Middle East Technical University - Department of Computer Engineering

# CENG 223

## Discrete Computational Structures

Fall '2018-2019

## Homework 1

Due Date: Oct 31,2018, 23:55
No late submissions !

# Question 1

1. Determine if the following compound propositions are a **tautology**, a **contradiction** or **neither one of them**. Construct a truth table for each proposition.

   (a) $((p \rightarrow q) \leftrightarrow (p \wedge \neg r)) \rightarrow \neg(q \wedge r)$

   (b) $\neg((p \vee q) \wedge (p \rightarrow q) \vee (q \rightarrow \neg p))$

2. Determine if the following predicate logic arguments are **valid** or **invalid**. Explain why you think the argument is **valid** or **invalid**. You do not need to make a formal proof for these questions. (**Hint:** Using counterexamples might be beneficial.)

   (a) $\exists x P(x) \wedge \exists x Q(x) \rightarrow \exists x (P(x) \wedge Q(x))$

   (b) $\forall x P(x) \rightarrow \exists x P(x)$

# Question 2

Show that $(\neg p \vee p) \rightarrow ((p \wedge \neg q) \rightarrow r)$ and $(q \vee r) \vee \neg p$ are logically equivalent. You should use tables 6, 7, and 8 given in pages 27 and 28 of your textbook.
In each step give the reference to the law **OR** the table.

# Question 3

Let $W(x)$ be "$x$ works in the lab", $Older(x, y)$ be "$x$ is older than $y$", $Phd(x)$ be "$x$ is a Phd. student", $Has\_CS\_Degree(x)$ be "$x$ has a CS degree", $Knows(x, y)$ be "$x$ knows $y$".

Use these predicates to express the following statements using quantifiers $\forall$ and $\exists$.

1. Everybody works in the lab has a CS degree.

2. All Phd. students working in the lab knows each other.

3. Cenk is the oldest person working in the lab.
4. Everyone working in the lab is a Phd. student except for Selen.
5. Not all the people working in the lab knows everyone working in the lab.
6. There are at most two Phd. students.
7. There are at least three people older than Gizem.
8. There is exactly one person who is doing Phd. and working in the lab.

# Question 4

Prove the following by using only the natural deduction rules for $\vee, \wedge, \rightarrow$, and $\neg$ introduction and elimination.
Any other rules/lemmas used should be proven by natural deduction as well.

$$(p \rightarrow r) \vee (q \rightarrow r) \vdash (p \wedge q) \rightarrow r$$

# Question 5

Prove the following by using only the natural deduction rules for $\vee, \wedge, \rightarrow$, and $\neg$ introduction and elimination.
Any other rules/lemmas used should be proven by natural deduction as well.

$$(\neg p \vee \neg q) \vdash (p \wedge q) \rightarrow r$$

# Question 6

Prove the following by using only the natural deduction rules for $\vee, \wedge, \rightarrow, \neg, \forall$, and $\exists$ introduction and elimination. Any other rules/lemmas used should be proven by natural deduction as well.

$$\forall x(P(x) \rightarrow (Q(x) \rightarrow R(x))), \exists x(P(x)), \forall x(\neg R(x)) \vdash \exists x(\neg Q(x))$$

# 1 Regulations

1. You have to write your answers to the provided sections of the template answer file given.
2. **Late Submission:** Not allowed!
3. **Cheating:** We have zero tolerance policy for cheating. People involved in cheating will be punished according to university regulations.
4. **Newsgroup:** You must follow the newsgroup `https://cow.ceng.metu.edu.tr/News/` for discussions and possible updates on a daily basis.
5. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

# 2 Submission

Submission will be done via COW. Download the given template answer file "the1.tex". When you finish your exam upload the .tex file with the same name to COW.
**Note:** You cannot submit any other files. Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex the1.tex
```