# The Shepard Tone and Higher-Order Multi-rate Synchronous Data-Flow Programming in SIG

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## 1 Exposition

- Main Theme: Sig in a Nutshell
- Countertheme: The Shepard Tone
- 2 Development
  - Higher-Order Stream Programming
  - Multi-rate Stream Programming
- Recapitulation
  - Putting Things Together
  - Conclusion

#### **Spoiler**

- Synchronous Functional Data-Flow Language
  - Similar but not quite FRP
  - New features under development
- Classical sound construction
  - Small but nontrivial synthesis problem
  - Ideal application of new features

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  - pull-based, ultra-low latency
- Applications: embedded control, simulation, audio, ...
- Backends: JVM (C, DSP, FPGA, . . . )



#### Context: The Sig Language

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- Layered design
  - Core language with simple compositional semantics
  - Functional frontend: ADTs, pattern matching, higher order
  - Advanced features: physical units, multi-rate



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  - Advanced features: physical units, multi-rate
- Semantics of higher layers by transformation



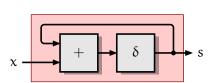
$$\begin{bmatrix} x \to s \\ s := 0 \, \mathring{g} \, (s + x) \end{bmatrix}$$

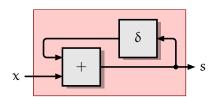
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#### **Basic Example: Cumulative Sum**

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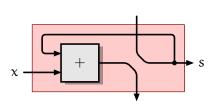


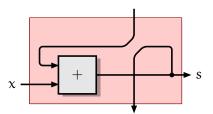


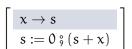
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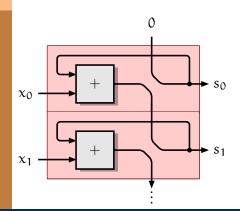
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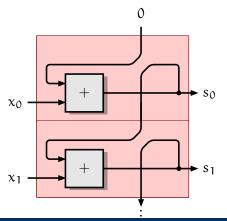




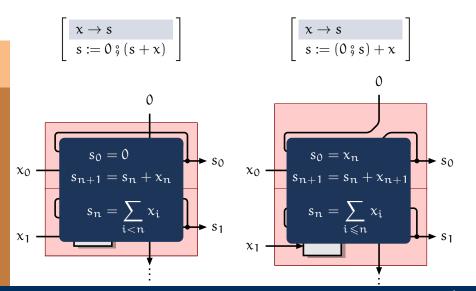


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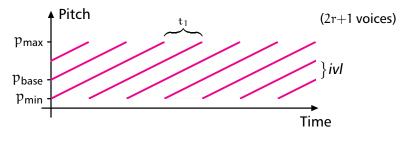


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- Auditory strange loop, analog of this...
  - or that...
- Small but nontrivial sound synthesis problem
  - Full Sig code in the paper!



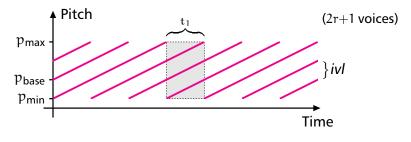


#### **Specification**



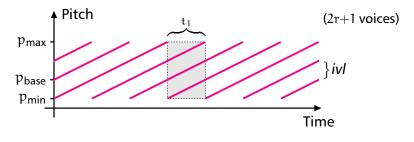


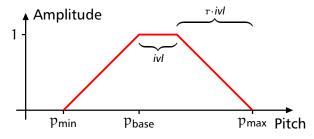
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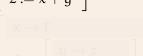
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  - But arbitrary number of stages
- Considered for adaptive high-performance computing (Kiselyov, Shan, and Kameyama 2012)

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#### No Stage Fright in Sig

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#### **Counterexample Defused**

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One system, many rates:

```
Audio waveform @ 44 kHz (CD), 96 kHz (studio)
Control parameters @ 1/64 audio, 1 kHz
  Event 24/quarter (MIDI), 120/minute (techno)
   Zero Initialization
```

# Multi-rate Audio Signal Processing

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- Asynchronous data flows both ways **Modulation** parameters from slow to fast **Aggregation** statistics from fast to slow

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#### Multi-rate Sig

$$\begin{bmatrix} x \to y \\ y := 0 \, \text{;} \, y + x * \mathbf{dt} \end{bmatrix}$$

- SIG programs have implicit rate
  - Components reusable at different rates (polydromic)
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- Exceptions by explicit resampling
  - Rate inequations
  - Fixed conversion factors



$$\underbrace{\mathcal{R}(amp)}_{\mathsf{R}_1} \leqslant \underbrace{\mathcal{R}(phase) = \mathcal{R}(out)}_{\mathsf{R}_2}$$

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$$\begin{bmatrix} amp \rightarrow & \\ amp := amp \end{bmatrix}$$
 # 
$$\begin{bmatrix} \rightarrow out \\ out := amp^+ * sin phase \\ phase := 0 \cdot phase + freq * dt \end{bmatrix}$$

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$$\begin{bmatrix} amp \rightarrow amp^- \\ amp^- := amp \end{bmatrix}$$
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- Slicing into synchronous subcomponents
- Buffered asynchronous data flow behind the scenes
  - Well-defined scheduling rules

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## The Shepard Tone in Sig

- Straightforward architecture
  - Three tiers, loosely corresponding to three rates
  - Two-stage (curried) config/runtime separation each
- Not quite trivial code
  - 27 LoC, as detailed in the paper
  - References to primitives not discussed here

## Top-Down Walkthrough

- Ensemble maintains an array of live voices (functions)
  - shift at rate R<sub>1</sub> driven by external clock
  - outputs mixed together

```
shepard 2 := \#[ clk \ 1 : void \longrightarrow s out : real]
 where
    make
                       := \#[k: int -> m out := $voice(k * ivl, ivl / dt(clk 1))]
    ensemble @ clk 1 := \%make(seq(-r, +r)); shiftr(\%make(-r), ensemble)
                       := sum(ensemble(upsample(clk 1, res)))
    s out
```

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- Voice modulates oscillator (amp+freq)
  - linear pitch increase
  - quantized at rate R<sub>2</sub> driven by subdivided clock

```
voice := #[ init pitch , ascent : real ->
 voice 2 := #[clk 2 : void -> v out : real
   where
      pitch @ clk 2 := init pitch ; pitch + ascent * dt
                   := $($osci(0))( $envelope(pitch), base freq * exp(pitch))
      v out
```

## Top-Down Walkthrough

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- Voice modulates oscillator (amp+freq)
  - linear pitch increase
  - quantized at rate R<sub>2</sub> driven by subdivided clock
- Oscillator maintains phase continuity
  - envelope, waveform global function-valued parameters
  - sample at audio rate R<sub>3</sub> driven by RT audio system

```
osci := #[ init phase : real ->
 osci 2 := \#[amp, freq : real \rightarrow o out : real]
   where
      phase := init phase ; phase + upsample(freq) * dt
      o out := upsample(amp) * $wave(phase)
```

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- Demo with manual Java coding
  - Simulated code generation
  - Against actual API; binary compatible



14/15

- Many things to do
  - Make rates work
  - Whole-program analysis & optimization
  - Hard real-time runtime environments



# **Concluding Stuff**

- Many things to do
  - Make rates work
  - Whole-program analysis & optimization
  - Hard real-time runtime environments
- Take-home message:

Multi-rate higher-order programming rocks

- Dynamic reconfiguration of signal processing networks
- Applications ranging from trivial to hugely complex
- Clean compositional semantics & reliability essential

## **SIG Bibliography I**



Trancón y Widemann, B. and M. Lepper (2011). "tSig — Towards Semantics for a Functional Synchronous Signal Language". In: 16. Kolloquium Programmiersprachen und Grundlagen der Programmierung (KPS 2011). Ed. by H. Kuchen and M. Müller-Olm. Arbeitsberichte des Instituts für Wirtschaftsinformatik 132. Universität Münster, pp. 163–168.



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## **Sig Bibliography II**



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