ESE 505 - SPRING 2015 - HOMEWORK #2
DUE 4-Feb-2015

Complex Arithmetic You should be sure you can do these problems by hand. Struggling with complex arithmetic won't make for a pleasant semester.

1. Write each of the following in the form $a + jb$, where $j = \sqrt{-1}$.

   a. $(1+j)(1-2j)$

   b. $\dfrac{(1+8j)}{-2j}$

   c. $\dfrac{(3+4j)}{(3-4j)}$

   Answers: (a) $3-j$, (b) $-4+0.5j$, (c) $-\dfrac{7}{25}+\dfrac{24}{25}j$

2. Write each of the following in the form $re^{j\theta}$, where $r > 0$ and θ is in radians. Again, we need to be able to do these by hand.

   a. $(-3+3j)(2+2j)$

   b. $\dfrac{(4+4j)}{-2j}$

   c. $\dfrac{(4+4j)}{(2-2j)}$

   Answers: (a) $12e^{j\pi}$, (b) $2\sqrt{2}e^{j3\pi/4}$, (c) $2e^{j\pi/2}$

ODE Review & Laplace practice For the following problems, you may use computational software (e.g. Wolfram Alpha) if it helps. The important thing is to conceptually understand the *forms* of the solutions and where the numerical values--particularly the characteristic values--in these forms come from. On a quiz or exam, you might be asked to pick the correct solution form several alternatives. If you understand the concepts, these exam questions will not be difficult.

3. Solve the following ordinary differential equation. Do it once the "regular" way, and once with Laplace Transforms. Notice that the characteristic values (λ) from the "regular" way are the "poles" in the Laplace way.

$$\frac{d^2y}{dt^2}+5\frac{dy}{dt}+4y=0,\ y(0)=0,\ \frac{dy}{dt}(0)=3$$

Answer: $y(t) = e^{-t} - e^{-4t}$. If you think of the system as representing a mass-spring-damper, can you give a physical interpretation of the result? That is in some ways more important than doing the math.

4. Solve the following system of ordinary differential equations. Do it once the "regular" way and once with Laplace transforms.

$$\frac{d\underline{x}}{dt} = \frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}\underline{x},\ \underline{x}(0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

Answer: $\underline{x}(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + \begin{pmatrix} -1 \\ 4 \end{pmatrix} e^{-4t}$. You should notice that this problem is the same as the previous problem, with $x_1(t) = y(t)$ and $x_2(t) = \dfrac{dy}{dt}$.

For Laplace transforms, we start by writing the equation in the following form $\dot{\underline{x}} = A\underline{x}$ and $y = C\underline{x}$, with $A = \begin{bmatrix} 0 & 1 \\ -4 & -5 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, then the Laplace Transform becomes $y = C\left(sI - A\right)^{-1}\underline{x}(0)$. Since the eigenvalues of *A* are the values of *s* that make $\left(sI - A\right)$ singular, we see here the connection between eigenvalues and poles.

5. Repeat problem 3 for the following.

$$\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 169y = 0,\ y(0) = 2,\ \frac{dy}{dt}(0) = -10$$

Answer: $y(t) = 2e^{-5t}\cos\left(12t\right)$. When you use Laplace, things get kind of messy here.

6. Repeat problem 4 for the following:

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} 0 & 1 \\ -169 & -10 \end{bmatrix}\underline{x},\ \underline{x}(0) = \begin{pmatrix} 2 \\ -10 \end{pmatrix}$$

Answer: $\underline{x}(t) = \begin{pmatrix} 2 \\ -10 \end{pmatrix} e^{-5t}\cos\left(12t\right) + \begin{pmatrix} 0 \\ -24 \end{pmatrix} e^{-5t}\sin(12t)$. The most important thing here is that you see where "5" and "12" come from. Don't worry if you aren't yet totally happy about using the (complex) eigenvectors of the matrix and complex coefficients. Maybe the Laplace transform way is simpler here.

Applied Circuit & Controls Analysis There is a really cool magnetic levitation lab project at Berkeley. Check out this student video: https://www.youtube.com/watch?v=62AYgiB8EvQ.

Now check out this document: http://www-inst.eecs.berkeley.edu/~ee128/fa10/Labs/Lab4-Fa10.pdf.

The document suggests that the analog Controller (in the red box in Figure 3) corresponds to the following transfer function: $K_c\left(\frac{1+s/z}{1+s/p}\right)$.

7. Can you relate the real-valued parameters Kc, z, and p to the values of R1, R2, and C in the circuit?
8. Can you convert any other elements in Figure 3 into mathematical representations on a block diagram? We are looking for something with a little more detail than Figure 2, and with transfer functions, in place of words. You won't know any numbers, of course, but perhaps just the forms of the transfer functions (many of which will just be simple gains). If you manage to convert everything, you might be able to derive the form of the closed-loop response. This is a bit of a stretch at this stage in our work, but see what you can do.