My set theory exercises

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Contents

Chapter 1

Introduction

1.1 Elementary Set Theory

Let A, B, C be sets

1.1.1

If $a \notin A \setminus B$ and $a \in A$, show that $a \in B$

Because $a \notin A \setminus B$, we follow that $x \in B$ or $x \notin A$. Since $x \in A$, we follow that $x \in B$, as desired.

1.1.2

Show that if $A \subseteq B$, then $C \setminus B \subseteq C \setminus A$

Let $c \in C \setminus B$. Then we follow that $c \in C$ or $c \notin B$. Since $A \subseteq B$, we follow that $c \notin B$ implies that $c \notin A$. Thus we follow that $c \in C \setminus B$ implies that $c \in C \setminus A$. Therefore $C \setminus B \subseteq C \setminus A$.

1.1.3

Suppose $A \setminus B \subseteq C$. Show that $A \setminus C \subseteq B$.

Suppose that $a \in A \setminus C$. Then we follow that $a \in A$ and $a \notin C$.

Given that $A \setminus B \subseteq C$ and $A \notin C$, we follow that $a \notin A \setminus B$. Thus $a \notin A$ or $a \in B$. Since $a \in A$, we follow that $a \in B$. Thus

$$a \in A \setminus C \to a \in B$$

$$A \setminus C \subseteq B$$

as desired.

1.1.4

Suppose $A \subseteq B$ and $A \subseteq C$. Show that $A \subseteq B \cap C$

Suppose that $a \in A$. Then we follow that $a \in B$ and $a \in C$. Thus $a \in B \cap C$. Therefore we follow that $A \subseteq B \cap C$.

1.1.5

Suppose $A \subseteq B$ and $B \cap C = \emptyset$. Show that $A \in B \setminus C$

Suppose that $a \in A$. Then we follow that $a \in B$ and since $B \cap C = \emptyset$, we follow that $a \notin C$. Thus $a \in B \setminus C$ by definition. Therefore $A \subseteq B \setminus C$.

1.1.6

Show that $A \setminus (B \setminus C) \subseteq (A \setminus B) \cup C$. Suppose that $a \in A \setminus (B \setminus C)$. Then we follow that $a \in A$ and $a \notin B \setminus C$. Thus $a \notin B$ and $a \in C$. Thus we follow that $a \in A \setminus B$ or $a \in C$. Thus $A \setminus (B \setminus C) \subseteq (A \setminus B) \cup C$ as desired.

1.1.7

Let $P(x)$ be the property $x > \frac{1}{x}$. Are the assertions $P(2)$, $P(-2)$, $P(\frac{1}{2})$ $P(\frac{-1}{2})$ true or false .

$$2 > \frac{1}{2} \rightarrow P(2) = true$$
$$-2 < \frac{-1}{2} \rightarrow P(-2) = false$$

last two are reversed.

1.1.8

Sow that each of the following sets can be expressed as an interval

$$a)(-3, 3)$$
$$b)(-3, \infty)$$
$$c)(-3, 3)$$

all of them follow from order properties of real numbers.

1.1.9

Express the following sets as truth sets

$$A = \{1, 4, 9, 16, 25, ...\} \iff A = \{x \in N : x = y^2 \textit{ for some } y \in N\}$$
$$B = \{..., -15, -10, -5, 0, 5, ...\} \iff A = \{x \in N : x = 5y \textit{ for some } y \in N\}$$

Rest are also trivial, not gonna go deep here