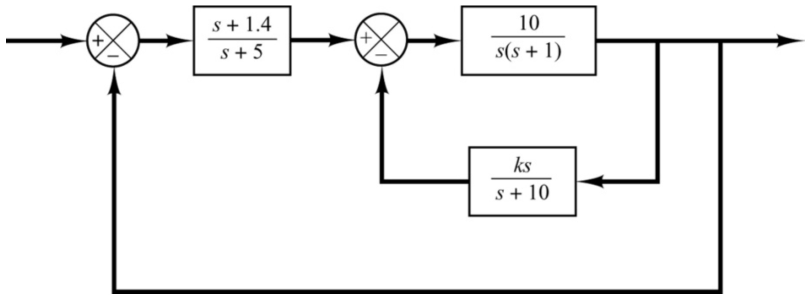
ESE 406 / ESE 505 / MEAM 513 - HOMEWORK #6
DUE 20-Feb-2013 (with Late Pass 25-Feb-2013)



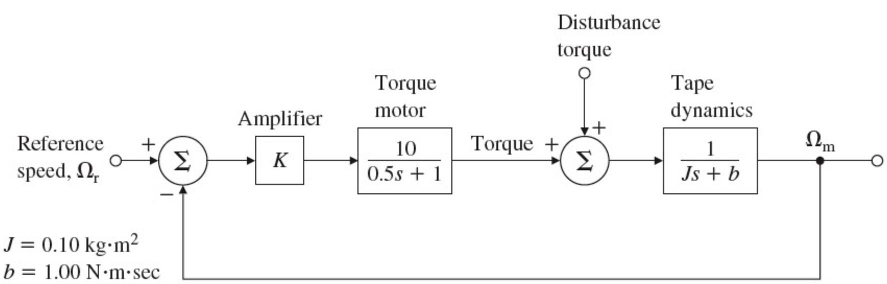
Problem 1 This problem is based on problem B-6-27 in Ogata, which considers the closed-loop controller shown above. We want to find the maximum value of the gain, k, for which all of the closed loop poles have a damping ratio of at least 0.5. (Poles on the negative real axis are considered to satisfy this requirement.) You may use MATLAB for this problem.

a. Find the closed-loop transfer function.

b. Create the root locus for k. (Hint: write the closed-loop denominator from part (a) in the form "D(s) + k N(s)".)

c. Find the value of k that places the oscillatory poles at a damping ratio of 0.5. What is the natural frequency of theses poles for this value of k?

d. Plot the closed-loop step response for this value of k.



Problem 2 This problem is based on problem 4.26 in the textbook by Franklin, et al., which considers the closed-loop control system shown above (Note that the values of "*J*" and "*b*" are given in the lower left corner above). We will replace the amplifier (K) in the problem with a PID controller, and then we will pick the 3 gains in our PID controller one at a time, following the procedure discussed in lecture this week.

i. Find the closed-loop transfer functions for $\frac{\Omega_M}{\Omega_R}$ and $\frac{\Omega_M}{W}$, where "W" is the disturbance torque.

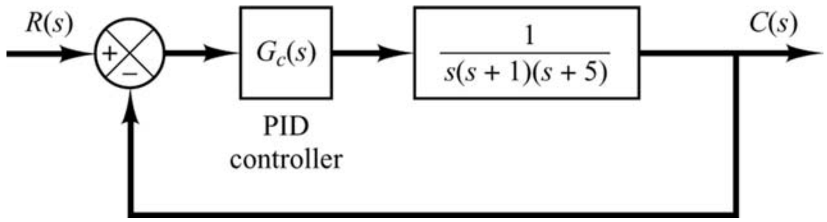
ii. Suppose that we have determined that for a sufficiently fast response, we want the closed-loop natural frequency to be at least 45 rad/sec. With zero derivative and integral gain, use a root locus to determine the minimum proportional gain required to achieve this requirement. What is the damping ratio of the poles with this gain?
Answer: $K_P \sim 10$; $\zeta \sim 0.13$

iii. Let's keep the proportional gain you found in part (ii), but now add derivative feedback to get some damping. If we add too much damping, the initial response will be slow, so we typically try for closed-loop $0.5 < \zeta < 0.7$. Use a root locus to find the K_D that will achieve $\zeta = 0.7$

iv. Now let's add integral feedback. Keeping the values of K_P and K_D from the previous step, use a root-locus to choose a non-zero value of K_I such that the damping ratio of the oscillatory poles is no less than 0.5. Submit the root locus for K_I.

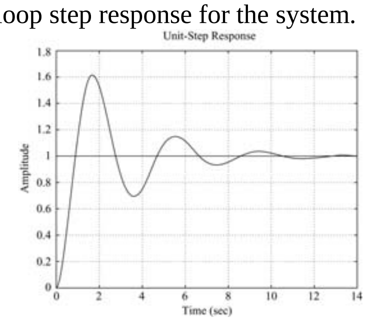
v. Find the closed-loop unit step response for the input (Ω_R) and disturbance (*W*). Submit these 2 graphs.

<u>Problem 3</u> Read Section 8.2 on Ziegler-Nichols tuning rules for PID controllers. This is a "turnkey" alternative to the sort of adjustment we did in the previous problem. Then consider the following control system:



i. Using proportional feedback only, what is the gain that results in neutral stability (poles on the imaginary axis). With what period will the closed-loop neutrally damped oscillations occur? *Answer*: K_{CR}~30 and P_{CR} ~ 2.8 sec.

ii. Use the Ziegler-Nichols Second-Method to design a PID controller for this system. Make a plot of the closed-loop step response for the system. *Answer*: graph below.



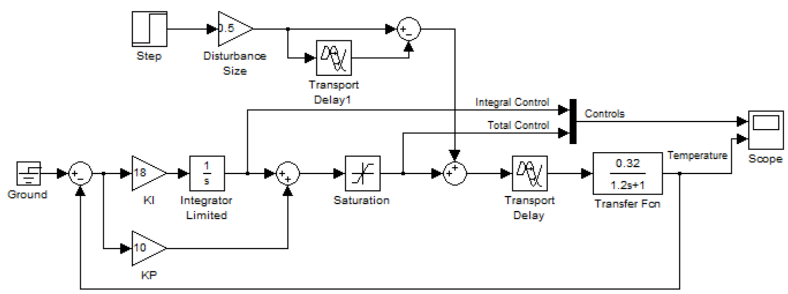
iii. As noted in the textbook, Ziegler-Nichols always yields a compensator of the following form:

$$G_C(s) = K\frac{(s+a)^2}{s}$$

where $K = 0.075K_{CR}P_{CR}$ and $a = \dfrac{4}{P_{CR}}$. Fix the value of "a" at the value given by the tuning rules and then make a root-locus plot to show how the poles move with K. Notice that the damping ratio of the poles is not a strong function of K for values of K close to the chosen value. In order to improve the damping ratio, we have to move the double zero in the numerator (at *s*=-*a*) closer to the origin. Can you find values of *K* and *a* that yield overshoot of less 20 percent and settling time of less than 4 seconds? (There are lots of possible answers.)

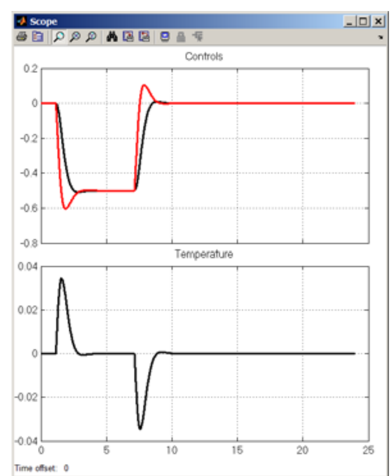
Ziegler-Nichols rules can be a convenient way to find an initial set of reasonable gains from which to begin the sort of tuning that we did in the previous problem.

Problem 4 We mentioned in lecture many times that "integrator windup" can be a big problem for control systems. Let's explore this phenomenon using the SIMULINK model of the Glasshouse. This model has been provided for you on Blackboard. Here is what it looks like:



In this version of the model, gains and dynamics have been converted so that time is measured in hours, for convenience. The disturbance gain at the top of the model can be adjusted to look at the effect of different amplitudes of disturbance. The "transport delay" at the top of the model allows the disturbance to be applied for 6 hours before it is turned back off.

Notice that the control is limited by the "saturation" node in the center of the model. This node limits the total control to +/- 1 unit. This is a crude representation of authority limits on our heater. (Remember that we are linearized about some nominal operating condition, so "negative heating" corresponds to downward adjustments of the heat; a symmetric limit on control is roughly like assuming we trimmed at 50% power.)

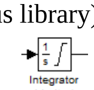


Run the simulation as initially configured, with a disturbance magnitude of 0.5. The scope output should look like the figure at right. This is what you would expect. Once the integrator has eliminated the disturbance, it is like we have reached a new trim condition. The removal of the disturbance generates a symmetric response in the opposite direction, and the integrator goes back to nominally zero control.

Now double the disturbance magnitude to 1.0 and repeat the simulation. You should see that the temperature decrease when the disturbance is removed is actually larger than the initial increase! The integrator is asking for more control than is available. This is called "integrator windup". Notice that with a disturbance of 1.0, the system is just barely able to reject the disturbance, so the integrator windup stops before the disturbance is removed.

Repeat the simulation again with a disturbance of 1.5. Now there is insufficient control to reject the disturbance and the integrator is running away until the disturbance is removed. This is a very dangerous situation, and you can see the resulting temperature change when the disturbance is removed is very large, as the integrator has to "unwind" from the large negative value it reached.

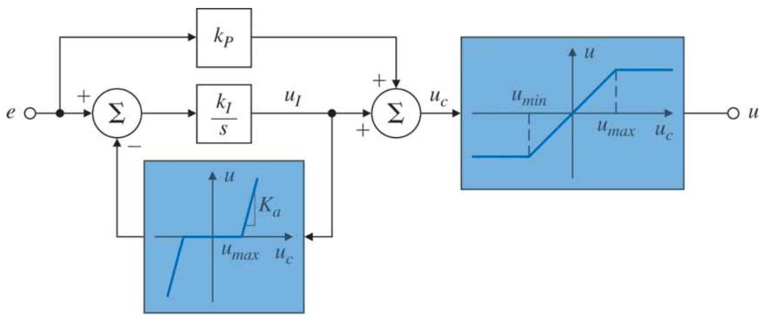
The integrator we used in this simulink model is called a "Limited Integrator" (from the Continuous library).

Double-click on the integrator and check the box next to "limit output". The icon for the integrator, 

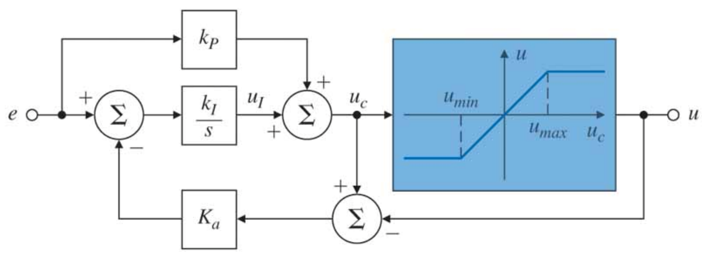
, now shows that the integrator is limited. Now repeat the simulation with the disturbance of 1.5. Be sure you understand what is happening here. Integral control is rarely safe without careful integrator windup protection. Limiting the integral output is just one of many design schemes to deal with windup. Some other schemes from the book by Franklin, et al., are shown on the following page.

<u>WHAT TO SUBMIT</u>: Implement the architecture shown on page 14 of the lecture notes from (11-Feb-2013) with the limited integrator. Submit a screen-capture of your simulink model and the scope output disturbance of 1.5.

Various integrator anti-windup schemes are shown below.



(a)



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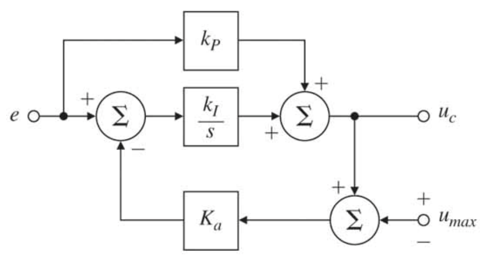

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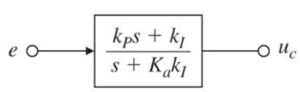

(d)