ESE 406 - SPRING 2012
HOMEWORK #5
DUE 15-Feb-2012 (with Late Pass 20-Feb-2012)

Problem 1 Solve problem 4.29 in the textbook.

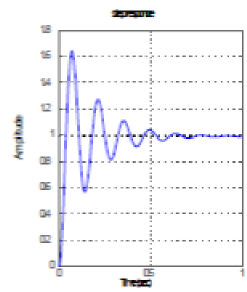
Answers: (a) $\dfrac{Y(s)}{W(s)} = \dfrac{-1500s}{s^2 + 60(1+10k_p)s + 600k_I}$; (b) $k_I = 12$, $k_p = 0.1$

Problem 2 We will work a modified version of problem 4.26 in the textbook. ***You only have to submit the 2 plots described in part (vi)***.

i. First, let's replace the amplifier (K) in the problem with a PID controller. Find the closed-loop transfer functions for $\dfrac{\Omega_M}{\Omega_R}$ and $\dfrac{\Omega_M}{W}$, where "W" is the disturbance torque.

ii. Work part (a) of the problem, using only proportional control.

Answer: $e_{ss} \le 0.01$, $k_p \ge 9.9$ pick $k_p = 10.$



iii. Using the value of proportional gain you found in part (ii), work part (b) in the textbook. That is, find the pole locations and make a graph of the response to a unit step command input. You should also make a graph showing the response to a unit step disturbance input. Look at the graphs and be sure they make sense.

Answer: $\omega_n = \sqrt{2020} \simeq 45$, $\zeta = \dfrac{12}{2\sqrt{2020}} \cong 0.13$ & figure at right.

iv. The response with only Kp is obviously too oscillatory to be of much use in this application. We need derivative feedback to get some damping. The specifications in part (c) of the textbook correspond to requiring that $\sigma > 46$ (real part left of -46 in complex plane) and $\zeta > 0.7$. Find the minimum values of Kp and Kd that will satisfy these requirements. Plot the updated unit step responses.

v. Now let's add integral feedback. Keeping the values of Kp and Kd from the previous step, choose a non-zero value of Ki such that the damping ratio of the oscillatory poles is no less than 0.5. You may do this by trial-and-error, if you like. (One way to do this is to enter the polynomial for the closed-loop denominator you found in step (i) above, and use MATLAB's "roots" command.) Make a final plot of the unit step responses.

vi. Please combine all 3 of the command step responses onto one plot and all 3 of the disturbance step responses on another plot. Put a legend on the plots that includes the values of the gains, and be sure to use different line types and colors so the plot will look nice on a black-and-white printout. (These should look like the examples in lecture). This is all you have to submit for this problem!

Problem 3 As we have discussed on several occasions in the lecture, treatment of pure time delay is an extremely important aspect of control system analysis and design. The Laplace transform of a pure time delay is

$$G_T(s) = e^{-Ts}$$

However, because our analysis tools depend on transfer functions being expressed as a ratio of two polynomials in *s*, it is often useful to approximate the time delay using a rational polynomial. If we assume that the delay, T, is small compared to the time scales of the dynamics that are of primary interest in our design, so that Ts is small, we could expand the transfer function in a Taylor series, keeping only a single term to get

$$G_T(s) \approx G_1(s) = 1 - Ts$$

Alternative, we could write the time delay as

$$G_T(s) = \frac{1}{e^{Ts}}$$

And now expanding the exponential in a Taylor series, we get

$$G_T(s) \approx G_2(s) = \frac{1}{1+Ts}$$

Yet another approach is to split the difference, writing the time delay as

$$G_T(s) = \frac{e^{-Ts/2}}{e^{Ts/2}}$$

Now again using a first-order Taylor series, this time for both the numerator and denominator gives

$$G_T(s) \approx G_3(s) = \frac{1 - \frac{Ts}{2}}{1 + \frac{Ts}{2}}$$

G_3 is called the first-order "Pade approximation" for time delay. Higher-order Pade approximations retain more terms in the numerator and denominator[1].

ESE 406 : Include the first-order Pade approximation for delay in the dynamics of the Glasshouse by replacing $\frac{A}{\tau s+1}$ that we used as the plant model in lecture with $\frac{AG_3(s)}{\tau s+1}$. Find the value of Kp that will result in neutral stability, using only proportional feedback with the architecture shown on page 7 of the notes from last week (6-Feb-2012). You should now understand why we sometimes say that proportional feedback with delay behaves a little bit like having a negative derivative gain...

ESE 505 / MEAM 513 : Do the ESE 406 problem for G1, G2, and G3. That is, find the values of Kp (if any) that result in neutral stability. Use SIMULINK to find the correct value of Kp that results in neutral stability. For "fun", you could try higher-order Pade approximations to get a feeling for how accurate they are. It is very common to use a 4th-order Pade approximation.

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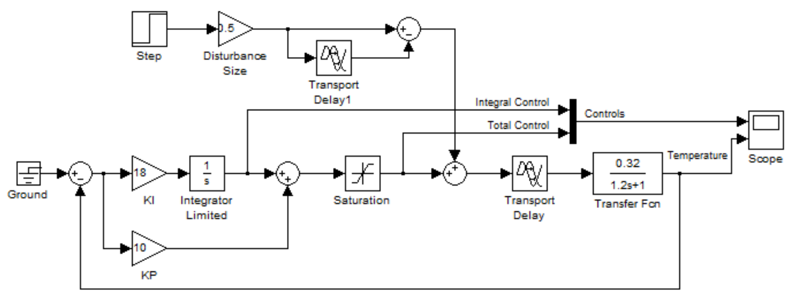
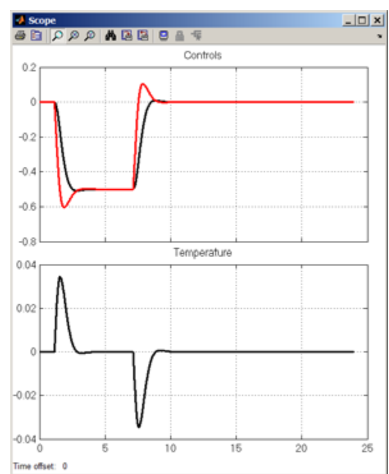
[1] Note, though, that expanding the numerator ($e^{-Ts/2}$) & denominator ($e^{Ts/2}$) in higher-order Taylor series does *not* generate the higher-order Pade approximations of time delay. To see this, compare the coefficients of the 3rd order Pade approximation, calculated with MATLAB's `pade` command, with the terms in the Taylor series. Wikipedia has an informative and concise explanation of how Pade approximants are calculated. You don't have to submit anything for this footnote.

Problem 4 We mentioned in lecture many times that "integrator windup" can be a big problem for control systems. Let's explore this phenomenon using the SIMULINK model of the Glasshouse. This model has been provided for you on Blackboard. Here is what it looks like:



In this version of the model, gains and dynamics have been converted so that time is measured in hours, for convenience. The disturbance gain at the top of the model can be adjusted to look at the effect of different amplitudes of disturbance. The "transport delay" at the top of the model allows the disturbance to be applied for 6 hours before it is turned back off.

Notice that the control is limited by the "saturation" node in the center of the model. This node limits the total control to +/- 1 unit. This is a crude representation of authority limits on our heater. (Remember that we are linearized about some nominal operating condition, so "negative heating" corresponds to downward adjustments of the heat; a symmetric limit on control is roughly like assuming we trimmed at 50% power.)

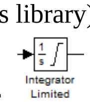


Run the simulation as initially configured, with a disturbance magnitude of 0.5. The scope output should look like the figure at right. This is what you would expect. Once the integrator has eliminated the disturbance, it is like we have reached a new trim condition. The removal of the disturbance generates a symmetric response in the opposite direction, and the integrator goes back to nominally zero control.

Now double the disturbance magnitude to 1.0 and repeat the simulation. You should see that the temperature decrease when the disturbance is removed is actually larger than the initial increase! The integrator is asking for more control than is available. This is called "integrator windup". Notice that with a disturbance of 1.0, the system is just barely able to reject the disturbance, so the integrator windup stops before the disturbance is removed.

Repeat the simulation again with a disturbance of 1.5. Now there is insufficient control to reject the disturbance and the integrator is running away until the disturbance is removed. This is a very dangerous situation, and you can see the resulting temperature change when the disturbance is removed is very large, as the integrator has to "unwind" from the large negative value it reached.

The integrator we used in this simulink model is called a "Limited Integrator" (from the Continuous library).

Double-click on the integrator and check the box next to "limit output". The icon for the integrator, 

, now shows that the integrator is limited. Now repeat the simulation with the disturbance of 1.5. Be sure you understand what is happening here. Integral control is rarely safe without careful integrator windup protection. Limiting the integral output is just one of many design schemes to deal with windup.

<u>**WHAT TO SUBMIT**</u>: Implement the architecture shown on page 18 of the glasshouse lecture notes (6-Feb-2012) with the limited integrator. Submit a screen-capture of your simulink model and the scope output disturbance of 1.5.