# My linear algebra exercises

Evgeny Markin

2022

# Contents

# Preface

Exercises are from "Linear algebra done right" by Sheldon Axler, 3rd ed. I've already read this book before and completed some exercises from it. Right now I want to brush up the material once again, put all the proofs on a more durable material than paper and to prepare myself to what's gonna happen afterwards.

# Chapter 1

# 1.A $R^n$ and $C^n$

## 1

*Suppose $a$ and $b$ are real numbers, not both $0$. Find real nuber $c$ and $d$ such that*

$$1/(a+bi) = c+di$$

$$\frac{1}{a+bi} = c+di$$

$$\frac{1}{a+bi} - c - di = 0$$

$$\frac{a-bi}{(a+bi)(a-bi)} = c+di$$

$$\frac{a-bi}{(a^2+b^2)} = c+di$$

$$\frac{a}{a^2+b^2} - \frac{b}{a^2+b^2}i = c+di$$

Thus $c = \frac{a}{a^2+b^2}$ and $d = -\frac{b}{a^2+b^2}$

## 2

*Show that*

$$\frac{-1+\sqrt{3}i}{2}$$

*is a cube root of* 1 *(meaning that its cube equals 1)*

$$(\frac{-1+\sqrt{3}i}{2})^3 = \frac{(-1+\sqrt{3}i)^3}{8} = \frac{(-1+\sqrt{3}i)(-1+\sqrt{3}i)^2}{8} = \frac{(-1+\sqrt{3}i)(1-2\sqrt{3}i-3)}{8} =$$

$$= \frac{(-1+\sqrt{3}i)(-2-2\sqrt{3}i)}{8} = \frac{2+2\sqrt{3}i-2\sqrt{3}i+6}{8} = \frac{8}{8} = 1$$

as desired.

## 3

*Find two distinct square roots of $i$*

Square root of $i$, I assume, is a number, whose square is equal to $i$. Suppose that $(a+bi)^2 = i$. It follows that

$$(a+bi)^2 = a^2 + 2abi - b^2$$

So if we set

$$a = b = 1/\sqrt{2}$$

this equation holds. Also it holds for

$$a = b = -1/\sqrt{2}$$

maxima seems to agree with me on this one

## 4

*Show that $\alpha+\beta = \beta+\alpha$ for all $\alpha, \beta \in \boldsymbol{C}$*

Let $\alpha = a_1 + b_1 i$ and $\beta = a_2 + b_2 i$. It follows

$$\alpha+\beta = a_1+b_1i+a_2+b_2i = a_2+b_2i+a_1+b_1i = \beta+\alpha$$

as desired.

## 5

*Show that $(\alpha+\beta)+\lambda = \alpha+(\beta+\lambda)$ for all $\alpha, \beta, \lambda \in \boldsymbol{C}$*

Let $\alpha = a_1 + b_1 i$ , $\beta = a_2 + b_2 i$, $\lambda = a_3 + b_3 i$. It follows that

$$\alpha+(\beta+\lambda) = a_1+b_1i+(a_2+b_2i+a_3+b_3i) = (a_1+b_1i+a_2+b_2i)+a_3+b_3i = (\alpha+\beta)+\lambda$$

## 6

*Show that* $(\alpha\beta)\lambda = \alpha(\beta\lambda)$

$\alpha+(\beta+\lambda) = (a_1+b_1i)((a_2+b_2i)+(a_3+b_3i)) = ((a_1+b_1i)(a_2+b_2i))+(a_3+b_3i) = (\alpha\beta)\lambda$

## 7

*Show that for every* $\alpha \in \boldsymbol{C}$ *there exists a unique* $\beta \in \boldsymbol{C}$ *such that* $\alpha + \beta = 0$

Suppose that there exist two different $\beta_1 \neq \beta_2$ such that $\alpha + \beta_1 = 0$ and $\alpha + \beta_2 = 0$. It follows that

$$\beta_1 = \beta_1 + 0 = \beta_1 + \alpha + \beta_2 = \alpha + \beta_1 + \beta_2 = 0 + \beta_2 = \beta_2$$

which is a contradiction. Therefore there exists only one unique $\beta$.

## 8

*Show that for every* $\alpha \in \boldsymbol{C}$ *with* $\alpha \neq 0$ *there exists a unique* $\beta \in \boldsymbol{C}$ *such that* $\alpha\beta = 1$

Suppose that it is not true and there exist two different $\beta_1 \neq \beta_2$ such that

$$\alpha\beta_1 = 1 \textit{ and } \alpha\beta_2 = 1$$

it follows then that

$$\beta_1 = 1 * \beta_1 = \alpha\beta_2\beta_1 = \alpha\beta_1\beta_2 = 1 * \beta_2 = \beta_2$$

which is a contradiction. Therefore there exists only one unique $\beta$.

## etc

The rest of the section is the repetition of this kind of stuff. That is a lot of writing, and not a lot of thinking, so I'll skip it. I don't ususally like to skip sections, but I have aa feeling, that I've completed this thing on paper somewhere, and there is not much reason to rewrite it here.

# Chapter 2

# 1B Definition of Vector Space

j