My topology exercises

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Contents

Preface

Those are my solutions for the James Munkres' "Topology", 2nd edition.

Part I

General Topology

Chapter 1

Set Theory and Logic

1.1 Fundamental Concepts

1.1.1

Check distributive and DML laws
GOTO set theory book

1.1.2

Determine which of the following are true.
(a) - impl
(b) - impl
(c) - true
(d) - rimpl
(e) - $\subseteq$, true if $B \subseteq A$.
(f) - $\supseteq$; $A - (B - A) = A$.
(g) - true
(h) - $\supseteq$
(i) - true
(j) - true
(k) - false
(l) - true
(m) - $\subseteq$
(n) - true
(o) - true
(p) - true
(q) - $\supseteq$

1.1.3

(a) Write a contrapositive and converse of the following statement: "If $x < 0$, then $x^2 - x > 0$" and determine which ones are true

Contrapositive:

$$x^2 - x \leq 0 \Rightarrow x \geq 0$$

Converse

$$x^2 - x > 0 \Rightarrow x < 0$$

Contrapositive is correct, converse is incorrect $(2^2 - 2 > 0)$

(b) Do the same for the statement $x > 0 \Rightarrow x^2 - x > 0$

Contrapositive:

$$x^2 - x \leq 0 \Rightarrow x \leq 0$$

Converse

$$x^2 - x > 0 \Rightarrow x > 0$$

Contrapositive is false $(1^2 - 1 = 0)$; Converse is also false $((-2)^2 - (-2) = 6)$.

1.1.4

Let A and B be the sets of real numbers. Write the negation of each of the following statements:

(a)

$$(\exists a \in A)(a^2 \notin B)$$

(b)

$$(\forall a \in A)(a^2 \notin B)$$

(c)

$$(\exists a \in A)(a^2 \in B)$$

(d)

$$(\forall a)(a \notin A \Rightarrow a^2 \notin B)$$

1.1.5

Let A be a nonempty collection of sets. Determine the truths of each of the following and their converses

(a)

$$x \in \bigcup A \Leftrightarrow (\exists B \in A)(x \in B)$$

(b)

$$x \in \bigcup A \Leftarrow (\forall B \in A)(x \in B)$$

(c)

$$x \in \bigcap A \Rightarrow (\exists B \in A)(x \in B)$$

(d)

$$x \in \bigcap A \Leftrightarrow (\forall B \in A)(x \in B)$$

1.1.6

Skip

1.1.7

skip

1.1.8

GOTO set theory book

1.1.9

Formulate DML for arbitrary unions and intersections

$$A \setminus \bigcap (B) = \bigcup (A \setminus B)$$
$$A \setminus \bigcup (B) = \bigcap (A \setminus B)$$

For the proof goto set theory or real analisys book

1.1.10

(a, b, d) are true

1.2 Functions

1.2.1

Let $f : A \to B$. Let $A_0 \subseteq A$ and $B_0 \subseteq B$.

(a) Show that $A_0 \subseteq f^{-1}[f[A_0]]$ and that equality holds if f is injective.

Suppose that $x \in A_0$. We follow that there exists $\langle x, y\rangle \in f$ for some $y \in f[A_0]$. Therefore there exists $\langle y, x\rangle \in f^{-1}$. Because $y \in f[A_0]$, we follow that $x \in f^{-1}[f[A_0]]$. Therefore $A_0 \subseteq f^{-1}[f[A_0]]$.

Suppose that f is injective. Suppose that there exists $x_0 \in f^{-1}[f[A_0]]$ such that $x_0 \notin A_0$. We follow that $\langle y, x_0\rangle, \langle y, x\rangle, \in f^{-1}$, therefore $\langle x_0, y\rangle, \langle x, y\rangle \in f$, and because $x_0 \neq x$ we follow that we've got a contradiction.

((b)

pretty simular to (a)

This chapter practicly mirrors the content of my set theory course, without going into much detail. Gonna skip it for now, and will come back if the need arises.