ESE 406/505 & MEAM 513 - SPRING 2013
HOMEWORK #7
DUE 20-Mar-2013 (Monday, 25-Mar-2013 with late pass)

1. Consider the following "lead compensator":

$$G_C(s) = 5\frac{s+2}{s+10}$$

   a. Suppose this compensator was derived by low-pass filtering a PD compensator. That is, suppose it is equivalent to

$$G_C(s) = \frac{1}{(\tau s+1)}\left(K_D s + K_P\right)$$

   What are the corresponding values of K_P, K_D, and τ?

   Answers: $K_P = 1$, $K_D = 0.5$, and $\tau = 0.1$

   b. What is the discrete-time transfer function, $G_C[z]$, that corresponds to $G_C(s)$ for a sample time $T = 0.05\sec$?

   Answer: $G_C[z] = \dfrac{4.2z - 3.8}{z - 0.6}$

   c. What is the different equation that corresponds to the discrete-time transfer function?

   Answer: $u[k] = 0.6u[k-1] + 4.2e[k] - 3.8e[k-1]$

   d. Copy and paste the code below into a text editor. Fill in the missing code to implement this compensator in an ARDUINO microcontroller of the sort that we have been using in the project. Print out and submit the completed subroutine.

```
double Gc_Function(double e) {
  double u;

//
// e = error at current iteration
// u = control command at current iteration
//

  return u;
}
```

2. This is Problem 8.3 from the textbook by Franklin, *et al*.

The one-sided z-transform is defined as

$$F(z) = \sum_{0}^{\infty} f(k) z^{-k}.$$

(a) Show that the one-sided transform of $f(k+1)$ is $\mathcal{Z}\{f(k+1)\} = zF(z) - zf(0)$.

(b) Use the one-sided transform to solve for the transforms of the Fibonacci numbers generated by the difference equation $u(k+2) = u(k+1) + u(k)$. Let $u(0) = u(1) = 1$. [*Hint:* You will need to find a general expression for the transform of $f(k+2)$ in terms of the transform of $f(k)$].

(c) Compute the pole locations of the transform of the Fibonacci numbers.

(d) Compute the inverse transform of the Fibonacci numbers.

(e) Show that, if $u(k)$ represents the kth Fibonacci number, then the ratio $u(k+1)/u(k)$ will approach $(1+\sqrt{5})/2$. This is the golden ratio valued so highly by the Greeks.

Answers:

a. Theorem shown in class & notes

b. $\mathcal{Z}\{f(k+2)\} = z^2 F(z) - z^2 f(0) - zf(1)$ & $U(z) = \dfrac{z^2}{z^2 - z - 1}$

c. $z = \dfrac{1 \pm \sqrt{5}}{2} = 1.618, \ -0.618$

d. $u(k) = 1,\ 1,\ 2,\ 3,\ 5, \cdots$ (the Fibonacci numbers!)

e. Partial fraction expansion of U[z] gives

$$u(k) \quad = \quad \left(\frac{5+\sqrt{5}}{10}\right)\left(\frac{1+\sqrt{5}}{2}\right)^k + \left(\frac{5-\sqrt{5}}{10}\right)\left(\frac{1-\sqrt{5}}{2}\right)^k$$

for large k the second term is $\cong 0$, and the ratio of $u(k+1)$ to $u(k)$ is $\alpha_1 = (1+\sqrt{5})/2$.

3. Consider the 3rd-order plant $G_P(s) = \dfrac{1}{s(s+1)(s+5)}$ with unity-gain feedback and a proportional compensator.

   a. Determine the proportional feedback gain, K_{CR}, required to reach neutral stability, and the expected period, P_{CR}, of the resulting oscillations. Use Simulink to confirm that this gain does indeed result in neutral stability.

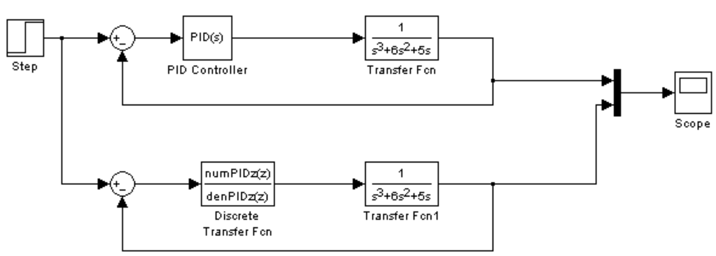
      Answers: $K_{CR} = 29.85$ & $P_{CR} = 2.8$

   b. Now use the Ziegler-Nichols "ultimate sensitivity method", to set the gains of a PID controller. Use simulink to examine the closed-loop response to a step input.

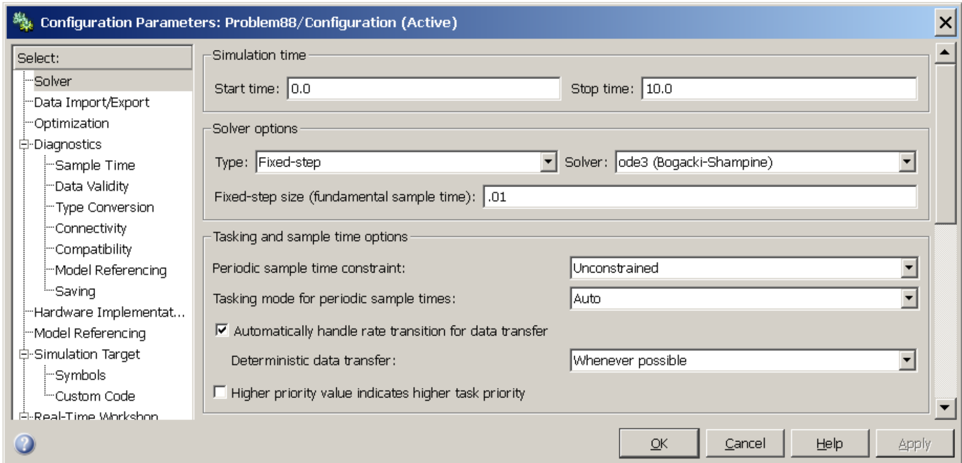
      Answers: $K_P = 17.9$, $K_D = 6.3$, $K_I = 12.8$. The closed-loop response is somewhat lightly damped.

   c. Now let's convert to discrete-time. Let's start with a sample time of 0.04 seconds. Use the "Tustin" method to convert the continuous-time PID compensator into a discrete-time compensator (you may use matlab's C2D function to get the discrete-time filter coefficients). What is the discrete-time transfer function of your digital compensator? Use Simulink to compare the closed-loop step responses of the total system with both the analog and discrete compensators. Submit a graph comparing the responses.

      Hints: Use a discrete-time transfer function from the "discrete" library to put your digital PID compensator in your Simulink model. You might use a model that looks like this:



      In order to get Simulink to automatically handle the mix of discrete-time and continuous-time blocks in the same model, you have to choose the "Automatically handle rate transition" option under the Configuration Parameters dialog. You should also be sure to set a fixed time step such that the sample rate is an integral multiple of the time step:



(Answers: $\dfrac{332.1z^2 - 627.5z + 296.3}{z^2 - 1}$ & discrete-time response very closely matches analog design.)

d. Finally, increase the sample time to 0.2 seconds and again compare step responses of the analog and discrete-time implementations.

Answers: $\dfrac{81.98z^2 - 123z + 46.16}{z^2 - 1}$ & discrete-time response is now unstable.

e. In your simulink model, insert "Transport Delay" in the continuous time system until the closed-loop becomes unstable. How much delay is required to reach neutral stability? We will learn how to predict and design for digital delay soon...