Stability Robustness
Gain & Phase Margins

ESE 505 & MEAM 513

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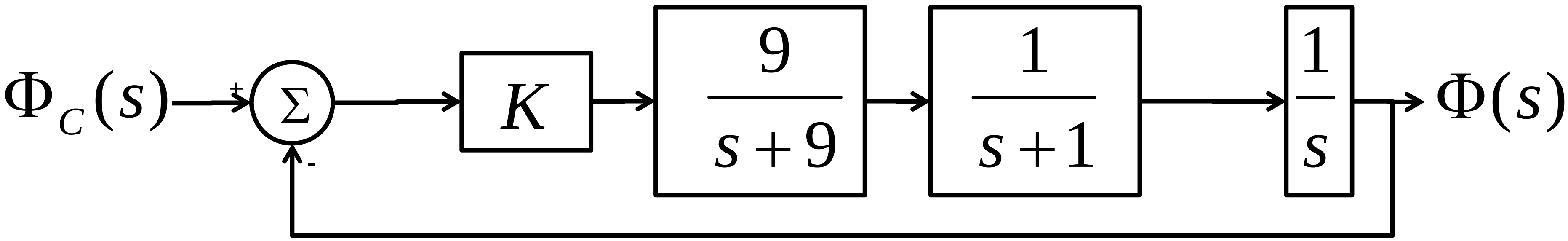
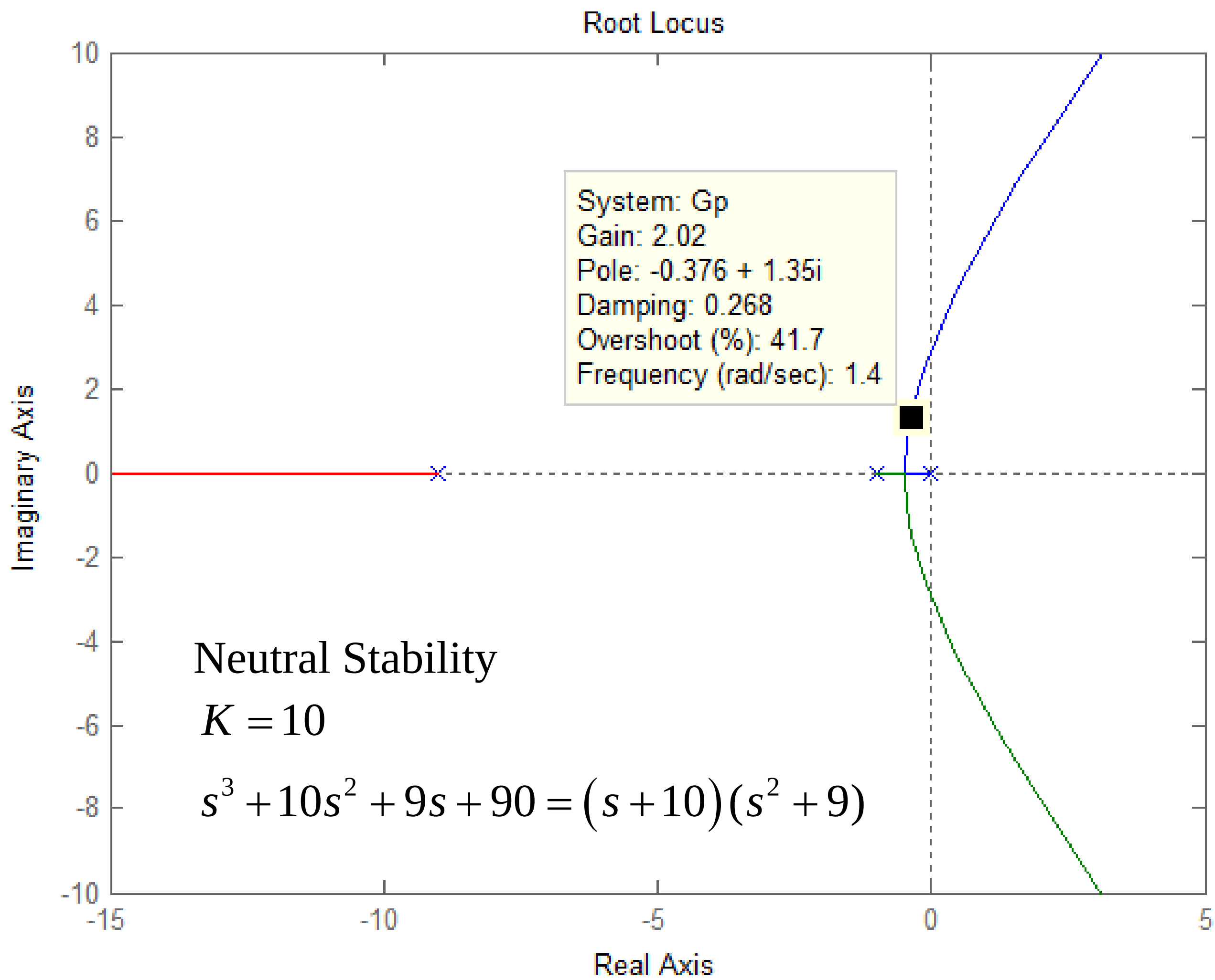
Example : Airplane Roll Control



$$\frac{\Phi(s)}{\Phi_C(s)} = \frac{9K}{s(s+1)(s+9)+9K}$$

$$\Delta_{CL}(s) = s(s+1)(s+9)+9K = 0$$

Root Locus with Proportional Feedback

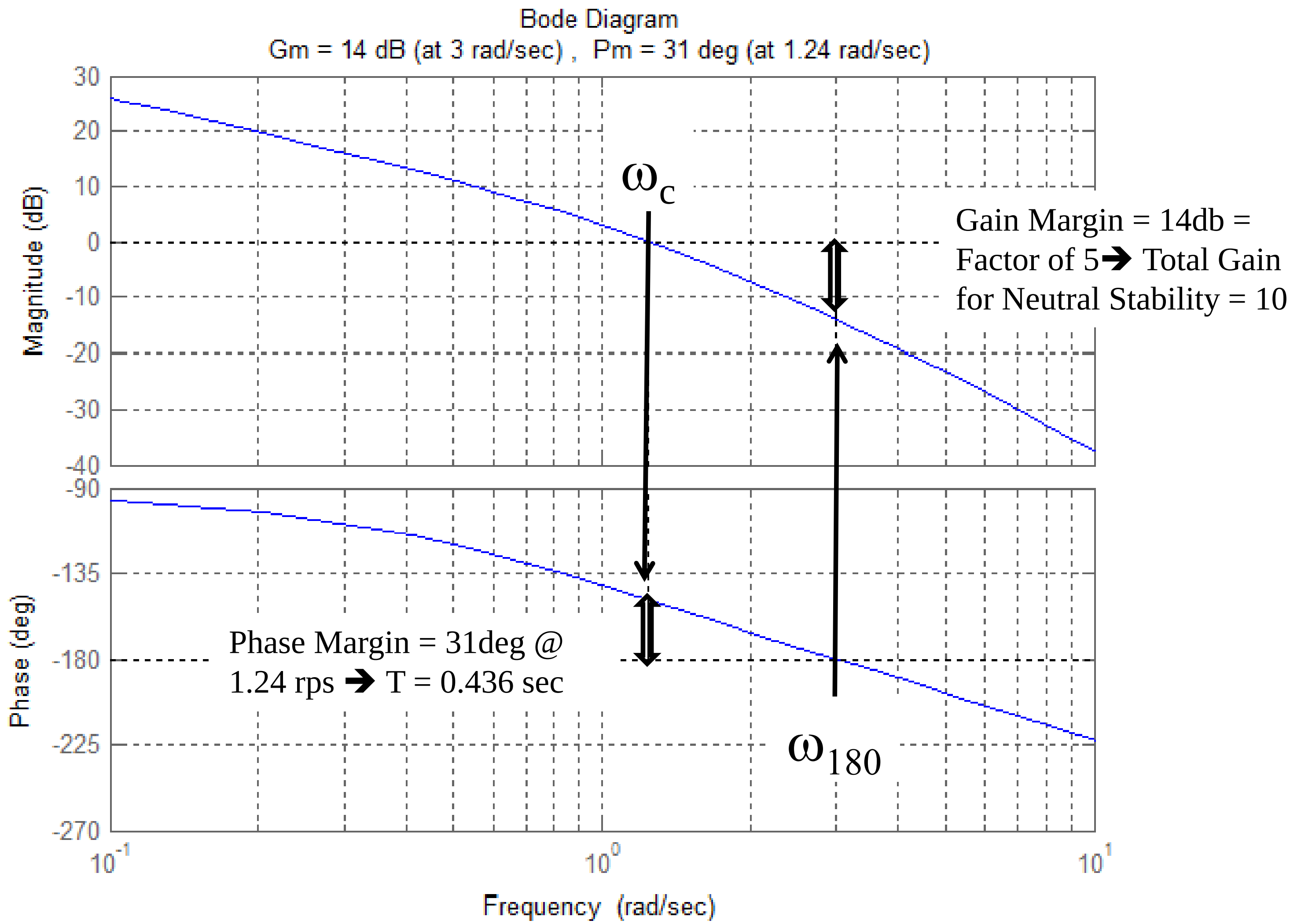


Neutral Stability

$$K = 10$$

$$s^3 + 10s^2 + 9s + 90 = (s+10)(s^2+9)$$

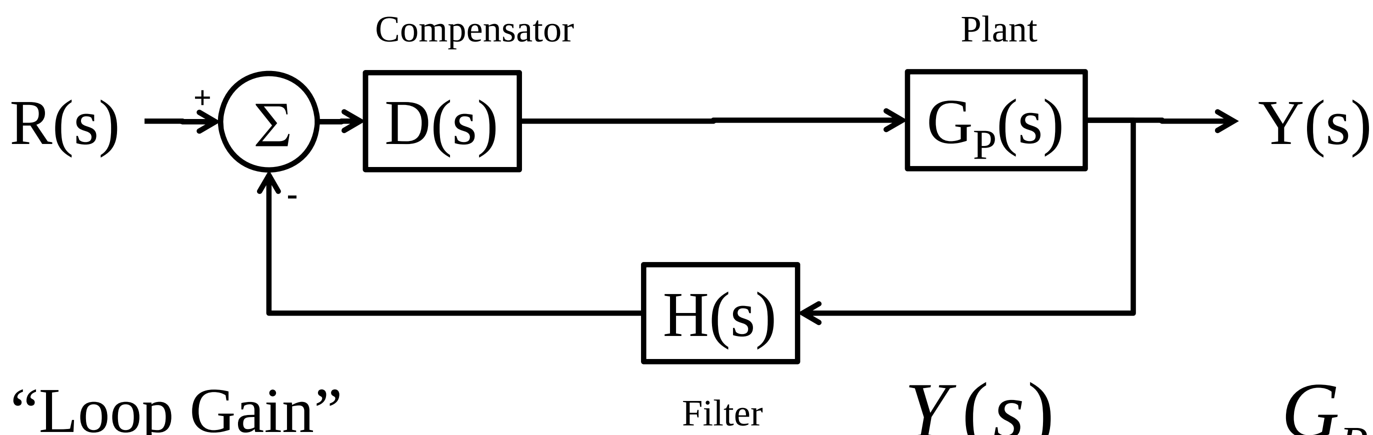
Loop Gain Bode Plot (with K=2)



What is Robustness?

- Engineers Should Avoid Lay Definitions
  - "Showing Vigor, Strength or Firmness"
  - Note that Strength & Stiffness Are Very Different Requirements in Mechanical Design
  - We Want Something Precise
- Robustness = Ability to Maintain Desired Characteristics When Operating Conditions Change
  - Performance Robustness = Desirable Characteristics (e.g. Settling Time) in Command Response Maintained
  - Stability Robustness = Closed-Loop System Remains Stable
- How to Measure Robustness
  - Parametric (Structured) Analysis = Change Parameter in System Model to Find How Sensitive System is to Variations
  - Unstructured Analysis = Apply Generic Change (e.g. Gain & Phase Change to Plant Bode Plot)

Recall : Closed-Loop Neutral Stability Condition



"Loop Gain"

$G(s) = G_P(s)D(s)H(s)$

$$\frac{Y(s)}{R(s)} = \frac{G_P(s)D(s)}{1+G_P(s)D(s)H(s)}$$

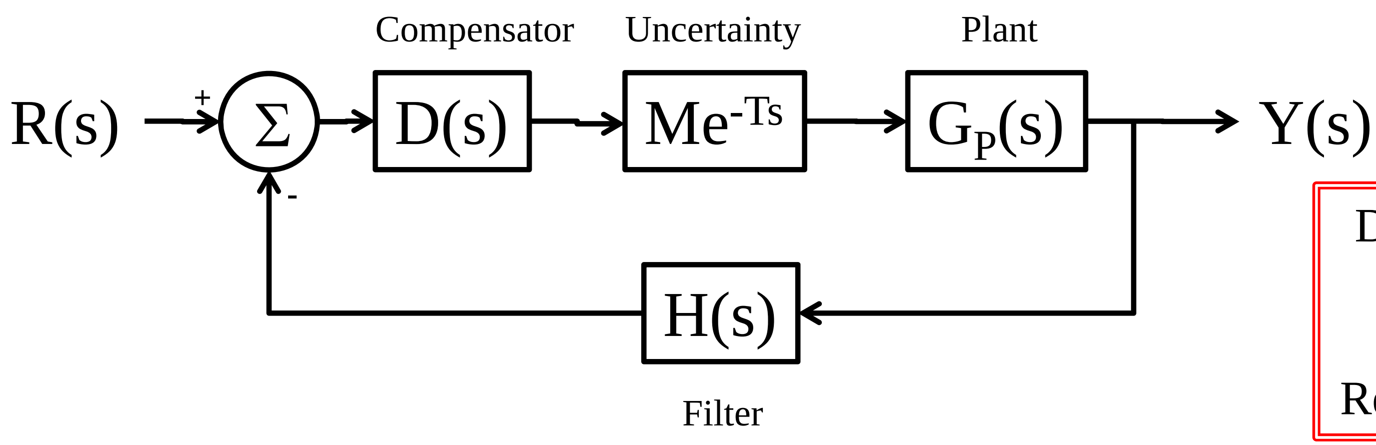
Closed-Loop Poles = Roots of…

$$\Delta_{CL}(s) = 1 + G_P(s)D(s)H(s) = 1 + G(s) = 0$$

Neutral Stability (Poles on Imaginary Axis) If…

$$\Delta_{CL}(s) = 1 + G(j\omega) = 0 \Rightarrow G(j\omega) = -1$$

"Margins" = Allowance for Model Errors



Difference Between Actual Plant & Model Plant Represented by Me^{-Ts}

- Uncertainty Represented by Some Unknown Change in Gain (M) and Delay (T)
- With No Delay (T=0), Max Allowable Value of M is Given by Gain Margin
- With No Gain (M=1), Max Allowable Value of T is Given by Phase Margin
  - Actually, $\omega_c T$ = Phase Margin

"Stability Margins" From Loop Gain Bode Plot