ESE 406 - SPRING 2012
HOMEWORK #4
DUE 8-Feb-2011 (with Late Pass 13-Feb-2011)

Problem 1 You should be able to solve all parts of problems 3.19 through 3.22 in the textbook. Submit solutions to the following problems from the textbook:

a. **ESE 406** - Problem 3.20(a). Answer: $$\frac{Y}{R} = \frac{G_1}{1+G_1} + G_2.$$

ESE 505 & MEAM 513 - Problem 3.20(b). Answer: $$\frac{Y}{R} = G_7 + \frac{G_1G_3G_4G_6}{(1+G_1G_2)(1+G_4G_5)}$$

b. Problem 3.21(b). Answer: $$\frac{Y}{R} = \frac{b_1s^2 + b_2s + b_3}{s^3 + a_1s^2 + a_2s + a_3}.$$
Note that there are infinitely many possible block diagrams that correspond to a given transfer function. This particular form of block diagram is called the "observer canonical form". The coefficients of the numerator and denominator polynomials appear as gains. This form is discussed in Chapter 7.

Problem 2 Work problem 3.23, all parts, in the textbook. Note that an overshoot of 25% corresponds to a damping ratio of 0.4.

ESE 406 - For part (c), simply take the Laplace transform of the equations in parts (a) and (b), with zero initial conditions, and substitute to eliminate the current.

ESE 505 & MEAM 513 - For part (c), first write a state-space version of the equations. Then take the Laplace transform, with zero initial conditions, to get the transfer function.

Answers:

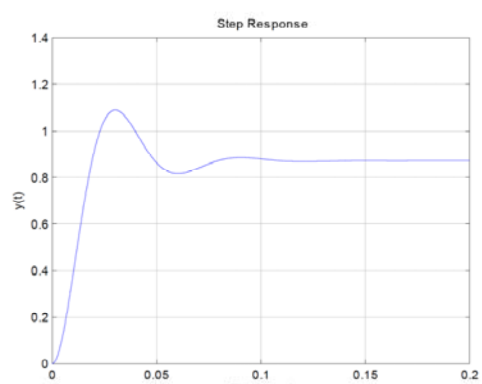
a) $$v_1(t) = L\frac{di}{dt} + Ri + \frac{1}{C}\int i(t)dt.$$

b) $$v_2(t) = \frac{1}{C}\int i(t)dt.$$

c) $$\frac{v_2(s)}{v_1(s)} = \frac{\frac{1}{sC}}{sL + R + \frac{1}{sC}} = \frac{1}{s^2LC + sRC + 1}$$

d) $40\ \Omega$

Problem 3 (**ESE 406 only**) Work problem 3.25 in the textbook. The overshoot and settling time specifications correspond to damping ratio of 0.4 and natural frequency of about 114 rad/sec. Verify your design by plotting a step response in MATLAB. What is the closed-loop steady-state gain of the system, in terms of K and a? Check your answer by looking at the step response graph. Answer: a=67; K=113. See graph at right for step response.



Problem 3 (**ESE 505 & MEAM 513 only**) A macro-economic model[1] for changes in the total contribution, $y(t)$, of one sector (for example, automotive industry) of the economy to total GDP can be written as:

$$\frac{dy(t)}{dt} = [a + u(t)]\, y(t) - \frac{1}{F}[y(t)]^2$$

where a and F are positive constants, and $u(t)$ is an input that depends on "the rates of change of variables shifting the sector's demand and supply equations, e.g. real income, real money, the real wage rate, the real price of capital, technological change, etc."

a. If the trim control is some positive constant u_o, find an expression for the non-zero trim output, y_o. Assume that the system is trimmed at this condition for parts (b) and (c).

b. Find the linear ODE that describes the relationship between small perturbations in control and output.

c. Sketch the time response of the perturbation in output to a small positive perturbation in the input.

Problem 4 You don't have to submit anything for this problem. It is just a MATLAB & SIMULINK familiarization exercise. Please download the files HW04.m & HW04sim.mdl from blackboard. Go through the HW04.m script and run it. You will need to understand how the commands in that script work in many future homework assignments. Use the internet, MATLAB's help system, or office hours if you need help.

[1] Zellner, Arnold, "My Experiences with Nonlinear Dynamic Models in Economics," *Studies in Nonlinear Dynamics and Econometrics*, Volume 6, Issue 2, 2002.