ROMA TRE
UNIVERSITÀ DEGLI STUDI
Dipartimento di Fisica “E. Amaldi”

A Monte Carlo code for accreting sources

Francesco Tamborra

supervisor: Giorgio Matt **in collaboration with Stefano Bianchi**

Outline

- The physics of AGN
- The Monte Carlo approach
- The code
- Some preliminary results
- Future developments

WORK IN PROGRESS

The physics of AGN – The Unified Model

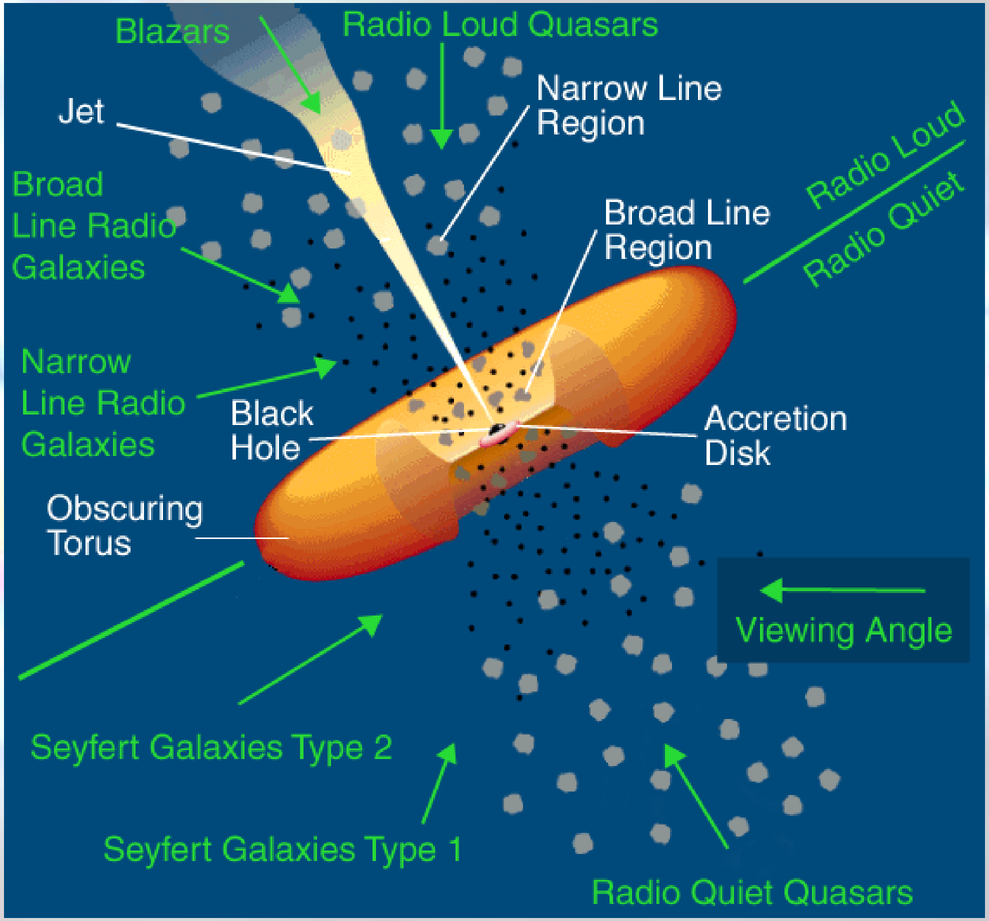
$L \approx 10^{42} - 10^{48}$ erg/s

SMBH $\approx 10^{6} - 10^{8}\ M_{\odot}$

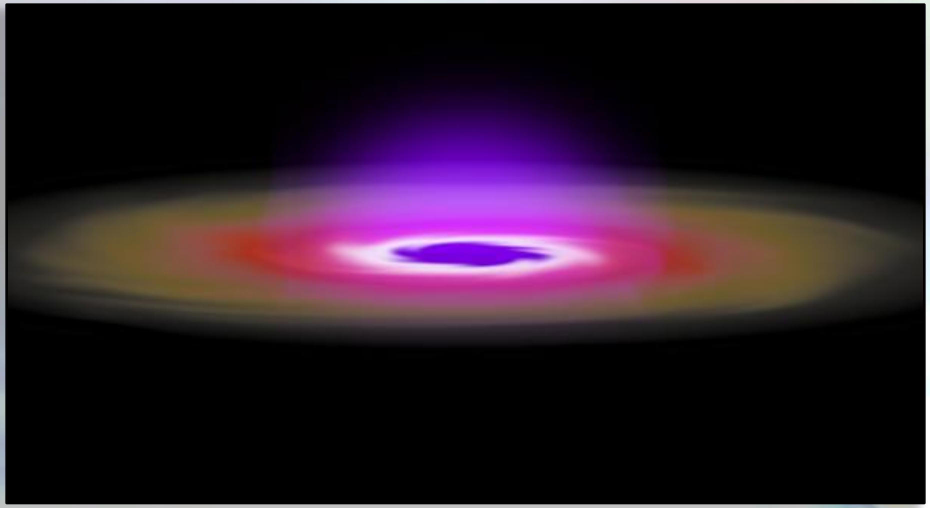
$L = \eta \cdot M_{dot} \cdot c^2$

$\eta \approx 0.06 - 0.4$
$(\eta_{pp} \approx 0.007)$

The Unified Model
Antonucci, 1993



The physics of AGN – The accretion disc



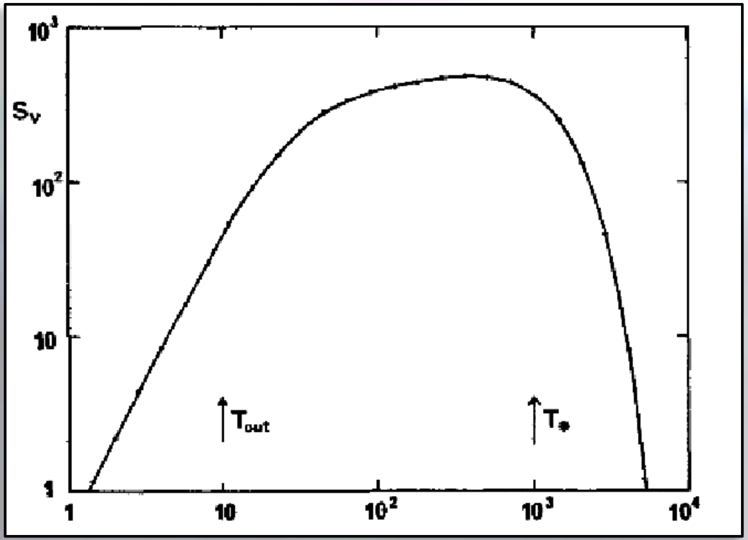
Optically thick, geometrically thin disc

$$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')\, ds'.$$

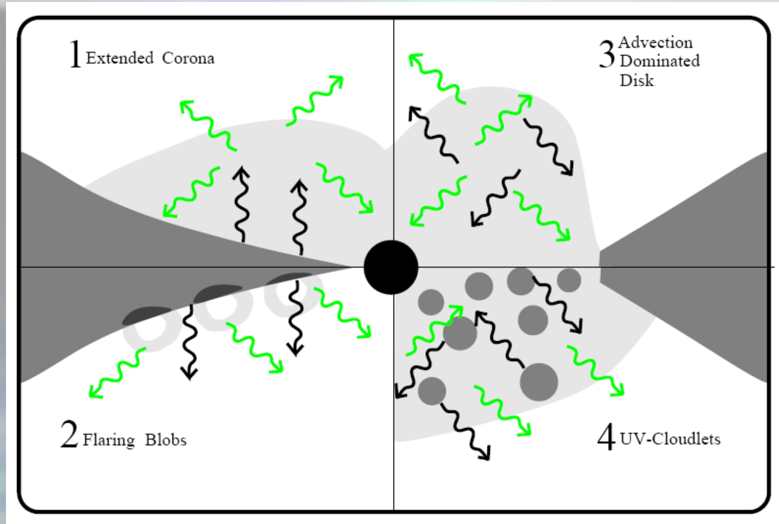
$$l_\nu = \frac{1}{\alpha_\nu} = \frac{1}{n\sigma_\nu}.$$

Shakura & Sunyaev, 1973

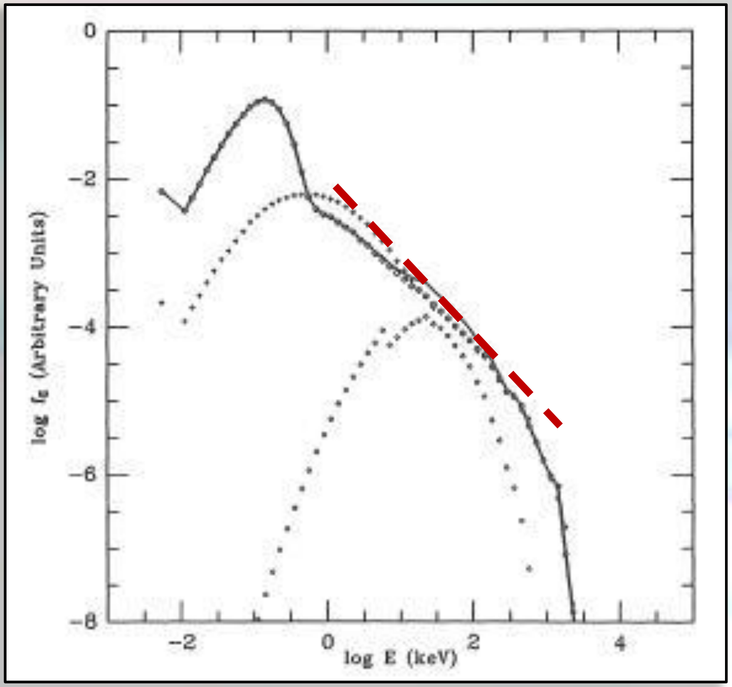
$$T_S(R) = \left[\frac{3GM\dot{m}}{8\pi R^3\sigma}\left(1 - \sqrt{\frac{R_0}{R}}\right)\right]^{\frac{1}{4}}$$



The physics of AGN – The X-rays production



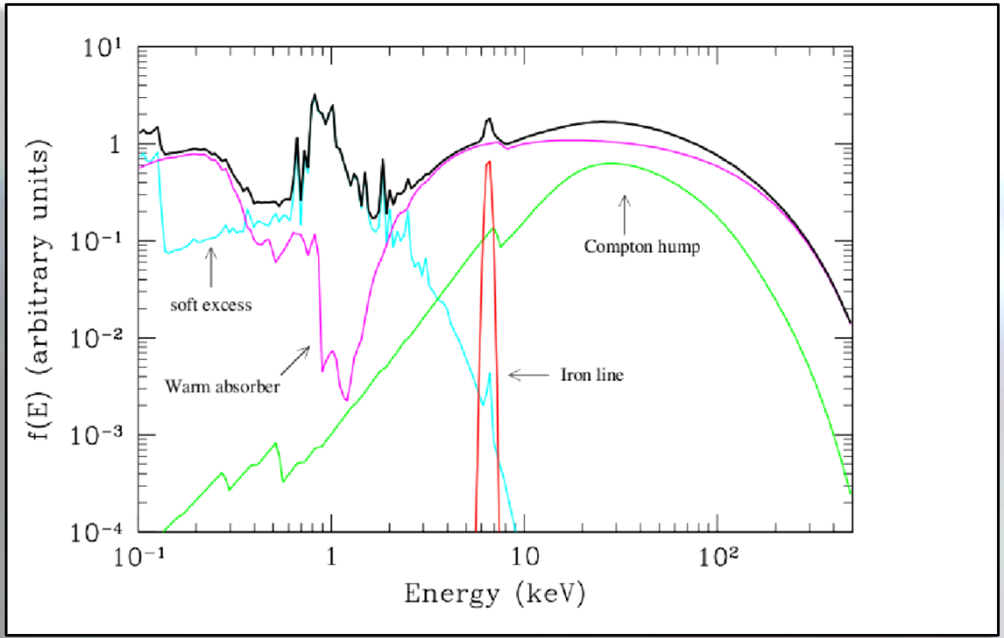
Haardt & Maraschi, 1991



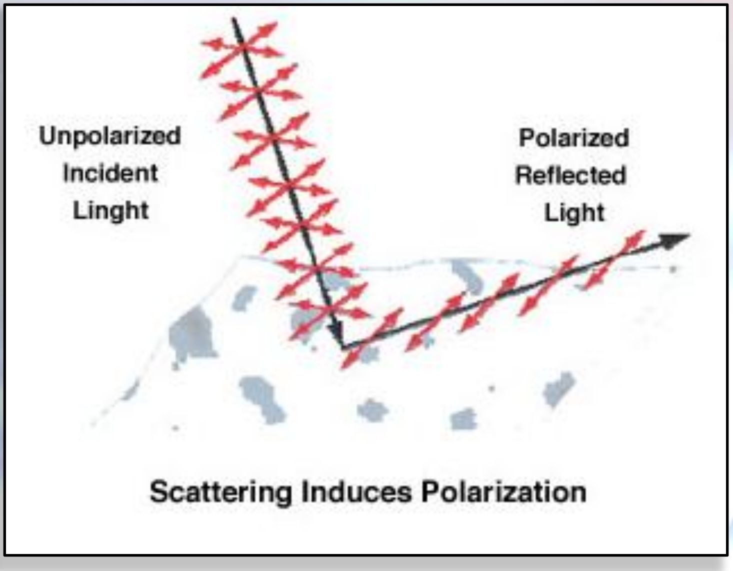
$\tau = 0.1$

$kT_e \approx 250\ KeV$

$kT_{BB} = 50\ eV$



The physics of AGN – The polarization



Stokes parameters

$$I \equiv \mathcal{E}_1^2 + \mathcal{E}_2^2 = \mathcal{E}_0^2$$
$$Q \equiv \mathcal{E}_1^2 - \mathcal{E}_2^2 = \mathcal{E}_0^2 \cos 2\beta \cos 2\chi$$
$$U \equiv 2\mathcal{E}_1 \mathcal{E}_2 \cos(\phi_1 - \phi_2) = \mathcal{E}_0^2 \cos 2\beta \sin 2\chi$$
$$V \equiv 2\mathcal{E}_1 \mathcal{E}_2 \sin(\phi_1 - \phi_2) = \mathcal{E}_0^2 \sin 2\beta.$$

$$I^2 = Q^2 + U^2 + V^2$$

Degree of polarization

$$\Pi \equiv \frac{I_{\rm pol}}{I} = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}.$$

Degree of polarization & polarization angle for linearly polarized photons

$$\Pi = \frac{\sqrt{Q^2 + U^2}}{I},$$

$$\chi = \tfrac{1}{2} \arctan \frac{U}{Q}.$$

The Monte Carlo approach

by inversion

$$R = \frac{\int_{t_1}^{t} f(t')\,dt'}{\int_{t_1}^{t_2} f(t')\,dt'}.$$

$\Rightarrow$ $t(R)$

by rejecton method
(von Neumann)

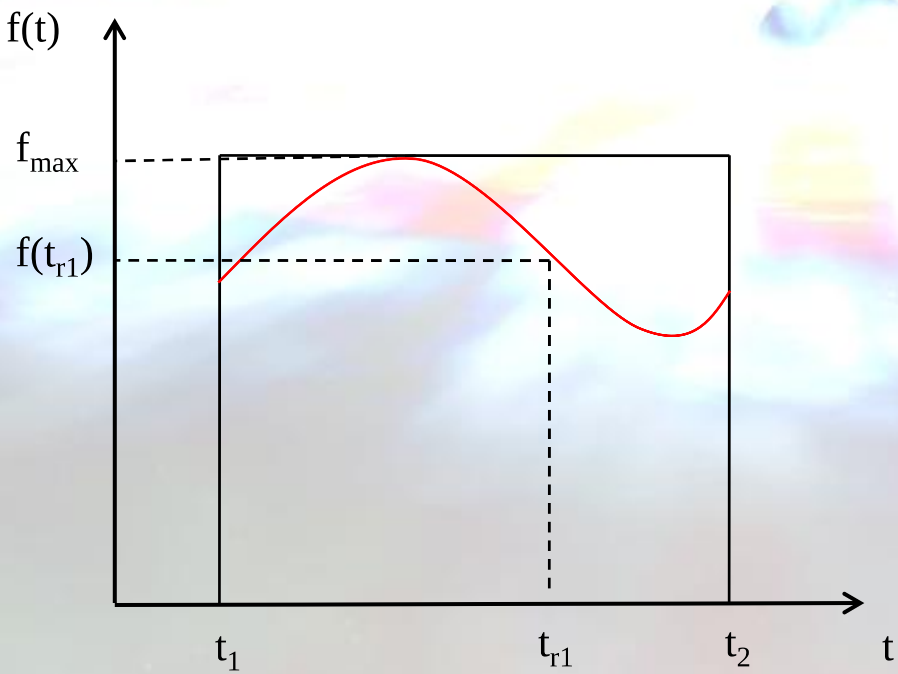
- calculate the maximum of the distribution, f_{max}
- extract a random value, t_{r1}, between t_1 and t_2
- extract a random number, r_2, between 0 and 1

IF $(r_2 * f_{max}) < f(t_{r1})$ THEN

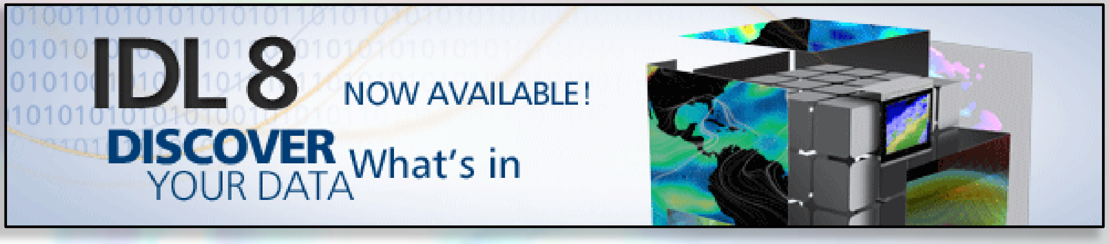
- accept t_{r1} as a realization of $f(t)$

ELSE

- reject and repeat the sampling



The Code




```
main.pro

    ShakSun.pro
    Chandra.pro
    Init_Direction.pro
    Renorm.pro
    Stokes.pro
    ControllerDisc.pro
    MFP.pro
    Controllers.pro
    MaxRel.pro
    Lorentz.pro
    CrossSec.pro
    InvComp.pro
    Sdriection.pro
```

`vector oriented, image processing`

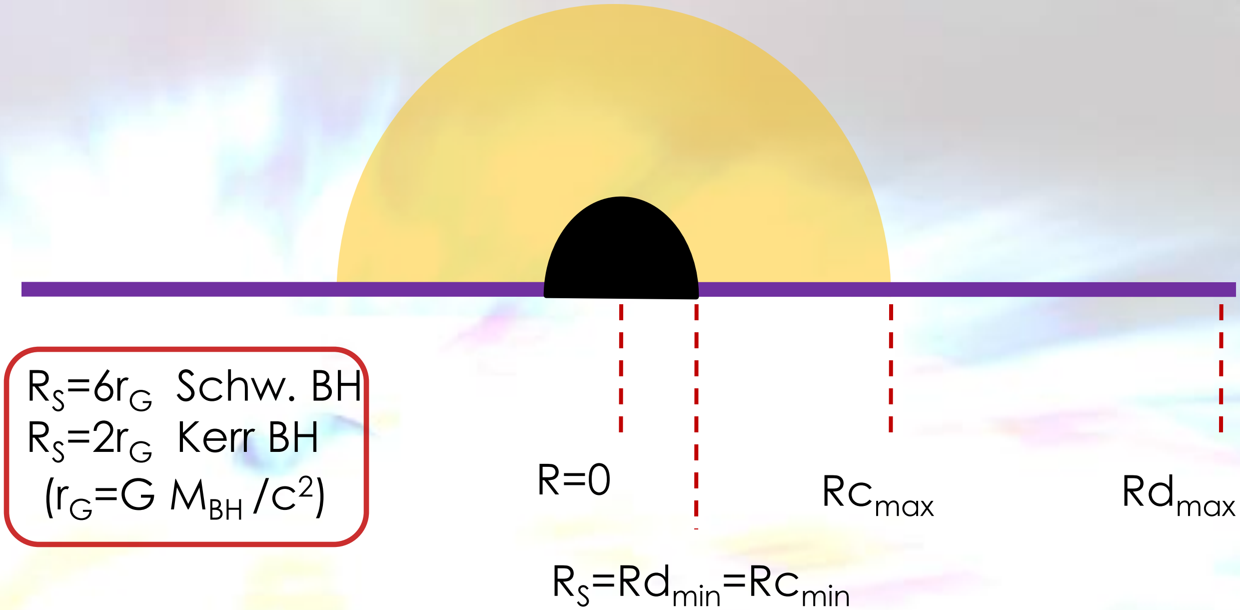
- modular
- fully special-relativistic

The Code – Initial parameters

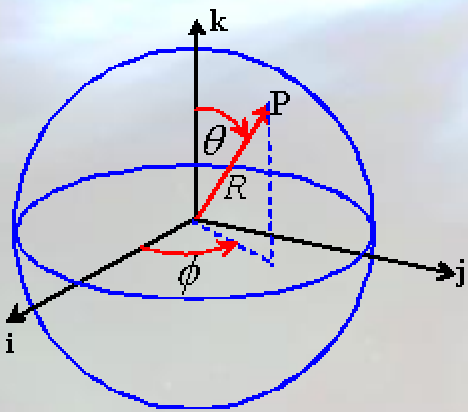
Input parameters:

- $M_{BH} = 10^8 M_{\odot}$
- $M_{dot} \approx 10^{-8} M_{\odot} / yr$
- $\tau = (0.1 - 10)$
- $(n_e = \tau / Rc_{max} \cdot \sigma_T)$
- $kT_e = (50 - 250)$ *KeV*

The geometry:



photons loop



Initial position:

$\varphi_{0(d)} \in [0, 2\pi]$

$\theta_{0(d)} = \pi/2$

$R_{0(d)} \in [Rd_{min}, Rd_{max}]$ *

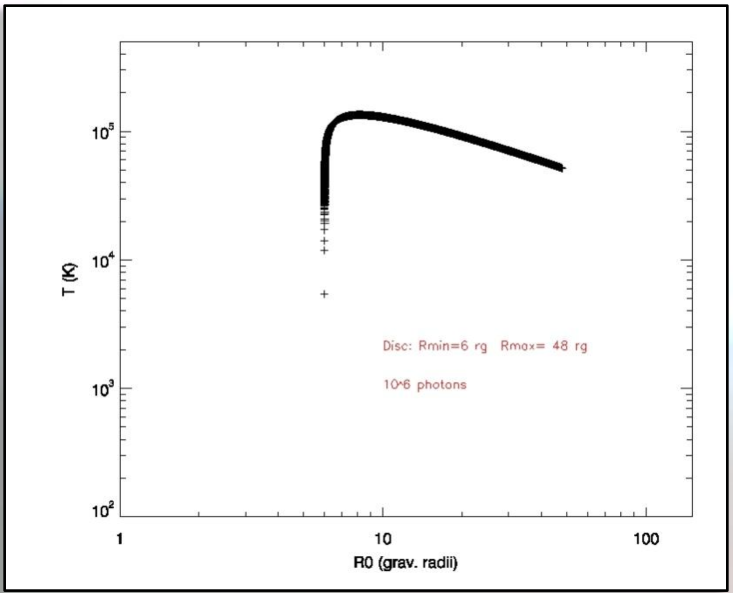
The Code – Initial energy

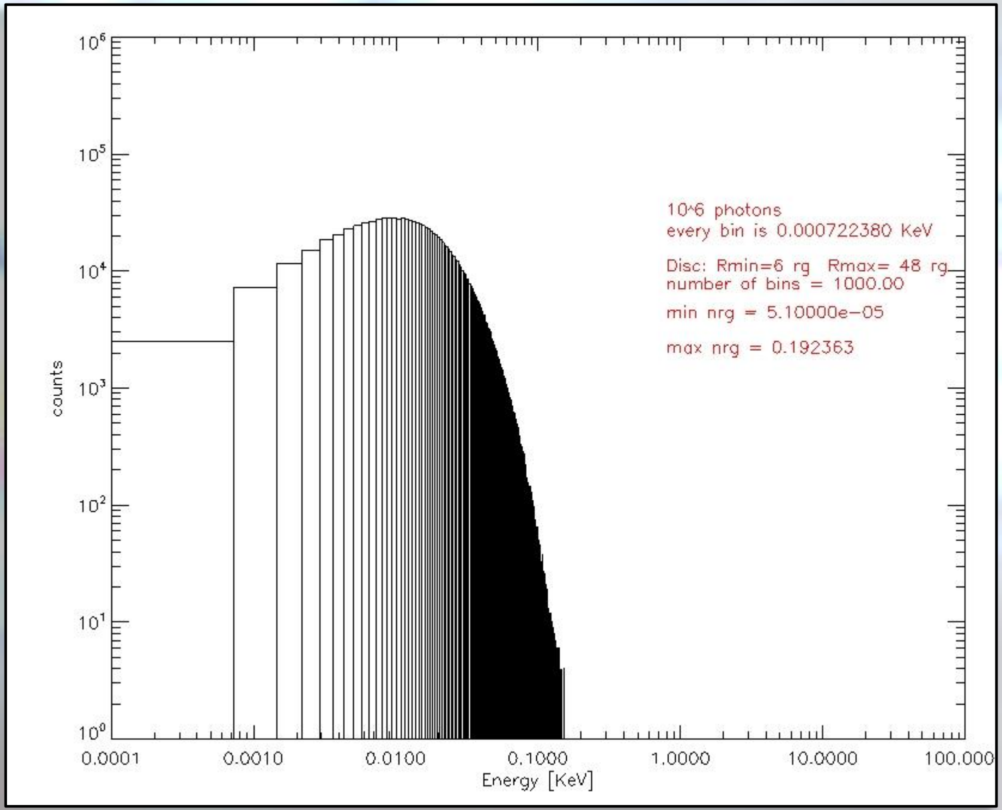
ShakSun.pro

$$T_S(R) = \left[\frac{3GM\dot{m}}{8\pi R^3 \sigma}\left(1 - \sqrt{\frac{R_0}{R}}\right)\right]^{\frac{1}{4}}$$

$, R_0 = Rd_{min}$

$T_{max} = (49/36) Rd_{min}$





The Code – Initial direction

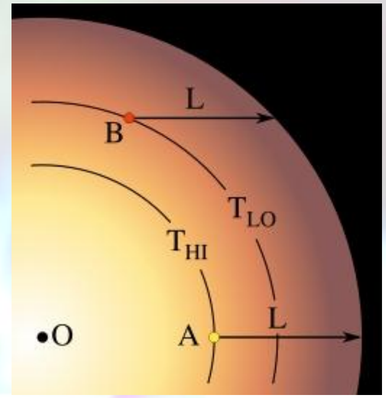
```
Chandra.pro
Init_Direction.pro
```

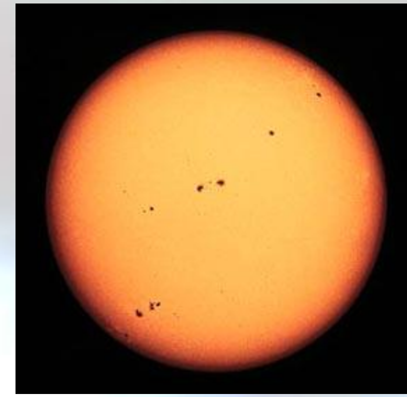
$\varphi_{d(d)} = \text{ran}\cdot 2\pi$

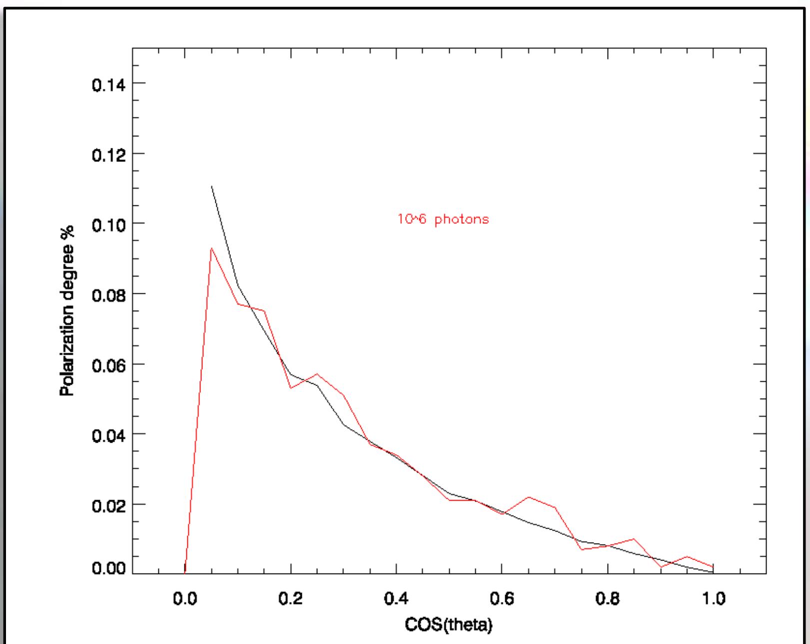
$\theta_{d(d)}$ extracted according to Chandrasekhar limb darkening law

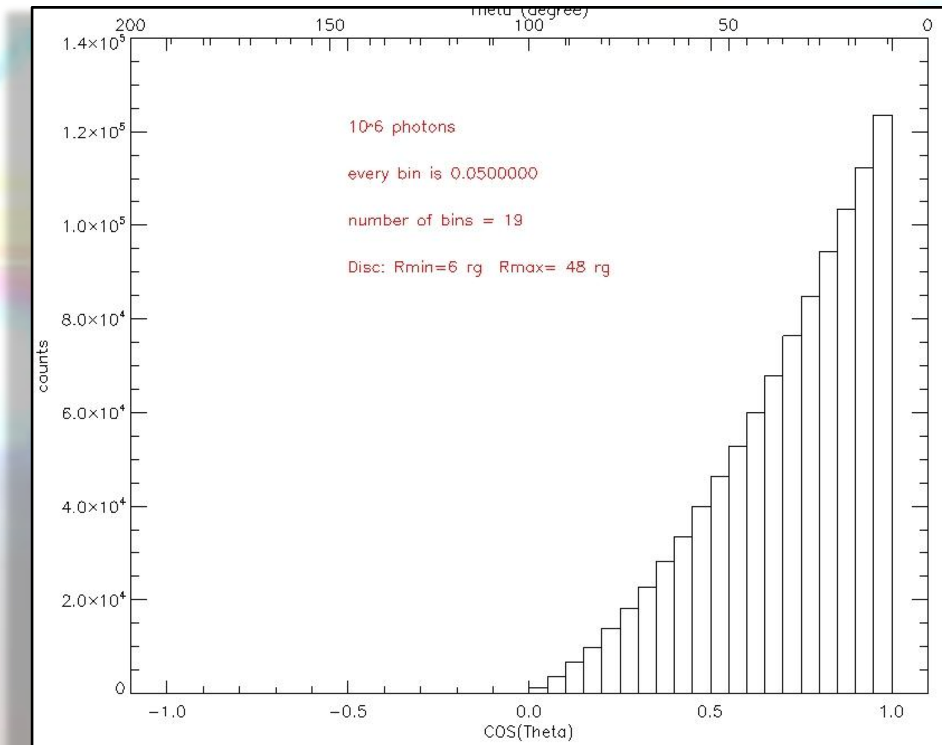
$$F(\mu) \approx \mu\cdot(1 + 2.06\cdot\mu)$$

$$\mu = \cos(\theta_{d(d)})$$









The Code – Initial polarization

Stokes.pro

IF ran < pol THEN

$$P_x = \frac{1}{|\vec{P}|}\{D_x \sin\Theta \cos\Phi - P_{x,0}\}$$

$$P_y = \frac{1}{|\vec{P}|}\{D_y \sin\Theta \cos\Phi - P_{y,0}\}$$

$$P_z = \frac{1}{|\vec{P}|}\{D_z \sin\Theta \cos\Phi - P_{z,0}\}.$$

ELSE

random polarization

$$\vec{P} = \frac{1}{|\vec{P}|}(\vec{P}_0 \times \vec{D}) \times \vec{D},$$

Angel, 1969

Matt et al., 1996

$$\vec{Q}_+ = \frac{1}{\sqrt{1-D_z^2}}(-D_y, D_x, 0)$$

$$\vec{Q}_- = \frac{1}{\sqrt{1-D_z^2}}(-D_xD_z, -D_yD_z, 1-D_z^2)$$

$$\vec{U}_+ = \frac{1}{\sqrt{2(1-D_z^2)}}(-D_y - D_xD_z, D_x - D_yD_z, 1-D_z^2)$$

$$\vec{U}_- = \frac{1}{\sqrt{2(1-D_z^2)}}(D_y - D_xD_z, -D_x - D_yD_z, 1-D_z^2)$$

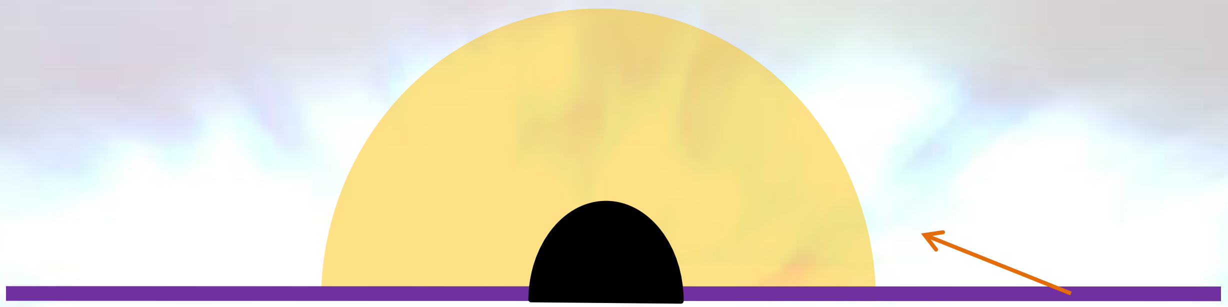
Stokes parameters for the j-th photon

$$Q_j = (\vec{Q}_+ \cdot \vec{P}_j)^2 - (\vec{Q}_- \cdot \vec{P}_j)^2$$

$$U_j = (\vec{U}_+ \cdot \vec{P}_j)^2 - (\vec{U}_- \cdot \vec{P}_j)^2.$$

The Code – Controllers & Relativistic Maxwell-Boltzmann distribution

ControllerDisc.pro



---------------------- scattering loop ----------------------

MaxRel.pro

$$f(\gamma) = \frac{y}{4\pi K_2(y)} e^{-y\gamma}, \quad y = \frac{m_e c^2}{kT_e},$$

Poutanen & Vilhu, 1993

$\beta = 1 - (1 / \gamma^2) \Rightarrow \beta_x, \beta_y, \beta_z$

The Code – Lorentz boost

`Lorentz.pro`

$$\Lambda = \begin{bmatrix} \gamma & -\beta_x\gamma & -\beta_y\gamma & -\beta_z\gamma \\ -\beta_x\gamma & 1+(\gamma-1)\frac{\beta_x^2}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_y}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_z}{\beta^2} \\ -\beta_y\gamma & (\gamma-1)\frac{\beta_y\beta_x}{\beta^2} & 1+(\gamma-1)\frac{\beta_y^2}{\beta^2} & (\gamma-1)\frac{\beta_y\beta_z}{\beta^2} \\ -\beta_z\gamma & (\gamma-1)\frac{\beta_z\beta_x}{\beta^2} & (\gamma-1)\frac{\beta_z\beta_y}{\beta^2} & 1+(\gamma-1)\frac{\beta_z^2}{\beta^2} \end{bmatrix}$$

$k_{0(d)} = (2\pi\nu/c^2\ , k_x, k_y, k_z)$ ⇒ $k_{0(e)} = \Lambda\, k_{0(d)}$ $\quad (\nu, \theta_d, \varphi_d)_{(e)}$

$p_{0(d)} = (0\ , p_x, p_y, p_z)$ ⇒ $p_{temp(e)} = \Lambda\, p_{0(d)}$

$$f' = \left(0, f'^i - \frac{f'^0 k'^i}{k'^0}\right)$$

Connors et al., 1980

$$p_{j(e)} = p_{temp\ j+1(e)} - (p_{temp\ 0(e)} \cdot k_{j+1(e)} / k_{0(e)})$$

The Code – Cross Section & Mean Free Path

CrossSec.pro

Klein-Nishina total cross section

$$\sigma=\sigma_T\cdot\frac{3}{4}\left[\frac{1+x}{x^3}\left\{\frac{2x(1+x)}{1+2x}-\ln(1+2x)\right\}+\frac{1}{2x}\ln(1+2x)-\frac{1+3x}{(1+2x)^2}\right]$$

$, x = h\nu_{(e)} / m_e c^2$

$$\sigma\approx\sigma_T\left(1-2x+\frac{26x^2}{5}+\cdots\right),\qquad x\ll 1,$$

MFP.pro

$l = (\sigma_{KN} \cdot n_e)^{-1}$

$P = e^{-s/l} \Rightarrow s, \theta_{d(d)}, \varphi_{d(d)}$

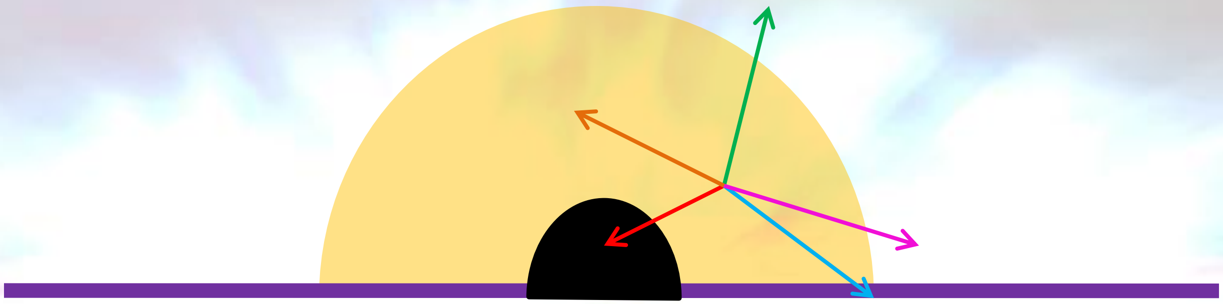
starting point

arrival point

$$\left\{\begin{array}{l}\varphi_{0(d)}\\ \theta_{0(d)}\\ R_{0(d)}\end{array}\right. \Rightarrow \left\{\begin{array}{l}\varphi'_{(d)}\\ \theta'_{(d)}\\ R'_{(d)}\end{array}\right.$$

The Code – Controllers

Controllers.pro



the photon can:

- fall in the BH
- fall on the disc
- fall on the disc, in future
- escape

- continue its travel in the corona

The Code – Inverse Compton

InvComp.pro

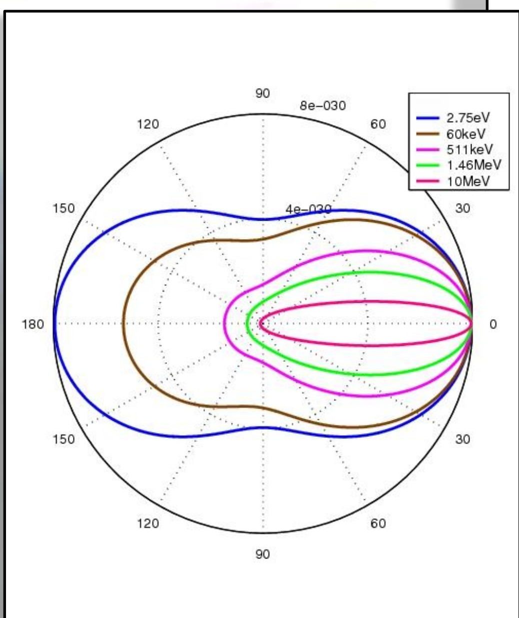
$$\frac{d\sigma_{\mathrm{KN,U}}}{d\Omega} = \frac{1}{2} r_0^2 \beta^2 [\beta + \beta^{-1} - \sin^2\Theta],$$

$$\beta = \frac{E}{E_0} = \frac{1}{1 + (E_0/m_e c^2)(1 - \cos\Theta)}.$$

θ_{scatt}

$$R = \frac{\int_{t_1}^{t} f(t')\,dt'}{\int_{t_1}^{t_2} f(t')\,dt'}.$$

$$R = \left(\frac{\mathrm{Eps0}\ (-1 + \mathrm{mu})\ (4 + \mathrm{Eps0}\ (10 - 6\,\mathrm{mu} + \mathrm{Eps0}\ (\mathrm{Eps0} + 2\ (-2 + \mathrm{mu})^2 - \mathrm{Eps0}\,\mathrm{mu})))}{(1 + \mathrm{Eps0} - \mathrm{Eps0}\,\mathrm{mu})^2} + (4 - 2\ (-2 + \mathrm{Eps0})\ \mathrm{Eps0})\ \mathrm{Log}[1 + \mathrm{Eps0} - \mathrm{Eps0}\,\mathrm{mu}] \right) \Big/ \left(2 \left(-\frac{2\,\mathrm{Eps0}\ (2 + \mathrm{Eps0}\ (1 + \mathrm{Eps0})\ (8 + \mathrm{Eps0}))}{(1 + 2\,\mathrm{Eps0})^2} + (2 - (-2 + \mathrm{Eps0})\ \mathrm{Eps0})\ \mathrm{Log}[1 + 2\,\mathrm{Eps0}] \right) \right)$$



φ_{scatt}

$$(2\pi R - \Phi)(\beta + \beta^{-1} - \sin^2\Theta) + \sin^2\Theta \sin\Phi \cos\Phi = 0,$$

The Code – final direction

Sdriection.pro

(in the RF of the electron!!)

$$D_x = \frac{1}{|\vec{D}|}\{D_{x,0}\cos\Theta + P_{x,0}\sin\Theta\cos\Phi + \sin\Theta\sin\Phi(D_{y,0}P_{z,0} - D_{z,0}P_{y,0})\}$$

$$D_y = \frac{1}{|\vec{D}|}\{D_{y,0}\cos\Theta + P_{y,0}\sin\Theta\cos\Phi + \sin\Theta\sin\Phi(D_{z,0}P_{x,0} - D_{x,0}P_{z,0})\}$$

$$D_z = \frac{1}{|\vec{D}|}\{D_{z,0}\cos\Theta + P_{z,0}\sin\Theta\cos\Phi + \sin\Theta\sin\Phi(D_{x,0}P_{y,0} - D_{y,0}P_{x,0})\}$$

Matt et al., 1996

IF ran < Π_p THEN

$$P_x = \frac{1}{|\vec{P}|}\{D_x\sin\Theta\cos\Phi - P_{x,0}\}$$

$$P_y = \frac{1}{|\vec{P}|}\{D_y\sin\Theta\cos\Phi - P_{y,0}\}$$

$$P_z = \frac{1}{|\vec{P}|}\{D_z\sin\Theta\cos\Phi - P_{z,0}\}.$$

$$\Pi_p = 2\frac{1 - \sin^2\Theta\cos^2\Phi}{\beta + \beta^{-1} - 2\sin^2\Theta\cos^2\Phi}.$$

$$\beta = \frac{E}{E_0} = \frac{1}{1 + (E_0/m_e c^2)(1 - \cos\Theta)}.$$

ELSE

random polarization

$$\vec{P} = \frac{1}{|\vec{P}|}(\vec{P}_0 \times \vec{D}) \times \vec{D},$$

Angel, 1969

The Code – Anti Transformation & Binning

Lorentz.pro

$-\beta_x, -\beta_y, -\beta_z \Rightarrow$

$$k'_{(d)} = \Lambda\, k'_{(e)}$$

$$p'_{temp(d)} = \Lambda\, p'_{(e)} \qquad p'_{j(d)} = p'_{temp\ j+1(d)} - (p'_{temp0(d)} \cdot k'_{j+1(d)} / k'_{0(d)})$$

binning:

- energy
- θ
- φ
- **Q_j**
- **U_j**

$$I = N$$

$$Q = \sum_{j=1}^{N} Q_j$$

$$U = \sum_{j=1}^{N} U_j,$$

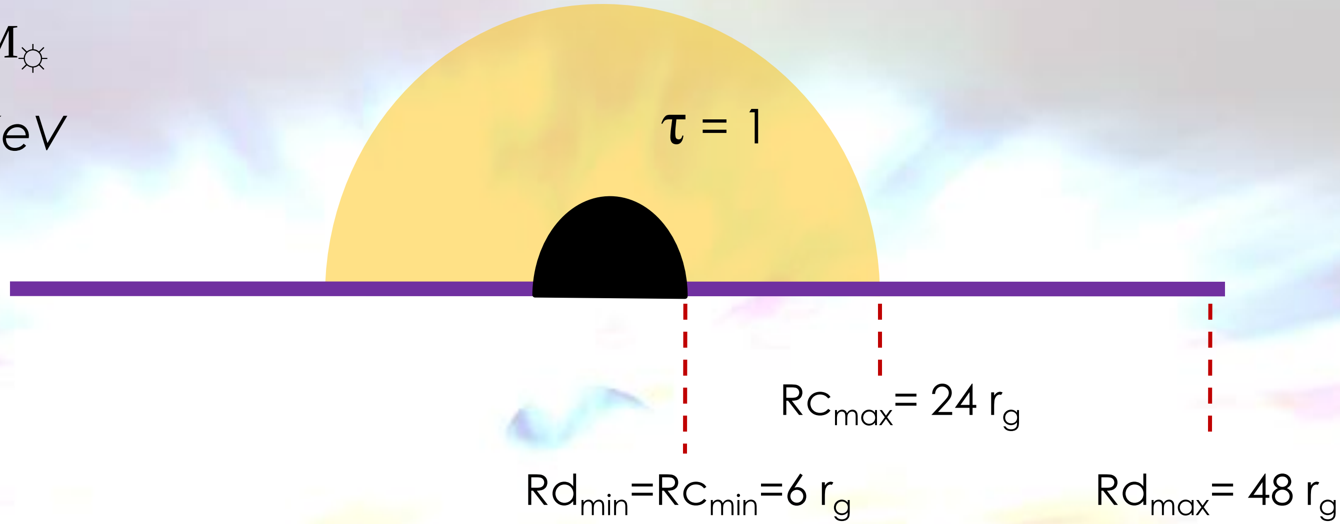
$$\Pi = \frac{\sqrt{Q^2 + U^2}}{I},$$

$$\chi = \tfrac{1}{2}\arctan\frac{U}{Q}.$$

Some preliminary results

$M_{BH} = 10^8\, M_{\odot}$

$kT_e = 100\ KeV$

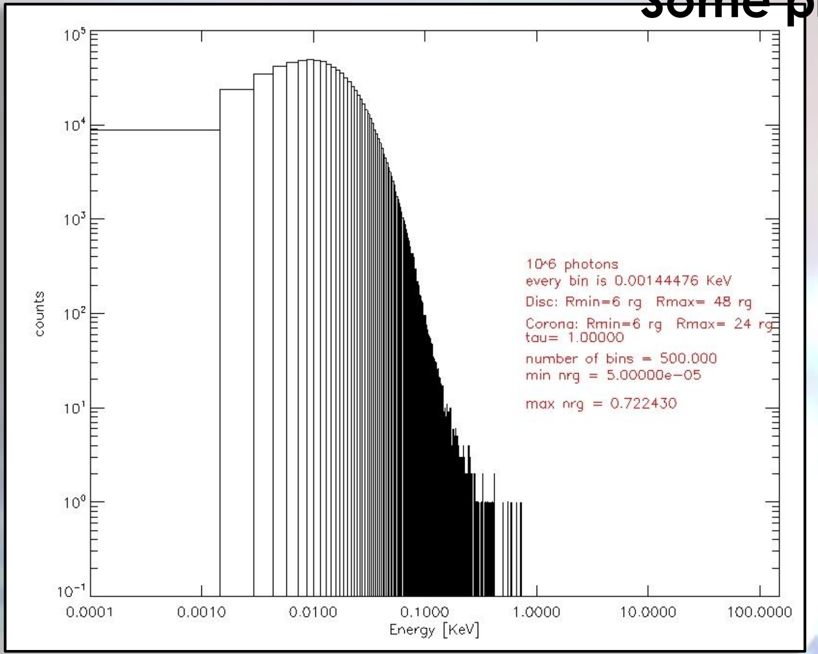


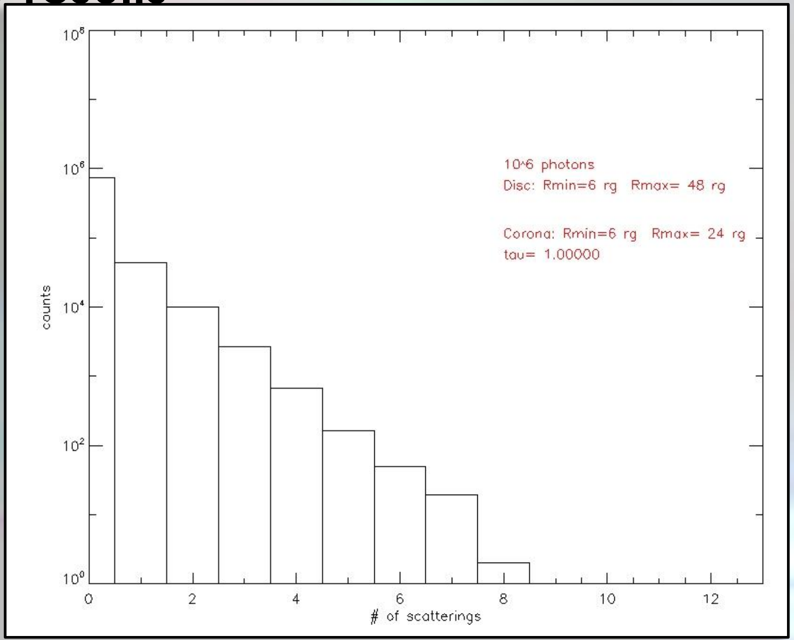
$N_{ph} = 10^6$

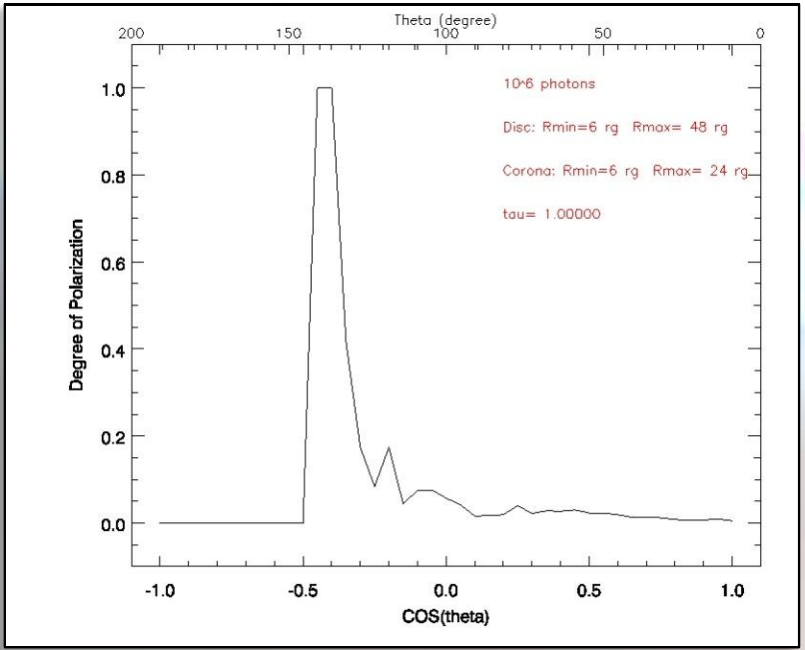
fallen in the BH	**0,4%**
fallen on the disc	**9,5%**
fallen on the disc, later	**9,7%**
escaped w/o scatterings	**74,6%**

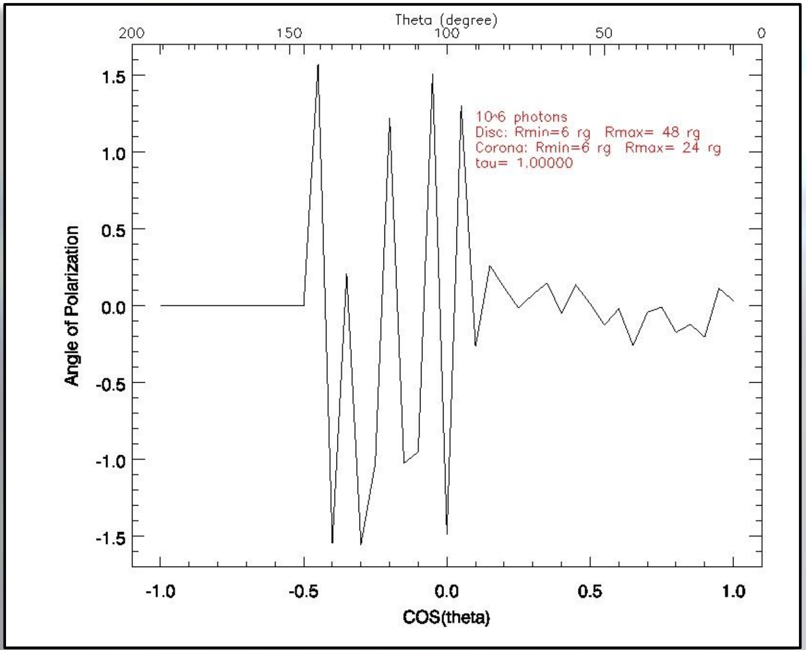
total number of scatterings $\approx 3 \cdot 10^5$

Some preliminary results





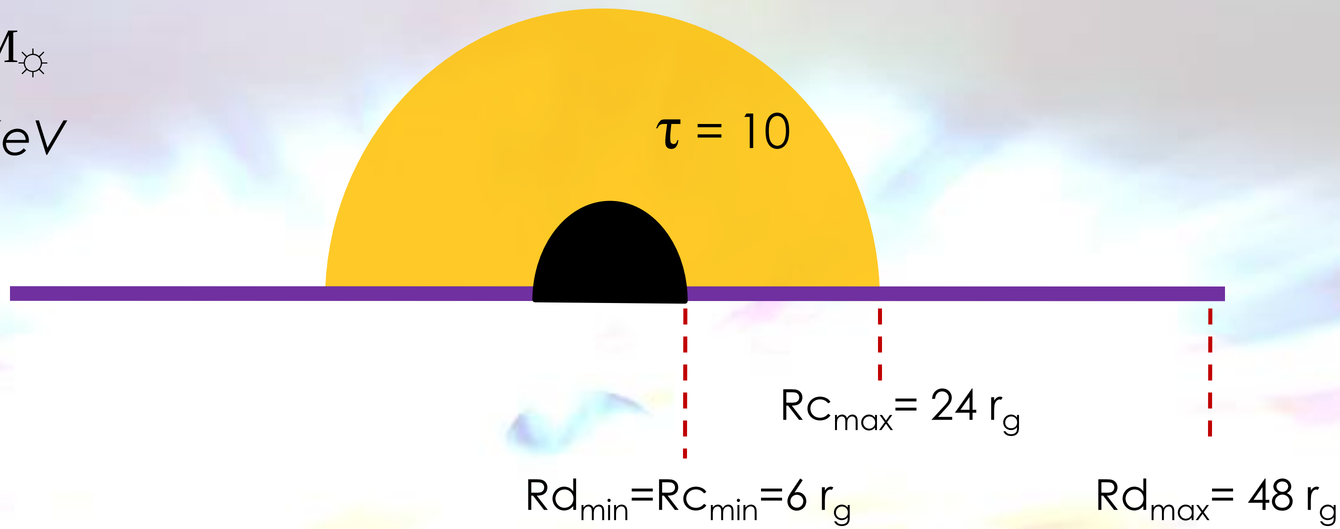




Some preliminary results

$M_{BH} = 10^8 M_{\odot}$

$kT_e = 100\ KeV$

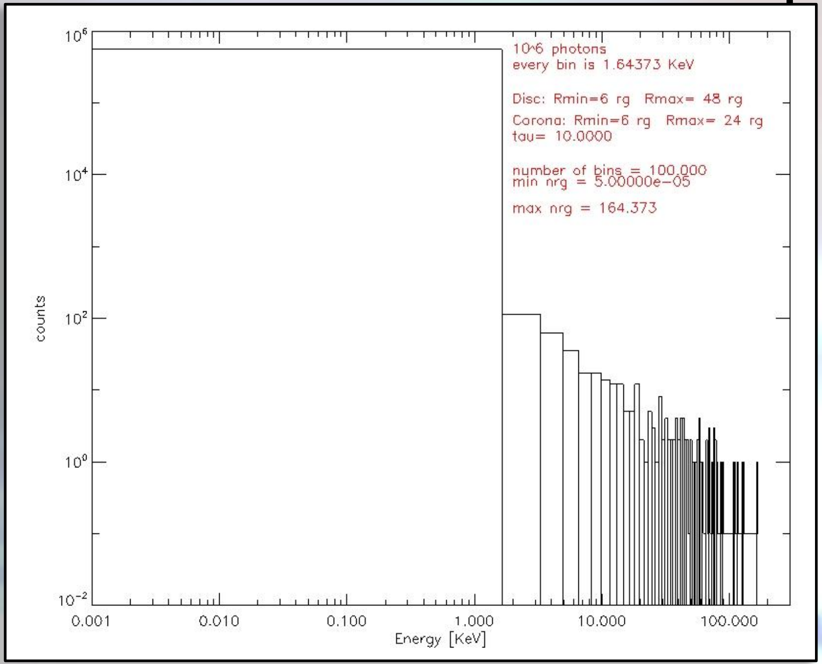


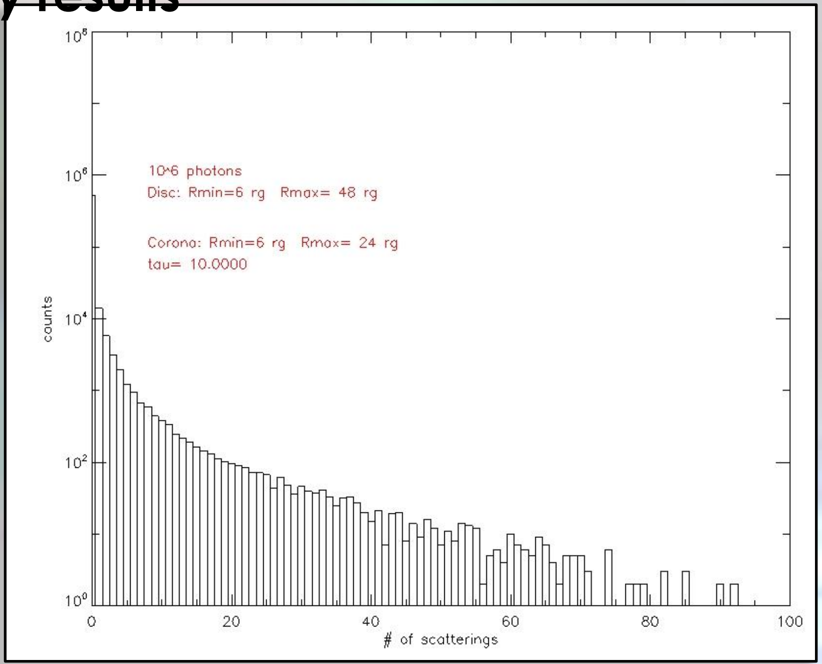
$N_{ph} = 10^6$

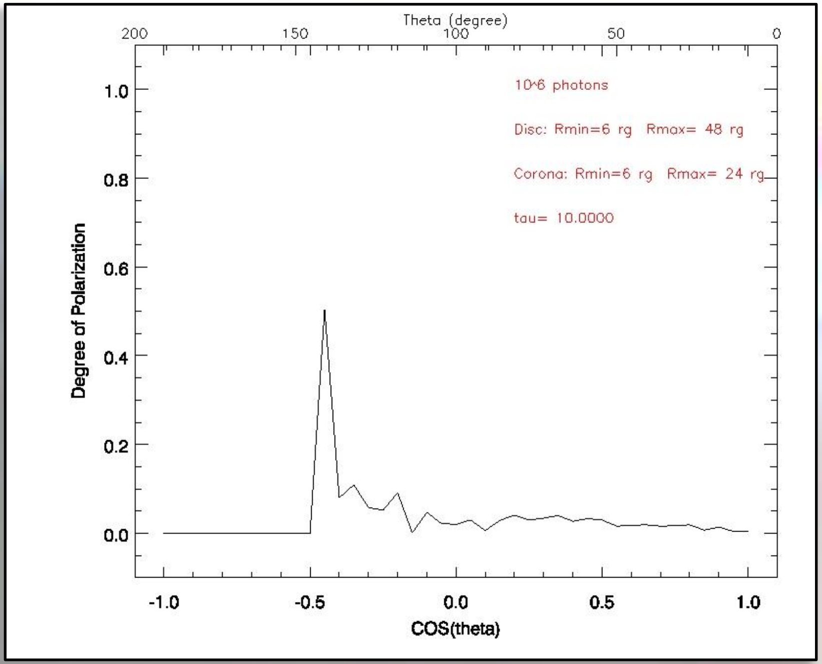
fallen in the BH	0,9%
fallen on the disc	40%
fallen on the disc, later	2,7%
escaped w/o scatterings	53,2%

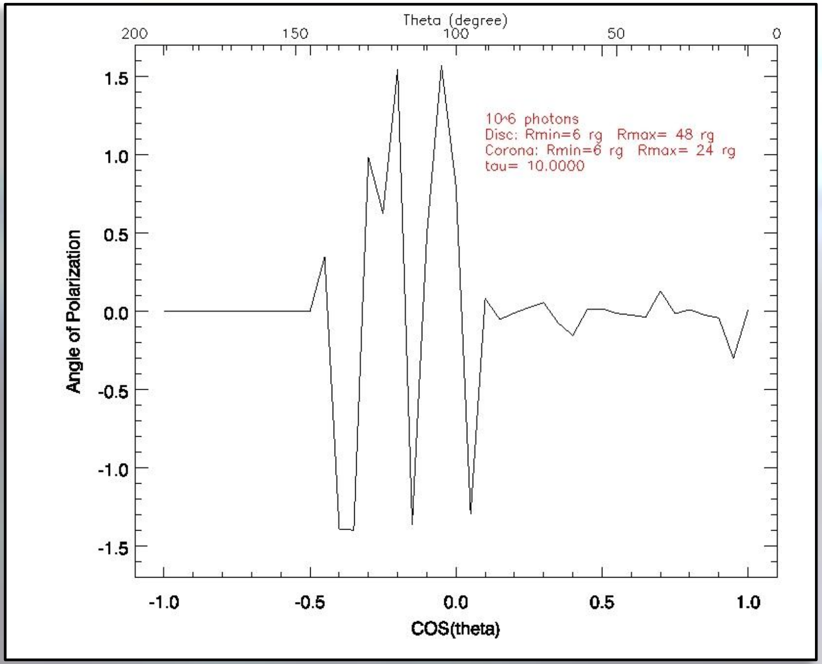
total number of scatterings $\approx 9 \cdot 10^5$

Some preliminary results



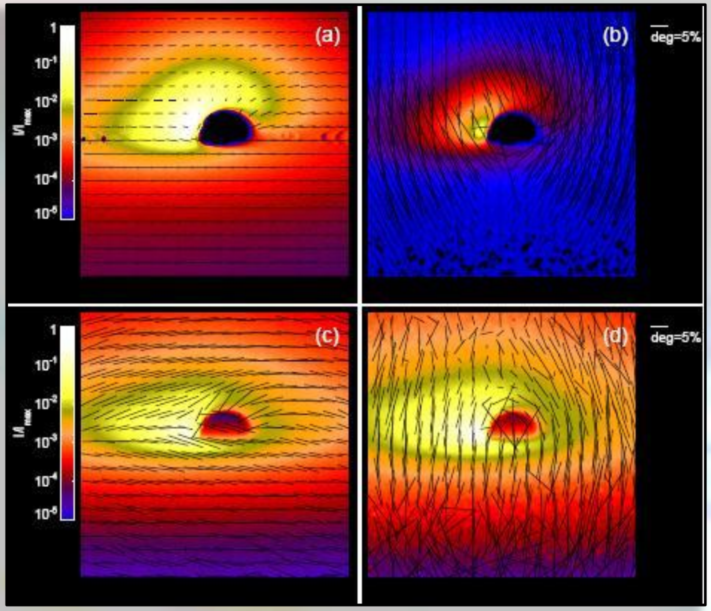






Previous work

Schnittman & Krolik, 2009



Polarization

General relativity

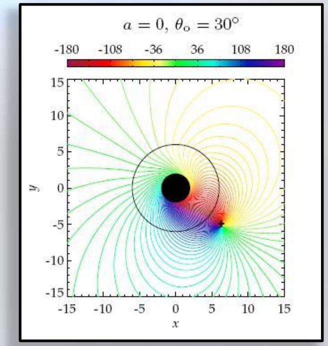
Thomson scattering

Maxwell-Boltzmann distribution (?)

Future developments

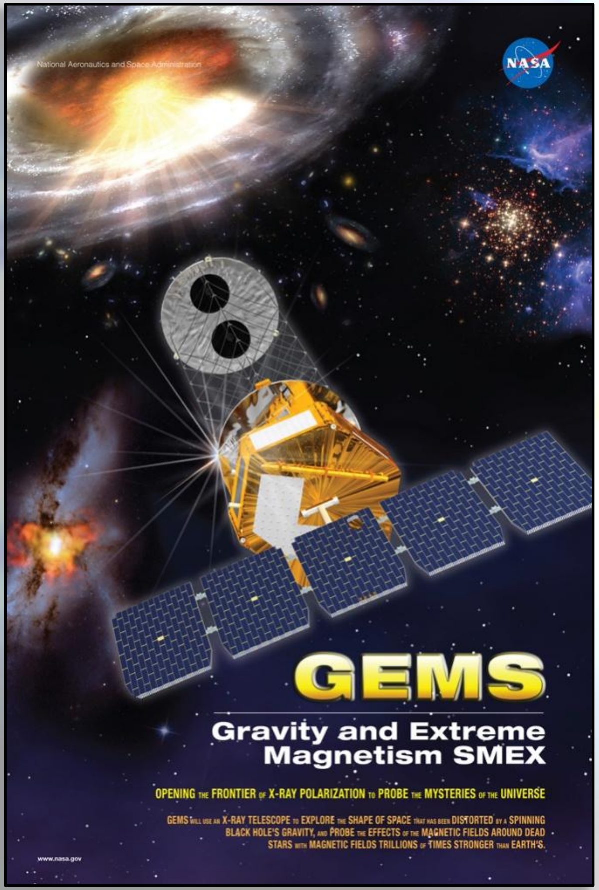
- finish it!
- include reflection from the disc
- include general relativity (spin)

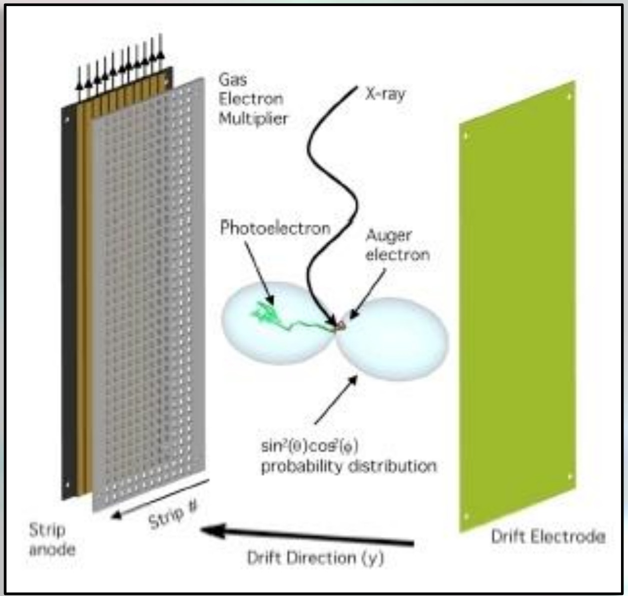
ray-tracing code



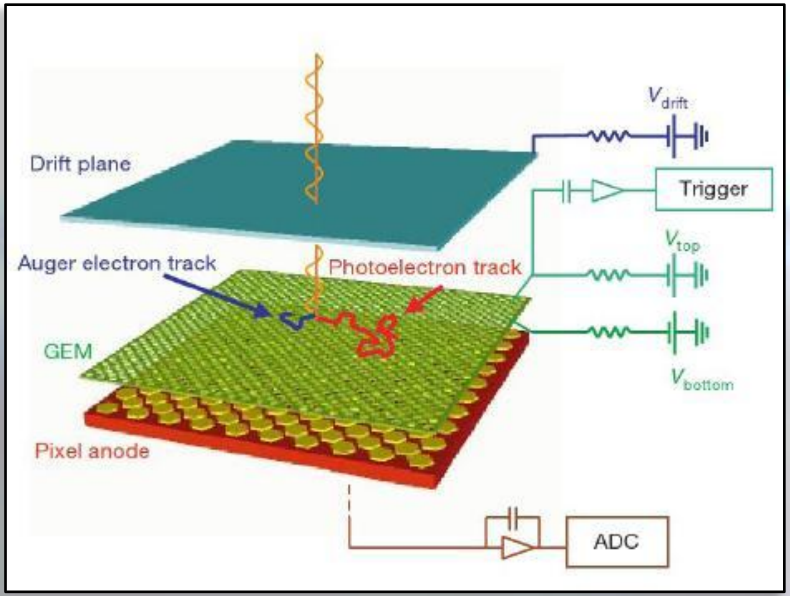
Observations

launch scheduled in 2014





Time Projection Chamber



Gas Pixel Detector