ESE 406 - SPRING 2011
HOMEWORK #3
DUE 9-Feb-2011 (14-Feb-2011 with late pass)

Problem 1 You should be able to solve all parts of problems 2.10 through 2.17 in the textbook. Submit solutions to the following problems from the textbook:

a. Problem 2.12. Answer: $$\frac{V_{out}}{V_{in}} = \frac{\frac{10^7}{s+1}}{\frac{10^7}{s+1} - 1} = \frac{10^7}{-s-1+10^7} \cong \frac{-10^7}{s-10^7}$$
Note: The denominator "s+1" in the non-ideal op-amp model is not at all realistic, because it suggests that the op-amp will have a time response that looks like e^{-t}. In fact, actual non-ideal op-amps respond very much faster than this. But even with much faster dynamics, connecting the op-amp output to the "+" input terminal will generally result in an unstable system. This is why op-amp circuits are always constructed with the "feedback" connected to the "-" input terminal.

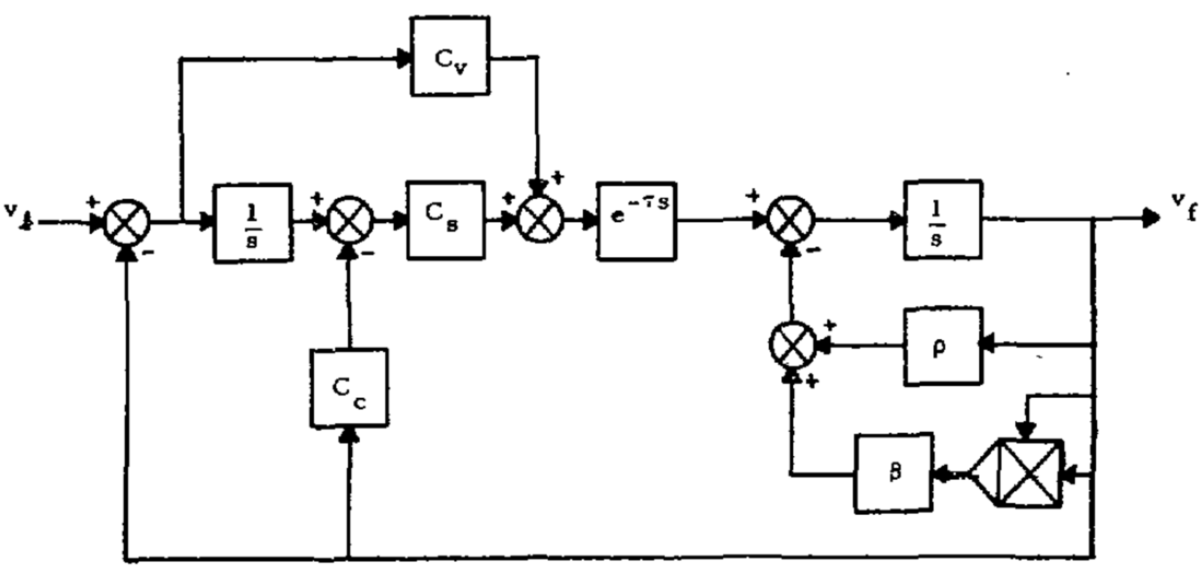
b. Problem 2.15 (b) and (d). Answers: (b) $-\frac{R_f}{R_2}\frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$; (d) $\frac{s^2+\frac{1}{R^2C^2}}{s^2+\frac{4}{RC}s+\frac{1}{R^2C^2}}$.

Problem 2 You should be able to solve all parts of problems 3.19 through 3.22 in the textbook. Submit solutions to the following problems from the textbook:

c. Problem 3.20(b). Answer: $$\frac{Y}{R} = G_7 + \frac{G_1G_3G_4G_6}{(1+G_1G_2)(1+G_4G_5)}$$

d. Problem 3.21(d). Answer: $$\frac{Y}{R} = \frac{D+DBH+AB}{1+BH+GD+GBDH+GAB}$$

Problem 3 Engineers working with systems having a human operator sometimes use models of human behavior which utilize control theory. One such application is a model of a driver controlling the speed of her car to follow the car in front of her[1]:



[1] Burnam, *et al.*, "Identification of Human Driver Models in Car Following," *IEEE Transactions on Automatic Control*, Vol AC-19, No. 6, December 1974. See also Iaonnou and Chien, "Autonomous Intelligent Cruise Control," *IEEE Transactions on Vehicular Technology*, Vol. 42, No. 4, November 1993.

The input to the model is the speed of the lead vehicle, V_l. The output is the speed of the following vehicle, V_f. The control is taken as a commanded acceleration. The researchers are essentially assuming that the driver will quickly learn the relationship between displacement of the gas pedal and the vehicle acceleration, and will adjust her feedback gains accordingly. This is a convenient assumption, because it liberates us from the details of the engine and fuel control, although it also omits possible extra dynamics by supposing that the commanded and actual acceleration are the same. The structure of this block diagram can be shown to provide an "optimal" balance between good tracking of the lead vehicle and judicious use of acceleration.

Explanations and numerical values (identified from real driving data!) for the parameters are presented in the following table:

Cv	0.5 sec^{-1}	Velocity feedback gain of the human driver
Cs	1.64 sec^{-2}	Position feedback gain of the human driver
Cc	1.14 sec	"California Rule" gain of human driver (increases desired inter-vehicle spacing at higher speed)
τ	0.09 sec	Transport delay in action of human operator
ρ	0.005 sec^{-1}	Linear friction coefficient of vehicle dynamics
β	0.00025 m^{-1}	Quadratic (aerodynamic drag) coefficient of vehicle dynamics

What should we do with the $e^{-\tau s}$ block? This represents pure time delay:

$$y(t) = u(t-\tau) \rightarrow Y(s) = e^{-\tau s}\, U(s)$$

(You should be able to show where this comes from by writing out the definition of the Laplace transform of $u(t-\tau)$ and then using a change-of-variable to perform the integration.) Transport delay (also called "time delay") occurs very often in models of control systems. We have already seen it in our Ping-Pong-Poise model to represent the effects of our discrete-time control implementation. You might recall that we asserted at that time that time delay tends to degrade closed-loop system stability. Time delay is also often used to represent dynamics that we don't know but might have an effect that looks kind of like delay.

In any case, this transfer function ($e^{-\tau s}$) is inconvenient, because we want all of our transfer functions to be ratios of polynomials. Very often, the exact representation of transport delay is replaced by a "Pade approximant". You can read about Pade approximations in Section 5.6.3 of the textbook, or google it to get some details. Here, we will settle for a first-order Pade approximation (equation 5.92 in the text):

$$e^{-\tau s} \approx \frac{-\frac{\tau}{2}s+1}{\frac{\tau}{2}s+1}$$

One more thing before we actually do something with this block diagram. Notice that the vehicle dynamics contain a non-linear term—the aerodynamic drag is proportional to the square of velocity—the input to the gain β is the product of velocity with itself. This is fine for time simulation (e.g. running the model in simulink), but don't be confused:

$$L\left\{v_f^2(t)\right\} \neq V_f^2(s)$$

A block diagram with both “s” blocks and nonlinear elements is a trap! The model can only be used in the time domain. If we want to do block-diagram algebra on this model, we need to replace the nonlinear aerodynamics by a linear approximation. To do this, we use the block diagram to figure out the ODE that describes the dynamics of the follower vehicle:

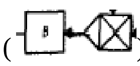
$$\frac{dv_f}{dt} = -\rho v_f - \beta v_f^2 + u$$

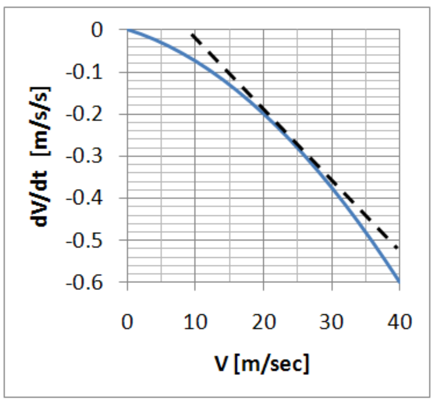
where u is the acceleration command, including the driver delay. The “trim” (equilibrium) equation is:

$$u_o = +\rho v_o + \beta v_o^2$$

And the linearized version of the equation is:

$$\frac{d\Delta v_f}{dt} = -\rho \Delta v_f - 2\beta v_{f_o} \Delta v_f + \Delta u$$

If we assume a steady speed of 25 m/sec (just under 60 mph), we find $2\beta v_{f_o}$ is about 0.0125 sec^{-1}. Notice that both ρ and $2\beta v_{f_o}$ multiply Δv_f. So, if we simply eliminate the nonlinear element () and replace ρ by $\gamma \triangleq \rho + 2\beta v_{f_o}$, we have a linearized block diagram valid for small perturbations from trim. Combined with the replacement of the transport delay with a Pade approximation, we can now apply our new skills at block-diagram manipulation to get a complete system transfer-function between the lead-vehicle velocity and the following vehicle velocity.



As an aside, when we linearize a system like this, we are replacing some algebraic nonlinearity with a locally linear approximation. In this case, we are replacing a nonlinear drag with a linear approximation, as shown in the figure at left. It looks like this approximation will be quite good if the speed remains between 20 and 30 m/sec, and it won’t be too bad even at speeds between 10 and 40 m/s.

WHAT SHOULD I SUBMIT?!

a. Do block-diagram algebra to find $\frac{V_f(s)}{V_l(s)}$. Use symbols (Cs, Cv, Cc, γ, τ), not numerical values.

b. Using the given numerical values, find an equation for the step response of the system. You may use matlab or other computer program to help with partial-fraction expansion. Implement the linearized block-diagram in simulink and compare (in a pretty plot) your exact answer with the simulation to be sure you have it right.

c. Increase the value of τ in your simulink model to experiment with the effects of time delay on closed-loop stability. For what value of τ does the closed-loop system become unstable (oscillations that grow in time)?