TOBB ETU COMPUTER ENGINEERING DEPARTMENT

BİL 133 Combinatorics and Graph Theory

HOMEWORK 3 (25 Points)

Due Date: June 11, 2018

1 [3 POINTS] DRAWING PARSE TREES OF PROPOSITIONAL LOGIC FORMULAS

Draw the corresponding parse trees of the following propositional logic formulas:

- $[1 Point]$ $p \to (\neg\neg q \vee (q \to p))$
- $[1 Point]$ $(((s \to (r \vee l)) \vee ((\neg q) \wedge r)) \to ((\neg(p \to s)) \to r))$
- $[1 Point]$ $(p \to q) \wedge (\neg r \to (q \vee (\neg p \wedge r)))$

2 [2 POINTS] VALIDITY AND SATISFIABILITY OF FORMULAS

We call a formula **valid** if it always computes T(true), no matter which truth values we choose for propositional atoms. We call a formula **satisfiable** if it computes T for at least one set of truth values for propositional atoms.
Is the formula $\neg((q \to \neg p) \wedge (p \to r \vee q))$ valid? Is it satisfiable?

3 [3 POINTS] INTRODUCING NEW CONNECTIVES TO PROPOSITIONAL LOGIC

Let $*$ be a new logical connective such that $p * q$ does *not* hold iff(if and only if) p and q are either both false or both true.

- $[1 Point]$ Write down the truth table for $p * q$

- [$1Point$] Write down the truth table for $(p * p) * (q * q)$

- [$1Point$] You should know $*$ already as a logic gate in circuit design. What is it called?

4 [4 POINTS] MATHEMATICAL INDUCTION WITH WEIRD IMPLICATIONS

We spent a great deal of time on mathematical induction. Everybody shall be comfortable with the following idea:
If $P(1)$ is true, and $P(i) \rightarrow P(i+1)$ is true for all positive integers i, then $P(i)$ is true for all positive integers i.
Sometimes, we may not have such implications handy but have only a weird set of implications at hand. Assume we have the following information about the property P on positive integers:

- $P(1)$ and $P(2)$ are true,

- $P(n) \rightarrow P(n-1)$ is true for all $n > 1$, and

- $P(n) \wedge P(2) \rightarrow P(2n)$ for all positive integers n.

Is it possible for us to conclude that $P(n)$ is true for all integers n, from that much of information about property P? Justify your claim.

5 [3 POINTS] ON REASONING

Assume that you are asked you to prove that $\mathscr{F}(3n)$ is even for all $n \geq 1$, where $\mathscr{F}(n)$ is the n^{th} Fibonacci number.
Further assume that you performed the following set of actions. You computed the value of $\mathscr{F}(n)$ for thousands of different values of n. In all these instances, you noticed that $\mathscr{F}(n)$ is even if and only if n is divisible by 3. You then claimed that the thousands of observations you made constitutes as a proof of correctness of the given assertion.

- [$1Points$] What kind of reasoning you used in the above process?

- [$1Points$] Is your methodology scientific?

- [$1Points$] Is the scientific methodology welcomed in our class, or is it treated as garbage?

6 [3 POINTS] MATHEMATICAL INDUCTION IN ACTION

Prove that $\mathscr{F}(3n)$ is even for all $n \geq 1$, where $\mathscr{F}(n)$ is the n^{th} Fibonacci number.

7 [4 POINTS] ON SYNTAX OF PREDICATE LOGIC

Let m be a constant, f a function symbol with one argument and S and B two predicate symbols, each with two arguments. Which of the following strings are formulas in predicate logic? Specify a reason for failure for strings which aren't.

- $[0,5 Points]$ $S(m, x)$
- $[0,5 Points]$ $B(m, f(m))$
- $[0,5 Points]$ $f(m)$
- $[0,5 Points]$ $B(B(m, x), y)$
- $[0,5 Points]$ $S(B(m), z)$
- $[0,5 Points]$ $(B(x, y) \rightarrow (\exists z S(z, y)))$
- $[0,5 Points]$ $(S(x, y) \rightarrow S(y, f(f(x))))$
- $[0,5 Points]$ $(B(x) \rightarrow B(B(x)))$

8 [3 POINTS] TRANSLATION FORM ENGLISH TO THE LANGUAGE OF PREDICATE LOGIC

Use the predicates

$$A(x, y) : x \text{ admires } y$$

$$B(x, y) : x \text{ attended } y$$

$$P(x) : x \text{ is a professor}$$

$$S(x) : x \text{ is a student}$$

$$L(x) : x \text{ is a lecture}$$

and the constant

$$m : \text{ Mary}$$

to translate the following declarative sentences into the language of predicate logic:

- $[0,5 Point]$ Mary admires every professor. (Hint: The answer is *not* $\forall x A(m, P(x))$ since it is not even a formula!)
- $[0,5 Point]$ Some professor admires Mary.
- $[0,5 Point]$ Mary admires herself.

- $[0,5 Point]$ No student attended every lecture.
- $[0,5 Point]$ No lecture was attended by every student.
- $[0,5 Point]$ No lecture was attended by any student.