My algorithms exercises

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Preface

Exercises for Introduction to Algorithms by Cormen et al., 4th ed. It has some exercises, that should be written down, moslty in math and whatnot.

Part I

Appendix: Mathematical Background

Chapter 1

Summations

1-1

Prove that $\sum_{k=1}^{n} O(f_k(i)) = O(\sum_{k=1}^{n} f_k(i))$

Short answer:

$$\sum cg(x) = c \sum g(x)$$

Long answer:

Suppose that $g \in O(f_k(i))$. It follows that there exists n_i and c_i such that $0 \leq g(n) \leq cf_i(n)$. Thus we can pick $n = \max\{n_0, n_1, ...\}$ and $c = \max\{c_0, c_1, ...\}$. We know that both n and c will work all of functions f_k. Therefore by linearity of summations

$$\sum_{k=1}^{n} O(f_k(i)) = \sum_{k=1}^{n} cf_k(i) == c\sum_{k=1}^{n} f_k(i) == O(\sum_{k=1}^{n} f_k(i))$$

(notation is a little abused and there is nothing is rigorously proven, but it'll do).

1-2

Find a simple formula for $\sum_{k=1}^{n} (2k-1)$.

$$\sum_{k=1}^{n} (2k-1) = \sum_{k=1}^{n} (2k) - \sum_{k=1}^{n} (1) = 2\sum_{k=1}^{n} (k) - n = 2\frac{n(n+1)}{2} - n = n(n+1) - n = n^2$$

1-3

Interpret the decimal number $111,111,111$ *in light of equation A.6*

$$111,111,111 = \sum_{k=0}^{9} 10^k = \frac{10^{10}-1}{10-1}$$

1-4

Evaluate the infinite series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - ...$

The series converges absolutely to 2, so we are free to do anything with it.

$$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - ... = \sum_{k=0}^{\infty} \frac{1}{2}^{2k} - \sum_{k=0}^{\infty} \frac{1}{2}^{1+2k} = \sum_{k=0}^{\infty} \frac{1}{2}^{2k} - \frac{1}{2} \sum_{k=0}^{\infty} \frac{1}{2}^{2k} = \left(1 - \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{1}{2}^{2k} =$$

$$= \left(1 - \frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{1}{4}^{k} = \left(1 - \frac{1}{2}\right) \frac{1}{1 - \frac{1}{4}} = \frac{1}{2} * \frac{4}{3} = \frac{2}{3}$$

1.5

Let $c \geq 0$ *be a constant. Show that* $\sum_{k=1}^{n} k^c = \Theta(n^{c+1})$

$$\Theta(\sum_{k=1}^{n} k^c) = \sum_{k=1}^{n} \Theta(k^c) =$$

1-9

Show that $\sum_{k=0}^{\infty} (k-1)/2^k = 0$

$$\sum_{k=0}^{\infty} (k-1)/2^k = \sum_{k=0}^{\infty} (k-1)/2^k = \sum_{k=1}^{\infty} (k-1)/2^k - \frac{1}{2^0} = \sum_{k=1}^{\infty} (k-1)/2^k - 1 =$$