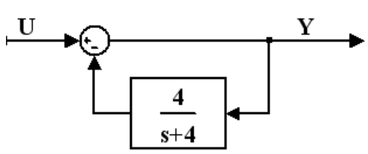
NAME _______________________

ESE505 & MEAM 513 ONLY - SPRING 2013 – Final EXAM
CLOSED NOTES & CLOSED BOOK

- Choose the one best answer for each question by *circling the letter*.
- A correct answer is worth 2 points.
- No answer is worth 0 points.
- An incorrect answer is worth -1 point. Random guessing will lower your grade, on average.

1. A transfer function, $G(z)$, represents a stable discrete-time system if…

   A. …all of the poles of $G(z)$ lie in the left half-plane.
   B. …all of the poles of $G(z)$ lie inside the unit circle.
   C. …$|G(z)| < 1$ when $|z| < 1$.
   D. …$|G(z)| \to 0$ as $|z| \to \infty$.



2. The transfer function corresponding to the above block diagram is…

   A. $\dfrac{Y(s)}{U(s)} = \dfrac{s}{s+4}$
   B. $\dfrac{Y(s)}{U(s)} = \dfrac{s+4}{s+8}$
   C. $\dfrac{Y(s)}{U(s)} = \dfrac{-1}{s+4}$
   D. None of the above.

3. Which of the following is LEAST ACCURATE about a compensator with transfer function $G_C(s) = \dfrac{s+1}{s+0.2}$?

   A. It is a lag compensator.
   B. It is typically used to increase the loop gain at low frequency.
   C. It has the side benefit of providing a small increase in the phase margin of most systems.
   D. It can be implemented readily on a digital controller.

4. Which of the following is LEAST ACCURATE about a compensator with transfer function $G_C(s) = \dfrac{s^2+3s+81}{s^2+12s+81}$?

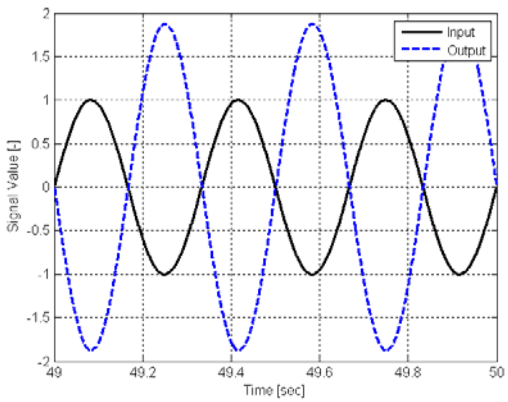
   A. It is a notch filter with notch frequency of 9 rps.
   B. The minimum gain of the filter is -4 dB.
   C. It is typically used to avoid destabilizing a lightly damped mode in the open-loop plant.
   D. It can be implemented readily on a digital controller (Tustin's method with pre-warping is usually recommended).

5. Which of the following is LEAST ACCURATE about a compensator with transfer function $G_C(s) = \dfrac{8s+1}{s+1}$?

   A. It is a lead compensator.
   B. It is typically used to improve low-frequency disturbance rejection.
   C. It is equivalent to a PD compensator where the derivative feedback has been passed through a first-order low-pass filter to avoid infinite gain at high frequency.
   D. It can be implemented readily on a digital controller.

6. A very good student runs a digital controller for an inverted pendulum with a sample rate of 16 milliseconds (0.016 sec). She observes 2.5 Hz noise in her sampled measurements of pendulum angle, but the high-speed video of the pendulum motion does not show any movement at this frequency. The most reasonable inference would be…

   A. The extra 16ms of time delay has probably caused her closed-loop system to go unstable.
   B. The noise is aliased 60 Hz electrical noise.
   C. Her system cannot work unless she purchases a much faster microcontroller.
   D. All of the above.



7. To which differential equation could the long-term response to sinusoidal input shown above correspond?

   A. $\frac{dy}{dt}+10y=40u(t-0.11)$
   B. $\frac{dy}{dt}=40u(t)$
   C. $\frac{d^2y}{dt^2}-\frac{dy}{dt}=40u(t)$
   D. Any of the above could correspond to this response.

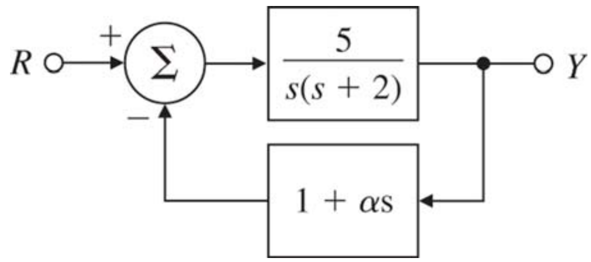
8. The system $\dot{\underline{x}}=A\underline{x}+B\underline{u}$ is *controllable* if...

   A. ...control can be used to achieve any desired final state vector in finite time.
   B. ...proportional state feedback, $\underline{u}=-K\underline{x}$, can be used to arbitrarily place the closed-loop eigenvalues.
   C. ...the matrix $\begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$ has rank n.
   D. All of the above.

9. Which of the following is LEAST CORRECT concerning the linear estimator / observer $\dot{\hat{\underline{x}}}=A\hat{\underline{x}}+B\underline{u}+L\left(y-C\hat{\underline{x}}\right)$?

   A. If the output of the estimator is used in a proportional state-feedback design, the control will never encounter the authority limits (Saturation Principle).
   B. If (A,C) is observable, the gain matrix L can be used to place the closed-loop eigenvalues of the estimator.
   C. It is a dynamic system and is typically implemented with a digital computer.
   D. It requires knowledge of the plant control input, $\underline{u}$

All of the problems in this column relate to the following block diagram:



10. Which of the following is the transfer function, $\frac{N(s)}{D(s)}$, that we should pass to MATLAB's `rlocus` command to show how the poles vary with $0\le\alpha\le\infty$?

    A. $\frac{5s}{s^2+2s}$
    B. $\frac{5s}{s^2+2s+5}$
    C. $\frac{5+5\alpha s}{s^2+(2+5\alpha)s+5}$
    D. None of the above.

11. Which of the following is the "loop transfer function," G(s), that we use to make a bode plot for assessing the stability margins?

    A. $\frac{5+5\alpha s}{s^2+2s}$
    B. $\frac{5}{s^2+(2+5\alpha)s+5}$
    C. $\frac{5+5\alpha s}{s^2+(2+5\alpha)s+5}$
    D. None of the above.

12. With $\alpha=0$, the phase margin of the system will be...

    A. ...negative, because the closed-loop system will be unstable.
    B. ...approximately 45 degrees.
    C. ...nearly 90 degrees.
    D. ...infinite, because the lack of derivative feedback keeps the loop gain low.

13. In the train project, which of the following is MOST ACCURATE about the consequences of removing the explicit 0.1-second delay in the microcontroller?

    A. It would have enabled a modest increase in gains and slightly better overall performance.
    B. It would have enabled the gains to be at least doubled, because the motor and train dynamics were very fast compared to the 0.1-second delay time.
    C. It would have required the gains to be reduced by at least 50% due to the windup effect associated with Tustin pre-warping.
    D. We would no longer have needed integral control to achieve zero steady error.

14. "Lyapunov's Second (or Direct) Method" ...

    A. ...is used to assess stability of an equilibrium of a nonlinear system of ODEs.
    B. ...is especially useful because it does not require the solution of the nonlinear ODEs.
    C. ...relies on having a Lypunov function, $V(\underline{x})$, which is not always easy to find.
    D. ...all of the above.

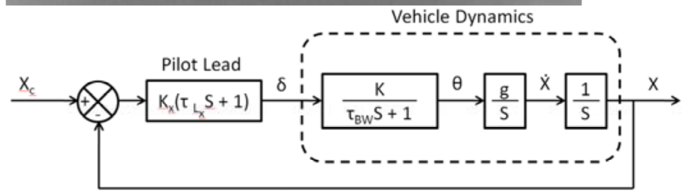
15. What is the *difference equation* for a digital implementation of a derivative compensator, $u = K_D \dfrac{de}{dt}$, if we use Tustin's method, $s \rightarrow \dfrac{2}{T}\dfrac{z-1}{z+1}$?

    A. $u[k] = \dfrac{K_D}{T}\left(e[k] - e[k-1]\right)$
    B. $u[k] = -u[k-1] + \dfrac{2K_D}{T}\left(e[k] - e[k-1]\right)$
    C. $u[k] = u[k-1] - \dfrac{2K_D}{T}e[k]$
    D. None of the above.

$$u[k] = 0.6u[k-1] + 2.0e[k] - 1.6e[k-1]$$

16. The difference equation shown above corresponds to which discrete transfer function?

    A. $G(z) = \dfrac{1.6 - 2.0z^{-1}}{0.6 - z^{-1}}$
    B. $G(z) = \dfrac{2.0 - 1.6z^{-1}}{1 - 0.6z^{-1}}$
    C. $G(z) = 0.6 + 2.0z^{-1} - 1.6z^{-2}$
    D. None of the above.



17. A recent paper[1] presented the block diagram shown above to describe how a pilot controls the position of a helicopter during aerial refueling. The input is desired position, X_c, and the output is the actual position, X. The pilot is directly "measuring" the error, X_c-X, by visually judging the distance between the fueling boom and the supply drogue. The block labeled "pilot lead" is a simple model of a control strategy the pilot might use to track a desired trajectory. Which of following is correct regarding the simple representation of the pilot control strategy shown?

    A. It is equivalent to PI control.
    B. It is equivalent to PD control.
    C. It is equivalent to a notch filter.
    D. It is equivalent to full-state feedback with a dynamic estimator.

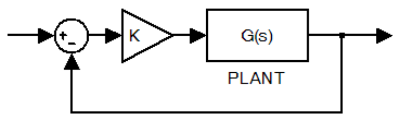
18. Which of the following conditions *MUST* be true in order for the closed-loop system to be stable?

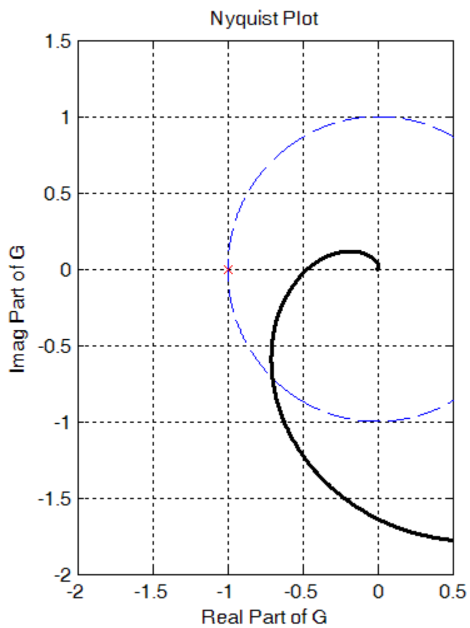
    A. $\tau_{Lx} > \tau_{BW}$
    B. $\tau_{Lx} < \tau_{BW}$
    C. $K_X < gK$
    D. $K_X > gK$

19. Which of the following is LEAST CORRECT concerning "Tustin's Method"?

    A. It is a useful way to convert a continuous-time transfer function into a discrete-time transfer function.
    B. It preserves the exact discrete-time stability boundary.
    C. The resulting discrete-time frequencies *exactly* match the corresponding continuous-time frequencies.
    D. It can be derived by considering trapezoidal integration as an approximation to "1/s".

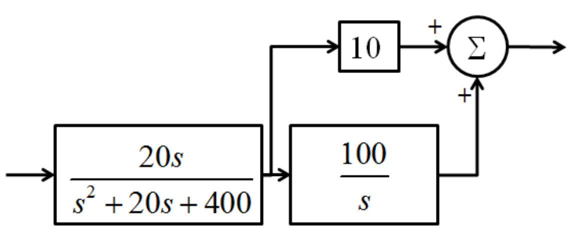
[1] Kashawlic, *et al.*, "MH-47G DAFCS Helicopter Aerial Refueling Control Laws," *AHS Forum*, 2011.





20. The Nyquist plot shown above corresponds to the open-loop stable plant G(s). If the plant is used with proportional feedback as shown in the block diagram above, for what value of K will this system be neutrally stable?

    A. ...the closed-loop would be unstable with K=2.
    B. ...the system would have about 90 deg of phase margin with K ~ 0.6.
    C. ...the bode plot of G(s) would have a phase of -270 deg at very high frequencies.
    D. ...all of the above.



21. The figure above shows a compensator adapted from a wind turbine drivetrain feedback system[2]. The compensator shown appears to be...

    A. ...a bandpass filter with PI compensation.
    B. ...a notch filter with PI compensation.
    C. ...a low-pass filter with PI compensation.
    D. ...an LQR full-state feedback controller.

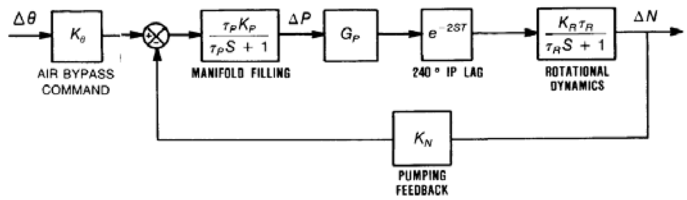
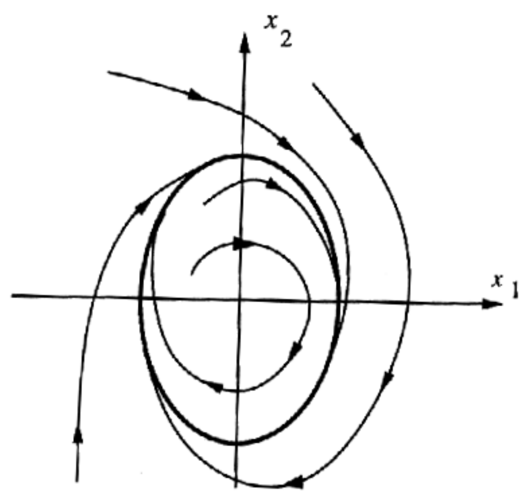


Fig. 2. Block diagram of linearized engine model.

22. The figure above[3] shows a simplified model of an internal combustion engine. The unlabelled block in the center, G_P, contains no dynamics--it is simply the "delayed pressure torque gain." Which of the following is MOST ACCURATE about this system?

    A. The "manifold filling" is a notch filter.
    B. The block diagram represents a non-linear system because of the e^{-2sT} block describing the "induction power stroke delay."
    C. Assuming all of the parameters in the block diagram have positive values, the closed-loop system will be stable for sufficiently small values of G_P and unstable for sufficiently large values of G_P.
    D. All of the above.



23. The figure above shows...

    A. The phase portrait of a 2-state nonlinear dynamic system.
    B. An unstable equilibrium at the origin.
    C. A stable limit cycle.
    D. All of the above.

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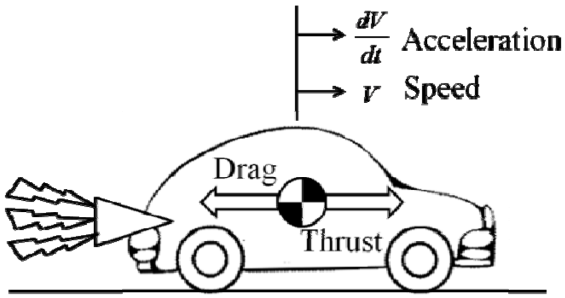
[2] Mandic, et al, "Mechanical Stress Reduction in Variable Speed Wind Turbine Drivetrains", IEEE 978-1-4577-0541-0/11.

[3] Cook & Powell, "Modeling of an IC Engine for Control Analysis," *IEEE Control Systems Magazine*, 1988.



The questions on this page pertain to a cruise control design for the rocket-propelled car shown above. The mass of the car is m=400kg. The thrust in Newtons is 800*u, where u=throttle (control). The drag in Newtons is $2V^2$, when V is measured in meters per second.

24. Writing Newton's second law (F=ma) for this car results in which of the following ODEs?

    A. $800\frac{dV}{dt} = V^2 + 400u$

    B. $400\frac{dV}{dt} = -2V^2 + 800u$

    C. $400\frac{dV}{dt} = -800V^2 + 2u$

    D. None of the above.

25. If the trim speed of the car, V_o, is to be 20 m/s, what is the corresponding trim control?

    A. $u_o = 1$

    B. $u_o = 800$

    C. $u_o = 0$

    D. None of the above.

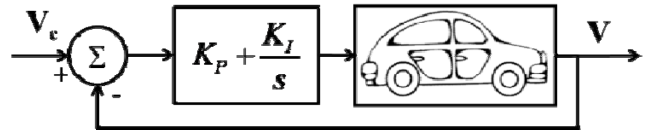
26. The transfer function for small changes in speed and small changes in control, relative to trim, is

    A. $\frac{\Delta V(s)}{\Delta U(s)} = \frac{800}{s+400}$

    B. $\frac{\Delta V(s)}{\Delta U(s)} = \frac{800}{400s+2}$

    C. $\frac{\Delta V(s)}{\Delta U(s)} = \frac{2}{s+0.2}$

    D. None of the above.



27. Which of the following corresponds to the closed-loop transfer function for the system shown above?

    A. $\frac{V(s)}{V_C(s)} = \frac{2K_P s + 2K_I}{s(s+0.2) + 2K_P s + 2K_I}$

    B. $\frac{V(s)}{V_C(s)} = \frac{K_I}{s(s+0.2) + K_I}$

    C. $\frac{V(s)}{V_C(s)} = \frac{2K_P s + 2K_I}{(s+0.2) + 2K_P s + 2K_I}$

    D. $\frac{V(s)}{V_C(s)} = \frac{2K_P(s+0.2)}{s(s+0.2) + 2K_I}$

28. Which of the following is CORRECT concerning the steady velocity response to a steady force disturbance for the controller shown above?

    A. The steady response will be non-zero because the plant is first-order.
    B. The steady response will be zero because of the integral feedback.
    C. The steady response will be unbounded because the closed-loop system is unstable.
    D. None of the above.

29. Which of the following pole locations most closely corresponds to requirements of settling time equal to 5 seconds and natural frequency equal to 2 rps?

    A. $-0.5 \pm 0.5j$

    B. $-1.5 \pm 3j$

    C. $-0.9 \pm 1.8j$

    D. $5 \pm 2j$

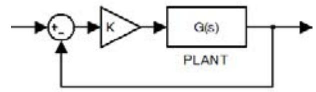
30. Which of the following gain settings most closely achieves the desired closed-loop poles from the previous problem?

    A. $K_P = 0.8$ & $K_I = 2.0$

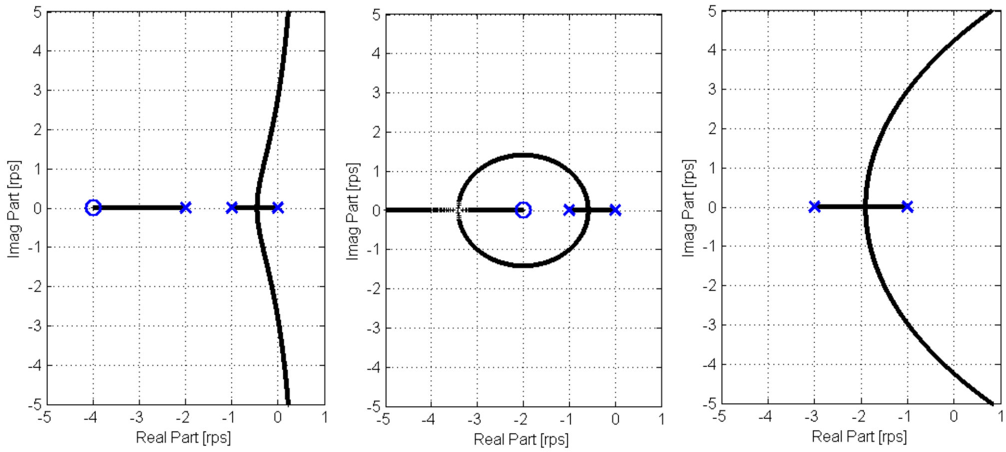
    B. $K_P = 2.0$ & $K_I = 0.1$

    C. $K_P = 0.4$ & $K_I = 4.5$

    D. $K_P = 0$ & $K_I = 1.0$

The questions on this page refer to the root loci for three different systems of the form 

A B C



31. In which case is the closed-loop system stable for all positive values of the gain K?

    A. System A
    B. System B
    C. System C
    D. All of the Above

32. In which case would oscillations with a period of just over 2 seconds correspond to neutral stability?

    A. System A
    B. System B
    C. System C
    D. All of the Above

33. Which system would have an infinite gain margin, regardless of the value of K?

    A. System A
    B. System B
    C. System C
    D. None of the Above

34. Which system would have an infinite phase margin, regardless of the value of K?

    A. System A
    B. System B
    C. System C
    D. None of the Above

35. For the system C, there are poles and/or zeros which are not visible in the region of the locus shown above. Which “missing” (not shown on the locus) elements of G(s) might reasonably be expected to explain the general shape (topology) of the locus?

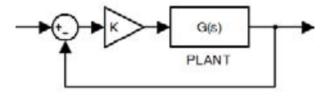
    A. G(s) having pure time delay, represented by a Pade approximation on the locus.
    B. G(s) having a zero in the right-half plane
    C. G(s) having an additional pole in the left-half plane.
    D. All of the above are reasonable guesses at the missing elements.

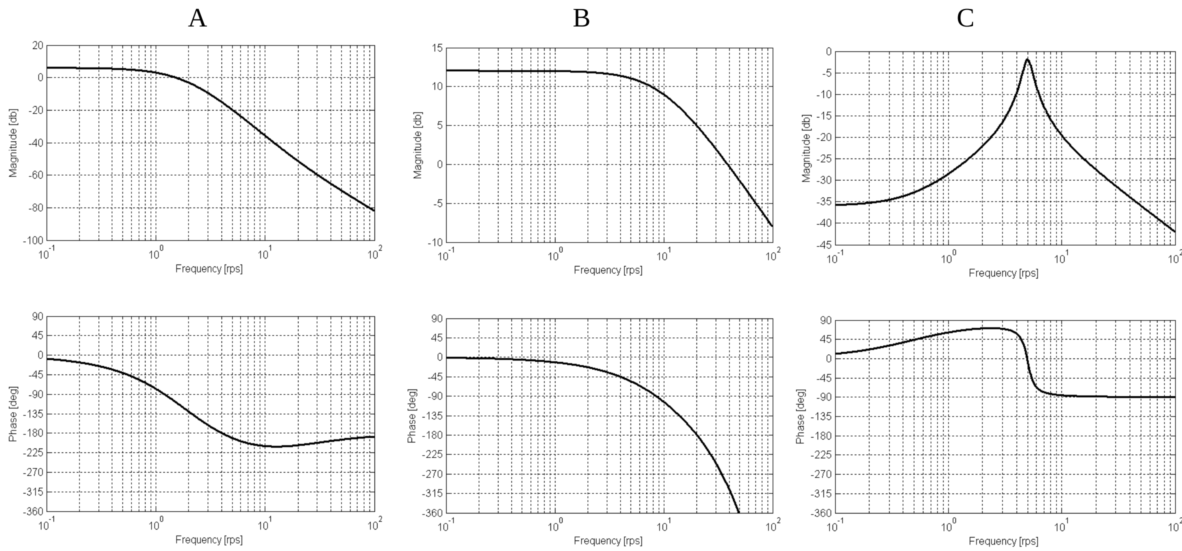
36. Assuming a gain is chosen for each system to yield stable closed loops, in which case will the closed-loop system NOT have zero steady-state error for a step input?

    A. System A
    B. System B
    C. System C
    D. All of the Above

Sikorsky's S-92 Fly-by-Wire Test Aircraft
(No Question, Just a Cool Photo)

The questions on this page consider the consequences of placing a plant in a unity-gain feedback system with proportional compensation, as shown at right. For all three of the plants whose frequency responses are shown below, all of the finite poles and zeros are in the left-half plane and are within the range of frequencies shown in the bode plots. Any time delay is shown exactly. Note that the frequency and phase scales are fixed, but the magnitude scales vary, to show more details.





37. With K=1, what is a reasonable estimate of the gain margin of System A?
    - A. About 15 dB
    - B. About 6 dB
    - C. About -6 dB
    - D. None of the above.

38. With K=1, what is a reasonable estimate of the phase margin of System A?
    - A. About 135 degrees
    - B. About 80 degrees
    - C. About 45 degrees
    - D. None of the above

39. If a positive value of K were chosen for each system to ensure closed-loop stability, which system would have zero steady-state error for a unit step input?
    - A. System A
    - B. System B
    - C. System C
    - D. None of the Above

40. Which system will be closed-loop stable for any positive value of K?
    - A. System A
    - B. System B
    - C. System C
    - D. None of the Above

41. Which system will be closed-loop stable when K=1 with an arbitrary additional time delay?
    - A. System A
    - B. System B
    - C. System C
    - D. None of the Above

42. Which system will be closed-loop *unstable* for K=1?
    - A. System A
    - B. System B
    - C. System C
    - D. None of the Above

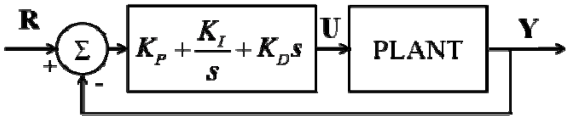
43. Which system will be closed-loop stable but have poor steady-state tracking with K=1?
    - A. System A
    - B. System B
    - C. System C
    - D. All of the Above

44. If proportional feedback were *replaced* by derivative feedback on System B, what would be the derivative gain for neutral stability?
    - A. About 2.5
    - B. About 0.25
    - C. About 0.025
    - D. About 0.0025

For all of the problems in this column, the plant is described by the following differential equation:

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} + 4y = 2u$$

A PID controller is to be designed using unity feedback, as shown below:



45. Which of the following is MOST ACCURATE about the effects of derivative feedback with K_I=0?

    A. Derivative feedback is required to ensure stability.
    B. Derivative feedback will decrease the natural frequency of the closed-loop poles.
    C. Derivative feedback will decrease the damping ratio of the closed-loop poles.
    D. All of the above.

46. Suppose the controller gains are chosen to ensure a stable closed-loop system. Which of the following is MOST ACCURATE about the effects of proportional feedback with K_I=0?

    A. Proportional feedback is required to ensure stability.
    B. Proportional feedback will increase the natural frequency of the closed-loop poles.
    C. Proportional feedback will increase the damping ratio of the closed-loop poles.
    D. All of the above.

47. Suppose the proportional and derivative gains are chosen to ensure a stable closed-loop system with K_I=0. Which of the following is MOST ACCURATE about adding non-zero integral feedback?

    A. It is required to ensure zero steady-state tracking error.
    B. For fixed values of the other gains, increasing integral feedback gain will decrease the damping ratio of the closed-loop poles.
    C. If there is a possibility of control saturation, it would typically be important to implement some sort of anti-windup protection for the integral feedback.
    D. All of the above.

48. Suppose we decided to drop the PID control of the previous three problems and attempt to use modern control. If we wanted to represent the plant using state-space, with $x_1 = \frac{dy}{dt}$ and $x_2 = y$, which of the following system matrices would be correct?

    A. $A = \begin{bmatrix} -4 & 2 \\ 1 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
    B. $A = \begin{bmatrix} 4 & 1 \\ -1 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$
    C. $A = \begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$ & $B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$
    D. None of the above.

49. Which of the following full-state feedback matrices, with $u = -K\underline{x}$, would result in closed-loop poles with damping ratio of 0.5 and natural frequency of 3 rps?

    A. $K = \begin{bmatrix} 0.5 & 3 \end{bmatrix}$
    B. $K = \begin{bmatrix} 2 & 2.5 \end{bmatrix}$
    C. $K = \begin{bmatrix} 0 \\ 2.5 \end{bmatrix}$
    D. None of the above.

50. Instead of choosing the state feedback matrix, K, to place the closed-loop poles at desired locations, as we did in the last problem, we can choose K using LQR design. Which of the following is LEAST ACCURATE concerning LQR design?

    A. "LQR" stands for linear quadratic regulator.
    B. LQR is based on minimizing a weighted scalar combination of state and control activity.
    C. LQR is a simple replacement for just about everything we learned this semester and can be usefully applied by blinding "turning the crank" on a given problem.
    D. LQR has excellent stability robustness properties when the full state is available by direct measurement.