2014-03-17 CONTROLS <u>RECAP</u>

1. SYSTEM DESCRIBED BY ODE(S) [505/513: LINEARIZE]

2. LAPLACE TRANSFORM ⇒ $Y(s) = G(s)\,U(s)$

$$G(s) = \text{"TRANSFER FUNCTION"} = \frac{N(s)}{D(s)} = \frac{N(s)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

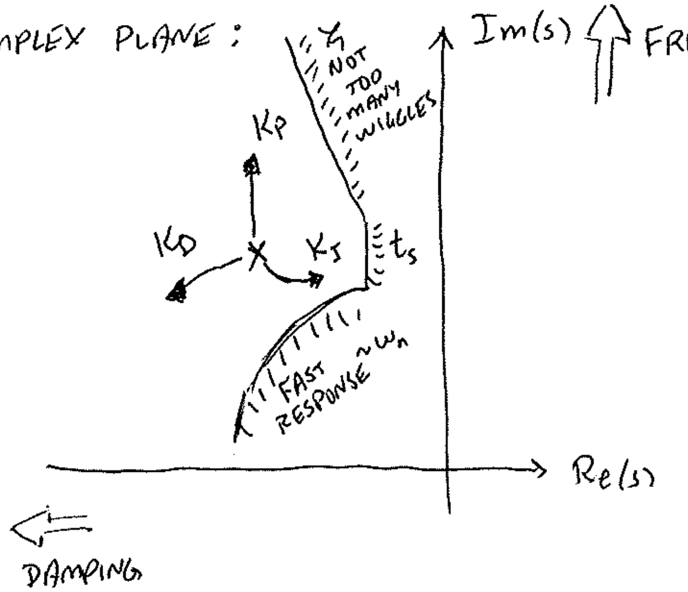
3. TIME RESPONSE

$$Y(s) = \frac{C_1}{s-p_1} + \frac{C_2}{s-p_2} + \cdots + \frac{C_n}{s-p_n} + \langle \text{STUFF LIKE } U(s) \rangle$$

$\hookrightarrow U(s) = \frac{1}{s} \Rightarrow \frac{G(0)}{s}$

$$y(t) = C_1 e^{p_1 t} + C_2 e^{p_2 t} + \cdots + \underbrace{G(0)}_{\text{STEADY RESPONSE}}$$

4. THINK ABOUT TRANSIENT RESPONSE IN TERMS OF WHERE p_k ARE IN COMPLEX PLANE:



PLOT OF CLOSED-LOOP POLE LOCATIONS AS WE VARY K IS CALLED A "ROOT LOCUS."

FREQUENCY RESPONSE ⇒ $u(t) = A\sin(\omega t) \Rightarrow U(s) = \frac{A\omega}{s^2+\omega^2}$ (M,ϕ) LIKE POLAR COORDS

$$Y(s) = \frac{C_1}{s-p_1} + \cdots + \frac{C_n}{s-p_n} + \frac{C_{cos}\, s + C_{sin}\,\omega}{s^2+\omega^2}$$

$$y(t) = \underbrace{C_1 e^{p_1 t} + \cdots + C_n e^{p_n t}}_{\text{NOW, WE DON'T WORRY ABOUT THIS PART!}} + \underbrace{C_{cos}\cos(\omega t) + C_{sin}\sin(\omega t)}$$

$C_{sin} = MA\cos\phi$

$C_{cos} = MA\sin\phi$

$$= M\cdot A\cdot\sin(\omega t + \phi) = MA\left[\cos\phi\,\sin(\omega t) + \sin\phi\,\cos(\omega t)\right]$$

<u>MAGIC</u>: $\boxed{Me^{j\phi} = G(j\omega)}$ ← SEE NOTES | TEXTBOOK FOR DETAILS | <u>NOTE</u>: $M = M(\omega)$, $\phi = \phi(\omega)$

(USUALLY TAKE $A=1$)

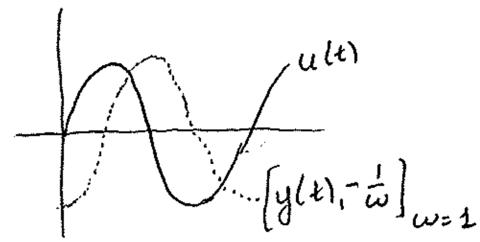
① $G(s) = \frac{1}{s}$, $u = \sin(\omega t)$

WE DONT CARE ABOUT THIS PART $\rightarrow C_1 e^{\sigma t}$

$\Downarrow$ $\Downarrow$

$$\frac{dy}{dt} = u(t) \Longrightarrow y(t) = -\frac{1}{\omega}\cos(\omega t) + \frac{1}{\omega}$$

$$= \frac{1}{\omega}\sin(\omega t - 90^\circ) + \frac{1}{\omega}$$



$M = \frac{1}{\omega}$

$\phi = -90^\circ$

$$G(j\omega) = \frac{1}{j\omega} = \frac{1}{\omega}(-j) = \underbrace{\frac{1}{\omega}}_{M}\,\underbrace{e^{-j\pi/2}}_{\phi = -90^\circ}$$

② $G(s) = \frac{1}{\tau s + 1}$ $u(t) = \sin(\omega t)$

$= \frac{\sigma}{s+\sigma}$

$$y(t) = \underbrace{C_1 e^{-\sigma t} + \overbrace{\frac{1}{1+\left(\frac{\omega}{\sigma}\right)^2}}^{\rightarrow \frac{\sigma^2}{\sigma^2+\omega^2}}\left[\sin(\omega t) - \left(\frac{\omega}{\sigma}\right)\cos(\omega t)\right]}_{\text{WOLFRAM ALPHA!}}$$

<u>CONSIDER 3 CASES</u>

$\omega << \sigma \Rightarrow \left(\frac{\omega}{\sigma}\right) << 1 \Rightarrow y(t) \rightarrow \sin(\omega t) = u(t)$

$\omega >> \sigma \Rightarrow \left(\frac{\omega}{\sigma}\right) >> 1 \Rightarrow y(t) \rightarrow -\frac{\sigma}{\omega}\cos(\omega t) = \frac{\sigma}{\omega}\sin(\omega t - 90^\circ)$

THIS LOOKS LIKE AN INTEGRATOR : $\frac{\sigma}{s} \simeq G(s)$

$\omega = \sigma \Rightarrow y(t) \rightarrow \frac{1}{2}\left[\sin(\omega t) - \cos(\omega t)\right] = \frac{\sqrt{2}}{2}\sin(\omega t - 45^\circ)$

$$G(j\omega) = \frac{\sigma}{j\omega + \sigma} = \frac{1}{1 + j\frac{\omega}{\sigma}} = \begin{cases} 1 & \omega << \sigma \\ \frac{\sqrt{2}}{2}e^{-j\pi/4} & \omega = \sigma \\ \frac{\sigma}{\omega}e^{-j\pi/2} & \omega >> \sigma \end{cases}$$

$$= \frac{\sigma}{\sigma^2 + \omega^2}(\sigma - j\omega)$$ COMPARE TO $y(t)$ ABOVE

THIS IS A LOW-PASS FILTER!

<u>NOTE</u> $\frac{dy}{dt} + \sigma y = \sigma u$

$y = M\sin(\omega t + \phi) \Rightarrow \frac{dy}{dt} = \omega M\cos(\omega t + \phi)$

$$\frac{\left\|\frac{dy}{dt}\right\|}{\|y\|} \sim \omega$$

THIS HELPS US UNDERSTAND THE RESULTS... (INTUITION)

2014-03-19 CONTROLS BODE PLOTS

WE WANT TO PLOT $M(\omega)$ & $\phi(\omega)$.

CUSTOMARY TO USE LOG-LOG PLOT FOR $M(\omega)$ & LOG-LINEAR PLOT FOR $\phi(\omega)$.

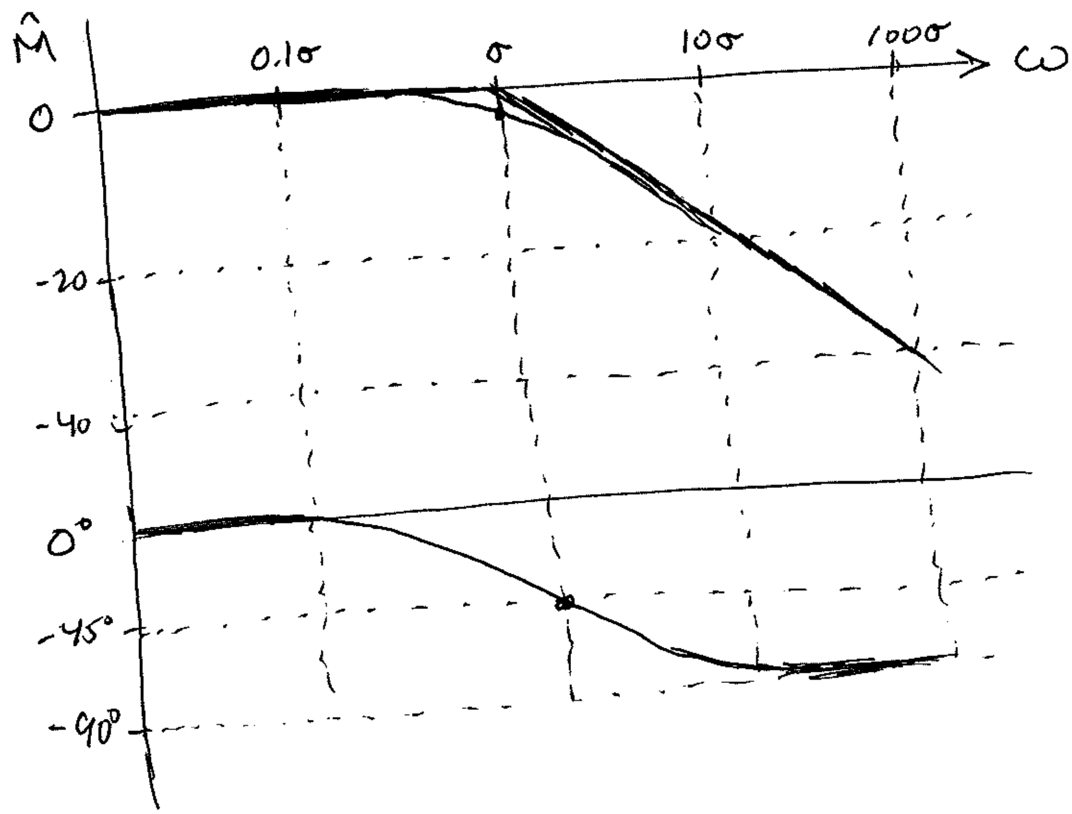
$$\hat{M} = 20 \log_{10} M \qquad [\hat{M}] = \text{decibels (dB)}$$

* CAREFUL! $10 \log_{10} X$ USED IN SOME CASES (TYPICALLY IF X MEASURES POWER)

LETS MAKE BODE PLOT OF OUR 1^{st} ORDER SYSTEM

$$G(s) = \frac{\sigma}{s+\sigma}$$

ω	M	$\hat{M}$	ϕ
$\omega << \sigma$	1	0	$0°$
$\omega = \sigma$	$\frac{\sqrt{2}}{2}$	-3	$-45°$
$\omega >> \sigma$	$\frac{\sigma}{\omega}$	$-20 * \log(\frac{\omega}{\sigma})$	$-90°$

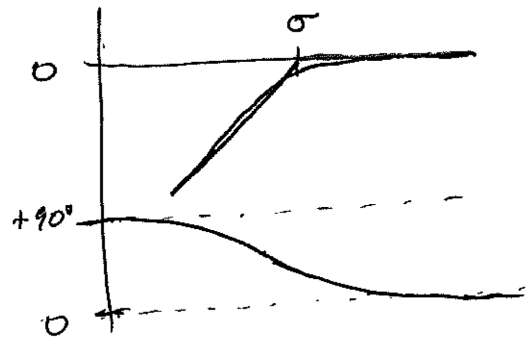


2014-03-19 CONTROLS BODE PLOTS

HOW ABOUT A HIGH-PASS FILTER

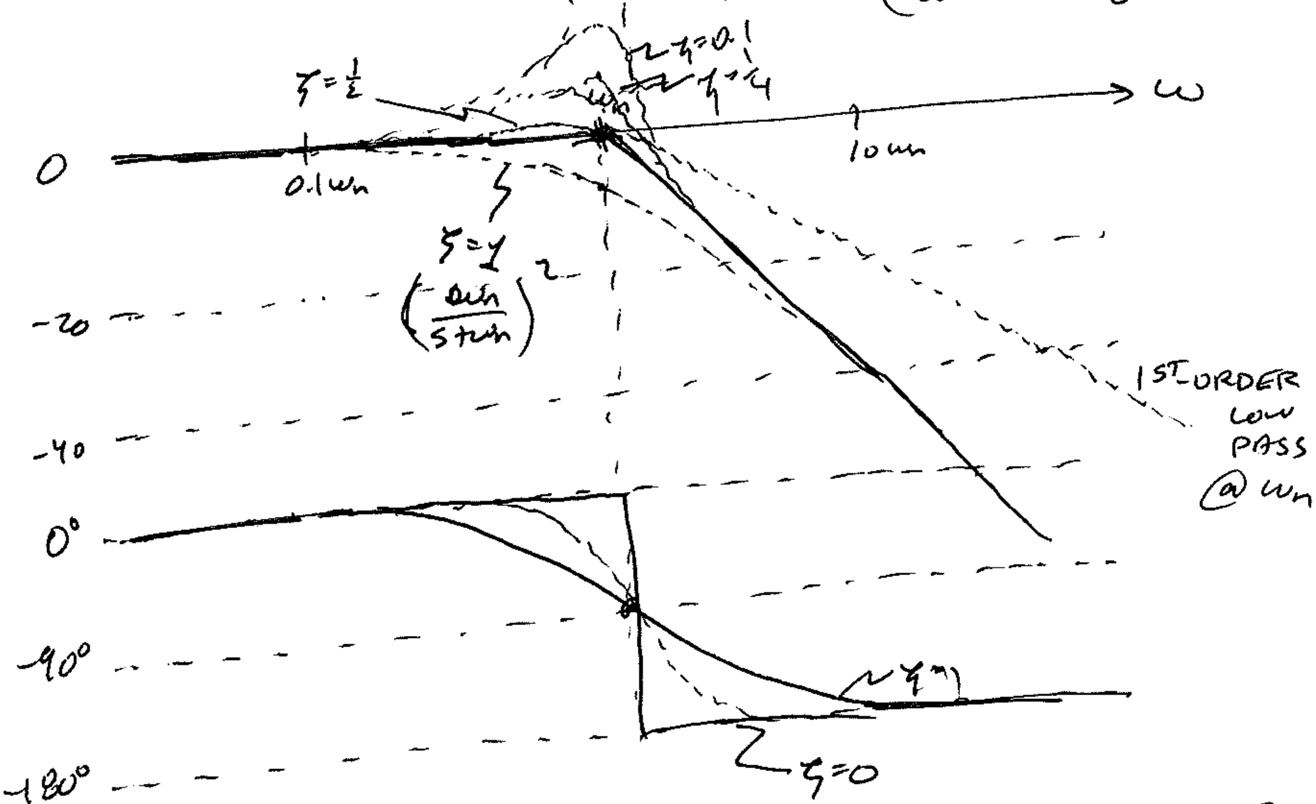
$$G(s) = \frac{s}{s+\sigma}$$

DETAILS UP TO YOU!



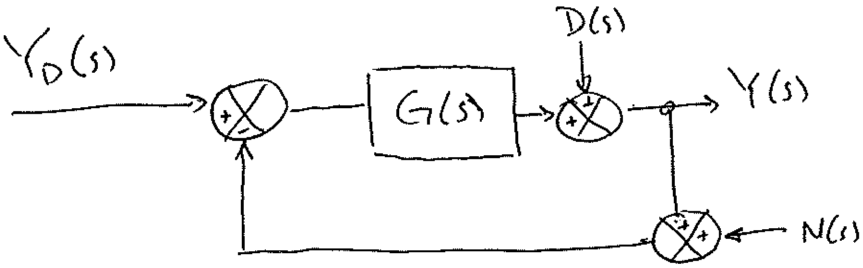
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ω	M	$\hat{M}$	ϕ
$\omega \ll \omega_n$	1	0	$0°$
$\omega = \omega_n$	$\frac{1}{2\zeta}$		$-90°$
$\omega \gg \omega_n$	$\left(\frac{\omega_n}{\omega}\right)^2$	$-40\log\left(\frac{\omega}{\omega_n}\right)$	$-180°$



NOW WE WILL MAKE ALL OUR BODE PLOTS WITH MATLAB.

2014-03-19 CONTROLS FREQUENCY RESPONSE DESIGN



$$Y(s) = \frac{G}{1+G}\left[Y_D(s) - N(s)\right] + \frac{1}{1+G(s)} D(s)$$

REQUIREMENTS

1. TRACKING $[Y = Y_D]$ $\Rightarrow |G|$ BIG
2. DISTURBANCE REJECTION $\left[\frac{Y}{D} \text{ SMALL}\right]$ $\Rightarrow |G|$ BIG
3. NOISE SUPPRESSION $\left[\frac{Y}{N} \text{ SMALL}\right]$ $\Rightarrow |G|$ SMALL
4. STABILITY $\Rightarrow [1 + G \neq 0]$ $\Rightarrow |G|$ SMALL WHEN $\angle G \approx -180°$