$$V\left(y,B\right)=\rho_D\log\left\{\exp\left[\frac{V^d(y)}{\rho_D}\right]+\exp\left[\frac{V^r(y,B)}{\rho_D}\right]\right\} \tag{1}$$

$$\Pr\left(d=1|y,B\right)=\frac{\exp\left[\frac{V^d(y)}{\rho_D}\right]}{\exp\left[\frac{V^d(y)}{\rho_D}\right]+\exp\left[\frac{V^r(y,B)}{\rho_D}\right]}=\frac{1}{1+\exp\left[\frac{V^r(y,B)-V^d(y)}{\rho_D}\right]} \tag{2}$$

$$V^d\left(y\right)=u\left[h\left(y\right)\right]+\beta\mathbb{E}_{y'|y}\left\{\gamma V\left(y',0\right)+(1-\gamma)V^d\left(y'\right)\right\} \tag{3}$$

$$W\left(y,B,B'\right)=u\left[y-\kappa B+q\left(y,B'\right)\left(B'-(1-\delta)B\right)\right]+\beta\mathbb{E}_{y'|y}V\left(y',B'\right) \tag{4}$$

$$V^r\left(y,B\right)=\rho_B\log\sum_{B'}\exp\left[\frac{W\left(y,B,B'\right))}{\rho_B}\right] \tag{5}$$

$$\Pr\left(B'=x\middle|y,B\right)=\frac{\exp\left[\frac{W\left(y,B,x\right))}{\rho_B}\right]}{\sum_i\exp\left[\frac{W\left(y,B,i\right))}{\rho_B}\right]} \tag{6}$$

$$q\left(y,B'\right)=\frac{1}{1+r}\mathbb{E}_{y'|y}\Pr\left(d=0\middle|y',B'\right)\left[\kappa+(1-\delta)\sum_{B''}\Pr\left(B''|y',B'\right)q\left(y',B''\right)\right] \tag{7}$$

$$u\left(c\right)=\frac{c^{1-\sigma}-1}{1-\sigma} \tag{8}$$

$$h\left(y\right)=y-\max\{0,\lambda_0 y+\lambda_1 y^2\} \tag{9}$$

$$\log y'=-(1-\rho)\frac{\sigma_y^2}{2\left(1-\rho^2\right)}+\rho\log y+\sigma_y\varepsilon,\quad \varepsilon\sim\mathcal{N}(0,1) \tag{10}$$