Middle East Technical University | Department of Computer Engineering

CENG 223

Discrete Computational Structures

Fall '2018-2019

Homework 4

Due date: 19 December 2018, 23:55

Question 1

Use generating functions to answer the questions below. Explain each step briefly.

a. Assume that you have infinite source of three types of candies: red,blue and green. How many ways are there to select 10 candies in such a way: green candies are in even numbers and less then five, red candies are more than three, blue candies are in odd numbers?

b. Answer the same question assuming you have only five of each of types in hand.

c. Find sequence corresponding to generating function $F(x) = \frac{7x^2}{(1-2x)(1+3x)}$

d. Solve the recurrence $s_n = 8s_{n-1} + 10^{n-1}$ with initial condition $s_1 = 9$

Question 2

How many of the numbers up to n are prime? The Inclusion-Exclusion Principle offers a useful way to calculate when n is large. Use Inclusion-Exclusion to count the number of composite (nonprime) positive integers up to n. Subtracting this from n then gives the number of primes.

Let C_n be the set of composite numbers in $(1..n]$, and let A_m be the set of numbers in the interval $(m..n]$ that are divisible by m. Note that A_m is empty set when $m \geq n$. So

$$C_n = \bigcup_{i=2}^{n-1} A_i \tag{1}$$

a. Verify that if $m|k$, then $A_k \subseteq A_m$

b. Explain why the right-hand side of Equation 1 equals

$$\bigcup_{primes\ p \leq \sqrt{n}} A_p \tag{2}$$

c. Explain why $|A_m| = \lfloor n/m \rfloor - 1$ for $m \geq 2$

d. Consider any two relatively prime numbers $a, b \leq n$. What is the one number in $(A_a \cap A_b) - A_{ab}$?

e. Let P be a finite set of at least two primes. Give a simple formula for

$$\left| \bigcap_{p \in P} A_p \right| \tag{3}$$

f. Use the Inclusion-Exclusion principle to obtain a formula for $|C_{45}|$ in terms the sizes of intersections among the sets A_2, A_3, A_5.

g. Use this formula to find the number of primes up to 45.

Question 3

a. $\ll$ is a relation on $\mathbb{Z}^2$ defined as: $(a, b) \ll (c, d)$ if and only if either $a < c$ or else $a = c$ and $b \leq d$. Prove that $\ll$ is transitive.

b. $\propto$ is a relation on the set of all functions from $\mathbb{R}$ to $\mathbb{R}$, defined as: $f \propto g$ if and only if there is a $k \in \mathbb{R}$ such that $f(x) = g(x)$ for every $x \geq k$. Prove that $\propto$ is an equivalence relation.

1 Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.

2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.

3. **Late Submission: Not allowed**

4. **Cheating: We have zero tolerance policy for cheating**. People involved in cheating will be punished according to the university regulations.

5. **Newsgroup:** You must follow the newsgroup (news.ceng.metu.edu.tr) for discussions and possible updates on a daily basis.

6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

2 Submission

Submission will be done via COW. Download the given template file, "hw4.tex", when you finish your exam upload the .tex file with the same name to COW.
Note: You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex hw4.tex
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