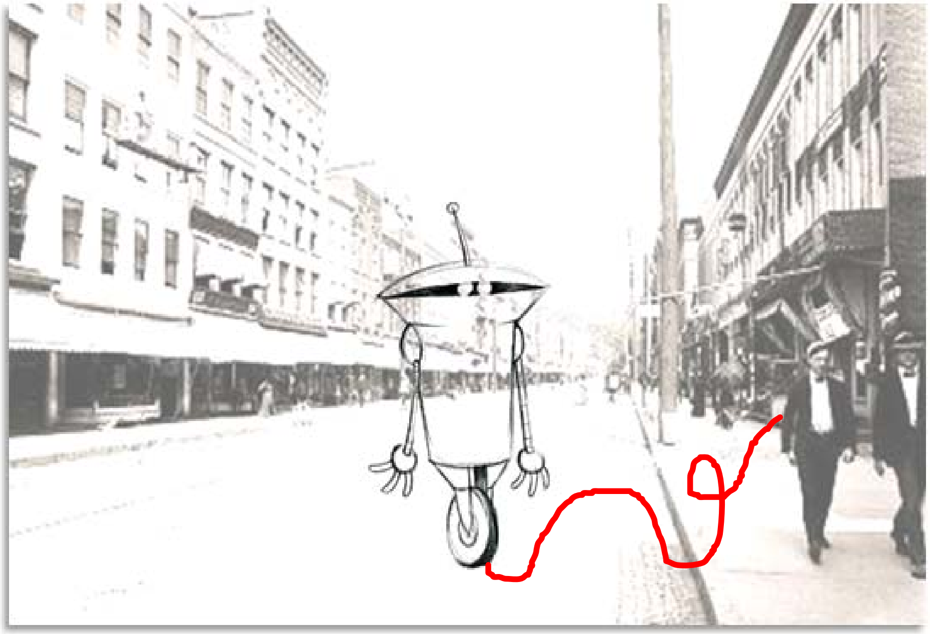
Lecture 19: Motion planning (6)
Probabilistic Roadmaps (PRMs)



Topics:

- Probabilistic roadmaps
  - Sampling strategies
  - Neighbors
- Visibility-based PRMs

Reading:

- Choset: 7
- LaValle: 5

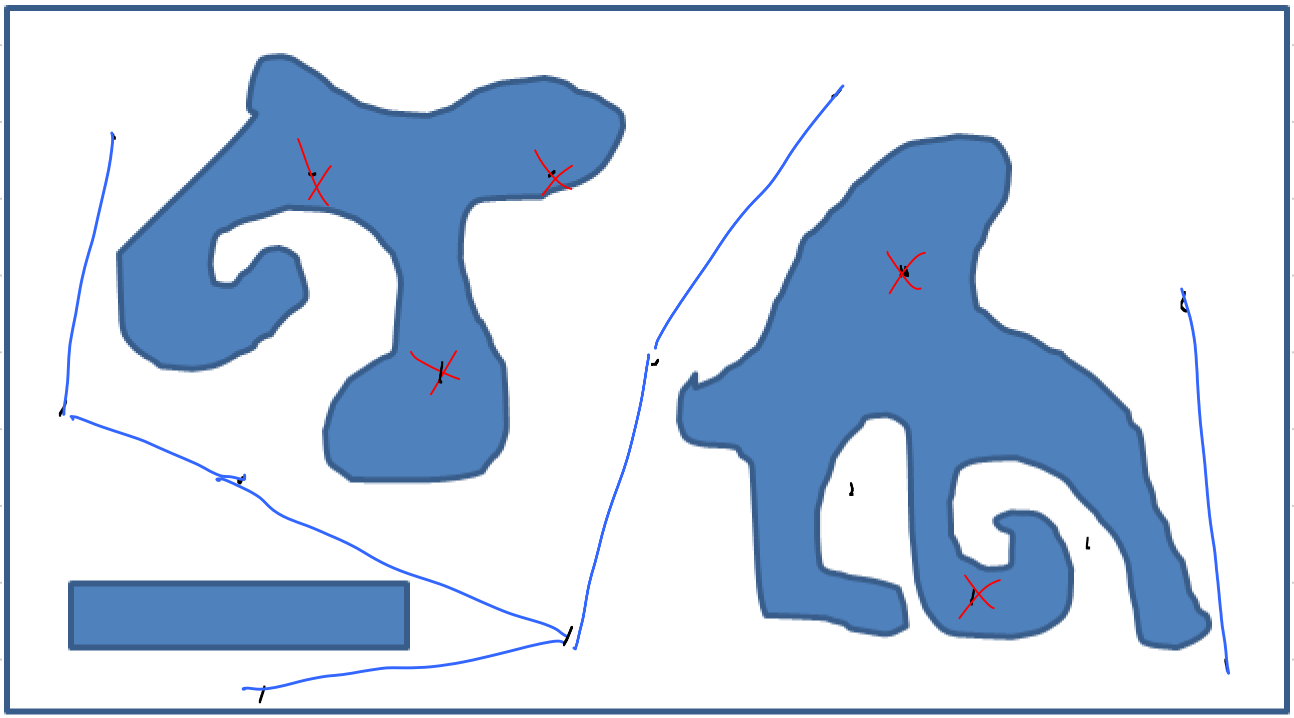
Motion planning

Given: q_{goal}, $q_{1:t}$

sometimes: map, q_0

Find: $u_{1:t}$ s.t. $q_t = q_{goal}$

Basic idea: Sample → check → connect



probabilistic / Resolution complete

Building a PRM

given: ability to check whether $q \in Q_{free}$, n

find: PRM: $G = (V, E)$

Init: $V = \emptyset$ $E = \emptyset$

while $|V| < n$

$\boxed{\text{sample}}$ $q \in Q$

if $q \in Q_{free}$ then $V = V \cup \{q\}$

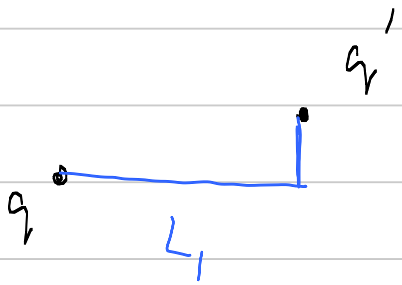
$\forall q \in V$

N_q = set of $\boxed{\text{neighbors}}$ of q

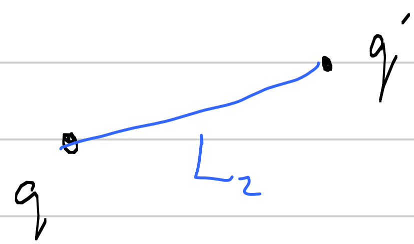
$\forall q' \in N_q$: if $\boxed{(q, q') \in Q_{free}}$ and $(q, q') \notin E$

then $E = E \cup \{(q, q')\}$

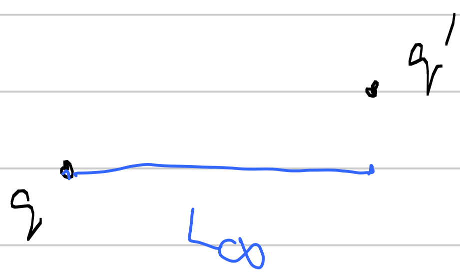
L_p norm: $\rho(q, q') = \left(\sum_{i=1}^{n} |q_i - q_i'|^p \right)^{1/p}$



$L_1 = \sum_{i=1}^{n} |q_i - q_i'| =$ manhattan distance



$L_2 =$ Euclidean distance



$L_\infty = \max_i \{ |q_i - q_i'| \}$

Sampling

random

deterministic

* random : given PDF over Q

Example: Uniform $\quad \rho(q) = \dfrac{1}{\text{volume of } Q}$

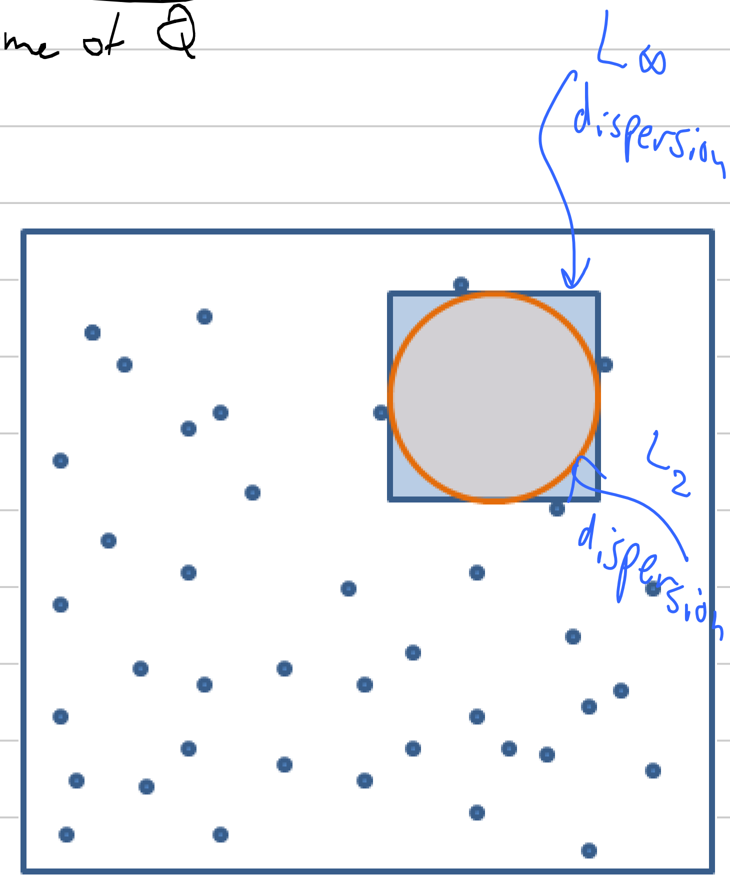
* Deterministic

Dispersion: for P samples in Q

$$\delta(P) = \sup_{q \in Q} \left\{ \min_{p \in P} \rho(q, p) \right\}$$

largest area without samples

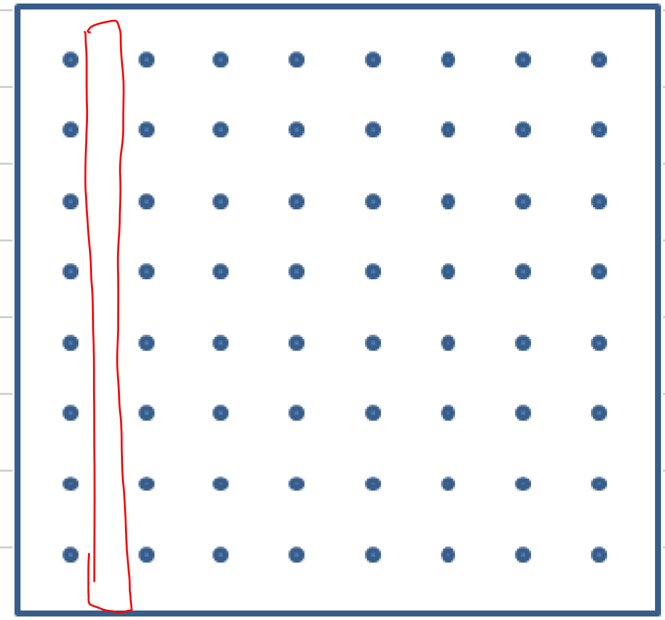
want low dispersion



- low dispersion: Grid

partition Q into P cubes, with a sample in each cube

divide each axis into $\lfloor P^{\frac{1}{2}} \rfloor$



Discrepancy: P samples $|P| = k$

R = set of all axis aligned rectangles in Q

$\mu(\cdot)$ = volume

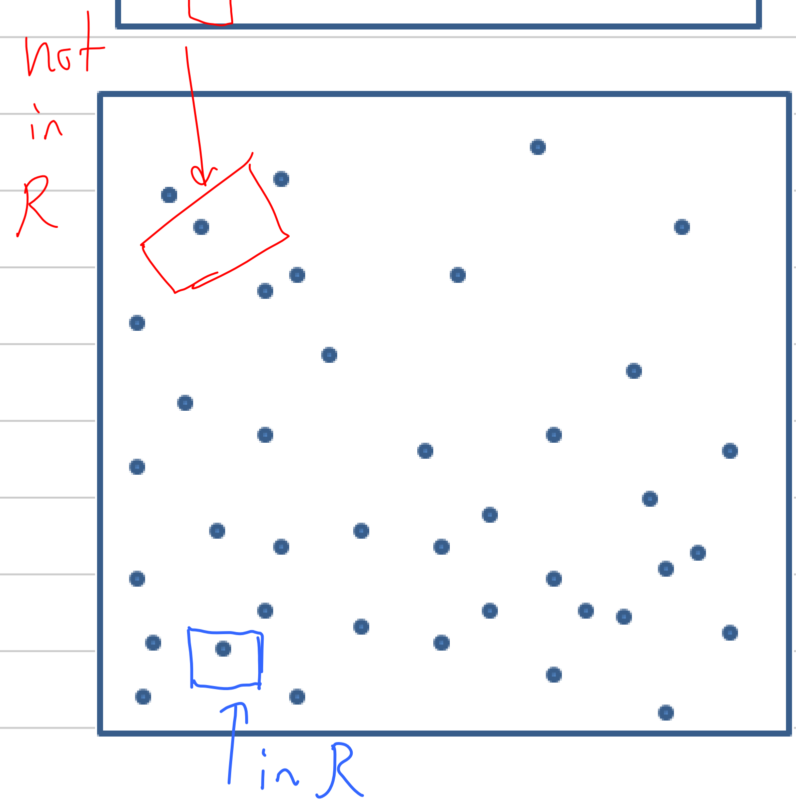
$$D(P,R) = \sup_{R \in \mathcal{R}} \left| \frac{\mu(R)}{\mu(Q)} - \frac{P \cap R}{k} \right|$$

how "uniform" the sampling is

fraction of workspace

"# of samples in R"

# of samples



want low discrepancy

- low discrepancy: Halton sequance

n-dimensions

choose n relatively prime numbers $p_1, \dots, p_n$

for sample i:

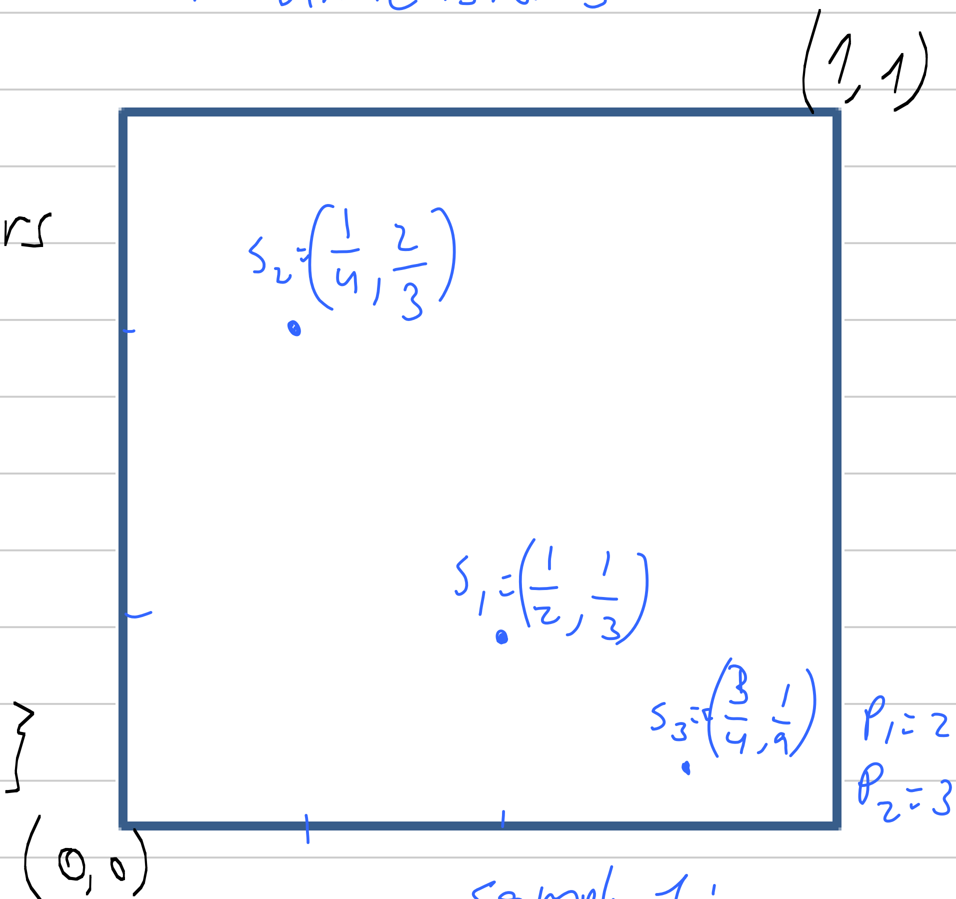
represent i in base p_m

$m = 1, \dots, n$

$$i = \sum_j a_j p_m^j \qquad a_j \in \{0, \dots, p_m - 1\}$$

$$r(i, p_m) = \sum_j a_j \, p_m^{-(j+1)}$$

$$\text{sample } i = \left(r(i, p_1), r(i, p_2), \dots, r(i, p_n) \right)$$



sample 1:

$1 = 1 \cdot 2^0 + 0 \cdot 2^1 + \dots$

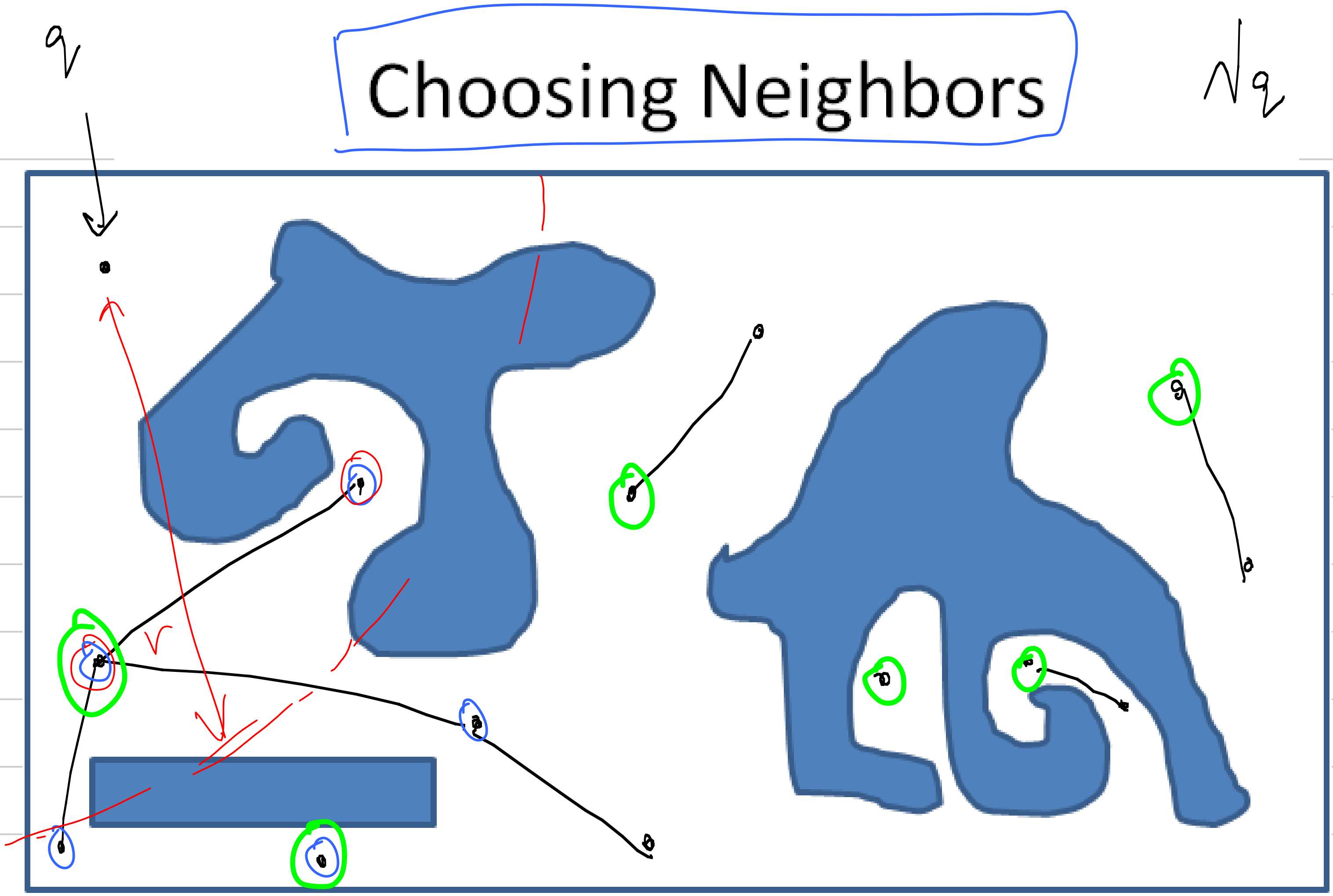
$1 = 1 \cdot 3^0 + 0 \cdot 3^1 + \dots$

$r(1, 2) = \frac{1}{2}$

$r(1, 3) = \frac{1}{3}$

- Hemmersly sequance (choset 7, 1, 3)

Choosing Neighbors



- k-nearest neighbors $k=5$
- radius r around q
- component: only unconnected components

Connecting Neighbors

$(q, q') \in E$ iff $(q, q') \in Q_{free}$

idea: fine sampling of path
(check points along path)

- Incremental

- Subdivision