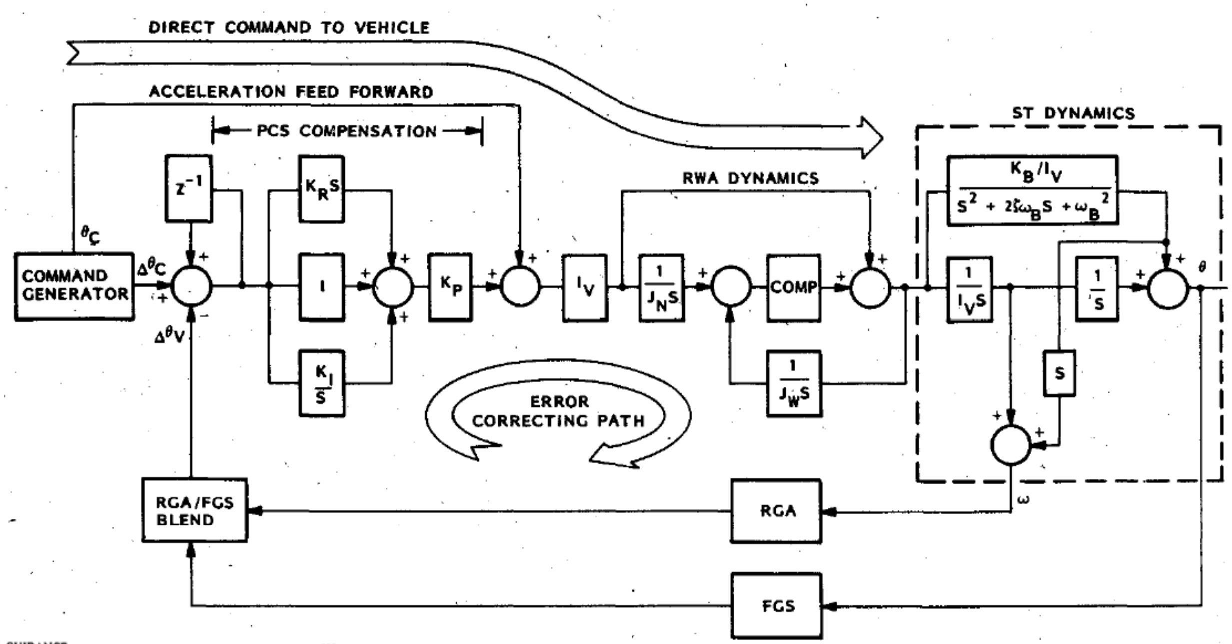
Digital Implementation



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AIAA 80-1784R

Space Telescope Pointing Control System

ESE 505 & MEAM 513

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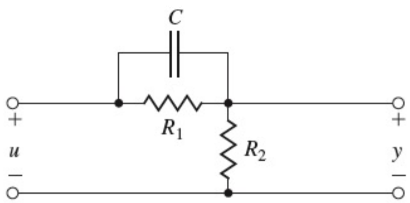
How Do We Implement Dynamic Controller?

- We Have Seen Many Dynamic Compensators in Our Control System Designs
  - PID Controllers
  - First-Order Lag (Low-Pass) Filters
  - First-Order Washout (High-Pass) Filters
  - Notch Filters
  - Lead & Lag Compensators

- How Do We Implement These Designs?
  - Analog Computers (Op-Amps + Resistors + Capacitors)
  - Digital Computers (Software)

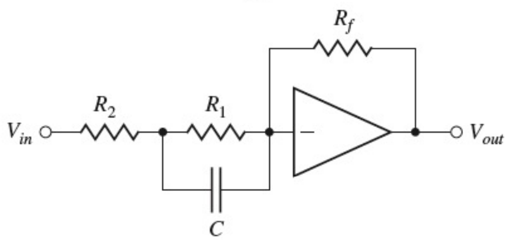
Analog Implementation (Prob. 2.15)



(a)

Lead Filter

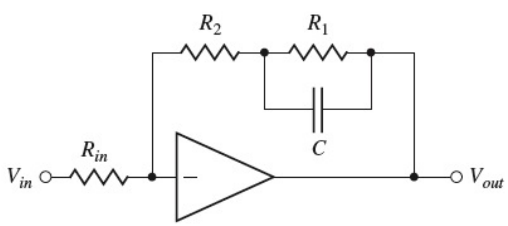
$$\frac{Cs + \frac{1}{R_1}}{Cs + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$



(b)

Lead Filter

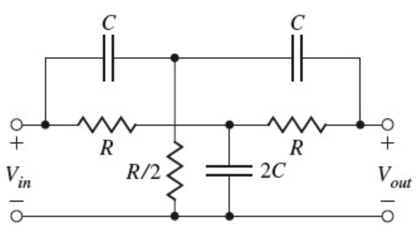
$$-\frac{R_f}{R_2}\frac{s + \frac{1}{R_1 C}}{s + \frac{1}{R_1 C} + \frac{1}{R_2 C}}$$



(c)

Lag Filter

$$-\frac{R_2}{R_{in}}\frac{s + \frac{1}{R_2 C} + \frac{1}{R_1 C}}{s + \frac{1}{R_1 C}}$$



(d)

Notch Filter

$$\frac{s^2 + \frac{1}{R^2 C^2}}{s^2 + \frac{4}{RC}s + \frac{1}{R^2 C^2}}$$

What Does Digital Control Software Look Like?

1. Process Input(s) From Sensors (A/D Conversion)

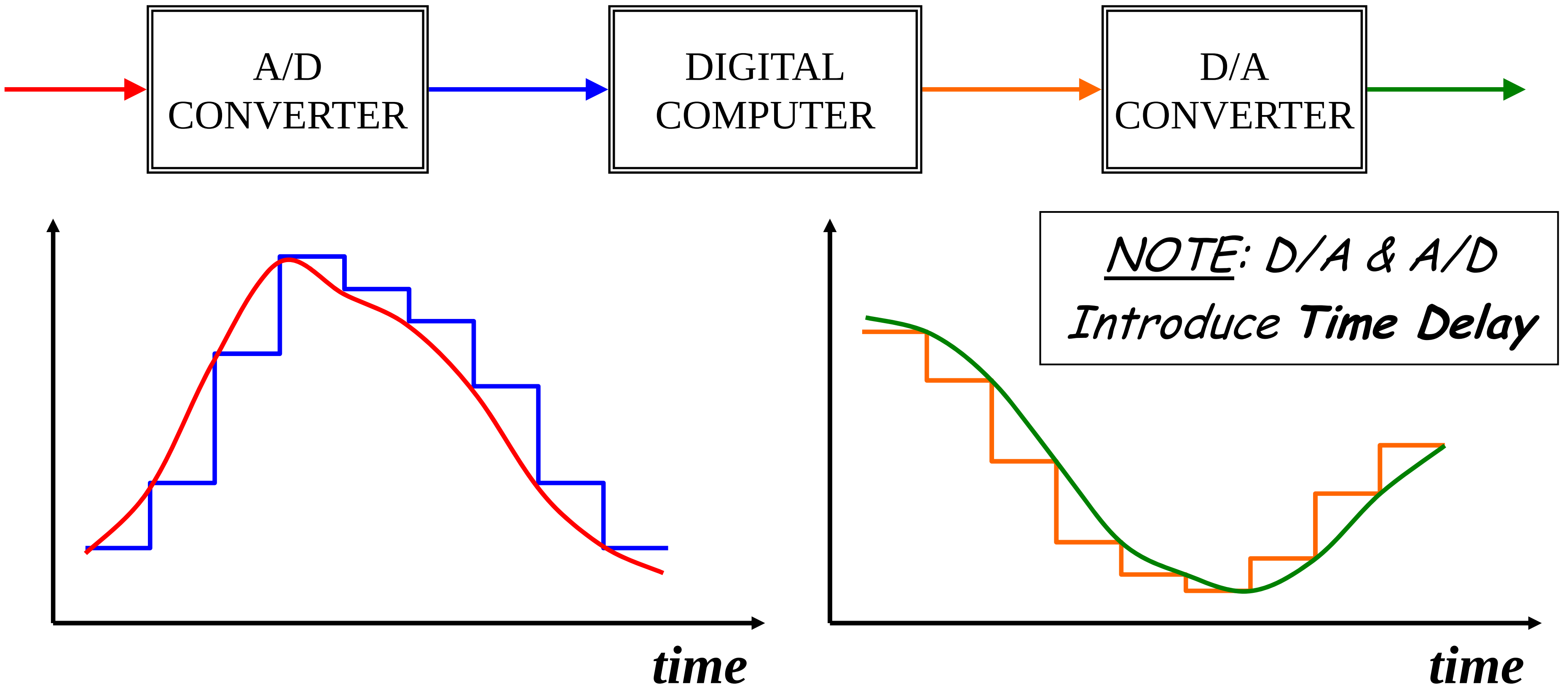
2. Compute “New Value” of Output
   (Discrete-Time Version of Dynamic Compensation)

3. Send Output(s) to Actuators (D/A Conversion)

4. Wait For Clock Pulse (“Fixed Frame Rate” = T)

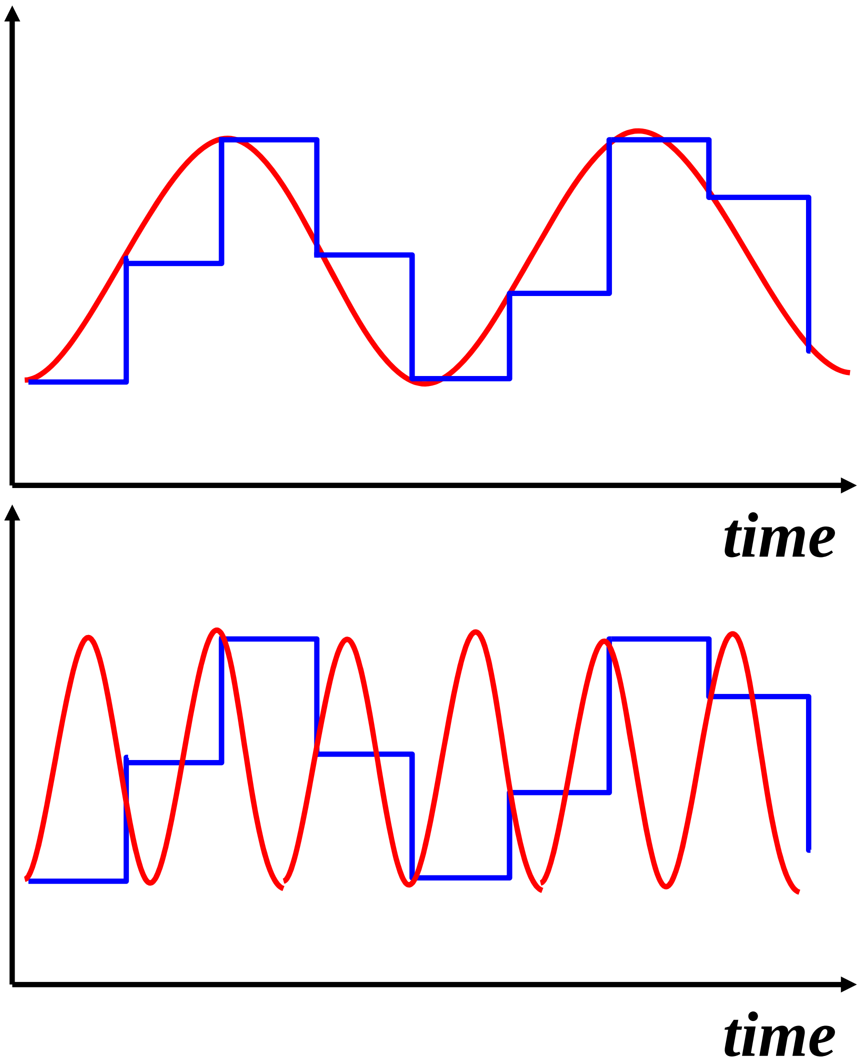
5. Go to Step 1 (Repeat Calculations every T Seconds)

Key Aspects of Digital Implementation



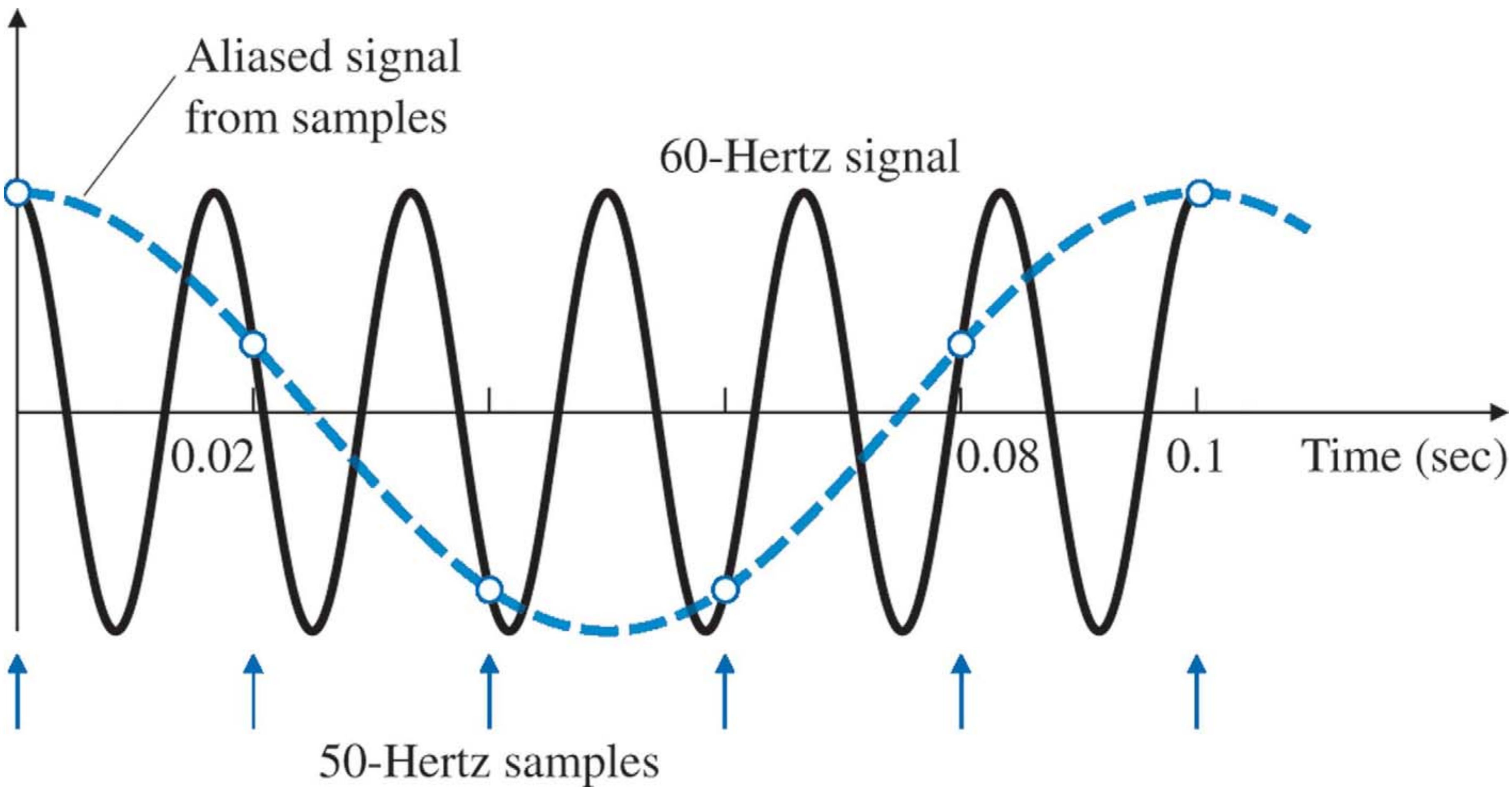
- Sampling Frequency = Samples / Second
- Resolution = Number of Digital Values Per Analog Volt

Nyquist Frequency & Aliasing



- Must Have 2 Samples Per Cycle to “See” Analog Oscillations
- “Nyquist Frequency” = (Sample Frequency)/2
- Frequencies Above Nyquist Frequency Are “Aliased” to Frequency Below Nyquist Frequency

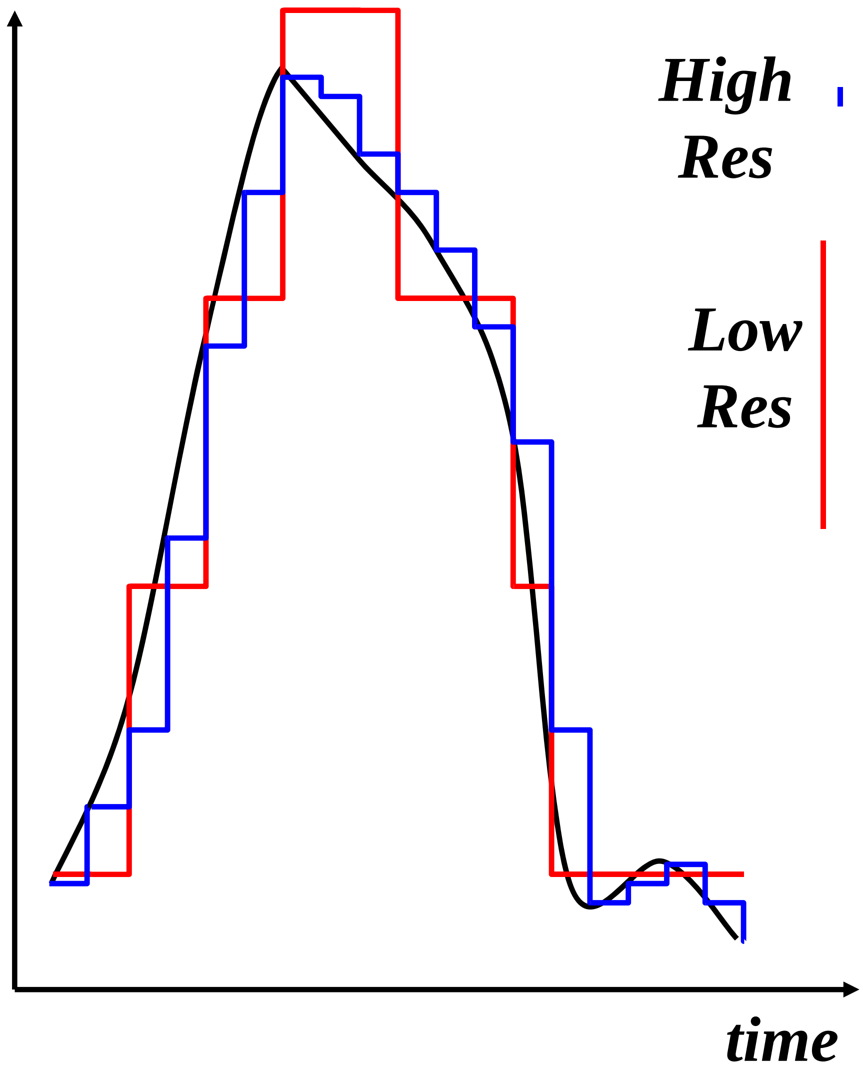
Textbook's Quantitative Aliasing Example



Implications of Aliasing & D/A

- MUST Use Analog Low-Pass Filter Before Sampling!
  - Once Sampled, It is Too Late—Cannot Distinguish Aliased Signal from Lower-Frequency Signal
  - Analog Low-Pass Filter with Cut-off Frequency Below Nyquist Frequency (How Far Below = Important Design Decision)
- Note Implications of Digital Control
  - D/A & A/D Processes Introduce Effective Time Delay = Phase Loss at High Frequency
  - Feedback Always (& Feed-Forward Usually) Also Have Anti-Aliasing Low-Pass Filters = More Phase Loss at High Frequency
- For High-Bandwidth Feedback with Good Stability, Want Small Phase Loss = Fast Sample Frequency

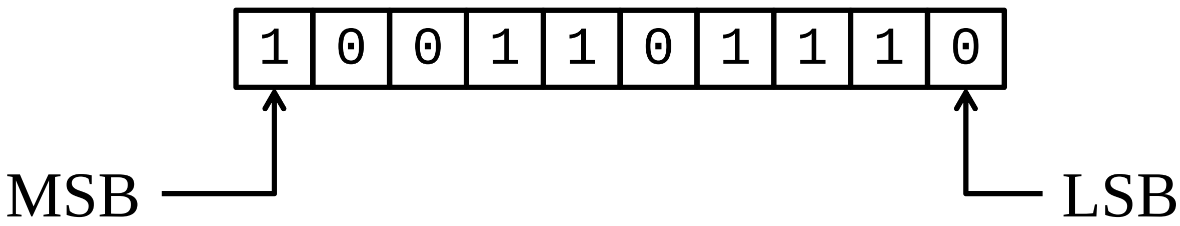
Resolution ("Bits") & Amplification



- Analog-to-Digital (A/D) Conversion Over Hardware-Dependent Voltage Range
  - 0V to +5V
  - -10V to +10V
- Range Divided into 2^n Digital Values (n from Hardware)
  - n=8 → 256
  - n=10 → 1024
  - n=16 → 65536
- Must Amplify Analog Signal To Get Good Digital Precision!
  - 8-bit A/D with +/-5V Range Yields Resolution of 10/256 = 0.039V
  - Analog Signal +/- 0.1V → Only 5 Values Possible! (0, +/- 0.039, +/- 0.078)

More About Resolution

- Typical 10-Bit A/D Conversion



- MSB = “Most Significant Bit” Represents Half of Range
- LSB = “Least Significant Bit” is Resolution
  - Cannot Control More Accurately Than This!
  - Often Expect Hysteresis +/- 1 Bit in Response
  - Sometimes Use Deadband in Control to Avoid Chatter
- Caution : Fast Sample Rates (Generally Good) Can Cause Problems with Finite Resolution
  - Approximate Derivatives Can Show Zero Value For Most Cycles, Followed By HUGE Values on Change of LSB!

How to “Compute Dynamic Compensation” ?

Example : Integral Feedback

Continuous Time

$$E(s) \longrightarrow \boxed{D(s) = \frac{K_I}{s}} \longrightarrow U(s)$$

$$\frac{du}{dt}(t) = K_I e(t) \Rightarrow u(t) = K_I \int_0^t e(\tau)d\tau$$

$$u(t) = u(t-T) + K_I \underbrace{\int_{t-T}^{t} e(\tau)d\tau}$$

How Do We Approximate This
Integral If We Only Have e(t)
at Discrete Instants (t=kT)?

Continuing With Integral Feedback Example…

Discrete Time $u(kT) \triangleq u[k]$

Careful with notation: "k" denotes index of iteration through the software loop

- e[k] is the input to the filter on iteration k
- u[k] is the output of the filter on iteration k
- u[k] may not make the actuator move a time delay of almost T compared to when e[k] was recorded

$$u[k] = u[k-1] + K_I \int_{t-T}^{t} e(\tau)\,d\tau$$

$$u[k] = u[k-1] + K_I e[k] T$$

Rectangular Integration

$$u[k] = u[k-1] + K_I \frac{\left(e[k] + e[k-1]\right)}{2} T$$

Trapezoidal Integration

General Case : Digital Filter

- We Need to Save Previous Values of Inputs to and Outputs from the Filter
- We Form New Output As Linear Combination of Previous Values and New Value of Input

$$u[k] = -\sum_{j=1}^{j=n} a_j u[k-j] + \sum_{j=0}^{j=m} b_j e[k-j] \quad \textit{Difference Equation}$$

$$\underline{x}[k+1] = A\underline{x}[k] + Be[k]$$
$$u[k] = C\underline{x}[k] + De[k]$$

State-Space Representation

- Implement Analog Design → Choose Coefficients to Approximate Continuous Transfer Function
- Direct Digital Design → Choose Coefficients Based on Discrete-Time Analysis (We Won't Do This)

We'll Do Analog (s) Design w/ Digital (z) Implementation

- Design Analog Compensators with Laplace Transforms
- Every Transfer Function Corresponds to Some ODE
- Find An Equivalent Difference Equation to Approximate Input-Output Behavior of ODE
- Easy Way = Use Discrete-Time Equivalent of Laplace to Represent Difference Equation

→ Z Transforms ←

Z-Transform = Discrete Counterpart of Laplace

$$Y[z] = \mathbb{Z}\{y[k]\} = \sum_{k=0}^{k=\infty} z^{-k} y[k]$$

Compare:

$$Y(s) = L\{y(t)\} = \int_{t=0}^{t=\infty} e^{-st} y(t) dt$$

Example: Unit Step

$$y[k] = 1 \Rightarrow Y[z] = \sum_{k=0}^{k=\infty} z^{-k} = 1 + z^{-1} + z^{-2} + \cdots$$

$$\sum_{k=0}^{k=\infty} z^{-k} = 1 + z^{-1} + z^{-2} + \cdots = \frac{1}{1 - z^{-1}} = \frac{z}{z-1}$$

Actually, This Infinite Sum Converges Only if |z|<1, but We Will Ignore This Restriction As We Did with Convergence Limits for Laplace

Exponential Continuous Time → Geometric Discrete Time

$$y(t) = e^{\lambda t} \Rightarrow Y(s) = \frac{1}{s-\lambda}$$

$$y[k] = y(kT) = e^{\lambda T k}$$ Equivalent Discrete-Time System

$$a \triangleq e^{\lambda T}$$

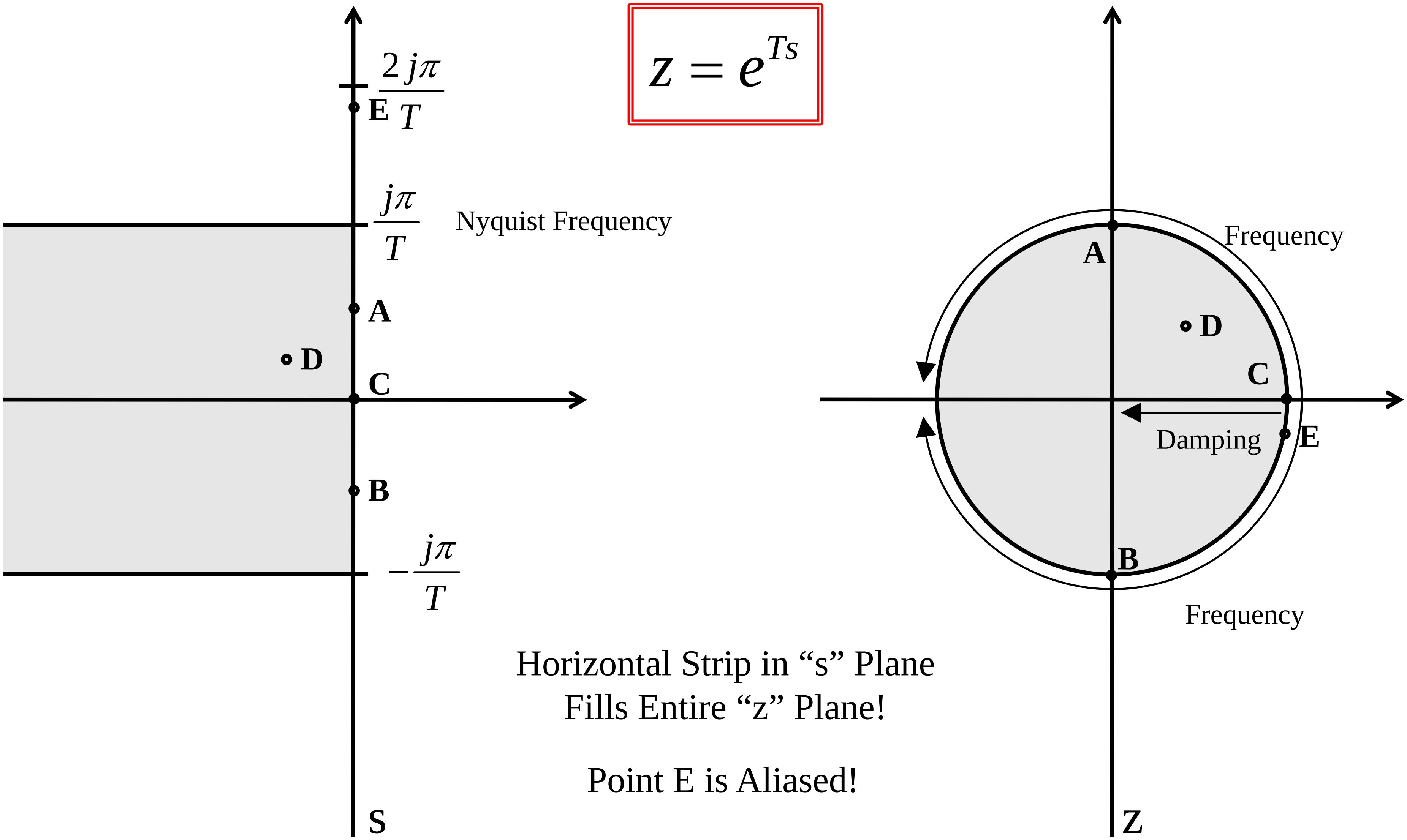
$$y[k] = a^k \Rightarrow Y[z] = \sum_{k=0}^{k=\infty} \left(az^{-1}\right)^k = 1 + az^{-1} + \cdots = \frac{1}{1-az^{-1}}$$

Pole @ s=λ in Continuous Time → Pole @ z=a=$e^{\lambda T}$ in Discrete Time

$$\boxed{z = e^{Ts}}$$

Continuous Poles → Discrete Poles

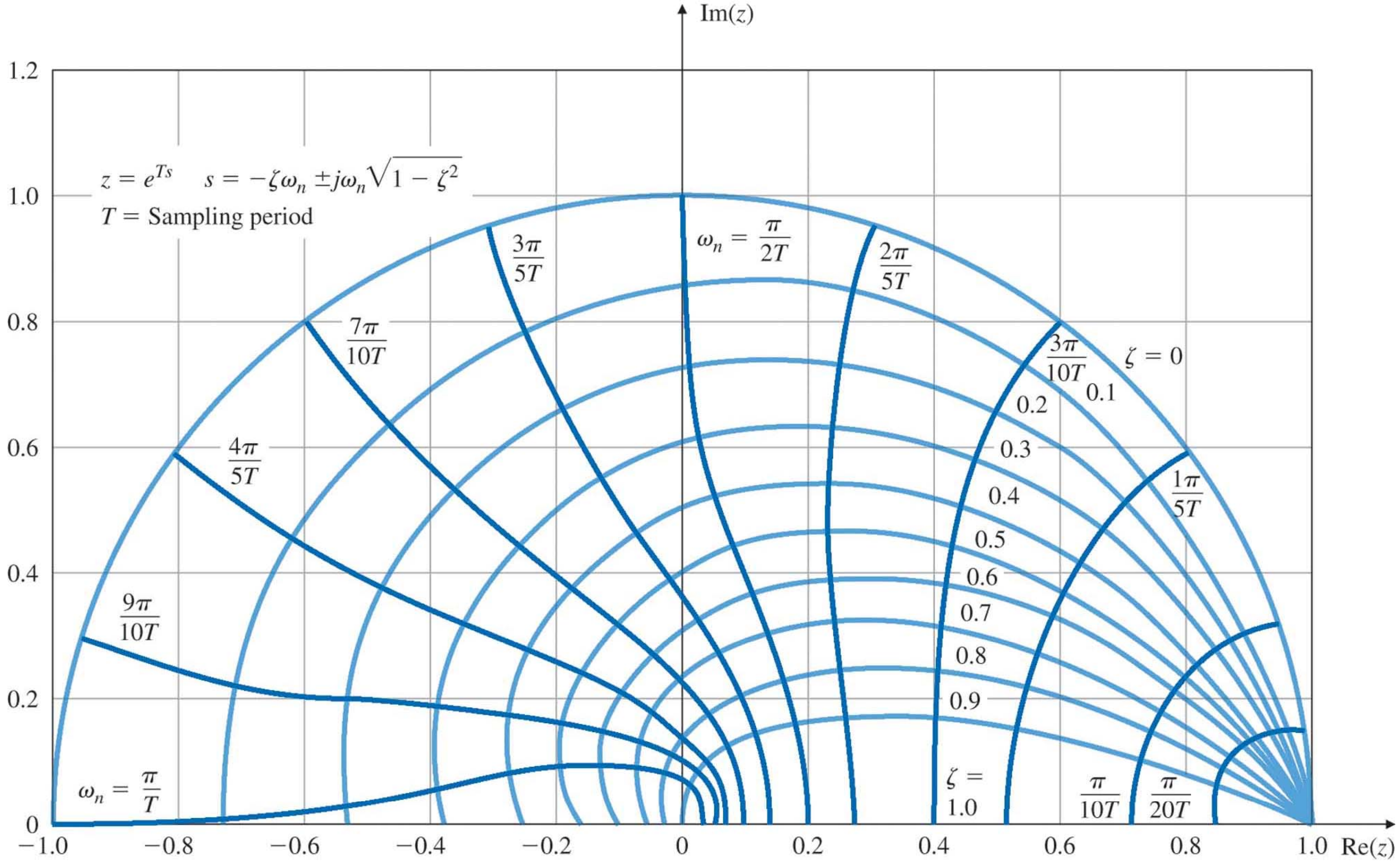
$$z = e^{Ts}$$



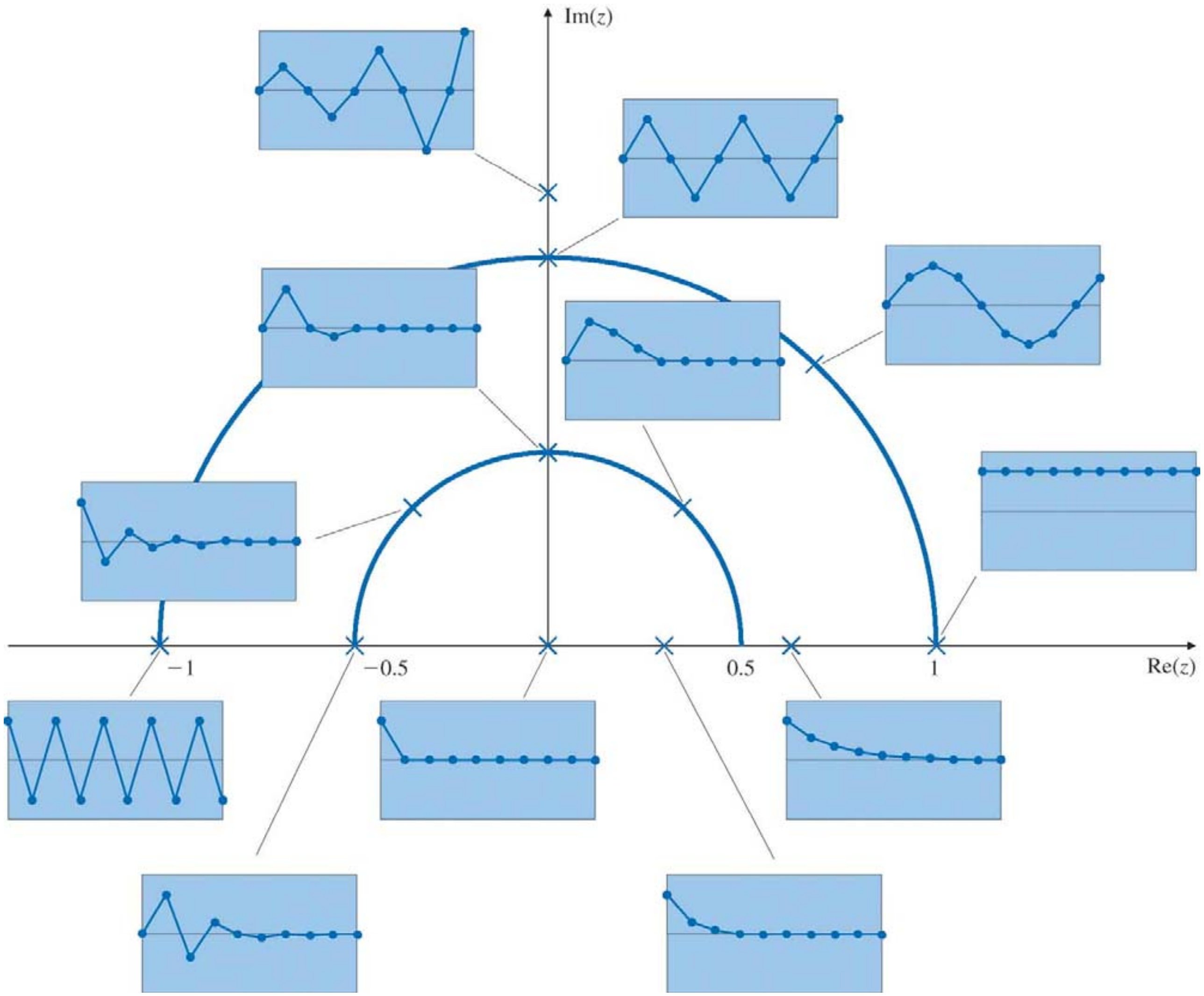
Horizontal Strip in "s" Plane
Fills Entire "z" Plane!

Point E is Aliased!

Mapping of Stable Poles in s Plane to z Plane



Discrete-Time Sequences in Z Plane



Some Z-Transform Theorems

Final Value Theorem

$$\lim_{k\to\infty} y[k] = \lim_{z\to 1}\left(z-1\right)Y[z]$$

Initial Value Theorem (obvious from definition)

$$y[0] = \lim_{z\to\infty} Y[z]$$

Time Shift (Compare to Differentiation/Integration for Laplace)

$$Z\left\{y[k+1]\right\} = \sum_{k=0}^{\infty} y[k+1]z^{-k} = z\sum_{k=1}^{\infty} y[k]z^{-k} = zY[z] - zy[0]$$

Transfer Functions & Solving Difference Equations

Laplace: (d/dt) → s

Z-Transform: [k-1] → z^{-1}

$$Y[z] = \frac{b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}}{1 + a_1 z^{-1} + \cdots + a_n z^{-n}} U[z] = H[z] U[z]$$

Partial-Fraction Expansion $$Y[z] = \sum_{i=1}^{n} \frac{\alpha_i}{1 + \beta_i z^{-1}}$$

Pole Locations $$\beta_i = r_i e^{j\varphi_i}$$

Now Return to Digital Implementation…

$$U(s) = \frac{K_I}{s} E(s) \qquad \frac{du}{dt}(t) = K_I e(t) \Rightarrow u(t) = K_I \int_0^t e(\tau) d\tau$$

$$u[k] = u[k-1] + K_I e[k] T$$

Rectangular Integration

$$U[z] = K_I T \frac{z}{z-1} E[z] \qquad s \rightarrow \frac{1}{T} \frac{z-1}{z} \qquad z \rightarrow \frac{1}{1-Ts}$$

- Naïve Approach Often Used by Students or on Simple Control Systems
- Reasonable for $|Ts| << 1$ (Fast Sample Rate Compared to System Dynamics)

"Tustin's" Method (Trapezoidal Integration)

$$u[k] = u[k-1] + K_I \frac{\left(e[k] + e[k-1]\right)}{2} T$$

Trapezoidal Integration

$$U[z] = K_I T \frac{1}{2} \frac{z+1}{z-1} E[z]$$

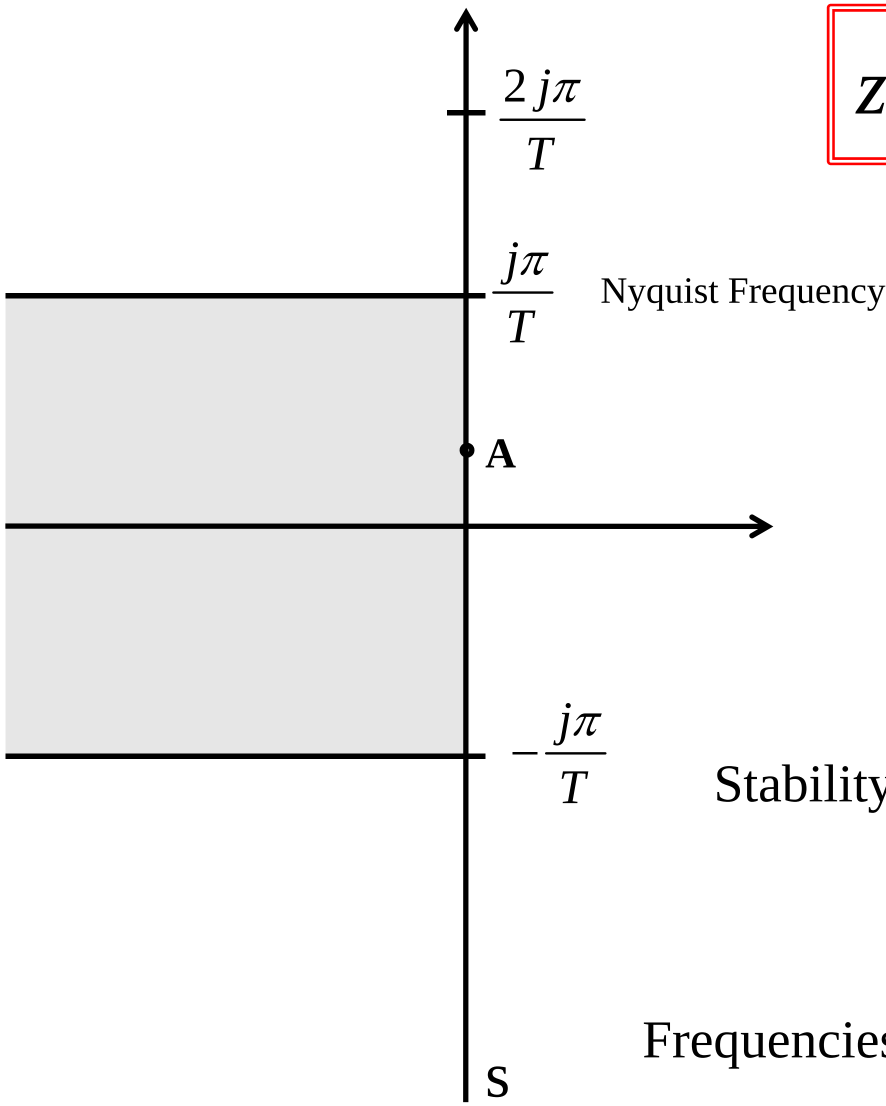
$$s \rightarrow \frac{2}{T} \frac{z-1}{z+1}$$

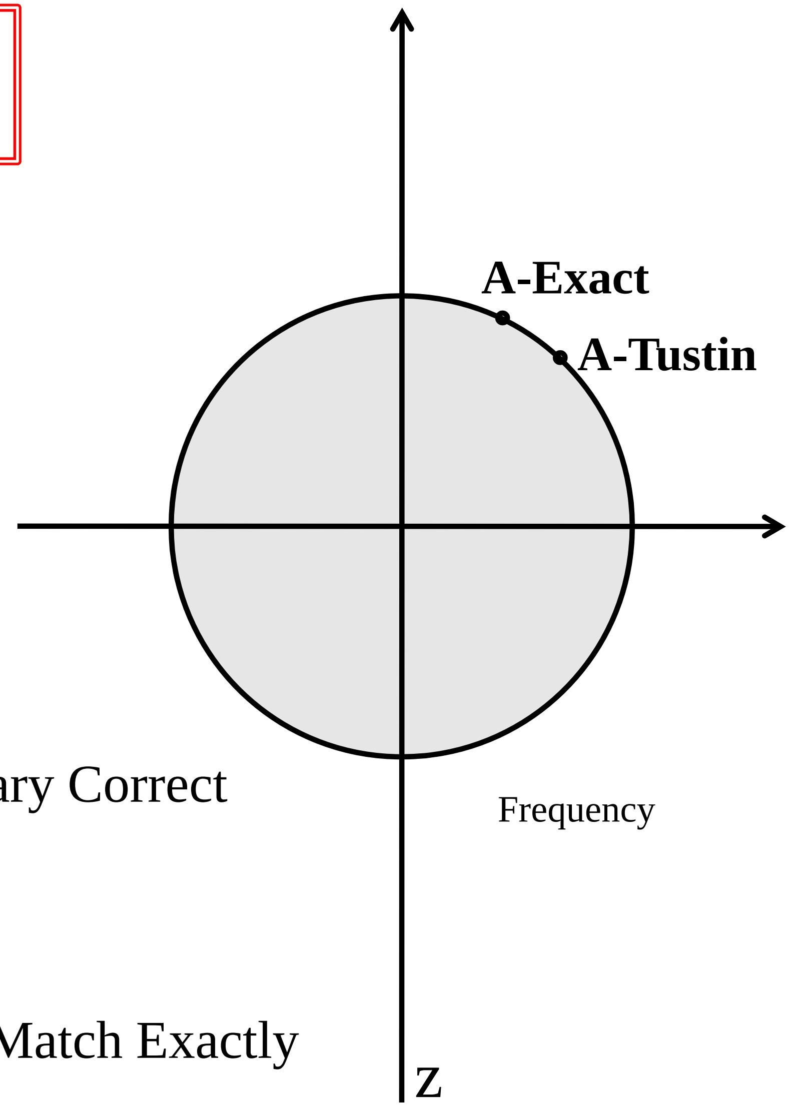
$$z \rightarrow \frac{1 + \frac{T}{2} s}{1 - \frac{T}{2} s}$$

- Equivalent to First-Order Pade Approximation of Delay
- Correct Mapping of LHP to Inside Unit Circle (Stability)

Exact vs. Tustin Mapping

$$\boxed{z = e^{Ts}}$$





Stability Boundary Correct

But…

Frequencies Don’t Match Exactly

General Bi-Linear : Tustin with Pre-Warping

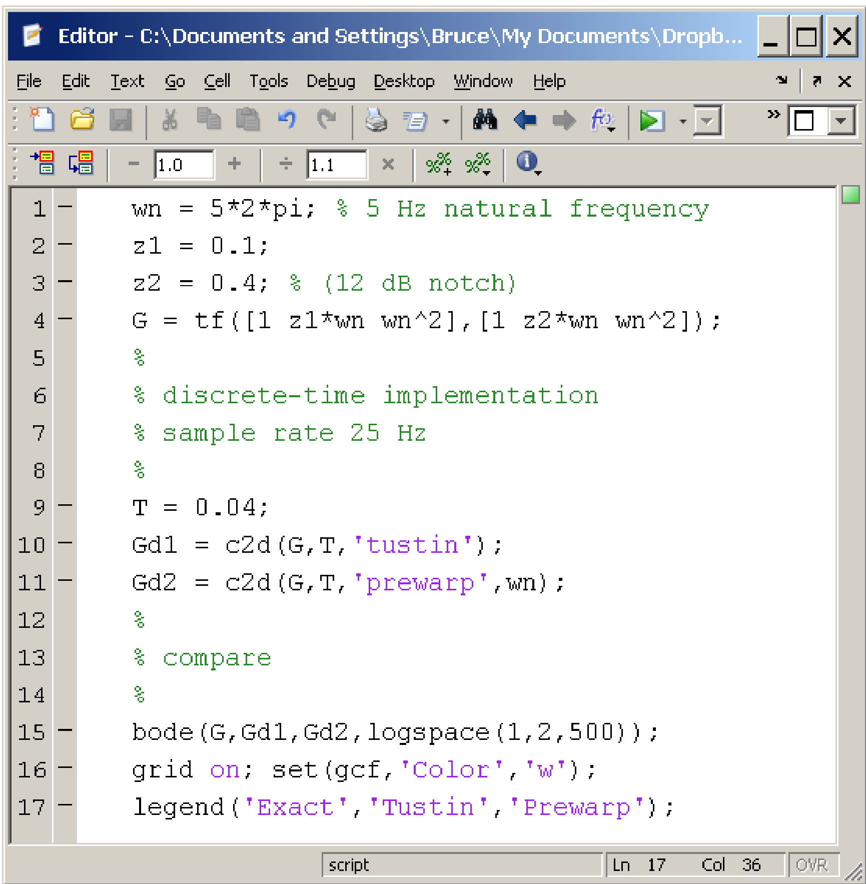
$$s \to \gamma \frac{z-1}{z+1} \qquad \text{OR} \qquad z \to \frac{\gamma + s}{\gamma - s}$$

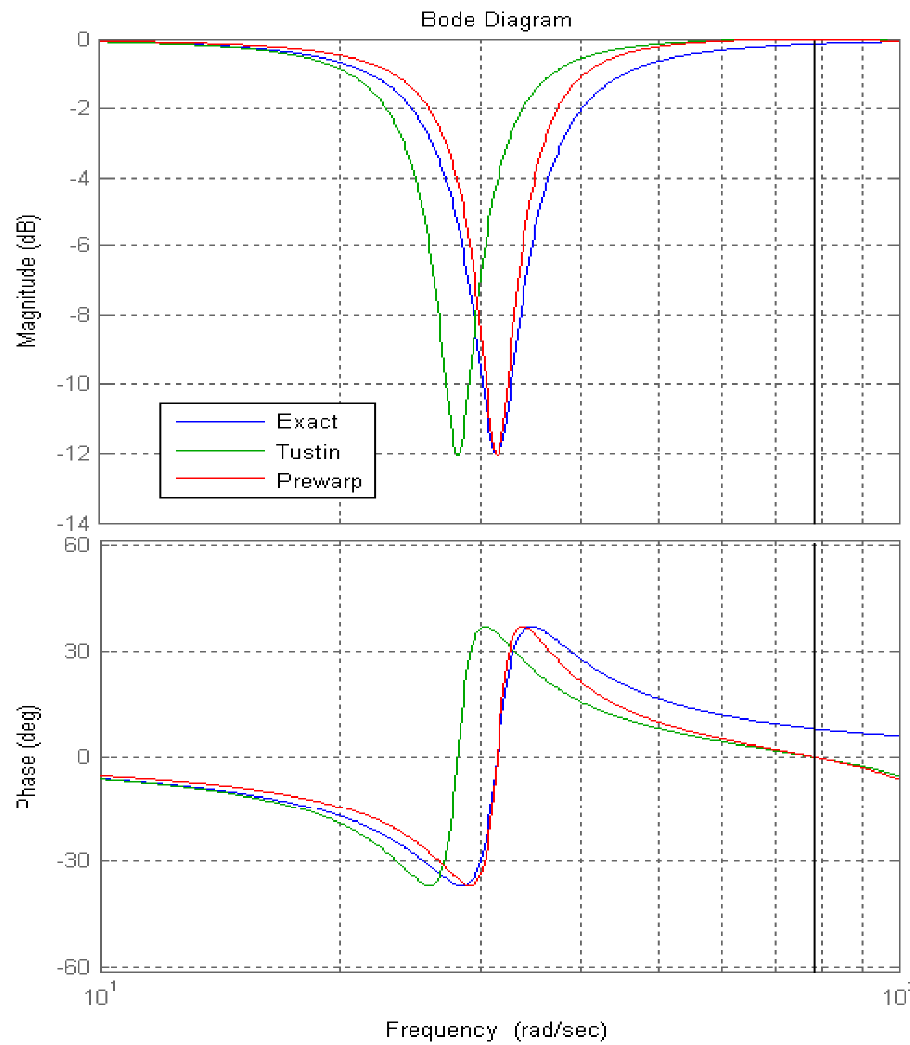
Choose γ to Get Exact Match @ s=jω

$$\frac{\gamma + j\omega}{\gamma - j\omega} = e^{j\omega T} \Rightarrow \gamma = \frac{\omega}{\tan\left(\frac{\omega T}{2}\right)}$$

- Very Important for Notch Filters
- Easy to Implement with MATLAB

Example in MATLAB





Summary of Conversion Process

- Do Analog Compensator Design → Gc(s)
  - `Gc = tf(num,den);`

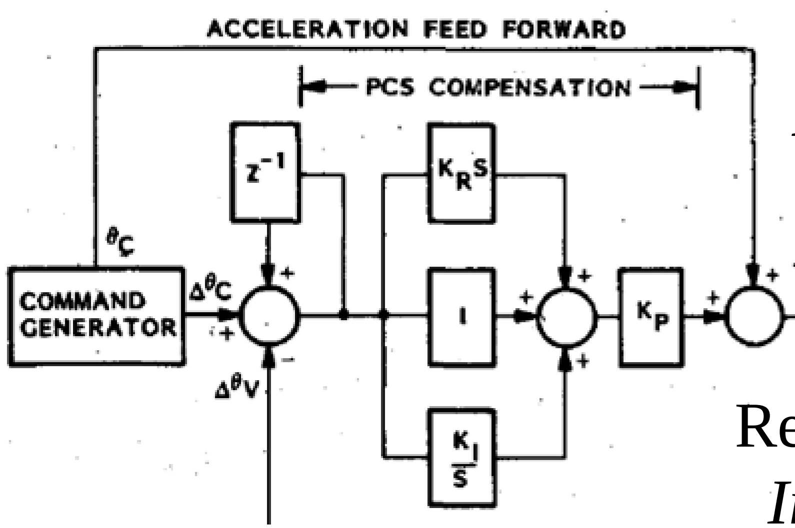
- Convert to Discrete-Time Compensator → Gd(z)
  - `Gd = c2d(Gc,Ts,method);`

- Convert to State-Space Representation
  - `[numd,dend]=tfdata(Gd,'v');`
  - `[A,B,C,D]=tf2ss(num,den);`

- Write Code to Implement Controller

What Is Going On in Satellite Block Diagram?

The input to the control system shown in Fig. 4 is the command generator acceleration and incremental position commands, rate gyro assembly "incremental" angles per 25 ms and the fine guidance sensor angle output for attitude. The rate gyro assembly data can be used for both rate and short-term attitude. The control system uses position, rate, and integral compensation. A digital filter is used in the rate path to suppress Space Telescope structural modes. The optical telescope assembly modal parameter values are large and require suppression to maintain adequate stability margins.



$$E[z] = z^{-1}E[z] + \Delta\theta_C[z] - \Delta\theta_V[z]$$

$$E[z] = \frac{z}{z-1}\left(\Delta\theta_C[z] - \Delta\theta_V[z]\right)$$

Rectangular Integration of Attitude Error *Increments* →Input to PID Controller!