In scripts within this directory that start with `airy-`, Chebyshev spectral methods are used to numerically approximate the solution to the Sturm-Liouville problem (where λ is an eigenvalue):

$$-\frac{d^2y}{dx^2} + xy = \lambda y, \text{ on } x \in [0, \infty].$$

With the boundary conditions $y(0) = y(\infty) = 0$. Both such scripts use the Chebyshev extrema grid that has been transformed to an approximation of the semi-infinite domain.

`airy-linear.jl` uses a linear transformation of this grid to approximate the semi-infinite domain. The transformed domain goes from 0 to 870.

`airy-rat.jl` uses a rational transformation of this grid to approximate the semi-infinite domain, with $L = 366$ and the transformation:

$$x = L\left(\frac{1+x'}{1-x'}\right)$$

where x' is the variable of the Chebyshev extrema grid and x is the variable of the transformed grid. I should clarify, however, that in `airy-rat.jl` the variable of the transformed grid is represented as y and the extrema grid is represented as x.

The analytical solutions to this problem are:

$$y_n(x) = a_n \mathrm{Ai}(x - \lambda_n)$$

where the eigenvalues, λ_n, are the negative of the zeros of the Airy $\mathrm{Ai}(x)$ function and a_n are arbitrary constants.

Out of the two methods, the linear transformation yields the most accurate eigenvalues with almost 5,400 possible with b 870 and $N = 10000$, whilst the rational transformation yields at best 2,700 accurate eigenvalues with the same N value (which is achieved at $L = 366$).