My real analysis exercises

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4.4.1

Show that $f(x) = x^3$ is continuous on all of $\mathbf{R}$.

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In order to show, that f is continous we need to show, that $\forall \epsilon \in \mathbf{R}$ $\exists \delta$ s.t.

$$|x - c| < \delta \rightarrow |f(x) - f(c)| < \epsilon$$

Let's rewrite the first formula

$$|f(x) - f(c)| = |x^3 - c^3| = |(x - c)(x^2 + cx + c^2)| = |x - c||x^2 + cx + c^2|$$

We can put $|x - c|$ can be as small as we want it to be. Therefore we need an upper bound for $|x^2 + cx + c^2|$