My topology exercises

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2023

Contents

Preface

Those are my solutions for the James Munkres' "Topology", 2nd edition.

Part I

General Topology

Chapter 1

Set Theory and Logic

1.1 Fundamental Concepts

1.1.1

Check distributive and DML laws
GOTO set theory book

1.1.2

Determine which of the following are true.
(a) - impl
(b) - impl
(c) - true
(d) - rimpl
(e) - $\subseteq$, true if $B \subseteq A$.
(f) - $\supseteq$; $A - (B - A) = A$.
(g) - true
(h) - $\supseteq$
(i) - true
(j) - true
(k) - false
(l) - true
(m) - $\subseteq$
(n) - true
(o) - true
(p) - true
(q) - $\supseteq$

1.1.3

(a) Write a contrapositive and converse of the following statement: "If $x < 0$, then $x^2 - x > 0$" and determine which ones are true

Contrapositive:

$$x^2 - x \leq 0 \Rightarrow x \geq 0$$

Converse

$$x^2 - x > 0 \Rightarrow x < 0$$

Contrapositive is correct, converse is incorrect $(2^2 - 2 > 0)$

(b) Do the same for the statement $x > 0 \Rightarrow x^2 - x > 0$

Contrapositive:

$$x^2 - x \leq 0 \Rightarrow x \leq 0$$

Converse

$$x^2 - x > 0 \Rightarrow x > 0$$

Contrapositive is false $(1^2 - 1 = 0)$; Converse is also false $((-2)^2 - (-2) = 6)$.

1.1.4

Let A and B be the sets of real numbers. Write the negation of each of the following statements:

(a)

$$(\exists a \in A)(a^2 \notin B)$$

(b)

$$(\forall a \in A)(a^2 \notin B)$$

(c)

$$(\exists a \in A)(a^2 \in B)$$

(d)

$$(\forall a)(a \notin A \Rightarrow a^2 \notin B)$$

1.1.5

Let A be a nonempty collection of sets. Determine the truths of each of the following and their converses

(a)

$$x \in \bigcup A \Leftrightarrow (\exists B \in A)(x \in B)$$

(b)

$$x \in \bigcup A \Leftarrow (\forall B \in A)(x \in B)$$

(c)

$$x \in \bigcap A \Rightarrow (\exists B \in A)(x \in B)$$

(d)

$$x \in \bigcap A \Leftrightarrow (\forall B \in A)(x \in B)$$

1.1.6

Skip

1.1.7

skip

1.1.8

GOTO set theory book

1.1.9

Formulate DML for arbitrary unions and intersections

$$A \setminus \bigcap(B) = \bigcup(A \setminus B)$$
$$A \setminus \bigcup(B) = \bigcap(A \setminus B)$$

For the proof goto set theory or real analisys book

1.1.10

(a, b, d) are true

1.2 Functions

1.2.1

Let $f : A \to B$. Let $A_0 \subseteq A$ and $B_0 \subseteq B$.

(a) Show that $A_0 \subseteq f^{-1}[f[A_0]]$ and that equality holds if f is injective.

Suppose that $x \in A_0$. We follow that there exists $\langle x, y\rangle \in f$ for some $y \in f[A_0]$. Therefore there exists $\langle y, x\rangle \in f^{-1}$. Because $y \in f[A_0]$, we follow that $x \in f^{-1}[f[A_0]]$. Therefore $A_0 \subseteq f^{-1}[f[A_0]]$.

Suppose that f is injective. Suppose that there exists $x_0 \in f^{-1}[f[A_0]]$ such that $x_0 \notin A_0$. We follow that $\langle y, x_0\rangle, \langle y, x\rangle, \in f^{-1}$, therefore $\langle x_0, y\rangle, \langle x, y\rangle \in f$, and because $x_0 \neq x$ we follow that we've got a contradiction.

((b)

pretty simular to (a)

This chapter practicly mirrors the content of my set theory course . Gonna skip it for now, and will come back if the need arises.

Chapter 2

Topological Spaces and Continous Functions

2.1 Topological Spaces

2.2 Basis for a Topology

2.2.1

Let X be a topological space; Let A be a subset of X. Suppose that for each $x \in A$ there is an open set U containing x such that $U \subseteq A$. Show that A is open in X.

Let U_x be a indexed set such that $U_x = \{U \in \mathcal{T} : (\exists x \in A)(x \in U)$