ESE 406 - SPRING 2013 - HOMEWORK #1
Math Review
DUE 16-Jan-2013 (Late Pass 23-Jan-2013)

"Textbook" Problems These aren't really textbook problems, but they could be.

1. [5 points each] Write each of the following in the form $a + jb$, where $j = \sqrt{-1}$. Do these by hand, not electronically (no calculator nor computer).

   a. $(1+j)(1-2j)$

   b. $\dfrac{(1+8j)}{-2j}$

   c. $\dfrac{(3+4j)}{(3-4j)}$

   Answers: (a) $3-j$, (b) $-4+0.5j$, (c) $-\dfrac{7}{25}+\dfrac{24}{25}j$

2. [5 points each] Write each of the following in the form $re^{j\theta}$, where $r > 0$ and θ is in radians. Do these by hand, not electronically (no calculator nor computer).

   a. $(-3+3j)(2+2j)$

   b. $\dfrac{(4+4j)}{-2j}$

   c. $\dfrac{(4+4j)}{(2-2j)}$

   Answers: (a) $12e^{j\pi}$, (b) $2\sqrt{2}e^{j3\pi/4}$, (c) $2e^{j\pi/2}$

3. [15 points] Solve the following ordinary differential equation and make a graph of the solution:

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+4y=0,\ y(0)=0,\ \frac{dy}{dt}(0)=3$$

   Answer: $y(t)=e^{-t}-e^{-4t}$. The graph is up to you. It is instructive to attempt to sketch graphs by hand, but please make a nice quantitative graph (graph paper or computer) to submit.

4. [20 points] Solve the following system of ordinary differential equations.

$$\frac{d\underline{x}}{dt} = \frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}\underline{x},\ \underline{x}(0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

Answer: $\underline{x}(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}e^{-t} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}e^{-4t}$. It should be obvious that this problem is the same as the previous problem, with $x_1(t) = y(t)$ and $x_2(t) = \frac{dy}{dt}$.

5. [15 points] Solve the following ordinary differential equation and make a graph of the solution:

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 169y = 0,\ y(0) = 2,\ \frac{dy}{dt}(0) = -10$$

Answer: $y(t) = 2e^{-5t}\cos(12t)$. The graph is up to you.

6. [20 points] Solve the following system of ordinary differential equations.

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -169 & -10 \end{bmatrix}\underline{x},\ \underline{x}(0) = \begin{pmatrix} 2 \\ -10 \end{pmatrix}$$

Answer: $\underline{x}(t) = \begin{pmatrix} 2 \\ -10 \end{pmatrix}e^{-5t}\cos(12t) + \begin{pmatrix} 0 \\ -24 \end{pmatrix}e^{-5t}\sin(12t)$. The most important thing here is that you see where "5" and "12" come from. Don't worry if you aren't yet totally happy about using the eigenvectors of the matrix.