The problem being solved in this directory is:

$$\ddot{\theta} = -\frac{g}{l}\cos\theta$$

integrating both sides with respect to θ yields:

$$\begin{aligned} \frac{\dot{\theta}^2}{2} &= -\frac{g}{l}\sin\theta + C \\ \dot{\theta} &= \pm\sqrt{-\frac{2g}{l}\sin\theta + C} \\ \implies C &= \dot{\theta}_0^2 \\ \therefore \dot{\theta} &= \pm\sqrt{\dot{\theta}_0^2 - \frac{2g}{l}\sin\theta} \end{aligned}$$

where $\dot{\theta}_0$ is $\dot{\theta}$ when $\sin\theta = 0$ (therefore $\theta = n\pi$ where $n \in \mathbb{Z}$). t can therefore be computed as:

$$t = \pm\int_{\theta_0}^{\theta_1} \frac{d\theta}{\sqrt{\dot{\theta}_0^2 - \frac{2g}{l}\sin\theta}}.$$

In this repository, the initial conditions are:

$$\theta(t=0) = \dot{\theta}(t=0) = 0.$$

In other words, the pendulum bob starts at the positive x axis with zero velocity and moves solely under the influence of gravity. If we imagine a pendulum subject to these conditions, it becomes clear that theta will range from $-\pi$ (the bob being right on the negative x-axis) to 0. If we wish to determine the period of θ (i.e. the value of χ such that $\theta(t+\chi) = \theta(t)\ \forall t$), we must set $\theta_0 = 0$, $\theta_1 = -\pi$ and multiply our final result by two (as our result will only reflect how long it takes to go from the positive x axis to the negative x axis, not how long it will take to make the return trip)

$$
\begin{aligned}
\chi &= -2\int_0^{-\pi} \frac{d\theta}{\sqrt{-\frac{2g}{l}\sin\theta}} \\
&= 2\int_{-\pi}^{0} \frac{d\theta}{\sqrt{-\frac{2g}{l}\sin\theta}}.
\end{aligned}
$$

Above we chose the negative on the $\pm$ sign because otherwise we will get a negative value for t, and we are choosing to keep time positive. It is impossible to solve this integral analytically and use it to express θ in terms of t, therefore we are reduced to using numerical methods to approximate θ. The three numerical methods used in this directory are:

- `ode78` from the ODE.jl Julia module.
- Runge-Kutta 4th order method.
- The Newton-Kantorovich method to linearize the problem, and then a Chebyshev spectral method to approximate the solution to the linearized version of the problem.

Out of these, only the Newton-Kantorovich method likely needs further explanation. To linearize the problem, we used $\theta_{i+1} = \theta_i + \Delta_i$, substuting θ_{i+1} into our original equation yields:

$$
\begin{aligned}
\ddot{\theta}_i + \ddot{\Delta}_i &= -\frac{g}{l}\cos\left(\theta_i + \Delta_i\right) \\
&\approx -\frac{g}{l}\left(\cos\theta_i - \sin\theta_i \Delta_i\right) \\
\therefore\ \ddot{\Delta}_i - \frac{g}{l}\sin\theta_i\Delta_i &= -\ddot{\theta}_i - \frac{g}{l}\cos\theta_i
\end{aligned}
$$