# Summary of results for CIR\_alpha\_1\_beta\_1\_sigma\_0.25\_x0\_3

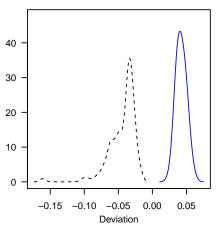
June 9, 2020

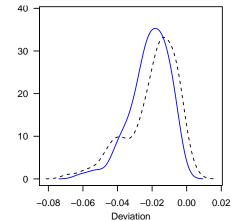
$$M = 10$$

#### m = 1

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

#### Posterior mean





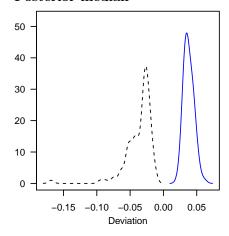


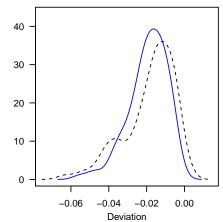
Based on up to 100 results.

Table 1: RMSE

	beta	sigma2
Euler	0.049	0.025
Milstein	0.043	0.024

#### Posterior median







Based on up to 100 results.

Table 2: RMSE

	beta	sigma2
Euler	0.042	0.023
Milstein	0.039	0.022

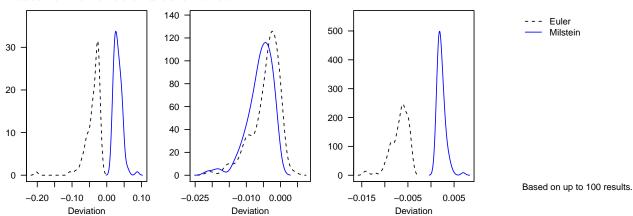


Table 3: RMSE

	beta	sigma2	covariance
Euler	0.047	0.006	0.007
Milstein	0.034	0.007	0.002

#### Number of iterations and effective sample size

	numIterations mean	$numIterations\ sd$	multivarESS mean	${\it multivarESS sd}$
Euler	25526341	1420153	2800847	178293
Milstein	7701884	519918	839945	61821

	ARparam mean	ARparam sd
$\overline{\mathrm{td}}_{-}\mathrm{E}$	0.481	0.01
$td\_M$	0.484	0.01

 $\mathbf{m}=\mathbf{2}$  mean of # of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

total # of negative proposals:

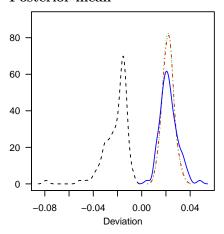
DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
0	19	2	0

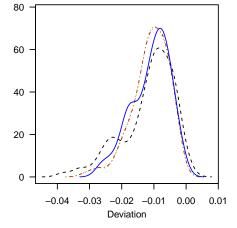
ratio of negative proposals and number of iterations:

	DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
min	0	0e+00	0e+00	0
max	0	5e-07	3e-07	0
median	0	0e+00	0e+00	0
mean	0	0e+00	0e+00	0

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

#### Posterior mean





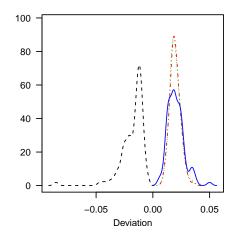


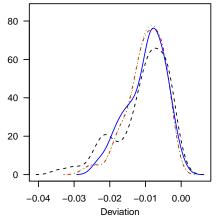
Based on up to 100 results.

Table 8: RMSE

	beta	sigma2
MBE-E	0.024	0.014
$\mathrm{MBE}\text{-}\mathrm{M}$	0.022	0.012
MBM-M	0.023	0.013
DBM-M	0.022	0.012

#### Posterior median





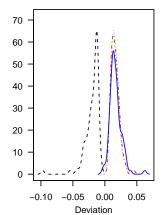


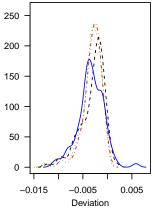
Based on up to 100 results.

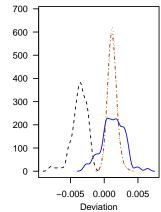
Table 9: RMSE

	beta	sigma2
MBE-E	0.021	0.013
MBE-M	0.020	0.011
MBM-M	0.021	0.011
DBM-M	0.020	0.011

#### Posterior variance and covariance







--- MBE-E --- MBE-M --- MBM-M

Based on up to 100 results.

Table 10: RMSE

	beta	sigma2	covariance
MBE-E	0.023	0.004	0.004
$\mathrm{MBE}\text{-}\mathrm{M}$	0.017	0.004	0.001
MBM-M	0.019	0.004	0.002
$\mathrm{DBM} ext{-}\mathrm{M}$	0.017	0.004	0.001

#### Number of iterations and effective sample size

	numIterations mean	numIterations sd	multivarESS mean	multivarESS sd
MBE-E	8703286	405038	620479	34308
$\mathrm{MBE}\text{-}\mathrm{M}$	2928042	110520	202089	10295
MBM-M	208360	12831	14837	1266
DBM-M	2827604	101161	203710	10969

	ARpath mean	ARpath sd	ARparam mean	ARparam sd
MBE-E	0.960	0.005	0.439	0.009
$\mathrm{MBE}\text{-}\mathrm{M}$	0.956	0.005	0.440	0.009
MBM-M	1.000	0.000	0.440	0.009
$\mathrm{DBM} ext{-}\mathrm{M}$	0.972	0.005	0.440	0.009

m=5  $$\rm{mean}$$  of  $\rm{\#}$  of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

total # of negative proposals:

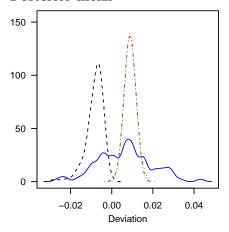
DBM_td_M_pd_M	MB_td_E_pd_E	$MB\_td\_M\_pd\_E$	MB_td_M_pd_M
0	1	0	0

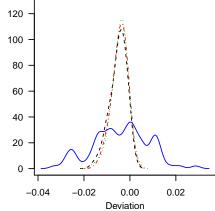
ratio of negative proposals and number of iterations:

	DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
min	0	0e+00	0	0
max	0	2e-07	0	0
median	0	0e+00	0	0
mean	0	0e+00	0	0

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

#### Posterior mean





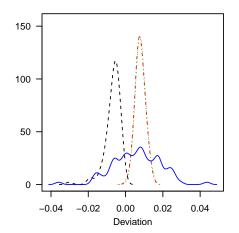


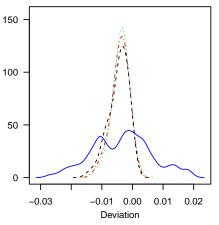
Based on up to 100 results.

Table 15: RMSE

	beta	sigma2
MBE-E	0.009	0.006
MBE-M	0.009	0.005
MBM-M	0.014	0.013
DBM-M	0.009	0.005

#### Posterior median





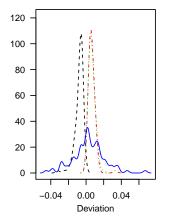
--- MBE-E .... MBE-M --- MBM-M

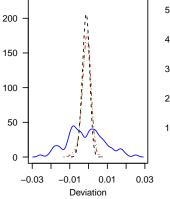
Based on up to 100 results.

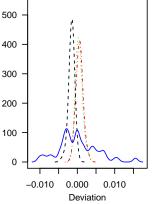
Table 16: RMSE

	beta	sigma2
MBE-E	0.008	0.006
$\mathrm{MBE}\text{-}\mathrm{M}$	0.008	0.005
MBM-M	0.014	0.010
DBM-M	0.008	0.005

#### Posterior variance and covariance







MBM-M DBM-M

--- MBE-E ···· MBE-M

Based on up to 100 results.

Table 17: RMSE

	beta	sigma2	covariance
MBE-E	0.009	0.002	0.002
$\mathrm{MBE} ext{-}\mathrm{M}$	0.007	0.003	0.001
MBM-M	0.018	0.010	0.005
$\mathrm{DBM}\text{-}\mathrm{M}$	0.008	0.002	0.001

#### Number of iterations and effective sample size

	numIterations mean	numIterations sd	multivarESS mean	multivarESS sd
MBE-E	6985100	265716	207675	11536
$\mathrm{MBE}\text{-}\mathrm{M}$	1590029	52271	45518	2850
MBM-M	39749	2797	1092	193
DBM-M	1556197	37966	45555	2561

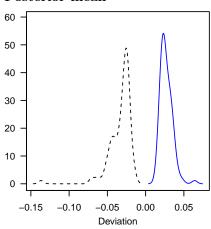
	ARpath mean	ARpath sd	ARparam mean	ARparam sd
MBE-E	0.972	0.003	0.336	0.007
$\mathrm{MBE}\text{-}\mathrm{M}$	0.959	0.004	0.336	0.007
MBM-M	0.991	0.001	0.336	0.008
DBM-M	0.973	0.003	0.336	0.007

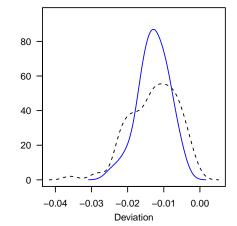
M = 20

m = 1

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

#### Posterior mean





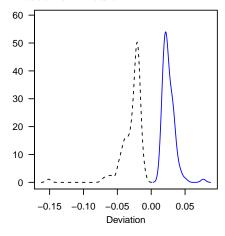


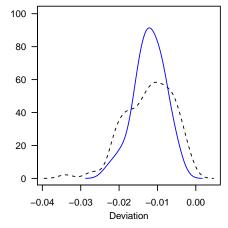
Based on up to 100 results.

Table 20: RMSE

	beta	sigma2
Euler	0.037	0.014
Milstein	0.028	0.014

#### Posterior median



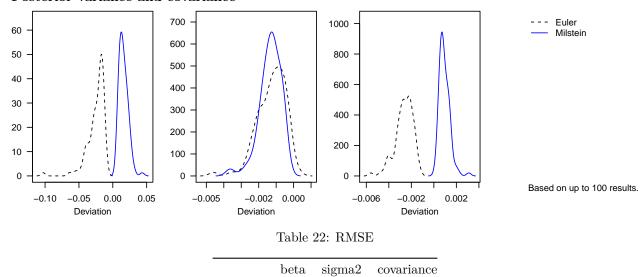




Based on up to 100 results.

Table 21: RMSE

	beta	sigma2
Euler Milstein	0.034 $0.027$	0.014 0.013



Number of iterations and effective sample size

 $\operatorname{Euler}$ 

Milstein

	numIterations mean	numIterations sd	multivarESS mean	multivarESS sd
Euler	24235310	873174	2875908	134151
Milstein	4710795	133551	551467	23891

0.001

0.001

0.003

0.001

0.028

0.017

	ARparam mean	ARparam sd
$\overline{\mathrm{td}}_{-}\mathrm{E}$	0.419	0.009
$td\_M$	0.420	0.009

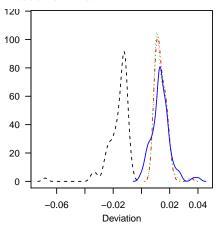
 $\mathbf{m}=\mathbf{2}$  mean of # of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

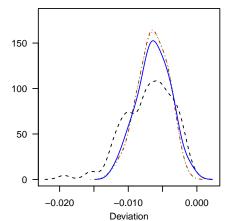
total # of negative proposals:

DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
0	0	0	0

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

#### Posterior mean





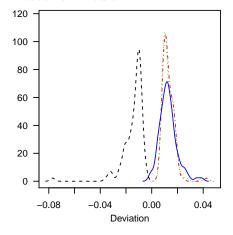
MBE-E MBE-M MBM-M DBM-M

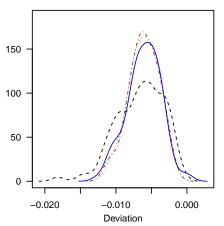
Based on up to 100 results.

Table 26: RMSE

	beta	sigma2
MBE-E	0.018	0.008
$\mathrm{MBE} ext{-}\mathrm{M}$	0.014	0.007
MBM-M	0.015	0.007
DBM-M	0.014	0.007

#### Posterior median



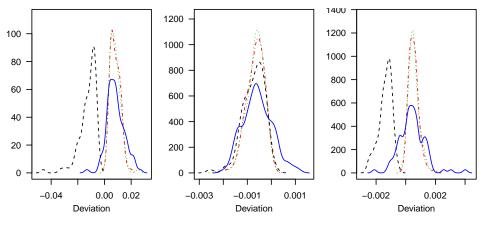




Based on up to 100 results.

Table 27: RMSE

	beta	sigma2
MBE-E	0.017	0.007
$\mathrm{MBE}\text{-}\mathrm{M}$	0.014	0.006
MBM-M	0.014	0.007
DBM-M	0.014	0.006



--- MBE-E .... MBE-M — MBM-M .--- DBM-M

Based on up to 100 results.

Table 28: RMSE

	beta	sigma2	covariance
MBE-E	0.014	0.001	0.001
$\mathrm{MBE}\text{-}\mathrm{M}$	0.009	0.001	0.001
MBM-M	0.011	0.001	0.001
DBM-M	0.009	0.001	0.001

#### Number of iterations and effective sample size

	numIterations mean	${\bf numIterations~sd}$	multivarESS mean	multivarESS sd
MBE-E	8463062	287951	492772	23810
MBE-M	1935561	43285	109873	5009
$\mathrm{MBM} ext{-}\mathrm{M}$	179130	6669	10264	736
DBM-M	1883743	43523	109705	4979

	ARpath mean	ARpath sd	ARparam mean	ARparam sd
MBE-E	0.975	0.003	0.363	0.008
MBE-M	0.972	0.003	0.364	0.007
MBM-M	1.000	0.000	0.364	0.007
DBM-M	0.981	0.003	0.364	0.007

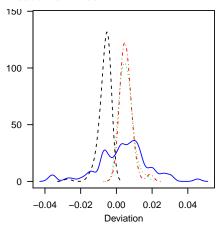
m=5  $$\rm{mean}$$  of  ${\rm \#}$  of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

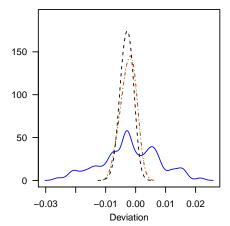
total # of negative proposals:

DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
0	0	0	0

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

#### Posterior mean





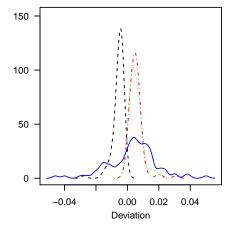
--- MBE-E --- MBE-M --- MBM-M --- DBM-M

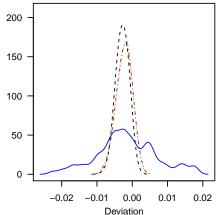
Based on up to 100 results.

Table 32: RMSE

	beta	sigma2
MBE-E	0.007	0.003
$\mathrm{MBE}\text{-}\mathrm{M}$	0.006	0.003
MBM-M	0.015	0.010
$\mathrm{DBM} ext{-}\mathrm{M}$	0.006	0.003

#### Posterior median



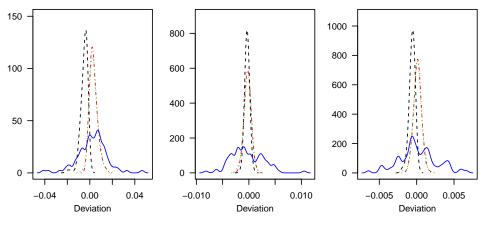


--- MBE-E .... MBE-M — MBM-M

Based on up to 100 results.

Table 33: RMSE

	beta	sigma2
MBE-E	0.007	0.003
$\mathrm{MBE}\text{-}\mathrm{M}$	0.007	0.003
MBM-M	0.016	0.009
DBM-M	0.007	0.003



--- MBE-E .... MBE-M — MBM-M ... DBM-M

Based on up to 100 results.

Table 34: RMSE

	beta	sigma2	covariance
MBE-E	0.006	0.000	0.001
$\mathrm{MBE}\text{-}\mathrm{M}$	0.005	0.001	0.001
MBM-M	0.013	0.003	0.002
DBM-M	0.005	0.001	0.000

#### Number of iterations and effective sample size

	numIterations mean	${\bf numIterations~sd}$	multivarESS mean	multivarESS sd
MBE-E	6758898	174695	130635	6279
$\mathrm{MBE}\text{-}\mathrm{M}$	950894	22130	17775	1232
MBM-M	37288	1714	662	98
DBM-M	934422	19399	17770	1377

	ARpath mean	ARpath $sd$	ARparam mean	$\mathbf{ARparam\ sd}$
MBE-E	0.983	0.002	0.258	0.005
MBE-M	0.975	0.003	0.258	0.005
${ m MBM} ext{-}{ m M}$	0.995	0.000	0.258	0.006
$\mathrm{DBM} ext{-}\mathrm{M}$	0.983	0.002	0.258	0.005

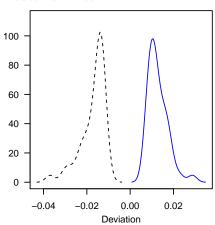
ARpath mean	ARnath sd	ARparam mean	ARparam sd
Anpam mean	Anpam su	Anparam mean	Anparam su

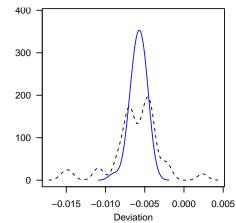
M = 50

m = 1

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

#### Posterior mean





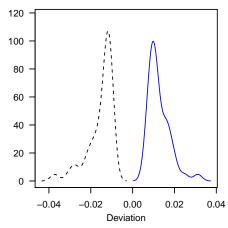


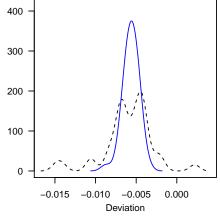
Based on up to 42 results.

Table 37: RMSE

	beta	sigma2
Euler	0.018	0.007
Milstein	0.014	0.006

#### Posterior median





--- Euler — Milstein

Based on up to 42 results.

Table 38: RMSE

	beta	sigma2
Euler	0.017	0.007
Milstein	0.014	0.006

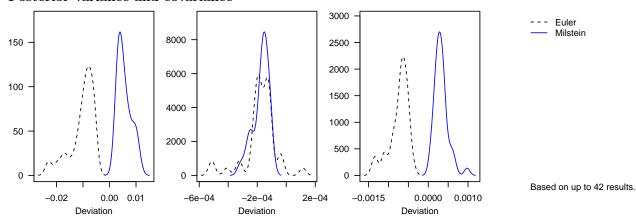


Table 39: RMSE

	beta	sigma2	covariance
Euler	0.011	0	0.001
Milstein	0.006	0	0.000

# Number of iterations and effective sample size

	numIterations mean	${\bf numIterations~sd}$	multivarESS mean	${\it multivarESS \ sd}$
Euler	22636393	606150	2564921	98128
Milstein	2151260	55226	240572	10524

	ARparam mean	ARparam sd
td_E	0.320	0.007
$td\_M$	0.321	0.006

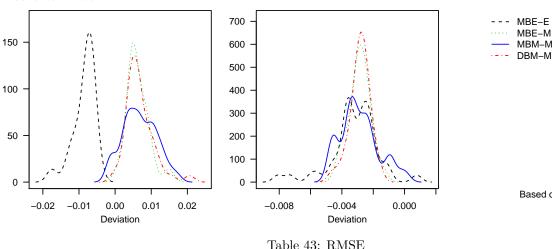
m = 2mean of # of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

total # of negative proposals:

DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
0	0	0	0

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

#### Posterior mean



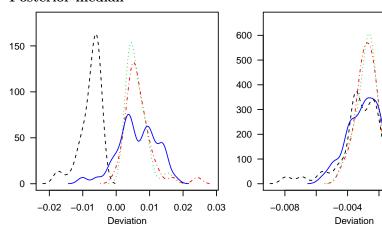
Based on up to 42 results.

MBM-M

Table 43: RMSE

	beta	sigma2
MBE-E	0.009	0.004
MBE-M	0.007	0.003
MBM-M	0.008	0.003
$\mathrm{DBM} ext{-}\mathrm{M}$	0.008	0.003

#### Posterior median



MBE-E MBE-M MBM-M DBM-M

Based on up to 42 results.

0.000

Table 44: RMSE

	beta	sigma2
MBE-E	0.008	0.003
$\mathrm{MBE}\text{-}\mathrm{M}$	0.007	0.003
MBM-M	0.009	0.003
DBM-M	0.008	0.003

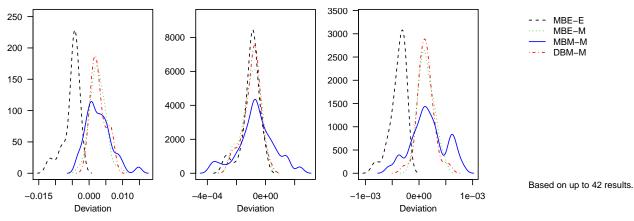


Table 45: RMSE

	beta	sigma2	covariance
MBE-E	0.006	0	0
$\mathrm{MBE} ext{-}\mathrm{M}$	0.003	0	0
MBM-M	0.005	0	0
DBM-M	0.004	0	0

#### Number of iterations and effective sample size

	numIterations mean	${\bf numIterations~sd}$	multivarESS mean	multivarESS sd
MBE-E	7669843	807004	282143	30905
$\mathrm{MBE}\text{-}\mathrm{M}$	951493	19843	33907	1575
MBM-M	161801	3126	5761	389
DBM-M	938174	23638	33939	1710

	ARpath mean	ARpath $sd$	ARparam mean	${\bf ARparam\ sd}$
MBE-E	0.986	0.001	0.26	0.005
$\mathrm{MBE}\text{-}\mathrm{M}$	0.984	0.002	0.26	0.005
${ m MBM} ext{-}{ m M}$	1.000	0.000	0.26	0.006
DBM-M	0.988	0.001	0.26	0.005

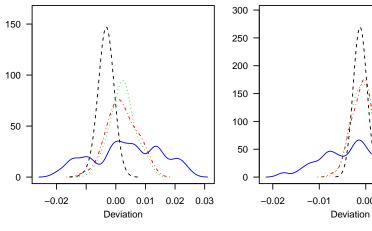
m = 5mean of # of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

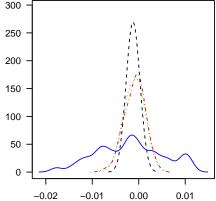
total # of negative proposals:

DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
0	0	0	0

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

#### Posterior mean





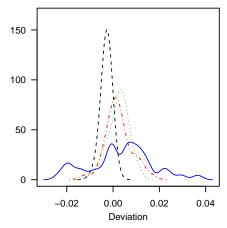
MBE-E MBE-M MBM-M DBM-M

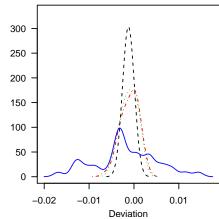
Based on up to 42 results.

Table 49: RMSE

	beta	sigma2
MBE-E	0.004	0.001
$\mathrm{MBE}\text{-}\mathrm{M}$	0.005	0.002
MBM-M	0.012	0.007
$\mathrm{DBM} ext{-}\mathrm{M}$	0.005	0.002

#### Posterior median







Based on up to 42 results.

Table 50: RMSE

	beta	sigma2
MBE-E	0.003	0.001
$\mathrm{MBE}\text{-}\mathrm{M}$	0.005	0.002
MBM-M	0.014	0.007
DBM-M	0.006	0.002

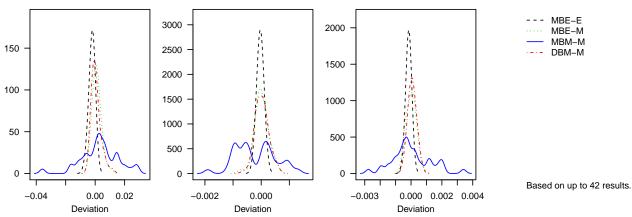


Table 51: RMSE

	beta	sigma2	covariance
MBE-E	0.003	0.000	0.000
MBE-M	0.003	0.000	0.000
MBM-M	0.013	0.001	0.001
DBM-M	0.003	0.000	0.000

#### Number of iterations and effective sample size

	numIterations mean	numIterations sd	multivarESS mean	multivarESS sd
MBE-E	5903543	192370	60842	3372
$\mathrm{MBE}\text{-}\mathrm{M}$	424196	17059	4273	366
MBM-M	34370	1234	363	55
DBM-M	423190	8194	4340	347

#### Acceptance rates

	ARpath mean	ARpath sd	ARparam mean	ARparam sd
MBE-E	0.990	0.001	0.172	0.004
$\mathrm{MBE}\text{-}\mathrm{M}$	0.986	0.001	0.172	0.004
${ m MBM} ext{-}{ m M}$	0.998	0.000	0.172	0.004
$\mathrm{DBM} ext{-}\mathrm{M}$	0.990	0.001	0.172	0.004

# # of missing results: 0

# Stan results (sampling from and optimizing the true posterior)

#### M = 10

# of missing results: 0

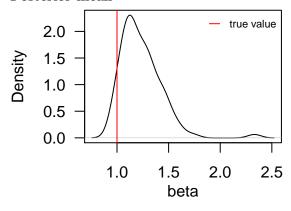
# Rhat > 1.01: 0 (out of 200)

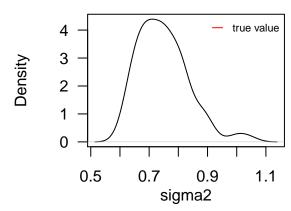
range of max. duration in seconds: (1070.5, 4113.8), median: 1659.4

median multivar ESS:  $6.44577\times10^5$ 

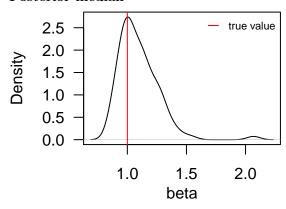
The following sections show density plots of the respective statistic calculated for the 100 simulated datasets.

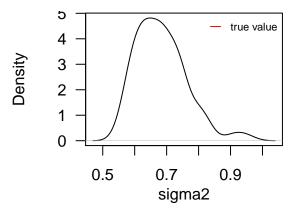
#### Posterior mean



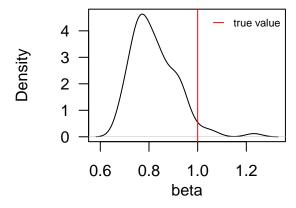


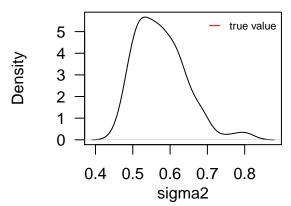
#### Posterior median

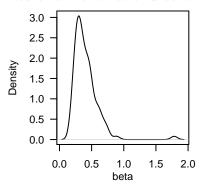


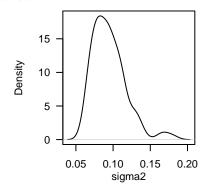


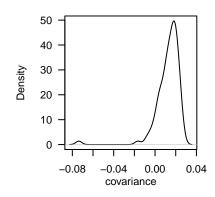
#### Optimized value



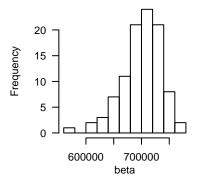


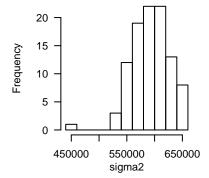


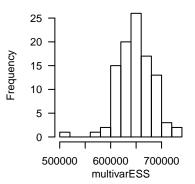




# Effective sample size







#### M = 20

# of missing results: 0

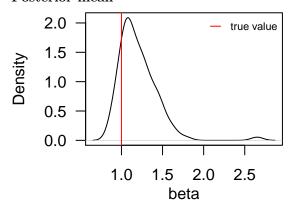
# Rhat > 1.01: 0 (out of 200)

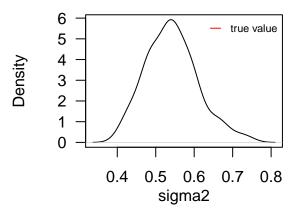
range of max. duration in seconds: (4262.5, 1.17888  $\times$   $10^4),$  median: 6564.5

median multivar ESS:  $6.90289\times10^5$ 

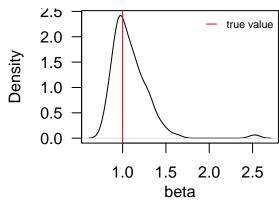
The following sections show density plots of the respective statistic calculated for the 100 simulated datasets.

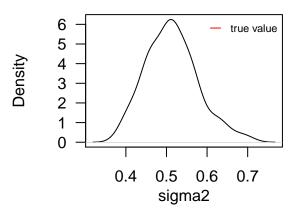
#### Posterior mean



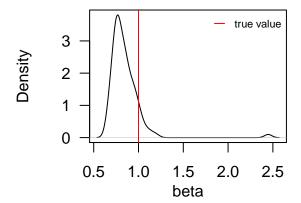


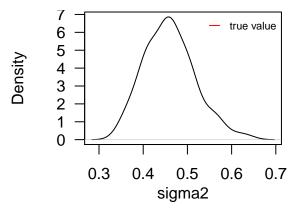
#### Posterior median

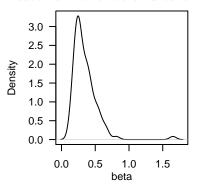


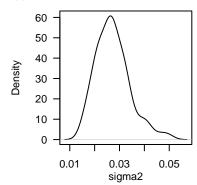


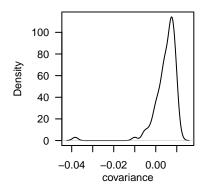
#### Optimized value



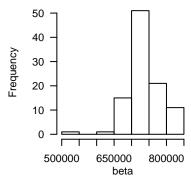


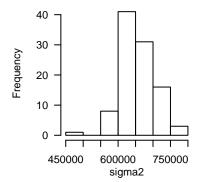


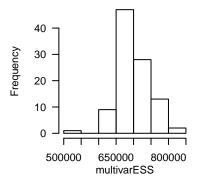




# Effective sample size







#### M = 50

# of missing results: 58

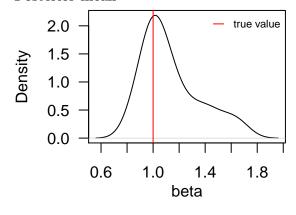
# Rhat > 1.01: 0 (out of 200)

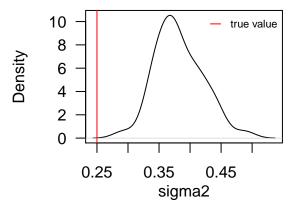
range of max. duration in seconds:  $(4.548 \times 10^4, 8.90532 \times 10^4)$ , median:  $6.24209 \times 10^4$ 

median multivar ESS:  $7.3862\times10^5$ 

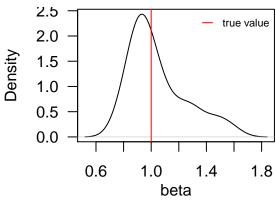
The following sections show density plots of the respective statistic calculated for the 100 simulated datasets.

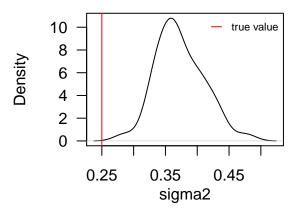
#### Posterior mean



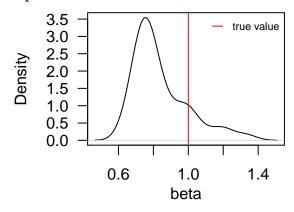


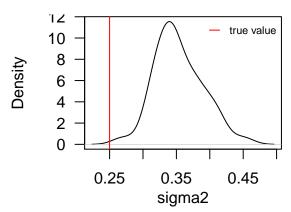
#### Posterior median

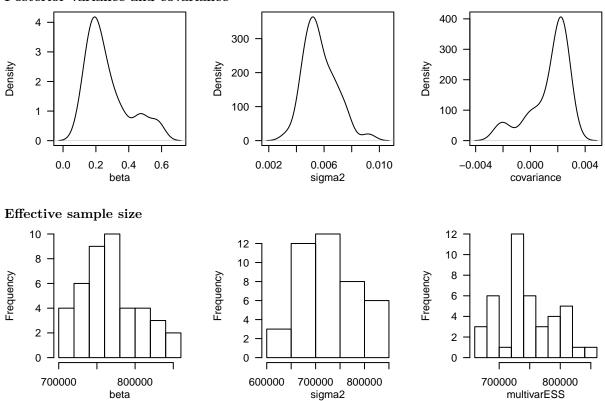




#### Optimized value







# of missing Stan results: 58