LINEAR REGRESSION MODELS W4315

Quiz QUESTIONS

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1. (30 points) Prove the Law of Total Probability. Suppose events $A_1, A_2, \cdots, A_n$ form a *partition* of Ω. That is, the events are mutually disjoint and their union is all of Ω. Then for any other event B, we have

$$P(B) = P(A_1) \times P(B|A_1) + P(A_2) \times P(B|A_2) + \cdots + P(A_n) \times P(B|A_n).$$

2. (40 points)

$$\begin{cases} x_1 + 2x_2 + 3x_3 &= \alpha \\ 3x_1 + 5x_2 - x_3 &= \beta \\ x_1 + x_2 - 7x_3 &= \gamma \end{cases}$$

Under what conditions of α, β, γ, the above equations have (a)unique solution (b)no solution (c)infinite solutions.

3. (30 points) Suppose

$$Y_i = X_i\beta + \epsilon_i,$$

where $\epsilon_i \overset{iid}{\sim} N(0, \sigma_1^2)$, $i = 1, 2, \cdots, n$ and $\epsilon_i \overset{iid}{\sim} N(0, \sigma_2^2)$, $i = n+1, n+2, \cdots, n+m$. Derive the Maximum Likelihood Estimator of β, σ_1^2 and σ_2^2.