My abstract algebra exercises

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Preface

This is another yet another attempt at making any progress with abstract algebra, this time with 'Fundamentals of Abstract Algebra' by Mark J. DeBonis. Hopefully this time I wont encounter any deal-breakers

Chapter 1

Background Material

1.1 Equivalence Relations

1.1.1

For the examples in Example 1.1, list three elements in each relation

$$a \equiv b; b \equiv b; c \equiv d$$

for the first one,

$$1 < 2 < 3 < 4$$

for the second one, and

$$\emptyset \subseteq \{0\} \subseteq \{0, 1\} \subseteq \{0, 1, 2\}$$

for the third one. The rest is trivial as well.

The majority of the exercises are pretty trivial or were dealt with before in my previous endeavours in various parts of math. Instead of repeating, I'm just gonna complete the ones, that I like the most

1.1.13

Suppose a relation $\equiv$ on a set A has the following two properties:

(a) For all $a \in A, a \equiv a$.

(b) For all $a, b, c \in A, a \equiv b \wedge b \equiv c \Rightarrow c \equiv a$.

Prove that $\equiv$ is an equivalence relation on A

Part (a) gives us reflexivity. If $a \equiv b$, then we follow that $b \equiv b$ by part (a), and thus by part (b) we've got $b \equiv a$, which is symmetry. Now we follow that $c \equiv a$ implies that $a \equiv c$, which when appended to the end of part (b) gives us trasitivity, which concludes the proof.

1.2 Functions

Notes

There's no reason why this chapter is following the previous chapter, they should be swapped in place. Also, definition of 'well-defined map' is nonsensical

All the material, that is presented in this section mirrors the content of the similar section in set theory course, so exercises are skipped. This also applies to all the material in this chapter

Chapter 2

Basic Group Theory

2.1 Definitions and Examples

Notes

Traditionally, the group axioms dont include the closure axioms on the account of the fact that it's taken care of in the definition of the binary operation.

2.1.1

Verify the group axioms for the structures given in Example 2.1, namely

a) $(Z, +)$ *is an abelian group*

$0 + a = a$ for all $a \in Z$, and for all