MEAM 620 Homework 1

Due: Monday, January 26, 11:59pm

1. Show (or disprove) that the A^* algorithm reduces (is equivalent in terms of computations) to: (a) Dijkstra's algorithm if $h = 0$; and (b) the depth-first search algorithm if h is the depth of the graph.

2. (a) Show that the set of rotation matrices forms a group. (b) Is the set of orthogonal matrices a group? Explain.

3. Write down the appropriate number of independent constraints on the 9 elements of a rotation matrix:

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

4. For a rotation matrix as defined in Problem 3, prove that:

$$R_{11} = \begin{vmatrix} R_{22} & R_{23} \\ R_{32} & R_{33} \end{vmatrix}$$

5. Consider a 2×2 rotation matrix which takes the form

$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix},$$

and another fixed but arbitrarily chosen 2×2 rotation matrix $\mathbf{S}$. Show that $\mathbf{R}^{-1}$ is a continuous function of $\mathbf{R}$ and $\mathbf{RS}$ is a continuous function of $\mathbf{R}$.

6. Starting from the second phase of project 1, you will need to model the dynamics of a quadrotor. The equations of motion (EOMs) are dependent on the orientation of the quadrotor. It is convenient to use a three-dimensional parameterization for the rotation. We will use use Euler angles. However, the direct use of Euler angles is not preferred when deriving the EOMs because they have singularities. Instead, we use rotation matrices. There are several ways to define the Euler angles. In our convention, we rotate along $Z - X - Y$ axes respectively to move from world frame to the body frame. In other words, first rotation is along the world-Z axis; then along X and the Y axes respectively. Find the rotation matrix which corresponds to $(yaw, roll, pitch) = (\psi, \phi, \theta) = (30, 15, 5)$ degrees of rotation along the axes $Z - X - Y$ respectively.