Introduction to PID Control – Part 2

ESE 505 & MEAM 513

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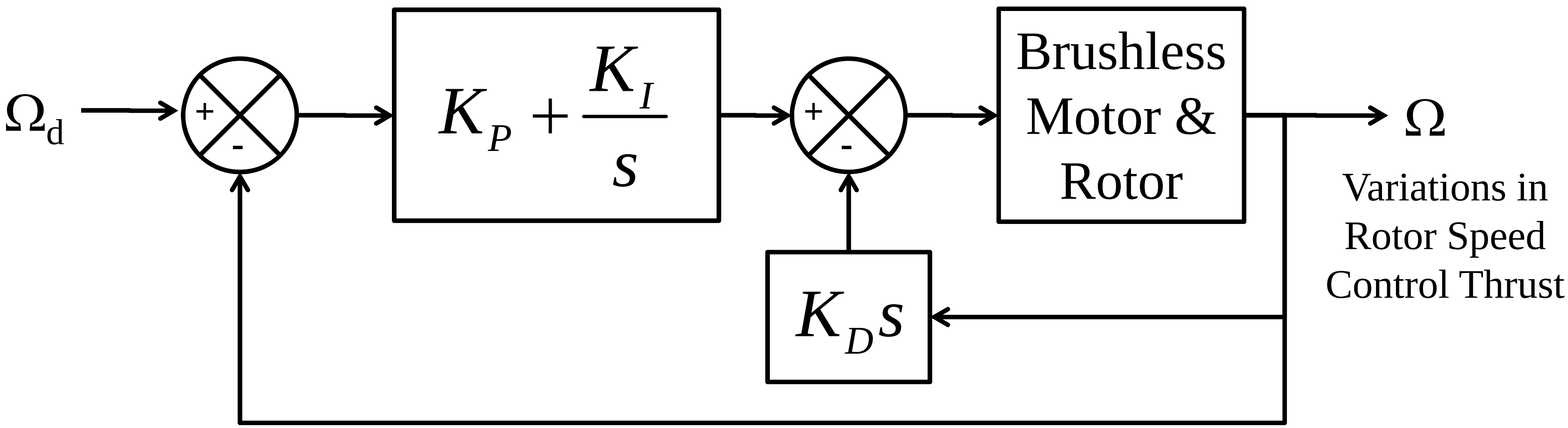
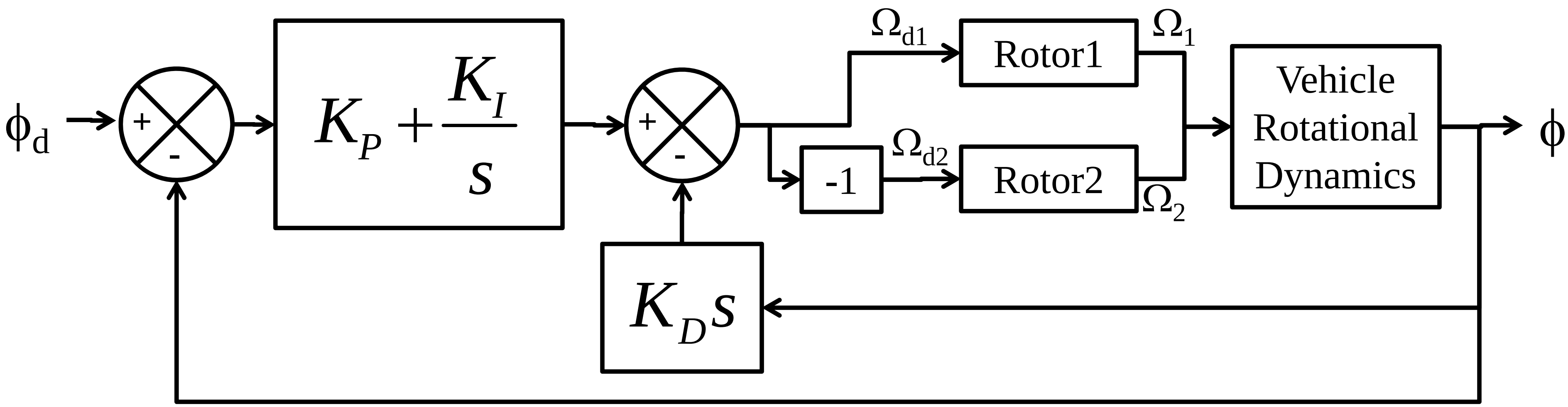
http://www.youtube.com/watch?v=WoMrPKCpWXw

“Inner Loop” : PID Control on Rotor Speed



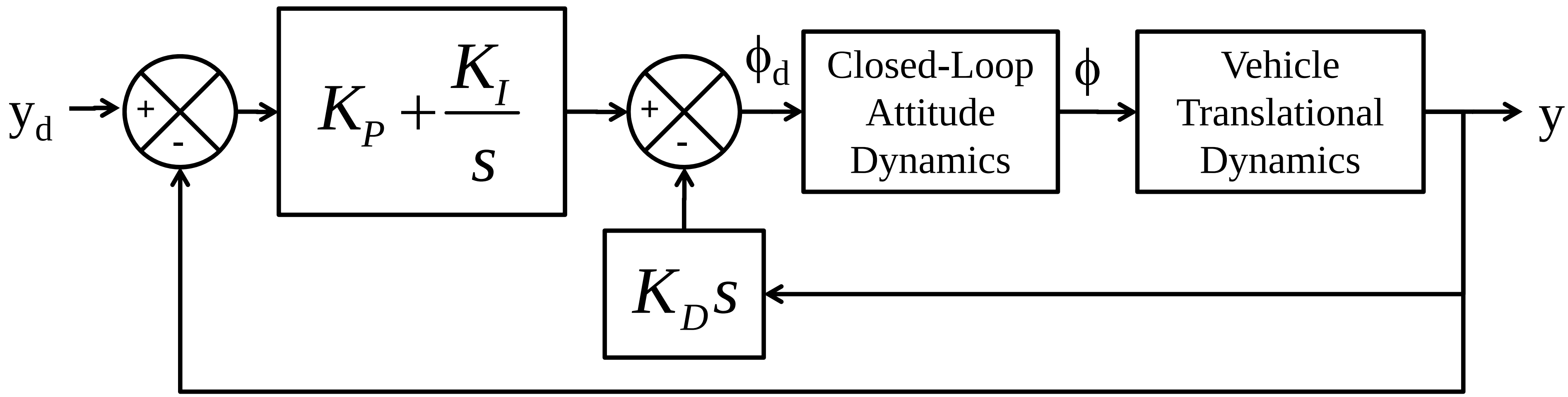
- Expect Motor Dynamics ~ First Order + Time Delay
- KP for High-Bandwidth Tracking
- KD for Stability (Only on Feedback?)
  - Poles Well Damped
  - Phase Not Too Close to -180 @ Crossover
- KI for Zero Steady Error (e.g. Due to Battery Drain)

“Middle Loop” : PID Control on Attitude



- Expect Rotational Dynamics ~ Double Integrator
- “Rotor1” & “Rotor2” Include Inner-Loop PID Control
- KP For Tracking (Lower Bandwidth Than Motor Loop)
- KD for Stability
- KI for Zero Steady Error (e.g. Due to Mass Offset)
  - CAREFUL! Sensor Not Perfect!

"Outer Loop" : PID Control on Position

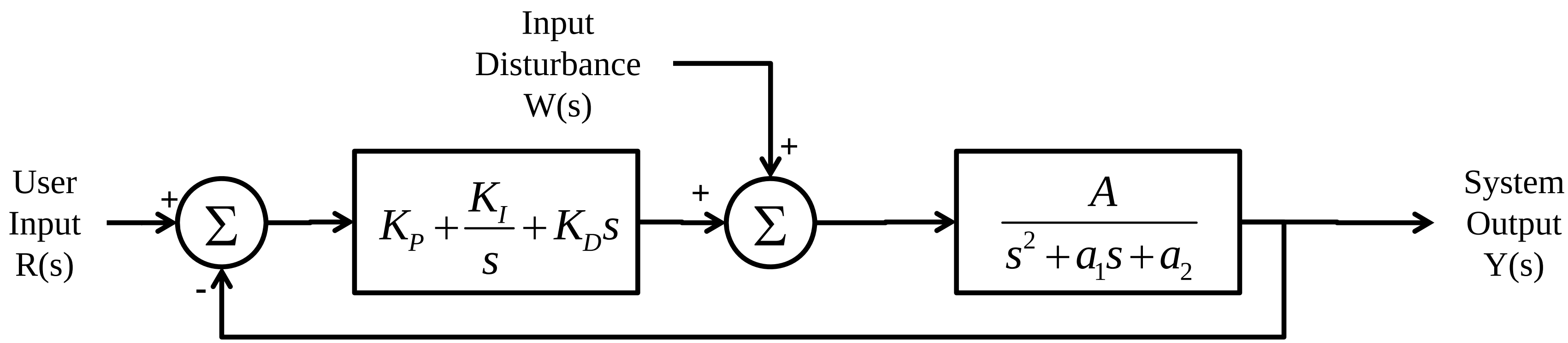


- Expect Translational Dynamics ~ Double Integrator
- KP For Tracking (Lower Bandwidth Than Attitude Loop)
- KD for Stability
- KI for Zero Steady Error

Some Interesting Details / Questions…

- Steady Rotor Speed Management?
  - “Middle Loop” Cartoon Commanded CHANGES in Rotor Speed, Relative to the Value Required for Hover
  - PID on Desired Altitude?
- Rotor Speed Measurement from Brushless “Electronic Speed Controller” (ESC) Timing
- Attitude Measurement from Complementary Filter
  - At High Frequency: Use Integral of Angular Velocity
  - At Low Frequency: Use Linear Acceleration Measured on Aircraft
- Attitude Integral Feedback Authority Limits?
- Position Measurement?
- Trajectory Adjustment to Avoid Walls?

PID Control on Second-Order Plants



$$\frac{Y(s)}{R(s)} = \frac{A\left(K_D s^2 + K_P s + K_I\right)}{s^3 + \left(a_1 + AK_D\right)s^2 + \left(a_2 + AK_P\right)s + K_I}$$

$$\frac{Y(s)}{W(s)} = \frac{As}{s^3 + \left(a_1 + AK_D\right)s^2 + \left(a_2 + AK_P\right)s + K_I}$$

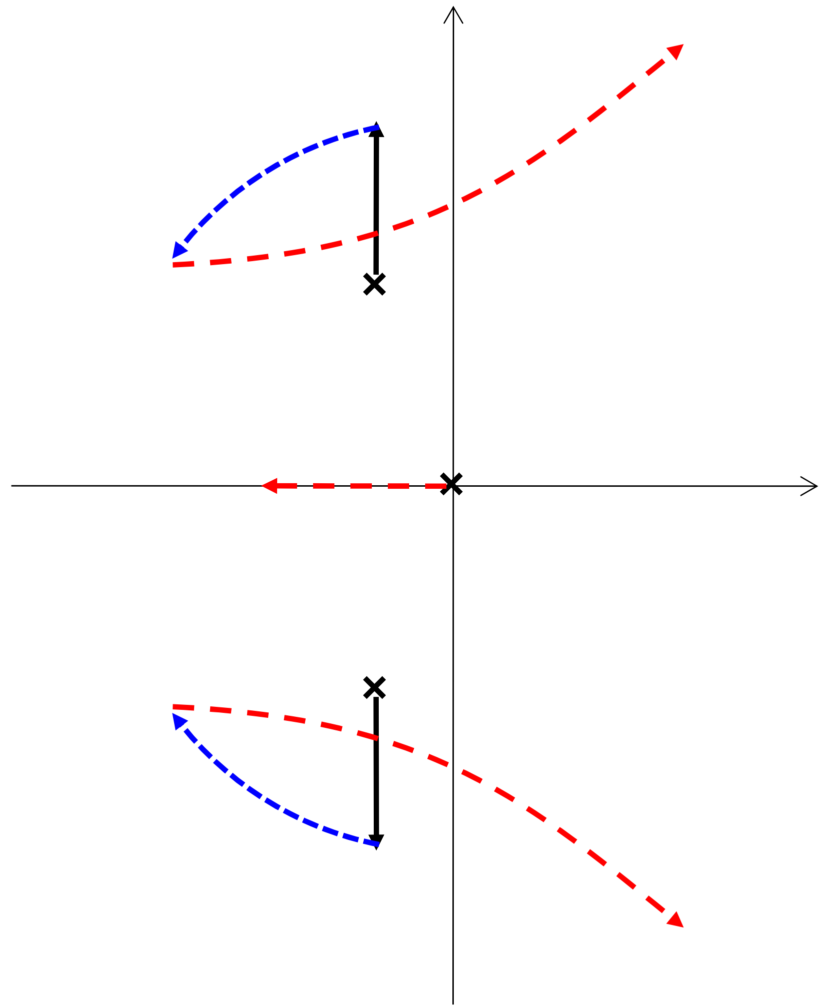
Feedback Effects on Pole Locations

$$\Delta_{CL}(s) = s^3 + \left(a_1 + AK_D\right)s^2 + \left(a_2 + AK_P\right)s + K_I$$

- Consider First $K_I = 0$
  - Pole @ s=0 Cancels with Zero in Numerators
  - K_P Controls Natural Frequency (Quicker Response)
  - K_D Controls Damping Ratio (Shorter Settling Time & Less Overshoot)
  - Higher-Order Dynamics and/or Time Delay Limit Max Gains
- Need $K_I > 0$ For Tracking & Disturbance Rejection
  - $K_I > 0$ Required for Type 1 Tracking & Disturbance Rejection
  - BUT... $K_I > 0$ Tends to Push Poles to Right (Less Stable)

Cartoon : PID Effects on Pole Locations

- Increase K_P with K_I=0 & K_D=0
- Increase K_D with K_P Fixed & K_I=0
- Increase K_I with K_P & K_D Fixed



Quantitatively Making This Drawing is Called "Root Locus"

We'll Study This More Soon…

Example : PID Control of DC Motor Position

PID & DC MOTOR POSITION | 2012-02-08 | 1/1

GOVERNING EQUATIONS OF DC MOTOR:

$$L\frac{di}{dt} = e - Ri - K\Omega \qquad \text{(VOLTAGE)}$$

$$J\frac{d\Omega}{dt} = Ki + Q_{EXT} \qquad \text{(TORQUE)}$$

$$\frac{d\theta}{dt} = \Omega \qquad \text{(POSITION)}$$

⊗ IGNORE "FAST" CURRENT DYNAMICS ⇒ LET $L \to 0$

$$\Rightarrow \quad i = \frac{1}{R}e - \frac{K}{R}\Omega \Rightarrow Js\Omega = \frac{K}{R}e - \frac{K^2}{R}\Omega + Q_{EXT}$$

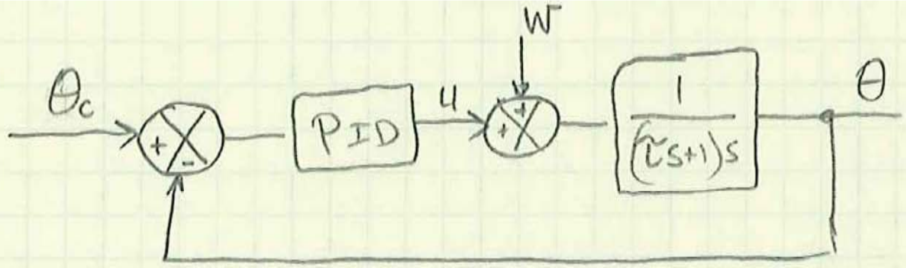
$$\Rightarrow \left(Js^2 + \frac{K^2}{R}s\right)\theta(s) = \frac{K}{R}e + Q_{EXT}$$

$$\Rightarrow \theta(s) = \frac{\frac{1}{K}e(s) + \frac{R}{K^2}Q_e(s)}{(\tau s + 1)s} = \frac{u(s) + w(s)}{(\tau s + 1)s}$$

$$u \triangleq \frac{1}{K}e \qquad \text{(VOLTAGE SCALED TO STEADY } \Omega\text{)}$$

$$w = \frac{R}{K^2}Q_{EXT} \qquad \text{(TORQUE " " " ")}$$

PID Control of DC Motor Position (Cont.)



THIS IS A CLASSIC PROBLEM!

$$\Theta = \frac{K_D s^2 + K_P s + K_I}{\tau \Delta_{CL}(s)} \theta_c + \frac{s}{\tau \Delta_{CL}(s)} W$$

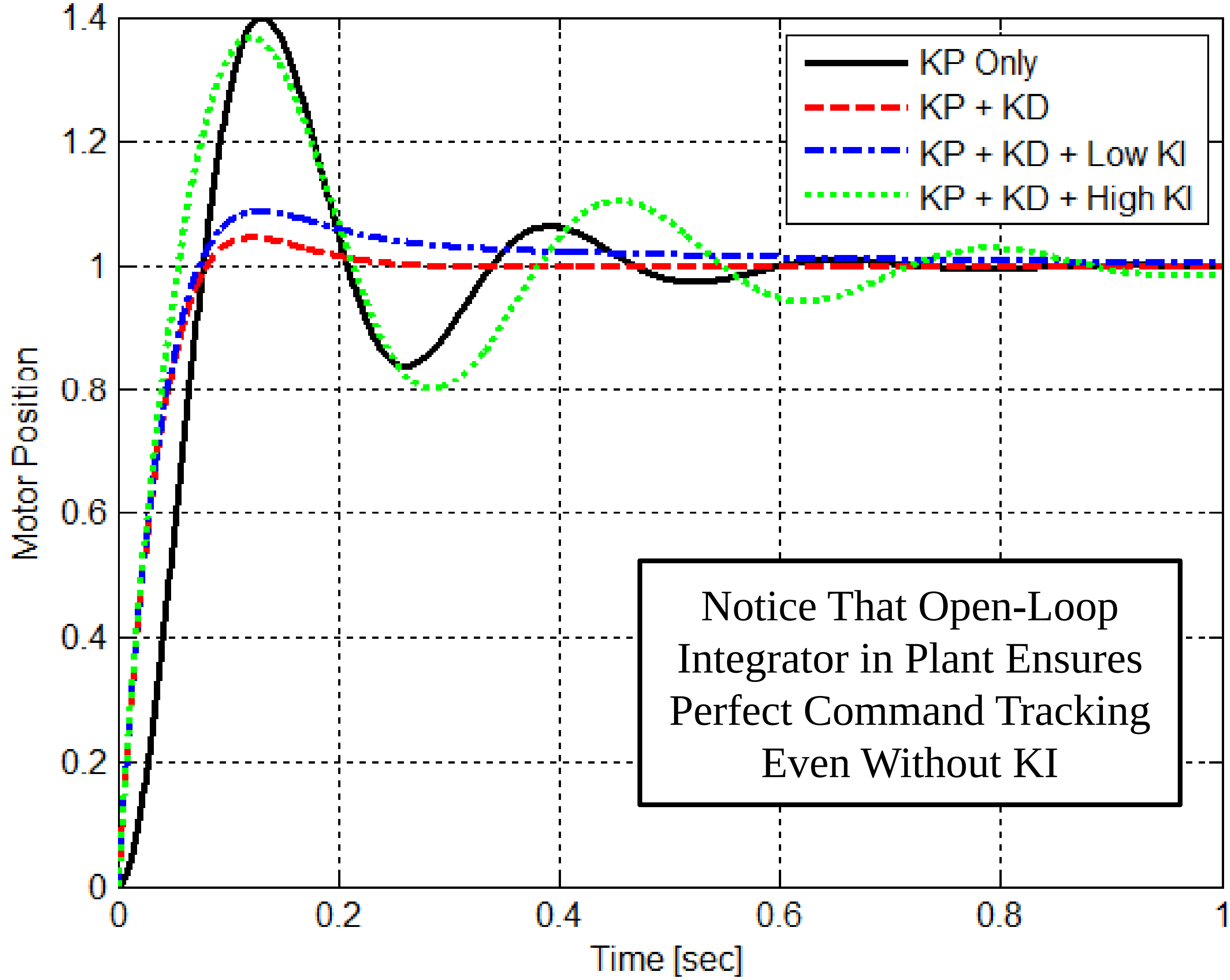
$$\Delta_{CL} = s^3 + \frac{K_D+1}{\tau} s^2 + \frac{K_P}{\tau} s + \frac{K_I}{\tau}$$

EXAMPLE: $\left.\begin{array}{l} \tau = \frac{1}{14} \\ K_P = \frac{625}{14} \end{array}\right\} \Rightarrow \omega_N = 25 \quad (K_I = 0)$

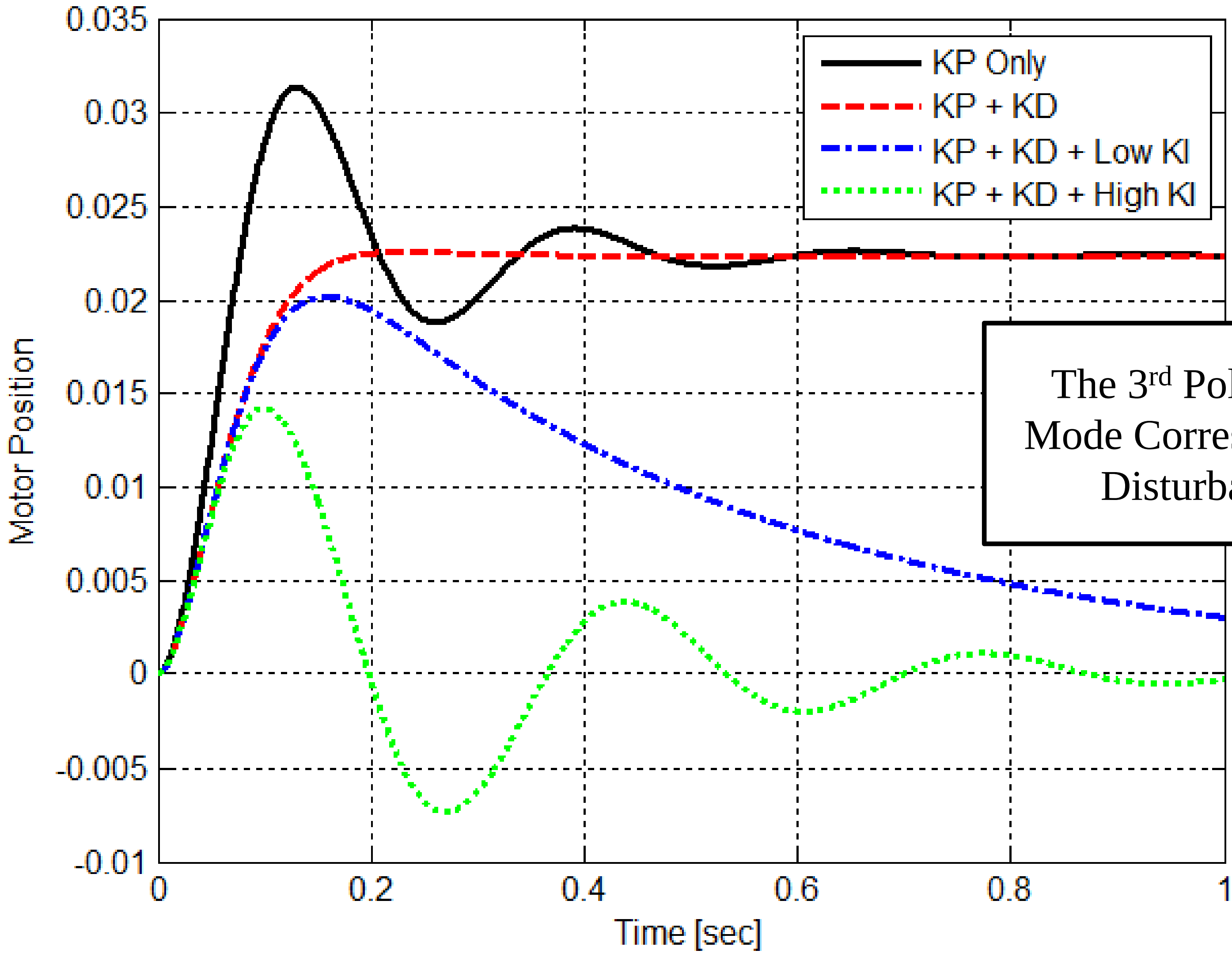
$K_D = 2 \Rightarrow \zeta = \frac{21}{25} \quad (K_I = 0)$

$$K_I = \begin{cases} 2K_P & \text{LOW} \\ 20K_P & \text{HIGH} \end{cases}$$

Example : DC Motor Position : Step Input



Example : DC Motor Position : Step Disturbance



The 3rd Pole is a First-Order Mode Corresponding to Steady Disturbance Rejection

A Few Words of Caution with PID Controllers

- PID Controllers Are Good First Choice for Many Simple Systems
  - Second Order Systems with Force-Like Actuation
  - First-Order Systems with Time Delay (Often Just Use PI)
- The "I" Introduces Very Important Complexities
  - Input to "I" Will Be Forced to Zero—Is This REALLY What You Want?!
  - Need Anti-Windup Mechanism to Prevent Controller Runaway When Actuator Reaches Authority Limit
  - How To Initialize a Redundant System After Single Power Failure?
  - "I" Cannot "Cancel" a Zero @ Origin in Plant (e.g., Cannot Control Airplane Speed Using Throttle!)
- There is No Substitute for Good Knowledge of Plant and Careful Selection of Compensation
- We Will Learn Alternatives That Often Work Better