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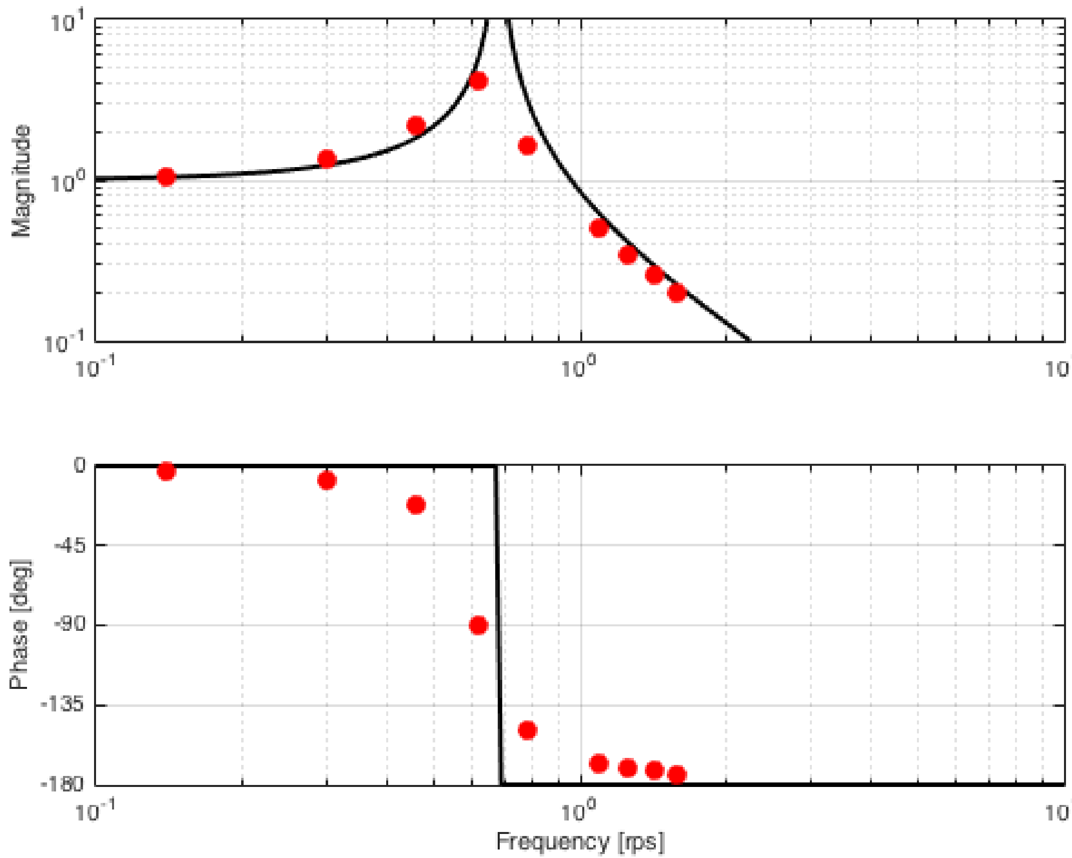
ESE 505 Homework 4

Transfer Function from Points

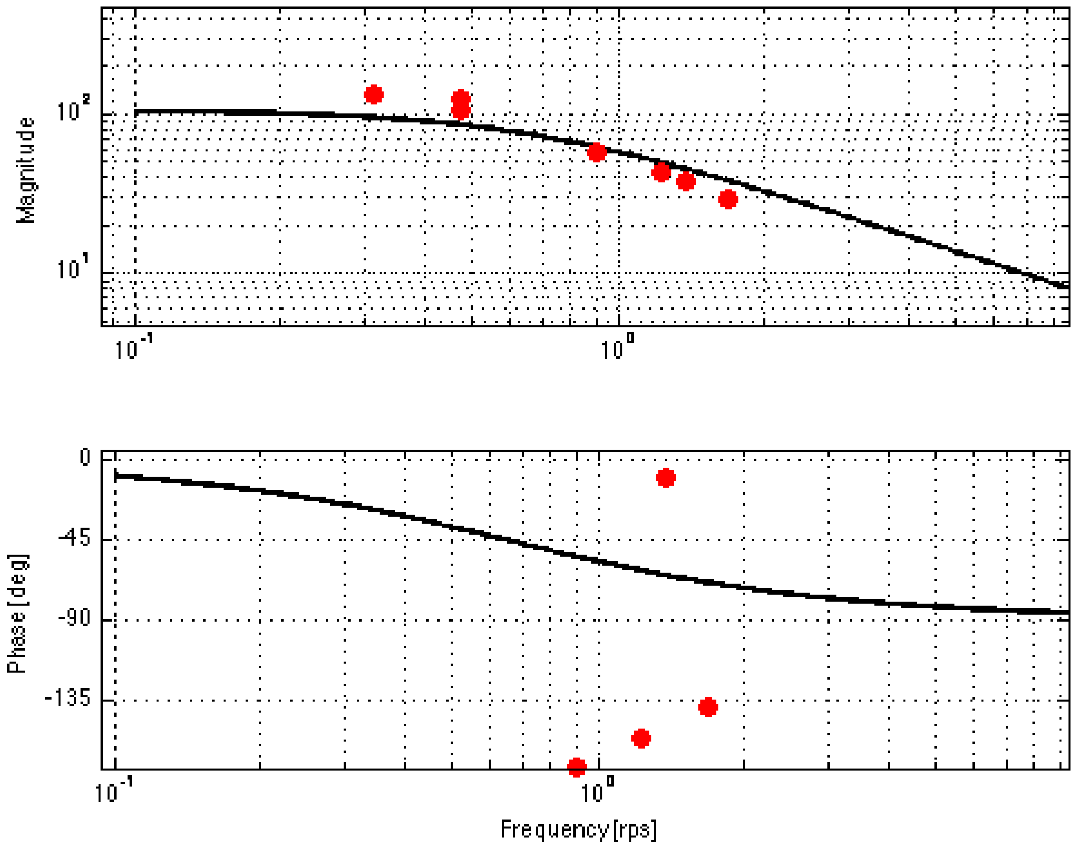
A

Figure 1: Find Transfer Function from Simulink



B

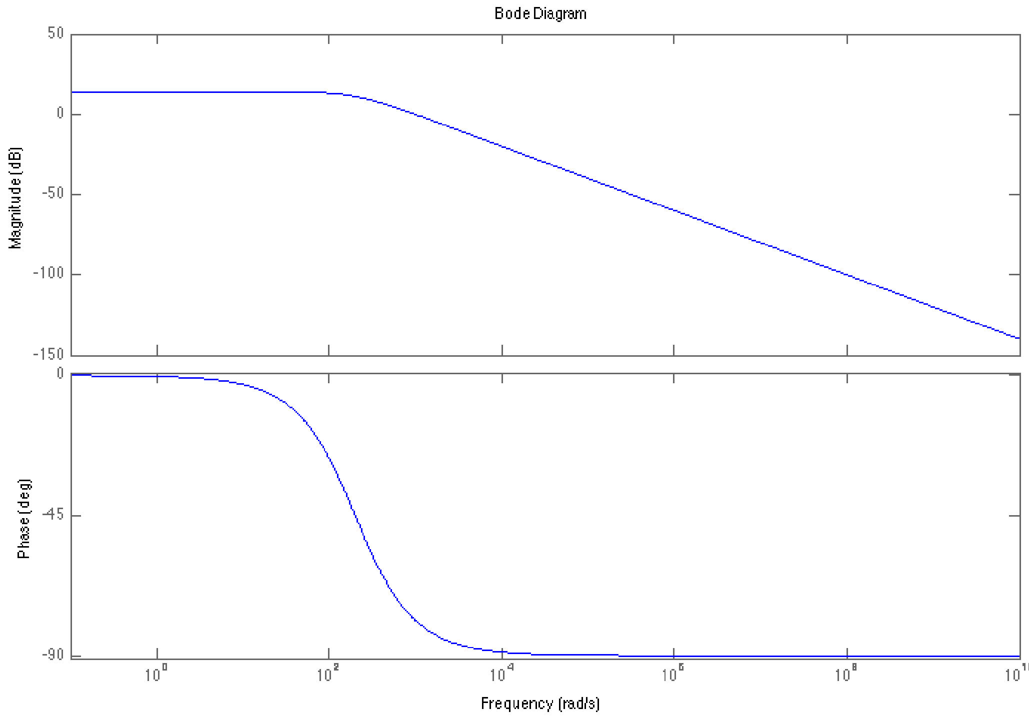
Figure 2: Find Transfer Function from Simulink



Creating Bode Plots from Transfer Functions

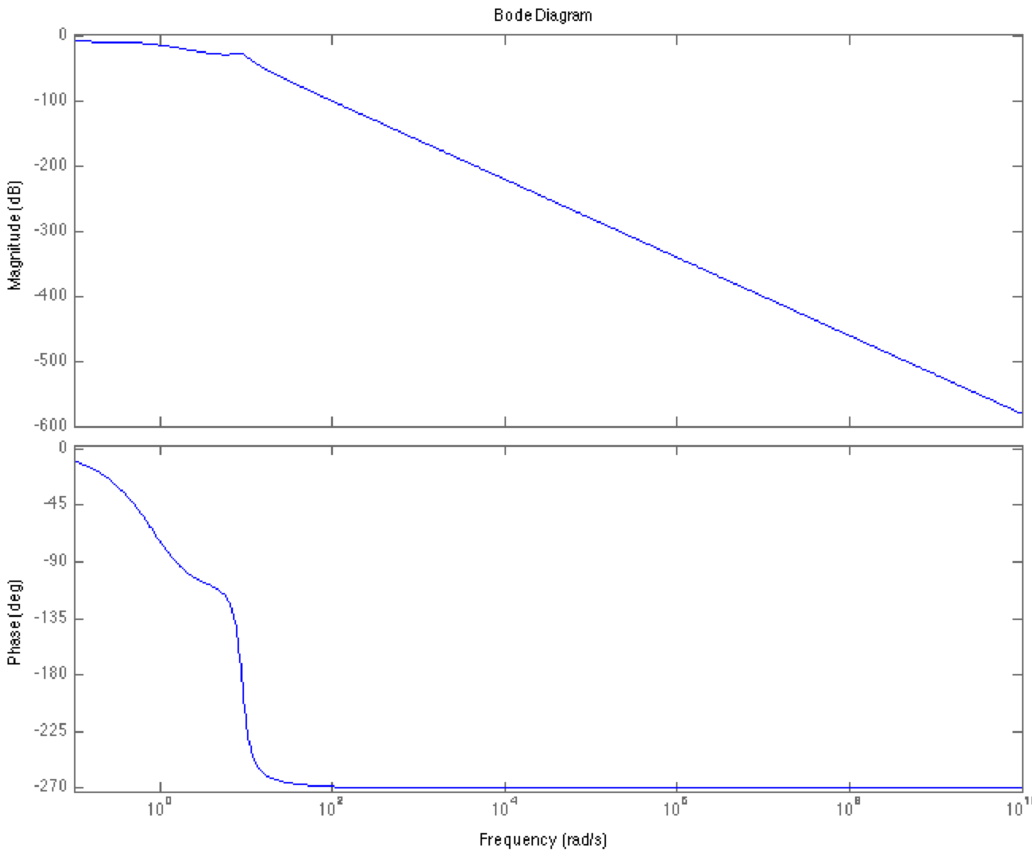
0.1 $G(s) = \frac{1000}{s+200}$

Figure 3: Bode plot of $G(s) = \frac{1000}{s+200}$



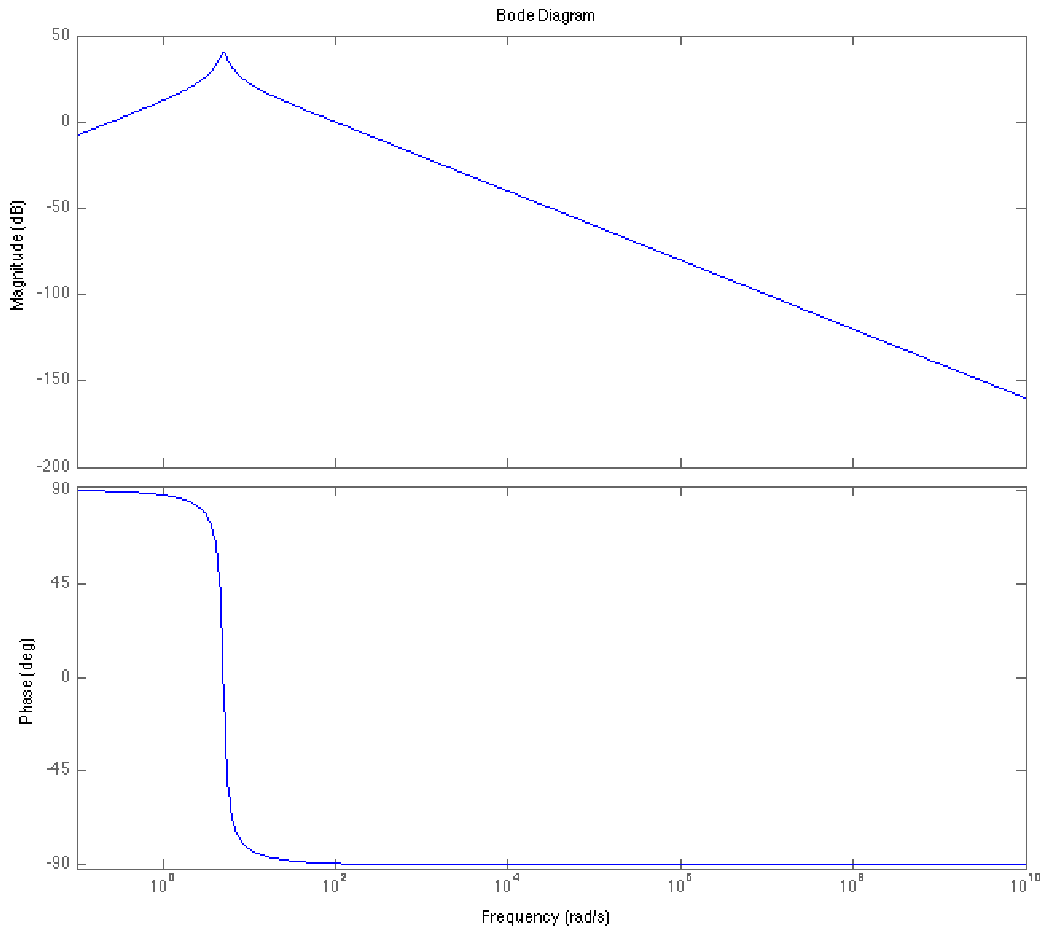
0.2 $G(s) = \frac{9s+27}{(s+1)^2+(s^2+3s+81)}$

Figure 4: Bode plot of $G(s) = \frac{9s+27}{(s+1)^2+(s^2+3s+81)}$



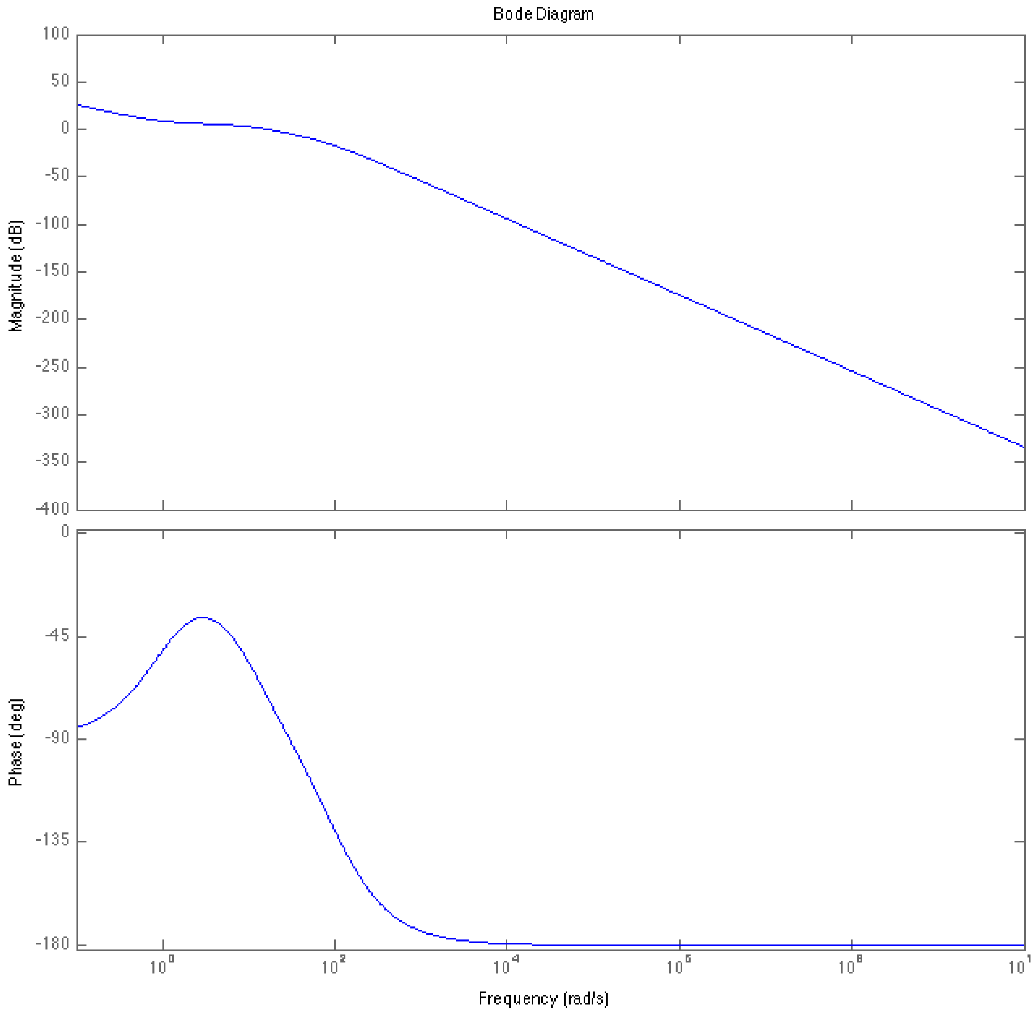
0.3 $G(s) = \frac{100s}{(s^2+s+25)}$

Figure 5: Bode plot of $G(s) = \frac{100s}{(s^2+s+25)}$



0.4 $G(s) = \frac{2000s+2000}{(s(s+10)(s+100)}$

Figure 6: Bode plot of$G(s) = \frac{2000s+2000}{(s(s+10)(s+100)}$



Controller Design

Ziegler Nicholas Ultimate Sensitivity Form

Zigerler Nichols form can be writter as

$$KG_c = K_p + \frac{K_d}{s} + K_i s$$

$$K_p = .6 * K_u, K_d = \frac{2K_p}{T_u s}, K_i = \frac{K_p T_u s}{8}$$

$$KG_c = .6K_u(\frac{2}{T_u s} + \frac{T_u s}{8})$$

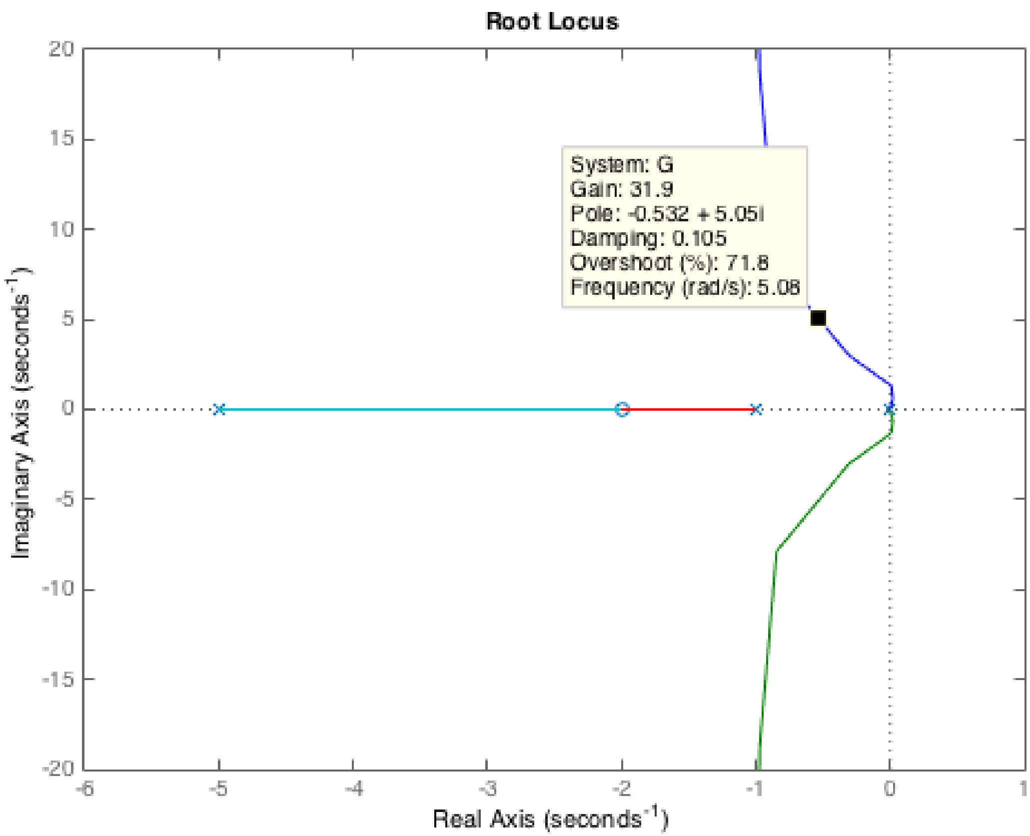
$$KG_C = \frac{.6K_u}{8T_u}(\frac{16 + s^2T_u^2}{s})$$

$$KG_C = \frac{.6K_u}{8T_u}\left(\frac{(T_u s + 4)^2}{s}\right)$$

Proportional Feed Back

Using a root locus with proportional feed back only we found that our $K_u = 30.$ and our $\omega_d = 2.23$, which when converted to find $T_u = 2.8$. Using these values to plug in for K_u andT_u, in our compensator function we have $K = 6.3$ and $a = 1.4$. When we plot the root locus of our transfer function with our PID controller included we have:

Figure 7: Root locus with PID controller



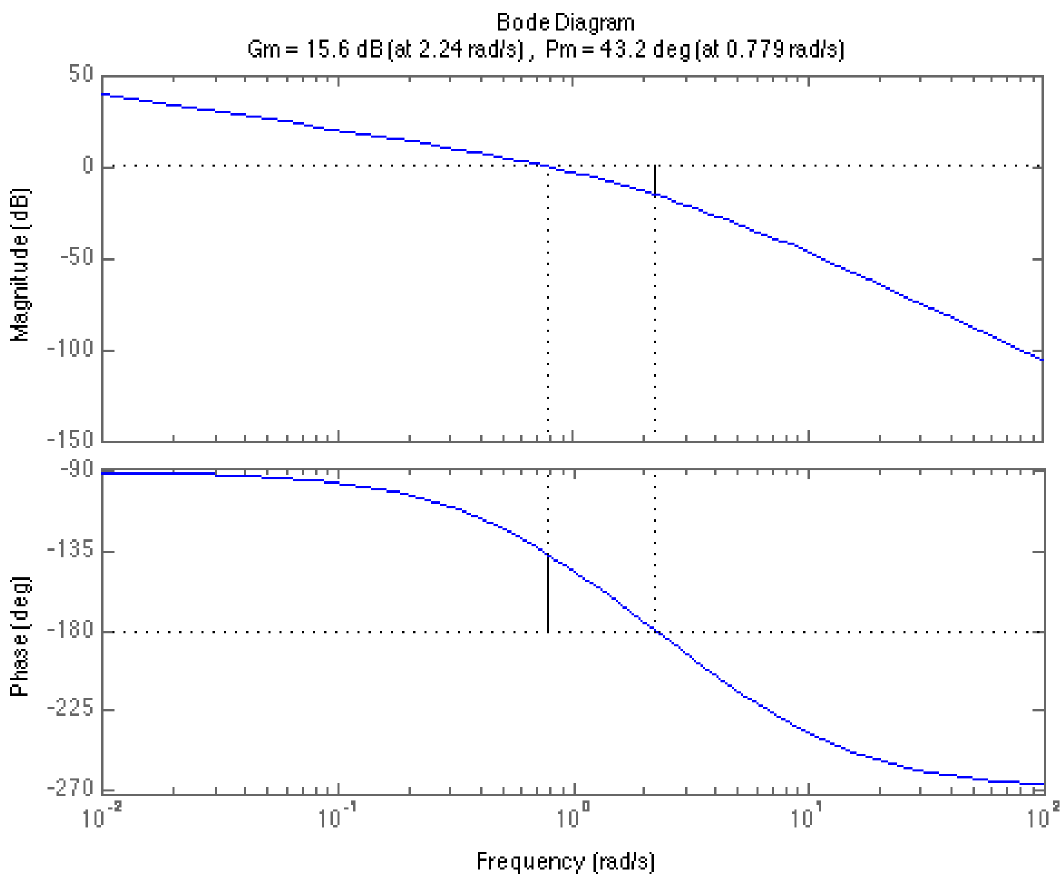
Using only Proportional feedback make a bode plot of $KG_cG_p(s)$ with K = 5 . What is the gain margin for this system?

For the system$KG_cG_p(s)$ we get use the transfer function

$$\frac{K}{s(s+1)(s+5}$$

Which gives us the following bode plot:

Figure 8: Bode Plot of Transfer function with only proportional gain



As you can see in 2 We have a gain margin of 15.6 dB, which is close to a magnitude of 6. So we can say that we can increase the gain by a factor of 5 to reach neutral stability. Like wise we see that $\omega_p hase$ is the same as the previous problem which implies the neutrally stable period of 2.28 seconds.

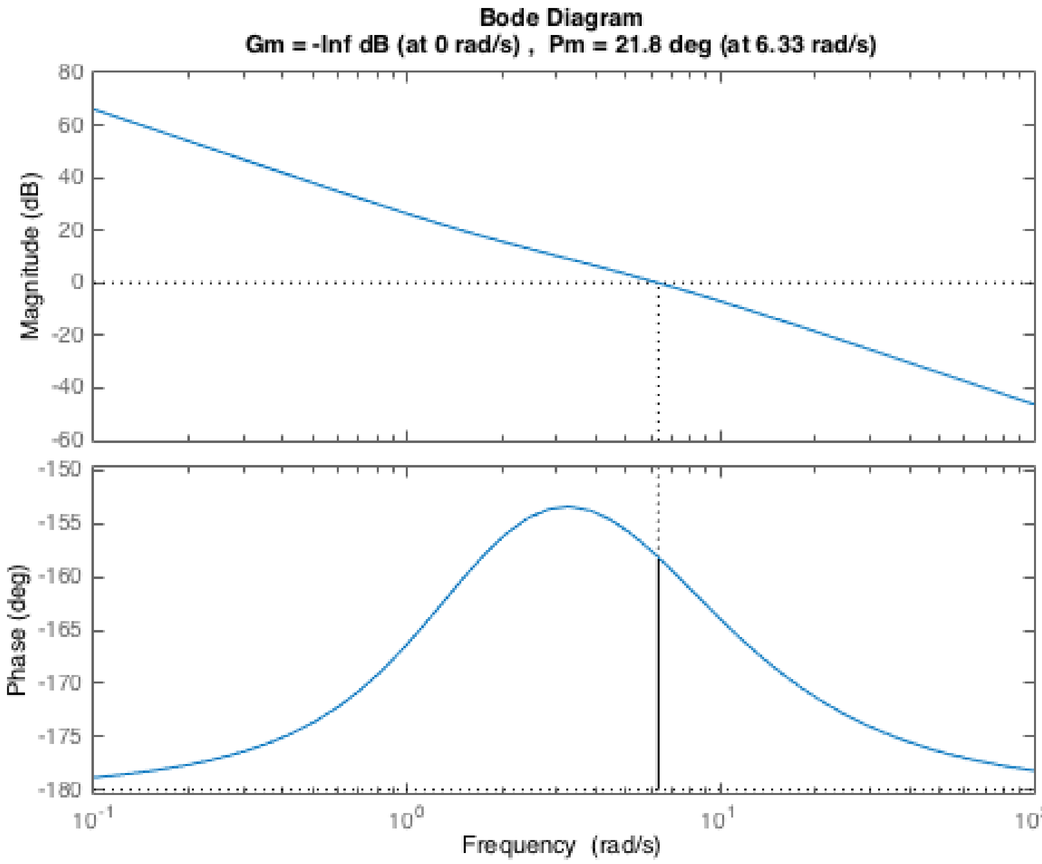
Bode plot of $KG_cG_p(s)$, for the Ziegler-Nichols design with the nominal gain.

Replacing K with the $\frac{.6K_u}{8T_u}\left(\frac{(T_us+4)^2}{s}\right)$ We have the new transfer function

$$\frac{.6K_u}{8T_u}\left(\frac{(T_us+4)^2}{s}\right)\frac{1}{s(s+1)(s+5}$$

In figure 3 We see that we have an infinite Gain margin because the system is stable for all values of K, and therefore never pass -180 degrees on the phase plot

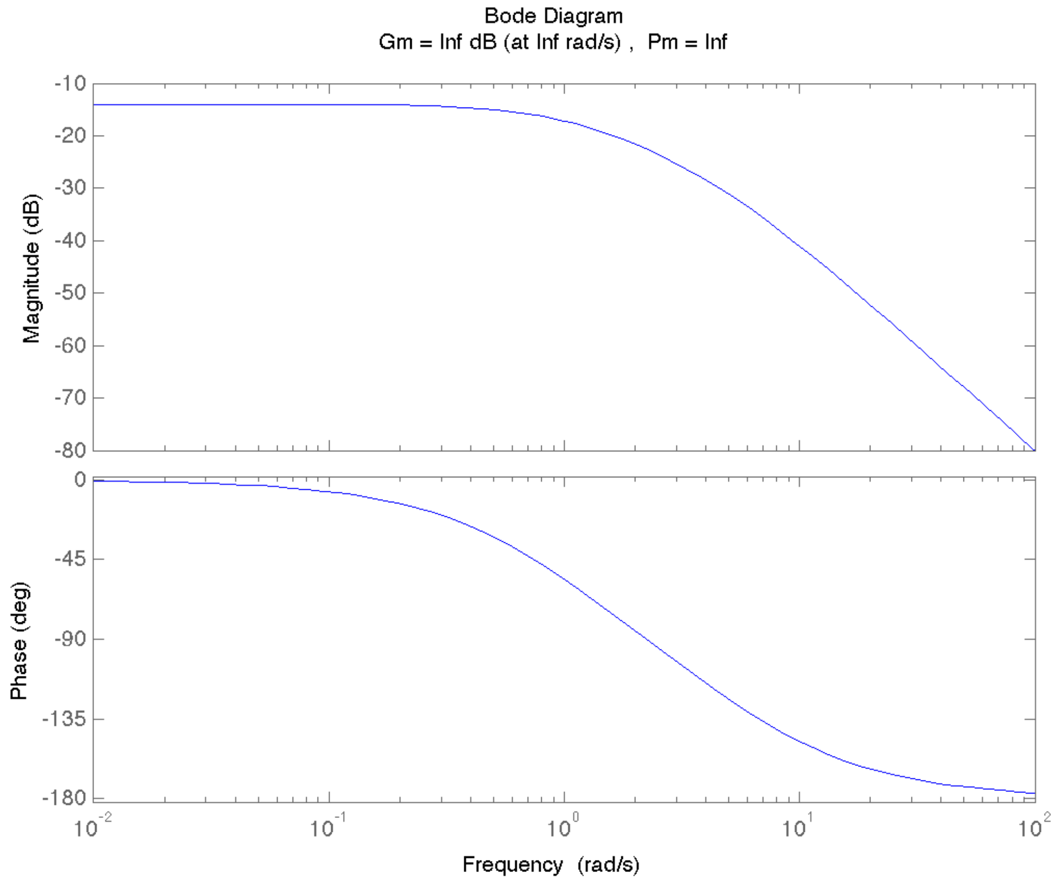
Figure 9: Bode plot of Open loop Transfer function with PID controller



find a value for a that would allow for 45 degrees of phase margin. What is the value of K that achieves this margin? Submit the bode plot that shows these values.

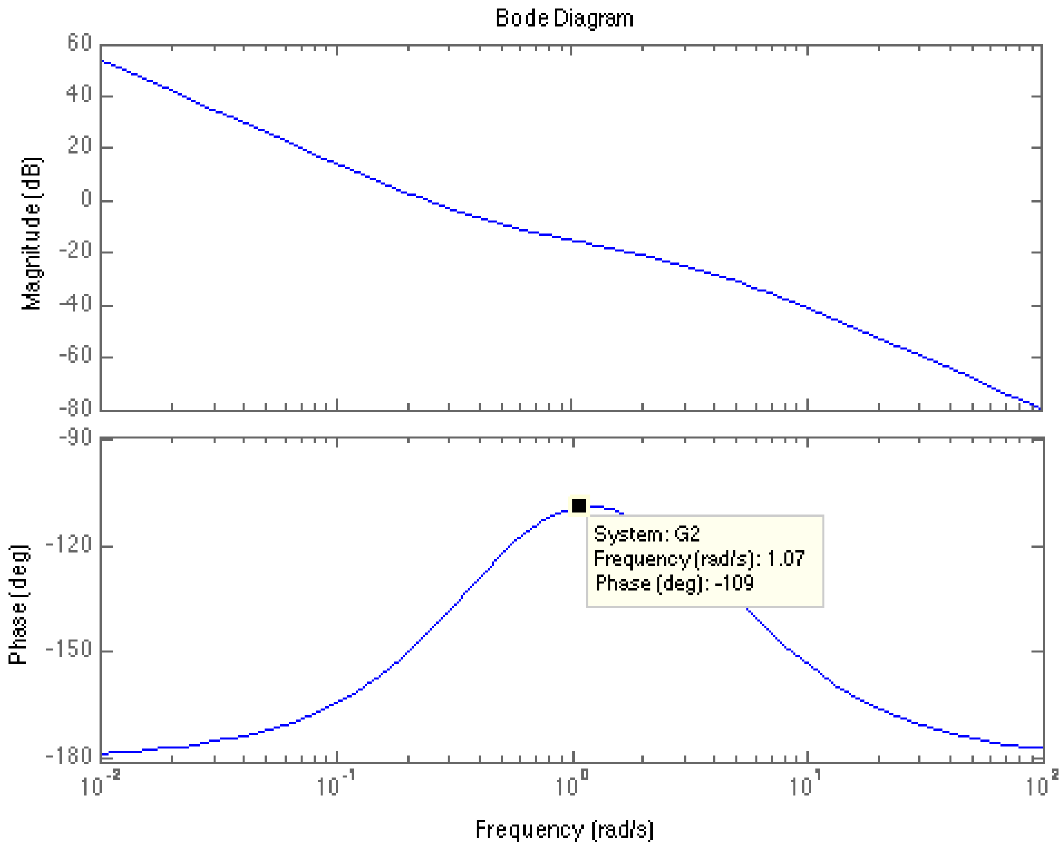
For this part I tried different values of a and saw how the phase margin shifted. For a = 0 Figure 4 we have an infinite phase and gain margin (gain never crosses 0 dB) and a maximum possible phase margin of 25 degrees

Figure 10: Bode plot of open loop transfer function PID with a =0



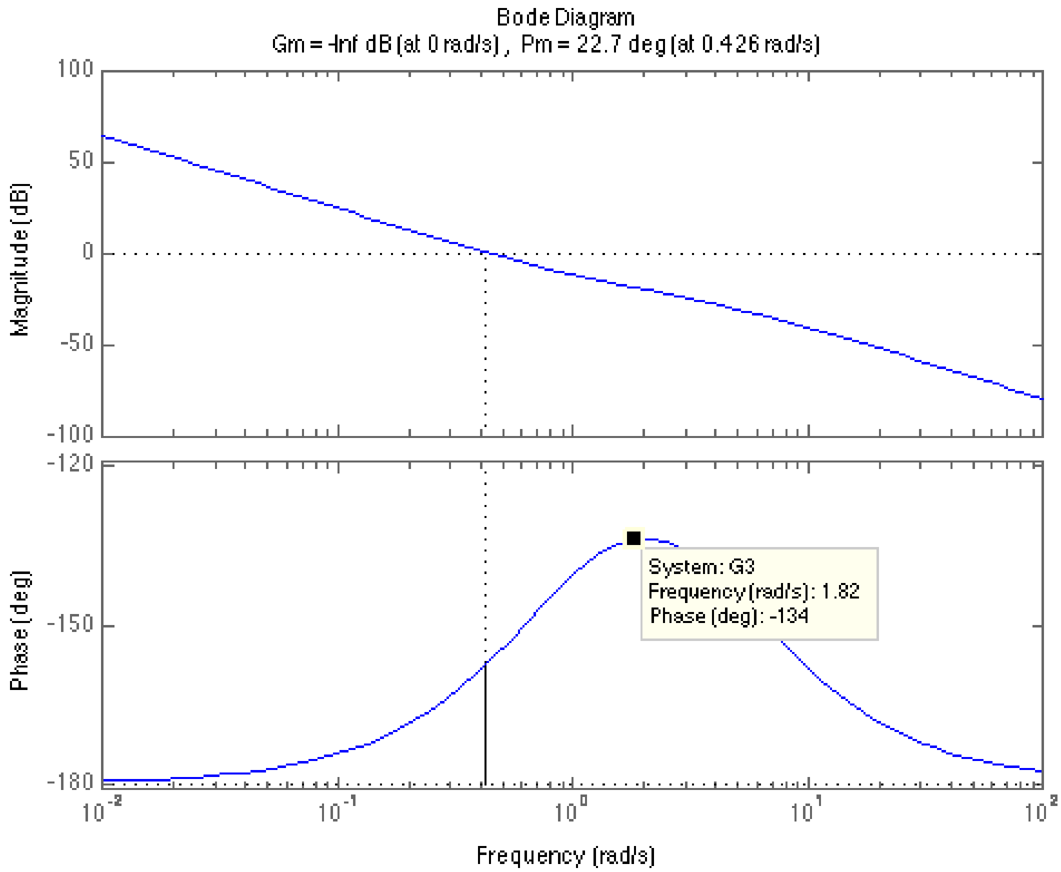
So I increased a to be .5 in figure 5 we see that we now have a maximum possible margin of 71 degrees

Figure 11: Bode plot of open loop transfer function PID with a = .5



The closest to the 45 degree maximum phase angle turned out to be a = .9 as seen in figure 6.

Figure 12: Bode plot of open loop transfer function PID with a = .9



Apply frequency-response analysis to the Black-Box system.

0.5 Bode plots of Analog and Digital System

For the Black box system for both the analog and digital implementations we can look at the gain margin to find our K_u and our $\omega_p hase$ to find T_u

Figure 13: Black Box Analog System $K_u = 12dB$ =4 $T_u = .1726$

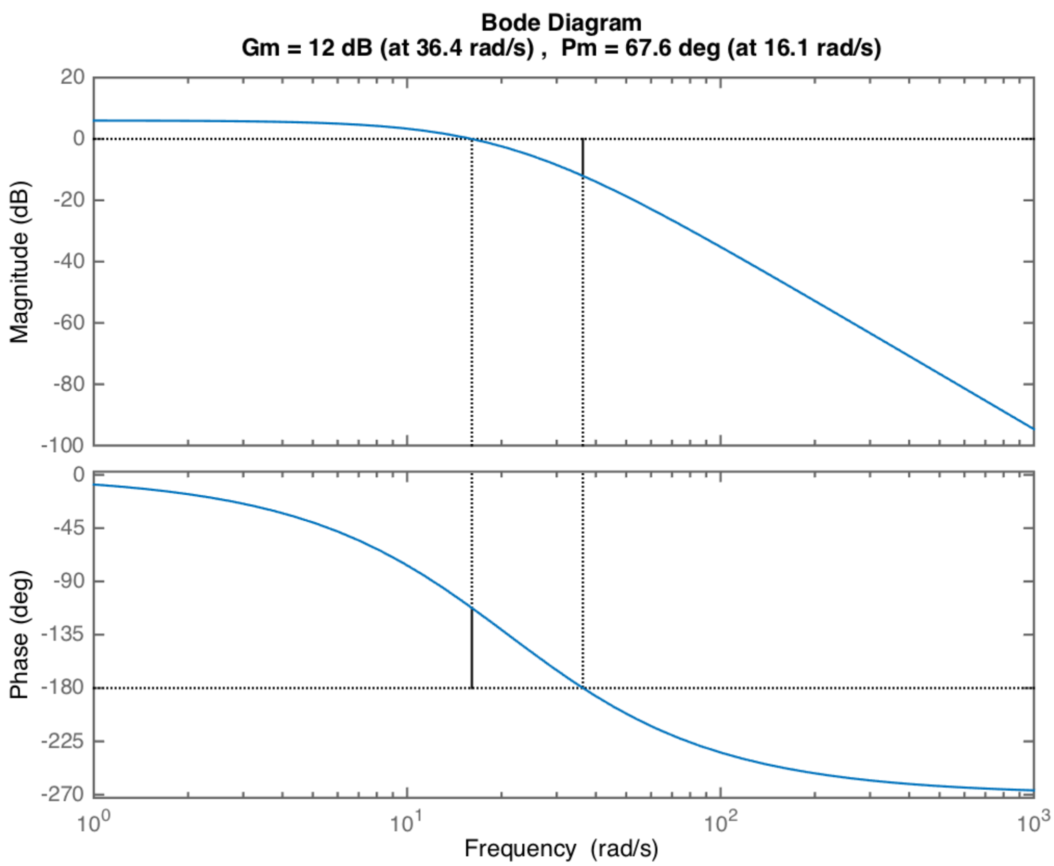
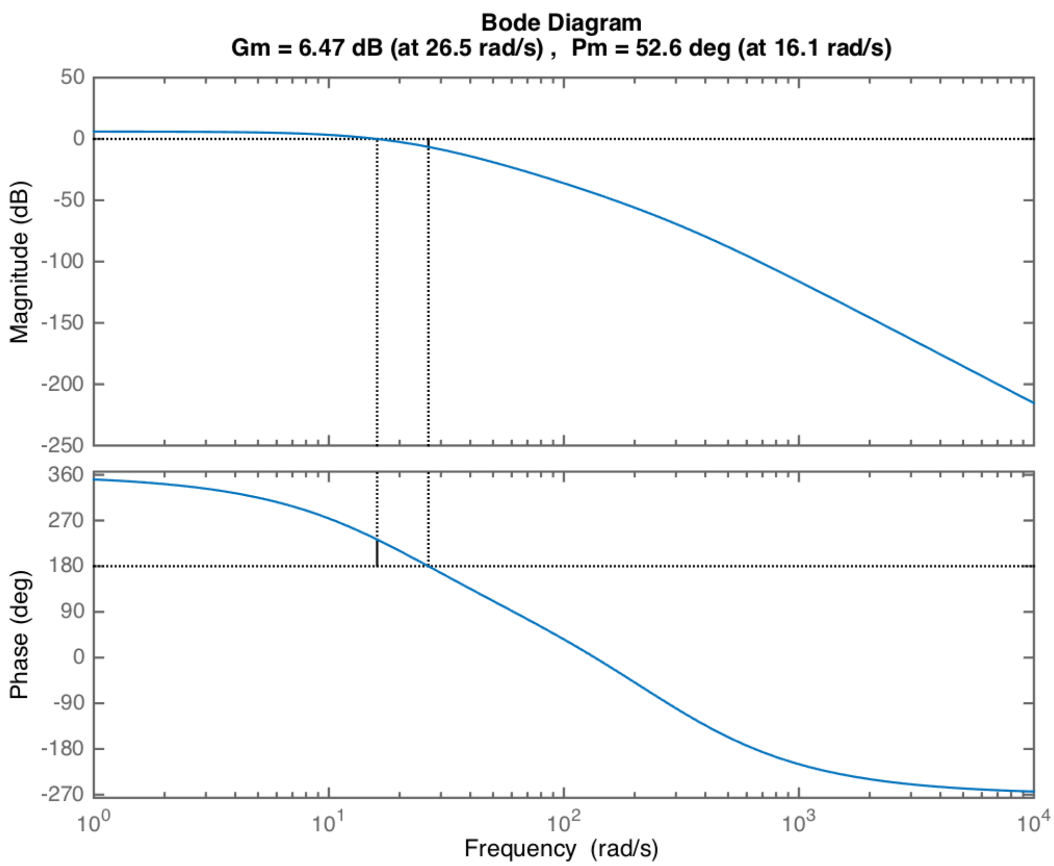


Figure 14: Black Box Digital System $K_u = 6.47dB$ =2.1 $T_u = .2371$s



Use Ziegler Nichols

Figure 15: Black Box Analog System Cross over Frequency =39.2 rad/s

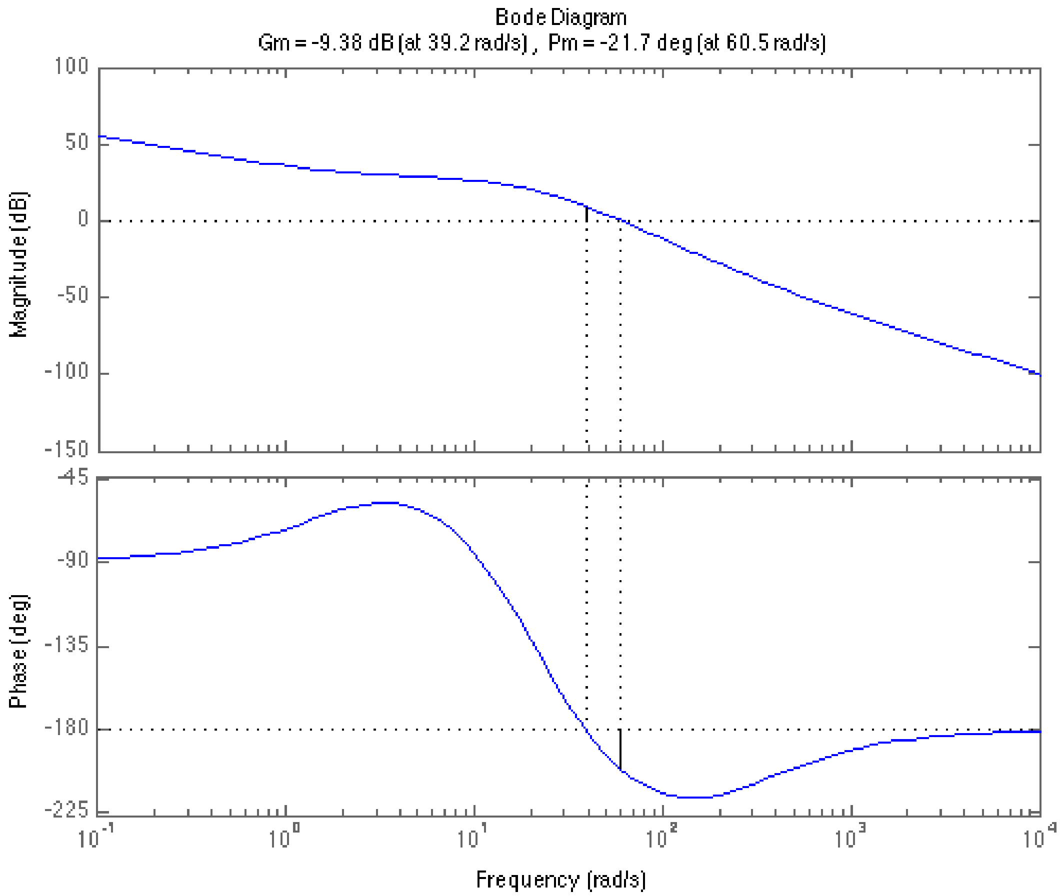
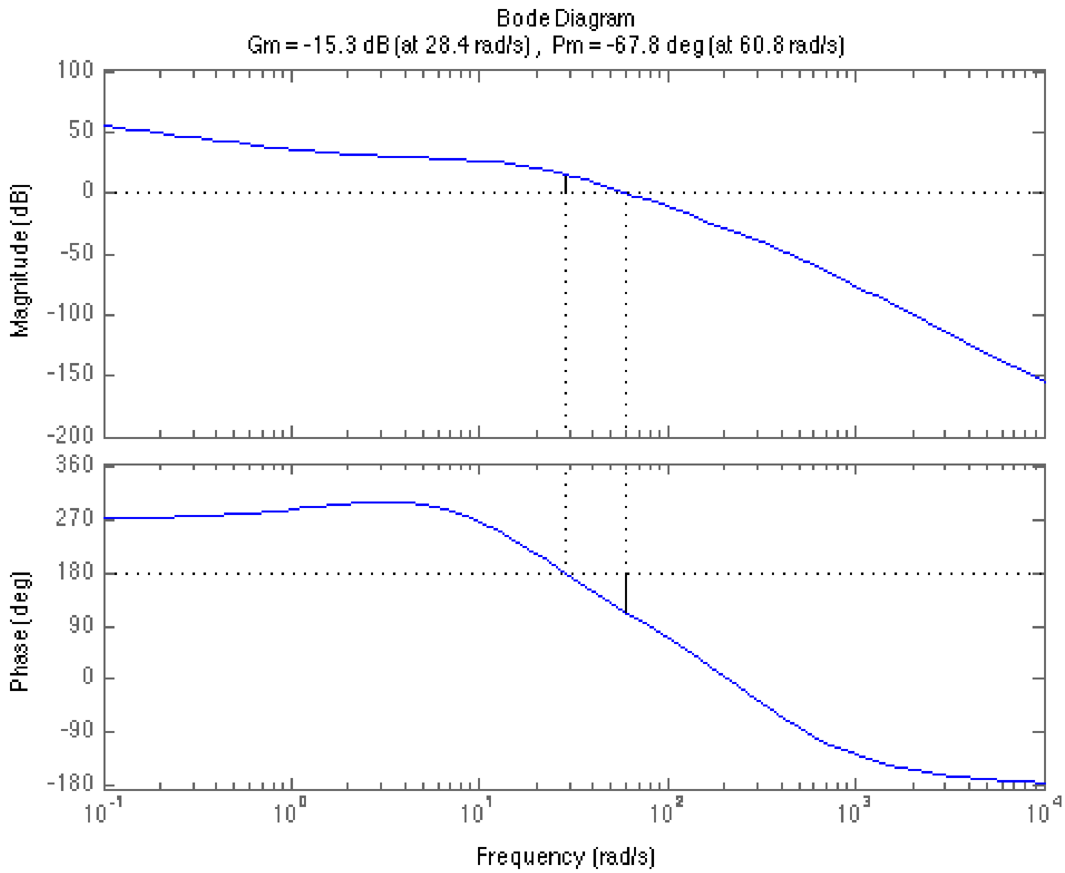


Figure 16: Black Box Digital System Cross over Frequency = 28.4 rad/s



Discuss the margins for the nominal design.

Since 0dB gain is past the 180 phase frequencies for both we should adjust the gains to have the crossover frequency less than the 180 margin frequency.

Figure 17: Black Box Analog System Adjusted Gains

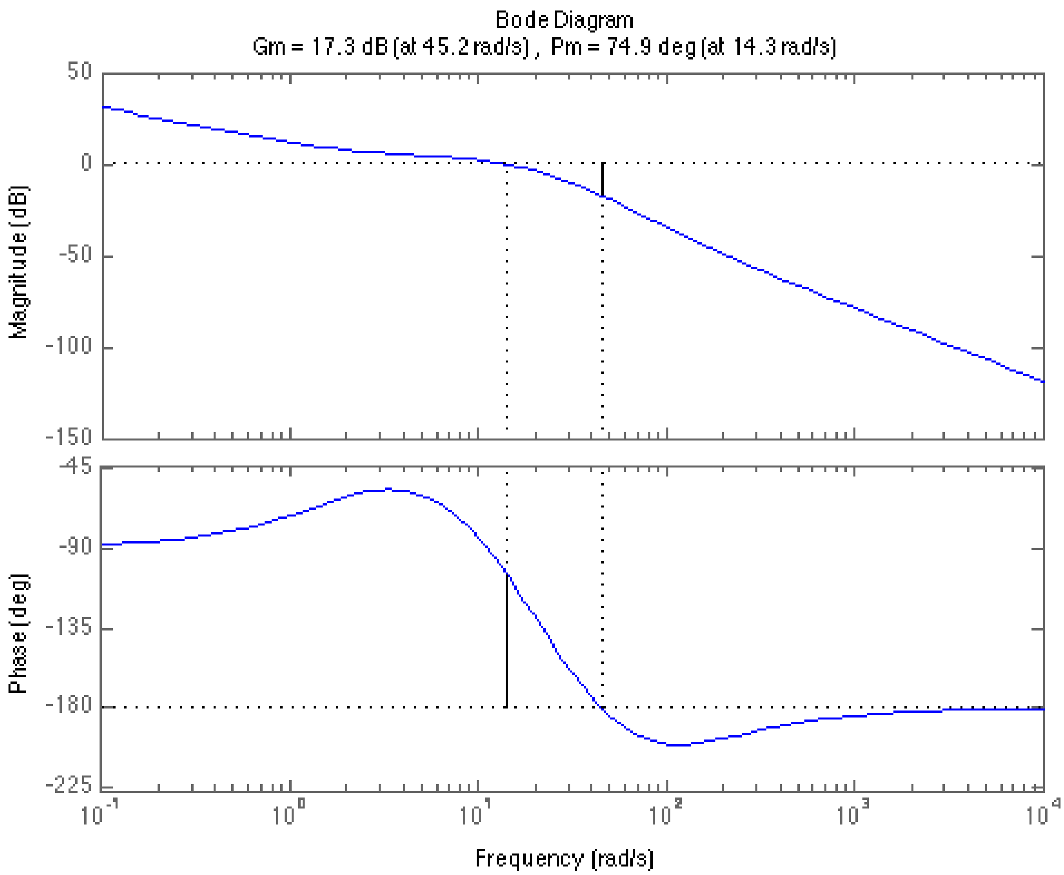
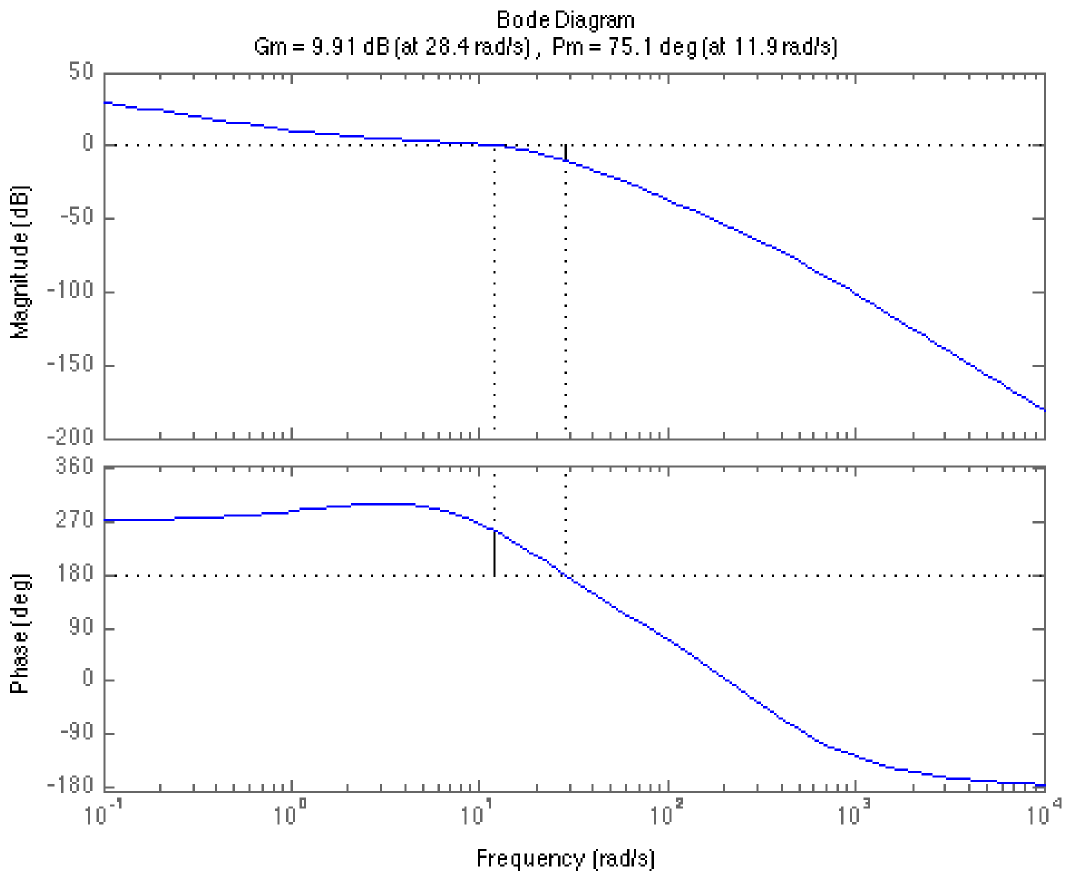


Figure 18: Black Box Digital System



12 Red Blue