ESE 505 & MEAM 513 - SPRING 2015
HOMEWORK #4
DUE Monday 2015-04-06

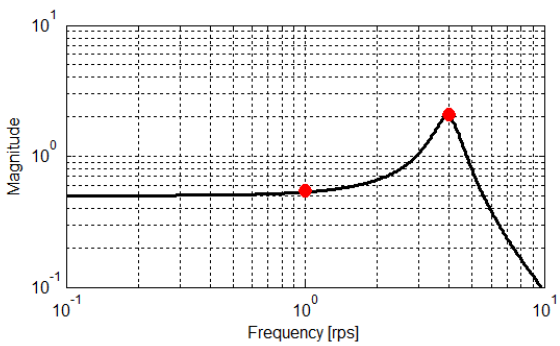
1. Download the file `HW04_Problem1.mdl`. Also download the file `HW04_P1.m` and complete all the missing information. In particular, you will need to supply at least 5 data points that you obtain by running the SIMULINK model for at least 5 different input frequencies. These represent "experimental" frequency response measurements.

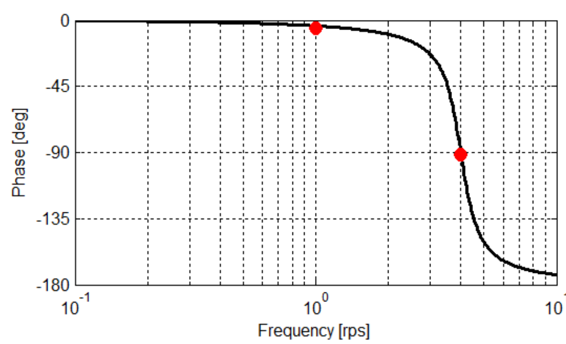
   The two transfer functions have the following forms:

   a. $G(s) = \dfrac{A}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

   b. $G(s) = \dfrac{Ce^{-Ts}}{(s+b)}$.

   After you make a good guess at the system dynamics, you should be able to get nice plots that look something like this (except your plots will have more data points):





2. Use MATLAB to make frequency response (bode) plots of the following transfer functions. If you want to understand what is going on, try to add asymptotes to your graphs by hand (or using a computer drawing tool to add lines representing the asymptotes). Good info at http://lpsa.swarthmore.edu/Bode/BodeHow.html

   a. $G(s) = \dfrac{1000}{(s+200)}$

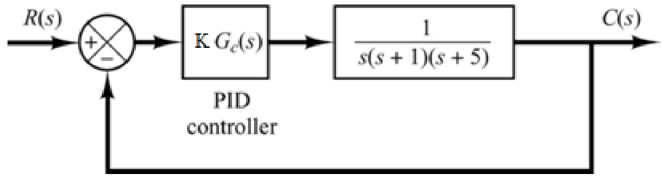
   b. $G(s) = \dfrac{9s+27}{(s+1)^2(s^2+3s+81)}$

   c. $G(s) = \dfrac{100s}{(s^2+s+25)}$

   d. $G(s) = \dfrac{2000s+2000}{s(s+10)(s+100)}$

Answers: Just use MATLAB's `bode` command for each system. Be sure to turn the grid on. For example, you can check (a) with `bode([1000], [1 200]);`

3. We will explore controller design for this system:



Below, we will refer to the plant as $G_P(s) = \dfrac{1}{s(s+1)(s+5)}$.

a. This is an optional exercise. Show that for any system, the Ziegler-Nichols "ultimate sensitivity method" PID gains result in a controller of the following form:

$$KG_c(s) = K\frac{(T_u s + 4)^2}{s}, \text{ where } K = \frac{0.6K_u}{8T_u}.$$

<u>Answer</u>: This is an exercise in algebraic manipulation.

b. Let's design a PID controller using Ziegler-Nichols "ultimate sensitivity method". Use the "old way" of finding K_u and T_u with a root locus for proportional feedback only. You don't have to submit this root locus.

Now make a root locus for the PID controller, showing the variation of closed-loop pole locations with as a function of the overall compensator gain, K.

Answer: K_u~30 and T_u ~ 2.8 sec $\rightarrow G_C(s) = K\dfrac{(s+a)^2}{s}$, with $K = 6.3$ and $a = 1.4$ rps.

c. Now let's think about the problem using frequency response tools. Starting with proportional feedback only, make a bode plot of $KG(s) = KG_C(s)G_P(s)$ with $KG_C(s) = K = 5$. What is the gain margin for this system? What does this imply about the proportional gain required to reach neutral stability? At what frequency does the phase cross –180 degrees? What does this imply about the period of neutrally stable oscillations when the gain is set to the critical value?

Answers: The gain margin is approximately 15dB, which implies that the gain could be increased by a factor of about 6, from 5 to K_u~30, for neutral stability.

The bode plot crosses –180 degrees at 2.24 rad/sec, which implies a period of oscillation of about T_u ~2.8 seconds

These gain and period numbers should be familiar from part (a).

d. Now make a bode plot of $KG(s) = KG_C(s)G_P(s)$, for the Ziegler-Nichols design with the nominal gain. Submit a bode plot which clearly shows the gain and phase margins for the system with this compensator. Explain what the margins mean.

Answers: There is no gain margin, because the phase never crosses –180 degrees. This corresponds to the result in part (a), where the root locus showed that we could increase K to arbitrarily large values without encountering instability. In a typical application, we would include another low-pass filter in the compensator to avoid passing noise to the control at high frequency. This would result in an additional drop of 90 degrees in phase and some corresponding upper limit on gain (finite gain margin).

The phase margin is 22 degrees at a frequency of 1.73 rad/sec. One way to think about phase margin is to ask, "how much time delay would we have to add to the system to reach neutral stability?" Because time delay affects only phase, not gain, we can compute the time delay from the phase margin and ω_{180}. In order for the system to be unstable due to unmodelled delay, there would have to be enough delay to cause a phase reduction of 22 degrees at ω_{180}=1.73 rad/sec. In other words, there would have to be 0.22 seconds of delay[1].

e. Notice that the maximum value of phase in the Ziegler-Nichols bode plot is about -153 degrees. This implies that the maximum phase margin is about 27 degrees[2], which is less than is typically required. Try to find a value for a that would allow for 45 degrees of phase margin. What is the value of K that achieves this margin? Submit the bode plot that shows these values.

Answers: $a \approx 0.9$ and $K \approx 10$. Other combinations are possible with smaller values of a.

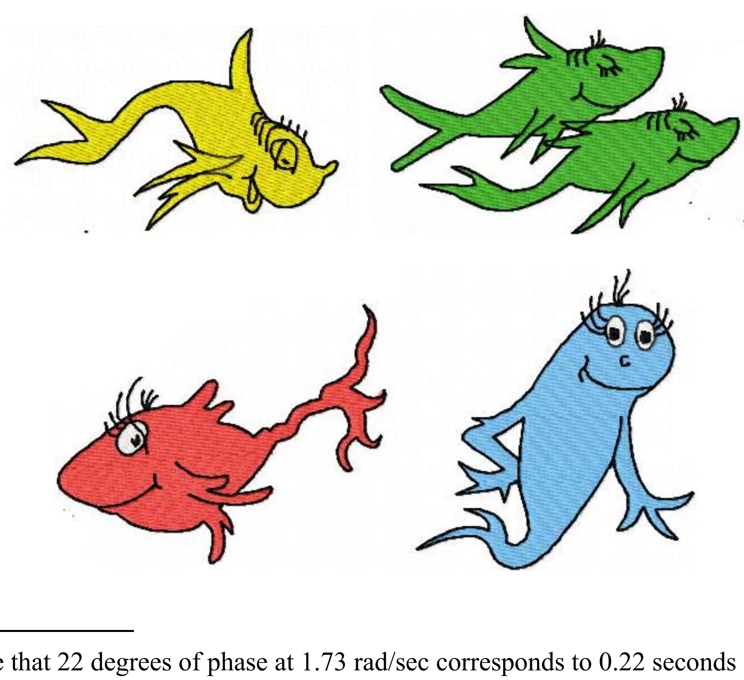
4. Apply frequency-response analysis to the Black-Box system. See homework #3 for a circuit drawing and transfer function of the plant.

   a. Start by making a single bode plot that shows both the analog system and the digital system, with the extra time delay[3] and low-pass filters. Show how to use these plots to find K_u and T_u for both cases. Be sure you can relate the results to the root locus work we did in homework #3.

   b. Use the values of K_u and T_u for both cases in the last problem to design Ziegler-Nichols PID controllers. Show the corresponding bode plots (one figure). What is the cross-over frequency?

   c. Discuss the margins for the nominal design. Compare to a root locus on K if you really want to show off your knowledge. Now adjust the Ziegler-Nichols PID designs to ensure 60 degrees of phase margin.

5. Apply frequency-response analysis to the One-Two-Red-Blue system. All of the details are up to you.



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[1] It is just a coincidence that 22 degrees of phase at 1.73 rad/sec corresponds to 0.22 seconds of delay. This comes from the fact that $(1.73)(180)/\pi \sim 100$.

[2] You should try to find the value of gain, K, that results in the maximum phase margin, when you don't change *a*.

[3] Remember that you don't have to use the Pade approximation for time delay when doing bode plots.