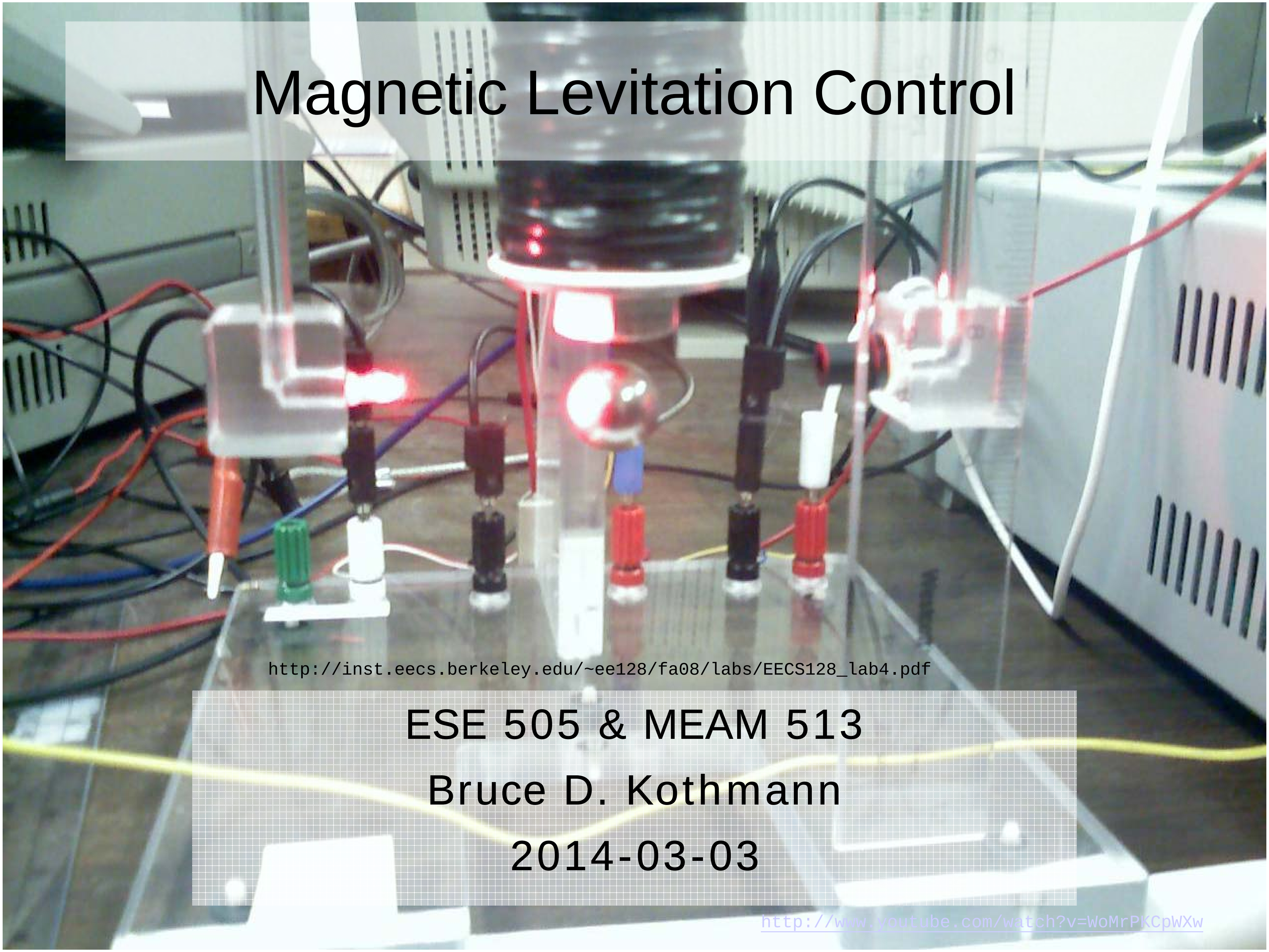


Magnetic Levitation Control

http://inst.eecs.berkeley.edu/~ee128/fa08/labs/EECS128_lab4.pdf

ESE 505 & MEAM 513

Bruce D. Kothmann

2014-03-03

http://www.youtube.com/watch?v=WoMrPKCpWXw

Magnetic Levitation Video & Requirements

http://www.youtube.com/watch?v=GVHwvMadvTg

- Academic System Requirements
  - Learn About PID Control
  - Learn About Electronics / Digital Control
  - Learn About Linearization
- Industrial System Requirements
  - Performance ~ “Small” Errors in Position
  - Disturbance Rejection ~ Zero Steady Error & “Fast” Response
  - Noise Suppression
  - Robustness
    - Performance = Errors Don’t Depend Strongly on Mass Being Levitated
    - Stability = Closed-Loop Stability Not Compromised by System Variations
- We Will Design to Usual Nominal Targets
  - Natural Frequency / Damping Ratio / Zero Steady Error

High-Level Block Diagram

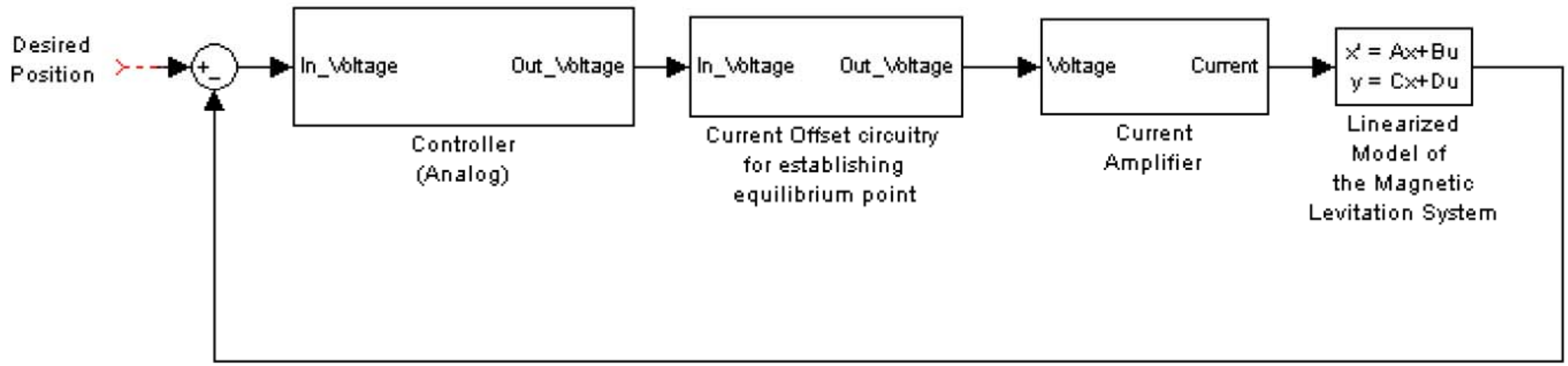


Figure 2: Block diagram of our system

Analog Implementation !

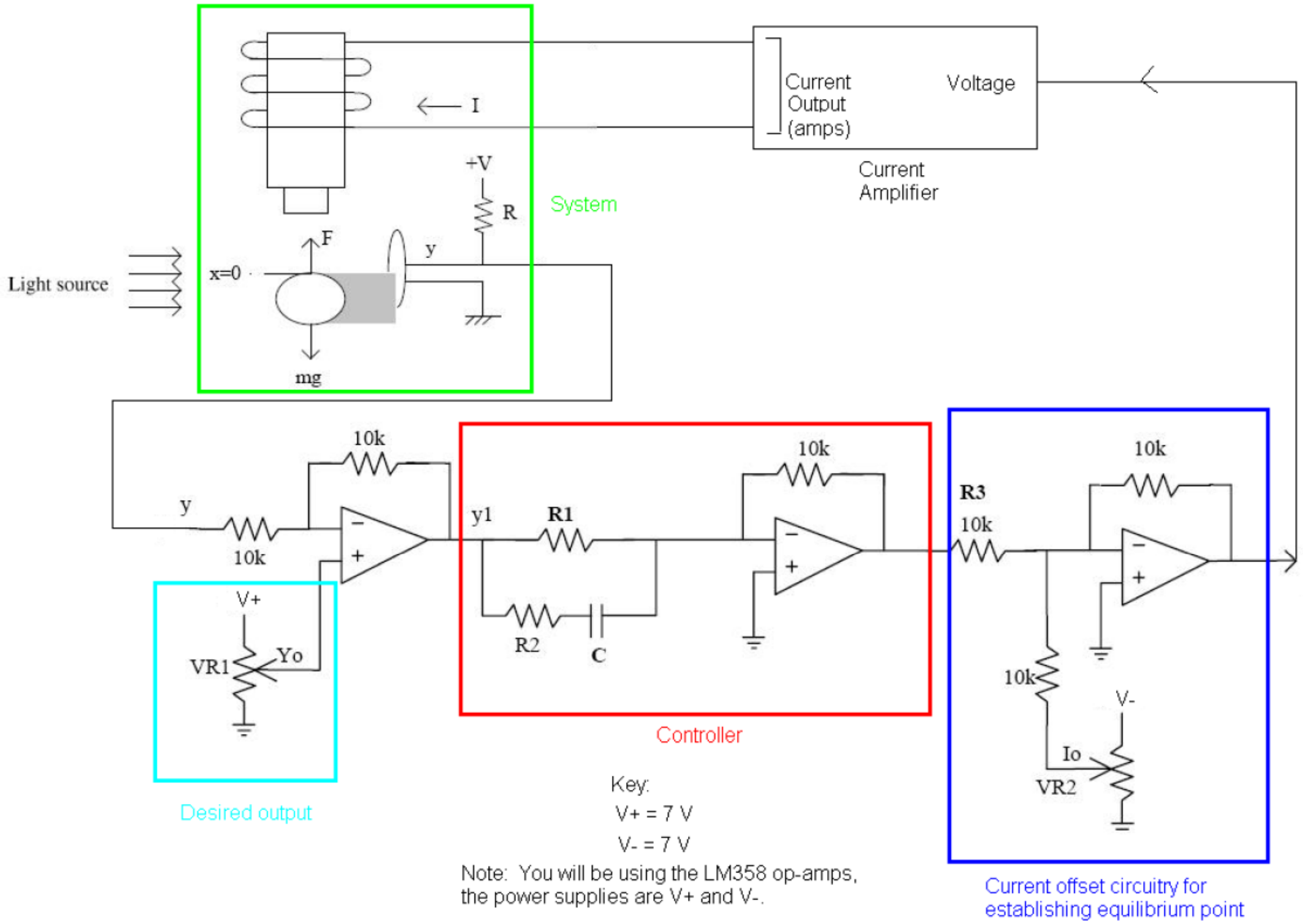
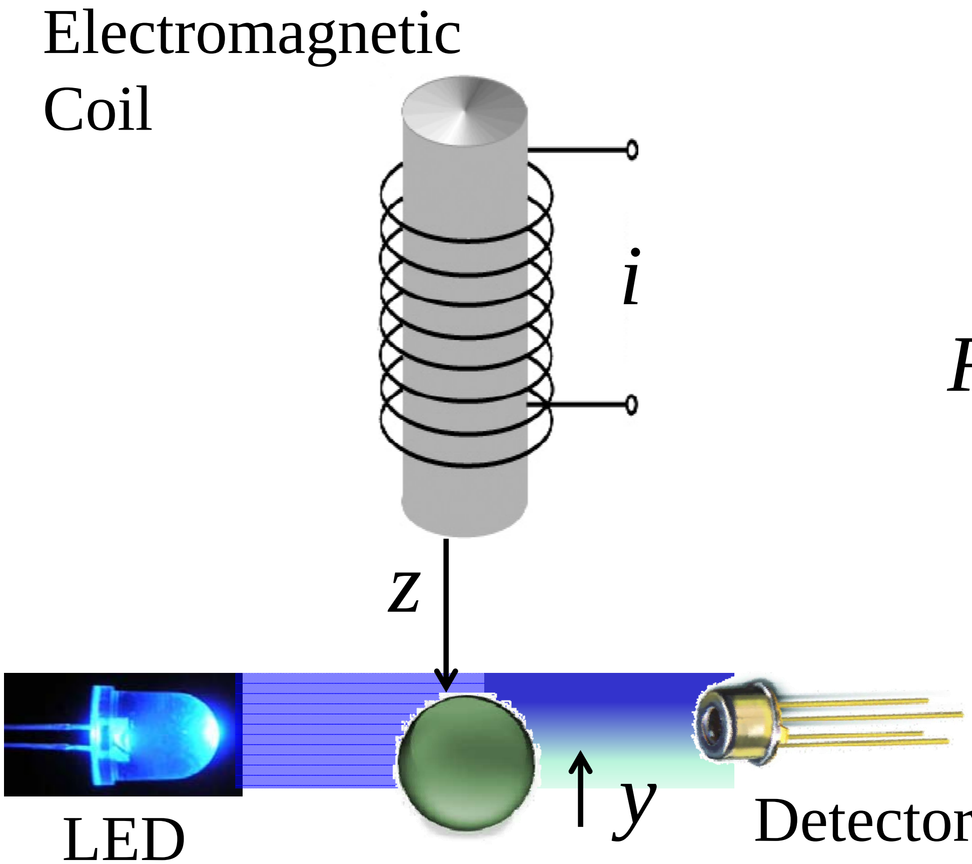


Figure 3: You will be picking values for R1, R2 and C for stable equilibrium

Equations of Motion



$$y = z_o - z$$

$$m\frac{d^2 z}{dt^2} = mg - F(z,i)$$

$$F(z,i) = \frac{\mu_o (Ni)^2 A}{4z^2} = k\frac{i^2}{z^2}$$

N = # Turns

μ_o = Permeability of Air

A = Pole Face Area

Force Model Adapted From
http://neil-mclagan.net/Maglev/NSM-CEVS-Chap-2.pdf

Model in State Space Form & Linearization

$$x_1 = z \qquad \frac{dx_1}{dt} = x_2$$

$$x_2 = \frac{dz}{dt}$$

$$u = i \qquad \frac{dx_2}{dt} = g - \frac{1}{m} F(x_1, u) = g - \frac{ku^2}{mx_1^2}$$

$$y = x_{1o} - x$$

TRIM

$$x_{2o} = 0$$

$$u_o^2 = \frac{mgx_{1o}^2}{k}$$

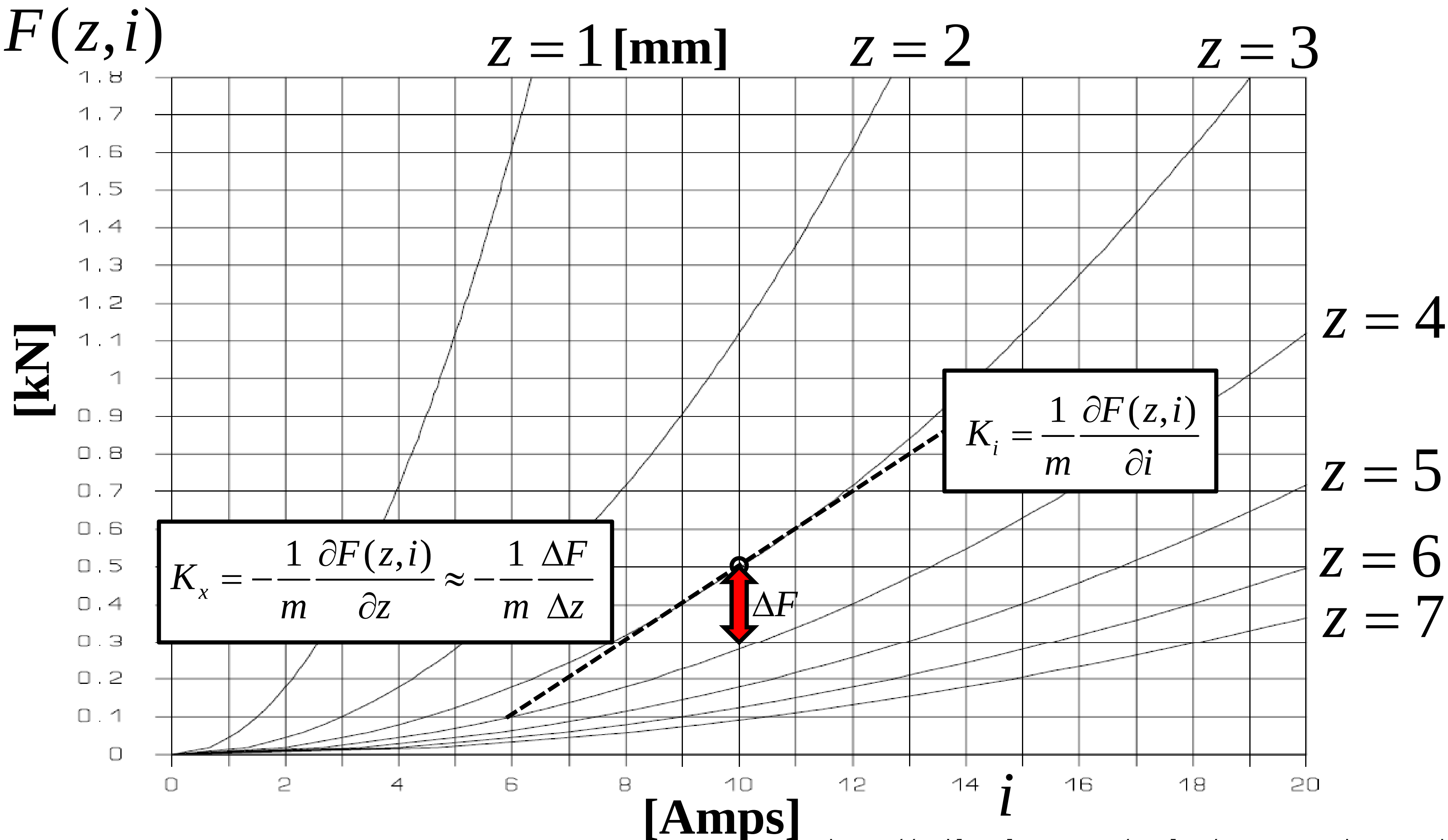
LINEARIZATION

$$[A] = \left.\begin{bmatrix} 0 & 1 \\ -\frac{1}{m}\frac{\partial F}{\partial x_1} & 0 \end{bmatrix}\right|_o = \begin{bmatrix} 0 & 1 \\ \frac{2ku_o^2}{mx_{1o}^3} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ K_x & 0 \end{bmatrix} \qquad [B] = \left.\begin{bmatrix} 0 \\ -\frac{1}{m}\frac{\partial F}{\partial u} \end{bmatrix}\right|_o = \begin{bmatrix} 0 \\ -\frac{2ku_o}{mx_{1o}^2} \end{bmatrix} = \begin{bmatrix} 0 \\ -K_i \end{bmatrix}$$

$$[C] = [-1 \quad 0] \qquad [D] = [0] \qquad K_x = -\frac{1}{m}\frac{\partial F(z,i)}{\partial z} = \frac{2ki_o^2}{mz_o^3} \qquad K_i = \frac{1}{m}\frac{\partial F(z,i)}{\partial i} = \frac{2ki_o}{mz_o^2}$$

$$\frac{Y(s)}{U(s)} = G_P(s) = C\left(sI - A\right)^{-1} B + D = \frac{K_i}{s^2 - K_x}$$

Magnetic Force (For Large-Scale Experiment)

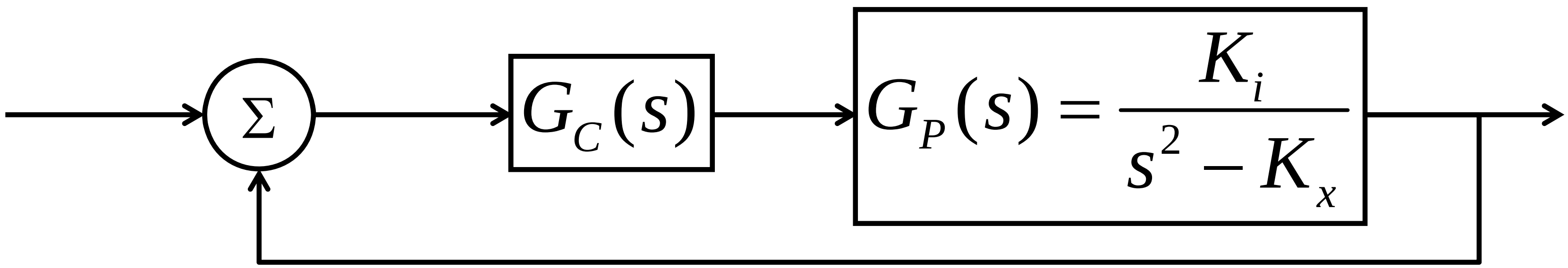


$$K_x = -\frac{1}{m}\frac{\partial F(z,i)}{\partial z} \approx -\frac{1}{m}\frac{\Delta F}{\Delta z}$$

$$K_i = \frac{1}{m}\frac{\partial F(z,i)}{\partial i}$$

http://neil-mclagan.net/Maglev/NSM-CEVS-Chap-2.pdf

Block Diagram & Compensators



- Start with Proportional Control

$$G_C(s) = K$$

- Lead Shaping = (P+D) Through Low-Pass Filter

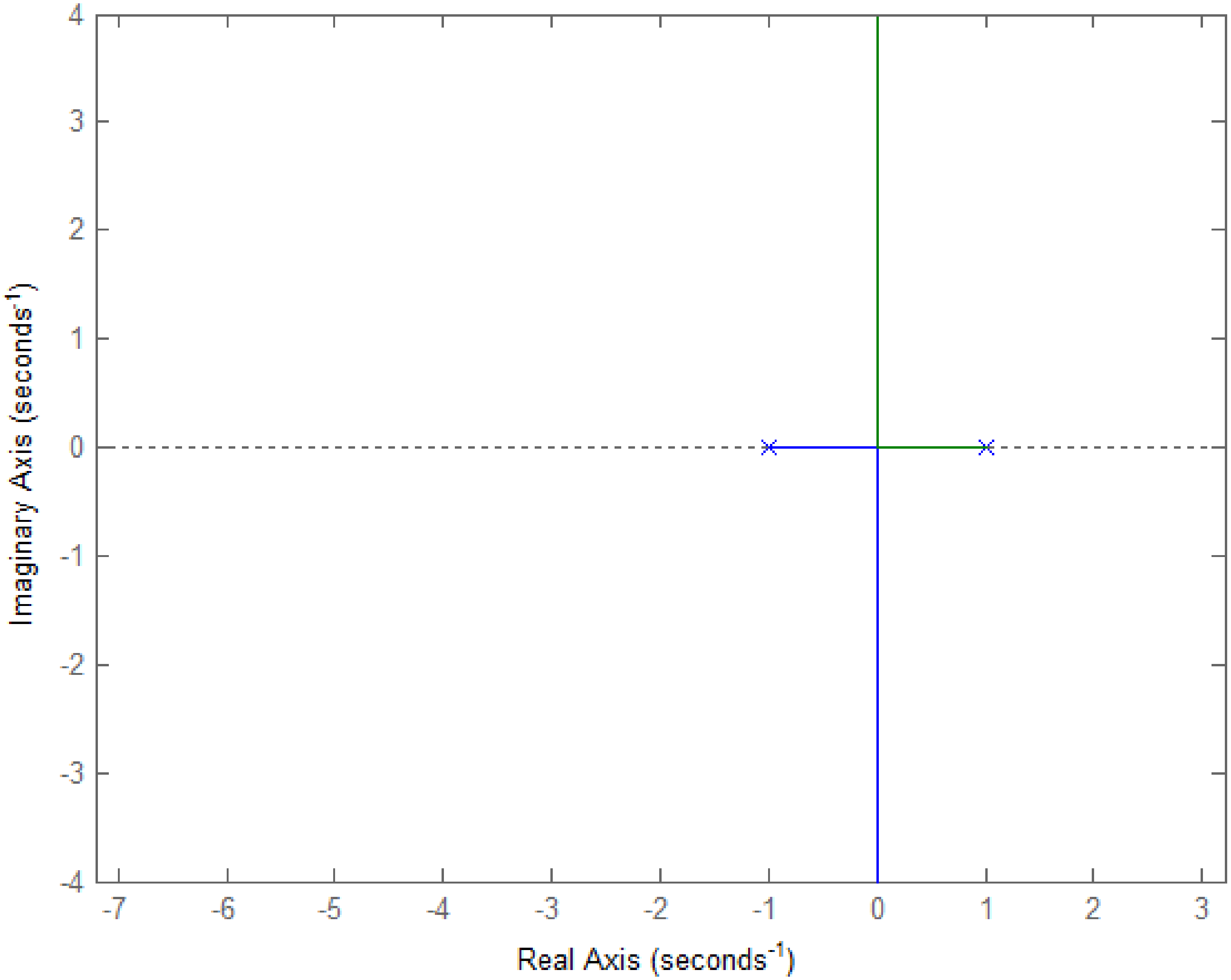
$$G_C(s) = K\frac{\tau_2 s + 1}{\tau_1 s + 1} \quad \tau_2 > \tau_1$$

- Lead-Lag = Generalization of P+I+D

$$G_C(s) = K\frac{\tau_2 s + 1}{\tau_1 s + 1}\frac{s + z}{s + p} \quad \tau_2 > \tau_1 \quad z > p$$

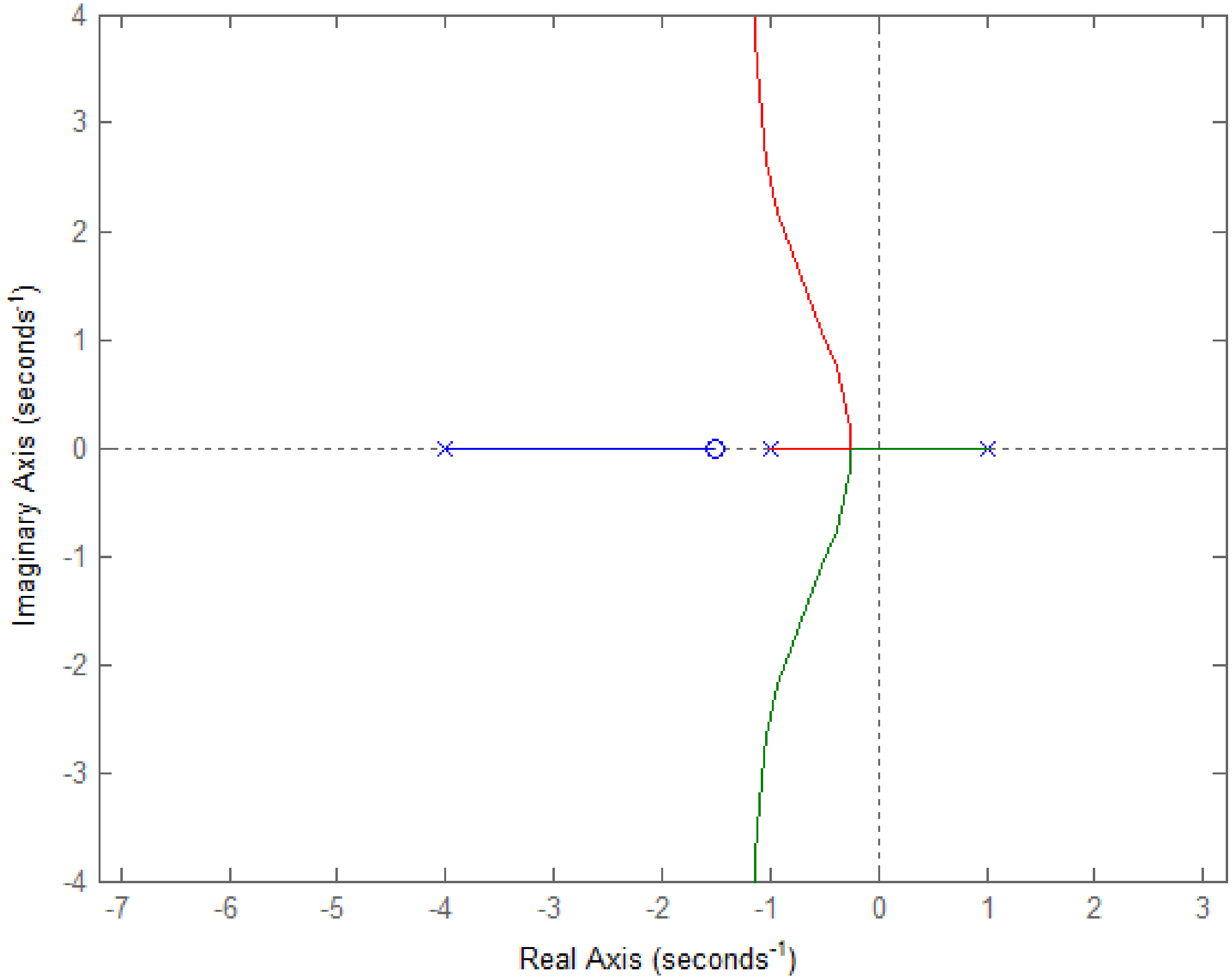
Typical Root Locus for Proportional Feedback

```
rlocus([1],[1 0 -1]);axis([-6 2 -4 4]); axis('equal'); set(gcf,'Color','w');
```



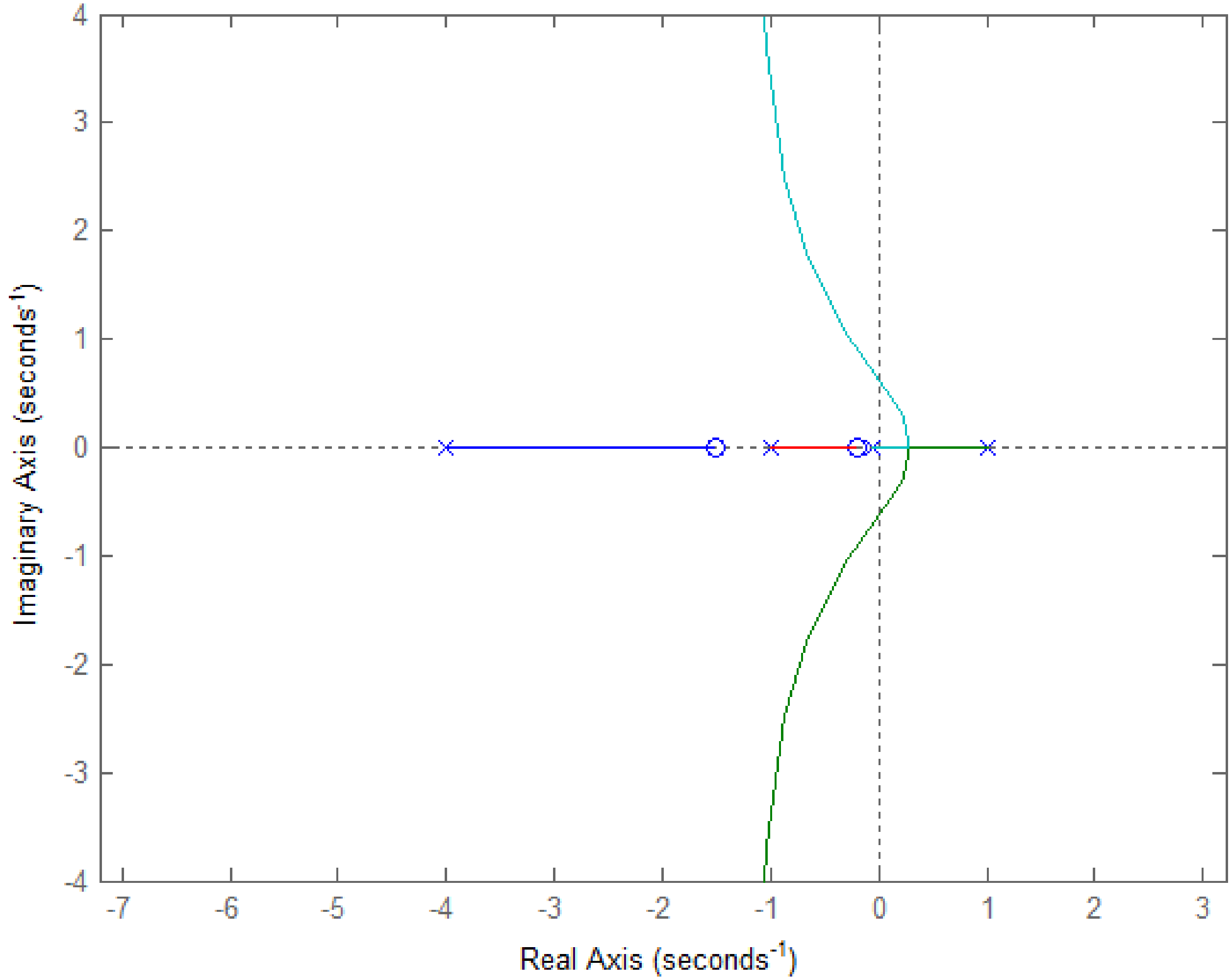
Typical Root Locus for Lead Compensation

```
rlocus([1 1.5],conv([1 0 -1],[1 4]));axis([-6 2 -4 4]); axis('equal'); set(gcf,'Color','w');
```



Typical Root Locus for Lead-Lag Compensation

```
rlocus(conv([1 1.5],[1 0.2]),conv([1 0 -1],conv([1 4],[1 0.05])));axis([-6 2 -4 4]); axis('equal'); set(gcf,'Color','w');
```



Closed-Loop Time Responses (K=10)