Lista 1

1) Sejam:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \end{bmatrix}_{2\times 3} \qquad C = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}_{3\times 1} \qquad E = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}_{2\times 3} \qquad D = \begin{bmatrix} 2 & -1 \end{bmatrix}_{1\times 2} \qquad F = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 2 \\ -2 & 0 & -2 \end{bmatrix}_{3\times 3}$$

Calcule:

a) $A+B$

$$\begin{bmatrix} -1 & 2 & 4 \\ 5 & 1 & 0 \end{bmatrix}$$

b) $A \cdot C$

$$\begin{bmatrix} (1\cdot -1)+(2\cdot 2)+(3\cdot 4) \\ (2\cdot -1)+(1\cdot 2)+(1\cdot 4) \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} +15 \\ -4 \end{bmatrix}$$

c) B.C

$$\begin{bmatrix} (-2\cdot-1)+(0\cdot2)+(1\cdot4) \\ (3\cdot-1)+(0\cdot2)+(4\cdot1) \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 2+0+4 \\ -3+0+4 \end{bmatrix} \rightarrow \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

d) D.A

$$\begin{bmatrix} (2\cdot1)+(-1\cdot2) & (2\cdot2)+(-1\cdot1) & (2\cdot3)+(-1\cdot1) \\ 0 & 3 & 7 \end{bmatrix}$$

e) D.D

$$\begin{bmatrix} (-1\cdot2) & (-1\cdot-1) \\ (2\cdot2) & (2\cdot-1) \\ (4\cdot2) & (4\cdot-1) \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} -2 & 1 \\ 4 & -2 \\ 8 & -4 \end{bmatrix}$$

f) D.B

$$\begin{bmatrix} (-2\cdot2)+(-1\cdot3) & (0\cdot2)+(-1\cdot0) & (2\cdot1)+(-1\cdot1) \end{bmatrix}$$

$$\begin{bmatrix} -7 & 0 & 1 \end{bmatrix}$$

g) -A

$$\begin{bmatrix} -1 & -2 & -3 \\ -2 & -1 & 1 \end{bmatrix}$$

h) -D

$$\begin{bmatrix} -2 & +1 \end{bmatrix}$$

i) E-F

$$\begin{bmatrix} 0 & 0 & 3 \\ 1 & 4 & 4 \\ 9 & 8 & 11 \end{bmatrix}$$

j) E+F

$$\begin{bmatrix} 2 & 4 & 3 \\ 7 & 6 & 8 \\ 5 & 8 & 4 \end{bmatrix}$$

k) $2E - F$

$$2 \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 2 \\ -2 & 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 6 \\ 5 & 9 & 10 \\ -16 & 16 & 20 \end{bmatrix}$$

l) $E - 3F$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} - 3 \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 2 \\ -2 & 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 & 0 \\ 9 & 3 & 6 \\ -6 & 0 & -6 \end{bmatrix} \rightarrow \begin{bmatrix} -2 & -4 & 3 \\ -5 & 2 & 0 \\ 13 & 8 & 15 \end{bmatrix}$$

m) $E \cdot F$

$$\begin{bmatrix} ((1.1)+(2.3)+(3.-2)) & (1.2+2.1+3.0) & (1.0+2.2+(3.-2)) \\ ((4.1)+(5.3)+(6.-2)) & (4.2+5.1+6.0) & (4.0+5.2+(6.-2)) \\ ((7.1)+(8.3)+(9.-2)) & (7.2+8.1+9.0) & (7.0+8.2+(9.-2)) \end{bmatrix}$$

$$\begin{bmatrix} 1+6+(-6) & 2+2+0 & 0+4+(-6) \\ 4+15+(-12) & 8+5+0 & 0+10+(-12) \\ 7+24+(-18) & 14+8+0 & 7+16+(-18 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & -2 \\ 7 & 13 & -2 \\ 13 & 22 & -2 \end{bmatrix}$$

n) F.A

O número de colunas da matriz do esquerda deve ser igual ao número de linhas da matriz da direita

a) -E

$$\begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{bmatrix}$$

f) F

$$\begin{bmatrix} -1 & -2 & 0 \\ -3 & -1 & -2 \\ +2 & 0 & +2 \end{bmatrix}$$

2)

$$A \times B \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 4 & 1 & 0 \\ 2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & -3 & 0 & 1 \\ 1 & 15 & 0 & -5 \\ -3 & 15 & 0 & -5 \end{bmatrix}$$

$$A \times C \begin{bmatrix} 1 & -3 & 2 \\ 2 & 1 & -3 \\ 4 & -3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 & -1 & -2 \\ 3 & -2 & -1 & -1 \\ 2 & -5 & -1 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -3 & 0 & 1 \\ 1 & 15 & 0 & 5 \\ -3 & 15 & 0 & 5 \end{bmatrix}$$

3) O produto de matrizes não é comutativo, não temos necessariamente $AB+BA=2AB$ e $A^2+BA=0$

4)

$$\begin{pmatrix} 2 & x^2 \\ 2x-1 & 0 \end{pmatrix} = A = A^T = \begin{pmatrix} 2 & 2x-1 \\ x^2 & 0 \end{pmatrix}$$

$x^2 = 2x-1$

$x^2-2x+1=0$

$\Delta = b^2 - 4ac$

$\Delta = -2^2 - 4\,1\,1$

$\Delta = 4 - 4$

$\Delta = 0$

$x = \frac{+2 \pm 0}{2}$

$x' = x'' = \frac{2}{2} = 1$

5)

$$\begin{bmatrix} 2x+3y & 3x+4y \\ 2z+3w & 3z+4w \end{bmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{cases} 2x+3y=1 \quad (-3) \\ 3x+4y=0 \quad (2) \end{cases}$$

$$-6x-9y=-3$$
$$6x+8y=0$$

$$-y=-3 \quad (-1)$$
$$+y=3$$

$$2x+3\cdot 3=1$$
$$2x+9=1$$
$$2x=-9+1$$
$$x=\frac{-8}{2}$$
$$\boxed{x=-4}$$

$$\begin{cases} x=-4 \\ y=3 \end{cases}$$

$$2z+3w=0 \quad (-3)$$
$$3z+4w=1 \quad (2)$$

$$-6z-9w=0$$
$$6z+8w=2$$

$$-w=2 \quad (-1)$$
$$\boxed{w=-2}$$

$$2z+3\cdot(-2)=0$$
$$2z=+6$$
$$z=\frac{6}{2}$$
$$\boxed{z=3}$$

$$\begin{cases} w=-2 \\ z=3 \end{cases}$$

6)

$$\begin{bmatrix} 3x-1 & x+y \end{bmatrix} = \begin{bmatrix} y & 35 \end{bmatrix}$$

$3x - 1 = y$
$x + y = 35$

$3x - y = +1 \quad (1)$
$x + y = 35 \quad (-3)$

$3x - y = 1$
$-3x - 3y = -105$

$104 \mid 4$
$24 \quad 26$
0

$-4y = -104 \quad (-1)$
$4y = 104$
$y = \frac{104}{4}$
$y = 26$

$3x - y = 1$
$3x - 26 = 1$
$3x = 26 + 1$
$x = \frac{27}{3}$
$x = 9$

$x = 9 \text{ e } y = 26$

7)

$$2\begin{bmatrix} x & y \\ z & -w \end{bmatrix} - \begin{bmatrix} 3 & x-y \\ z+w & 6+y \end{bmatrix} = \begin{bmatrix} x+y & 5 \\ 2w & 2w-z \end{bmatrix}$$

$$\begin{bmatrix} 2x & 2y \\ 2z & -2w \end{bmatrix} - \begin{bmatrix} 3 & x-y \\ z+w & 6+y \end{bmatrix} = \begin{bmatrix} x+y & 5 \\ 2w & 2w-z \end{bmatrix}$$

$$\begin{bmatrix} 2x-3 & 3y-x \\ z-w & -2w-6-y \end{bmatrix} = \begin{bmatrix} x+y & 5 \\ 2w & 2w-z \end{bmatrix}$$

$$\begin{cases} 2x-3=x+y \\ 3y-x=5 \end{cases}$$

$$x-y=3$$
$$3y-x=5$$

$$x-y=3$$
$$-x+3y=5$$

$$2y=8$$
$$y=\frac{8}{2}$$
$$\boxed{y=4}$$

$$x-y=3$$
$$x-4=3$$
$$x=4+3$$
$$\boxed{x=7}$$

$$\begin{cases} z-w=2w \\ -2w-6-y=2w-z \end{cases}$$

$$-3w+z=0 \quad (-4)$$

$$-4w+z=10 \quad (-3)$$

$$-12w+12=0$$
$$+12w-12z=-30$$

$$\boxed{z=-30}$$

$$-3w+z=0$$
$$-3w-30=0$$
$$-3w=30$$
$$w=\frac{30}{-3}$$
$$\boxed{w=-10}$$

8)

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$5x=1$$
$$\boxed{x=\frac{1}{5}}$$