ESE 406/505 & MEAM 513 - SPRING 2012
HOMEWORK #11
DUE by Wednesday 11-April-2012 (Late Pass Monday 16-April-2012)

1. Work problem 6.32 in the textbook. Add the following additional questions:

   (g) Replace the proportional compensator, "K", with a PID controller designed using the Ziegler-Nichols ultimate sensitivity tuning rules. What are the gain and phase margins of with the PID controller in the loop.

   (h) Now replace the PID controller with a lead-lag compensator of the form $K\frac{s+z}{s}\frac{T_1 s+1}{T_2 s+1}$, with $T_1 > T_2$. Be sure to satisfy these constraints with your design:

   - margins of at least PM=45° and GM=6dB;
   - loop gain less than -30dB at all frequencies above 10rps;
   - closed-loop damping ratio at least 0.35 on all poles.

   Try to maximize the loop gain at frequencies below 0.5 rps. Compare the value of the product Kz, which is the effective integral gain of your compensator, to the integral gain from Ziegler Nichols.

   Answers:

   a. K=80, when the closed-loop denominator is $\Delta_{CL} = \left(s^2+4\right)\left(s^2+9s+24\right)$.
   b. K=5.9.
   c. GM = (K for neutral stability) / (K nominal) = (80/5.9) = 13.5 = 22.6 db.
   d. K=30 for PM=65°. GM = (80/30) = 2.67 = 8.5 db. We might expect the damping ratio to be PM/100 ~ 0.65, but the actual damping ratio is more like 0.3. We have to be careful with the rough approximations sometimes.
   e. The root locus is the same, of course.
   f. The poles are the same but there is a zero in the closed-loop for figure (b), so we expect a bit faster response with more overshoot. Intuitively, in figure (b), we are low-pass filtering the measurement, so the feedback won't begin to reduce the control due to the response as quickly. This will result in more control initially, which gives faster response and larger overshoot.

2. Submit something that shows you are making progress on the design project. Simulink diagrams. Time histories. Short descriptions of what you are doing. The rest of the homework was made shorter than usual so that you would have time to make progress on the project. No bluffing here, please. Only submit something if you are actually making some progress.

3. (**ESE 505 / MEAM 513 Only**). This problem is "the exception that proves the rule." We said in lecture that "all" controls problems amount to figuring out how to get the loop gain

high at low frequency and then low at high frequency, without crossing 0dB too quickly. This is generally true, but we have already seen one problem this year where it wasn't true. Go back to homework #6 and remind yourself of the 747 yaw damper design. That design was well suited to the root-locus design technique, which we used to choose the best gain K=1.4. But we can also understand what the design accomplished in the frequency domain.

In particular, suppose we want to examine how the aircraft responds to gusty winds. In the state-space model, replace the sideslip state, β, which tells the direction the nose of the airplane is pointing relative to the direction it is flying, with $\beta + \beta_G$, where β_G represents changes in the wind direction due to gusts. Treat β_G as a new external variable. That is, we can now write the total aircraft yaw rate response as follows:

$$r = G_\delta(s)\delta + G_\beta(s)\beta_G$$

The feedback system generates rudder control from a combination of pilot input, δ_P, and feedback of yaw rate, as shown in the simulink drawing from HW#6.

Please submit three frequency response plots for this problem:

i. A loop bode plot for the feedback. Take a moment to study this graph. It doesn't look like our usual loop bode plot. The gain is above 0 dB only over a narrow region. Do you see how this is consistent with the design objectives? Identify the positive gain and phase margins on the graph. Careful--the "margin" command may give you negative margins, even though we know that the system is stable.

ii. The yaw rate response to pilot input, $\dfrac{r}{\delta_P}$, showing both open-loop and closed-loop frequency responses. Note that the spike associate with the lightly damped dutch roll mode has been suppressed by the feedback. We know that this suppression also could have been achieved using a notch filter on the pilot input.

iii. The yaw rate response to gust input, $\dfrac{r}{\beta_G}$, showing both the open-loop and closed-loop frequency responses. Again the dutch roll mode spike is removed. But we could not have done this with notch filters. Be sure you understand what is going on here.