

$$-\frac{d^2 y_n}{dx^2} + x y_n = \lambda_n y_n$$

on $0 \leq x \leq \infty$, with $y(0) = y(\infty) = 0$.

To this equation the solution is:

$$y_n = C_1 \text{Ai}(x + \lambda_n)$$

Where C_1 is an arbitrary constant, and λ_n are the zeros of the Airy function, $\text{Ai}(x)$, on the negative x axis (e.g. $\lambda_1 \approx -2.338$).

To numerically integrate this solution let us use a Chebyshev spectral method. If t_m comprises points on the Chebyshev extrema grid, that is:

$$t_m = -\cos\left(\frac{\pi m}{N}\right), \quad m \in [0, 1, 2, \dots, N]$$

then:

$$x_m = L \left(\frac{1+t_m}{1-t_m} \right)$$

This is a rational transformation from the Chebyshev extrema grid to the semi-infinite domain.