

```

In[1]:= r = 1;
m = 1;
g = 10;
x1[t_] = r * Cos[φ1[t]] * Sin[θ1[t]];
      |cosinus |sinus
y1[t_] = r * Sin[φ1[t]] * Sin[θ1[t]];
      |sinus |sinus
z1[t_] = r * Cos[θ1[t]];
      |cosinus
x2[t_] = x1[t] + r * Cos[φ2[t]] * Sin[θ2[t]];
      |cosinus |sinus
y2[t_] = y1[t] + r * Sin[φ2[t]] * Sin[θ2[t]];
      |sinus |sinus
z2[t_] = z1[t] + r * Cos[θ2[t]];
      |cosinus
T = m/2 * (x1'[t]^2 + y1'[t]^2 + z1'[t]^2) + m/2 * (x2'[t]^2 + y2'[t]^2 + z2'[t]^2);
V = m * g * (z1[t] + z2[t]);
L = T - V;
eqφ1 = D[D[L, φ1'[t]], t] - D[L, φ1[t]] // FullSimplify;
      |.. |oblicz pochodną |oblicz pochodną |uprość pełniej
eqφ1 /. {φ1[t] → φ1, φ1'[t] → φ1', φ1''[t] → φ1'', φ2[t] → φ2,
        φ2'[t] → φ2', φ2''[t] → φ2'', θ1[t] → θ1, θ1'[t] → θ1', θ1''[t] → θ1'',
        θ2[t] → θ2, θ2'[t] → θ2', θ2''[t] → θ2''} // TraditionalForm
      |tradycyjna forma
eqθ1 = D[D[L, θ1'[t]], t] - D[L, θ1[t]] // FullSimplify;
      |.. |oblicz pochodną |oblicz pochodną |uprość pełniej
eqθ1 /. {φ1[t] → φ1, φ1'[t] → φ1', φ1''[t] → φ1'', φ2[t] → φ2,
        φ2'[t] → φ2', φ2''[t] → φ2'', θ1[t] → θ1, θ1'[t] → θ1', θ1''[t] → θ1'',
        θ2[t] → θ2, θ2'[t] → θ2', θ2''[t] → θ2''} // TraditionalForm
      |tradycyjna forma
eqφ2 = D[D[L, φ2'[t]], t] - D[L, φ2[t]] // FullSimplify;
      |.. |oblicz pochodną |oblicz pochodną |uprość pełniej
eqφ2 /. {φ1[t] → φ1, φ1'[t] → φ1', φ1''[t] → φ1'', φ2[t] → φ2,
        φ2'[t] → φ2', φ2''[t] → φ2'', θ1[t] → θ1, θ1'[t] → θ1', θ1''[t] → θ1'',
        θ2[t] → θ2, θ2'[t] → θ2', θ2''[t] → θ2''} // TraditionalForm
      |tradycyjna forma
eqθ2 = D[D[L, θ2'[t]], t] - D[L, θ2[t]] // FullSimplify;
      |.. |oblicz pochodną |oblicz pochodną |uprość pełniej
eqθ2 /. {φ1[t] → φ1, φ1'[t] → φ1', φ1''[t] → φ1'', φ2[t] → φ2,
        φ2'[t] → φ2', φ2''[t] → φ2'', θ1[t] → θ1, θ1'[t] → θ1', θ1''[t] → θ1'',
        θ2[t] → θ2, θ2'[t] → θ2', θ2''[t] → θ2''} // TraditionalForm
      |tradycyjna forma

```

Out[14]//TraditionalForm=

$$\sin(\theta_1) \left(4 \theta_1' \cos(\theta_1) \phi_1' + 2 \sin(\theta_1) \phi_1'' + \sin(\phi_1 - \phi_2) \left(\sin(\theta_2) (\phi_2')^2 - \theta_2'' \cos(\theta_2) \right) + \right. \\ \left. 2 \theta_2' \cos(\theta_2) \phi_2' \cos(\phi_1 - \phi_2) + (\theta_2')^2 \sin(\theta_2) \sin(\phi_1 - \phi_2) + \sin(\theta_2) \phi_2'' \cos(\phi_1 - \phi_2) \right)$$

Out[16]//TraditionalForm=

$$2 \theta_1'' + \theta_2'' (\cos(\theta_1) \cos(\theta_2) \cos(\phi_1 - \phi_2) + \sin(\theta_1) \sin(\theta_2)) + \\ 2 \cos(\theta_1) \theta_2' \cos(\theta_2) \phi_2' \sin(\phi_1 - \phi_2) + (\theta_2')^2 (\sin(\theta_1) \cos(\theta_2) - \cos(\theta_1) \sin(\theta_2) \cos(\phi_1 - \phi_2)) + \\ \cos(\theta_1) \sin(\theta_2) \phi_2'' \sin(\phi_1 - \phi_2) - \cos(\theta_1) \sin(\theta_2) (\phi_2')^2 \cos(\phi_1 - \phi_2) - \sin(2 \theta_1) (\phi_1')^2 - 20 \sin(\theta_1)$$

Out[18]//TraditionalForm=

$$\sin(\theta_2) \left(\sin(\phi_1 - \phi_2) \left(\theta_1'' \cos(\theta_1) - \sin(\theta_1) (\phi_1')^2 \right) + 2 \theta_1' \cos(\theta_1) \phi_1' \cos(\phi_1 - \phi_2) + \right. \\ \left. (\theta_1')^2 (-\sin(\theta_1)) \sin(\phi_1 - \phi_2) + \sin(\theta_1) \phi_1'' \cos(\phi_1 - \phi_2) + 2 \theta_2' \cos(\theta_2) \phi_2' + \sin(\theta_2) \phi_2'' \right)$$

Out[20]/TraditionalForm=

$$\begin{aligned}
& \theta_1'' \cos(\theta_1) \cos(\theta_2) \cos(\phi_1) \cos(\phi_2) + \theta_1'' \cos(\theta_1) \cos(\theta_2) \sin(\phi_1) \sin(\phi_2) + \theta_1'' \sin(\theta_1) \sin(\theta_2) - \\
& 2 \theta_1' \cos(\theta_1) \cos(\theta_2) \phi_1' \sin(\phi_1 - \phi_2) + (\theta_1')^2 (\cos(\theta_1) \sin(\theta_2) - \sin(\theta_1) \cos(\theta_2) \cos(\phi_1 - \phi_2)) - \\
& \sin(\theta_1) \cos(\theta_2) \phi_1'' \sin(\phi_1 - \phi_2) - \sin(\theta_1) \cos(\theta_2) (\phi_1')^2 \cos(\phi_1 - \phi_2) + \theta_2'' - \sin(\theta_2) \cos(\theta_2) (\phi_2')^2 - 10 \sin(\theta_2)
\end{aligned}$$