ESE 406 Final Exam Cheat Sheet

NOTE: In Table, u(t) is unit step
(i.e. time domain f(t) = 0 for t < 0)

Function	Time domain $f(t) = \mathcal{L}^{-1}\{F(s)\}$	Laplace s-domain $F(s) = \mathcal{L}\{f(t)\}$
*n*th power (for integer *n*)	$\frac{t^n}{n!} \cdot u(t)$	$\frac{1}{s^{n+1}}$
*q*th power (for complex *q*)	$\frac{t^q}{\Gamma(q+1)} \cdot u(t)$	$\frac{1}{s^{q+1}}$
unit step	$u(t)$	$\frac{1}{s}$
delayed unit step	$u(t-\tau)$	$\frac{e^{-\tau s}}{s}$
ramp	$t \cdot u(t)$	$\frac{1}{s^2}$
*n*th power with frequency shift	$\frac{t^n}{n!}e^{-\alpha t} \cdot u(t)$	$\frac{1}{(s+\alpha)^{n+1}}$
exponential decay	$e^{-\alpha t} \cdot u(t)$	$\frac{1}{s+\alpha}$
exponential approach	$(1-e^{-\alpha t}) \cdot u(t)$	$\frac{\alpha}{s(s+\alpha)}$
sine	$\sin(\omega t) \cdot u(t)$	$\frac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t) \cdot u(t)$	$\frac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\alpha t) \cdot u(t)$	$\frac{\alpha}{s^2-\alpha^2}$
hyperbolic cosine	$\cosh(\alpha t) \cdot u(t)$	$\frac{s}{s^2-\alpha^2}$
Exponentially-decaying sine wave	$e^{-\alpha t}\sin(\omega t) \cdot u(t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
Exponentially-decaying cosine wave	$e^{-\alpha t}\cos(\omega t) \cdot u(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt.$$

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=0}^{\infty} x[n] z^{-n}.$$

$$t_r = \frac{1.8}{\omega_n}$$

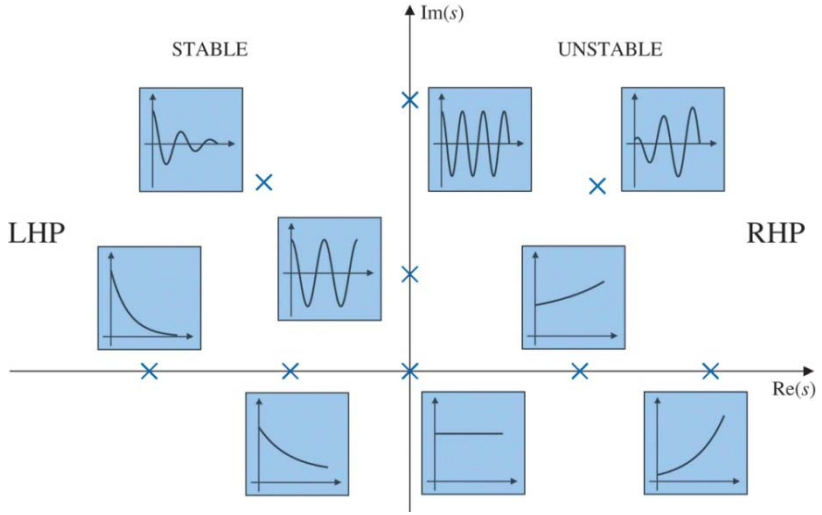
$$t_s = \frac{4.6}{\sigma}$$

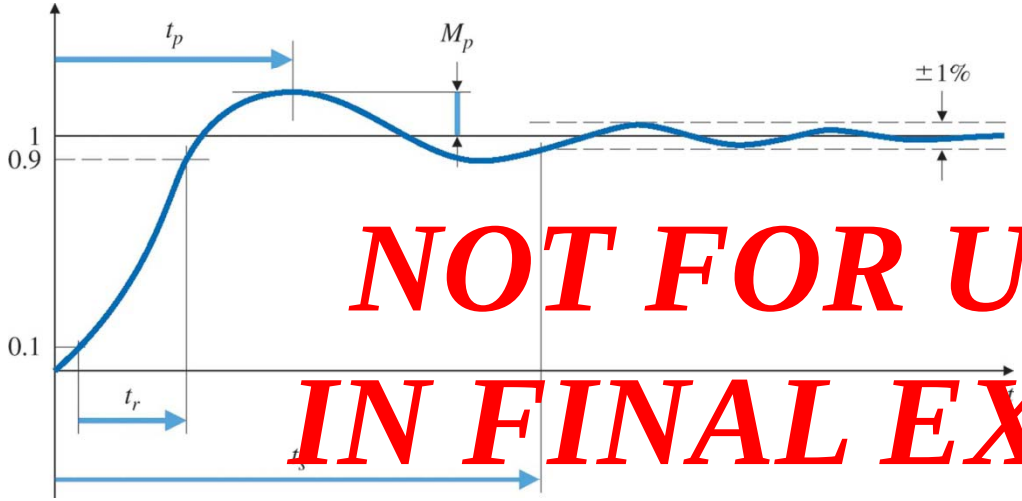
TABLE 8-1
Short Table of *z*-Transforms

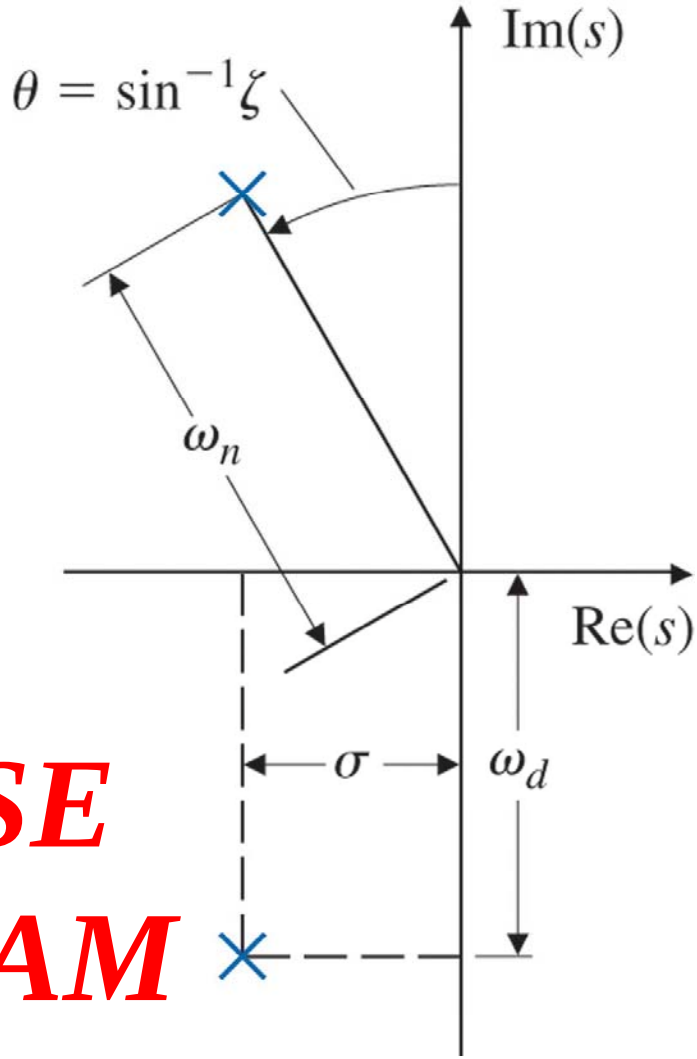
Transform Pair Number	Continuous-time Function $f(t)$ for $t > 0$	Sample Values $f(nT)$ for $n \geq 0$	*z*-Transform of $f(nT)$
1.	—	$f(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \triangleq \delta(n)$	1
2.	1 (unit step)	1	$\frac{1}{1-z^{-1}}$
3.	e^{-at}	$e^{-anT} = (e^{-aT})^n = K^n$	$\frac{1}{1-e^{-aT}z^{-1}} = \frac{1}{1-Kz^{-1}}$
4.	t	nT	$\frac{Tz^{-1}}{(1-z^{-1})^2}$
5.†	te^{-at}	nTe^{-anT}	$\frac{Te^{-aT}z^{-1}}{(1-e^{-aT}z^{-1})^2}$
6.†	$\sin bt$	$\sin bnT$	$\frac{(\sin bT)z^{-1}}{1-2(\cos bT)z^{-1}+z^{-2}}$
7.†	$\cos bt$	$\cos bnT$	$\frac{1-(\cos bT)z^{-1}}{1-2(\cos bT)z^{-1}+z^{-2}}$
8.†	$e^{-at}\sin bt$	$e^{-anT}\sin bnT$	$\frac{e^{-aT}(\sin bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$
9.†	$e^{-at}\cos bt$	$e^{-anT}\cos bnT$	$\frac{1-e^{-aT}(\cos bT)z^{-1}}{1-2e^{-aT}(\cos bT)z^{-1}+e^{-2aT}z^{-2}}$

†In transforms 5 through 9, e^{-aT} and bT can be replaced by constants, K_1 and K_2, respectively, as was done in transform 3. Convergence of the *z*-transform requires $|K_1| < 1$.

Stuff You Should Know (Not on Cheat Sheet)

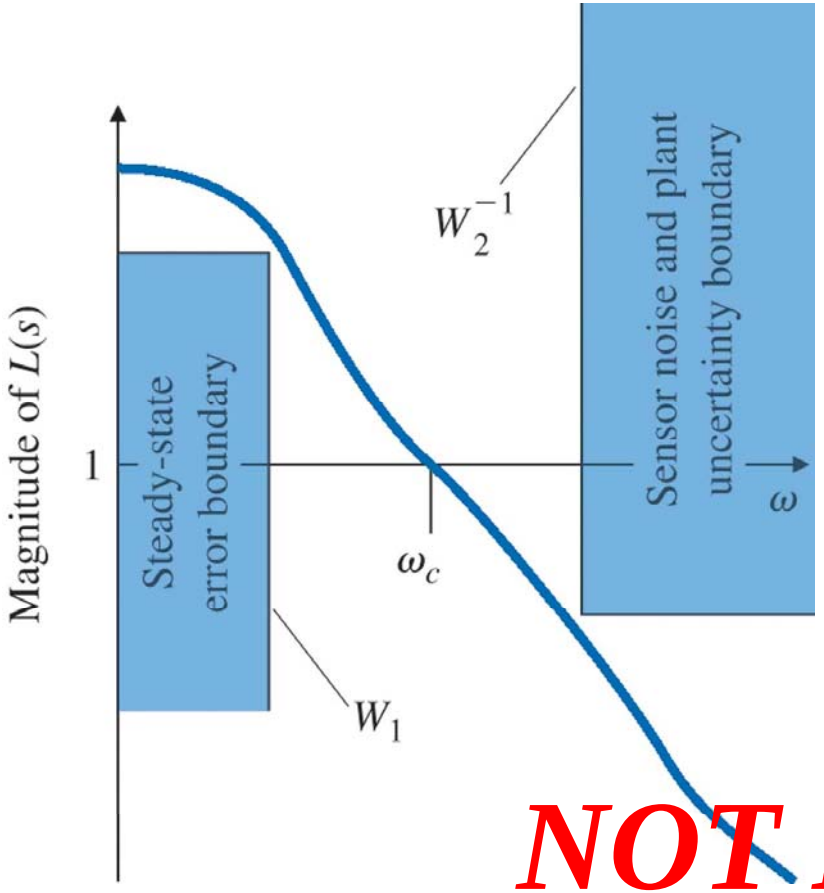


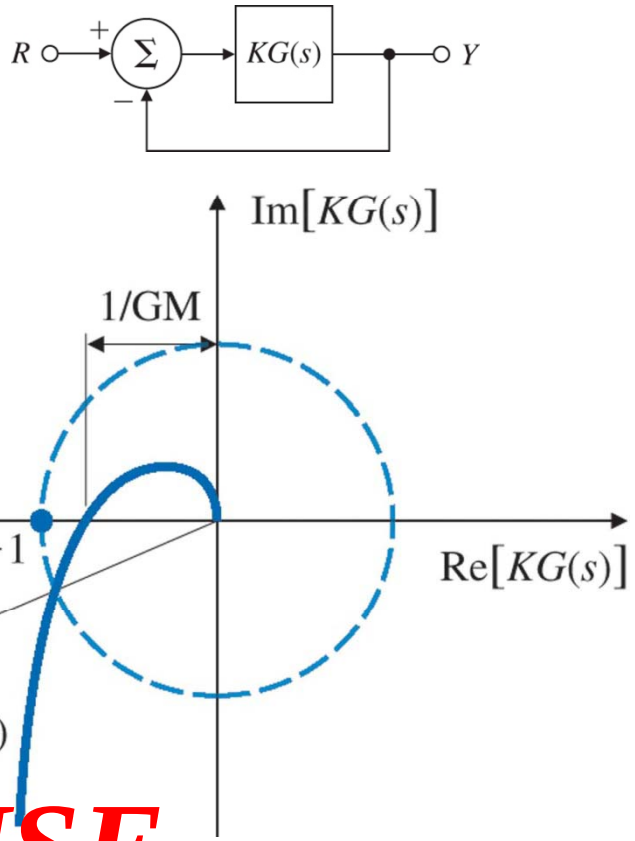




NOT FOR USE IN FINAL EXAM

More Stuff You Should Know (Not on Sheet)





NOT FOR USE IN FINAL EXAM