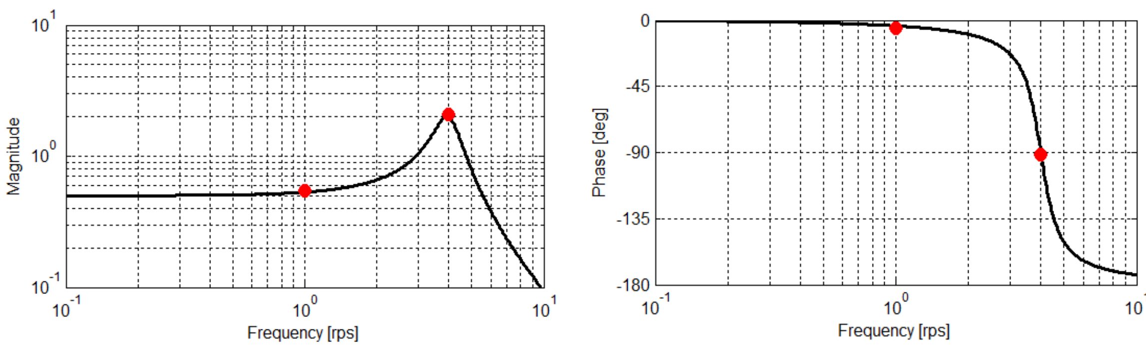
ESE 406/505 & MEAM 513 - SPRING 2013
HOMEWORK #8
DUE by Wednesday 2013-04-03 (Late Pass Monday 2013-04-08)

1. Download the file `HW08_Problem1.mdl`. Also download the file `HW08.m` and complete all the missing information. In particular, you will need to supply at least 5 data points that you obtain by running the simulation for at least 5 different input frequencies. These represent "experimental" frequency response measurements. You have reason to believe that the unknown systems have the following forms:

   a. $G(s) = \dfrac{A}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

   b. $G(s) = \dfrac{Ae^{-Ts}}{(s+\omega)}$.

   After you make a good guess at the system dynamics, you should be able to get nice plots that look something like this (except your plots will have more data points):



   In the next homework, we will learn how to do more complicated experiments that give us good estimates of frequency response even when there is a lot of noise in the system.

2. Use MATLAB to generate a bode plot for each of the following transfer functions. Print each plot and then *by hand sketch* the gain and phase asymptotes. For the phase asymptotes, simply use discrete jumps of 90 or 180 degrees for first-order or second-order poles and zeros, respectively. Be sure to identify the zero-frequency (DC) gain of the system if it is not infinite.

   a. $G(s) = \dfrac{2000}{2(s+200)}$

   b. $G(s) = \dfrac{9s+27}{(s+1)^2(s^2+3s+81)}$

c. $G(s) = \dfrac{100s}{\left(s^2 + s + 25\right)}$

d. $G(s) = \dfrac{2000s + 2000}{s\left(s+10\right)(s+100)}$

Answers: Just use MATLAB's `bode` command for each system. Be sure to turn the grid on.

3. Let's revisit Problem 3 from homework #6 and #7, which involved the design of a unit-feedback PID controller using Ziegler-Nichols tuning rules for the following plant:

$$G_P(s) = \frac{1}{s\left(s+1\right)(s+5)}$$

You should review the analysis we did in the previous assignments for this problem.

a. Starting with proportional feedback only, make a stability bode plot with $K_P = 5$. What is the gain margin for this system? What does this imply about the proportional gain required to reach neutral stability? At what frequency does the phase cross −180 degrees? What does this imply about the period of neutrally stable oscillations when the gain is set to the critical value?

Answers: The gain margin is approximately 15dB, which implies that the gain could be increased by a factor of about 6, from 5 to 30, for neutral stability.

The bode plot crosses −180 degrees at 2.24 rad/sec, which implies a period of oscillation of about 2.8 seconds.

b. When we applied Ziegler-Nichols tuning rules to this system, we obtained a compensator of the following form:

$$G_C(s) = K\frac{\left(s+a\right)^2}{s}$$

with $a = \dfrac{4}{P_{CR}} = 1.4$ and $K = 0.075K_{CR}P_{CR} = 6.3$. Make a bode plot of the loop transfer function, $G(s) = G_C(s)G_P(s)$, for the Ziegler-Nichols design. Submit a bode plot which clearly shows the the gain and phase margins for the system with this compensator. Explain what the margins mean.

Answers: There is no gain margin, because the phase never crosses −180 degrees. This corresponds to the result that you might recall from Homework #6, where the root locus showed that we could increase K to arbitrarily large values without encountering instability. In a typical application, we would include another low-pass filter in the compensator to avoid excessive noise at high frequency. This would result in an additional drop of 90 degrees in phase and some corresponding upper limit on gain (finite gain margin).

The phase margin is 22 degrees at a frequency of 1.73 rad/sec. In order for the system to be unstable due to unmodelled delay, there would have to be enough delay to cause a phase reduction of 22 degrees at 1.73 rad/sec. In other words, there would have to be 0.22 seconds of delay[1]. This is the amount of delay that was found to result in neutral stability for this system in Homework #7.

c. Notice that the maximum value of phase in the loop bode plot is about -153 degrees. This implies that the maximum phase margin is about 27 degrees, which is less than is typically required. Try to find a value for a that would allow for 45 degrees of phase margin. What is the value of K that achieves this margin? Submit the bode plot that shows these values.

   Answers: $a \approx 0.9$ and $K \approx 10$. Other combinations are possible with smaller values of a.

[1] It is just a coincidence that 22 degrees of phase corresponds to 0.22 seconds of delay. This comes from the fact that (1.73)(180)/π ~ 100.