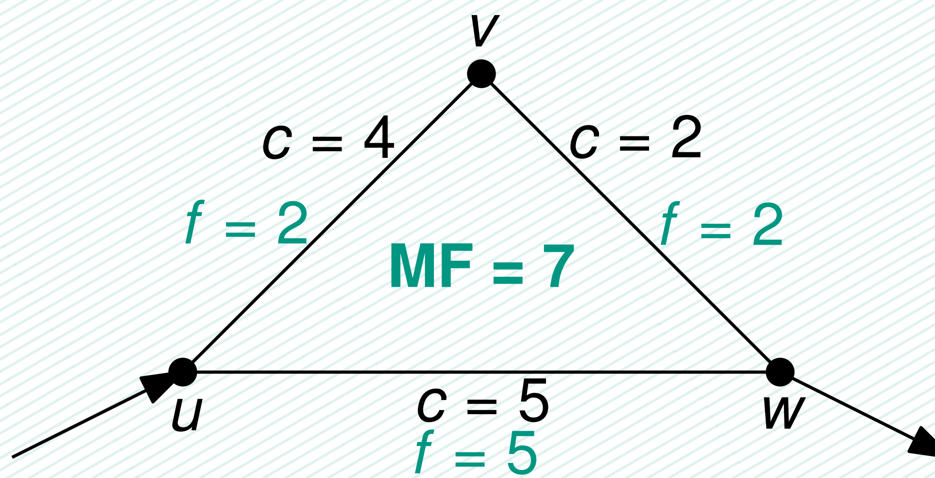
Institute of Theoretical Informatics
Algorithmics Group

Maximum FACTS Flow



physical model
(AC linearization)

lower bound

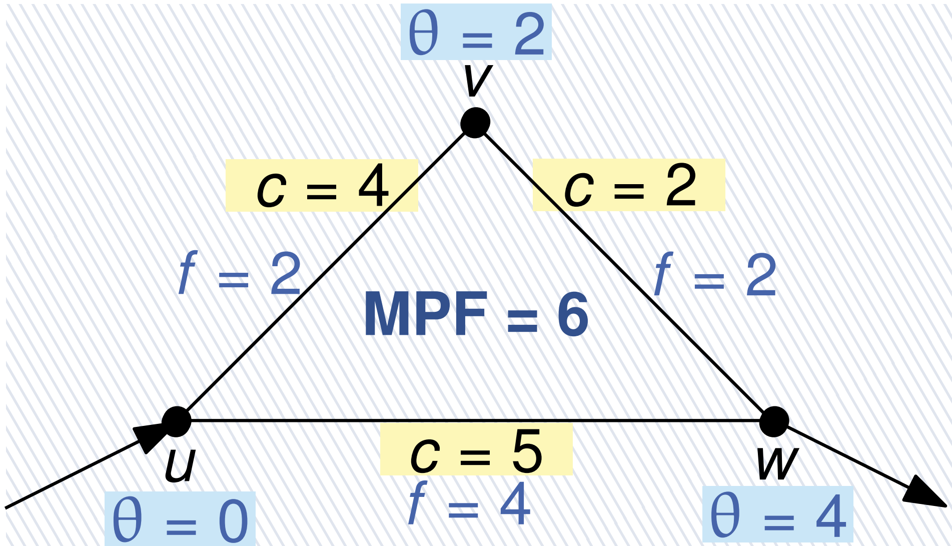


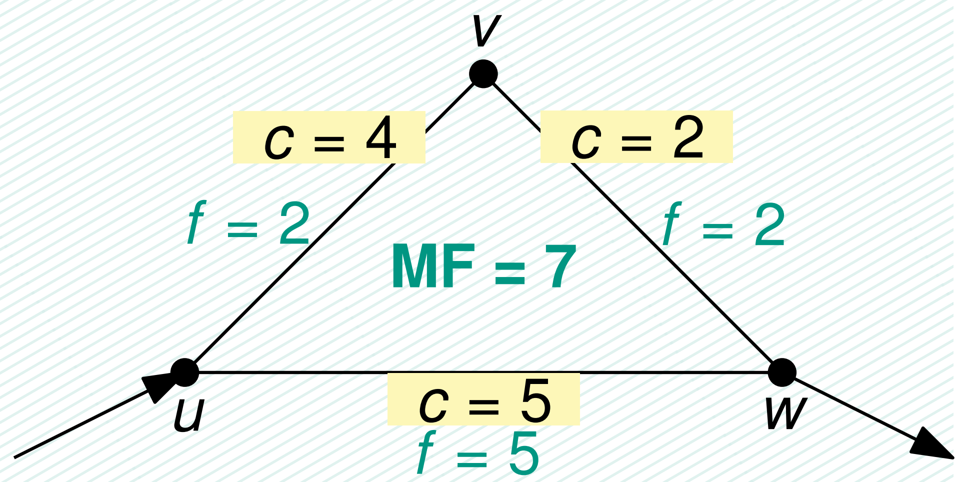
flow model

upper bound

Institute of Theoretical Informatics
Algorithmics Group

Maximum FACTS Flow





physical model
(AC linearization)

flow model

lower bound

branch capacity

upper bound

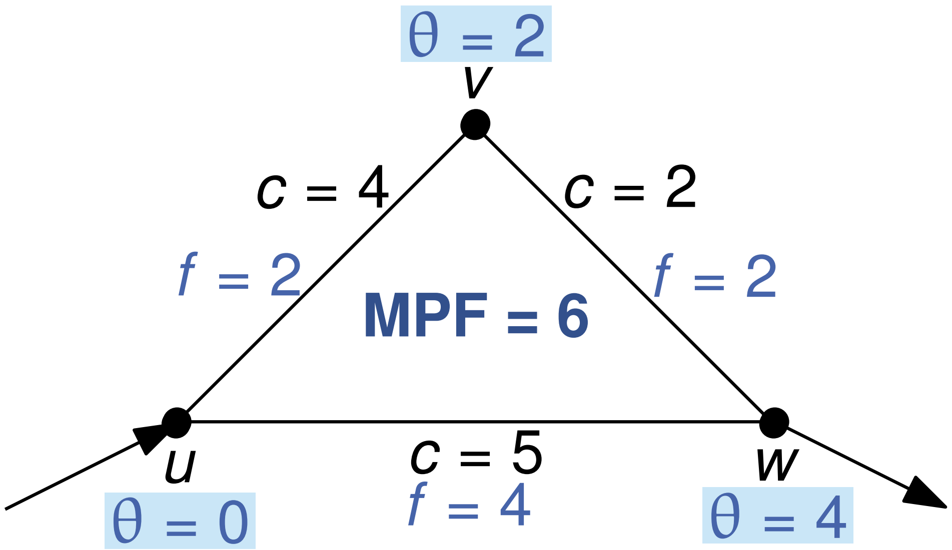
Kirchhoff's Current Law

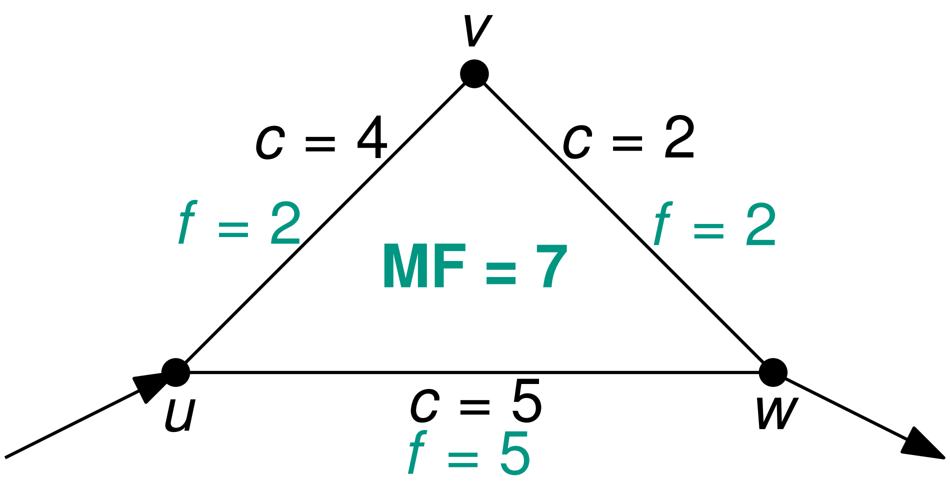
real power constraints

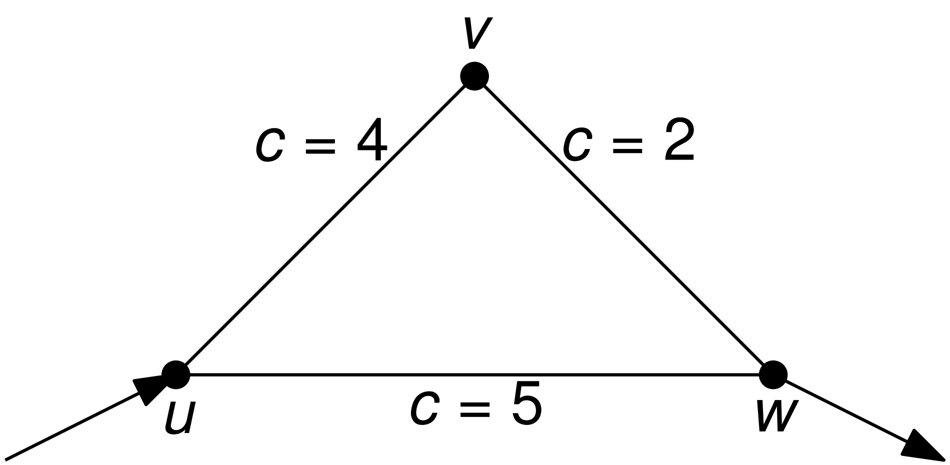
$$\forall (u, v) \in E\colon\ f(u, v) = b(u, v)(\theta(v) - \theta(u))$$

Institute of Theoretical Informatics
Algorithmics Group

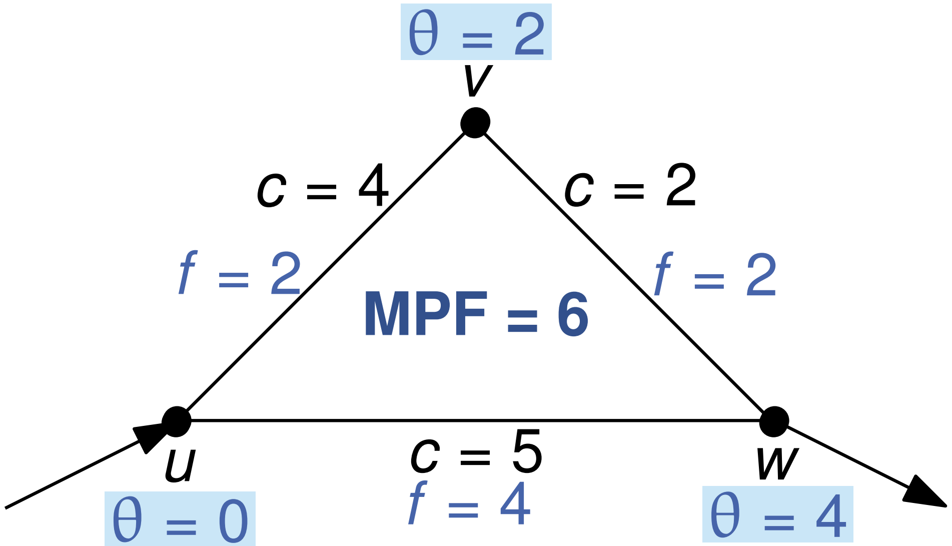
Maximum FACTS Flow

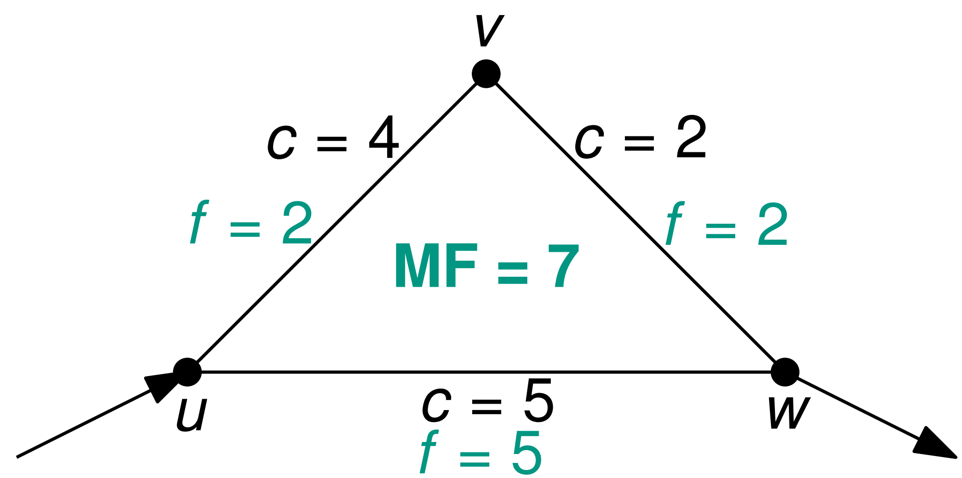


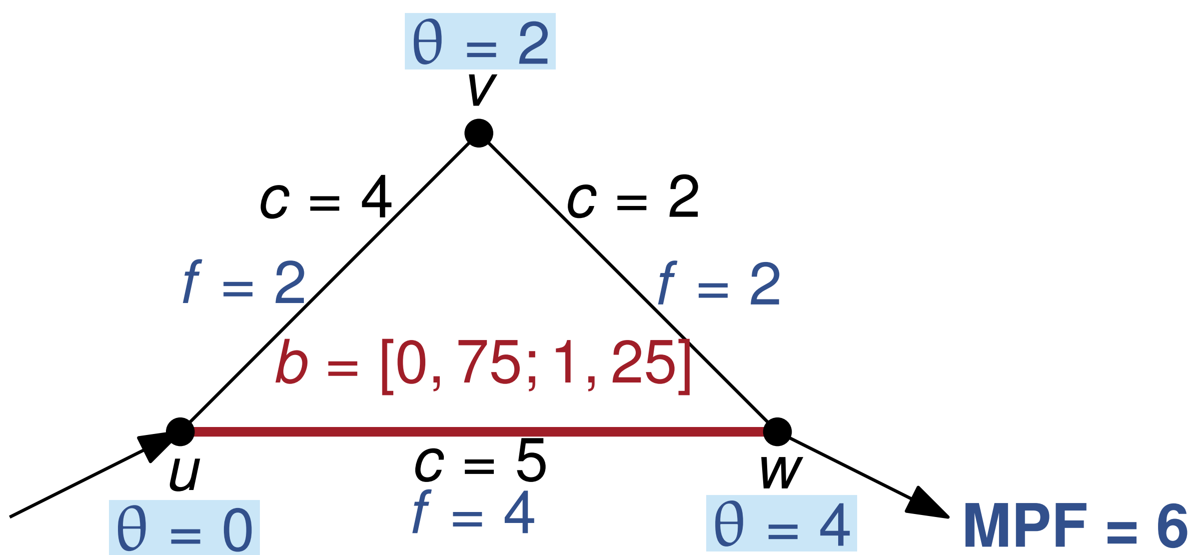




Maximum FACTS Flow

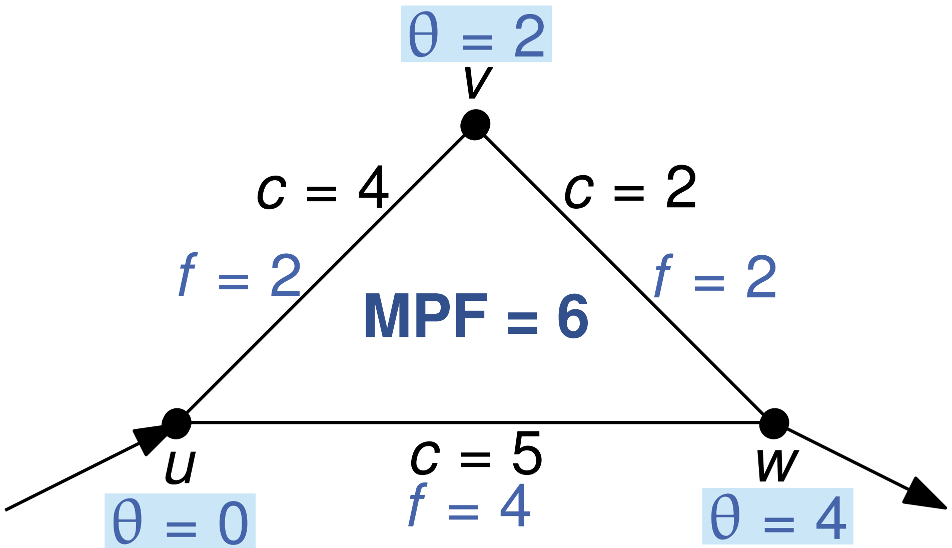


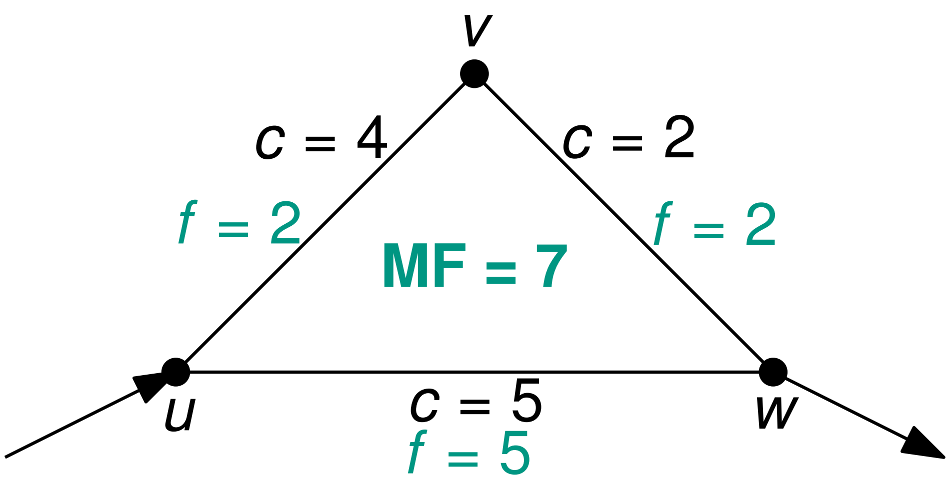


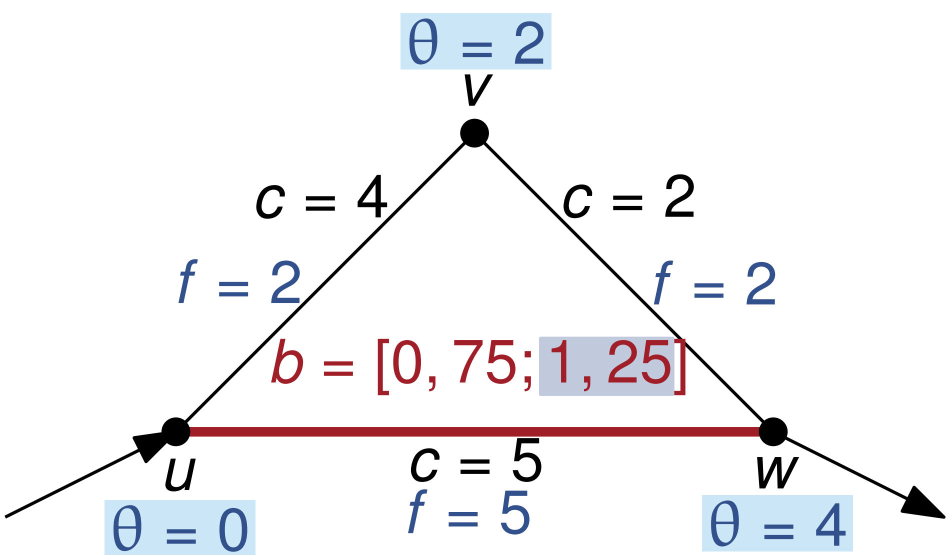


Institute of Theoretical Informatics
Algorithmics Group

Maximum FACTS Flow



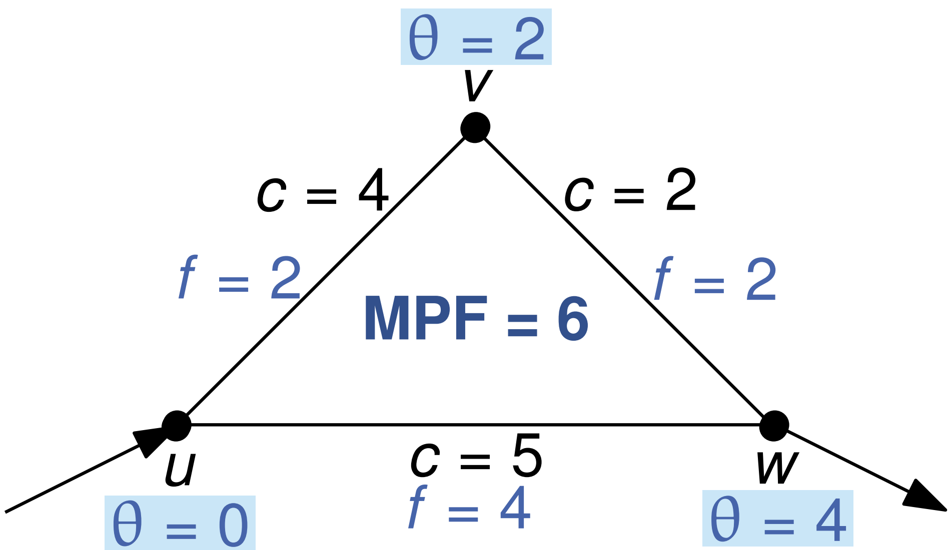


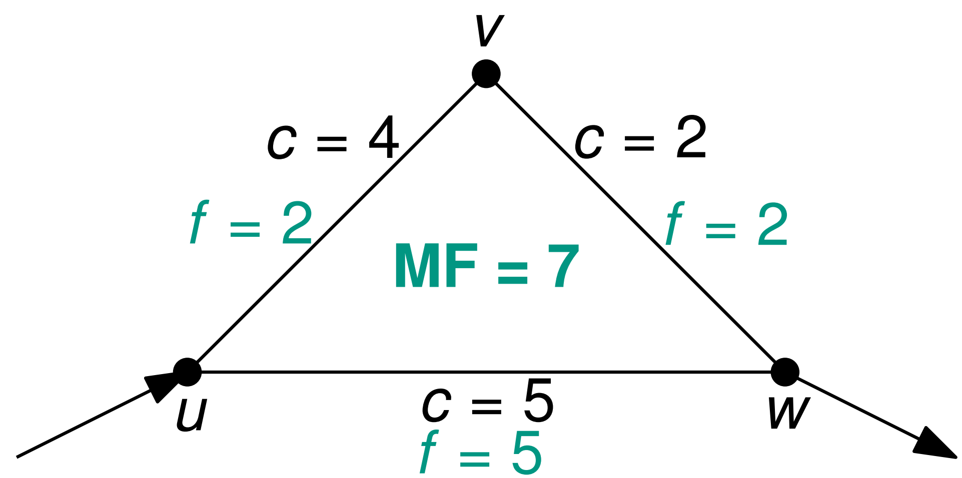


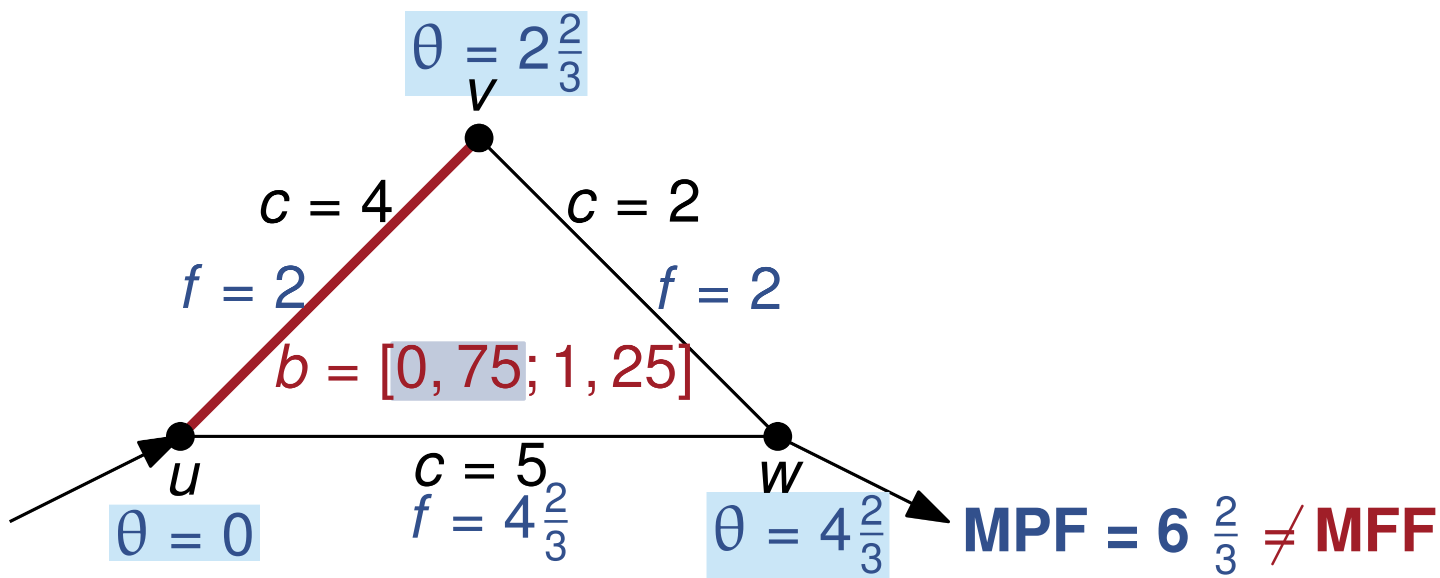
MPF = 7 = MFF

Institute of Theoretical Informatics
Algorithmics Group

Maximum FACTS Flow







Problem Statement: Maximum FACTS Flow

- maximum FACTS flow with k FACTS

$$\mathbf{MFF}(k) = \max_{E' \subseteq E, b} \mathbf{MPF}(G) \qquad |E'| = k$$

Institute of Theoretical Informatics
Algorithmics Group

Problem Statement: Maximum FACTS Flow

Karlsruhe Institute of Technology

- maximum FACTS flow with k FACTS

$$\mathbf{MFF}(k) = \max_{E' \subseteq E, b} \mathbf{MPF}(G) \qquad |E'| = k$$

- maximum FACTS flow

$$\mathbf{MFF}(G) = max_k \mathbf{MFF}(k)$$

Institute of Theoretical Informatics
Algorithmics Group

Problem Statement: Maximum FACTS Flow

Karlsruhe Institute of Technology

- maximum FACTS flow with k FACTS

$$\mathbf{MFF}(k) = \max_{E' \subseteq E, b} \mathbf{MPF}(G) \qquad |E'| = k$$

- maximum FACTS flow

$$\mathbf{MFF}(G) = max_k \mathbf{MFF}(k)$$

$$\forall (u, v) \in E \colon f(u, v) = \boxed{b(u, v)}(\boxed{\theta(v) - \theta(u)})$$

variables

Institute of Theoretical Informatics
Algorithmics Group

Problem Statement: Maximum FACTS Flow

- maximum FACTS flow with k FACTS

$$\mathbf{MFF}(k) = \max_{E' \subseteq E, b} \mathbf{MPF}(G) \qquad |E'| = k$$

- maximum FACTS flow

$$\mathbf{MFF}(G) = max_k \mathbf{MFF}(k)$$

$$\forall (u,v) \in E\colon f(u,v) = \boxed{b(u,v)}(\boxed{\theta(v) - \theta(u)})$$

variables

$$\boxed{b(u,v)}\boxed{\theta(v)} - \boxed{b(u,v)}\boxed{\theta(u)}$$

$$B(\boxed{b(u,v)}, \boxed{\theta(v)}) - B(\boxed{b(u,v)}, \boxed{\theta(u)})$$

$$\|$$

$$B(\boxed{b(u,v)}, \boxed{\theta(v)} - \boxed{\theta(u)})$$

Problem Statement: Maximum FACTS Flow

- maximum FACTS flow with k FACTS

$$\mathbf{MFF}(k) = \max_{E' \subseteq E, b} \mathbf{MPF}(G) \qquad |E'| = k$$

- maximum FACTS flow

$$\mathbf{MFF}(G) = max_k \mathbf{MFF}(k)$$

- minimum number of FACTS to reach the **MFF**

$$\mathbf{MNF}_{\mathbf{MFF}}(G) = \min_{\mathbf{MFF}(k) = \mathbf{MFF}(G)} k$$

Institute of Theoretical Informatics
Algorithmics Group

Relationship Optimal iFACTS Flow (OiFF)

physical model
(AC linearization)

flow model

Relationship Optimal iFACTS Flow (OiFF)

physical model
(AC linearization)

flow model

How to approach the graph theoretical flow?

Institute of Theoretical Informatics
Algorithmics Group

Relationship Optimal iFACTS Flow (OiFF)

physical model
(AC linearization)

flow model

How to approach the graph theoretical flow?

optimal iFACTS flow

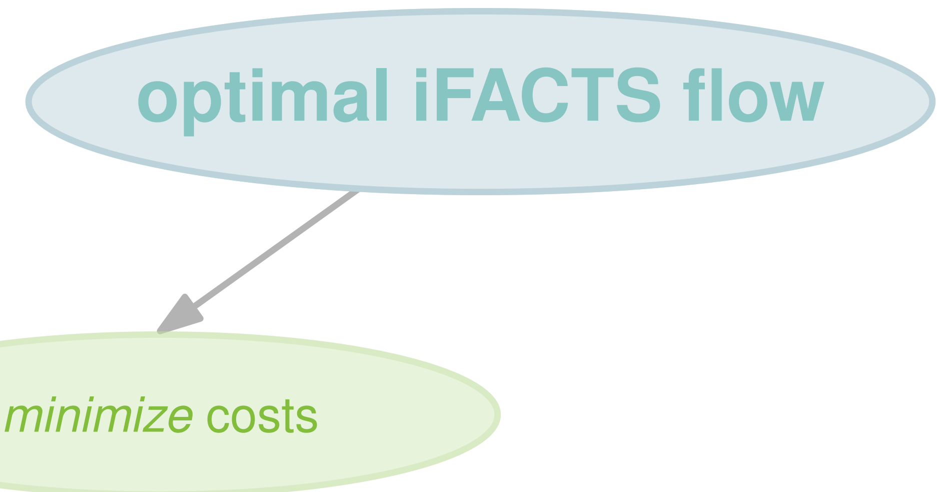
Institute of Theoretical Informatics
Algorithmics Group

Relationship Optimal iFACTS Flow (OiFF)

physical model
(AC linearization)

flow model

How to approach the graph theoretical flow?

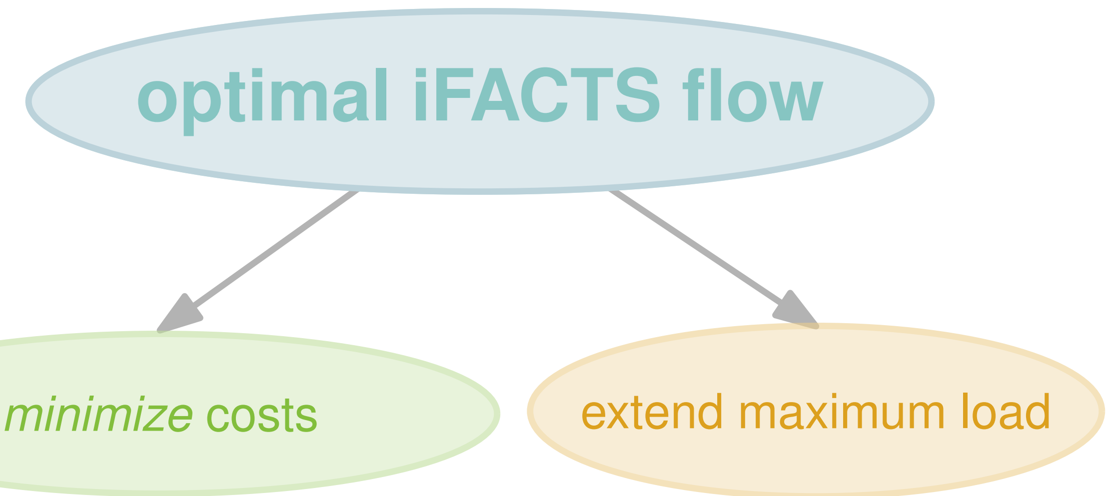


Relationship Optimal iFACTS Flow (OiFF)

physical model
(AC linearization)

flow model

How to approach the graph theoretical flow?



Relationship Optimal iFACTS Flow (OiFF)

physical model
(AC linearization)

flow model

How to approach the graph theoretical flow?

optimal iFACTS flow

minimize costs

extend maximum load

number of FACTS between
FFS and **FCS**

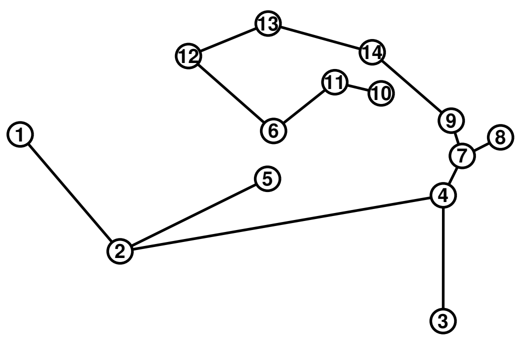
Institute of Theoretical Informatics
Algorithmics Group

Relationship Optimal iFACTS Flow (OiFF)

physical model
(AC linearization)

flow model

How to approach the graph theoretical flow?



optimal iFACTS flow

minimize costs

extend maximum load

number of FACTS between
FFS and **FCS**

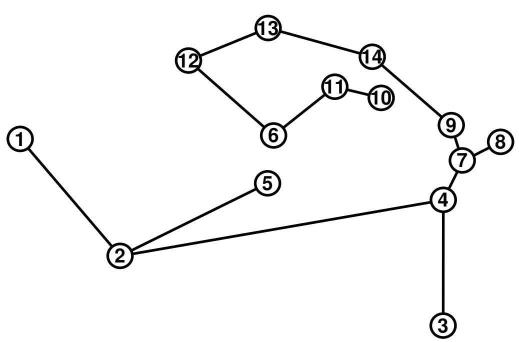
Institute of Theoretical Informatics
Algorithmics Group

Relationship Optimal iFACTS Flow (OiFF)

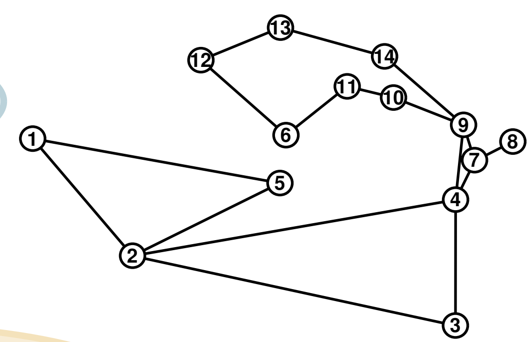
physical model
(AC linearization)

flow model

How to approach the graph theoretical flow?



optimal iFACTS flow



minimize costs

extend maximum load

number of FACTS between
FFS and **FCS**

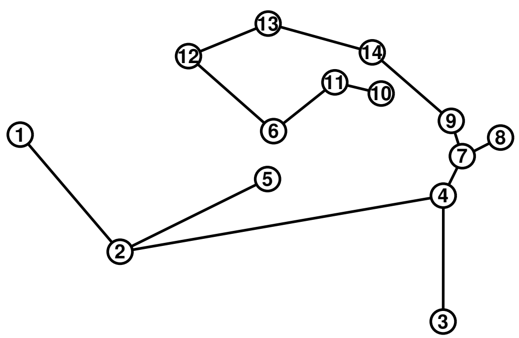
cacti, trees and series-parallel-graphs

Relationship Optimal iFACTS Flow (OiFF)

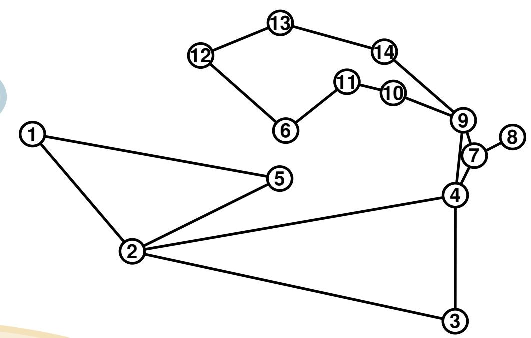
physical model
(AC linearization)

flow model

upper
lower
bound



optimal iFACTS flow



minimize costs

extend maximum load

number of FACTS between
FFS and **FCS**

cacti, trees and series-parallel-graphs

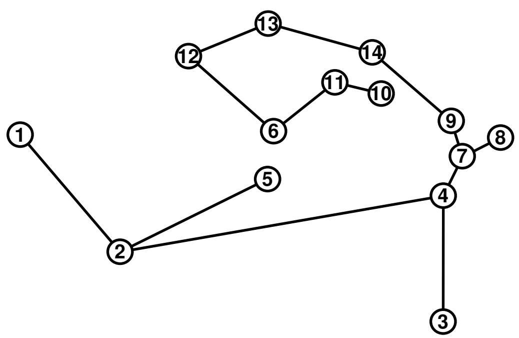
Relationship Optimal iFACTS Flow (OiFF)

physical model
(AC linearization)

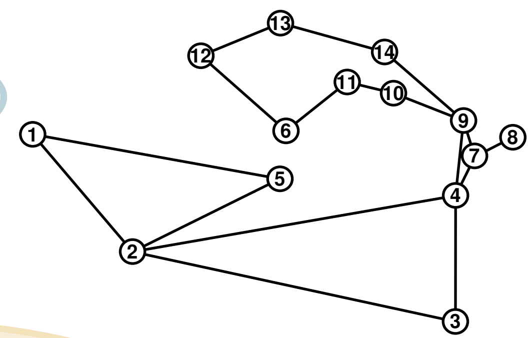
upper
lower
bound

flow model

upper
lower
bound



optimal iFACTS flow



minimize costs

extend maximum load

number of FACTS between
FFS and **FCS**

cacti, trees and series-parallel-graphs

Institute of Theoretical Informatics
Algorithmics Group

Relationship Optimal iFACTS Flow (OiFF)

Karlsruhe Institute of Technology

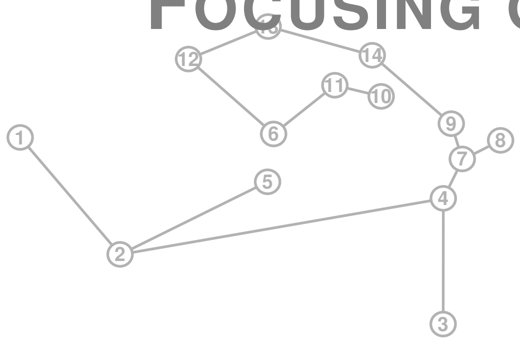
physical model
(AC linearization)

upper
lower
bound

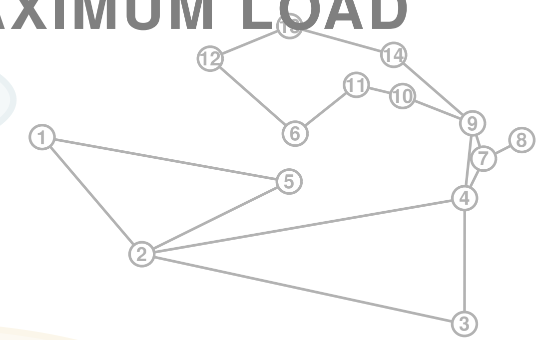
flow model

upper
lower
bound

FOCUSING ON THE EXTENSION OF THE MAXIMUM LOAD



optimal iFACTS flow



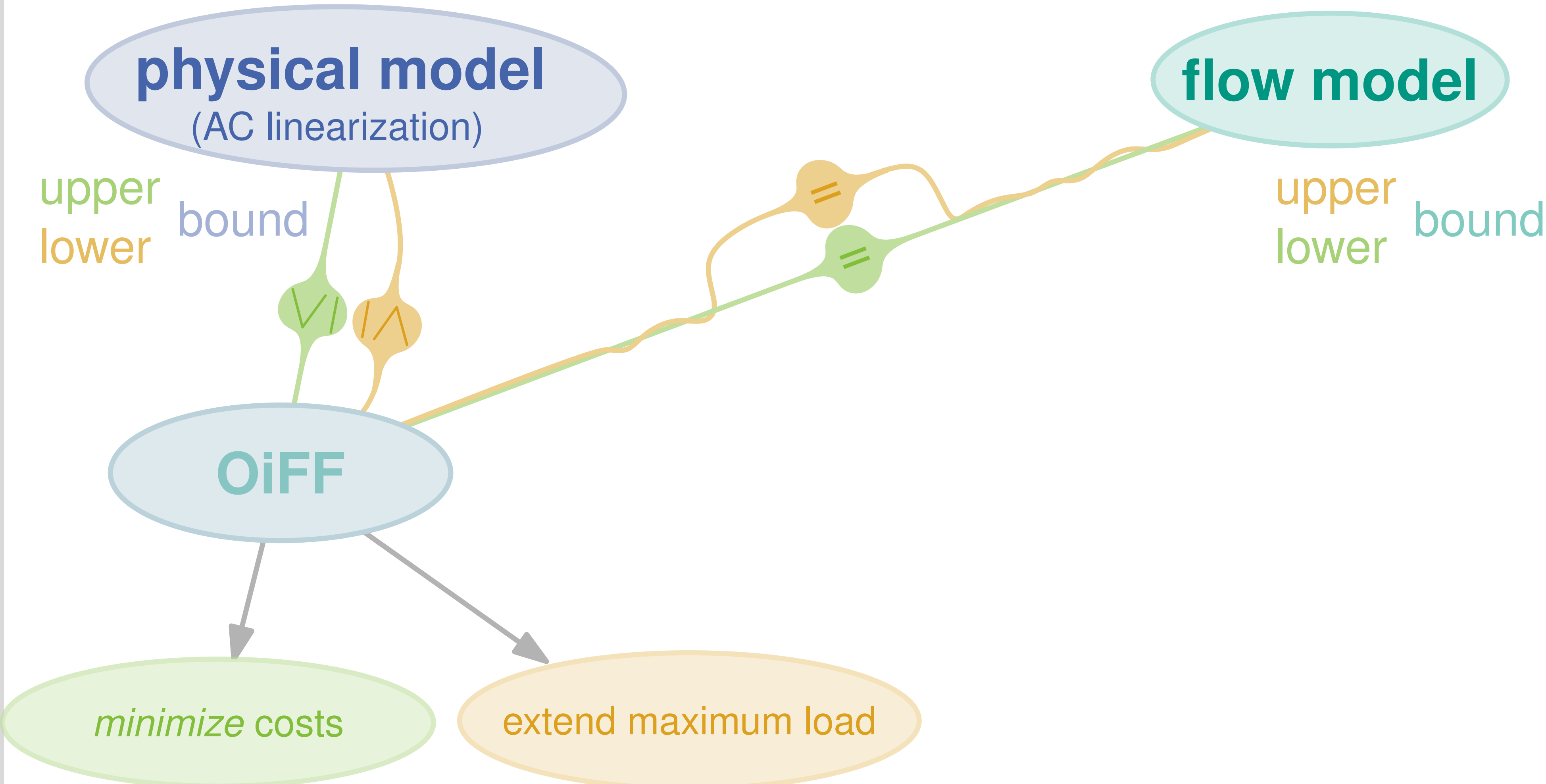
minimize costs

extend maximum load

number of FACTS between
FFS and **FCS**

cacti, trees and series-parallel-graphs

Conclusion

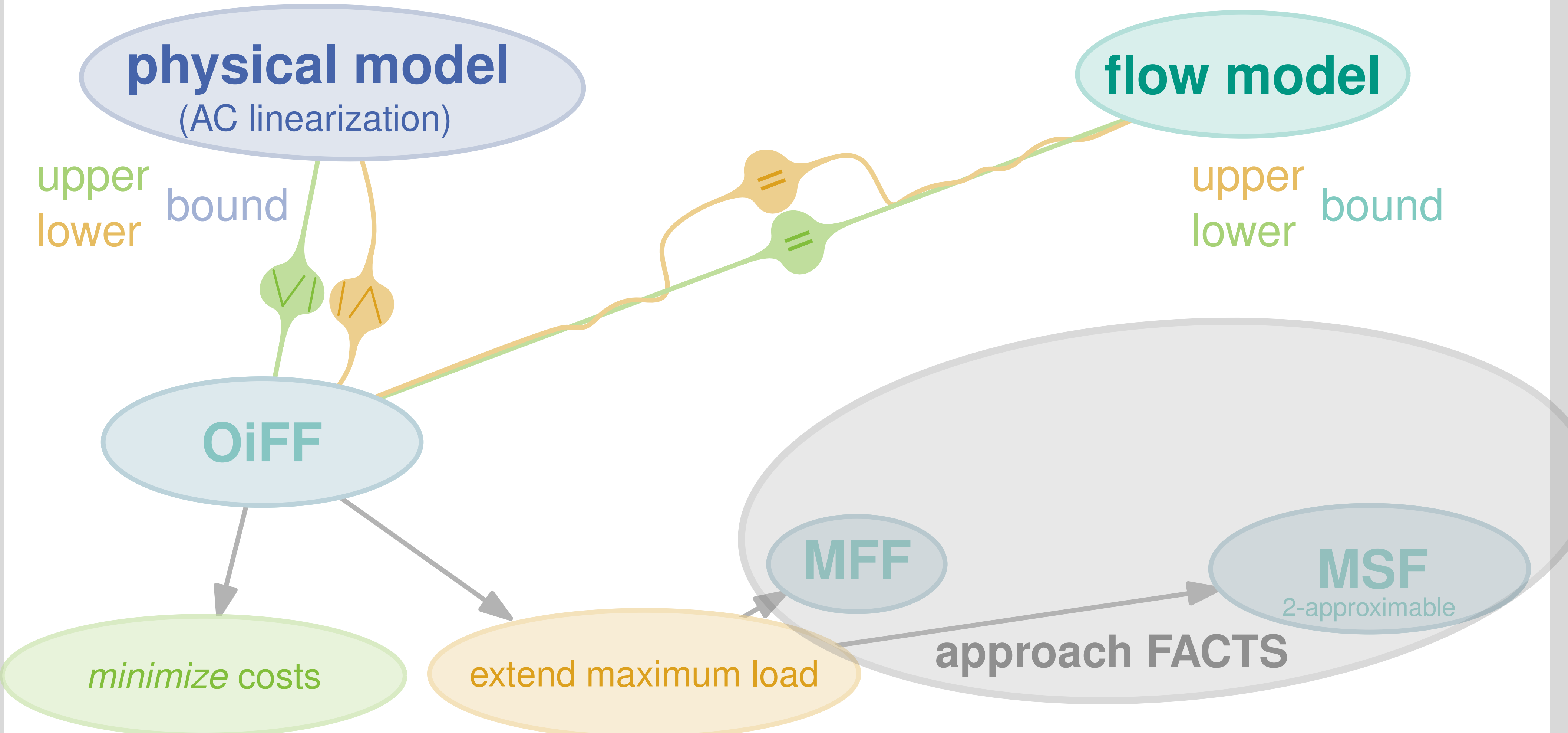


→ number of FACTS between **FFS** and **FCS**

→ cacti, trees and series-parallel-graphs

Institute of Theoretical Informatics
Algorithmics Group

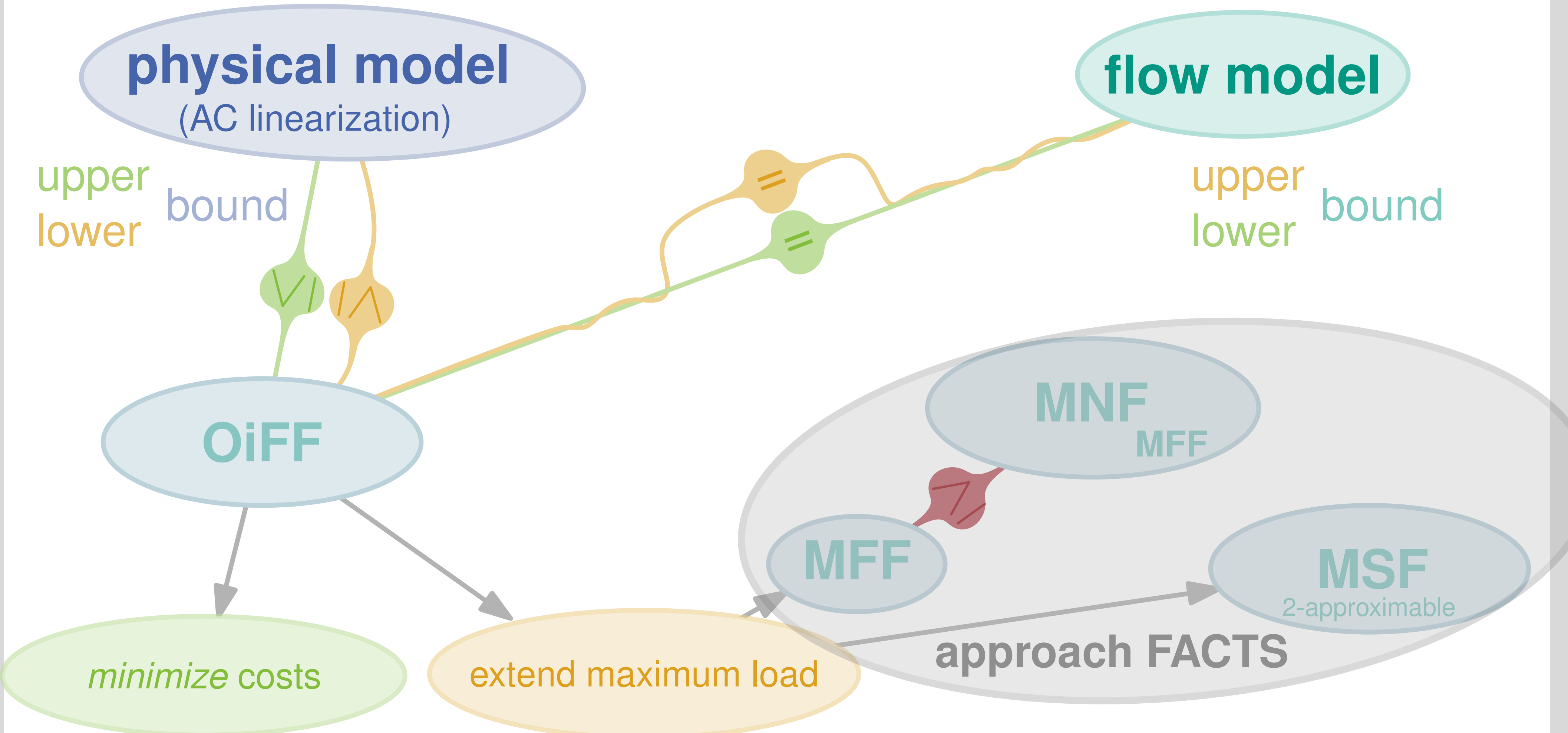
Conclusion



→ number of FACTS between **FFS** and **FCS**
→ cacti, trees and series-parallel-graphs

Institute of Theoretical Informatics
Algorithmics Group

Conclusion

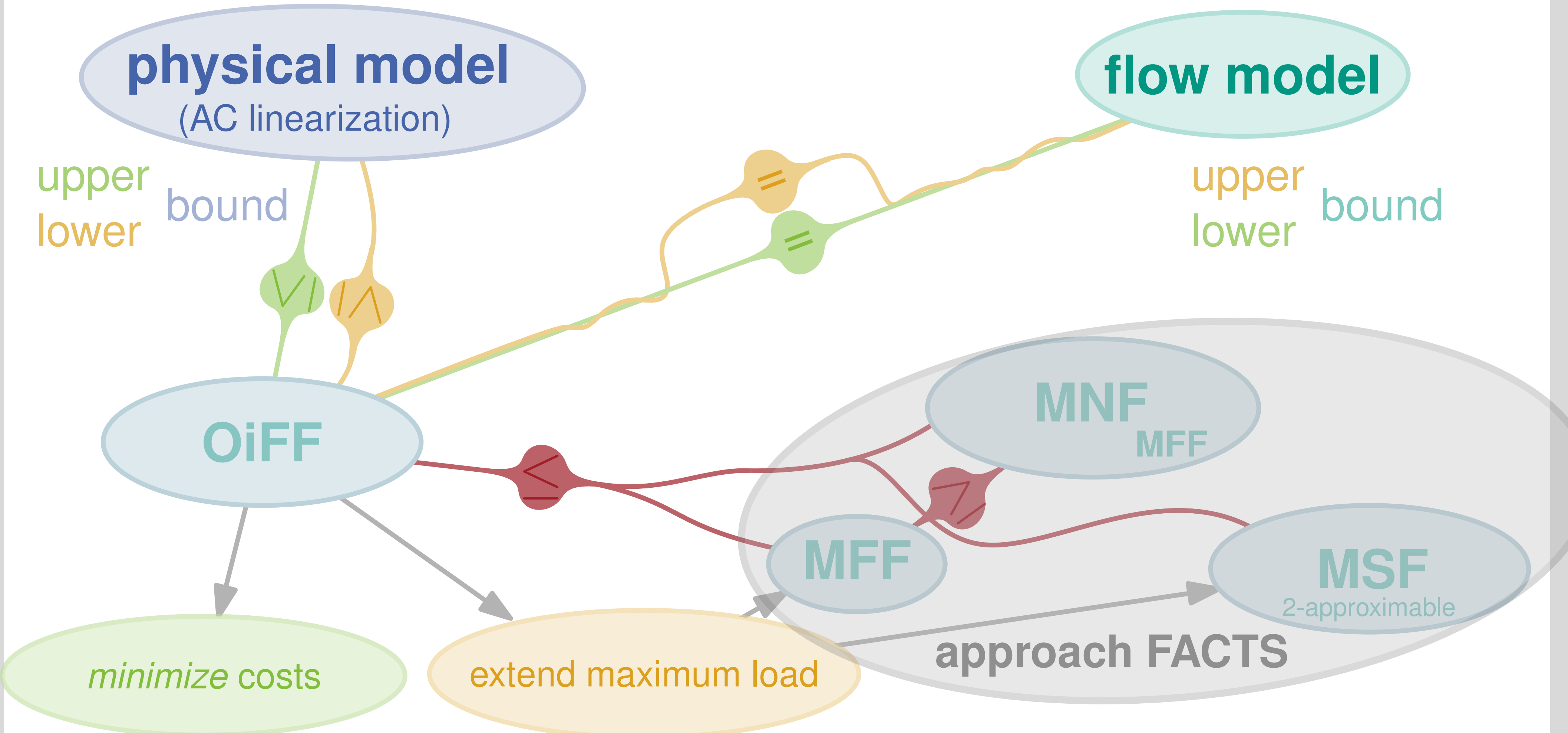


→ number of FACTS between **FFS** and **FCS**

→ cacti, trees and series-parallel-graphs

Institute of Theoretical Informatics
Algorithmics Group

Conclusion

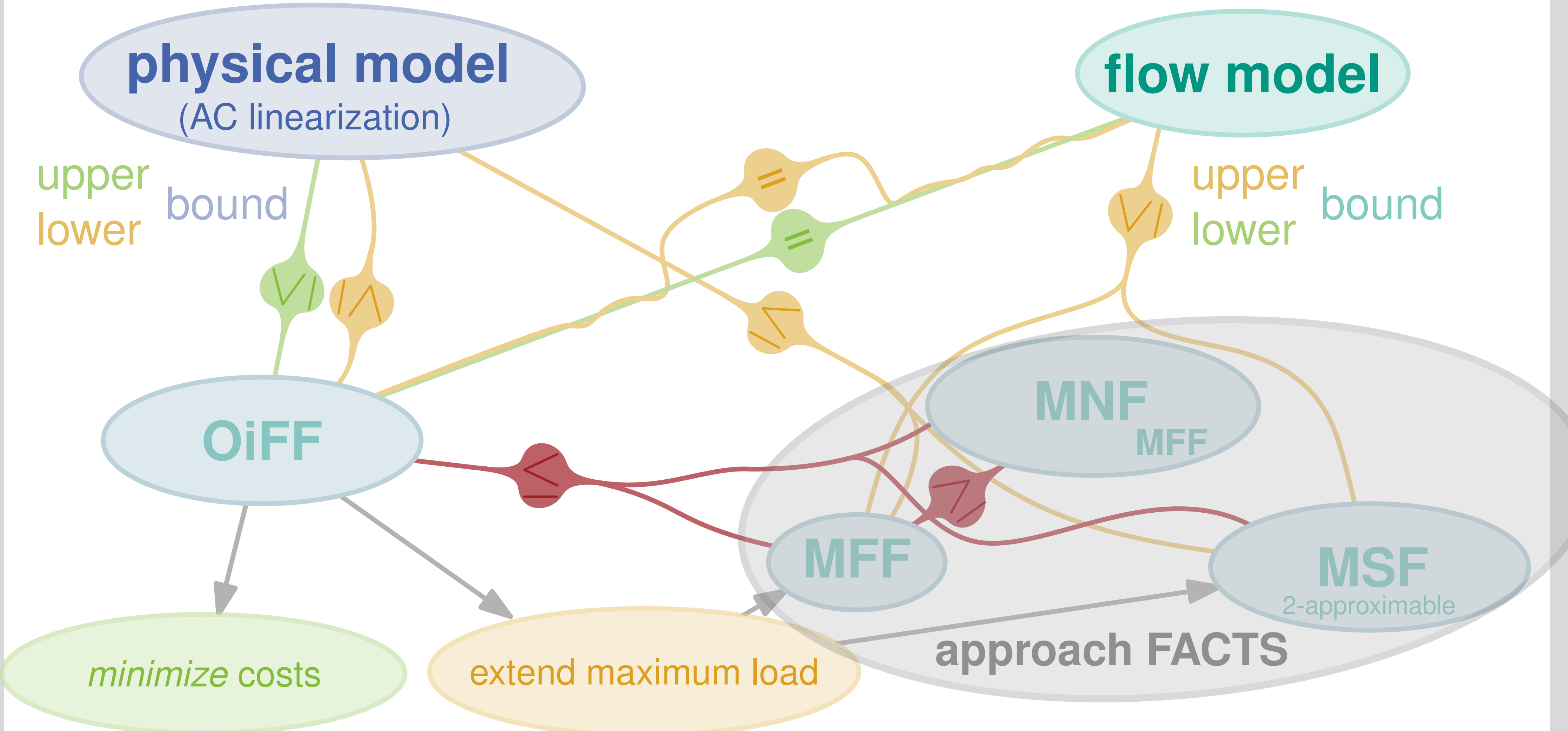


→ number of FACTS between **FFS** and **FCS**

→ cacti, trees and series-parallel-graphs

Institute of Theoretical Informatics
Algorithmics Group

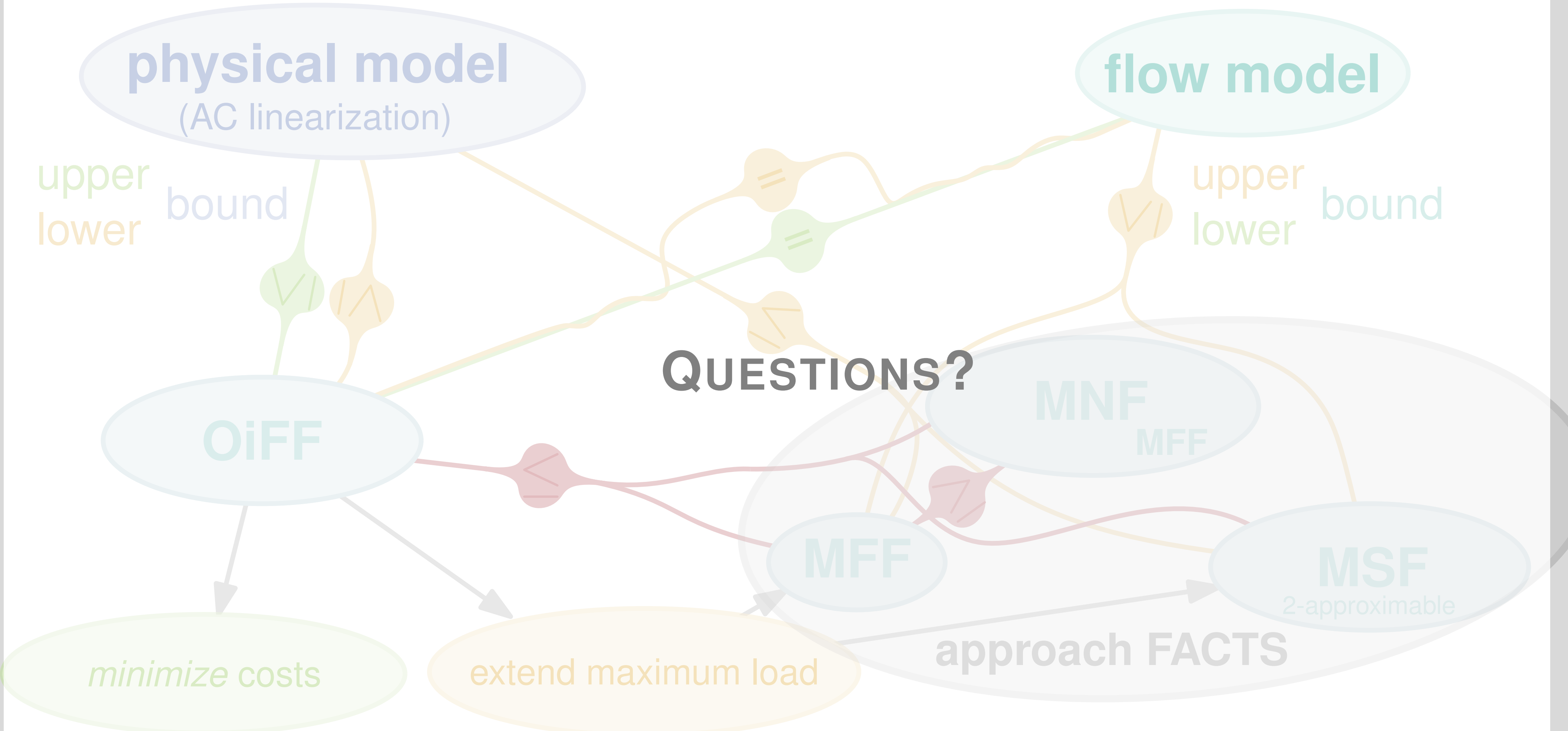
Conclusion



→ number of FACTS between **FFS** and **FCS**

→ cacti, trees and series-parallel-graphs

Conclusion



QUESTIONS?

→ number of FACTS between **FFS** and **FCS**

→ cacti, trees and series-parallel-graphs

Institute of Theoretical Informatics
Algorithmics Group