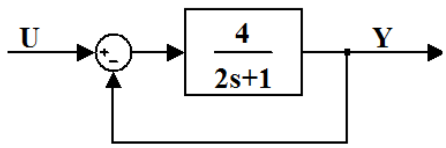
NAME ________________________

ESE505 / MEAM 513 - SPRING 2014 – FINAL EXAM

CLOSED NOTES & CLOSED BOOK : CALCULATORS (NO SMARTPHONES)

- Choose the one best answer for each question by *circling the letter*.
- A correct answer is worth 3 points.
- No answer is worth 1 points.
- An incorrect answer is worth 0 points. Random guessing will lower your grade, on average.
- You must work completely independently.



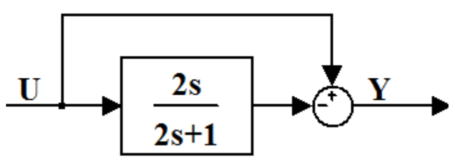
1. The transfer function corresponding to the above block diagram is…

A. $\frac{Y(s)}{U(s)} = \frac{2}{s+2.5}$

B. $\frac{Y(s)}{U(s)} = \frac{6s}{s+4}$

C. $\frac{Y(s)}{U(s)} = \frac{-2s+3}{2s+1.5}$

D. None of the above.



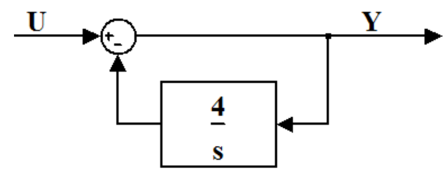
2. The transfer function corresponding to the above block diagram is…

A. $\frac{Y(s)}{U(s)} = \frac{2}{s}$

B. $\frac{Y(s)}{U(s)} = \frac{1}{s+2}$

C. $\frac{Y(s)}{U(s)} = \frac{1}{2s+1}$

D. None of the above.



3. The transfer function corresponding to the above block diagram is…

A. $\frac{Y(s)}{U(s)} = \frac{s-4}{s}$

B. $\frac{Y(s)}{U(s)} = \frac{s}{s+4}$

C. $\frac{Y(s)}{U(s)} = \frac{s-4}{s^2+4}$

D. None of the above.

4. The Nyquist frequency of a discrete-time process is...

A. ...the maximum frequency at which a digital computer can execute the control software.

B. ... equal to half the sampling frequency.

C. ...the frequency at which the amplitude of a signal is 3dB below the steady gain.

D. ...the frequency at which the gain of a digital compensator is 0dB (unity gain).

5. Which of the following is LEAST CORRECT concerning our study of discrete-time dynamic systems?

A. The discrete-time equivalent of a linear differential equation is a linear difference equation.

B. The discrete-time equivalent of the Laplace Transform is the Z-transform.

C. When a continuous-time signal is converted to discrete-time by sampling, the Laplace-transform pole is exactly mapped to a Z-transform pole according to $z = e^{Ts}$.

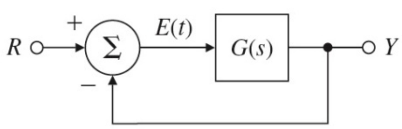
D. Aliasing won't exist as long as the sample time, T, is less than 16 ms (aka one "svedberg").

6. Which of the following is LEAST CORRECT concerning our study of discrete-time dynamic systems?

   A. The discrete-time equivalent of a linear differential equation is a linear difference equation.
   B. The discrete-time equivalent of the Laplace Transform is the Z-transform.
   C. When a continuous-time signal is converted to discrete-time by sampling, the Laplace-transform pole is exactly mapped to a Z-transform pole according to z=cos(πTs).
   D. The stability boundary for poles of discrete-time systems is the unit circle.

7. Which of the following is LEAST CORRECT concerning "Tustin's Method"?

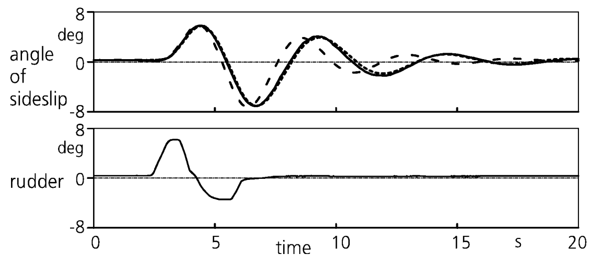
   A. It is a useful way to convert a continuous-time transfer function into a discrete-time transfer function.
   B. It preserves the exact discrete-time stability boundary.
   C. It exactly matches continuous-time frequencies.
   D. It can be derived by considering trapezoidal integration as an approximation to "1/s".



8. Suppose that the system shown in the figure above has reached neutral stability and exhibits sustained oscillations at some frequency, ω. From this observation, we can infer that…

   A. $\dfrac{G(j\omega)}{1+G(j\omega)} = 0$
   B. $G(j\omega) \to \infty$
   C. $G(j\omega) = -1$
   D. $G(j\omega) = 0$

All of the problems in this column refer to the following dynamic response:



9. The figure above[1] shows the sideslip response (top graph) of a Transall C-160 aircraft to a pilot rudder input (bottom graph). The graph includes flight data (solid line) and a linear model (dashed line). (Ignore the dotted line which is very nearly on top of the solid line.) Which of the following is the MOST ACCURATE inference about the stability of the system?

   A. The system is unstable without active stabilization by the pilot.
   B. The system is stable but lightly damped.
   C. The system is highly damped, most likely with 2 poles on the negative real axis.
   D. We can only judge stability from unit step inputs, not strange pilot inputs like this one.

10. Which of the following is the MOST ACCURATE inference about the differences between the model and the data?

   A. The model *over*-estimates the frequency of the poles that are clearly visible in the response.
   B. The model *under*-estimates the frequency of the poles that are clearly visible in the response.
   C. The model *over*-estimates the steady-state gain of the transfer function.
   D. The model *under*-estimates the steady-state gain of the transfer function.

---

[1] "Aerodynamic Modeling and System Identification from Flight Data—Recent Applications at DLR," *Journal of Aircraft*, 41:4, 2004.

Table 2 Eigenvalues with outer and inner feedback loops closed

$-0.0006 \pm 0.0140i$	Phugoid
$-2.0107 \pm 1.9866i$	Pitch attitude
$-1.9925 \pm 2.0038i$	Roll attitude
$-4.0000 + 0.0000i$	Yaw rate
$-4.0000 + 0.0000i$	Vertical velocity

11. The table above[2] shows the locations in the complex plane of the closed-loop poles[3] of a helicopter. Which of the following is the LEAST ACCURATE about the poles?

    A. The *Yaw rate* pole is first-order with a time constant of about $\tau = 0.25$ seconds.
    B. The *Phugoid* pole has a damping ratio $\zeta < 0.1$.
    C. The *Pitch attitude* pole has a damping ratio of about $\zeta = 0.7$.
    D. The Roll attitude pole has an oscillation period of $P = 2$ seconds.

12. If the poles listed in the table are the only poles in the system, how many states are there in the system?

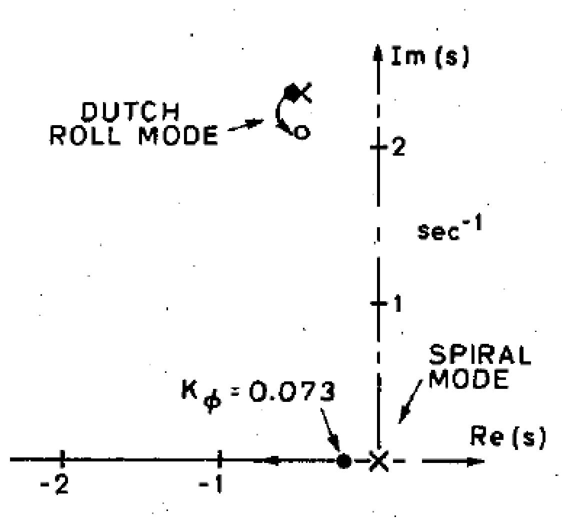
    A. 5
    B. 6
    C. 8
    D. 10

13. The same paper includes the following transfer function, which it says "represents the actuator dynamics":

$$\frac{\delta}{\delta_c} = \frac{30^2}{s^2 + 2(0.707)30s + 30^2}$$

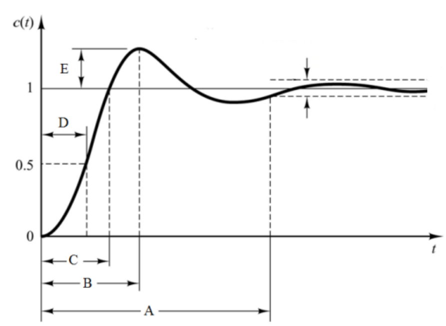
At about what frequency on a bode plot would this transfer function have a phase of -90 degrees?

    A. $\omega \approx 21$ rad/sec
    B. $\omega \approx 30$ rad/sec
    C. $\omega \approx 900$ rad/sec
    D. This transfer function never has phase equal to -90 degrees.



14. The figure above[4] shows the root locus varying the feedback gain K_ϕ. The solid dot (•) shows the pole locations for $K_\phi = 0.073$. Which of the following is the MOST ACCURATE inference about the effects of increasing the value of K_ϕ beyond 0.073?

    A. Larger values of K_ϕ eventually cause the "Dutch Roll Mode" to become unstable.
    B. Larger values of K_ϕ cause the damping ratio of the "Dutch Roll Mode" to exceed 1.0.
    C. Larger values of K_ϕ cause the "Spiral Mode" to have a smaller time constant.
    D. Larger values of K_ϕ cause the damping ratio of the "Spiral Mode" to approach 0.1.



15. In the space below, write the letter from the figure above that MOST ACCURATELY represents the *settling time* of a step response.

_____

[2] "Design of Flight Control Systems to Meet Rotorcraft Handling Qualities Specifications," *Journal of Guidance, Control and Dynamics*, 16:1, 1993.
[3] Poles are called "eigenvalues" in the table because the paper is using modern control design techniques.

[4] "New Concepts in Control Theory, 1959-1984," *Journal of Guidance, Control, and Dynamics*, 1985.

$$\frac{d}{dt}x = \begin{bmatrix} -0.02 & 0.005 & 2.4 & -32 \\ -0.14 & 0.44 & -1.3 & -30 \\ 0 & 0.018 & -1.6 & 1.2 \\ 0 & 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0.14 & -0.12 \\ 0.36 & -8.6 \\ 0.35 & 0.009 \\ 0 & 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 57.3 \end{bmatrix} x.$$

16. The equations above are part of an example presented in a seminal paper on Modern Control[5]. Which of the following is LEAST ACCURATE about these equations?

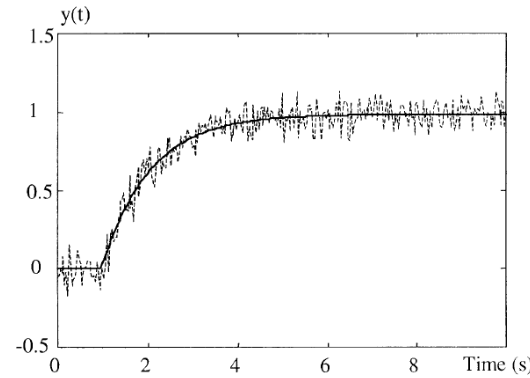
    A. The equations express a state-space model.
    B. There are 2 control inputs in the model.
    C. In the notation we used in class, the state output matrix is $C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 57.3 \end{bmatrix}$.
    D. Because the last row of the B matrix contains only zeros, control cannot affect the 4th state so the system is uncontrollable.

17. Which of the following is MOST CORRECT about lag compensation?

    A. It has the form $\frac{s+z}{s+p}$ with $z > p$.
    B. It is used to decrease the phase at high frequency to improve noise rejection.
    C. The name "lag compensator" comes from the observation that the magnitude of the frequency response plot of a lag compensator is less than unity (0dB).
    D. All of the above.

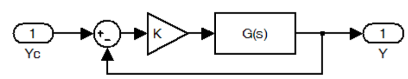
No Questions about Tesla Roadster!

All of the problems in this column refer to the following system:



18. The figure above[6] shows the response of a system to a unit step input. NOTE: The step is applied at $t=0$, NOT $t=1$. The dashed line is the measured response, which includes noise. The solid line is a model. Which of the following transfer functions is MOST LIKELY to be the equation for the model transfer function?

    A. $G(s) = \frac{e^{-s}}{s+1}$
    B. $G(s) = \frac{e^{-s}}{s}$
    C. $G(s) = \frac{1}{s^2+1}$
    D. $G(s) = \frac{s}{s+1}$



19. If the plant of the previous problem is included in the closed-loop proportional-gain system shown above, which of the following is MOST ACCURATE about the closed-loop stability?

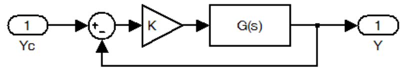
    A. The system is stable for all $K > 0$.
    B. The system is unstable for all $K > 0$.
    C. The system is stable only for $K > K_{MIN} > 0$.
    D. The system is unstable only for $K > K_{MAX} > 0$.

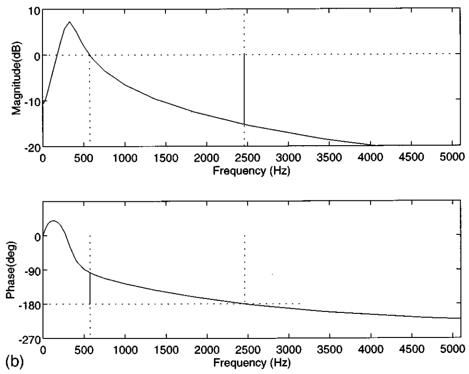
---

[5] "Multivariable Feedback Design: Concepts for a Classical / Modern Synthesis," *IEEE Transactions Automatic Control*, 1981.

[6] "Robust identification of first-order plus dead-time model from step response", *Control Engineering Practice*, 1999. Systems such as these are very common in heating and air conditioning applications. It isn't cheating if you get a hint about the right answer from the title of this paper.

All of the problems in this column refer to the following system:





20. The figure above[7] shows the bode plot of $KG(s)$ for the feedback system shown at top. Notice that the frequencies are in Hz, instead of our usual rad/sec, and the scaling on the frequency axis is linear. Which of the following is MOST ACCURATE about this system?

    A. The closed-loop system is stable.
    B. The closed-loop system is unstable.
    C. The stability of the closed-loop system depends on the input frequency.
    D. The stability of the closed-loop system depends on the initial conditions.

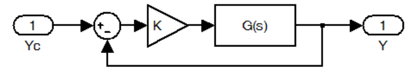
21. Continuing with the system of the previous problem, how much would the gain, K, need to be changed so that the system reaches neutral stability?

    A. The gain would have to be increased by about a factor of 5.
    B. The gain would have to be increased by about a factor of 2.
    C. The gain would have to be decreased by about a factor of 2.
    D. The gain would have to be decreased by about a factor of 5.

22. If the gain change of the previous problem were implemented, at what frequency would the neutrally stable oscillations occur?

    A. 575 Hz
    B. 2460 Hz
    C. 4000 Hz
    D. The instability would be a divergence, not an oscillation.

All of the problems in this column refer to the following system:



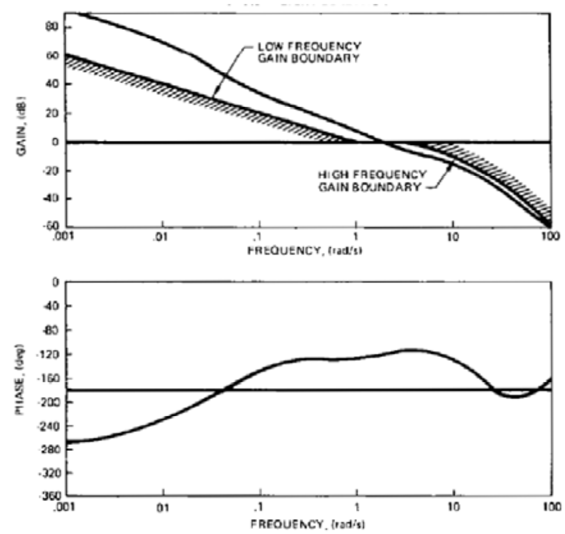


Fig. 22. Elevator open-loop frequency response for LQR design.

23. The figure above[8] shows the bode plot of $KG(s)$ for a system whose block diagram is shown at top. The phase margin is...

    A. ...about 20 degrees.
    B. ...about 60 degrees.
    C. ...about 120 degrees.
    D. ...exactly 180 degrees.

24. In the previous problem, which of the following is a REASONABLE justification for the "low frequency gain boundary" in the bode plot?

    A. Good tracking performance.
    B. Good disturbance suppression.
    C. Both of the above.
    D. Neither of the above.

25. Which of the following is a REASONABLE justification for the "high frequency gain boundary" in the bode plot?

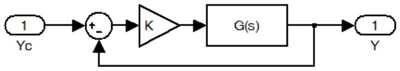
    A. Good noise rejection.
    B. Good stability robustness (there might be dynamics out there that we don't know about so we don't want to risk having high gain).
    C. Both of the above.
    D. Neither of the above.

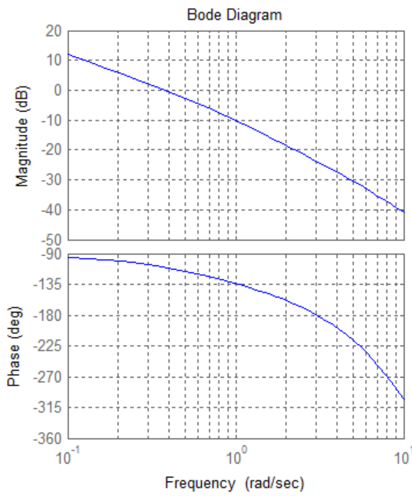
---

[7] "Implementation of an active headset by using the H-Infinity robust control theory", *Journal of the Acoustic Society of America*, 102:4, 1999.

[8] "Application of Modern Synthesis to Aircraft Control: Three Case Studies", *IEEE Transactions on Automatic Control*, 1986. Surprise! Another airplane control system!

All of the problems in this column refer to the following system:





26. The figure above shows the bode plot of $G(s)$ for the system shown at top. If $K = 3$, the phase margin is about...

    A. ... 130 degrees.
    B. ...80 degrees.
    C. ...50 degrees.
    D. ...not determined by the information in the figure.

27. For $K = 3$, how much additional time delay in $G(s)$ will result in neutral stability?

    A. About 0.1 seconds.
    B. About 0.2 seconds.
    C. About 0.5 seconds.
    D. About 1.0 seconds.

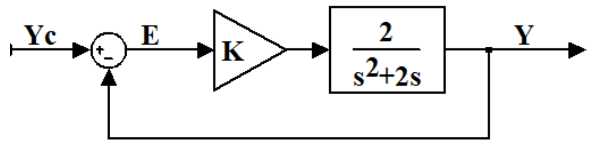
28. For $K = 3$, the steady-state response to a unit step input will be...

    A. ... 1 (perfect tracking).
    B. ... about 0.7.
    C. ... 0.25.
    D. ... 0.

29. If the proportional feedback were replaced by derivative feedback and the derivative gain were increased to the point of neutral stability, at what frequency would the oscillations occur?

    A. The instability would be divergent, not oscillatory.
    B. About 3 rad/sec.
    C. About 8 rad/sec.
    D. The system could not be made unstable by use of derivative feedback alone.

All of the problems in this column refer to the following block diagram:



30. Which of the following statements is MOST ACCURATE about the closed-loop stability of this system?

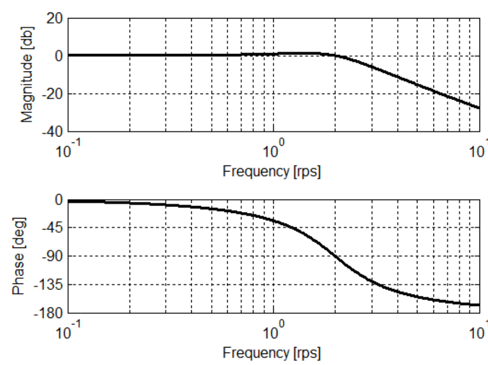
    A. The system will be stable only if $K \geq 4$.
    B. The system will be stable for all $K > 0$.
    C. The system will be unstable for all $K > 0$.
    D. The system will be unstable only if $K \geq 4$.

31. If we choose $K = 8$, what is the closed-loop damping ratio?

    A. $\zeta = -0.25$ (unstable)
    B. $\zeta = 0.25$
    C. $\zeta = 0.5$
    D. $\zeta > 1$ (two real poles)

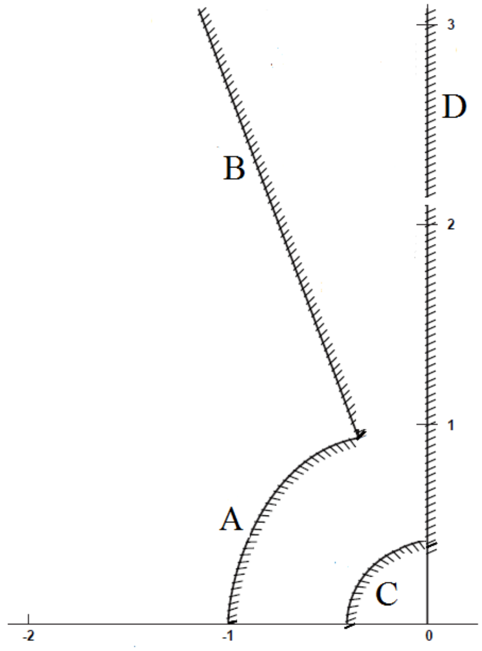
32. For approximately what value of K will the phase margin be 45 degrees?

    A. No $K > 0$ gives phase margin of 45 deg.
    B. $K \approx 1$
    C. $K \approx 3$
    D. $K \approx 12$



33. For what K does the figure above show the *closed-loop* frequency response, $\frac{Y}{Y_C}$?

    A. No $K > 0$ gives this closed-loop response.
    B. $K \approx 1$
    C. $K \approx 2$
    D. $K \approx 8$



34. The figure above shows some of the boundaries for pole locations that give various "Levels" of desirable response in a helicopter[9]. Which boundary corresponds to a minimum damping ratio, $\zeta > 0.35$?

   A. Boundary A
   B. Boundary B
   C. Boundary C
   D. Boundary D

35. Which boundary in the figure above corresponds to a minimum natural frequency of $\omega_n \geq 0.4$ rad/sec?

   A. Boundary A
   B. Boundary B
   C. Boundary C
   D. Boundary D

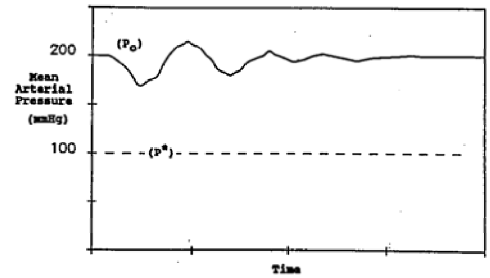
36. The difference equation,
   $u[k] = 0.8u[k-1] + 3.0e[k] - 2.4e[k-1]$,
   corresponds to which transfer function?

   A. $G(z) = 0.8z + 3.0 - 2.4z^{-1}$

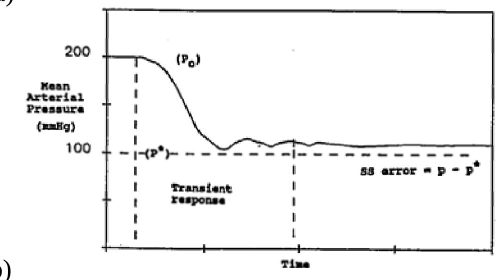
   B. $G(z) = \dfrac{3.0 - 2.4z^{-1}}{1 - 0.8z^{-1}}$

   C. $G(z) = \dfrac{1 - 3.0z^{-1}}{0.8 - 2.4z^{-1}}$

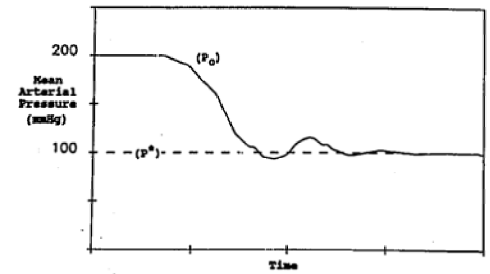
   D. $G(z) = z^{-2} - 0.8z^{-1} + 3.0 - 2.4z$



(a)



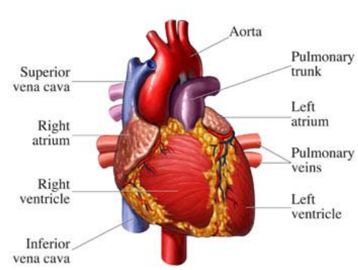
(b)



(c)

37. The figures above[10] show the closed-loop responses of 3 different feedback controllers attempting to change blood pressure from an initial value of 200 mmHg to a final value of 100 mmHg. The "plant" in the system has finite, non-zero steady gain. Which of the following is the MOST ACCURATE inference from these figures?

   A. The controller for figure (a) has zero steady gain, consistent with a high-pass filter.
   B. The controller for figure (b) has a finite steady gain, consistent with proportional feedback.
   C. The controller for figure (c) has infinite steady gain, consistent with integral feedback.
   D. All of the above are reasonable inferences.

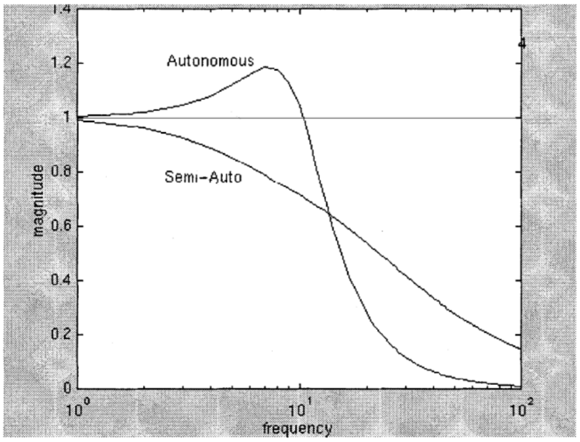


Middle School Science!

---

[9] *ADS-33E-PRF*, Figure 23, Lateral-Directional oscillatory requirements.

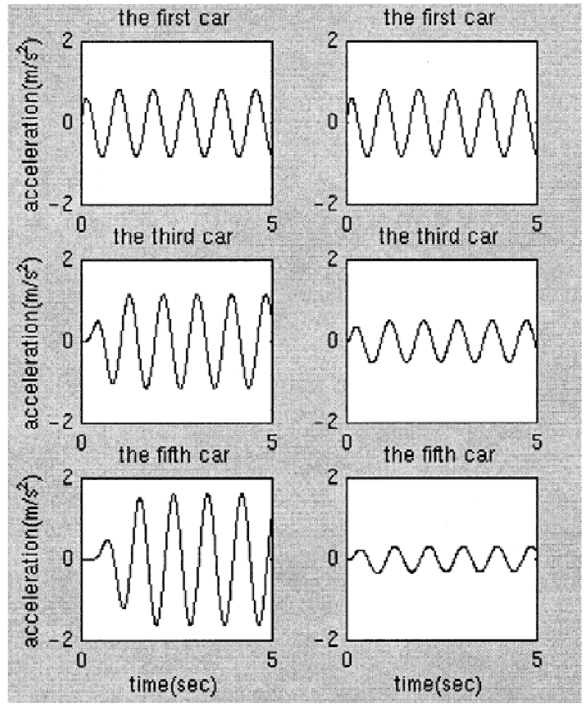
[10] "The Use of Computers for Controlling the Delivery of Anesthesia", *Anesthesiology*, 1992.



38. The upper figure above[11] shows the magnitude portion of the *closed-loop* bode plots of $G(s) = \dfrac{\delta_k(s)}{\delta_{k-1}(s)}$, where δ_k is the spacing *error* of the k[th] vehicle in a string of vehicles which are attempting to maintain constant inter-vehicle spacing. Notice that the bode plot uses linear scaling of magnitude. The "Autonomous" design has each vehicle operating independently. The "Semi-Autonomous" design has each vehicle receiving information only from the immediately preceding vehicle[12]. Which of the following is the MOST CORRECT inference from this figure?

A. The "Autonomous" design has a second-order pole at about 7 rad/sec with damping ratio less than 1.0.
B. The "Semi-Auto" design could be approximated by a first-order system with a pole at about 14 rps.
C. Both of the above.
D. Neither of the above.

This is just a fun photo--not directly related to answering these questions.



39. Continuing the previous problem...a string of vehicles has "string stability" if the spacing errors are guaranteed do not amplify as they propagate toward the tail of the string. In the figure above, the design on the right has "string stability" while the system on the left does not. Using insights from our study of discrete-time systems, which of the designs shown from the previous problem has "string stability"?

A. The "Autonomous" design.
B. The "Semi-Auto" design.
C. Both designs.
D. Neither design.

40. Which of the following is LEAST ACCURATE concerning LQR design?

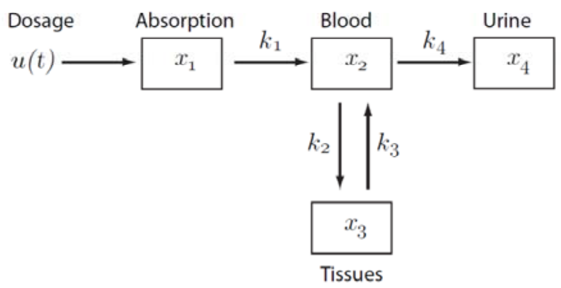
A. “LQR” stands for linear quadratic regulator.
B. LQR is based on minimizing

$$J = \int_0^\infty \left(x^T Q x + u^T R u \right) dt .$$

C. The solution to the LQR minimization problem is proportional state feedback of the form $u = -Kx$, where K is computed from the solution of the Algebraic Riccati equation.
D. LQR is especially useful because, unlike eigenvalue assignment, it can be applied to a state-space system even if (A, B) is not controllable.

---

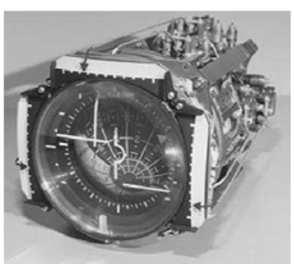
[11] "Semi-Autonomous Adaptive Cruise Control Systems", *IEEE Transactions on Vehicular Technology*, 51:5, 2002.
[12] Both of the designs shown here are attempts to reduce the communication required in the *platoon design* approach, in which each vehicle receives communication from the lead vehicle (k=1).



$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \\ \frac{dx_4}{dt} \end{bmatrix} = \begin{bmatrix} -k_1 & 0 & 0 & 0 \\ k_1 & -(k_2+k_4) & k_3 & 0 \\ 0 & k_2 & -k_3 & 0 \\ 0 & k_4 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

41. The figure above[13] shows a Pharmokinetic model for drug absorption, where the input is the drug dosage and the states are the concentrations of the drug in various parts of the body. Which of the following is LEAST ACCURATE about the system?

    A. This is a state-space model with 4 states and 1 control.
    B. There is at least one zero eigenvalue because the A matrix is singular.
    C. Regardless of the values of the reaction constants, k_i, the system is not controllable because the A matrix is singular.
    D. One eigenvector of the A matrix is $\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T$.



42. The attitude gyro used in the Apollo space program, shown above, measures the orientation of the spacecraft and is an example of what type of system element?

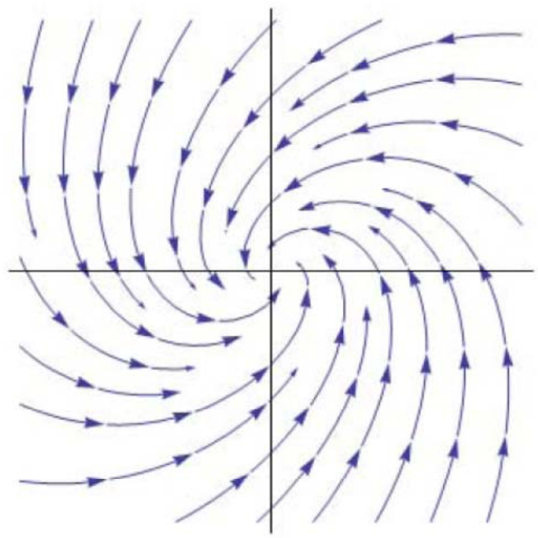
    A. Actuator
    B. Sensor
    C. Notch Filter
    D. None of the Above

43. What are the eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 \\ -25 & -6 \end{bmatrix}$?

    A. $\lambda = -6, -5$
    B. $\lambda = -3 \pm 4j$
    C. $\lambda = -6 \pm 5j$
    D. None of the above.

44. Is the system matrix pair $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ controllable?

    A. Yes.
    B. No.
    C. We need initial conditions to determine controllability.
    D. We need the C matrix to determine controllability.



45. The figure above[14] shows the Phase Portrait of a 2-state model of a species reaction network. Which of the following is the MOST ACCURATE inference from the figure?

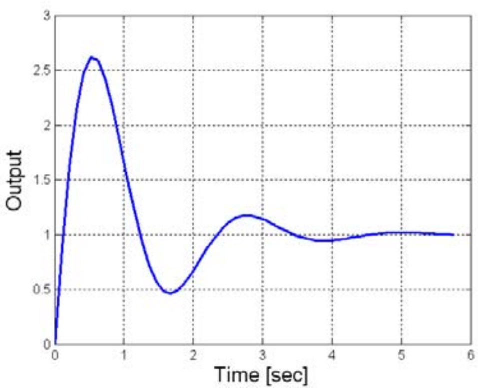
    A. The origin is a stable equilibrium.
    B. The origin is an unstable equilibrium.
    C. There are no equilibrium points in the region shown in the figure.
    D. The y-axis is a stable attractive manifold.

[13] *Control Theory for Bioengineers*, 2014.

[14] *Control Theory for Bioengineers*, 2014.

46. A "Lyapunov Function" is...

A. ...used the demonstrate the stability of a nonlinear dynamic system without having the solve the ODEs.
B. ...a static nonlinear input-output process that is used to stabilize a linear dynamic system empirically, without any analysis.
C. ...a method of scoring two points in basketball without getting the ball through the hoop.
D. All of the above.



47. Which of the following transfer functions is the BEST match to the unit step response shown above?

A. $\dfrac{9}{s^2+9}$
B. $\dfrac{9s+9}{s^2+2s+9}$
C. $\dfrac{9s}{s^2+2s+9}$
D. $\dfrac{9}{s^2+9s+9}$

48. Which of the following is NOT a reasonably likely use of a notch filter?

A. Removing a narrow-band noise from a feedback signal.
B. Preventing destabilization a lightly-damped high-frequency mode.
C. Getting a pizza delivered during a final exam.

Homemade Pizza!

49. The motion of a helicopter rotor blade in hovering flight is governed by the following differential equation:

$$\frac{d^2\beta}{dt^2}+\frac{\gamma\Omega}{8}\frac{d\beta}{dt}+\Omega^2\sin\beta\cos\beta=\frac{\gamma\Omega^2}{8}\vartheta$$

where $\vartheta(t)$ is the blade pitch angle (control input), $\beta(t)$ is the blade angle (system output), γ is the Lock number (a fixed parameter) and Ω is the rotor speed (a fixed parameter). What is the transfer function $G(s)=\dfrac{\Delta\beta(s)}{\Delta\vartheta(s)}$ for small perturbations from a trim condition of $\beta_o=0$?

A. $G(s)=\dfrac{\dfrac{\gamma\Omega^2}{8}}{s^2+\dfrac{\gamma\Omega}{8}s+\Omega^2}$

B. $G(s)=\dfrac{\dfrac{\gamma\Omega^2}{8}}{s^2+\dfrac{\gamma\Omega}{8}s+2\Omega^2}$

C. $G(s)=\dfrac{\dfrac{\gamma\Omega^2}{8}}{s^2+\dfrac{\gamma\Omega}{8}s-\Omega^2}$

D. $G(s)=\dfrac{\dfrac{\gamma\Omega^2}{8}s}{s^2-\dfrac{\gamma\Omega}{8}s-2\Omega^2}$

50. If the control system applies blade-pitch inputs to the system of the form $\vartheta(t)=A\sin\Omega t$, the long-term output will be of the form...

A. $\beta(t)=-A\sin(\Omega t)$
B. $\beta(t)=-A\cos(\Omega t)$
C. $\beta(t)=-\dfrac{\sqrt{2}}{2}\sin(\Omega t)$
D. None of the above.