Student Information

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Answer 1

1. (a) is a **tautology**.

Table 1: Truth table for Question 1.1.a

p	q	r	$\neg r$	$((p \to q)$	$\leftrightarrow$	$(p \wedge \neg r))$	$\to$	$\neg(q \wedge r)$
T	T	T	F	T	F	F	**T**	F
T	T	F	T	T	T	T	**T**	T
T	F	T	F	F	T	F	**T**	T
T	F	F	T	F	F	T	**T**	T
F	T	T	F	T	F	F	**T**	F
F	T	F	T	T	F	F	**T**	T
F	F	T	F	T	F	F	**T**	T
F	F	F	T	T	F	F	**T**	T

   (b) is a **contradiction**.

Table 2: Truth table for Question 1.1.b

p	q	$\neg p$	$\neg$	$((p \vee q)$	$\wedge$	$(p \to q)$	$\vee$	$(q \to \neg p))$
T	T	F	**F**	T	T	T	T	F
T	F	F	**F**	T	F	F	T	T
F	T	T	**F**	T	T	T	T	T
F	F	T	**F**	F	F	T	T	T

2. (a) $\exists x P(x) \wedge \exists x Q(x) \to \exists x (P(x) \wedge Q(x))$ is **invalid**.
   i. Let us say U is integers; $P(x)$ states "x is even", and $Q(x)$ states "x is odd".
   ii. The left hand side is T, and the right hand side is F.
   iii. Since $T \to F \equiv F$, the statement is **invalid**.

   (b) $\forall x P(x) \to \exists x P(x)$ is **valid**.
   i. Given logical statement can be translated as "for all x $P(x)$, there is at least one x such that $P(x)$."
   ii. Since each element inside $\exists x P(x)$ also exists inside $\forall x P(x)$; if $\forall x P(x)$ is T, then $\exists x P(x)$ also must be T. Similarly, if $\forall x P(x)$ is F, then $\exists x P(x)$ also must be F.
   iii. $T \to T \equiv T$, and $F \to F \equiv T$, the statement is **valid**.

Answer 2

$$
\begin{array}{rcl|l}
(\neg p \vee p) \rightarrow ((p \wedge \neg q) \rightarrow r) & \equiv & T \rightarrow ((p \wedge \neg q) \rightarrow r) & \text{By using } \textbf{Negation laws} \\
& \equiv & F \vee ((p \wedge \neg q) \rightarrow r) & \text{By using } \textit{Table 7} \\
& \equiv & ((p \wedge \neg q) \rightarrow r) \vee F & \text{By using } \textbf{Commutative laws} \\
& \equiv & (p \wedge \neg q) \rightarrow r & \text{By using } \textbf{Identity laws} \\
& \equiv & \neg(p \wedge \neg q) \vee r & \text{By using } \textit{Table 7} \\
& \equiv & \neg p \vee q \vee r & \text{By using } \textbf{De Morgan's laws} \\
& \equiv & \neg p \vee (q \vee r) & \text{By using } \textbf{Associative laws} \\
& \equiv & (q \vee r) \vee \neg p & \text{By using } \textbf{Commutative laws}
\end{array}
$$

Thus, $(\neg p \vee p) \rightarrow ((p \wedge \neg q) \rightarrow r)$ and $(q \vee r) \vee \neg p$ are logically equivalent.

Answer 3

1. $\forall x(W(x) \rightarrow Has\_CS\_Degree(x))$
2. $\forall x \forall y((Phd(x) \wedge W(x)) \wedge (Phd(y) \wedge W(y) \rightarrow Knows(x,y)))$
3. $\forall x((W(x) \wedge W(Cenk) \wedge (x \neq Cenk)) \rightarrow Older(Cenk, x))$
4. $\forall x((W(x) \wedge (x \neq Selen) \wedge W(Selen)) \rightarrow Phd(x))$
5. $\exists x \exists y((W(x) \wedge W(y)) \wedge \neg Knows(x,y))$
6. $\forall x \forall y \forall z(((Phd(x) \wedge Phd(y)) \wedge (x \neq y)) \rightarrow (Phd(z) \rightarrow ((z = x) \vee (z = y))))$
7. $\exists x \exists y \exists z(Older(x, Gizem) \wedge Older(y, Gizem) \wedge Older(z, Gizem) \wedge (x \neq y) \wedge (y \neq z))$
8. $\exists x \forall y((Phd(x) \wedge W(x)) \wedge ((Phd(y) \wedge W(y)) \rightarrow (y = x)))$

Answer 4

Table 3: Proof of $(p \to r) \vee (q \to r) \vdash (p \wedge q) \to r$

1	$(p \to r) \vee (q \to r)$	*premise*
2	$p \to r$	*assumption*
3	$p \wedge q$	*assumption*
4	p	$\wedge$e 3
5	r	$\to$e $2, 4$
6	$(p \wedge q) \to r$	$\to$i $3 - 5$
7	$(p \to r) \to ((p \wedge q) \to r)$	$\to$i $2 - 6$
8	$q \to r$	*assumption*
9	$p \wedge q$	*assumption*
10	q	$\wedge$e 9
11	r	$\to$e $8, 10$
12	$(p \wedge q) \to r$	$\to$i $9 - 11$
13	$(q \to r) \to ((p \wedge q) \to r)$	$\to$i $8 - 12$
14	$(p \wedge q) \to r$	$\vee$e $1, 7, 13$

Answer 5

Table 4: Proof of $(\neg p \vee \neg q) \vdash (p \wedge q) \to r$

1	$\neg p \vee \neg q$	*premise*
2	$p \wedge q$	*assumption*
3	p	$\wedge$e 2
4	q	$\wedge$e 2
5	$\neg p$	*assumption*
6	$\perp$	$\neg$e $3, 5$
7	r	$\perp$e 6
8	$\neg p \to r$	$\to$i $5 - 7$
9	$\neg q$	*assumption*
10	$\perp$	$\neg$e $4, 9$
11	r	$\perp$e 10
12	$\neg q \to r$	$\to$i $9 - 11$
13	r	$\vee$e $1, 5 - 7, 9 - 11$
14	$(p \wedge q) \to r$	$\to$i $2 - 13$

Answer 6

Table 5: Proof of $\forall x(P(x) \to (Q(x) \to R(x))), \exists x(P(x)), \forall x(\neg R(x)) \vdash \exists x(\neg Q(x))$

1	$\forall x(P(x) \to (Q(x) \to R(x)))$	*premise*
2	$\exists x(P(x))$	*premise*
3	$\forall x(\neg R(x))$	*premise*
4	$P(c) \to (Q(c) \to R(c))$	$\forall$e 1
5	$P(c)$	*assumption*
6	$Q(c) \to R(c)$	$\to$e $4, 5$
7	$\neg R(c)$	$\forall$e 3
8	$Q(c)$	*assumption*
9	$R(c)$	$\to$e $6, 8$
10	$\perp$	$\neg$e $7, 9$
11	$\neg Q(c)$	$\neg$i $8 - 10$
12	$\exists x \neg Q(x)$	$\exists$i 11
13	$\exists x \neg Q(x)$	$\exists$e $2, 5 - 12$