# Summary of results for GBM\_alpha\_1\_sigma\_2\_x0\_100

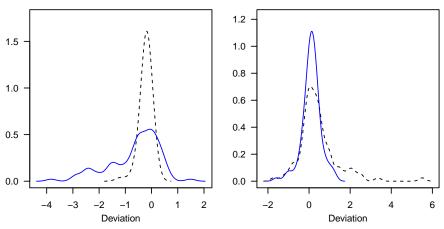
June 9, 2020

$$M = 10$$

### m = 1

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

### Posterior mean



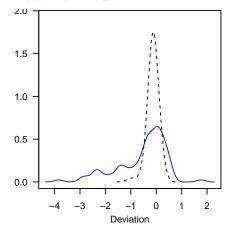
- - Euler - Milstein

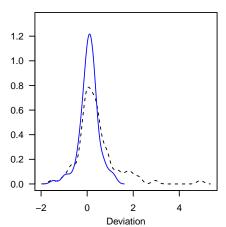
Based on up to 100 results.

Table 1: RMSE

	alpha	sigma2
Euler	0.304	1.059
Milstein	1.164	0.462

### Posterior median





- - Euler - Milstein

Based on up to 100 results.

Table 2: RMSE

	alpha	sigma2
Euler	0.230	0.945
Milstein	1.111	0.424

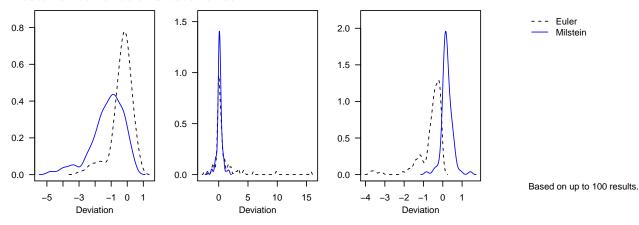


Table 3: RMSE

	alpha	sigma2	covariance
Euler	0.745	$2.254 \\ 0.501$	0.909
Milstein	1.586		0.346

## Number of iterations and effective sample size

	numIterations mean	$numIterations\ sd$	multivarESS mean	${\it multivarESS \ sd}$
Euler	26648991	888120	1218564	229564
Milstein	7487530	186164	145030	75716

	ARparam mean	ARparam sd
$\overline{\mathrm{td}_{-}\mathrm{E}}$	0.593	0.013
$td\_M$	0.395	0.087

 $\mathbf{m}=\mathbf{2}$  mean of # of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

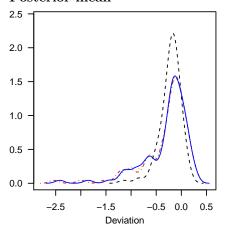
total # of negative proposals:

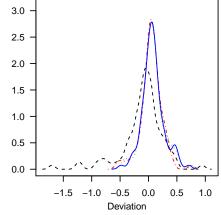
DBM_td_M_pd_M	MB_td_E_pd_E	$MB\_td\_M\_pd\_E$	${ m MB\_td\_M\_pd\_M}$
5407	1257639	601595	0

ratio of negative proposals and number of iterations:

	DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
min	0.0000000	0.0000012	0.0000007	0
max	0.0007643	0.0197556	0.0265252	0
median	0.0000000	0.0003267	0.0005302	0
mean	0.0000196	0.0013885	0.0021189	0

### Posterior mean





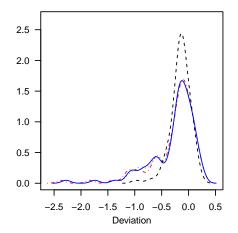


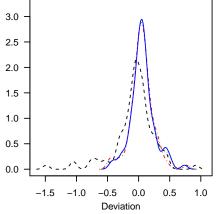
Based on up to 100 results.

Table 8: RMSE

	alpha	sigma2
MBE-E	0.299	0.384
$\mathrm{MBE}\text{-}\mathrm{M}$	0.538	0.211
MBM-M	0.539	0.206
DBM-M	0.561	0.195

### Posterior median





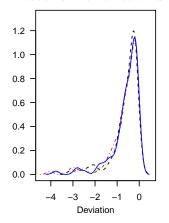


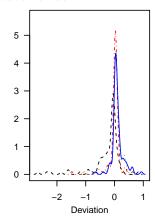
Based on up to 100 results.

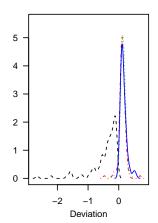
Table 9: RMSE

	alpha	sigma2
MBE-E	0.236	0.336
MBE-M	0.501	0.202
MBM-M	0.506	0.199
DBM-M	0.521	0.190

## Posterior variance and covariance







MBE-E MBE-M MBM-M DBM-M

Based on up to 100 results.

Table 10: RMSE

	alpha	sigma2	covariance
MBE-E	0.850	0.593	0.630
$\mathrm{MBE}\text{-}\mathrm{M}$	0.905	0.222	0.165
MBM-M	0.886	0.224	0.186
$\mathrm{DBM} ext{-}\mathrm{M}$	0.939	0.246	0.160

## Number of iterations and effective sample size

	numIterations mean	numIterations sd	multivarESS mean	multivarESS sd
MBE-E	9015192	279808	246491	45049
MBE-M	2830801	89107	28411	18188
MBM-M	332424	7726	6588	3104
$\mathrm{DBM} ext{-}\mathrm{M}$	2742274	59915	31756	21427

	ARpath mean	$\mathbf{ARpath}\ \mathbf{sd}$	ARparam mean	ARparam sd
MBE-E	0.784	0.055	0.530	0.010
$\mathrm{MBE}\text{-}\mathrm{M}$	0.704	0.084	0.442	0.063
MBM-M	1.000	0.000	0.443	0.062
$\mathrm{DBM} ext{-}\mathrm{M}$	0.752	0.082	0.443	0.062

m=5  $$\rm{mean}$$  of  $\rm{\#}$  of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 116

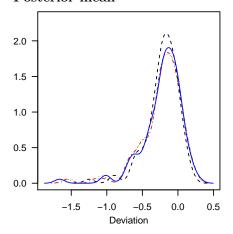
total # of negative proposals:

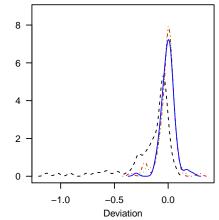
DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	${ m MB\_td\_M\_pd\_M}$
7	47467	23987	27

ratio of negative proposals and number of iterations:

	DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
min	0.0e+00	0.0000000	0.0000000	0.0000000
max	1.3e-06	0.0013412	0.0029764	0.0001385
median	0.0e+00	0.0000094	0.0000219	0.0000000
mean	0.0e+00	0.0000647	0.0001546	0.0000034

### Posterior mean





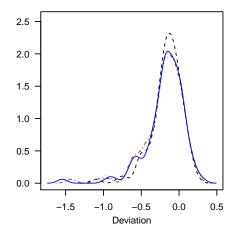


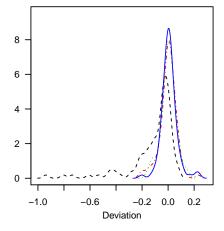
Based on up to 100 results.

Table 15: RMSE

	alpha	sigma2
MBE-E	0.314	0.273
$\mathrm{MBE}\text{-}\mathrm{M}$	0.359	0.060
MBM-M	0.355	0.065
DBM-M	0.364	0.080

### Posterior median





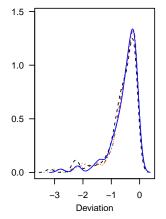


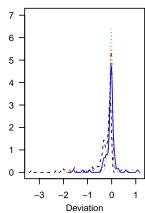
Based on up to 100 results.

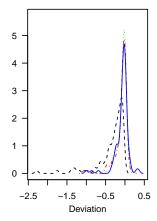
Table 16: RMSE

	alpha	sigma2
MBE-E	0.265	0.214
MBE-M	0.323	0.057
MBM-M	0.319	0.059
$\mathrm{DBM} ext{-}\mathrm{M}$	0.327	0.065

## Posterior variance and covariance









Based on up to 100 results.

Table 17: RMSE

	alpha	sigma2	covariance
MBE-E	0.827	0.654	0.503
$\mathrm{MBE}\text{-}\mathrm{M}$	0.738	0.218	0.194
MBM-M	0.717	0.283	0.192
$\mathrm{DBM} ext{-}\mathrm{M}$	0.739	0.296	0.201

## Number of iterations and effective sample size

	${\bf numIterations\ mean}$	${\bf numIterations~sd}$	${\it multivarESS mean}$	${\it multivarESS \ sd}$
MBE-E	7274387	201899	82976	14080
$\mathrm{MBE}\text{-}\mathrm{M}$	1544016	34696	10818	4108
MBM-M	82345	2641	844	251
DBM-M	1489759	40163	11825	4265

	ARpath mean	ARpath sd	ARparam mean	ARparam sd
MBE-E	0.851	0.035	0.399	0.007
$\mathrm{MBE}\text{-}\mathrm{M}$	0.782	0.053	0.380	0.011
MBM-M	0.939	0.034	0.380	0.012
$\mathrm{DBM} ext{-}\mathrm{M}$	0.833	0.045	0.380	0.010

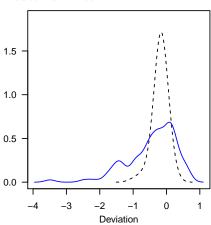
ARpath mean	ARpath sd	ARparam mean	ARparam sd
or or	op or o	op on- on	

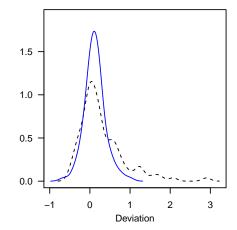
## M = 20

## m = 1

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

### Posterior mean





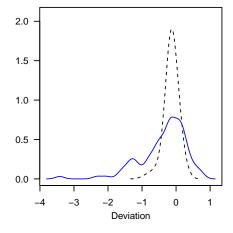


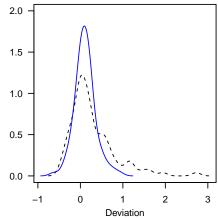
Based on up to 100 results.

Table 20: RMSE

	alpha	sigma2
Euler	0.282	0.638
Milstein	0.851	0.282

#### Posterior median



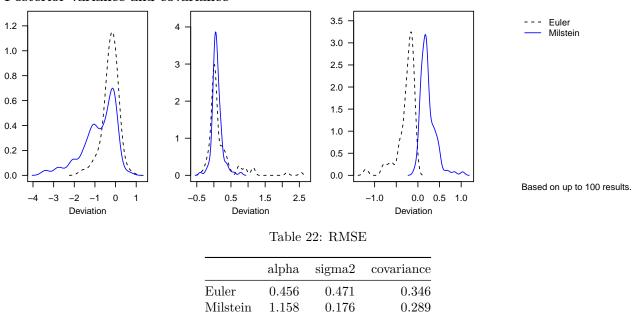




Based on up to 100 results.

Table 21: RMSE

	alpha	sigma2
Euler	0.244	0.600
Milstein	0.780	0.265



## Number of iterations and effective sample size

	numIterations mean	${\bf numIterations~sd}$	multivarESS mean	${\it multivarESS \ sd}$
Euler	25134301	879055	1273744	200463
Milstein	4454863	119491	146362	59491

	ARparam mean	ARparam sd
td_E	0.518	0.008
$td\_M$	0.425	0.060

 $\mathbf{m}=\mathbf{2}$  mean of # of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

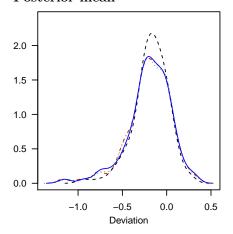
total # of negative proposals:

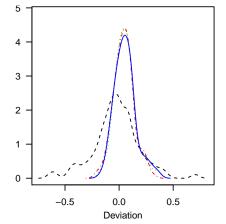
DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
0	21456	7042	0

ratio of negative proposals and number of iterations:

	DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
min	0	0.0000000	0.0000000	0
max	0	0.0006569	0.0008772	0
median	0	0.0000020	0.0000033	0
mean	0	0.0000253	0.0000397	0

### Posterior mean





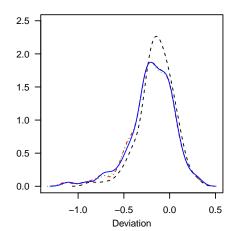


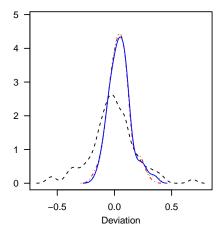
Based on up to 100 results.

Table 27: RMSE

	alpha	sigma2
MBE-E	0.266	0.211
$\operatorname{MBE-M}$	0.311	0.109
$\operatorname{MBM-M}$	0.315	0.112
DBM-M	0.318	0.101

### Posterior median





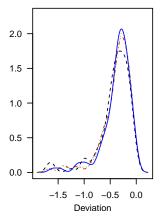


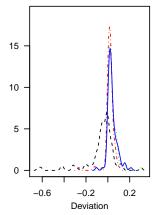
Based on up to 100 results.

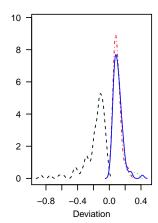
Table 28: RMSE

	alpha	sigma2
MBE-E	0.238	0.198
$\mathrm{MBE}\text{-}\mathrm{M}$	0.302	0.106
MBM-M	0.305	0.107
DBM-M	0.308	0.099

## Posterior variance and covariance







--- MBE-E MBE-M --- MBM-M DBM-M

Based on up to 100 results.

Table 29: RMSE

	alpha	sigma2	covariance
MBE-E	0.526	0.141	0.232
$\mathrm{MBE}\text{-}\mathrm{M}$	0.476	0.057	0.123
MBM-M	0.470	0.057	0.124
$\mathrm{DBM} ext{-}\mathrm{M}$	0.485	0.044	0.108

## Number of iterations and effective sample size

	numIterations mean	numIterations sd	multivarESS mean	multivarESS sd
MBE-E	8583614	217634	170827	33076
MBE-M	1816144	46540	24090	9090
MBM-M	300870	8498	6881	1437
$\mathrm{DBM} ext{-}\mathrm{M}$	1754024	174342	28089	8703

	ARpath mean	ARpath sd	ARparam mean	ARparam sd
MBE-E	0.842	0.031	0.442	0.006
$\mathrm{MBE}\text{-}\mathrm{M}$	0.799	0.040	0.417	0.012
MBM-M	1.000	0.000	0.417	0.012
$\mathrm{DBM} ext{-}\mathrm{M}$	0.839	0.034	0.417	0.011

m=5  $$\rm{mean}$$  of  ${\rm \#}$  of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

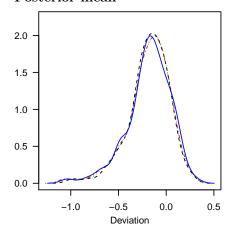
total # of negative proposals:

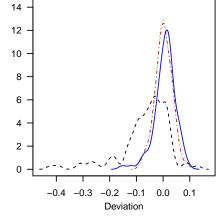
DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
0	91	40	0

ratio of negative proposals and number of iterations:

	DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
min	0	0.0e+00	0.00e+00	0
max	0	3.7e-06	2.44e-05	0
median	0	0.0e+00	0.00e+00	0
mean	0	1.0e-07	4.00e-07	0

### Posterior mean





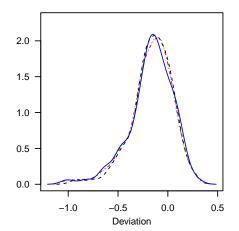


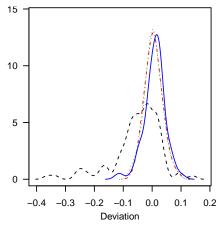
Based on up to 100 results.

Table 34: RMSE

	alpha	sigma2
MBE-E	0.277	0.113
$\mathrm{MBE}\text{-}\mathrm{M}$	0.288	0.031
MBM-M	0.292	0.040
DBM-M	0.291	0.031

### Posterior median





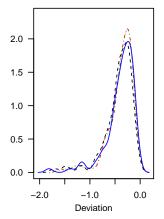


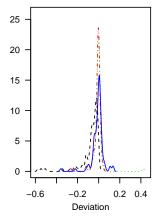
Based on up to 100 results.

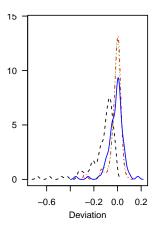
Table 35: RMSE

	alpha	sigma2
MBE-E	0.254	0.098
MBE-M	0.274	0.031
MBM-M	0.278	0.037
$\mathrm{DBM} ext{-}\mathrm{M}$	0.275	0.030

## Posterior variance and covariance







--- MBE-E MBE-M MBM-M DBM-M

Based on up to 100 results.

Table 36: RMSE

	alpha	sigma2	covariance
MBE-E	0.524	0.127	0.170
$\mathrm{MBE}\text{-}\mathrm{M}$	0.474	0.050	0.049
$\operatorname{MBM-M}$	0.492	0.058	0.067
DBM-M	0.472	0.037	0.055

## Number of iterations and effective sample size

	${\bf numIterations\ mean}$	${\bf numIterations~sd}$	${\it multivarESS mean}$	${\it multivarESS \ sd}$
MBE-E	6765054	702467	49885	8837
$\mathrm{MBE}\text{-}\mathrm{M}$	892487	22258	5033	1194
${ m MBM} ext{-}{ m M}$	78215	2858	573	114
$\mathrm{DBM} ext{-}\mathrm{M}$	879227	25268	5535	1179

	ARpath mean	ARpath sd	ARparam mean	ARparam sd
MBE-E	0.892	0.019	0.310	0.004
$\mathrm{MBE}\text{-}\mathrm{M}$	0.844	0.028	0.304	0.002
MBM-M	0.978	0.009	0.304	0.003
DBM-M	0.884	0.021	0.304	0.002

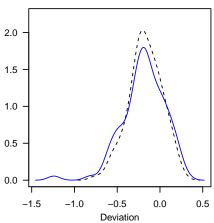
ARpath mean	ARpath sd	ARparam mean	ARparam sd
rricpauli ilicali	riipani sa	mican mean	riiparam sa

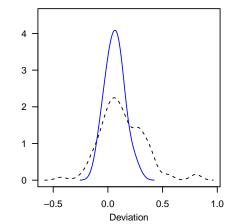
M = 50

m = 1

The following sections show density plots of the discrepancy between the respective statistic of the samples from the approximated posteriors (sampled with two-step MCMC) and the sample from the true posterior (sampled with Stan) calculated for the 100 simulated datasets.

### Posterior mean





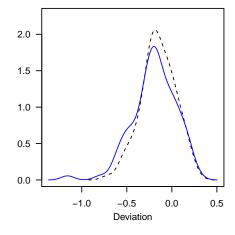


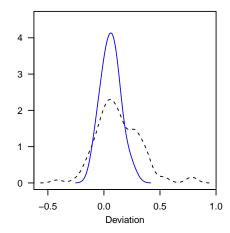
Based on up to 100 results.

Table 39: RMSE

	alpha	sigma2
	-	
Euler	0.259	0.240
Milstein	0.313	0.105

### Posterior median



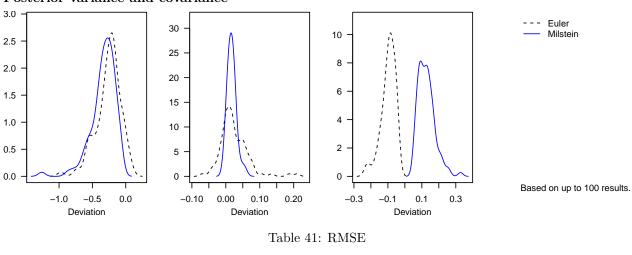




Based on up to 100 results.

Table 40: RMSE

	alpha	sigma2
Euler	0.244	0.234
Milstein	0.310	0.102



	alpha	sigma2	covariance
Euler	0.319	0.048	0.106
Milstein	0.384	0.021	0.136

## Number of iterations and effective sample size

	numIterations mean	${\bf numIterations~sd}$	multivarESS mean	${\it multivarESS \; sd}$
Euler	23195646	619312	1156494	108760
Milstein	2011421	51685	94538	10482

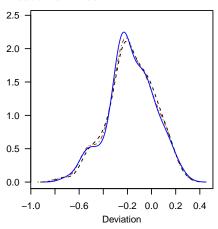
	ARparam mean	ARparam sd
td_E	0.398	0.003
$td\_M$	0.381	0.003

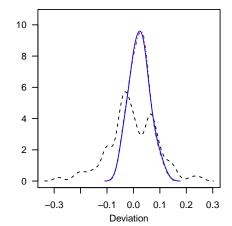
 $\mathbf{m}=\mathbf{2}$  mean of # of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

total # of negative proposals:

DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
0	0	0	0

#### Posterior mean





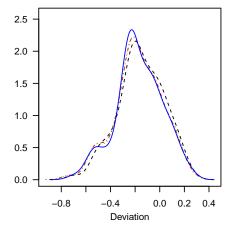
--- MBE-E .... MBE-M --- MBM-M DBM-M

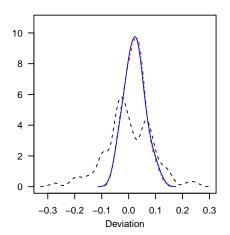
Based on up to 100 results.

Table 45: RMSE

	alpha	sigma2
MBE-E	0.253	0.091
MBE-M	0.259	0.044
MBM-M	0.257	0.045
$\mathrm{DBM} ext{-}\mathrm{M}$	0.260	0.044

### Posterior median







Based on up to 100 results.

Table 46: RMSE

	alpha	sigma2
MBE-E	0.242	0.089
$\mathrm{MBE}\text{-}\mathrm{M}$	0.258	0.043
MBM-M	0.255	0.044
DBM-M	0.258	0.043

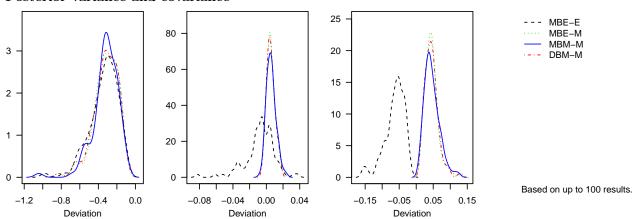


Table 47: RMSE

	alpha	sigma2	covariance
MBE-E	0.373	0.019	0.069
$\mathrm{MBE}\text{-}\mathrm{M}$	0.346	0.007	0.049
MBM-M	0.350	0.008	0.051
$\mathrm{DBM} ext{-}\mathrm{M}$	0.348	0.007	0.049

## Number of iterations and effective sample size

	numIterations mean	${\bf numIterations~sd}$	multivarESS mean	${\it multivarESS \ sd}$
MBE-E	7891530	455705	100013	12768
$\mathrm{MBE}\text{-}\mathrm{M}$	888981	32717	9416	1637
${ m MBM} ext{-}{ m M}$	250083	7199	3972	484
DBM-M	871113	31979	10625	1717

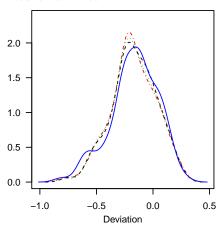
	ARpath mean	ARpath $sd$	ARparam mean	ARparam sd
MBE-E	0.898	0.011	0.320	0.002
MBE-M	0.872	0.014	0.313	0.001
$\mathrm{MBM} ext{-}\mathrm{M}$	1.000	0.000	0.313	0.001
$\mathrm{DBM} ext{-}\mathrm{M}$	0.899	0.011	0.313	0.001

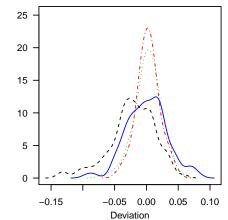
m=5  $$\rm{mean}$$  of  ${\rm \#}$  of switching to Euler for MB\_td\_Milstein\_pd\_Milstein: 0

total # of negative proposals:

DBM_td_M_pd_M	MB_td_E_pd_E	MB_td_M_pd_E	MB_td_M_pd_M
0	0	0	0

#### Posterior mean





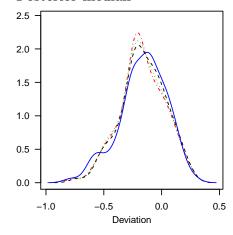
MBE-E MBE-M MBM-M DBM-M

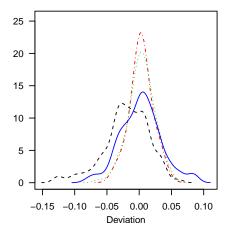
Based on up to 100 results.

Table 51: RMSE

	alpha	sigma2
MBE-E	0.258	0.042
MBE-M	0.262	0.021
MBM-M	0.269	0.032
$\mathrm{DBM} ext{-}\mathrm{M}$	0.263	0.017

### Posterior median







Based on up to 100 results.

Table 52: RMSE

	alpha	sigma2
MBE-E	0.249	0.040
$\mathrm{MBE}\text{-}\mathrm{M}$	0.258	0.020
MBM-M	0.261	0.032
DBM-M	0.258	0.018

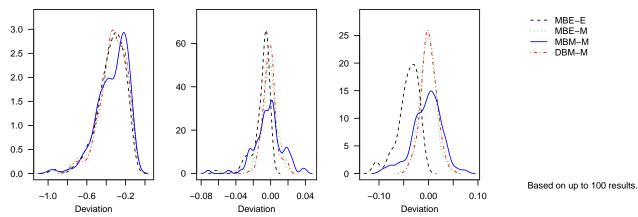


Table 53: RMSE

	alpha	sigma2	covariance
MBE-E	0.375	0.014	0.047
MBE-M	0.355	0.011	0.016
MBM-M	0.368	0.017	0.030
DBM-M	0.354	0.009	0.016

### Number of iterations and effective sample size

	numIterations mean	numIterations sd	multivarESS mean	multivarESS sd
MBE-E	5988019	188045	23951	2435
$\mathrm{MBE}\text{-}\mathrm{M}$	396953	11399	1388	190
MBM-M	69078	1868	285	47
DBM-M	393650	9266	1469	196

### Acceptance rates

	ARpath mean	ARpath sd	ARparam mean	ARparam sd
MBE-E	0.930	0.007	0.210	0.002
$\mathrm{MBE}\text{-}\mathrm{M}$	0.899	0.010	0.208	0.001
${ m MBM-M}$	0.993	0.002	0.208	0.002
$\mathrm{DBM} ext{-}\mathrm{M}$	0.926	0.008	0.208	0.001

# # of missing results: 0

## Stan results (sampling from and optimizing the true posterior)

### M = 10

# of missing results: 0

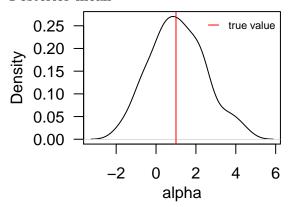
# Rhat > 1.01: 0 (out of 200)

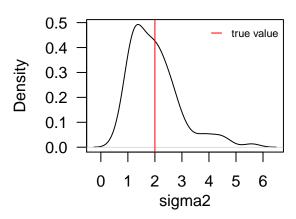
range of max. duration in seconds: (15.6, 17.2), median: 16.1

median multivar ESS:  $4.89745\times10^5$ 

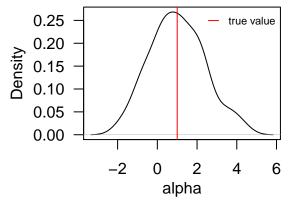
The following sections show density plots of the respective statistic calculated for the 100 simulated datasets.

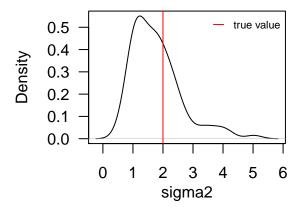
### Posterior mean



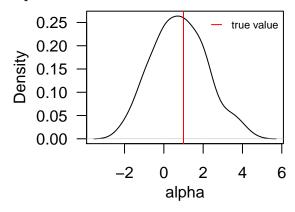


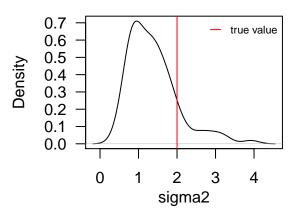
### Posterior median

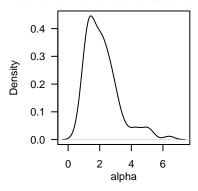


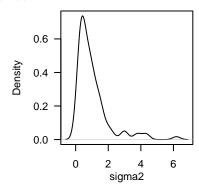


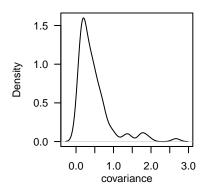
### Optimized value



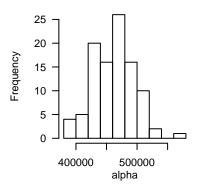


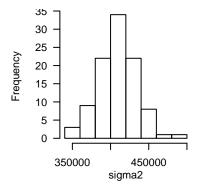


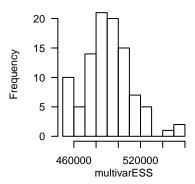




## Effective sample size







### M = 20

# of missing results: 0

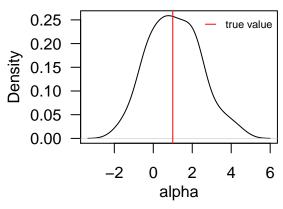
# Rhat > 1.01: 0 (out of 200)

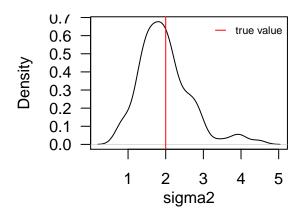
range of max. duration in seconds: (17.1, 19.9), median: 17.7

median multivar ESS:  $6.0174\times10^5$ 

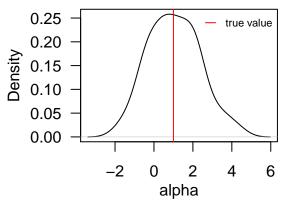
The following sections show density plots of the respective statistic calculated for the 100 simulated datasets.

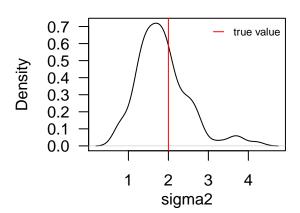
### Posterior mean



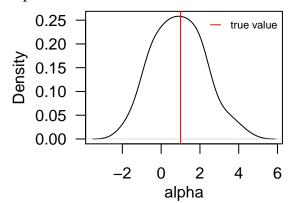


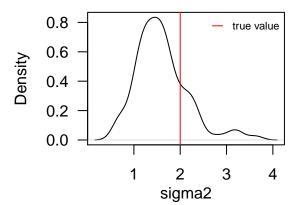
### Posterior median

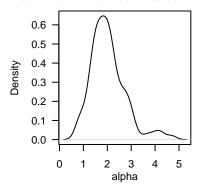


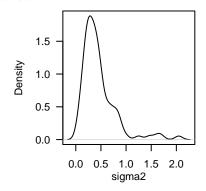


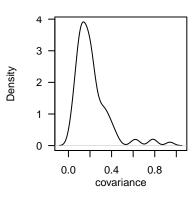
### Optimized value



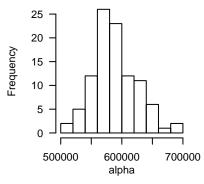


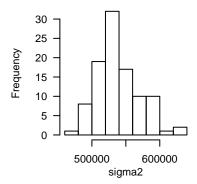


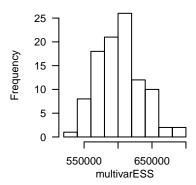




## Effective sample size







## M = 50

# of missing results: 0

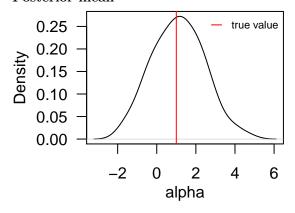
# Rhat > 1.01: 0 (out of 200)

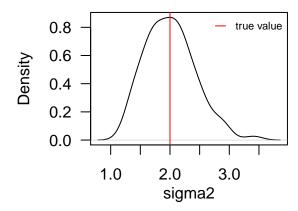
range of max. duration in seconds: (20.3, 26.1), median: 23.7

median multivar ESS:  $7.1865\times10^5$ 

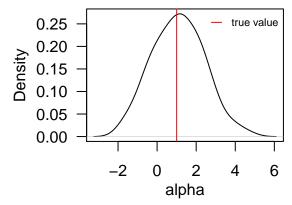
The following sections show density plots of the respective statistic calculated for the 100 simulated datasets.

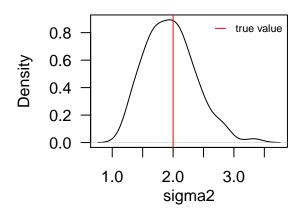
### Posterior mean



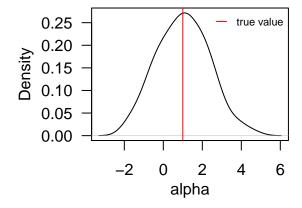


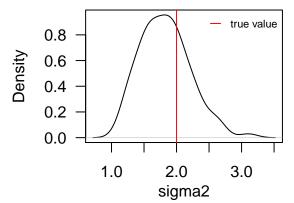
### Posterior median

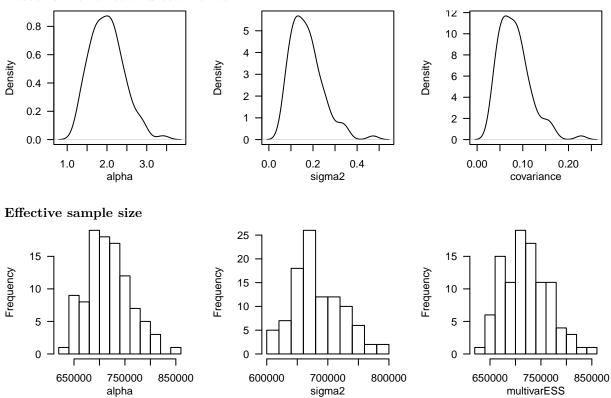




### Optimized value







# of missing Stan results: 0