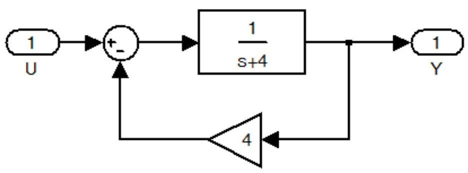
# ESE 406 - SPRING 2011 – MIDTERM EXAM : NAME _________________
# CLOSED NOTES & CLOSED BOOK

- Choose the one best answer for each question by *circling the letter*.
- A correct answer is worth 2 points.
- No answer is worth 0 points.
- An incorrect answer is worth -1 point.  Random guessing will lower your grade, on average.




1. Which of the following transfer functions corresponds to the block diagram shown above?

   A. $\dfrac{Y(s)}{U(s)} = \dfrac{4}{s+8}$

   B. $\dfrac{Y(s)}{U(s)} = \dfrac{1}{s+16}$

   C. $\dfrac{Y(s)}{U(s)} = \dfrac{1}{s+8}$

   D. None of the above.

2. Which of the following complex number is equal to $\dfrac{(1+j)}{(1-j)}$?

   A. $2+2j$

   B. $j$

   C. $-2j$

   D. None of the above.

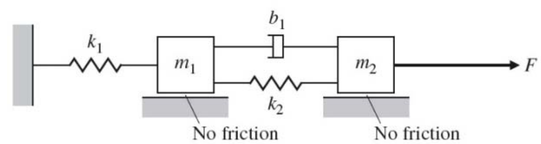
3. Which of the following is NOT correct concerning the differential equation
   $$\frac{d^2y}{dt^2} - 9y = \frac{du}{dt} - 9u$$

   A. The corresponding transfer function is $\dfrac{Y(s)}{U(s)} = \dfrac{s-9}{s^2-9}$

   B. The system described by the equation is unstable.

   C. The equation is second order.

   D. The differential equation is nonlinear.



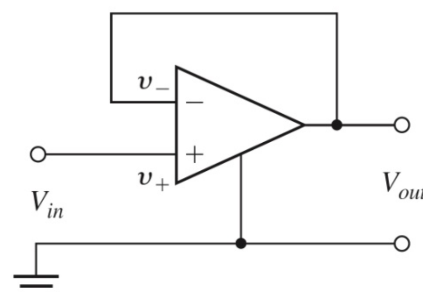

4. Which of the following is the correct force balance equation for $m_1$ in the figure above?

   A. $m_1 \dfrac{d^2x_1}{dt^2} = -k_1x_1 + k_2(x_2 - x_1) + b_1\left(\dfrac{dx_2}{dt} - \dfrac{dx_1}{dt}\right)$

   B. $m_1 \dfrac{d^2x_1}{dt^2} = -k_1x_1 - k_2x_1 - b_1\dfrac{dx_1}{dt}$

   C. $m_1 \dfrac{d^2x_1}{dt^2} = -k_1x_1 - k_2x_1 - b_1\dfrac{dx_1}{dt} + F$

   D. $m_1 \dfrac{d^2x_1}{dt^2} = -k_1x_1 + F$




5. Which of the following is correct concerning the “buffer” circuit shown above, assuming ideal op-amp characteristics?

   A. Most of the current output provided by the op-amp is “recycled” as it flows into the “-” terminal on the input side.

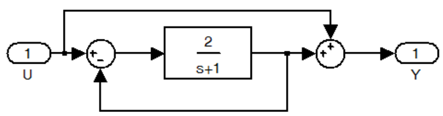
   B. The input, $V_{in}$, must be capable of supplying significant current in order for the circuit to work properly.

   C. The $V_+$ and $V_-$ voltages differ by the activation voltage of the op-amp, which is typically 1.7 volts.

   D. The circuit has a static input-output relationship, given by $V_{out} = V_{in}$.

6. Laplace transforms are very useful in control system design and analysis because…

   A. …they convert nonlinear equations into linear equations.
   B. …they convert differential equations into algebraic equations.
   C. …they convert unstable systems into stable systems.
   D. …All of the above.




7. Which of the following transfer functions corresponds to the block diagram shown above?

   A. $G(s) = \frac{s+3}{s+2}$
   B. $G(s) = \frac{s+2}{s+3}$
   C. $G(s) = \frac{s+5}{s+3}$
   D. None of the above.

8. A system is governed by the following differential equation:
   $$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = \frac{du}{dt} + 6u$$
   The transfer function for the system…:

   A. …depends on the initial conditions.
   B. …is not defined because the system is nonlinear.
   C. …is $\frac{Y(s)}{U(s)} = \frac{6s+1}{3s^2+4s+1}$
   D. …is $\frac{Y(s)}{U(s)} = \frac{s+6}{s^2+4s+3}$

9. A macro-economic model[1] for changes in the total contribution, $y(t)$, of one sector (for example, automotive industry) of the economy to total GDP can be written as:
   $$\frac{dy(t)}{dt} = [a+u(t)]\,y(t) - \frac{1}{F}[y(t)]^2$$
   where $a$ and $F$ are positive constants, and $u(t)$ is an input that depends on "the rates of change of variables shifting the sector's demand and supply equations, e.g. real income, real money, the real wage rate, the real price of capital, technological change, etc." Which of the following is the correct expression for the non-zero value of y(t) in trim?

   A. $y_o = \frac{1}{\sqrt{F}}$
   B. $y_o = (a+u_o)F$
   C. $y_o = -a-u_o$
   D. $y_o = \frac{-F}{(a+u_o)}$

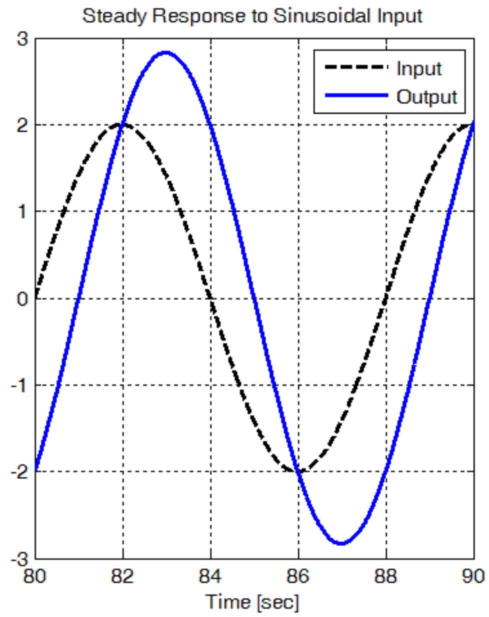
10. The linearized dynamics near the trim point of the previous problem can be written as $\frac{\Delta Y(s)}{\Delta U(s)} = \frac{K}{\tau s+1}$, with…

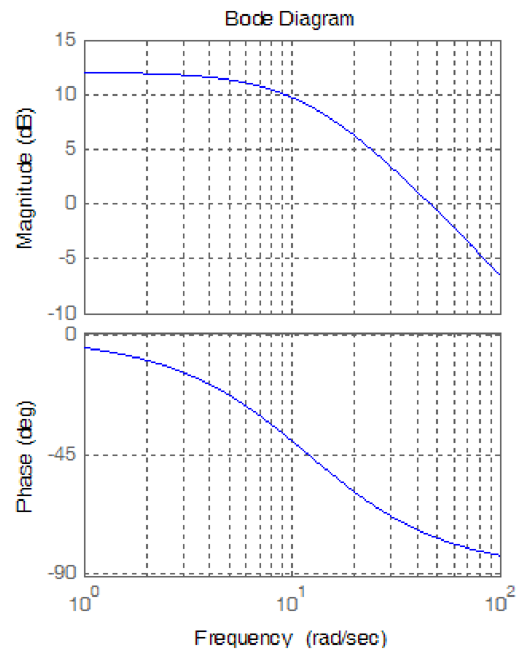
    A. …$\tau = (a+u_o)F$ & $K = 2$
    B. …$\tau = (a+u_o)^{-1}$ & $K = 2F$
    C. …$\tau = 2$ & $K = (a+u_o)$
    D. …None of the above.

11. Which of the following is NOT correct regarding the linear state-space equation $\dot{\underline{x}} = A\underline{x} + Bu$ with output $y = C\underline{x} + Du$

    A. The transfer function of the system is given by $C(sI-A)^{-1}B + D$.
    B. For our single-input, single-output systems, the D matrix is a scalar (1-by-1 matrix).
    C. The poles of the transfer function are eigenvalues of the *A* matrix.
    D. The zeros of the transfer function are the eigenvalues of the *C* matrix.

[1] Zellner, Arnold, "My Experiences with Nonlinear Dynamic Models in Economics," *Studies in Nonlinear Dynamics and Econometrics*, Volume 6, Issue 2, 2002.




12. What is the frequency of the sinusoidal input in the figure above?

    A. About 0.4 rps
    B. About 0.8 rps
    C. About 4 rps
    D. About 8 rps

13. On the bode plot of the transfer function corresponding to the figure above, what is the magnitude ( $\hat{M}$ ) at the frequency of this input?

    A. About 3 db
    B. About -3 db
    C. About 9 db
    D. About -9 db

14. On the bode plot of the transfer function corresponding to the figure above, what is the phase at the frequency of this input?

    A. About -45 deg
    B. About +45 deg
    C. About -90 deg
    D. About +90 deg

15. If the system were known to be of the form $G(s) = \dfrac{K}{\tau s + 1}$, what is the value of $K$?

    A. About 2.0
    B. About 1.0
    C. About 0.5
    D. About 0.1

16. The frequency response plot shown above might reasonably correspond to which transfer function?

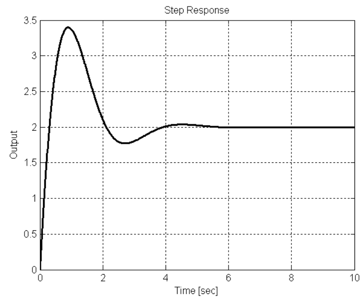
    A. $G(s) = \dfrac{12}{s+3}$
    B. $G(s) = \dfrac{48}{s+12}$
    C. $G(s) = \dfrac{12}{s+12}$
    D. None of the above.

17. For a system with the transfer function

    $$H(s) = \frac{Y(s)}{U(s)} = \frac{4(s+4)(s+2)}{(s^2+4s+16)(s+1)},$$

    the slope of the output *immediately following* a unit step input is…

    A. $\lim_{t \to 0^+} \dfrac{dy}{dt} = \infty$
    B. $\lim_{t \to 0^+} \dfrac{dy}{dt} = 0$
    C. $\lim_{t \to 0^+} \dfrac{dy}{dt} = 2$
    D. $\lim_{t \to 0^+} \dfrac{dy}{dt} = 4$




18. Which of the following transfer functions could reasonably be expected to correspond to the unit step response shown above?

A. $H(s) = \dfrac{8}{s^2 - 0.4s + 4}$

B. $H(s) = \dfrac{8s + 8}{s^2 + 2s + 4}$

C. $H(s) = \dfrac{s}{s^2 + 0.4s + 4}$

D. None of the above

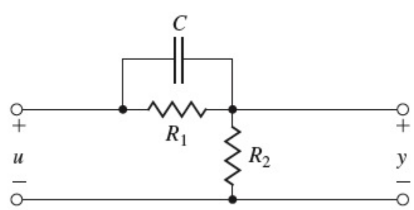
19. Which of the following is the correct expression for the Laplace Transform of $\tau \dfrac{dy}{dt} + 2y$?

A. $(\tau + 2s)Y(s) - \dfrac{dy}{dt}(0)$

B. $(\tau + 2)Y(s) - 2sy(0)$

C. $(\tau s + 2)Y(s) - \tau y(0)$

D. None of the above.



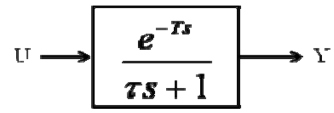

20. The transfer function, $\dfrac{Y(s)}{U(s)} = H(s)$, for the figure shown above is…

A. $H(s) = \dfrac{R_1 Cs + 1}{R_1 Cs + 1 + \dfrac{R_1}{R_2}}$

B. $H(s) = \dfrac{R_2 Cs}{R_1 R_2 Cs + 1}$

C. $H(s) = \dfrac{R_2}{R_1 Cs + 1}$

D. None of the above.




21. Which of the following differential equations corresponds to the system shown above?

A. $\tau \dfrac{dy(t-T)}{dt} + y(t-T) = u(t)$

B. $\tau \dfrac{dy(t)}{dt} + y(t) = e^{-t/T} u(t)$

C. $\tau \dfrac{dy(t)}{dt} + y(t) = u(t-T)$

D. None of the above is correct.

22. If a system has the transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{4s(s+4)(s+2)}{(s^2 + 4s + 16)(s+1)},$$

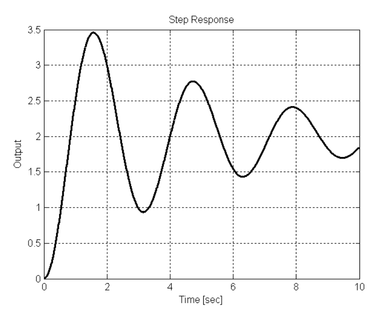
the steady-state value in response to a *unit ramp* input, $u(t) = t$ for $t > 0$, is…

A. $\lim_{t \to \infty} y(t) = \infty$

B. $\lim_{t \to \infty} y(t) = 4$

C. $\lim_{t \to \infty} y(t) = 2$

D. $\lim_{t \to \infty} y(t) = 0$




23. What is the most reasonable estimate of the damping ratio of the second-order system whose unit step response is shown above?

A. $\varsigma = 0.75$

B. $\varsigma = 0.4$

C. $\varsigma = 0.1$

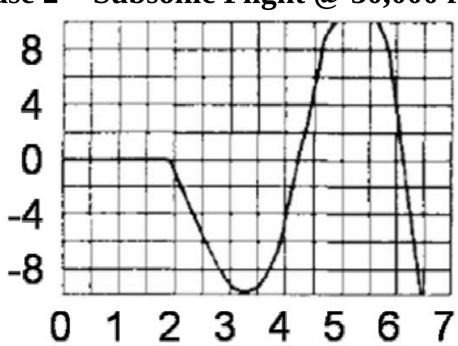
D. $\varsigma = 0$

The F-111 Ardvark aircraft (above) uses an adaptive control system to automatically update the control system parameters to account for changes in vehicle dynamics as a function of flight condition. A paper[2] describing this system included the graphs shown below of the pitch rate response [deg/sec] versus time [sec] for a unit step elevator input *for the open-loop system*. As you can see on the graphs, the step inputs were at different times for each flight condition. There is no pure time delay in the system. In each case, the engineers have confidence that the response can be well approximated by the following transfer function:

$$H(s) = \frac{Q(s)}{\delta_{ELEVATOR}(s)} = \frac{-(As+B)}{\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

**Case 1 = Supersonic Flight @ sea level**


**Case 2 = Subsonic Flight @ 30,000 feet**




[2] Ehlers & Smyth, "Adaptive Control Applications to Aerospace Vehicles", *Journal of Aircraft*, 1970

24. For Case 2, the most reasonable statement about damping ratio is…

    A. $\zeta < 0$ because the system is stable.
    B. $\zeta < 0$ because the system is unstable.
    C. $\zeta > 0$ because the system is unstable.
    D. $\zeta > 0$ because the system is stable.

25. For Case 2, the most reasonable value for A is…

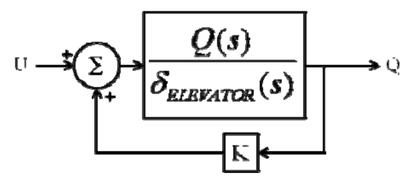
    A. $A = 0$
    B. $A = 3$
    C. $A = 6$
    D. $A = 12$

26. For Case 1, the most reasonable value for the natural frequency is…

    A. $\omega_n = 8\, rps$
    B. $\omega_n = 4\, rps$
    C. $\omega_n = 1.5\, rps$
    D. $\omega_n = 0.75\, rps$

27. For Case 1, the most reasonable value for $B$ is…

    A. $B = 130$
    B. $B = 75$
    C. $B = 2$
    D. $B = 0.016$

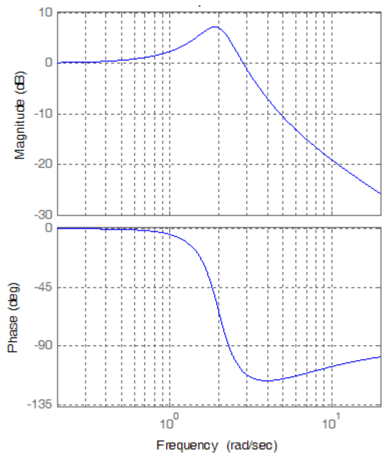



28. Suppose feedback control of the form shown above were used on the Aardvark. (The positive feedback is necessary because the open-loop transfer function has a "-" sign in the numerator.) With the given form of $H(s)$, which statement is MOST ACCURATE about the closed-loop system, applied to Case 1?

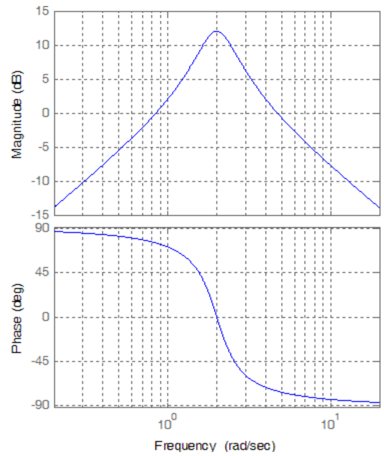
    A. The feedback guarantees that Q=U in steady state.
    B. For very large values of the gain, K, the system will have two unstable poles.
    C. The system is stable for all $K > 0$.
    D. None of the above is correct.

The following figures show the frequency responses for 3 different linear dynamic systems.

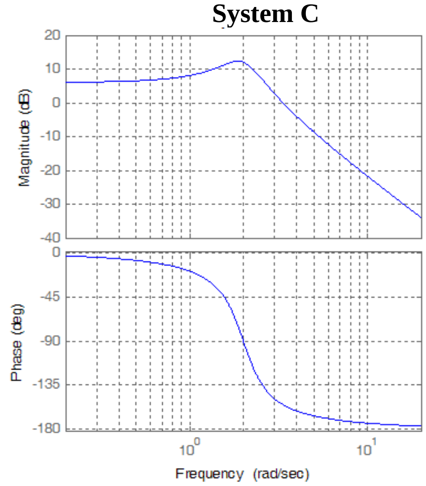
**System A**

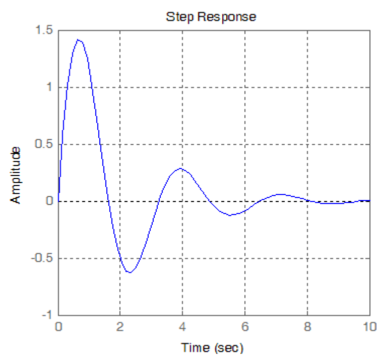



**System B**



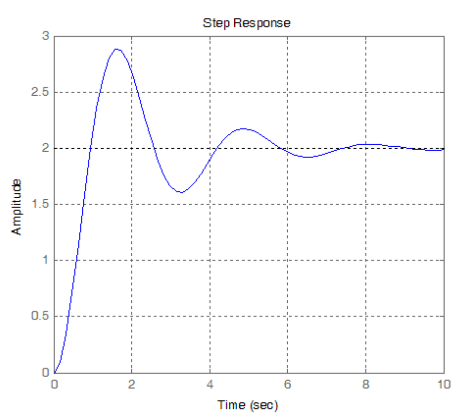

**System C**




29. Using the ideas we studied in class, which system could reasonably be inferred to have the unit step response shown above?

    A. System A
    B. System B
    C. System C
    D. None of the above.




30. Using the ideas we studied in class, which system could reasonably be inferred to have the unit step response shown above?

    A. System A
    B. System B
    C. System C
    D. None of the above.

31. Using the ideas we studied in class, which transfer function could reasonably be expected to have no zeros?

    A. System A
    B. System B
    C. System C
    D. None of the above.

*The following questions do not relate to the bode plots shown above.*

32. The *unit step* response for a system with transfer function $H(s) = \dfrac{4s}{s^2 - 4}$ is…

    A. None of the other answers.
    B. $\sin(2t)$
    C. $2\cos(2t)$
    D. $e^{2t} - e^{-2t}$

33. The *unit impulse* response for a system with transfer function $H(s) = \dfrac{4s}{s^2 + 4}$ is…

    A. $e^{2t} - e^{-2t}$
    B. $4\cos(2t)$
    C. $4\sin(2t)$
    D. None of the above