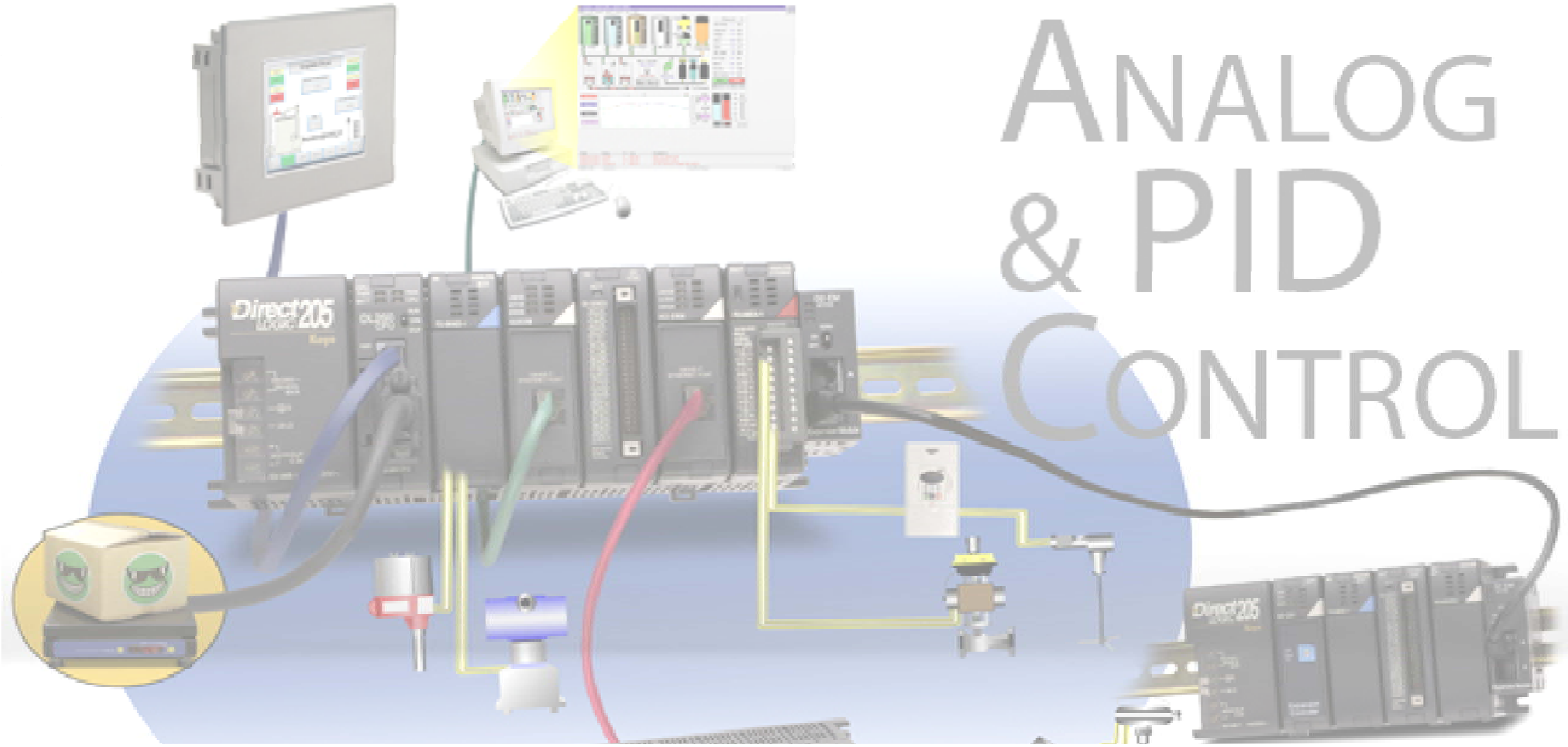
Introduction to PID Control – Part 1



ESE 505 & MEAM 513

Bruce D. Kothmann

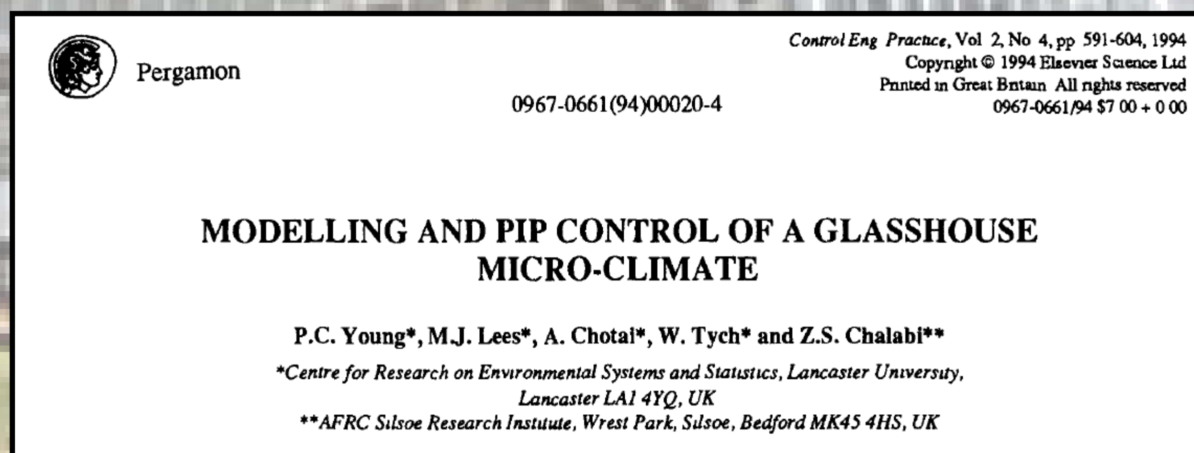
2014-02-17

Penn Engineering

First-Order Glasshouse Example

Suggestion for Motivated Students

- Read the Paper
- Write Down Stuff You Don't Understand
- Ask Lots of Questions!
- Read Again @ End of Semester!



Pergamon

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MODELLING AND PIP CONTROL OF A GLASSHOUSE MICRO-CLIMATE

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Temperature Dynamics Inside Glasshouse

- “Complex, non-linear model”
- “11 coupled, non-linear, first-order differential equations”
- “Used to design and evaluate a series of controllers for the internal climate of the glasshouse”
- “Necessary to develop a reduced-order, linearized control model which adequately describes the small perturbation dynamics of the system.”

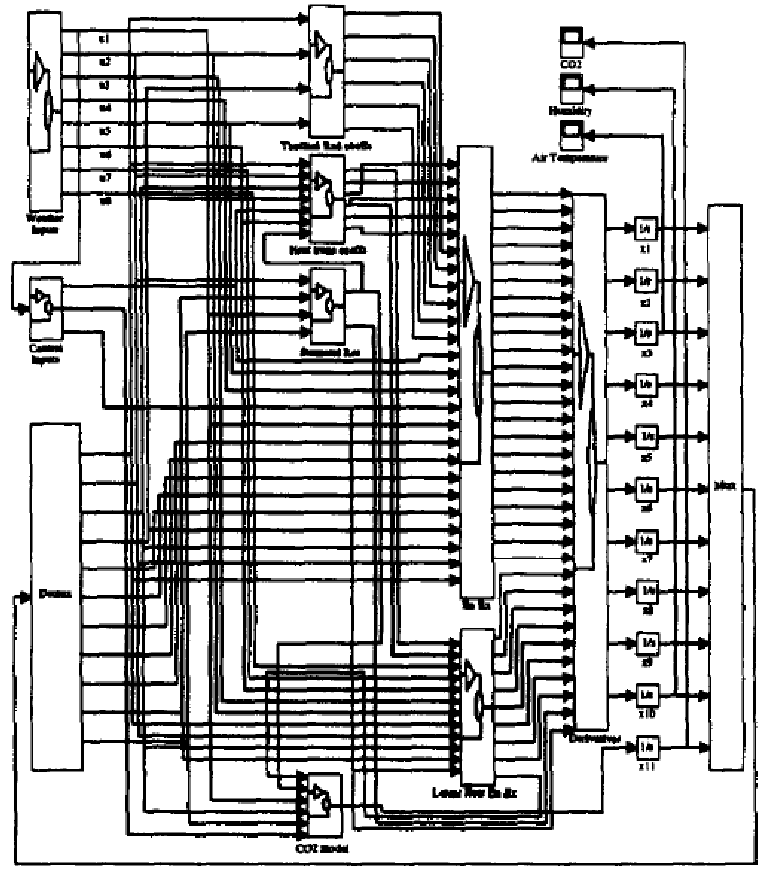
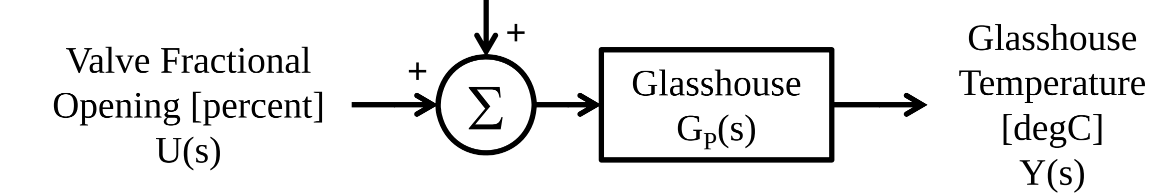


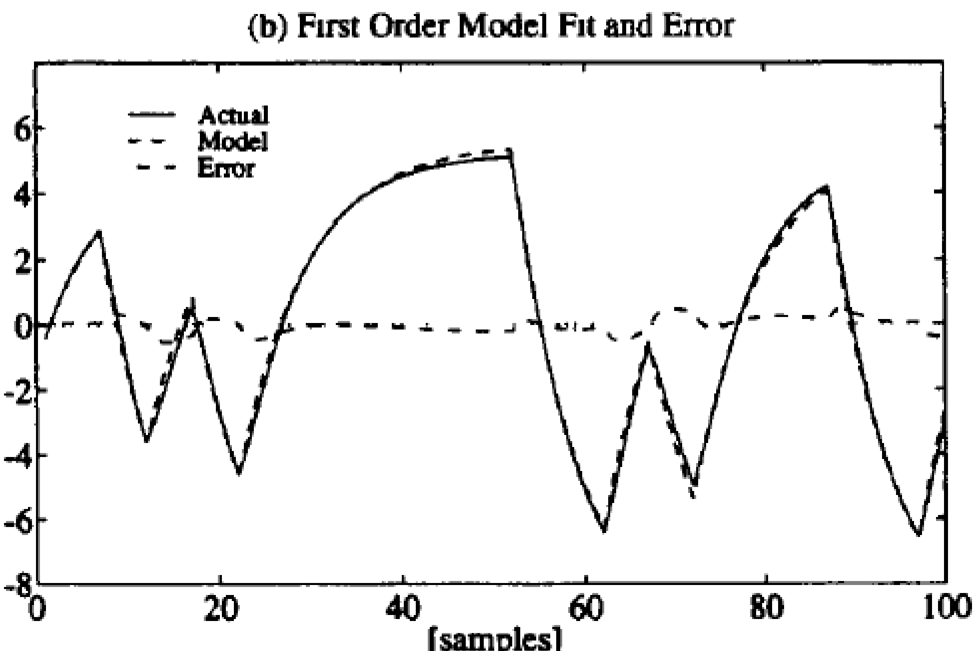
Fig 1 SIMULINK representation of the glasshouse climate model

Linear Approximation to Plant Dynamics

Complex Non-Linear Models Often Well-Approximated by Low-Order Linear Systems!

Time Delay Often Useful Approximation to Neglected Dynamics





1 sample = 600 seconds

$$G_P(s) = \frac{Ae^{-T_d s}}{\tau s + 1}$$

A = 0.32 degC/percent

Td = 600 sec (10 min)

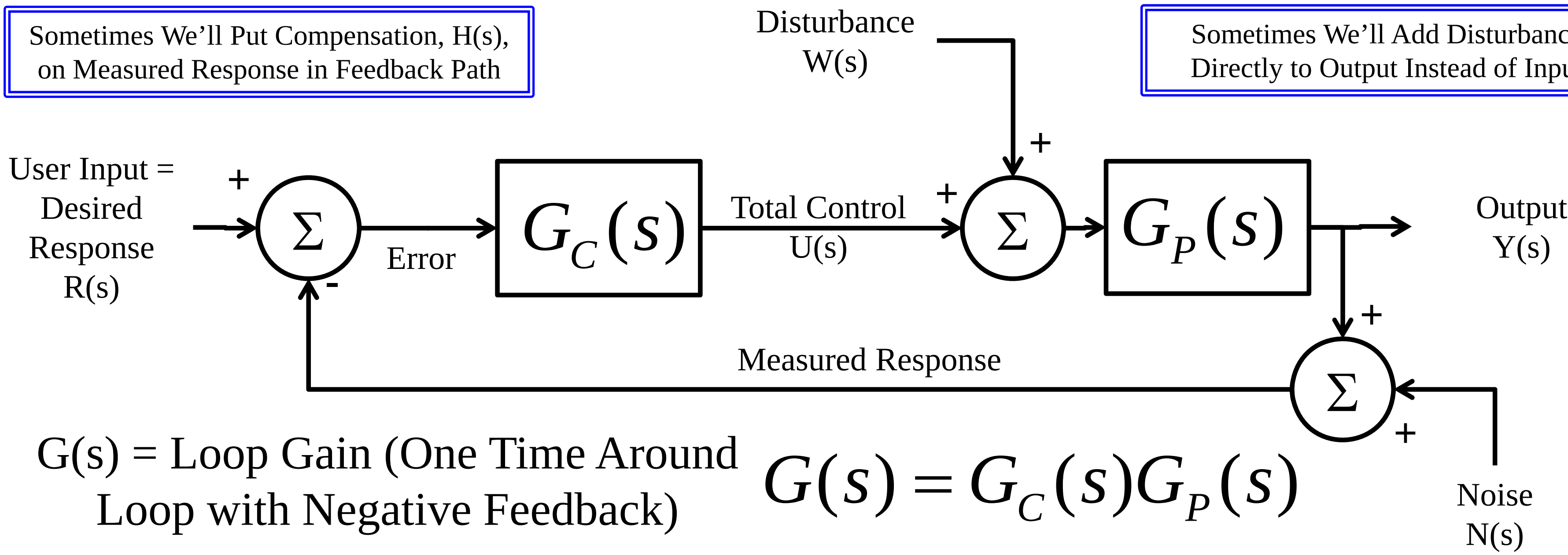
τ = 4440 sec (74 min)

We'll Simplify the Discussion by Ignoring Delay…

Design Requirements

- Perfect Temperature Regulation
  - We Interpret To Mean Zero Steady-State Error
- Reject Daily Temperature Swings (Solar Heating & Atmospheric Temperature)
  - Want Substantial Disturbance Rejection For 1-Per-Day Disturbances (Frequency = $2\pi/(24*3600)$ = 0.000072 rad/sec)
  - We'll Learn the Math to Analyze This Formally After Spring Break
- "Reduced Actuator Operation (to Minimize Wear)"
  - Roughly This Means: Don't Set Gains Higher Than Necessary to Meet Other Requirements
  - Many More Advanced Ideas Here → Google "Optimal Control" (Minimum Fuel Usage Very Important on Spacecraft)
- Fast Response → "Rise Time" ~ 1 Hour
  - I Made This Up, Based on Data in Paper ("Typical" for Tomatoes?)
- Very Little (No?) Overshoot
  - Usually Think of Overshoot Being Related to Closed-Loop Damping Ratio
  - We'll See Here That This Can Be A Little More Complicated

We'll Usually Study System Like This



G(s) = Loop Gain (One Time Around Loop with Negative Feedback)

$$G(s) = G_C(s)G_P(s)$$

$$Y(s) = \frac{G_C(s)G_P(s)}{1+G_C(s)G_P(s)}\left[R(s) - N(s)\right] + \frac{G_P(s)}{1+G_C(s)G_P(s)}W(s)$$

Note: All Architectures Have Same Loop Gain & Same Denominator = $\Delta_{CL}(s)$!

$$\Delta_{CL}(s) = 1 + G(s)$$

Let's Add Integral Feedback

P+I Feedback Very Common for First-Order Systems

$$G_C(s) = \frac{K_I}{s} + K_P \quad G_P(s) = \frac{A}{\tau s + 1}$$

$$Y(s) = \underbrace{\frac{A(K_P s + K_I)}{\tau s^2 + (AK_P + 1)s + AK_I}}\left[R(s) - N(s)\right] + \underbrace{\frac{As}{\tau s^2 + (AK_P + 1)s + AK_I}}W(s)$$

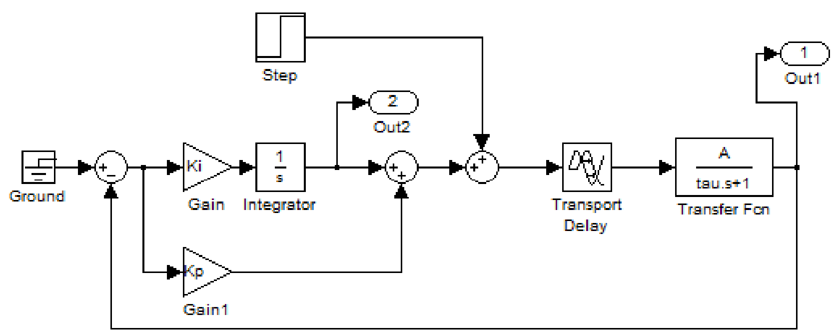
Steady-State Gain on Inputs Is Exactly Unity (Perfect Steady Tracking)

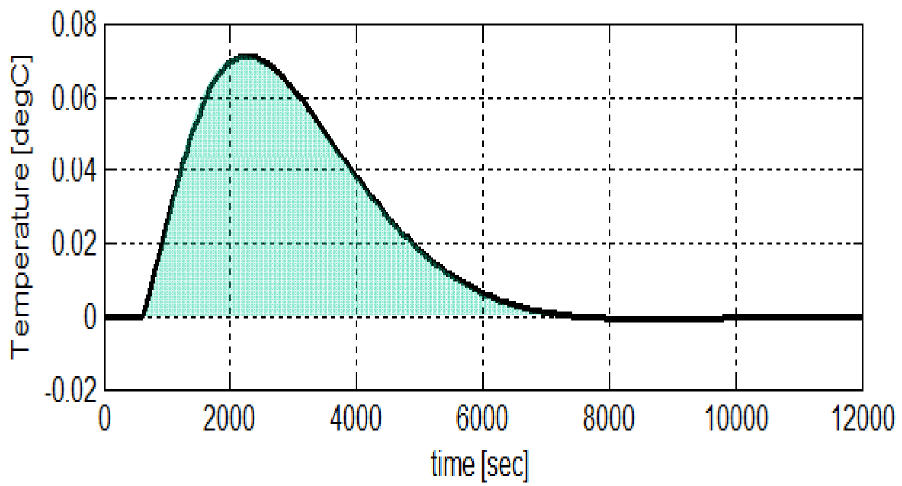
Steady-State Gain on Disturbances Is Exactly Zero (Perfect Steady Disturbance Rejection)

Derivative Feedback Often Not Necessary on First-Order Systems
(We'll Discuss Derivative Feedback Next Lecture)

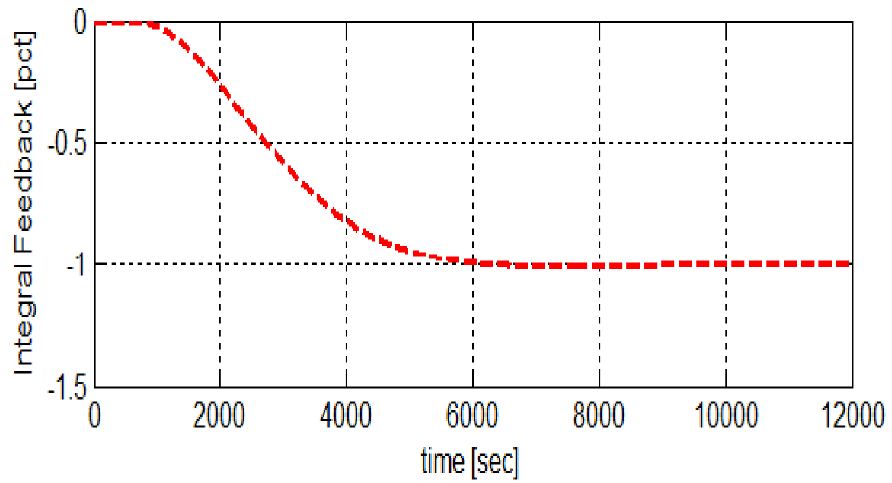
How Does Integral Feedback Work?

Unit Step Disturbance Response





(Area Under Temperature Curve) * (Integral Gain) = Control Needed to Cancel Steady Disturbance, Resulting in Zero Steady Error

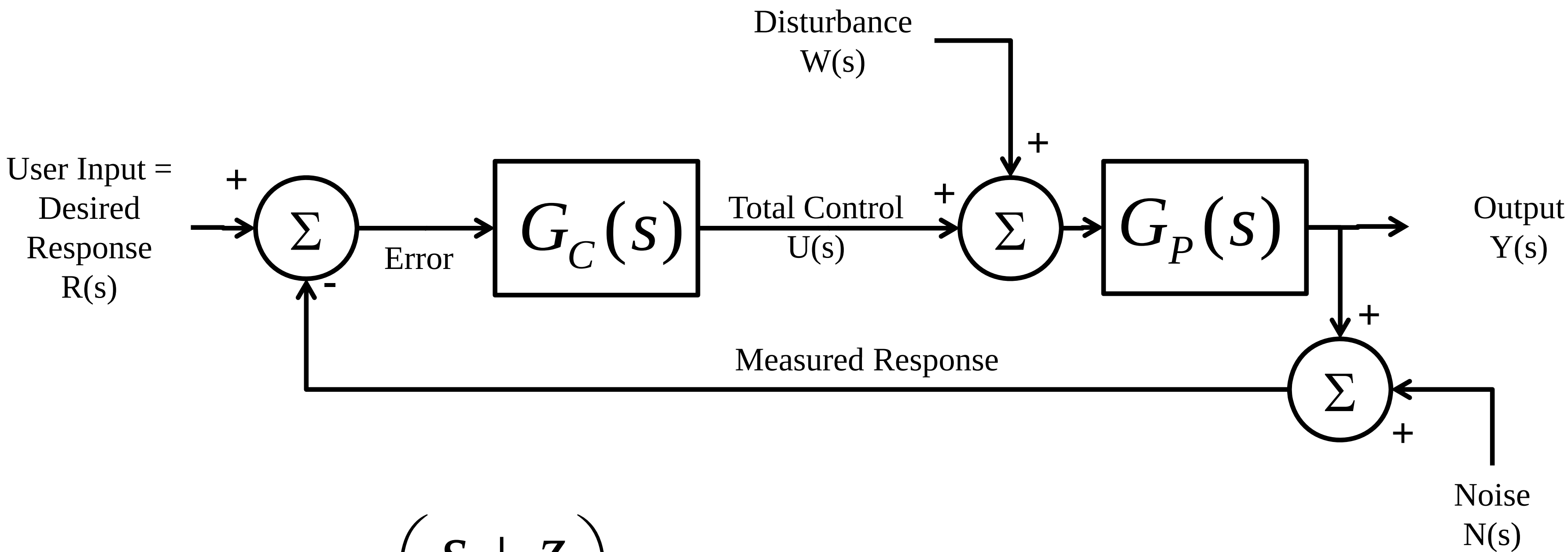


Proportional + Integral Feedback Comments

Closed-Loop Pole Locations $\Delta_{CL}(s) = \tau s^2 + \left(AK_P + 1\right)s + AK_I = 0$

- Theoretically, Can Place Closed-Loop Poles Anywhere We Want
  - KI Controls Natural Frequency
  - KP Controls Damping
- Practically, We Have Some Serious Limits
  - Thermal Control System Has Limited Heating Capability (Can't Add Energy Quickly Enough to Increase Temperature Arbitrarily Fast)
  - Remember Our "Reduced Actuator Operation" Requirement!
  - We Ignored Effects of Delay (Remember—Always Destabilizing!)
- Actual Design Chosen for Good Rejection of 1-per-day Disturbances, Quick Response & Low Actuation Use
  - KP = 9.9
  - KI = 0.0046 (~17X Larger Than with KI Alone without KP!)

Pade Approximation of Delay for Root Locus

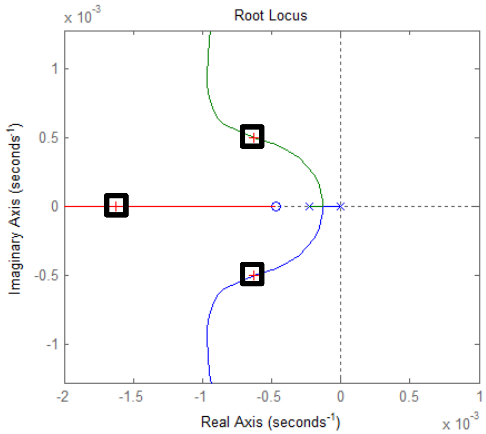


$$G_C(s) = K_P\left(\frac{s+z}{s}\right)$$

$$z = \frac{0.0046}{9.9}$$

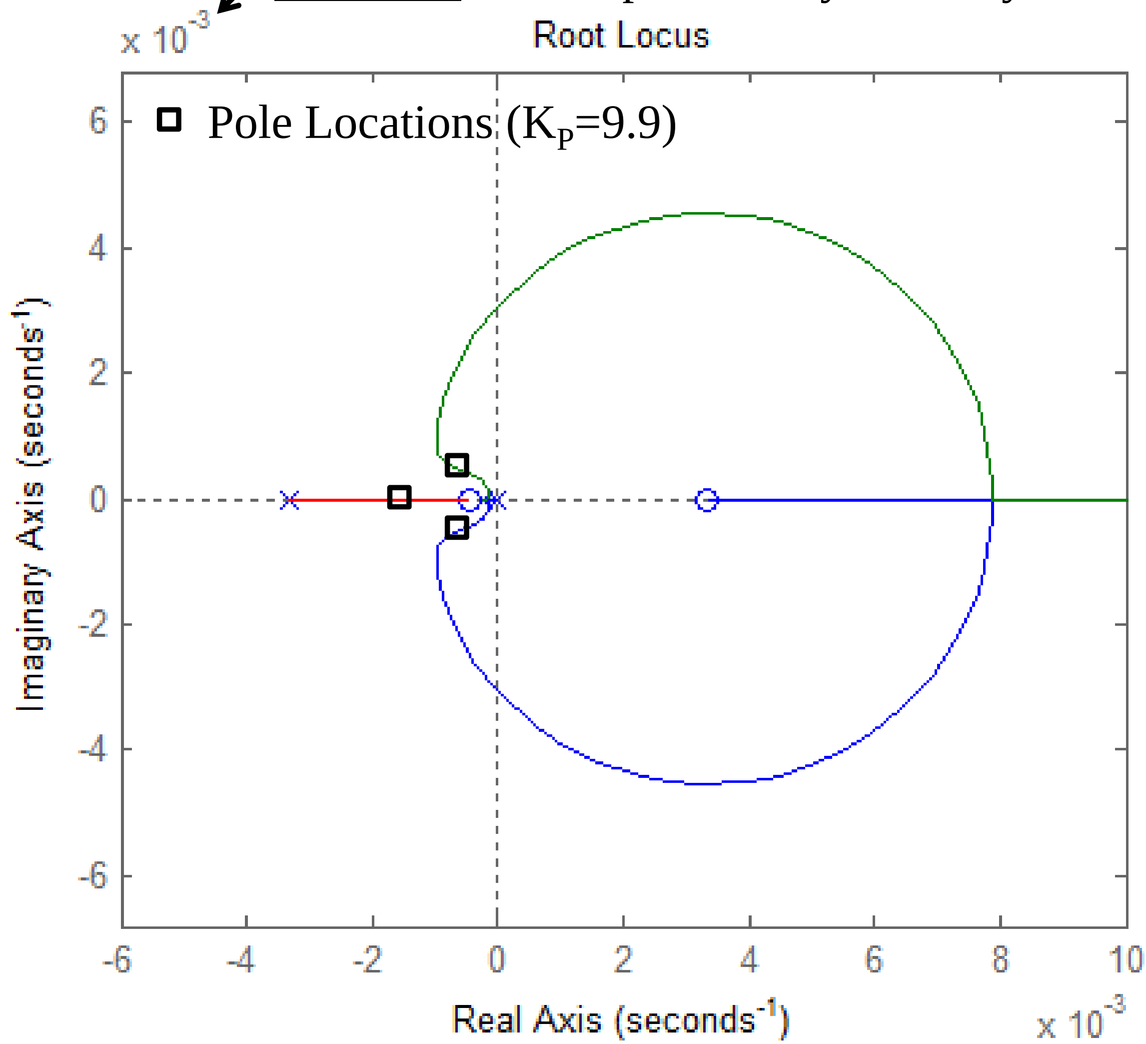
$$G_P(s) \approx \left(\frac{A}{\tau s+1}\right)\left(\frac{-0.5T_d s+1}{0.5T_d s+1}\right)$$

Root Locus on K_P (with $z = K_I/K_P$ Fixed)



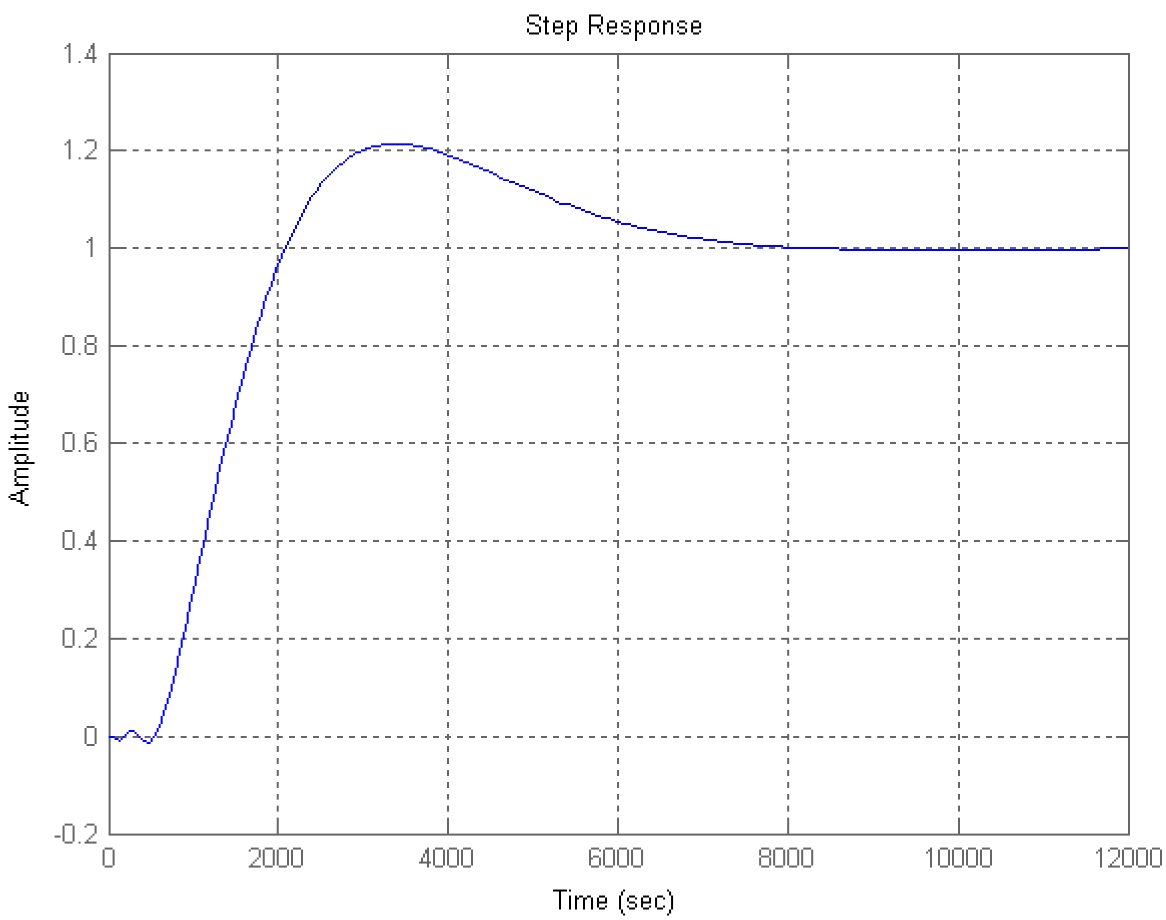

```
%
% plant model with Pade
%
Gp=tf(0.32,[4440 1]);
set(Gp,'InputDelay',600);
Gpx=pade(Gp);
%
% P+I compensator
%
Gc = tf([1 0.0046/9.9],[1 0]);
%
% root locus
%
rlocus(Gpx*Gc);
set(gcf,'Color','w');
axis([-0.006 0.01 -0.006 0.006]);
axis('equal');
```

RECALL: 0.007 rps ~ 100 cycles / day !

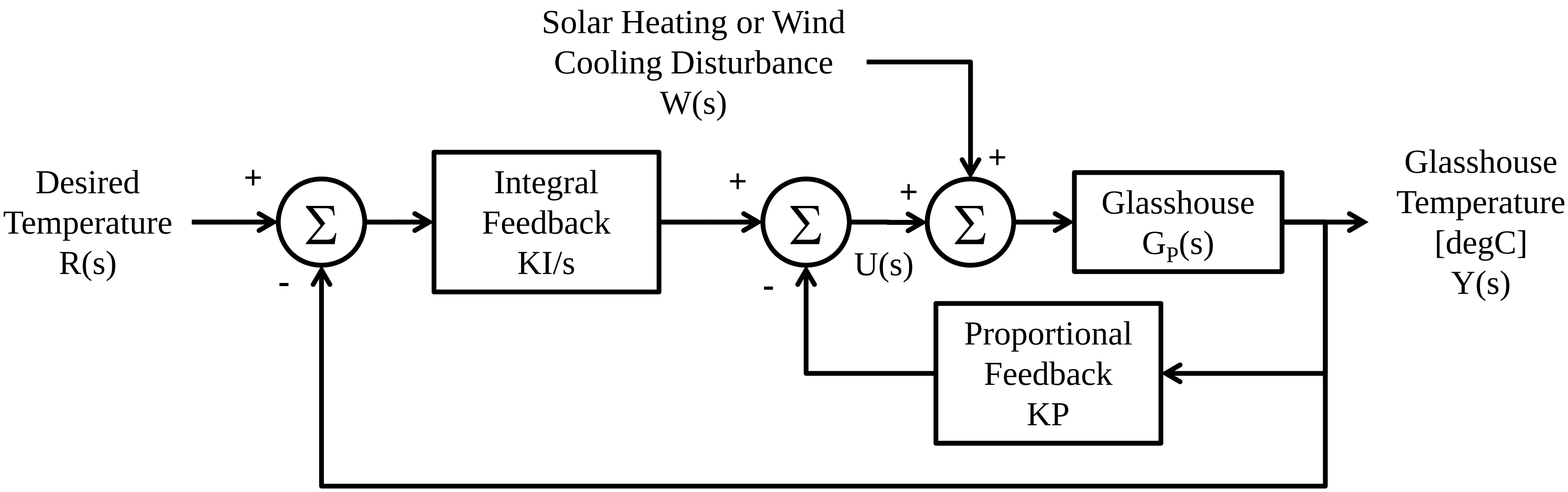


Step Response for Final Controller

- Oops! Zero in Closed-Loop Generates More Overshoot Than Desired, Despite Very High Damping Ratio!
- Very Fast Response Suggests Possible Over-Use of Actuator!

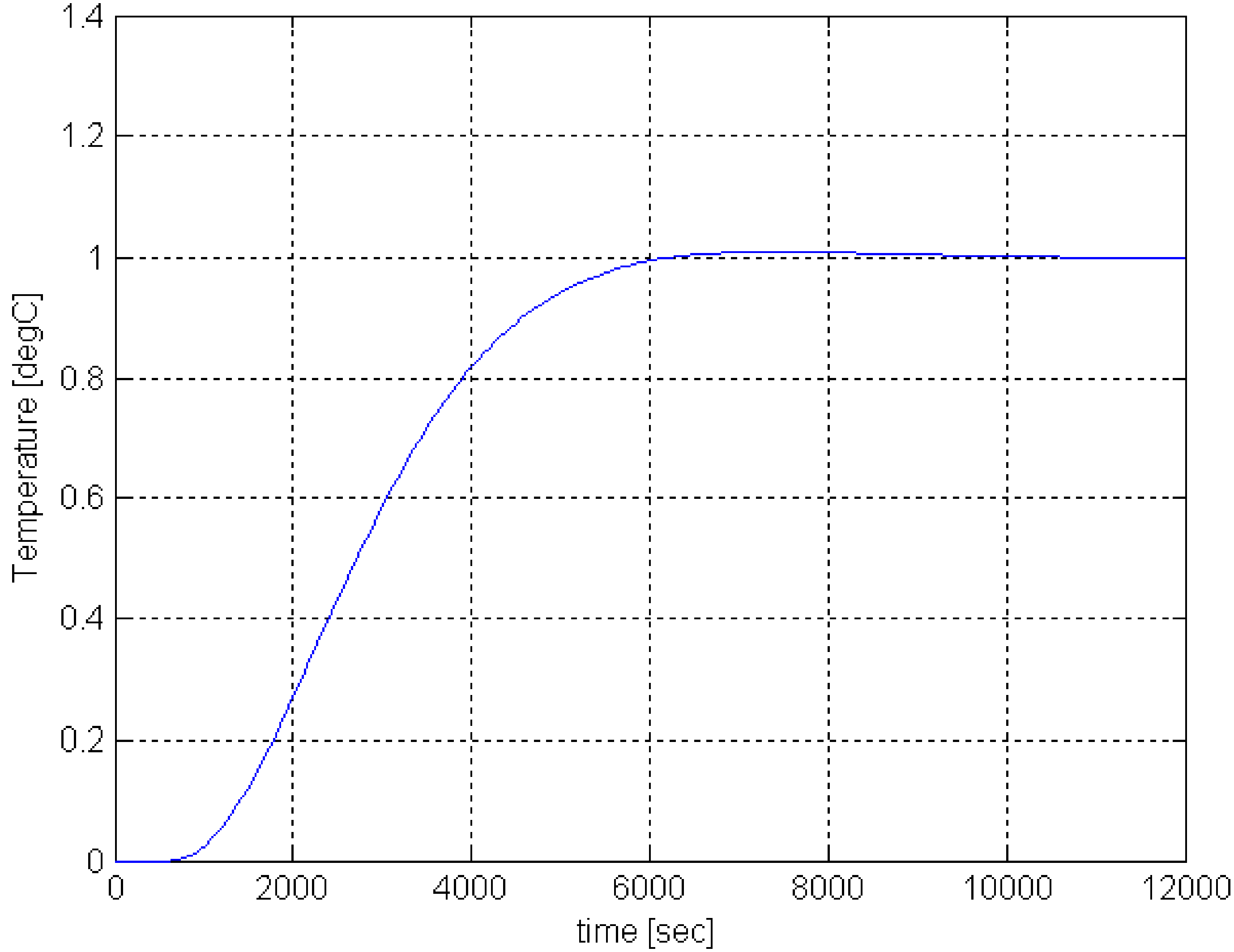


Simple Solution : Put KP on Feedback Only

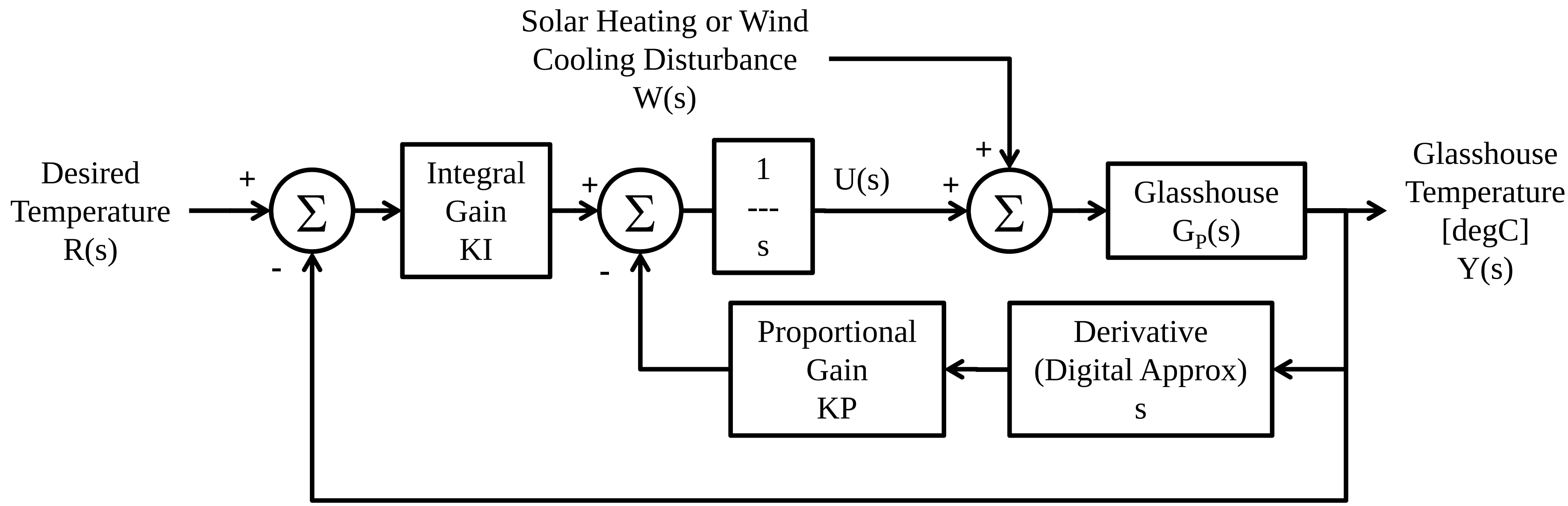


- Closed-Loop Denominator & Disturbance Rejection Unchanged
- Numerator of Y(s)/R(s) Has No Zero with This Architecture
  - Lower Actuator Usage (Integrator Has to Spool Up Following Step Command)
  - Overshoot Eliminated with Slightly Longer Rise Time
- Overshoot Issues Could Also Be Eliminated Using Feed-Forward
  - This is an Advantage of Architectures Like Explicit Model Following (Candidate #3)
  - But There is Added Complexity (→ Cost) in Implementation

Step Response with Modified Final Design



More Details of Actual Implementation…



- Proportional Feedback Implemented as Integrated Derivative!
- Integrator Output Limited
  – Prevents "Integrator Windup" With Persistent Steady Errors When Heating System Unable to Keep Up with Extreme Conditions
  – MANY Other Anti-Windup & Other Limiting Schemes Exist…

Actual Implementation Using Digital Computer

$$\frac{du}{dt} = K_I\left(y - r\right) + K_p \frac{dy}{dt}$$

$$\frac{u(t) - u(t - T_s)}{T_s} \approx K_I\left(y(t) - r(t)\right) + K_p \frac{y(t) - y(t - T_s)}{T_s}$$

$$u(t) = u(t - T_s) + T_s K_I\left(y(t) - r(t)\right) + K_p\left(y(t) - y(t - T_s)\right)$$

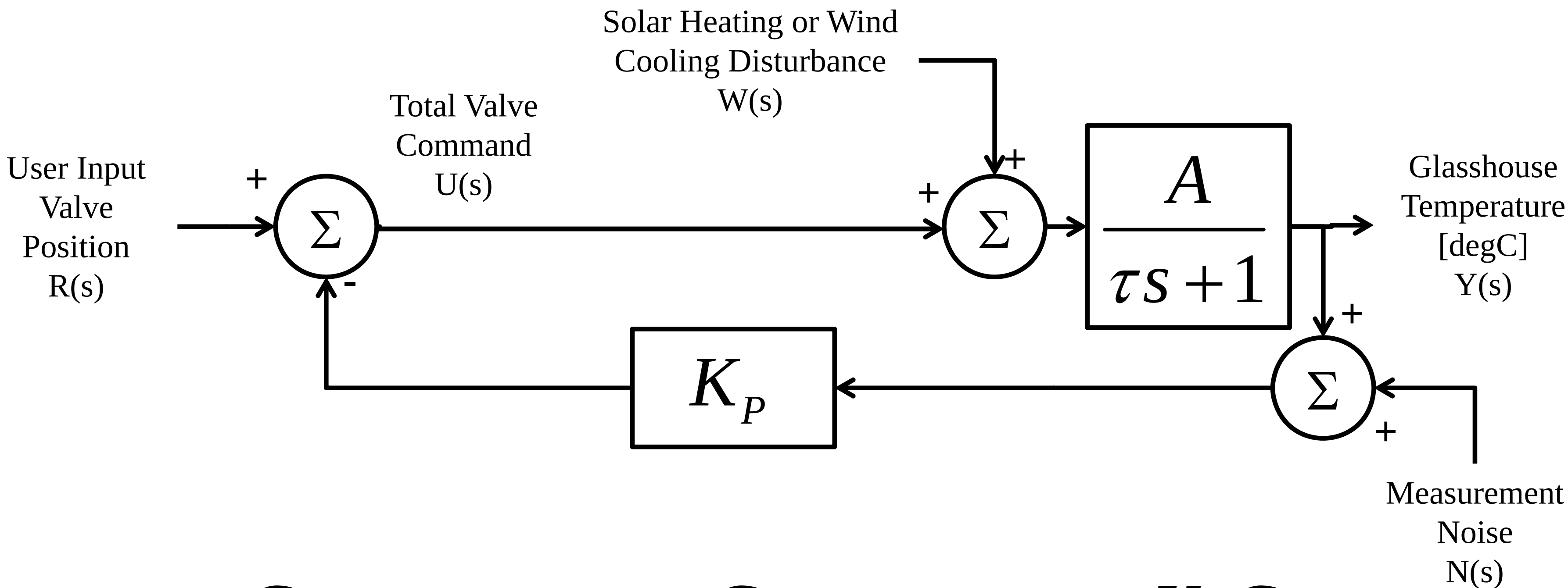
New Control = Old Control + Linear Combination of New Measurement, Old Measurement, New Command

We Need to Learn How to Do This Carefully!

Appendix

Other Possible Control Architectures

Alternate Architecture #1 : KP in Feedback Loop

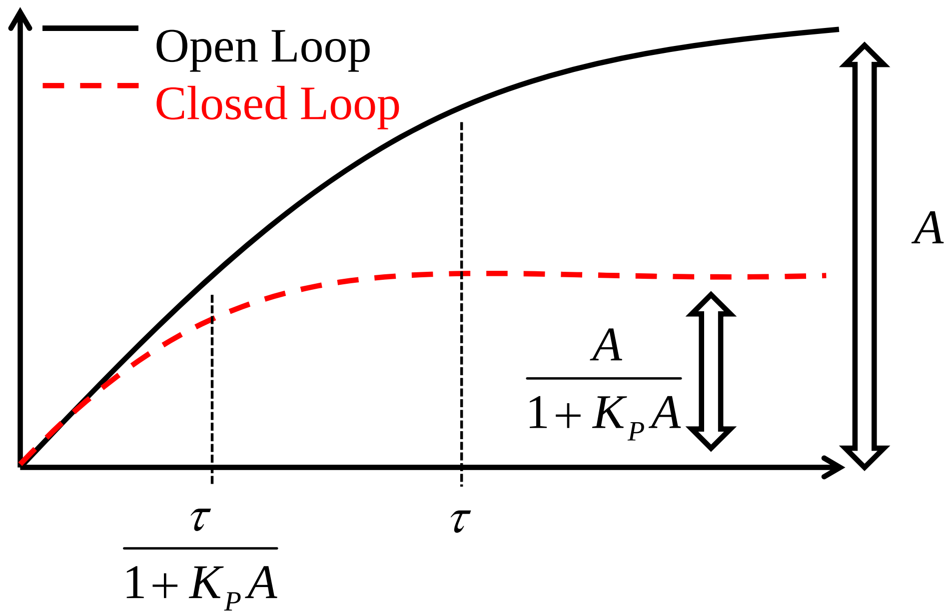


$$Y(s) = \frac{G_P}{1+K_P G_P} R(s) + \frac{G_P}{1+K_P G_P} W(s) - \frac{K_P G_P}{1+K_P G_P} N(s)$$

$$Y(s) = \frac{A}{\tau s+1+K_P A}\left[R(s)+W(s)\right] - \frac{A K_P}{\tau s+1+K_P A} N(s)$$

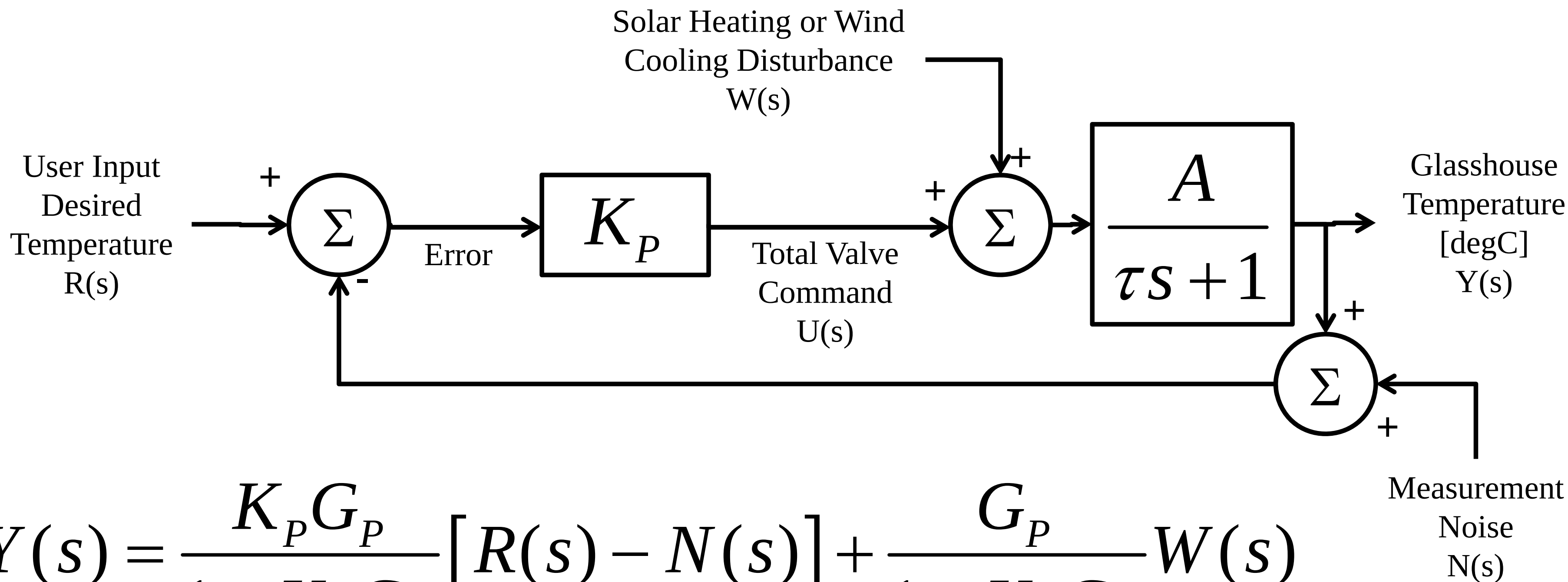
Comments on Architecture #1

Comparison of Unit Step Responses of Open-Loop & Closed-Loop Systems



- Proportional Feedback Moves Pole Left in Complex Plane → Faster Response Time
- High Gain Reduces Steady-State Response Amplitude to Both Inputs & Disturbances
  - User Needs to Know Gain When Setting Input to Reach Desired Temperature
- High Gain Results in Steady Response to Measurement Noise (Approaching 1deg/deg for Very Large Gain)

Alternate Architecture #2 : KP in Forward Loop

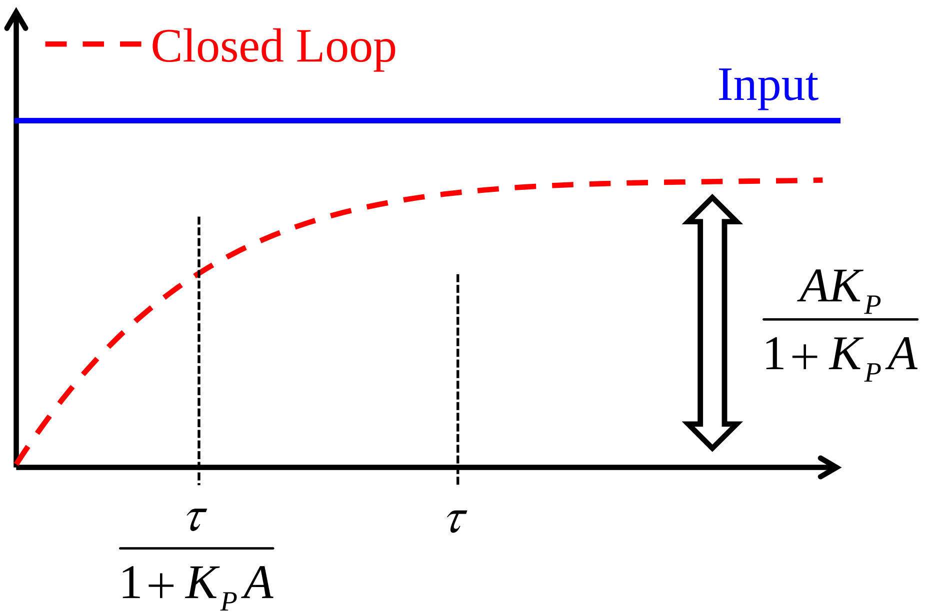


$$Y(s)=\frac{K_P G_P}{1+K_P G_P}\left[R(s)-N(s)\right]+\frac{G_P}{1+K_P G_P}W(s)$$

$$Y(s)=\frac{AK_P}{\tau s+1+K_P A}\left[R(s)-N(s)\right]+\frac{A}{\tau s+1+K_P A}W(s)$$

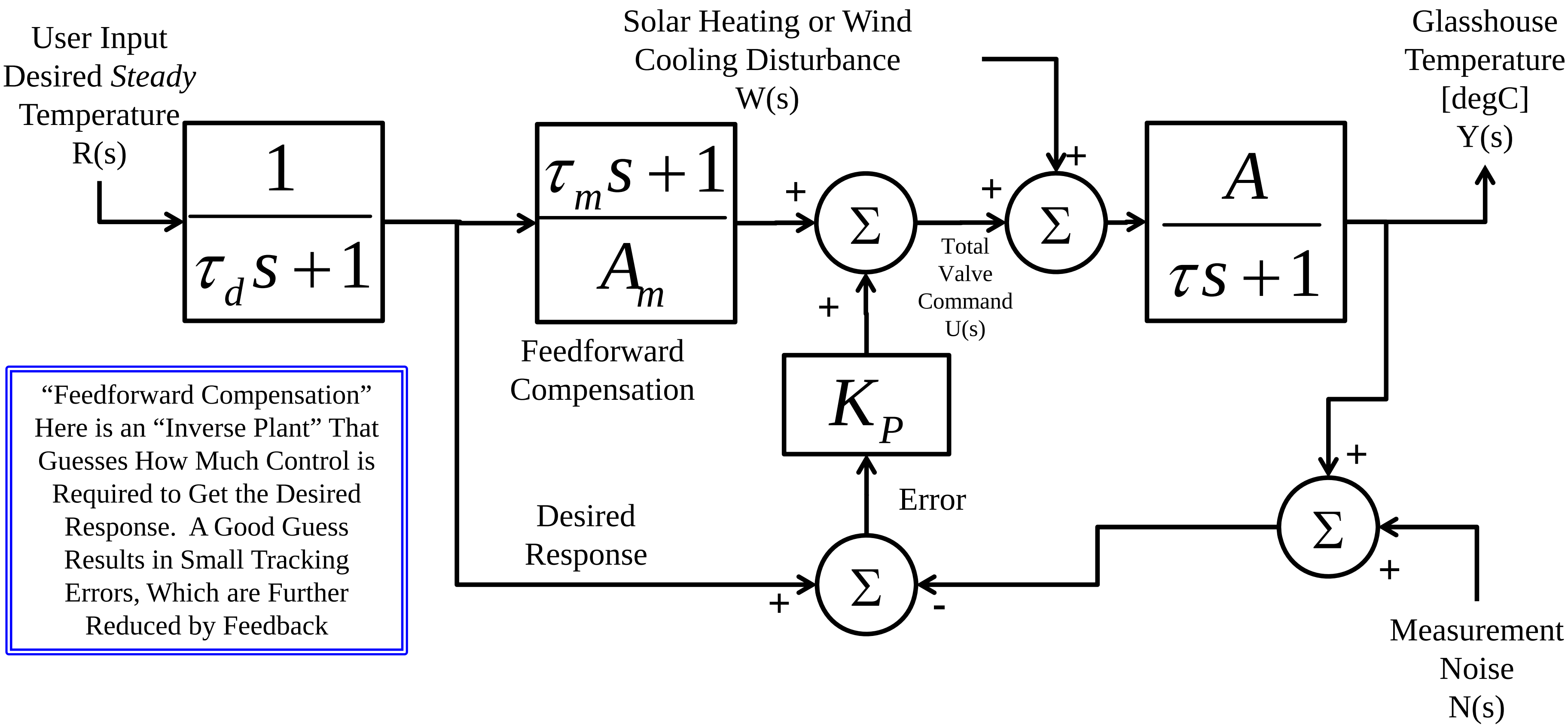
Comments on Architecture #2

Unit Step Responses of Closed-Loop System



- Proportional Feedback Moves Pole Left in Complex Plane → Faster Response Time (Same As Architecture #1)
- High Gain Reduces Steady-State Response Amplitude to Disturbances (Same As Architecture #1)
- High Gain Results in Same Steady Response to User Inputs & Measurement Noise (Approaching 1deg/deg for Very Large Gain)
- Still Don't Get "Perfect" Tracking to Desired Temperature

Alternate Architecture #3 : Explicit Model Following



"Feedforward Compensation" Here is an "Inverse Plant" That Guesses How Much Control is Required to Get the Desired Response. A Good Guess Results in Small Tracking Errors, Which are Further Reduced by Feedback

$$Y(s)=\left\{\left[\frac{A}{\tau s+1+K_P A}\right]\left[\frac{1}{\tau_d s+1}\right]\left[\frac{\tau_m s+1}{A_m}\right]+\left[\frac{AK_P}{\tau s+1+K_P A}\right]\left[\frac{1}{\tau_d s+1}\right]\right\}R(s)+\ldots$$

Disturbance & Noise Same As Architecture #2

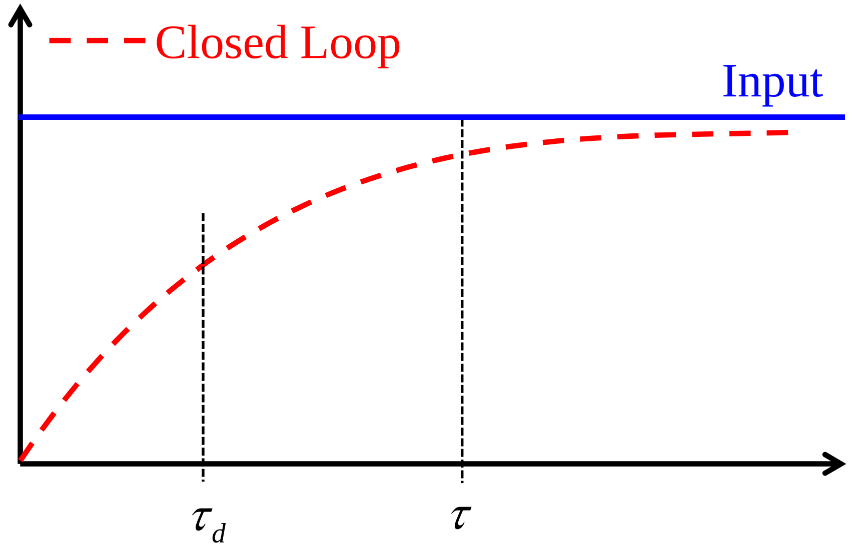
Comments on Architecture #3

Unit Step Response of Closed-Loop System

$A_m = A$ & $\tau_m = \tau$

$\Downarrow$

$$Y(s) = \left[\frac{1}{\tau_d s + 1}\right] R(s) + \ldots$$



- Good Guess at "Inverse Dynamics" → Very Good Response
  - Response Time Set Primarily by Command Model
  - Nearly Perfect Tracking (For Accurate Model & No Disturbances or Noise)
- High Gain Reduces Steady-State Response Amplitude to Disturbances (Same As Architecture #1 & #2)
- High Gain Results in Steady Response to Measurement Noise (Approaching 1deg/deg for Very Large Gain) (Same As #1 & #2)