

# Enrichment Problems

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## Introduction

I want to give you guy some problems that help in our fundamental understanding of discrete time systems.

## FIR Filter

An LTI system has a frequency response

$$H(e^{j\hat{\omega}}) = (1 - e^{j\pi/2}e^{-j\omega})(1 - e^{-j\pi/2}e^{-j\omega})(1 + e^{-j\omega})$$

The input to the system is

$$x[n] = 5 + 20 \cos(0.5\pi n + 0.25\pi) + 10\delta[n - 3]$$

Find the output of the system

## Another one

An LTI system has the difference equation

$$y[n] = -x[n] + 2x[n - 2] - x[n - 4]$$

I. Find the impulse response  $h[n]$  and plot it

II. Determine an equation for the frequency response  $H(e^{j\hat{\omega}})$  and express it in the form

$$R(e^{j\hat{\omega}})e^{-j\hat{\omega}n_0}$$

where  $n_0$  is an integer.

## Sample and Reconstruct

The signal  $x(t)$  is to be reconstructed by directly putting an ideal continuous to discrete converter in cascade with an ideal digital to discrete converter, each samples at a rate of  $f_s = 500$ (samples/sec).  $x(t)$  is defined as follows.

$$x(t) = 5 \cos(200\pi t) + 10 \cos(800\pi t + \frac{\pi}{3}) + 2 \cos(2200\pi t + \frac{\pi}{6})$$

I. Draw the block diagram of the entire process

II. Determine the output function  $y(t)$ .

## Discrete Fourier Transform

Find the  $N$  point DFT of  $u = [\sin(ja)]_{j=0}^{N-1}$ , in which  $N$  is a positive integer and  $a$  is a given constant number. To avoid trivialities suppose  $a$  is not an integer multiple of  $\pi$ .