In[1]:=
```
x1[r_, t_] = r * Cos[ϕ1[t]] * Sin[θ1[t]];
y1[r_, t_] = r * Sin[ϕ1[t]] * Sin[θ1[t]];
z1[r_, t_] = r * Cos[θ1[t]];
x2[r_, t_] = x1[r, t] + r * Cos[ϕ2[t]] * Sin[θ2[t]];
y2[r_, t_] = y1[r, t] + r * Sin[ϕ2[t]] * Sin[θ2[t]];
z2[r_, t_] = z1[r, t] + r * Cos[θ2[t]];
```

In[7]:=
```
T = m1 / 2 * (D[x1[r, t], t]^2 + D[y1[r, t], t]^2 + D[z1[r, t], t]^2) +
    m2 / 2 * (D[x2[r, t], t]^2 + D[y2[r, t], t]^2 + D[z2[r, t], t]^2);
V = g * (m1 * z1[r, t] + m2 * z2[r, t]);
L = T - V;
```

In[10]:=
```
eqϕ1 = D[D[L, ϕ1'[t]], t] - D[L, ϕ1[t]] // FullSimplify;
```

In[11]:=
```
eqϕ1 /. {ϕ1[t] → ϕ1, ϕ1'[t] → ϕ1', ϕ1''[t] → ϕ1'',
  ϕ2[t] → ϕ2, ϕ2'[t] → ϕ2', ϕ2''[t] → ϕ2'', θ1[t] → θ1, θ1'[t] → θ1',
  θ1''[t] → θ1'', θ2[t] → θ2, θ2'[t] → θ2', θ2''[t] → θ2''}
```

Out[11]=
$$r^2 \sin[\theta 1] \left(m2 \sin[\theta 2] \sin[\phi 1 - \phi 2] \left(\theta 2'\right)^2 + 2 \left(m1 + m2\right) \cos[\theta 1] \theta 1' \phi 1' + 2\, m2 \cos[\theta 2] \cos[\phi 1 - \phi 2] \theta 2' \phi 2' + m1 \sin[\theta 1] \phi 1'' + m2 \left(\sin[\phi 1 - \phi 2] \left(\sin[\theta 2] \left(\phi 2'\right)^2 - \cos[\theta 2] \theta 2''\right) + \sin[\theta 1] \phi 1'' + \cos[\phi 1 - \phi 2] \sin[\theta 2] \phi 2''\right)\right)$$

In[12]:=
```
eqθ1 = D[D[L, θ1'[t]], t] - D[L, θ1[t]] // FullSimplify;
```

In[13]:=
```
eqθ1 /. {ϕ1[t] → ϕ1, ϕ1'[t] → ϕ1', ϕ1''[t] → ϕ1'',
  ϕ2[t] → ϕ2, ϕ2'[t] → ϕ2', ϕ2''[t] → ϕ2'', θ1[t] → θ1, θ1'[t] → θ1',
  θ1''[t] → θ1'', θ2[t] → θ2, θ2'[t] → θ2', θ2''[t] → θ2''}
```

Out[13]=
$$\frac{1}{2} r \left(2\, m2\, r \left(\cos[\theta 2] \sin[\theta 1] - \cos[\theta 1] \cos[\phi 1 - \phi 2] \sin[\theta 2]\right) \left(\theta 2'\right)^2 + 4\, m2\, r \cos[\theta 1] \cos[\theta 2] \sin[\phi 1 - \phi 2] \theta 2' \phi 2' - \left(m1 + m2\right) \left(2\, g \sin[\theta 1] + r \sin[2\, \theta 1] \left(\phi 1'\right)^2 - 2\, r\, \theta 1''\right) + 2\, m2\, r \cos[\theta 1] \cos[\theta 2] \cos[\phi 1 - \phi 2] \theta 2'' + 2\, m2\, r \sin[\theta 2] \left(\sin[\theta 1] \theta 2'' + \cos[\theta 1] \left(-\cos[\phi 1 - \phi 2] \left(\phi 2'\right)^2 + \sin[\phi 1 - \phi 2] \phi 2''\right)\right)\right)$$

In[14]:=
```
eqϕ2 = D[D[L, ϕ2'[t]], t] - D[L, ϕ2[t]] // FullSimplify;
eqϕ2 /. {ϕ1[t] → ϕ1, ϕ1'[t] → ϕ1', ϕ1''[t] → ϕ1'',
  ϕ2[t] → ϕ2, ϕ2'[t] → ϕ2', ϕ2''[t] → ϕ2'', θ1[t] → θ1, θ1'[t] → θ1',
  θ1''[t] → θ1'', θ2[t] → θ2, θ2'[t] → θ2', θ2''[t] → θ2''}
```

Out[15]=
$$m2\, r^2 \sin[\theta 2] \left(-\sin[\theta 1] \sin[\phi 1 - \phi 2] \left(\theta 1'\right)^2 + 2 \cos[\theta 1] \cos[\phi 1 - \phi 2] \theta 1' \phi 1' + 2 \cos[\theta 2] \theta 2' \phi 2' + \sin[\phi 1 - \phi 2] \left(-\sin[\theta 1] \left(\phi 1'\right)^2 + \cos[\theta 1] \theta 1''\right) + \cos[\phi 1 - \phi 2] \sin[\theta 1] \phi 1'' + \sin[\theta 2] \phi 2''\right)$$

In[16]:=
```
eqθ2 = D[D[L, θ2'[t]], t] - D[L, θ2[t]] // FullSimplify;
```
oblicz pochodną | oblicz pochodną | uprość pełniej

```
eqθ2 /. {ϕ1[t] → ϕ1, ϕ1'[t] → ϕ1', ϕ1''[t] → ϕ1'',
  ϕ2[t] → ϕ2, ϕ2'[t] → ϕ2', ϕ2''[t] → ϕ2'', θ1[t] → θ1, θ1'[t] → θ1',
  θ1''[t] → θ1'', θ2[t] → θ2, θ2'[t] → θ2', θ2''[t] → θ2''}
```

Out[17]=
$$-m2\,r\left(g\,\text{Sin}[\theta 2]+r\left(\text{Cos}[\theta 2]\,\text{Cos}[\phi 1-\phi 2]\,\text{Sin}[\theta 1]-\text{Cos}[\theta 1]\,\text{Sin}[\theta 2]\right)\left(\theta 1'\right)^2+2\,r\,\text{Cos}[\theta 1]\,\text{Cos}[\theta 2]\,\text{Sin}[\phi 1-\phi 2]\,\theta 1'\,\phi 1'+r\,\text{Cos}[\theta 2]\,\text{Cos}[\phi 1-\phi 2]\,\text{Sin}[\theta 1]\left(\phi 1'\right)^2+r\left(\text{Cos}[\theta 2]\,\text{Sin}[\theta 2]\left(\phi 2'\right)^2-\left(\text{Cos}[\theta 1]\,\text{Cos}[\theta 2]\,\text{Cos}[\phi 1-\phi 2]+\text{Sin}[\theta 1]\,\text{Sin}[\theta 2]\right)\theta 1''-\theta 2''+\text{Cos}[\theta 2]\,\text{Sin}[\theta 1]\,\text{Sin}[\phi 1-\phi 2]\,\phi 1''\right)\right)$$

In[18]:=
```
eq1 = eqϕ1 == 0;
eq2 = eqθ1 == 0;
eq3 = eqϕ2 == 0;
eq4 = eqθ2 == 0;
Solve[{eq1, eq2, eq3, eq4}, {ϕ1''[t], ϕ2''[t], θ1''[t], θ2''[t]}] // FullSimplify
```
rozwiąż równanie | uprość pełniej

Out[22]=
$$\Big\{\Big\{\phi1''[t] \to \Big(2\,\Big(2\,m2\,r\,\text{Csc}[\theta1[t]]^2\,\Big(\text{Cot}[\theta1[t]]\,\text{Sin}[2\,\theta2[t]]\,\text{Sin}[\phi1[t]-\phi2[t]]+\text{Sin}[\theta2[t]]^2\,\text{Sin}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\Big)\,\theta1'[t]^2+r\,\text{Cot}[\theta1[t]]\,\Big(m2+8\,m1\,\text{Csc}[\theta1[t]]^2-m2\,\Big(\text{Cot}[\theta1[t]]^2-5\,\text{Csc}[\theta1[t]]^2+\text{Cos}[2\,\theta2[t]]\,\Big(-6+4\,\text{Csc}[\theta1[t]]^2\Big)+4\,\text{Cos}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\,\text{Sin}[\theta2[t]]^2+8\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Cot}[\theta1[t]]\,\text{Sin}[2\,\theta2[t]]\Big)\Big)\,\theta1'[t]\,\phi1'[t]+2\,m2\,\Big(2\,r\,\text{Csc}[\theta1[t]]^3\,\text{Sin}[\theta2[t]]\,\text{Sin}[\phi1[t]-\phi2[t]]\,\theta2'[t]^2-\Big(\text{Cot}[\theta1[t]]\,\text{Sin}[2\,\theta2[t]]\,\text{Sin}[\phi1[t]-\phi2[t]]+\text{Sin}[\theta2[t]]^2\,\text{Sin}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\Big)\,\Big(g\,\text{Cot}[\theta1[t]]\,\text{Csc}[\theta1[t]]-r\,\phi1'[t]^2\Big)+2\,r\,\text{Csc}[\theta1[t]]^3\,\text{Sin}[\theta2[t]]^3\,\text{Sin}[\phi1[t]-\phi2[t]]\,\phi2'[t]^2\Big)\Big)\Big/\Big(r\,\Big(-m2-8\,m1\,\text{Csc}[\theta1[t]]^2+m2\,\Big(\text{Cot}[\theta1[t]]^2-5\,\text{Csc}[\theta1[t]]^2+\text{Cos}[2\,\theta2[t]]\,\Big(-6+4\,\text{Csc}[\theta1[t]]^2\Big)+4\,\text{Cos}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\,\text{Sin}[\theta2[t]]^2+8\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Cot}[\theta1[t]]\,\text{Sin}[2\,\theta2[t]]\Big)\Big)\Big),$$

$$\phi2''[t] \to \Big(4\,\Big(4\,\Big(m1+m2\Big)\,r\,\text{Csc}[\theta2[t]]\,\text{Sin}[\theta1[t]]\,\text{Sin}[\phi1[t]-\phi2[t]]\,\theta1'[t]^2+2\,\Big(m2\,r\,\Big(\text{Cot}[\theta2[t]]\,\text{Sin}[2\,\theta1[t]]\,\text{Sin}[\phi1[t]-\phi2[t]]+\text{Sin}[\theta1[t]]^2\,\text{Sin}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\Big)\,\theta2'[t]^2+\Big(m1+m2\Big)\,\text{Csc}[\theta2[t]]\,\text{Sin}[\phi1[t]-\phi2[t]]\,\Big(-g\,\text{Sin}[2\,\theta1[t]]+2\,r\,\text{Sin}[\theta1[t]]^3\,\phi1'[t]^2\Big)\Big)+r\,\Big(\Big(-8\,m1-5\,m2+m2\,\text{Cos}[2\,\theta1[t]]+m2\,\Big(1+3\,\text{Cos}[2\,\theta1[t]]\Big)\,\text{Cos}[2\,\theta2[t]]\Big)\,\text{Cot}[\theta2[t]]+2\,m2\,\Big(4\,\text{Cos}[\theta2[t]]^2\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[2\,\theta1[t]]+\text{Cos}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\,\text{Sin}[\theta1[t]]^2\,\text{Sin}[2\,\theta2[t]]\Big)\Big)\,\theta2'[t]\,\phi2'[t]+2\,m2\,r\,\text{Sin}[\theta2[t]]^2\,\Big(\text{Cot}[\theta2[t]]\,\text{Sin}[2\,\theta1[t]]\,\text{Sin}[\phi1[t]-\phi2[t]]+\text{Sin}[\theta1[t]]^2\,\text{Sin}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\Big)\,\phi2'[t]^2\Big)\Big/\Big(r\,\Big(2\,\Big(8\,m1+5\,m2\Big)-m2\,\Big(2\,\text{Cos}[2\,\theta1[t]]+3\,\text{Cos}\Big[2\,\Big(\theta1[t]-\theta2[t]\Big)\Big]+2\,\text{Cos}[2\,\theta2[t]]+3\,\text{Cos}\Big[2\,\Big(\theta1[t]+\theta2[t]\Big)\Big]+8\,\text{Cos}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\,\text{Sin}[\theta1[t]]^2\,\text{Sin}[\theta2[t]]^2+8\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[2\,\theta1[t]]\,\text{Sin}[2\,\theta2[t]]\Big)\Big)\Big),\ \theta1''[t] \to$$

$$\Big(4\,\Big(g\,\Big(4\,m1+3\,m2+2\,m2\,\text{Cos}[2\,\theta2[t]]\,\text{Cos}[\phi1[t]-\phi2[t]]^2-m2\,\text{Cos}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\Big)\,\text{Sin}[\theta1[t]]-2\,g\,m2\,\text{Cos}[\theta1[t]]\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[2\,\theta2[t]]+r\,\Big(-m2\,\Big(\text{Cos}[\theta1[t]]\,\text{Cos}[2\,\theta2[t]]\,\Big(3+\text{Cos}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\Big)\,\text{Sin}[\theta1[t]]-2\,\text{Cos}[2\,\theta1[t]]\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[2\,\theta2[t]]+\text{Sin}[2\,\theta1[t]]\,\text{Sin}[\phi1[t]-\phi2[t]]^2\Big)\,\theta1'[t]^2+4\,m2\,\Big(-\text{Cos}[\theta2[t]]\,\text{Sin}[\theta1[t]]+\text{Cos}[\theta1[t]]\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[\theta2[t]]\Big)\,\theta2'[t]^2+\Big(\Big(2\,m1+m2-m2\,\text{Cos}[2\,\theta2[t]]\Big)\,\text{Sin}[2\,\theta1[t]]-2\,m2\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[\theta1[t]]^2\,\text{Sin}[2\,\theta2[t]]\Big)\,\phi1'[t]^2+4\,m2\,\text{Sin}[\theta2[t]]^2\,\Big(-\text{Cos}[\theta2[t]]\,\text{Sin}[\theta1[t]]+\text{Cos}[\theta1[t]]\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[\theta2[t]]\Big)\,\phi2'[t]^2\Big)\Big)\Big/\Big(r\,\Big(2\,\Big(8\,m1+5\,m2\Big)-m2\,\Big(2\,\text{Cos}[2\,\theta1[t]]+3\,\text{Cos}\Big[2\,\Big(\theta1[t]-\theta2[t]\Big)\Big]+2\,\text{Cos}[2\,\theta2[t]]+3\,\text{Cos}\Big[2\,\Big(\theta1[t]+\theta2[t]\Big)\Big]+8\,\text{Cos}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\,\text{Sin}[\theta1[t]]^2\,\text{Sin}[\theta2[t]]^2+8\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[2\,\theta1[t]]\,\text{Sin}[2\,\theta2[t]]\Big)\Big)\Big),$$

$$\theta2''[t] \to \Big(16\,\Big(m1+m2\Big)\,r\,\Big(\text{Cos}[\theta2[t]]\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[\theta1[t]]-\text{Cos}[\theta1[t]]\,\text{Sin}[\theta2[t]]\Big)\,\theta1'[t]^2+2\,m2\,r\,\Big(4\,\text{Cos}[2\,\theta2[t]]\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[2\,\theta1[t]]-\text{Sin}[2\,\theta2[t]]\,\Big(\text{Cos}[2\,\theta1[t]]\,\Big(3+\text{Cos}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\Big)+2\,\text{Sin}[\phi1[t]-\phi2[t]]^2\Big)\Big)\,\theta2'[t]^2+8\,\Big(m1+m2\Big)\,\Big(-g\,\text{Cos}[\theta2[t]]\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[2\,\theta1[t]]+2\,g\,\text{Cos}[\theta1[t]]^2\,\text{Sin}[\theta2[t]]+2\,r\,\text{Sin}[\theta1[t]]^2\,\Big(\text{Cos}[\theta2[t]]\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[\theta1[t]]-\text{Cos}[\theta1[t]]\,\text{Sin}[\theta2[t]]\Big)\,\phi1'[t]^2\Big)+4\,r\,\Big(-2\,m2\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[2\,\theta1[t]]\,\text{Sin}[\theta2[t]]^2+\Big(2\,m1+m2-m2\,\text{Cos}[2\,\theta1[t]]\Big)\,\text{Sin}[2\,\theta2[t]]\Big)\,\phi2'[t]^2\Big)\Big/\Big(r\,\Big(2\,\Big(8\,m1+5\,m2\Big)-m2\,\Big(2\,\text{Cos}[2\,\theta1[t]]+3\,\text{Cos}\Big[2\,\Big(\theta1[t]-\theta2[t]\Big)\Big]+2\,\text{Cos}[2\,\theta2[t]]+3\,\text{Cos}\Big[2\,\Big(\theta1[t]+\theta2[t]\Big)\Big]+8\,\text{Cos}\Big[2\,\Big(\phi1[t]-\phi2[t]\Big)\Big]\,\text{Sin}[\theta1[t]]^2\,\text{Sin}[\theta2[t]]^2+8\,\text{Cos}[\phi1[t]-\phi2[t]]\,\text{Sin}[2\,\theta1[t]]\,\text{Sin}[2\,\theta2[t]]\Big)\Big)\Big)\Big\}\Big\}$$