Coconut: Lecture notes

Josh Felmeden

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1 Hamming Codes

For any $m \geq 1$, the generalised Hamming code encodes up to $m-1$ data bits with m parity bits for a total of $2^m - 1$ respectively bits.

- The minimum distance $d = 3$ (respectively 4 for SECDED) for any value of m
- The i-th bit is a parity bit if i is a power of 2, otherwise a data bit.
- The parity bits are set such that the sum of all bits whose index i has a 1 at the j-th position when written out in binary equals 0.
- For the SECDED version, an additional initial bit 0 is set such that the sum of all the bits in the codeword is zero.

The parity bits in the generalized scheme is computed such that all of the 1s in the rows must sum to zero. For example:

0	1	2	3	4	5	6	7
0	0	0	0	1	1	1	1
0	0	1	1	0	0	1	1
0	1	0	1	0	1	0	1
Bit name	P_1	P_2	m_3	P_4	m_5	m_6	m_7

So, $P_4 + m_5 + m_6 + m_7 = 0$ where P_4 is a parity bit, and the rest are data bits.

We use the following rules:

1. $P_4 + m_5 + m_6 + m_7 = 0$
2. $P_2 + m_3 + m_6 + m_7 = 0$
3. $P_1 + m_3 + m_5 + m_7 = 0$

Let's revisit the table. If we have the sequence:

0	1	2	3	4	5	6	7
Ex 1	0	1	1	0	0	0	1
Bit name	P_1	P_2	m_3	P_4	m_5	m_6	m_7

Then, rules **1** and **2** are violated, but **3** is not. So, with the GHC decoding rule, we sum the indexes of violating parity bits. Since we know that P_1s bits are okay, but P_2, P_4 are not, we can therefore tell that m_6 is the issue, so if we flip this, then we'll probably be okay.