My topology exercises

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Contents

Preface

Those are my solutions for the James Munkres' "Topology", 2nd edition.

Part I

General Topology

Chapter 1

Set Theory and Logic

1.1 Fundamental Concepts

1.1.1

Check distributive and DML laws
GOTO set theory book

1.1.2

Determine which of the following are true.
(a) - impl
(b) - impl
(c) - true
(d) - rimpl
(e) - $\subseteq$, true if $B \subseteq A$.
(f) - $\supseteq$; $A - (B - A) = A$.
(g) - true
(h) - $\supseteq$
(i) - true
(j) - true
(k) - false
(l) - true
(m) - $\subseteq$
(n) - true
(o) - true
(p) - true
(q) - $\supseteq$

1.1.3

(a) Write a contrapositive and converse of the following statement: "If $x < 0$, then $x^2 - x > 0$" and determine which ones are true

Contrapositive:

$$x^2 - x \leq 0 \Rightarrow x \geq 0$$

Converse

$$x^2 - x > 0 \Rightarrow x < 0$$

Contrapositive is correct, converse is incorrect $(2^2 - 2 > 0)$

(b) Do the same for the statement $x > 0 \Rightarrow x^2 - x > 0$

Contrapositive:

$$x^2 - x \leq 0 \Rightarrow x \leq 0$$

Converse

$$x^2 - x > 0 \Rightarrow x > 0$$

Contrapositive is false $(1^2 - 1 = 0)$; Converse is also false $((-2)^2 - (-2) = 6)$.

1.1.4

Let A and B be the sets of real numbers. Write the negation of each of the following statements:

(a)

$$(\exists a \in A)(a^2 \notin B)$$

(b)

$$(\forall a \in A)(a^2 \notin B)$$

(c)

$$(\exists a \in A)(a^2 \in B)$$

(d)

$$(\forall a)(a \notin A \Rightarrow a^2 \notin B)$$

1.1.5

Let A be a nonempty collection of sets. Determine the truths of each of the following and their converses

(a)

$$x \in \bigcup A \Leftrightarrow (\exists B \in A)(x \in B)$$

(b)

$$x \in \bigcup A \Leftarrow (\forall B \in A)(x \in B)$$

(c)

$$x \in \bigcap A \Rightarrow (\exists B \in A)(x \in B)$$

(d)

$$x \in \bigcap A \Leftrightarrow (\forall B \in A)(x \in B)$$

1.1.6

Skip

1.1.7

skip

1.1.8

GOTO set theory book

1.1.9

Formulate DML for arbitrary unions and intersections

$$A \setminus \bigcap (B) = \bigcup (A \setminus B)$$
$$A \setminus \bigcup (B) = \bigcap (A \setminus B)$$

For the proof goto set theory or real analisys book

1.1.10

(a, b, d) are true

1.2 Functions