Graver basis

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Chapter 1

Introduction

The underlying problem we try to solve is the clasical *Integer program* (IP) that we formulate in the following way:

$$(IP) \equiv max\{c^t x : Ax = b, l \leq x \leq u, x \in \mathbb{Z}^n\}$$

$A \in \mathbb{Z}^{mxn}$, $b \in \mathbb{Z}^m$, $c \in \mathbb{Z}^n$, l and u lower and upper bounds for x

Despite its simplicity, it's well known the importance of IP. Several problems in diverse fields of the mathematics and algorithms admit an IP equivalent formulation (examples?). Unfortunately, IP is NP-Complete. This means that there is no efficient (polynomial) algorithm for solving IP in the general case (say general techniques and complexity?) and, therefore, knowing their importance and the lack of a general efficient algorithm for their resolution, there has been a great interest in restricted formulations of the problem and their resolution techniques.

In this project we present the concept of **Graver Basis** and its applications for solving the IP with, of course, the theoretical justification of this based on its properties. We apply this to a concrete IP formulation, the N-Fold IP and prove that it leads to a polynomial and efficient algorithm for this case.

Appendix A

Graver basis computation with 4ti2

Appendix B

IP resolution with Graver Basis example

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