ESE 406 Final Exam Cheat Sheet

Function	Time domain $f(t) = \mathcal{L}^{-1}\{F(s)\}$	Laplace s-domain $F(s) = \mathcal{L}\{f(t)\}$
nth power (for integer n)	$\dfrac{t^n}{n!} \cdot u(t)$	$\dfrac{1}{s^{n+1}}$
qth power (for complex q)	$\dfrac{t^q}{\Gamma(q+1)} \cdot u(t)$	$\dfrac{1}{s^{q+1}}$
unit step	$u(t)$	$\dfrac{1}{s}$
delayed unit step	$u(t-\tau)$	$\dfrac{e^{-\tau s}}{s}$
ramp	$t \cdot u(t)$	$\dfrac{1}{s^2}$
nth power with frequency shift	$\dfrac{t^n}{n!} e^{-\alpha t} \cdot u(t)$	$\dfrac{1}{(s+\alpha)^{n+1}}$
exponential decay	$e^{-\alpha t} \cdot u(t)$	$\dfrac{1}{s+\alpha}$
exponential approach	$(1 - e^{-\alpha t}) \cdot u(t)$	$\dfrac{\alpha}{s(s+\alpha)}$
sine	$\sin(\omega t) \cdot u(t)$	$\dfrac{\omega}{s^2+\omega^2}$
cosine	$\cos(\omega t) \cdot u(t)$	$\dfrac{s}{s^2+\omega^2}$
hyperbolic sine	$\sinh(\alpha t) \cdot u(t)$	$\dfrac{\alpha}{s^2-\alpha^2}$
hyperbolic cosine	$\cosh(\alpha t) \cdot u(t)$	$\dfrac{s}{s^2-\alpha^2}$
Exponentially-decaying sine wave	$e^{-\alpha t} \sin(\omega t) \cdot u(t)$	$\dfrac{\omega}{(s+\alpha)^2+\omega^2}$
Exponentially-decaying cosine wave	$e^{-\alpha t} \cos(\omega t) \cdot u(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega^2}$

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt.$$

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=0}^\infty x[n] z^{-n}.$$

$$t_r = \frac{1.8}{\omega_n}$$

$$t_s = \frac{4.6}{\sigma}$$

TABLE 8-1
Short Table of z-Transforms

Transform Pair Number	Continuous-time Function $f(t)$ for $t > 0$	Sample Values $f(nT)$ for $n \geq 0$	z-Transform of $f(nT)$
1.	—	$f(nT) = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases} \triangleq \delta(n)$	1
2.	1 (unit step)	1	$\dfrac{1}{1 - z^{-1}}$
3.	$e^{-\alpha t}$	$e^{-\alpha nT} = (e^{-\alpha T})^n = K^n$	$\dfrac{1}{1 - e^{-\alpha T}z^{-1}} = \dfrac{1}{1 - Kz^{-1}}$
4.	t	nT	$\dfrac{Tz^{-1}}{(1 - z^{-1})^2}$
5.[†]	$te^{-\alpha t}$	$nTe^{-\alpha nT}$	$\dfrac{Te^{-\alpha T}z^{-1}}{(1 - e^{-\alpha T}z^{-1})^2}$
6.[†]	$\sin bt$	$\sin bnT$	$\dfrac{(\sin bT)z^{-1}}{1 - 2(\cos bT)z^{-1} + z^{-2}}$
7.[†]	$\cos bt$	$\cos bnT$	$\dfrac{1 - (\cos bT)z^{-1}}{1 - 2(\cos bT)z^{-1} + z^{-2}}$
8.[†]	$e^{-\alpha t} \sin bt$	$e^{-\alpha nT} \sin bnT$	$\dfrac{e^{-\alpha T}(\sin bT)z^{-1}}{1 - 2e^{-\alpha T}(\cos bT)z^{-1} + e^{-2\alpha T}z^{-2}}$
9.[†]	$e^{-\alpha t} \cos bt$	$e^{-\alpha nT} \cos bnT$	$\dfrac{1 - e^{-\alpha T}(\cos bT)z^{-1}}{1 - 2e^{-\alpha T}(\cos bT)z^{-1} + e^{-2\alpha T}z^{-2}}$

[†]In transforms 5 through 9, $e^{-\alpha T}$ and bT can be replaced by constants, K_1 and K_2, respectively, as was done in transform 3. Convergence of the z-transform requires $|K_1| < 1$.

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