BİL 133 Combinatorics and Graph Theory

HOMEWORK 2 (25 Points)

Due Date: June 1, 2018

1 [5 POINTS] BITS AND PIECES

- [1$Point$] What kind of reasoning is used in mathematical induction? Inductive or deductive? Explain your answer in some detail.
- [1$Point$] How would Hal Abelson and Gerry Sussman answer to the question whether computer science is a science or not?
- [1$Point$] Give a real life example for the logical fallacy known as *argumentum ad populum*.
- [1$Point$] Given a real life example for the logical fallacy known as *argumentum ad hominem*.
- [1$Point$] What is the biggest crime one can commit in a university campus?

2 [3 POINTS] CONVERTING DECLARATIVE SENTENCES TO THE LANGUAGE OF PROPOSITIONAL LOGIC

Use $\neg, \wedge, \vee$, and $\rightarrow$ to express the following declarative sentences in propositional logic; in each case state what your respective propositional atoms p, q, etc. mean:

- [$1Point$] If you have read the lecture notes and if you have done the first three homework assignments, then you should be in good shape for the first exam; otherwise, you will have a problem.
- [$1Point$] Cancer will not be cured unless its cause is determined and a new drug for cancer is found.
- [$1Point$] If Smith has installed central heating, then he has sold his car, or he has not paid his mortgage.

3 [9 POINTS] ON NATURAL DEDUCTION

Prove the following sequents by using the calculus of propositional logic.

- [$1Points$] $(p \rightarrow r) \wedge (q \rightarrow r) \vdash p \wedge q \rightarrow r$
- [$1Points$] $q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$
- [$1Points$] $p \rightarrow q \vdash \big((p \wedge q) \rightarrow p\big) \wedge \big(p \rightarrow (p \wedge q)\big)$
- [$2Points$] $\vdash (p \rightarrow q) \rightarrow ((r \rightarrow s) \rightarrow (p \wedge r \rightarrow q \wedge s))$
- [$2Points$] $\vdash \neg p \rightarrow (p \rightarrow (p \rightarrow q))$
- [$2Points$] $q \vdash (p \wedge q) \vee (\neg p \wedge q)$

4 [5 POINTS] MORE PROPOSITIONAL LOGIC SEQUENTS TO BE PROVEN

Prove the following sequents by using the natural deduction rules:

- [$1Point$] $p \wedge \neg p \vdash \neg(r \rightarrow q) \wedge (r \rightarrow q)$
- [$1Point$] $\neg(\neg p \vee \neg q) \vdash p \wedge q$
- [$1Point$] $(p \rightarrow q) \rightarrow r, s \rightarrow \neg p, t, (\neg s \wedge t) \rightarrow q \vdash r$
- [$1Point$] $\vdash ((p \rightarrow q) \wedge (q \rightarrow p)) \rightarrow ((p \vee q) \rightarrow (p \wedge q))$
- [$1Point$] $\vdash ((p \rightarrow q) \rightarrow q) \rightarrow ((q \rightarrow p) \rightarrow p)$

5 [3 POINTS] AVOID PITFALLS WHEN USING MATHEMATICAL INDUCTION

This question is designed to check whether you still remember the material from BIL 132. Let us prove below that all horses are the same color by mathematical induction on the number of horses in a given set.

BASIS: If there is just one horse then it's the same color as itself, so the basis is trivial.

INDUCTIVE HYPOTHESIS: Horses $1,\ldots,n-1$ are the same color, and similarly horses $2,\ldots,n$ are the same color.

INDUCTIVE STEP: The middle horses, $\{2,\ldots,n-1\}$, can't change color when they are in different groups. So, horses 1 and n must be the same color as well, by transitivity. □

What, if anything, is wrong with this proof?