TOBB ETÜ, BİLGİSAYAR MÜHENDİSLİĞİ BÖLÜMÜ

BİL 133 Combinatorics and Graph Theory

HOMEWORK 5 (29 Points)

Due Date: July 3, 2017

1 [5 POINTS] FIRST RECURRENCE AND ITS SLIGHT GENERALIZATION

Assume that T is a function defined from the nonnegative exact powers of 2 to the set of nonnegative integers, i.e., $T : 2^k \rightarrow N$, where $k \in N$.(Notice that $N = \{0, 1, 2, \ldots\}$)

1.1 [2 POINTS] FIRST RECURRENCE

Solve the following recurrence by using the repertoire method. Notice that your solution should be of the form $T(n) = A(n) \cdot \alpha + B(n) \cdot \beta$.

- $T(1) = \alpha$
- $T(n) = 2 \cdot T(\frac{n}{2}) + \beta \cdot n$, for $n > 1$, and n is an exact power of 2.

(**Hint:** You may try to implement $T(n) = n \cdot \lg n$.)

1.2 [3 POINTS] SLIGHT GENERALIZATION

Solve the following recurrence by using the repertoire method. Notice that your solution should be of the form $T(n) = A(n) \cdot \alpha + B(n) \cdot \beta + C(n) \cdot \gamma$.

- $T(1) = \alpha$
- $T(n) = 2 \cdot T(\frac{n}{2}) + \beta \cdot n + \gamma$, for $n > 1$, and n is an exact power of 2.

2 [4 Points] Second Recurrence and Its Generalization

Assume that T is a function defined from the nonnegative exact powers of 3 to the set of nonnegative integers, i.e., $T : 3^k \to N$, where $k \in N$.(Notice that $N = \{0, 1, 2, \ldots\}$)

2.1 [2 Points] Second Recurrence

Solve the following recurrence by using the repertoire method. Notice that your solution should be of the form $T(n) = A(n)\cdot\alpha + B(n)\cdot\beta$.

- $T(1) = \alpha$
- $T(n) = 3\cdot T(\frac{n}{3}) + \beta\cdot n$, for $n > 1$, and n is an exact power of 3.

(**Hint:** You may try to implement $T(n) = n\cdot\lg n$.)

2.2 [2 Points] Slight Generalization

Solve the following recurrence by using the repertoire method. Notice that your solution should be of the form $T(n) = A(n)\cdot\alpha + B(n)\cdot\beta + C(n)\cdot\gamma$.

- $T(1) = \alpha$
- $T(n) = 3\cdot T(\frac{n}{3}) + \beta\cdot n + \gamma$, for $n > 1$, and n is an exact power of 3.

3 [4 Points] An Ugly Looking Recurrence Teaching the Importance of Playing with Small Instances

Solve the recurrence defined as:

- $Q_0 = \alpha$
- $Q_1 = \beta$
- $Q_n = \frac{1+Q_{n-1}}{Q_{n-2}}$ for $n > 1$.

(Hint: First show that $Q_4 = \frac{1+\alpha}{\beta}$).

Needless to say (but just in case), you have to prove that your solution to recurrence is correct by using mathematical induction.

4 [4 Points] Proof by Mathematical Induction

Prove that $n! > 2^n$ for $n \geq 4$.

5 [1 Point] Iverson's Convention

Simplify the expression $x \cdot ([x > 0] - [x < 0])$.

6 [1 Points] Evaluating a Simple Sum

What is the value of $\sum_k [1 \le j \le k \le n]$ as a function of j and n.

7 [5 Points] Summation Factor Method

Find a closed form for the recurrence given below:

- $T_0 = 5$
- $2 \cdot T_n = n \cdot T_{n-1} + 3 \cdot n!$, for $n > 0$.

8 [5 Points] Evaluating a Sum

Deduce the value of $\sum_{k=0}^{\infty} kx^k$ for $|x| < 1$.
(Hint: You may want to use perturbation followed by a differentiation).