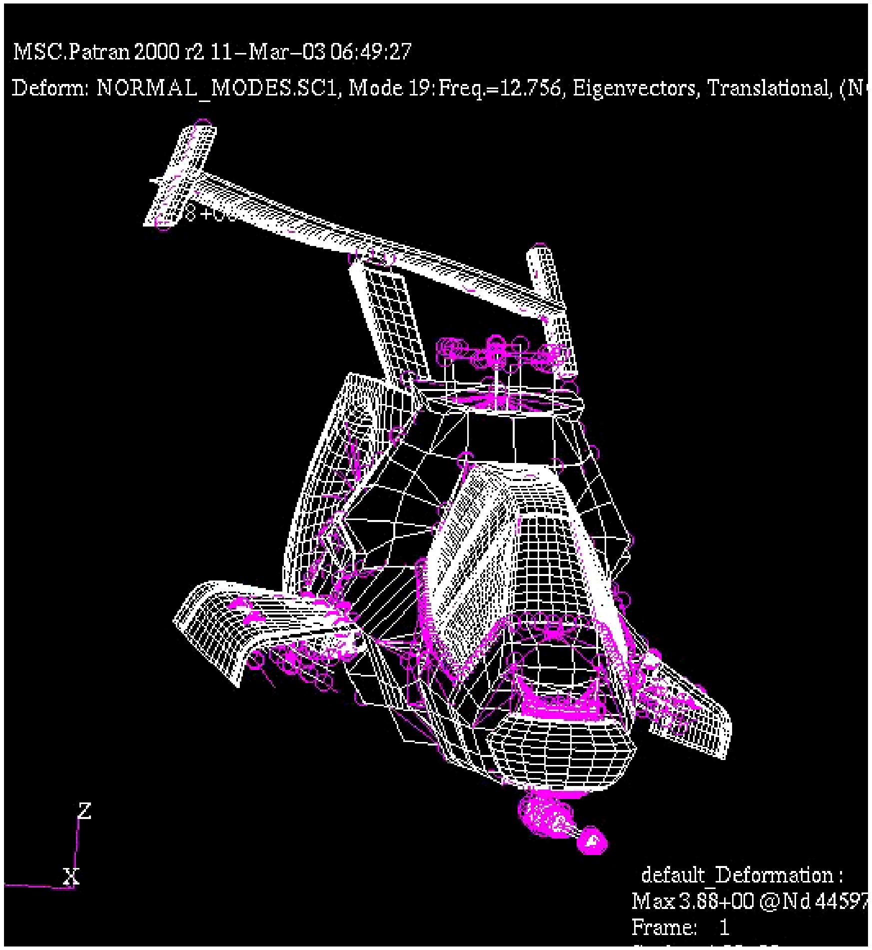
Frequency Response Design Example

ESE 505 & MEAM 513

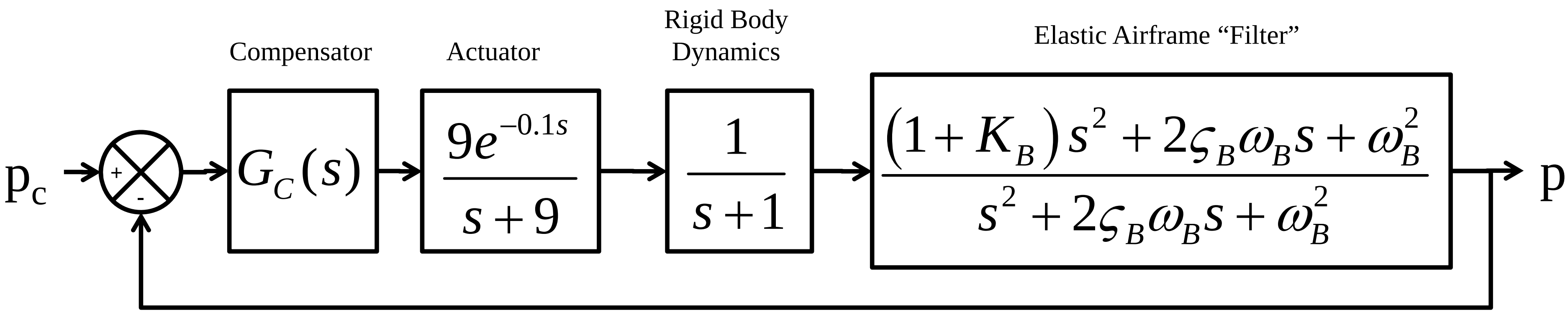
Bruce D. Kothmann

2014-04-02



Penn Engineering

Plant Description & Design Goals



- Overall Design Objective = Good Disturbance Rejection = High Velocity Error Constant = High Integral Gain in Compensator
- Some Design Constraints
  – Rigid Body Damping Ratio > ~ 0.3
  – Rigid Body Setting Time < ~ 2.5 sec
  – Phase Margin > ~ 45 Deg
  – Gain Margin > ~ 6 dB

Elastic Modes Very Common Challenge

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Space Telescope Pointing Control System

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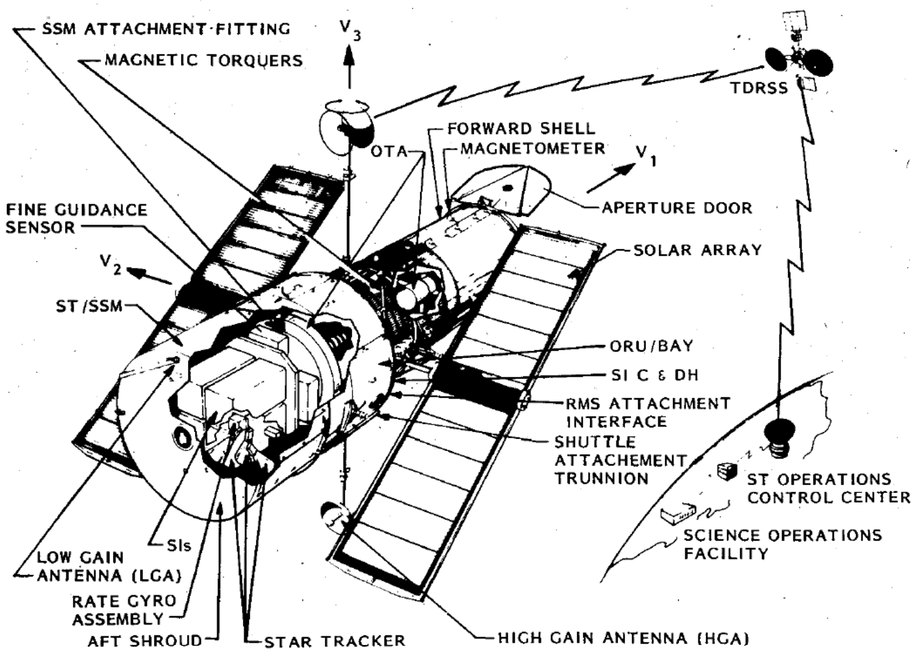
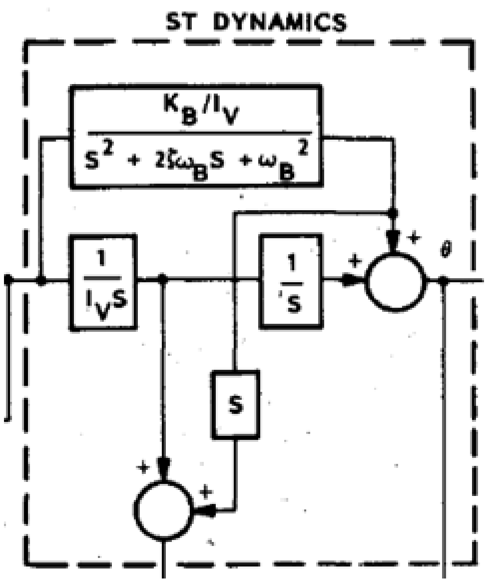
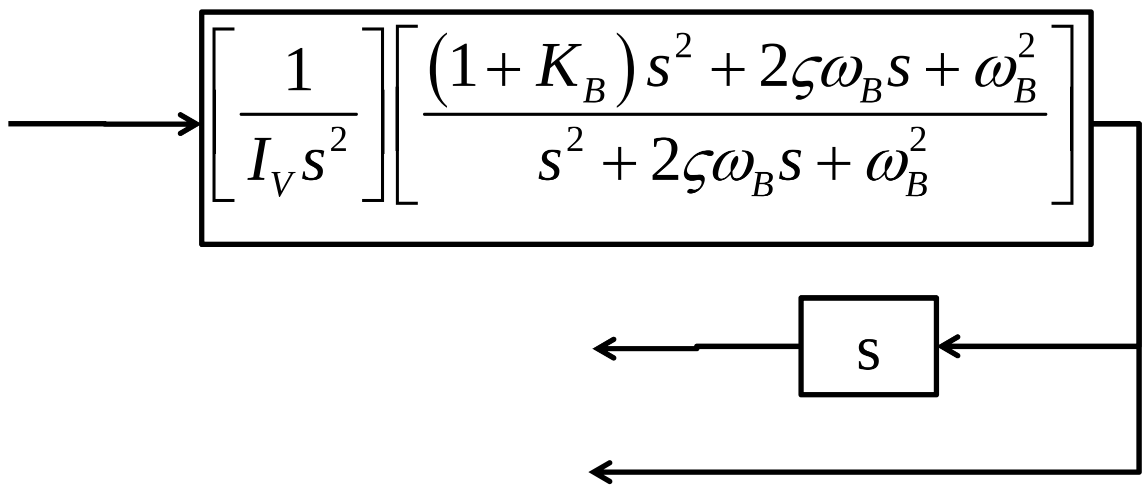


Space Telescope Dynamics

1) *Structural modes:* The solar array and optical telescope assembly modes have large modal coefficients. For example, the value of the solar array inertia about the Space Telescope center of mass is almost one-half that of the Space Telescope centerbody, which comprises the support systems module and optical telescope assembly. The control system sample rate and compensation are chosen to stabilize the modes. The command generator shapes maneuvers to limit structural excitations during maneuvers.

Modal Coefficient = K_B





Another Example : Space Station

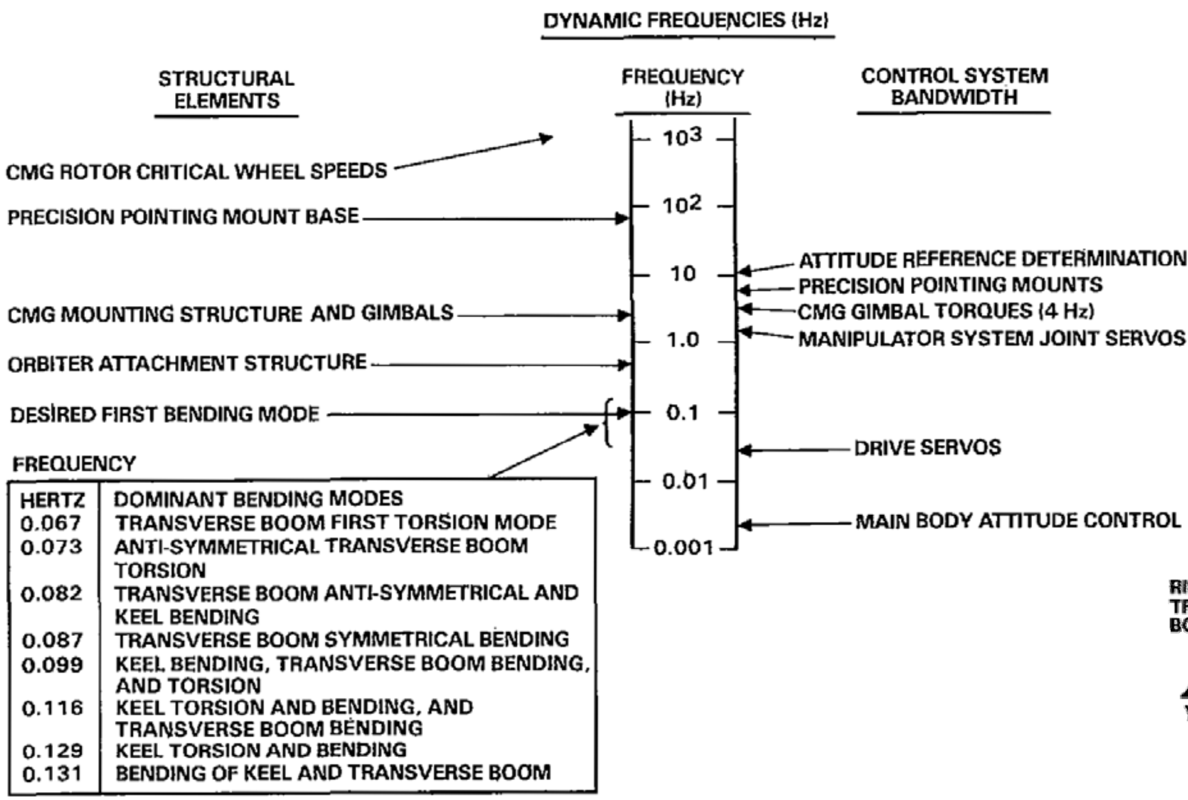


Fig. 3. Strong low-frequency structural modes close to desired alpha servo bandwidth and close to solar dynamic (exocentric) station controller bandwidth.

Pointing, Control, and Stabilization of Solar Dynamic Systems on a Space Station

Eric T. Falangas and Henry H. Woo

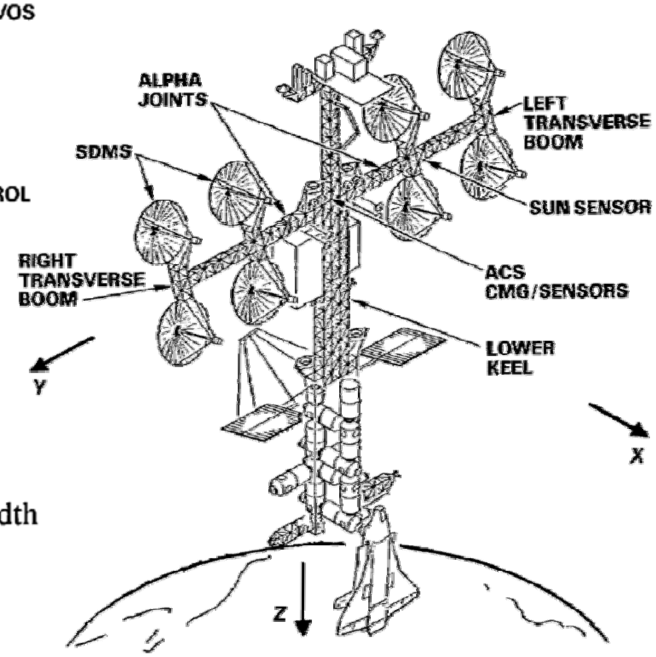
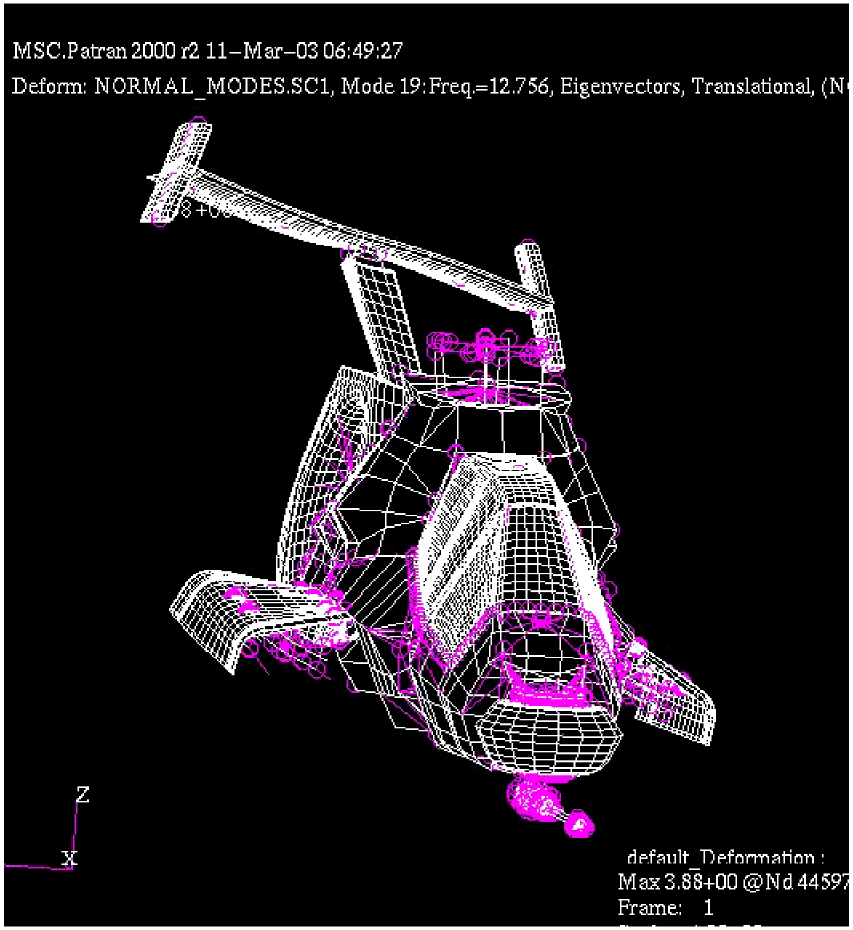


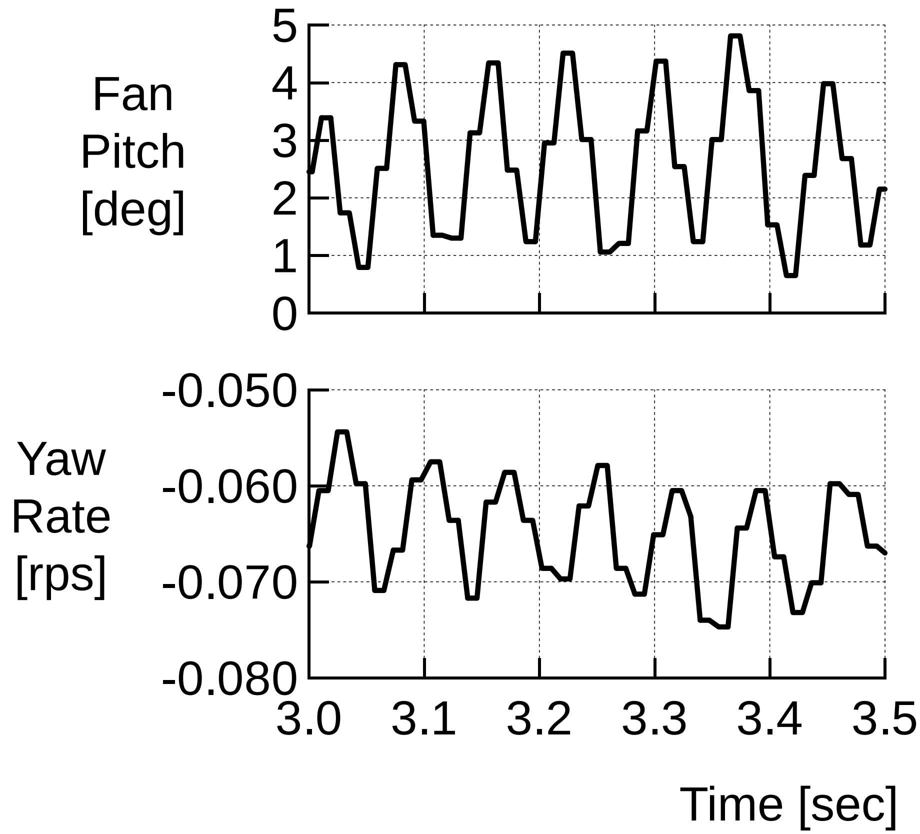
Fig. 1. Single-keel space station with solar dynamic system.

RAH-66 Comanche Aeroelastic Instability

Linear Instability With Actuator Rate Limit → Limit Cycle

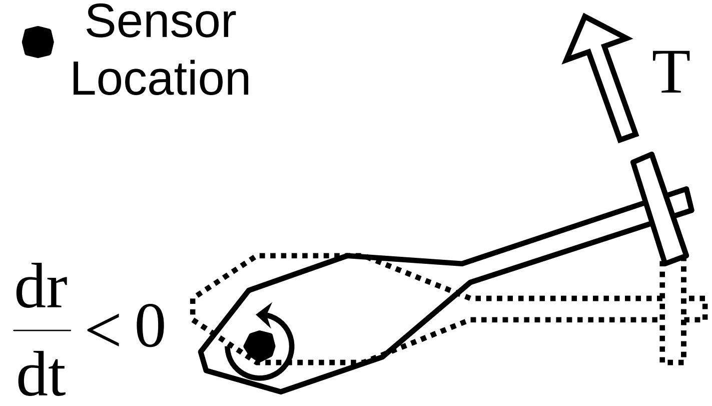


14-Hz Sustained Oscillation
90 Knots Descending Shallow Turn
MPFCS Mode - Yaw Rate Feedback Only

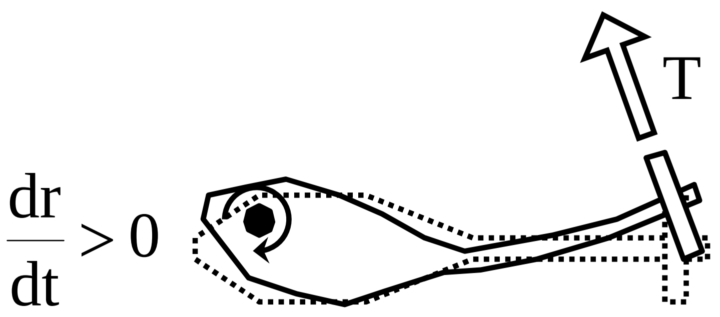


Simple Explanation of Feedback Instability

- Yaw Rate (r) Feedback to FANTAIL™ Thrust Required for Adequate Damping of Rigid-Body Modes
- Elastic Fuselage Response Can Change Sign of Response at High Frequency
- Effectively Positive Feedback Destabilizes Fuselage Elastic Mode
- Design: Linear Analysis Predicted Weaker Coupling

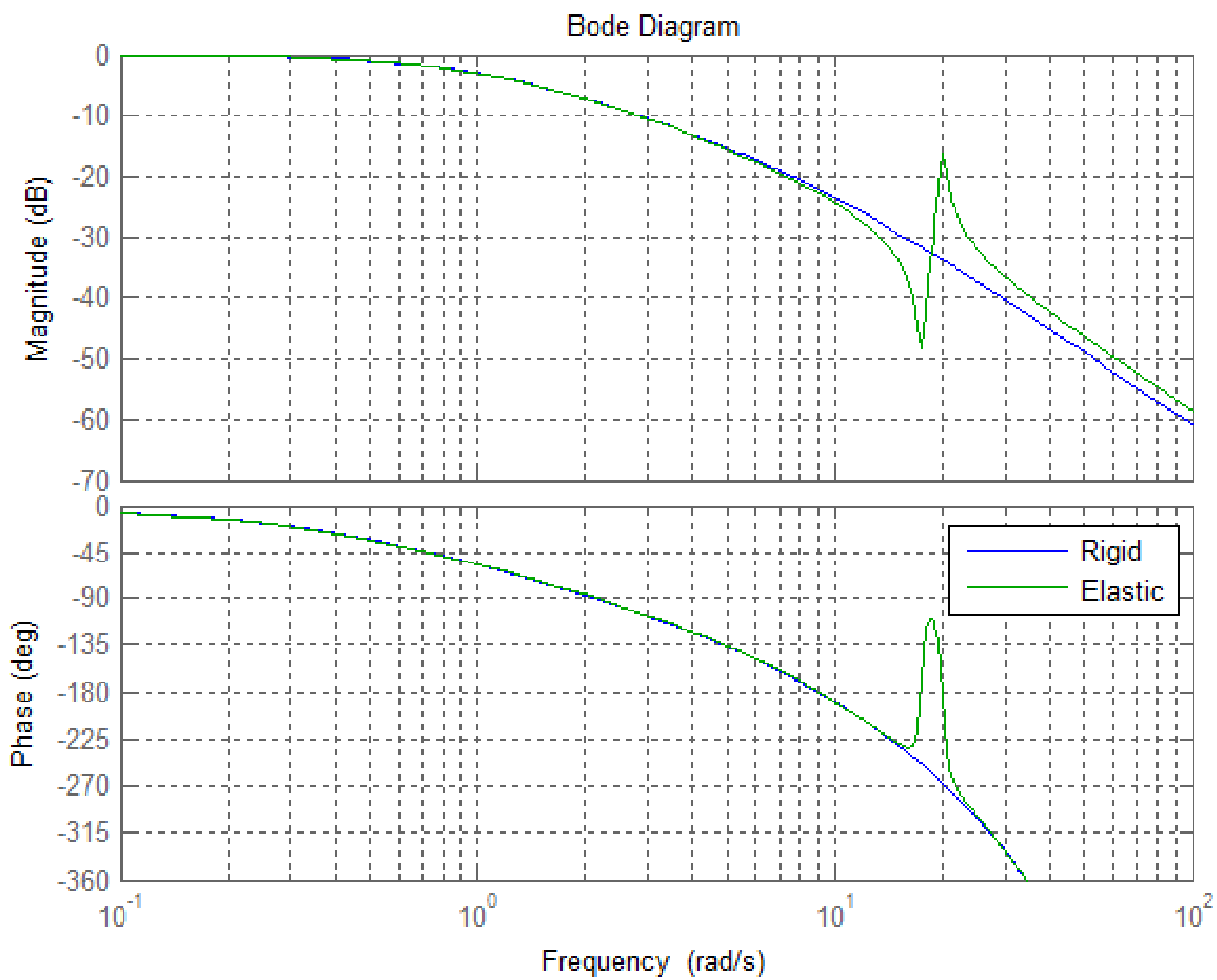


Rigid Fuselage Response

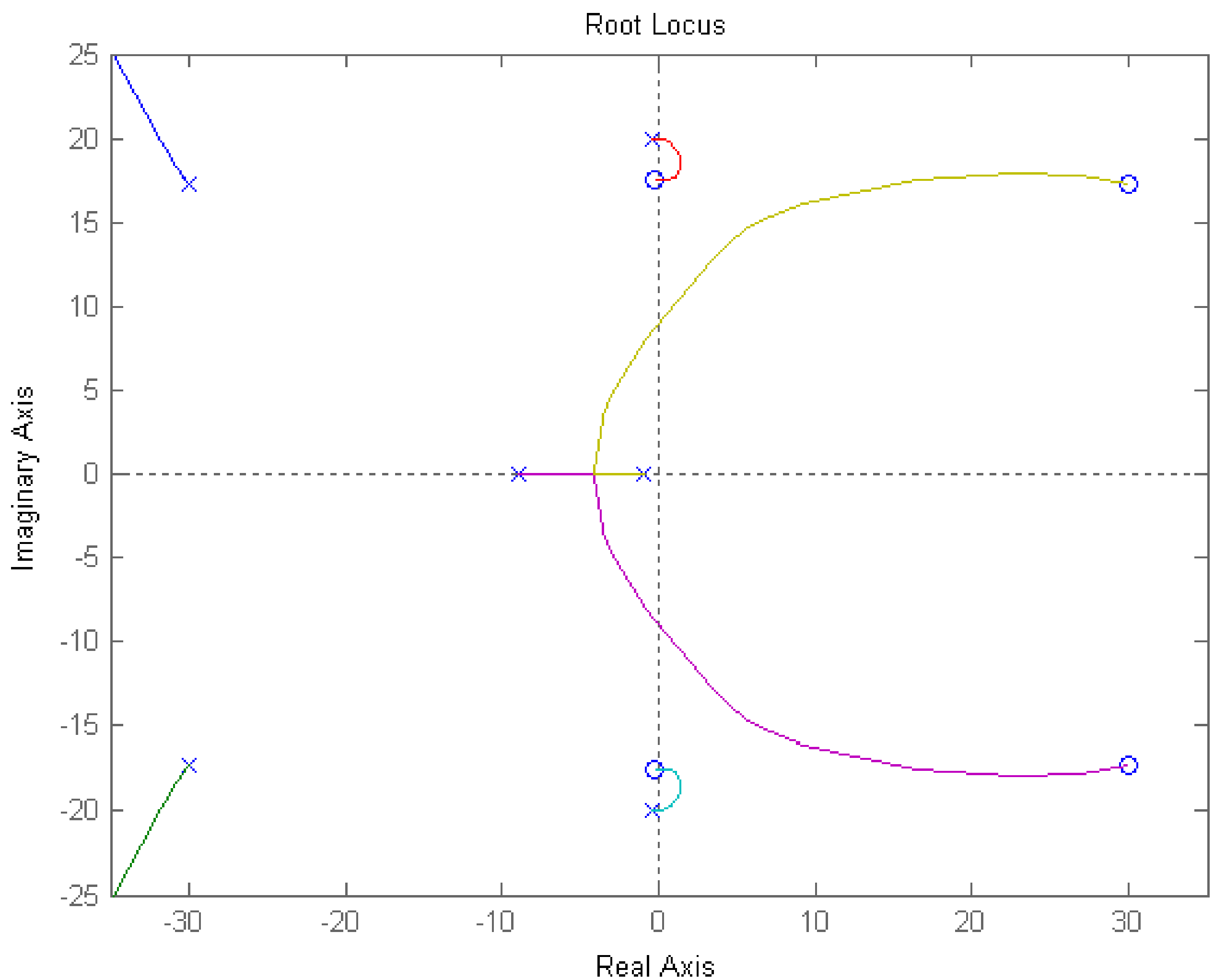


Elastic Fuselage Response

“Elastic”=Actuator + Rigid + Elastic Body Modes



Root Locus with Proportional Feedback

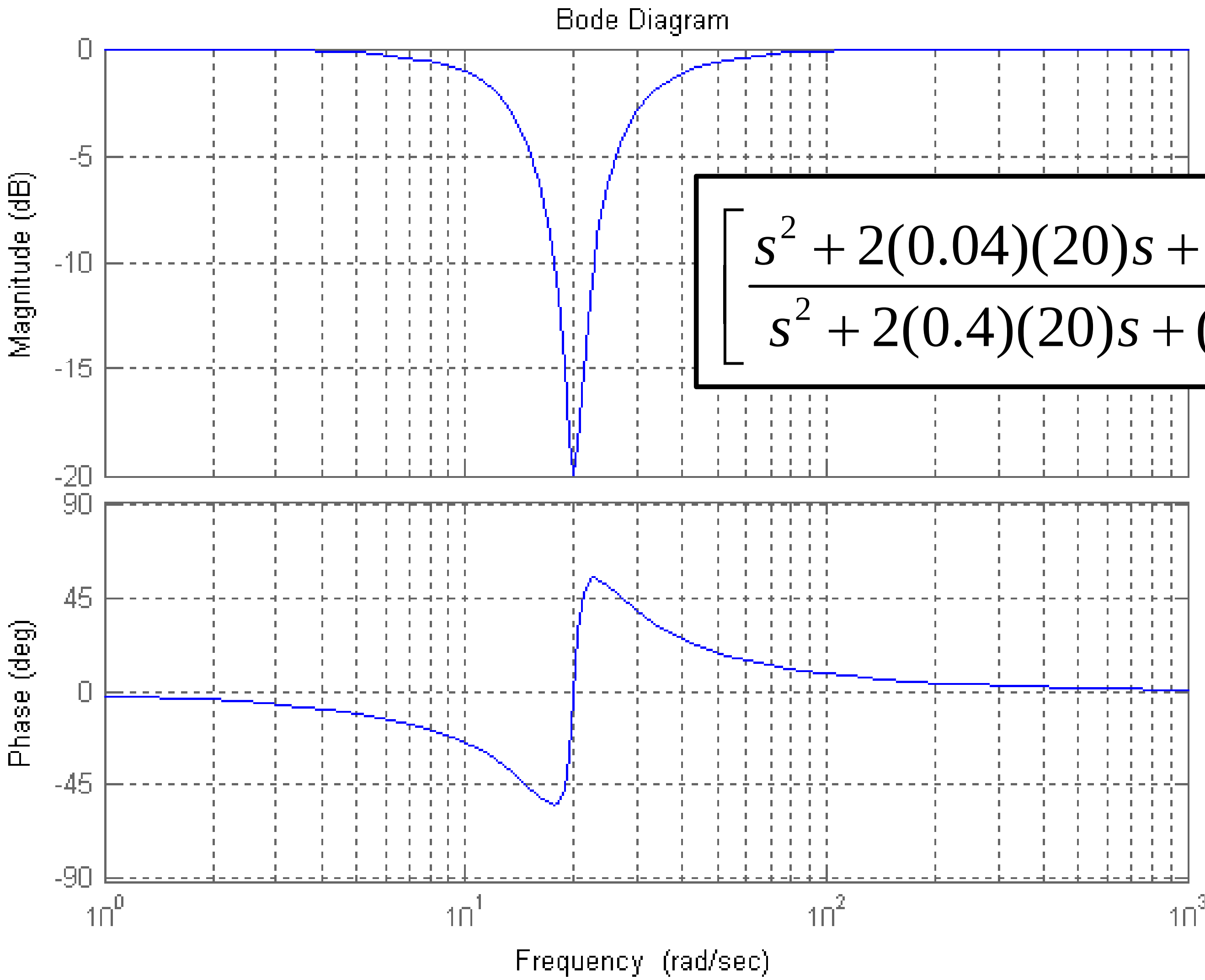


Elastic Modes Unstable for K > ~6

Rigid Body Modes Unstable for K > ~14

We Need to Do Something to Manage Elastic Modes!

Notch Filter

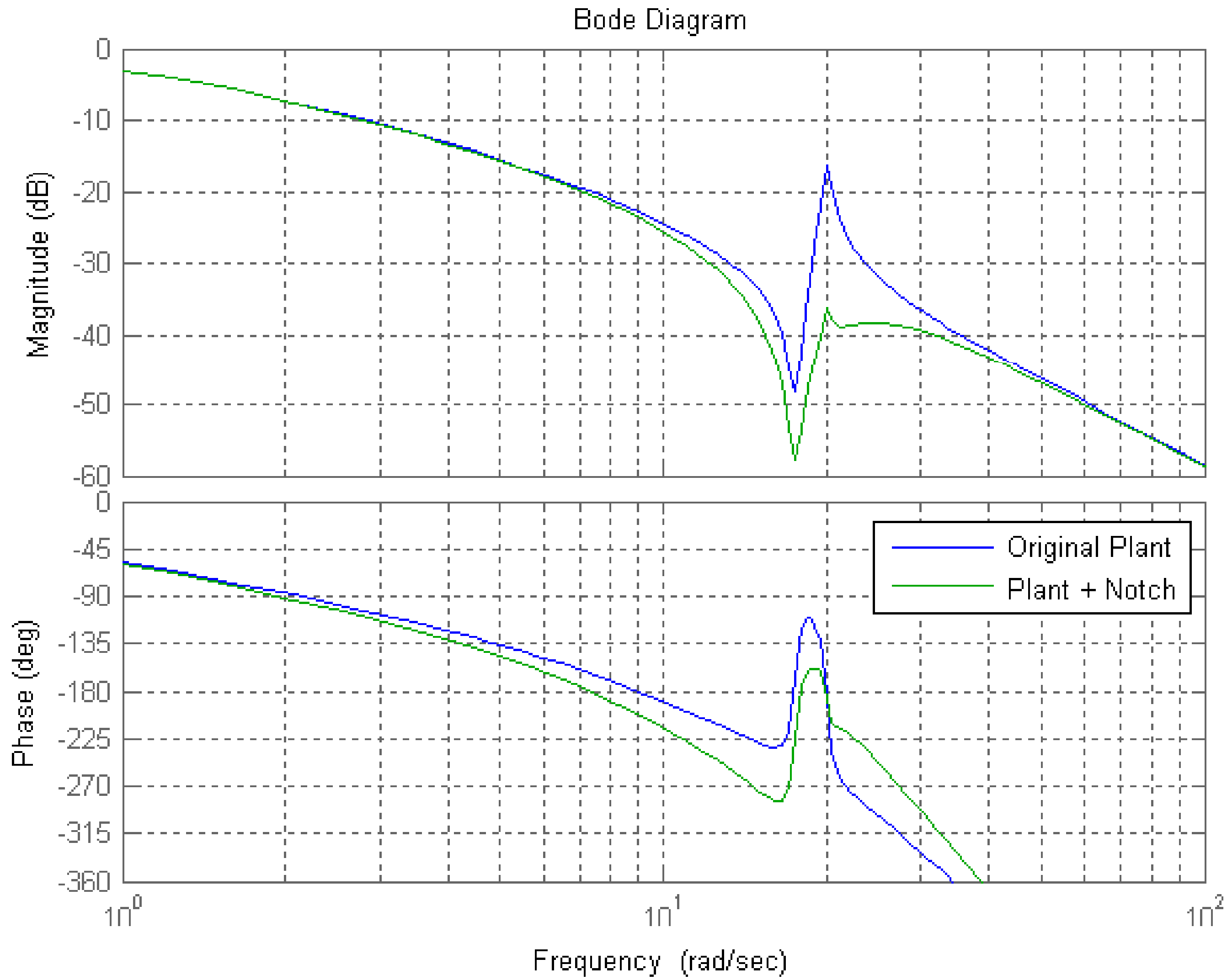


$$\left[\frac{s^2 + 2(0.04)(20)s + (20)^2}{s^2 + 2(0.4)(20)s + (20)^2}\right]$$

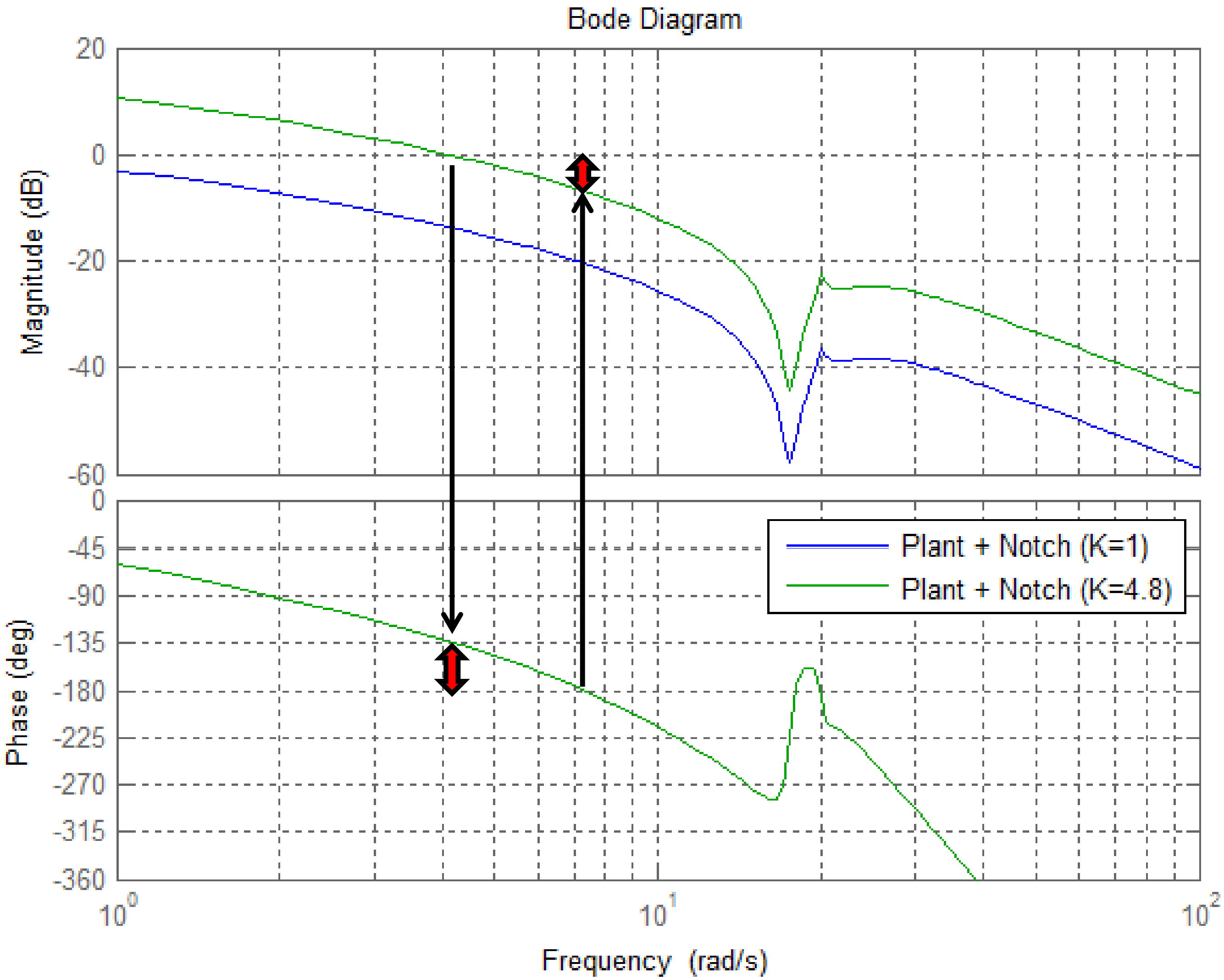
Start with a Deep & Moderately Wide Notch

(Conservative Design)

Effect of Notch on Loop Bode Plot (K=1)



Loop Bode Plot with Notch (K=1 vs. K=4.8)

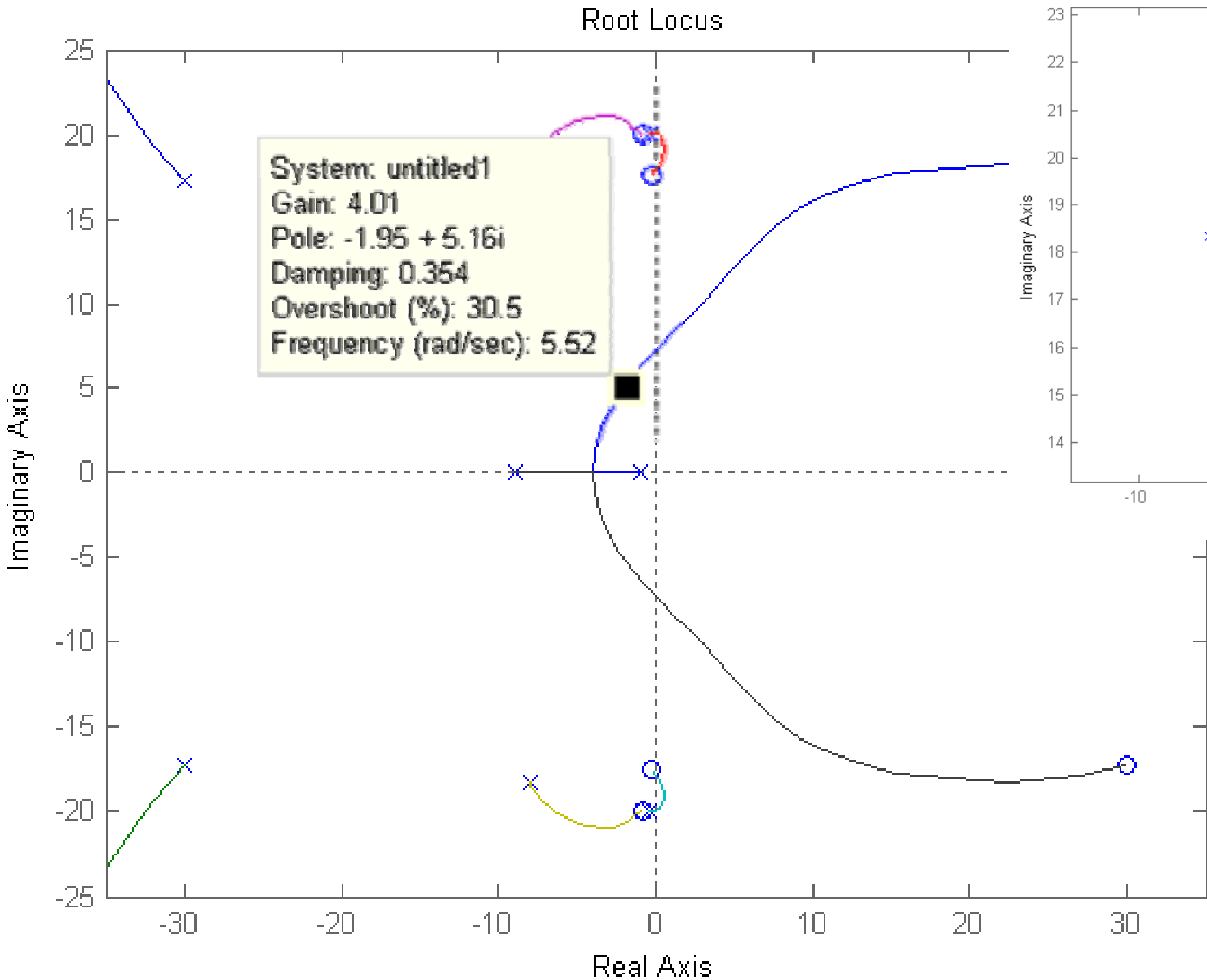


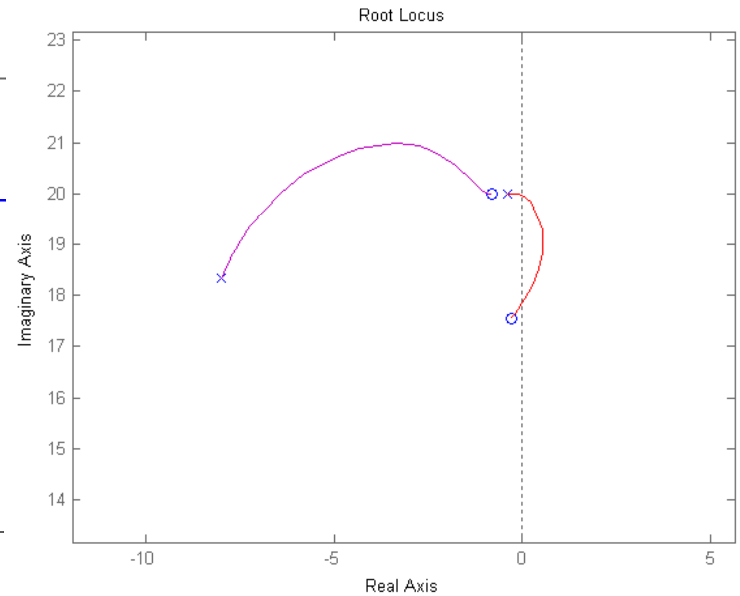
K=4.8

PM ~ 45 deg
@ 4.2 rps

GM ~ 6 dB
@ 7.3 rps

Effect of Notch on Root Locus





Elastic Modes Unstable
K > ~63

Rigid Modes Unstable
K > ~10

Damping Ratio ~ 35%
K=4

P+I = Special Form of "Lag Compensator"

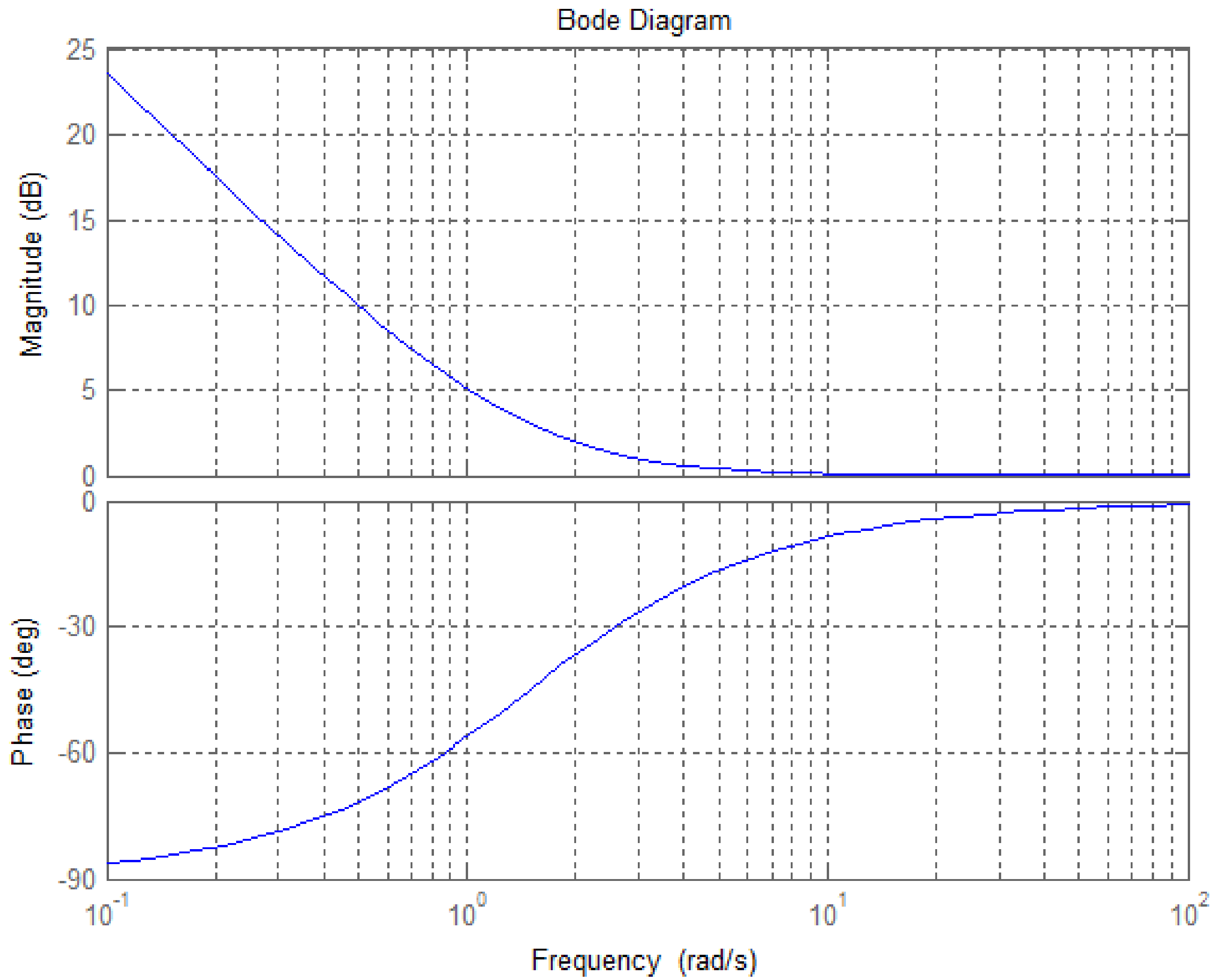
- General Form of Lag Compensator (z > p)

$$G_C = K\frac{s+z}{s+p}$$

- P+I Compensator → p = 0

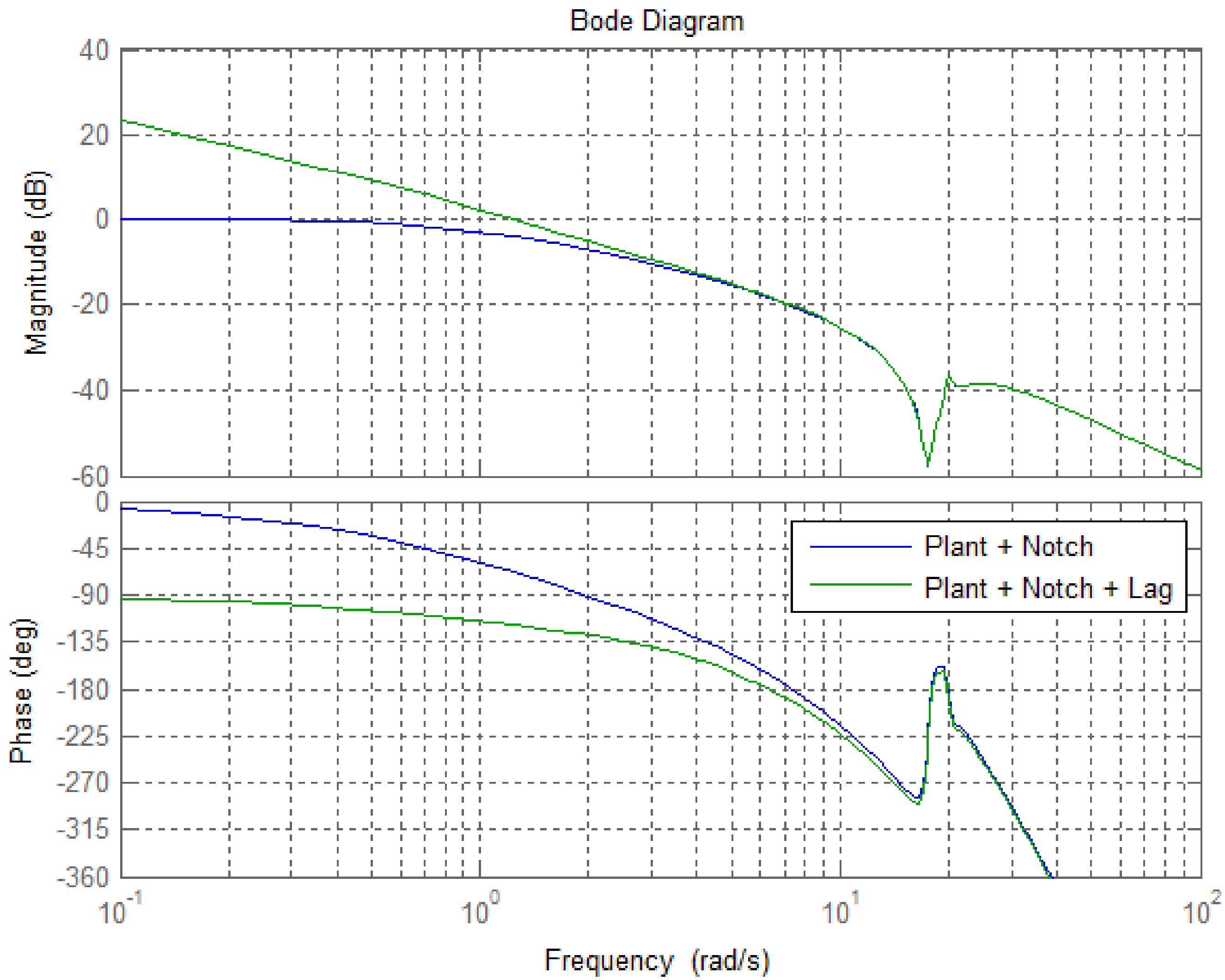
- Note: Gain → K @ Frequencies >> z
  - Design Crossover Region Without Lag Compensator
  - Add Lag Compensator with "z" Small Enough Not to "Mess Up" Bode Plot Near Crossover (Typically Want "z" Large for Tracking & Error Rejection Though)
  - Also Get Insight on Value of "z" From Root Locus

Bode Plot of Lag Filter (z=1.5, p=0)



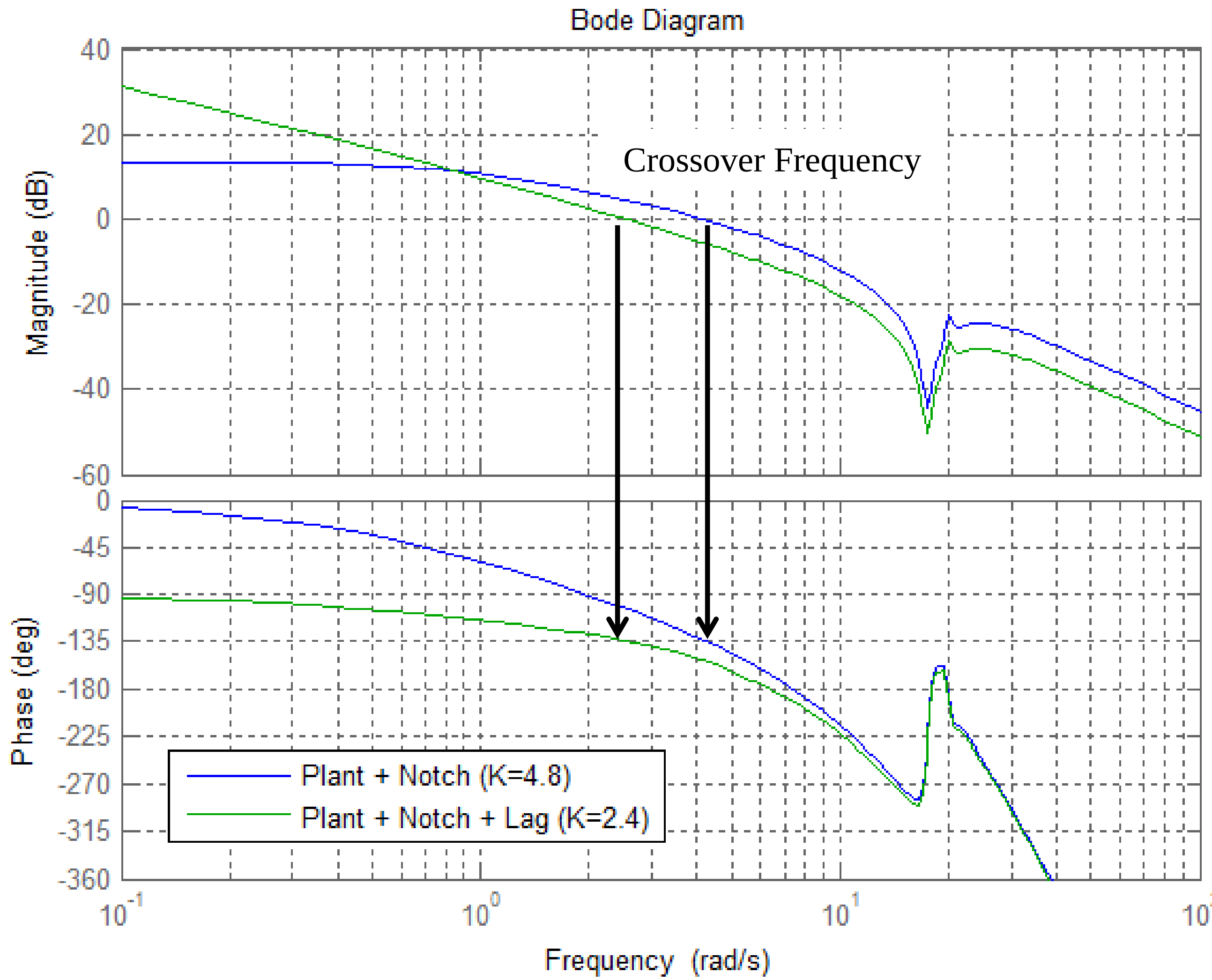
Notch vs. (Notch + Lag) (Both with K=1)

We "Pay For" the Higher Gain at Low Frequency (Better Tracking & Disturbance Rejection) with Lower Phase at Moderate Frequencies (Limit on K)

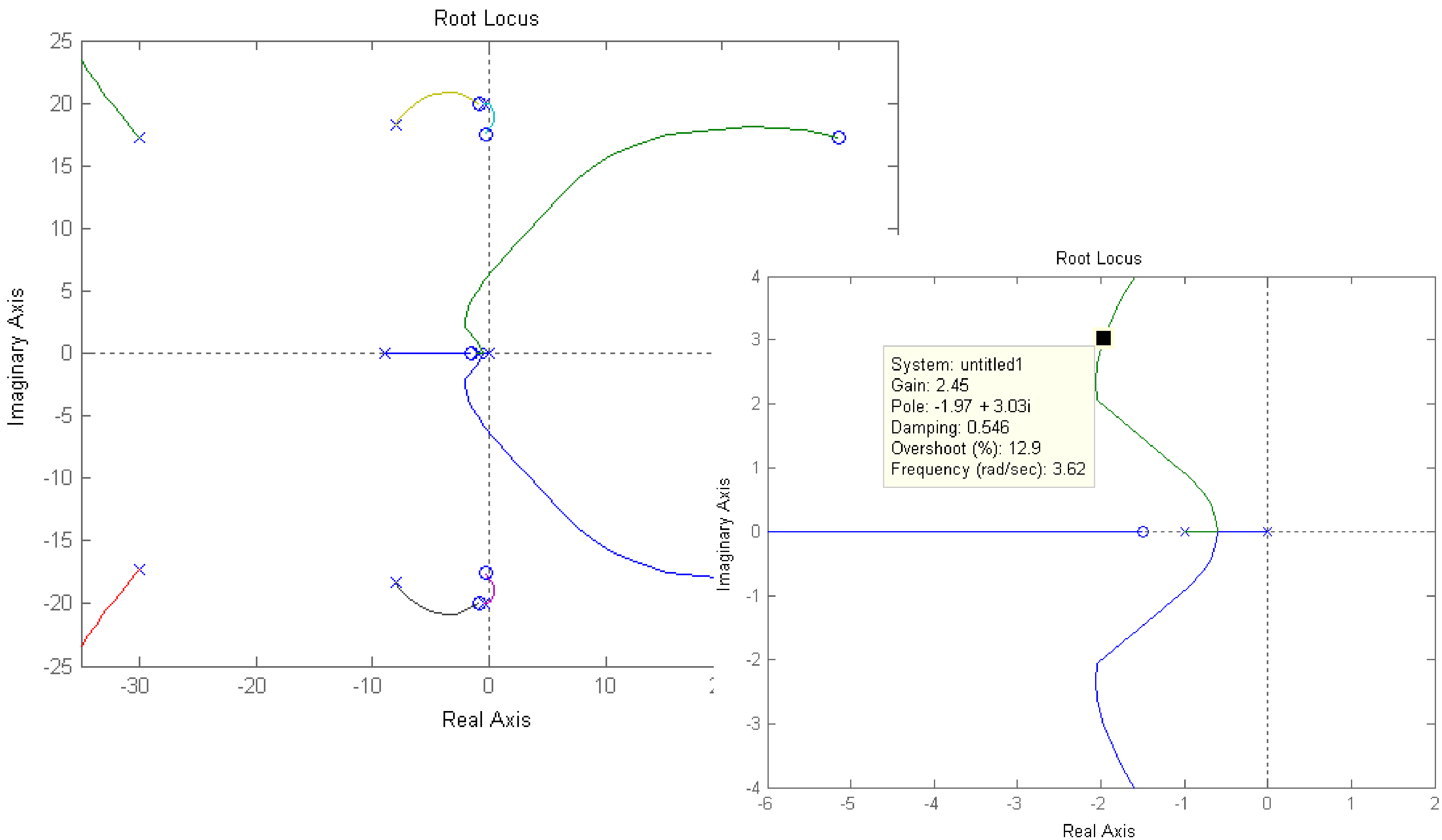


Notch vs. (Notch + Lag) (Both with PM=45 Deg)

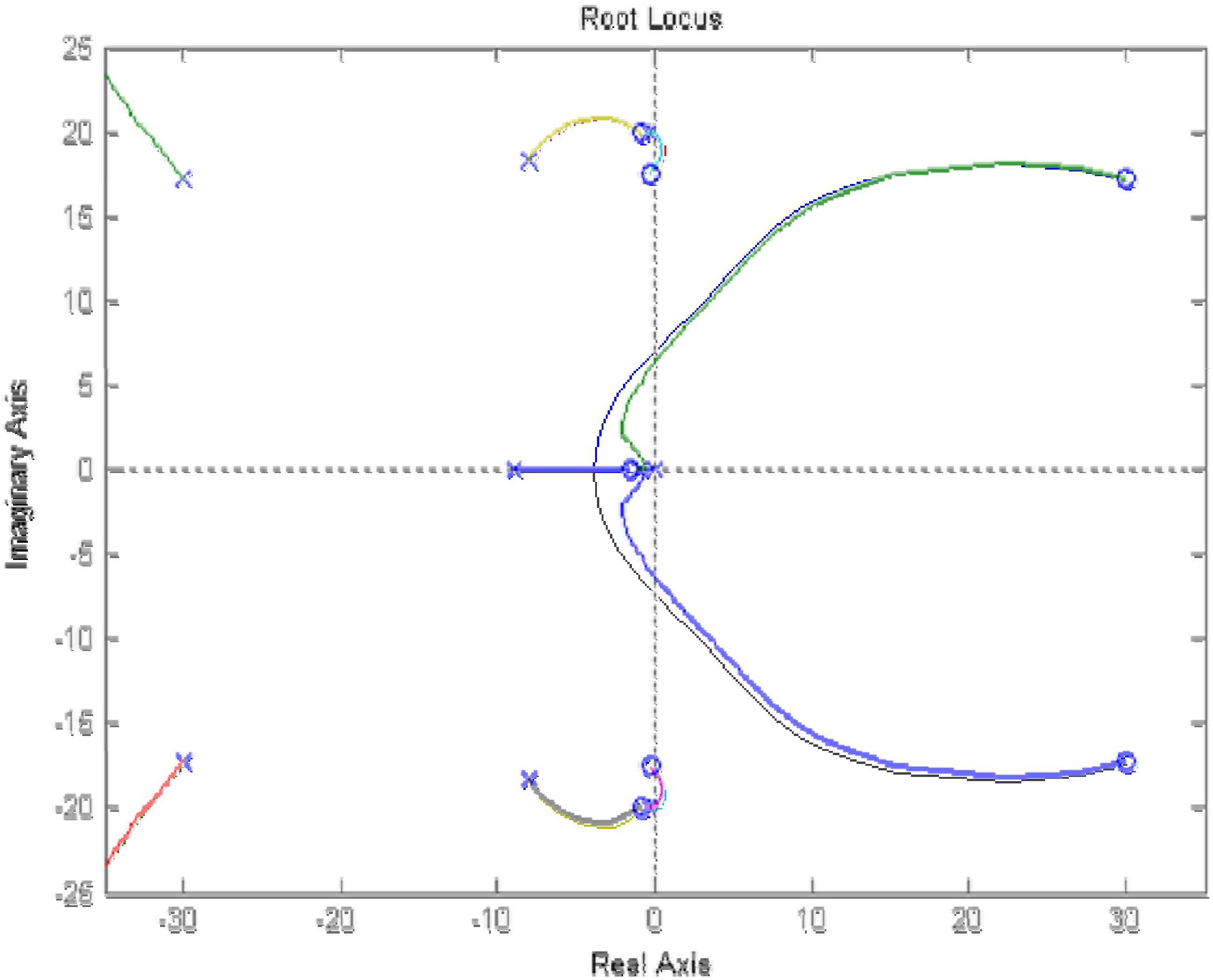
We "Pay For" the Higher Gain at Low Frequency (Better Tracking & Disturbance Rejection) with Lower Phase at Moderate Frequencies → Lower Crossover Frequency (~Bandwidth)



Root Locus with Lag Compensator (z=1.5, p=0)



Lag Compensator Design Difficult on Root Locus



Root Locus with Lag Compensator Very Similar to Root Locus for Proportional Feedback (Overlay Here)

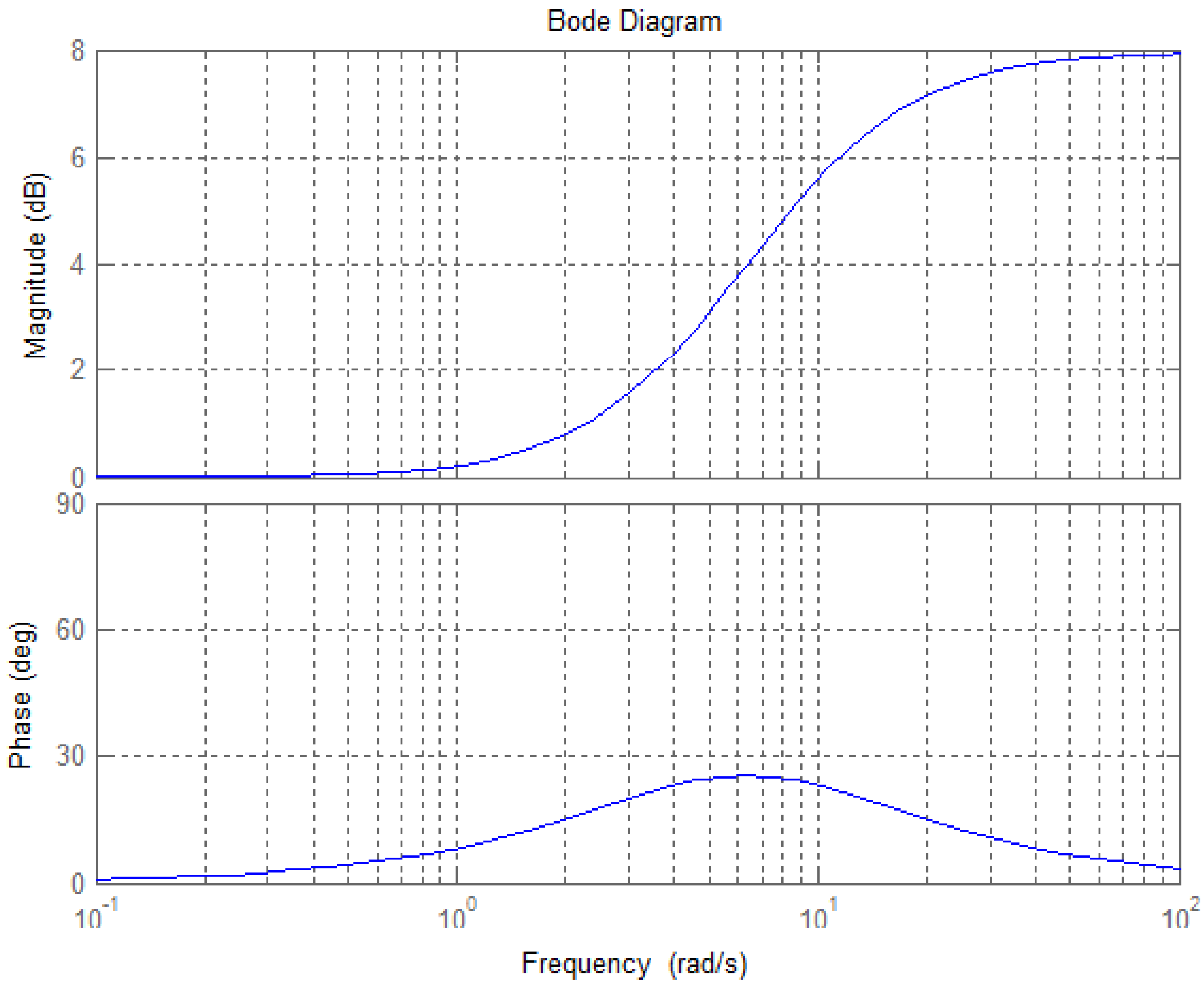
P+D = Special Form of Lead Compensator

- General Form of Lead Compensator (z < p)

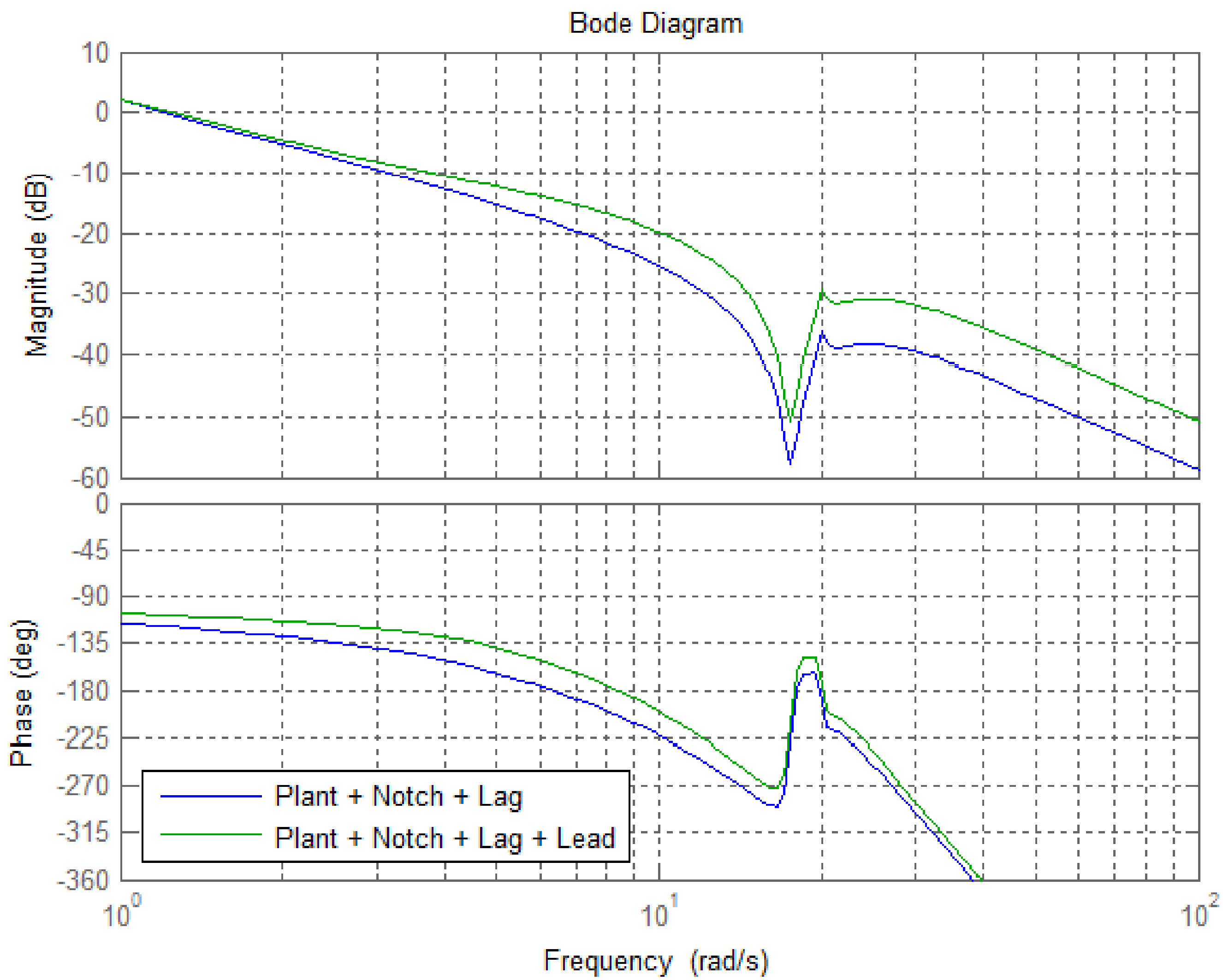
$$G_C = K\frac{\left(\frac{s}{z}+1\right)}{\left(\frac{s}{p}+1\right)}$$

- P+D Compensator ➔ p→Infinity (Usually Unrealistic)
- Note: Gain → K @ Frequencies << z
  - Design Low-Frequency (Tracking) Region Without Lead Compensator
  - Use Lead Compensator to Improve Stability Margins
  - Also Get Insight on Value of “z” From Root Locus

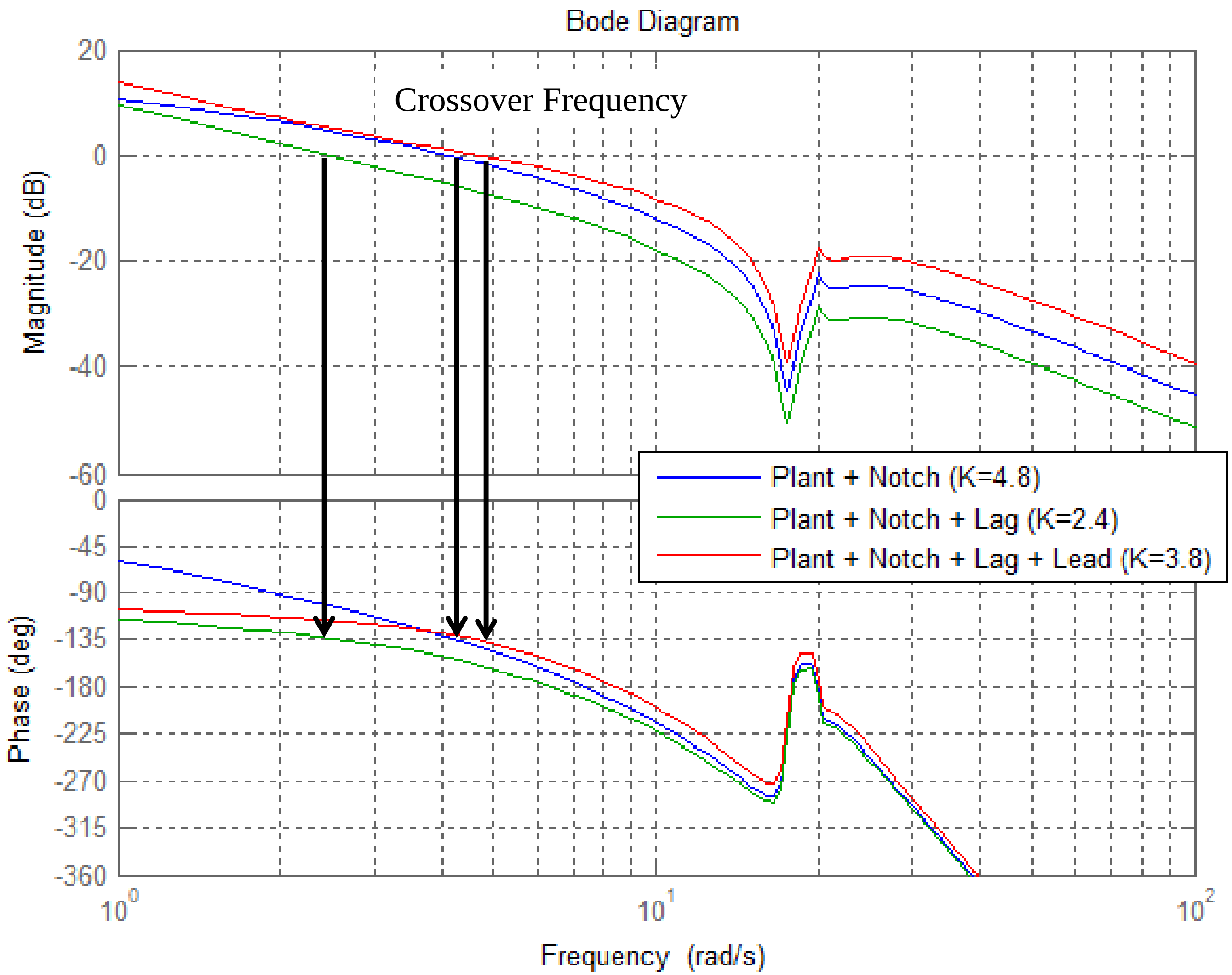
Bode Plot of Lead Filter (z=4, p=10)



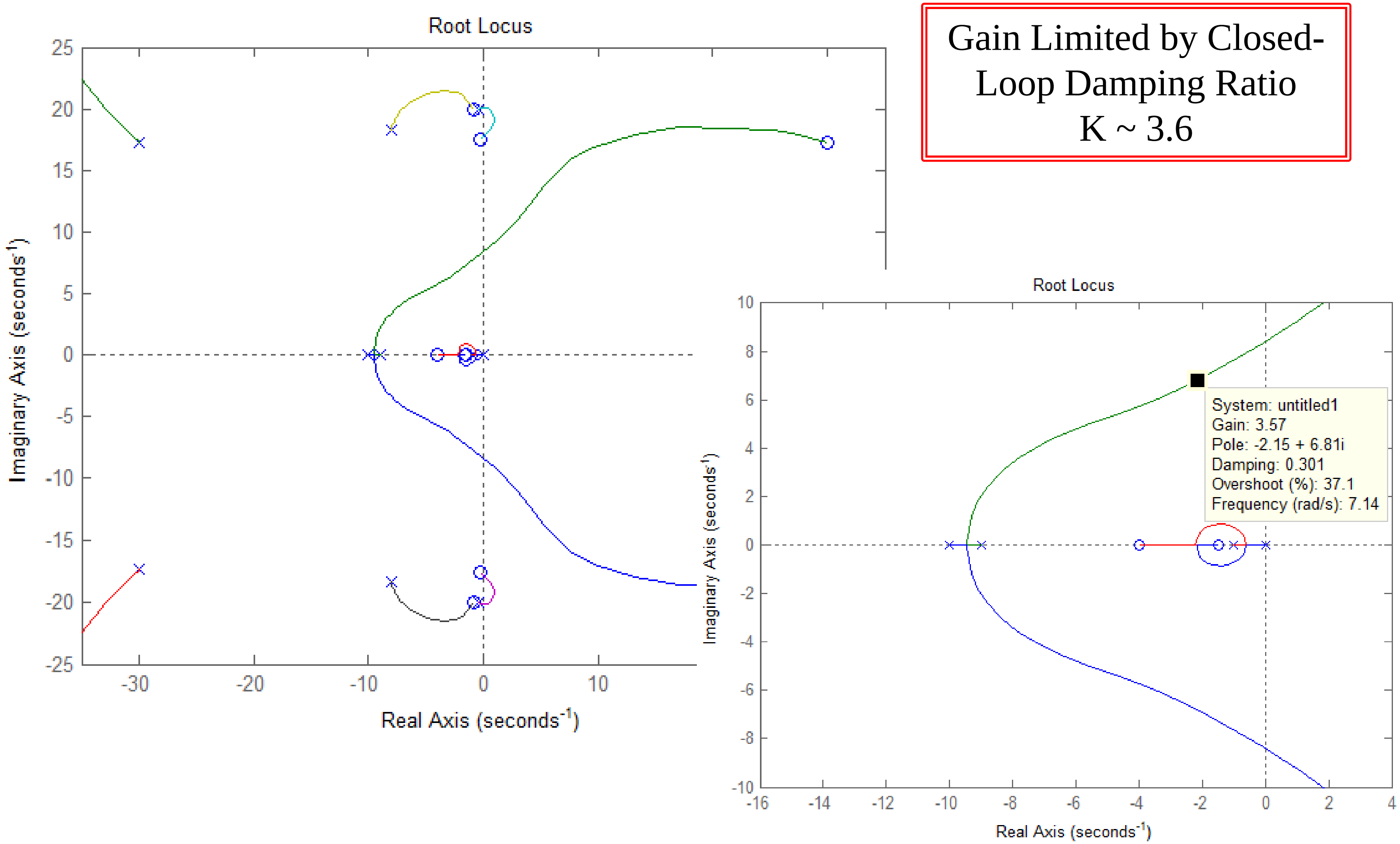
(Notch + Lag) vs. (Notch + Lag + Lead) (K=1)



Compensators Compared (All with PM=45)



New Root Locus (Notch + Lag + Lead)



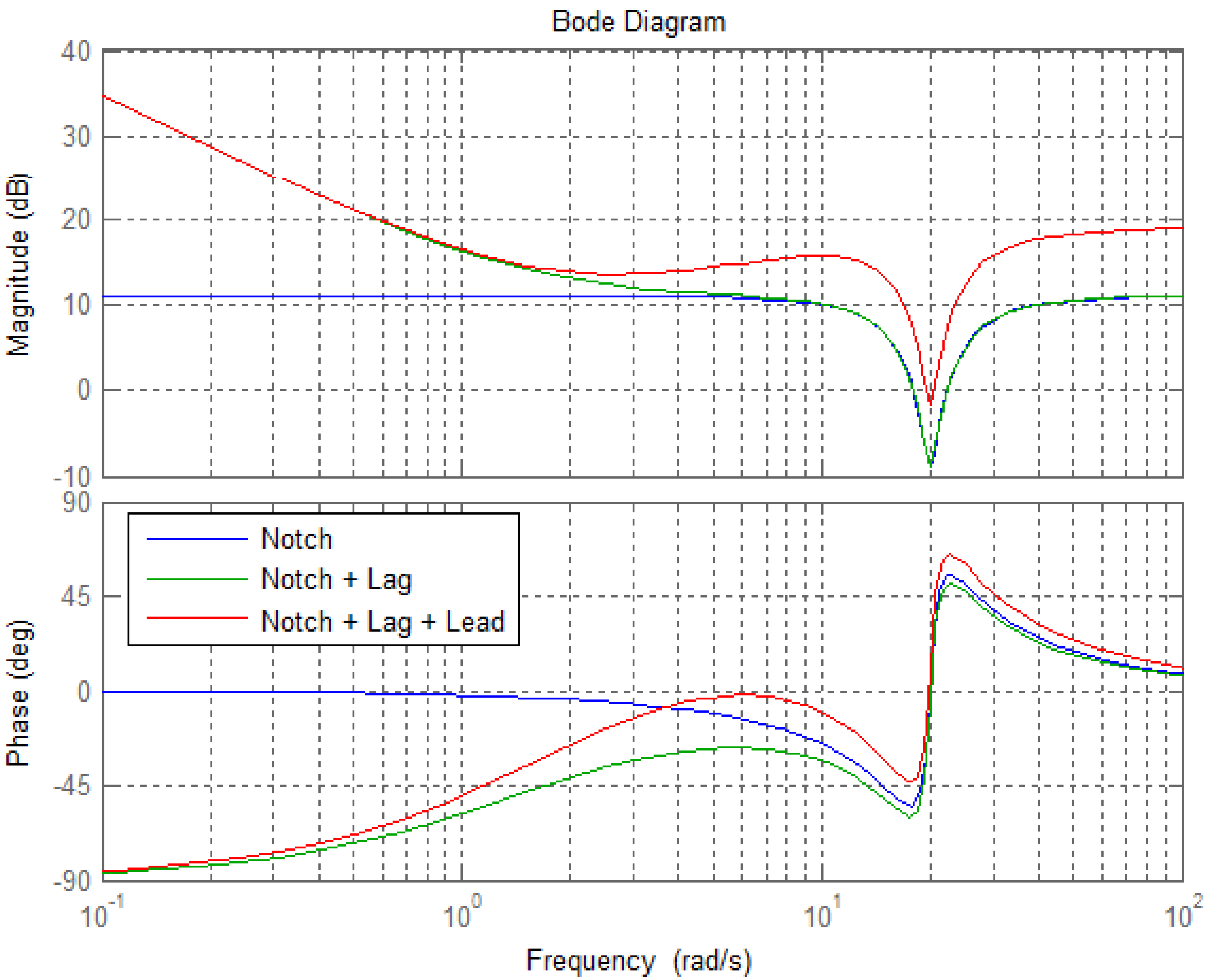
Gain Limited by Closed-Loop Damping Ratio
K ~ 3.6

Final Form of Compensator

$$G_C = K \frac{\left(\dfrac{s}{z_{LEAD}} + 1\right)}{\left(\dfrac{s}{p_{LEAD}} + 1\right)} \frac{\left(s + z_{LAG}\right)}{\left(s + p_{LAG}\right)} \left(\frac{s^2 + 2\varsigma_n \omega_n s + \omega_n^2}{s^2 + 2\varsigma_d \omega_n s + \omega_n^2}\right)$$

- Notch Filter to Prevent Destabilization of Elastic Modes
- Lag Compensator (~PI) for Good Tracking & Disturbance Rejection @ Low Frequency
- Lead Compensator for Quick Response & Good Stability Robustness (Allows Higher KI Gain)

Recap of Compensator Design (All with K=3.6)



- Notch to Eliminate Elastic Modes = High Frequency Stability Robustness
- Lag Compensation (P+I) for Good Tracking & Disturbance Rejection @ Low Frequency
- Lead Compensation for Better Stability → Higher Bandwidth
- These Elements are Quite Typical of Many Control System Designs