ESE 406 - SPRING 2012 - HOMEWORK #1
Math Review
DUE 18-Jan-2012 (Late Pass 23-Jan-2012)

"Textbook" Problems These aren't really textbook problems, but they could be.

1. [5 points each] Write each of the following in the form $a + jb$, where $j = \sqrt{-1}$. Do these by hand, not electronically (no calculator nor computer).

   a. $(1+j)(1-2j)$

   b. $\dfrac{(1+8j)}{-2j}$

   c. $\dfrac{(3+4j)}{(3-4j)}$

   Answers: (a) $3-j$, (b) $-4+0.5j$, (c) $-\dfrac{7}{25}+\dfrac{24}{25}j$

2. [5 points each] Write each of the following in the form $re^{j\theta}$, where $r > 0$ and θ is in radians. Do these by hand, not electronically (no calculator nor computer).

   a. $(-3+3j)(2+2j)$

   b. $\dfrac{(4+4j)}{-2j}$

   c. $\dfrac{(4+4j)}{(2-2j)}$

   Answers: (a) $12e^{j\pi}$, (b) $2\sqrt{2}e^{j3\pi/4}$, (c) $2e^{j\pi/2}$

3. [15 points] Solve the following ordinary differential equation and make a graph of the solution:

   $$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+4y=0,\ y(0)=0,\ \frac{dy}{dt}(0)=3$$

   Answer: $y(t)=e^{-t}-e^{-4t}$. The graph is up to you. It is instructive to attempt to sketch graphs by hand, but please make a nice quantitative graph (graph paper or computer) to submit.

4. [20 points] Solve the following system of ordinary differential equations.

$$\frac{d\underline{x}}{dt} = \frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}\underline{x},\ \underline{x}(0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

Answer: $\underline{x}(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-t} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}e^{-4t}$. It should be obvious that this problem is the same as the previous problem, with $x_1(t) = y(t)$ and $x_2(t) = \frac{dy}{dt}$.

5. [15 points] Solve the following ordinary differential equation and make a graph of the solution:

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 169y = 0,\ y(0) = 2,\ \frac{dy}{dt}(0) = -10$$

Answer: $y(t) = 2e^{-5t}\cos(12t)$. The graph is up to you.

6. [20 points] Solve the following system of ordinary differential equations.

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -169 & -10 \end{bmatrix}\underline{x},\ \underline{x}(0) = \begin{pmatrix} 2 \\ -10 \end{pmatrix}$$

Answer: $\underline{x}(t) = \begin{pmatrix} 2 \\ -10 \end{pmatrix}e^{-5t}\cos(12t) + \begin{pmatrix} 0 \\ 24 \end{pmatrix}e^{-5t}\sin(12t)$. The important thing here is that you see where "5" and "12" come from. Don't worry if you don't see how to use the eigenvectors of the matrix. We won't need that right away.