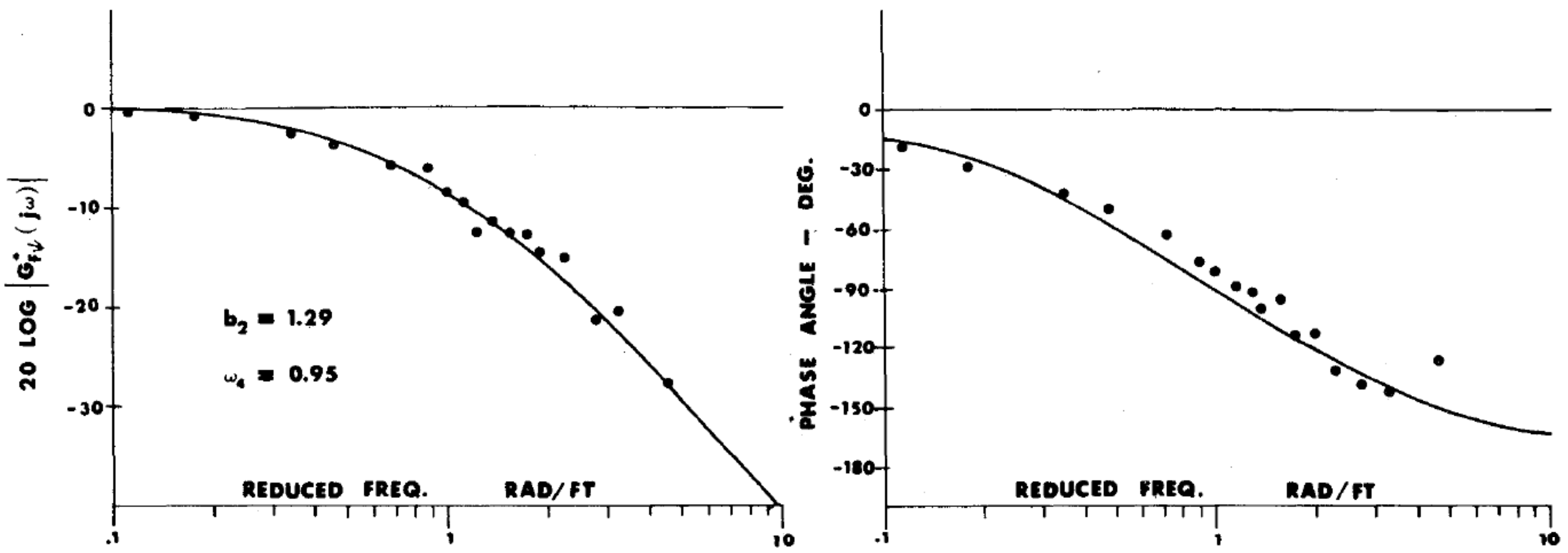
Bode Plots



b)

Fig. 4 Tire cornering force frequency response under cyclic yaw.

ESE 505 & MEAM 513

Bruce D. Kothmann

2014-03-19

Penn Engineering

VOL. 8, NO. 9, SEPTEMBER 1971 J. AIRCRAFT 689

Synthesis of Tire Equations for Use in Shimmy and Other Dynamic Studies

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Response to Sinusoidal Input & Bode Plot

$$u(t) = \sin(\omega t)$$

$$y(t) = M \sin(\omega t + \phi)$$

$$G(j\omega) = Me^{j\phi}$$

Response to Unit Sinusoidal Input of Frequency ω is Sinusoidal Output of Frequency ω with Magnitude M and Phase Shift ϕ, where $G(j\omega)=Me^{j\phi}$

Bode Plot = Standard Way of Graphing G(jω)

Use Logarithmic Units for Magnitude

$$\hat{M} = 20\log_{10}(M)$$

↑

"deci-bels" or dB

Also Use Logarithmic Scaling for Frequency

M	$\hat{M}$
0.5	–6
2	6
3	9

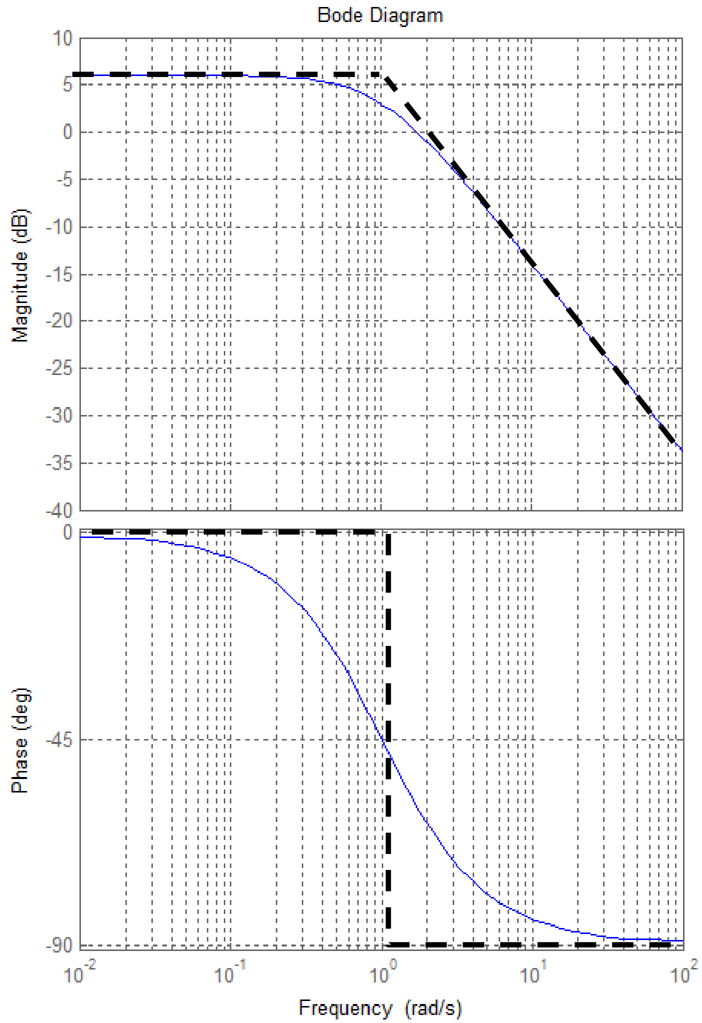
Example : First-Order Lag

$$G(s) = \frac{2}{s+1}$$

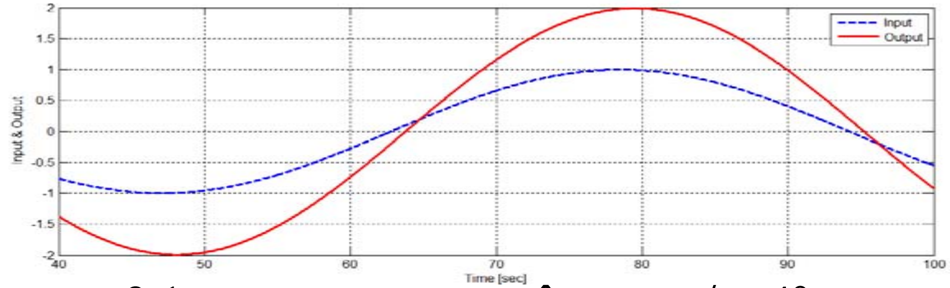
$$\omega \ll 1 \Rightarrow \begin{cases} \hat{M} = 6 \\ \phi = 0 \end{cases}$$

$$\omega \gg 1 \Rightarrow \begin{cases} \hat{M} = 6 - 20\log_{10}\omega \\ \phi = -90^\circ \end{cases}$$

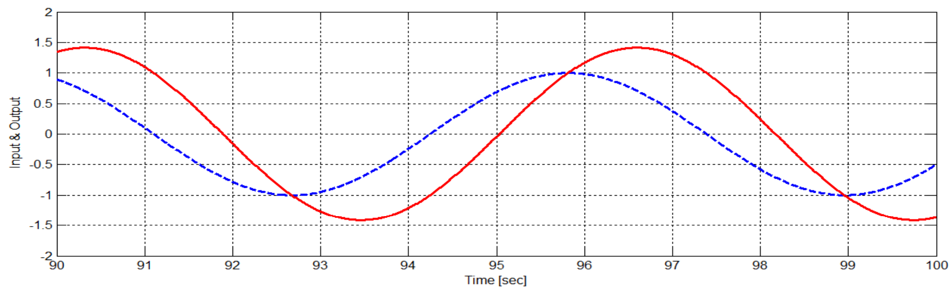
Draw Asymptotes & Sketch
Actual Curves



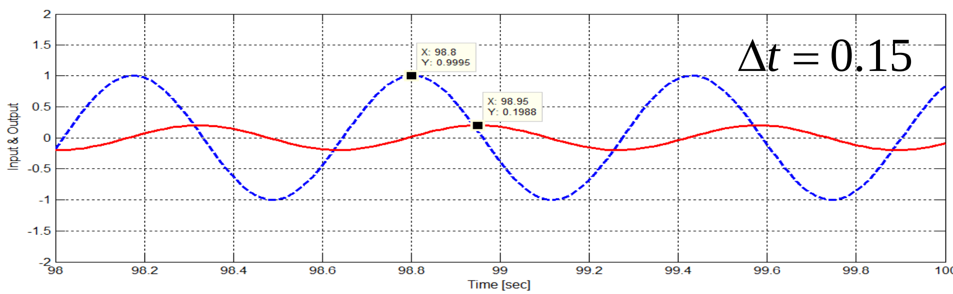
First-Order Lag : Time → Frequency



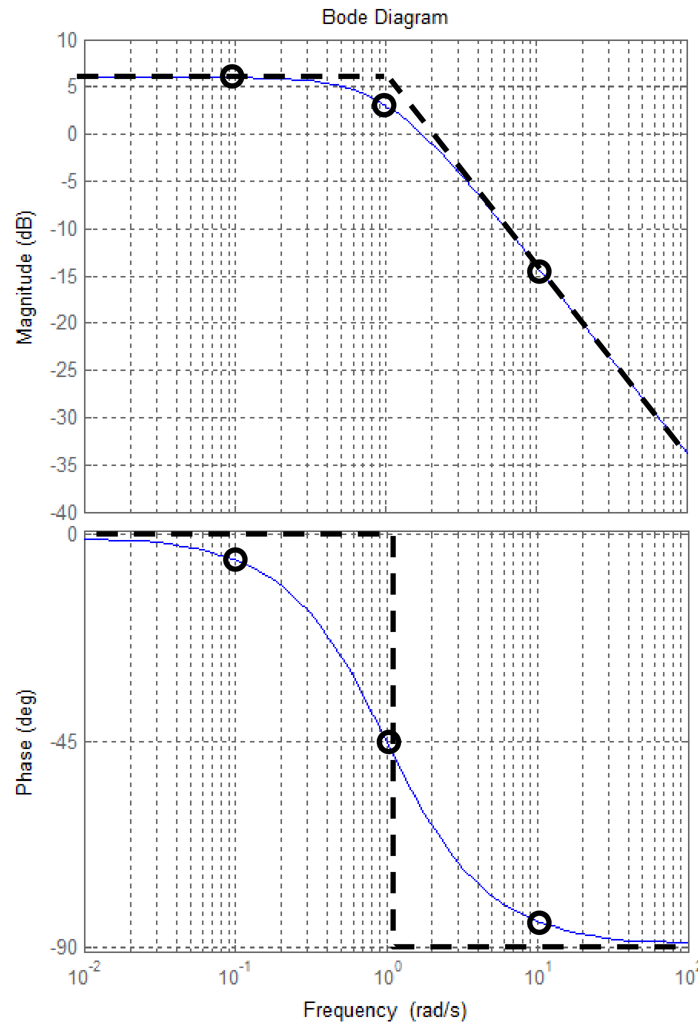
$\omega = 0.1 rps \qquad M \approx 2 \Rightarrow \hat{M} \approx 6 \qquad \phi \approx 4°$



$\omega = 1 rps \qquad M \approx 1.4 \Rightarrow \hat{M} \approx 3 \qquad \phi \approx -45°$



$\omega = 10 rps \qquad M \approx 0.2 \Rightarrow M \approx -14 \qquad \phi \approx -86°$



Example : High-Pass Filter

$$G(s)=\frac{6s}{s+2}=\underbrace{3s}_{G_1}\underbrace{\frac{1}{0.5s+1}}_{G_2}$$

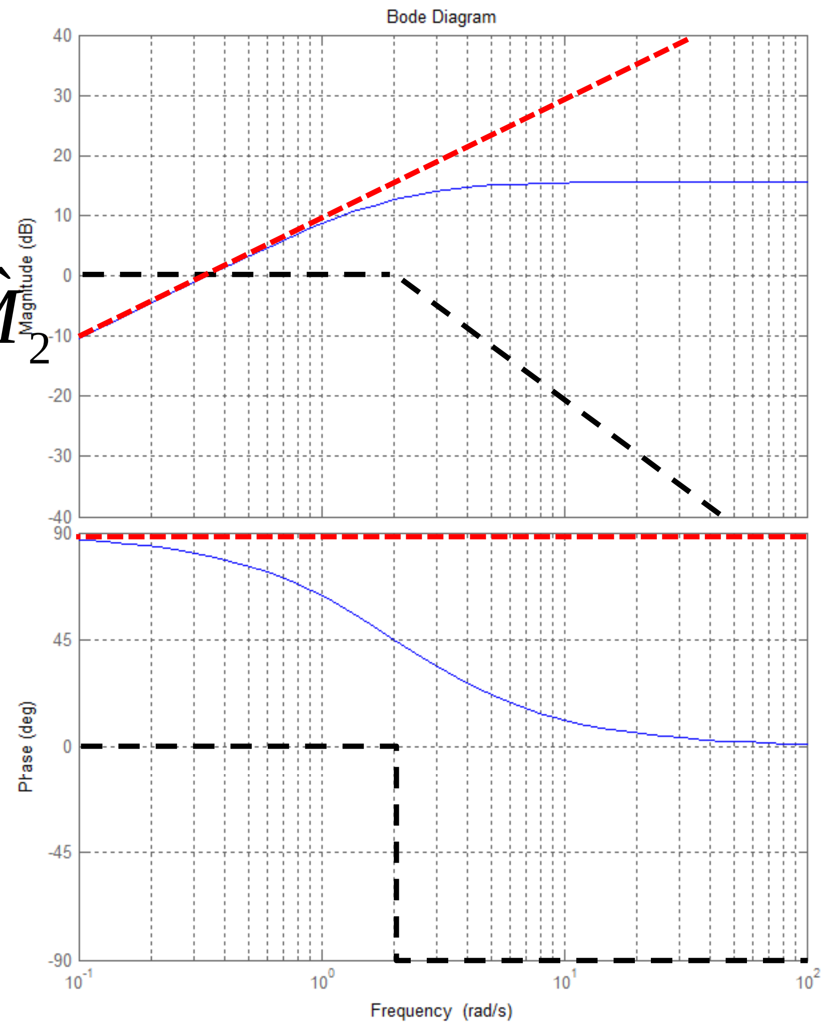
$$\hat{M}=20\log_{10}\left(M_1M_2\right)=\hat{M}_1+\hat{M}_2$$

$$\phi=\phi_1+\phi_2$$

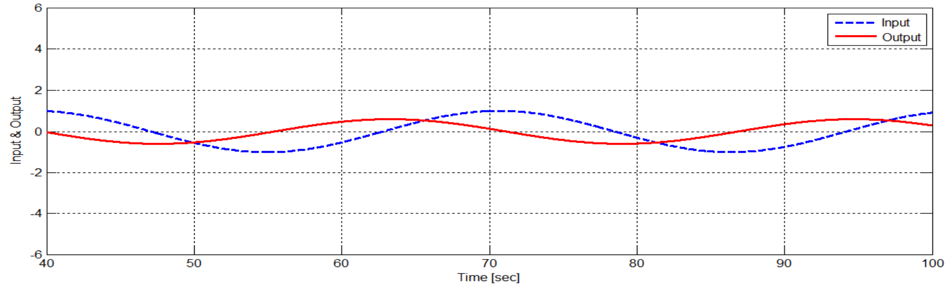
$$\hat{M}_1=9+20\log_{10}\left(\omega\right)$$

$$\phi=+90^\circ$$

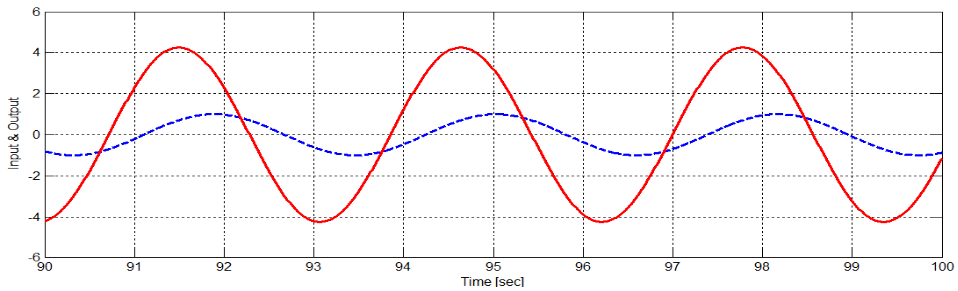
Draw Asymptotes For Each Term, Then Sum & Sketch Actual Curves



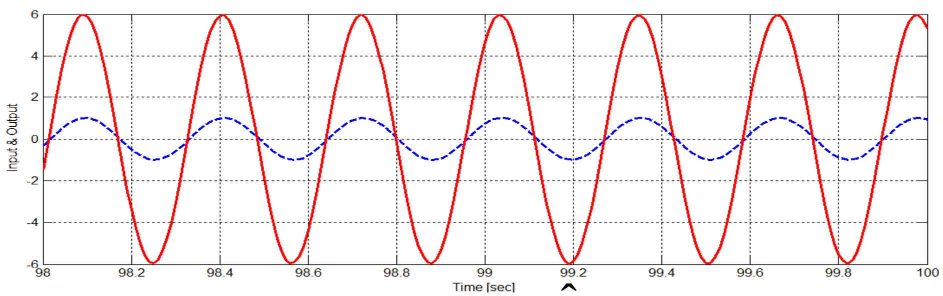
High-Pass Filter : Time → Frequency



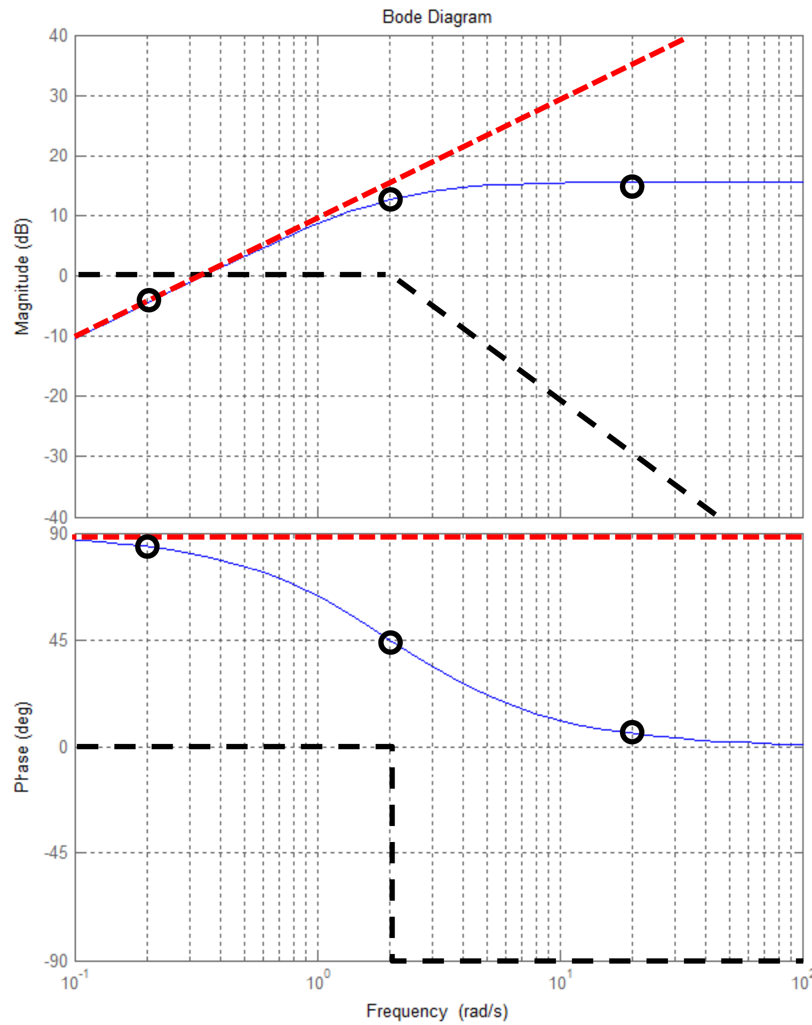
$\omega = 0.2rps \quad M \approx 0.6 \Rightarrow \hat{M} \approx -5 \quad \phi \approx 86°$



$\omega = 2rps \quad M \approx 4 \Rightarrow \hat{M} \approx 12 \quad \phi \approx 45°$



$\omega = 20rps \quad M \approx 6 \Rightarrow \hat{M} \approx 15 \quad \phi \approx 4°$



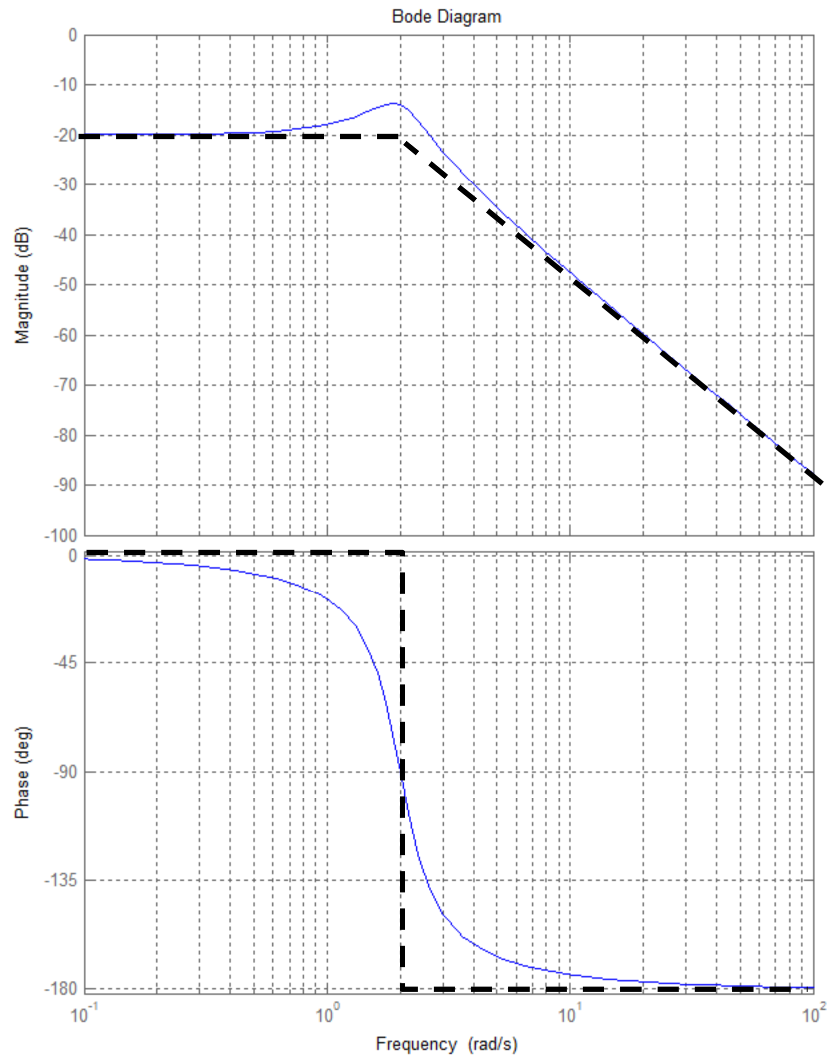
Example : Second-Order System

$$G(s) = \frac{0.4}{s^2 + s + 4}$$

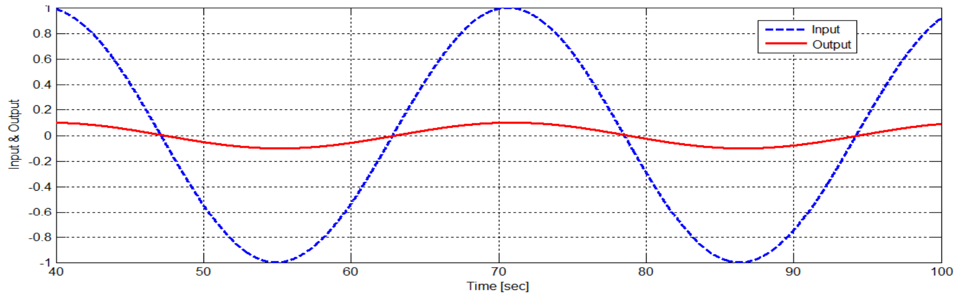
$$\omega \ll 2 \Rightarrow \begin{cases} \hat{M} = -20 \\ \phi = 0 \end{cases}$$

$$\omega = 2 \Rightarrow \begin{cases} \hat{M} = 20\log_{10}(0.2) = -14 \\ \phi = -90^\circ \end{cases}$$

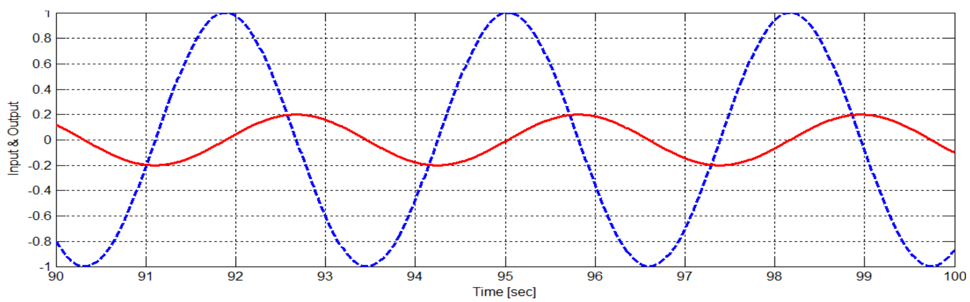
$$\omega \gg 2 \Rightarrow \begin{cases} \hat{M} = -8 - 40\log_{10}\omega \\ \phi = -180^\circ \end{cases}$$



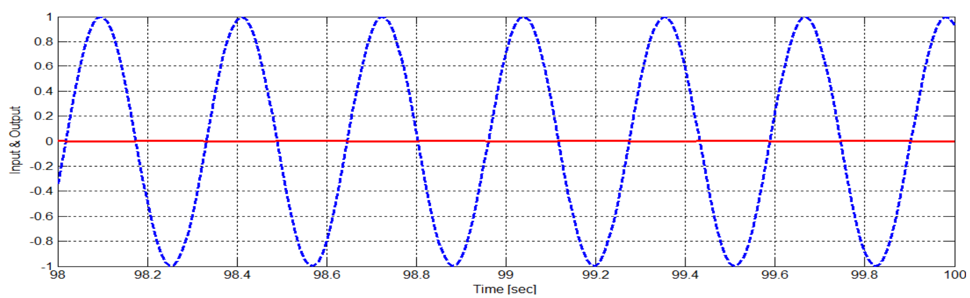
Second-Order System : Time → Frequency



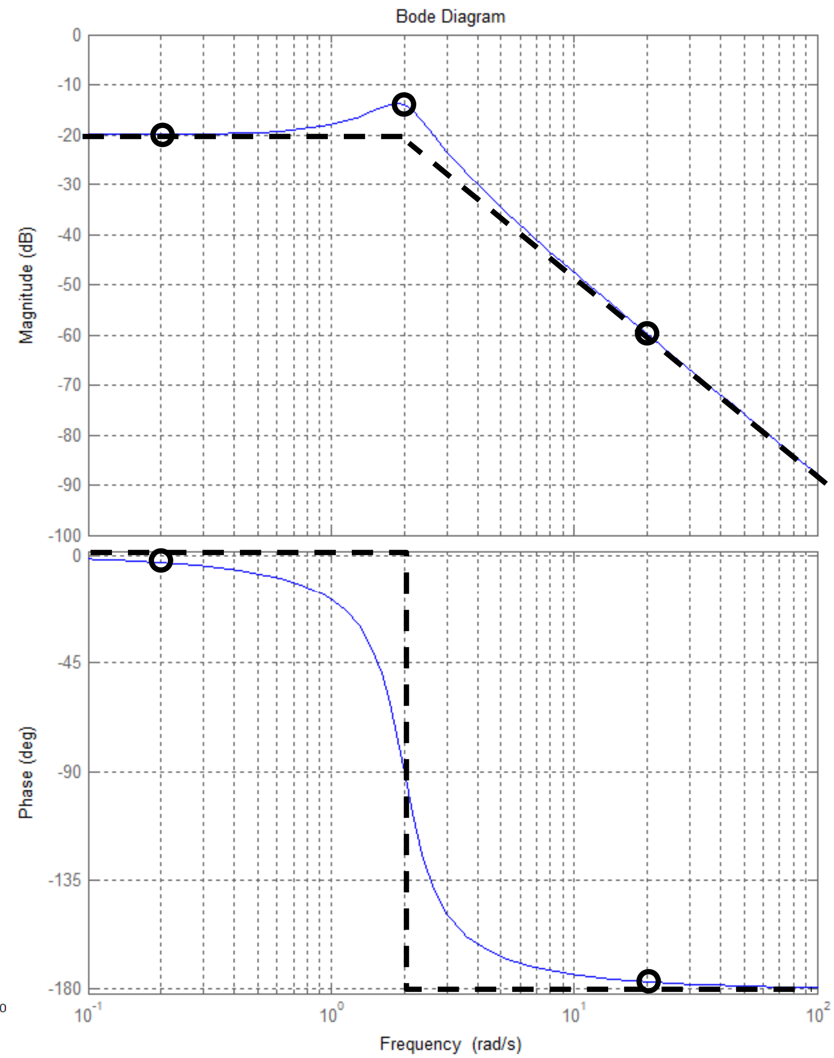
$\omega = 0.2rps \quad M \approx 0.1 \Rightarrow \hat{M} \approx -20 \quad \phi \approx -4^\circ$



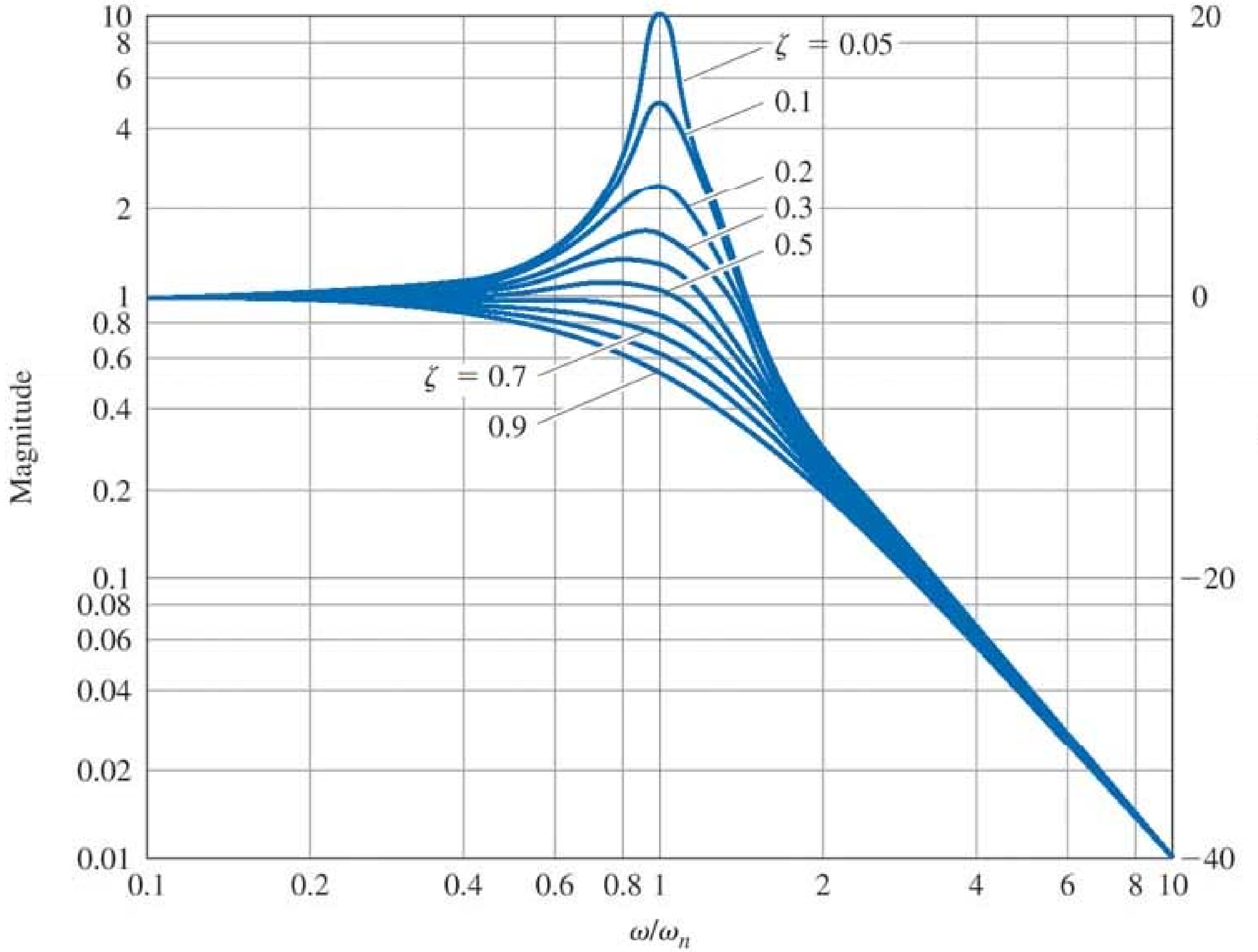
$\omega = 2rps \quad M \approx 0.2 \Rightarrow \hat{M} \approx -14 \quad \phi \approx 90^\circ$



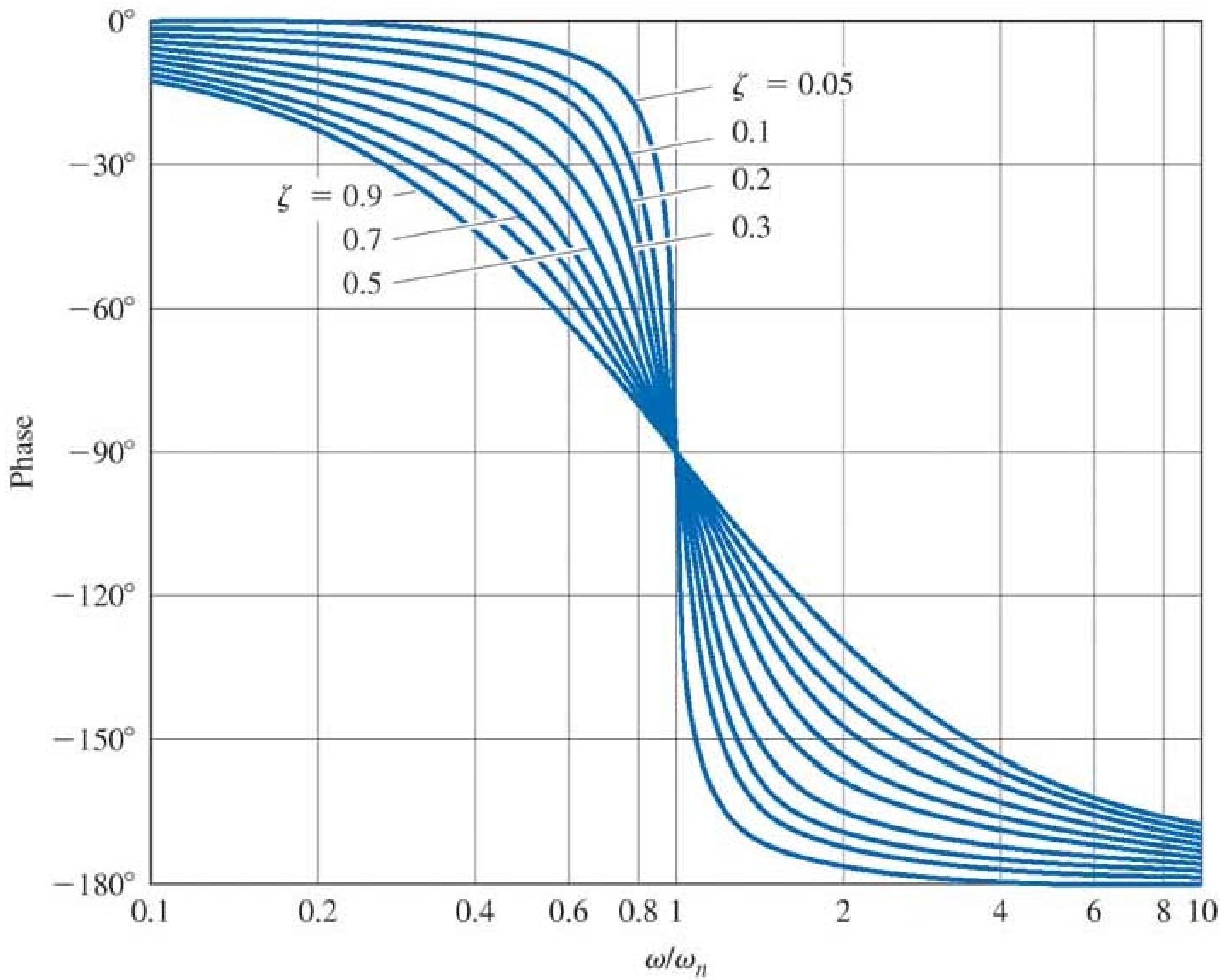
$\omega = 20rps \quad M \approx 0.001 \Rightarrow M \approx -60 \quad \phi \approx -176^\circ$



Damping Ratio Effect on Bode Plot (Magnitude)



Damping Ratio Effect on Bode Plot (Phase)



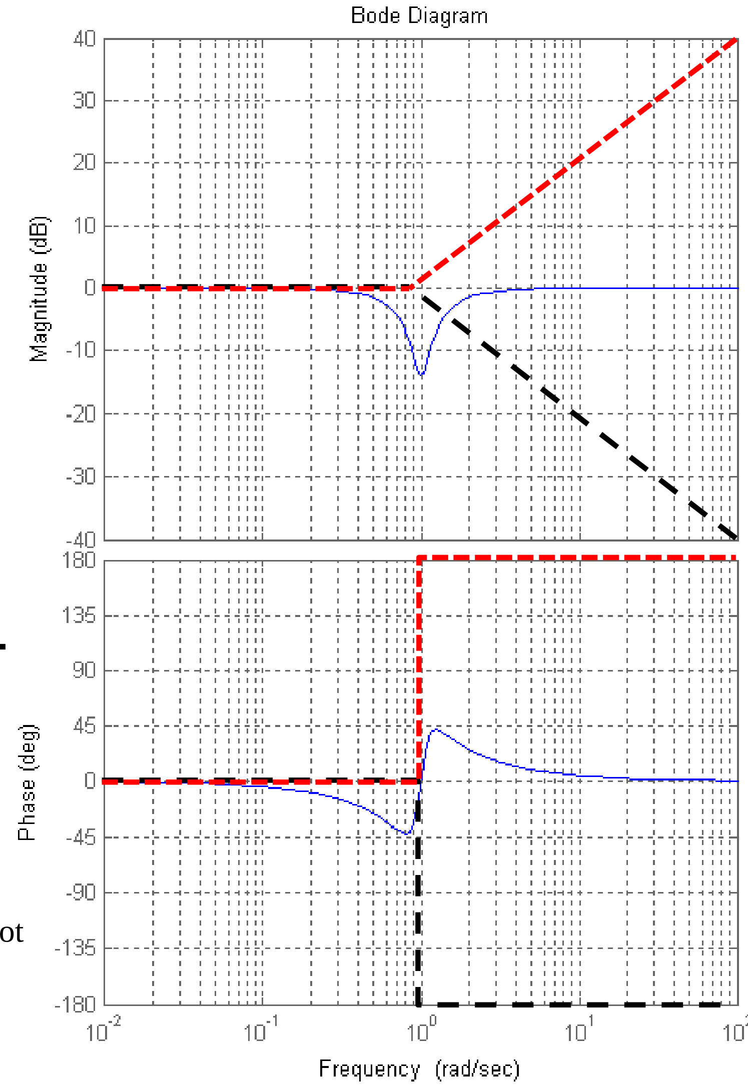
Example : Notch Filter

$$G(s) = \frac{s^2 + 0.2s + 1}{s^2 + s + 1}$$

$$\omega \ll 1 \Rightarrow \begin{cases} \hat{M} = 0 \\ \phi = 0° \end{cases}$$

$$\omega = 1 \Rightarrow \begin{cases} \hat{M} = 20\log_{10}(0.2) = -14 \\ \phi = 0° \end{cases}$$

$$\omega \gg 1 \Rightarrow \begin{cases} \hat{M} = 0 \\ \phi = 0° \end{cases}$$



Asymptotes Not Very Useful Here—Need Details!

General Approach

$$G(s) = \underbrace{K_o s^{N_Z}}_{\text{Every Transfer Function Has One Term That Looks Like This}} \cdots \underbrace{(\tau s + 1)^{\pm 1}}_{\text{Write All First-Order Poles \& Zeros Like This}} \cdots \underbrace{\left[\left(\frac{s}{\omega_n}\right)^2 + 2\varsigma \frac{s}{\omega_n} + 1\right]^{\pm 1}}_{\text{Write All Second-Order Poles \& Zeros Like This}} \cdots$$

N_Z= number of zeros @ origin (negative for poles @ origin)

K_o = Gain When All Other Terms Written In Form Shown

Example : 3rd-Order System with Zero

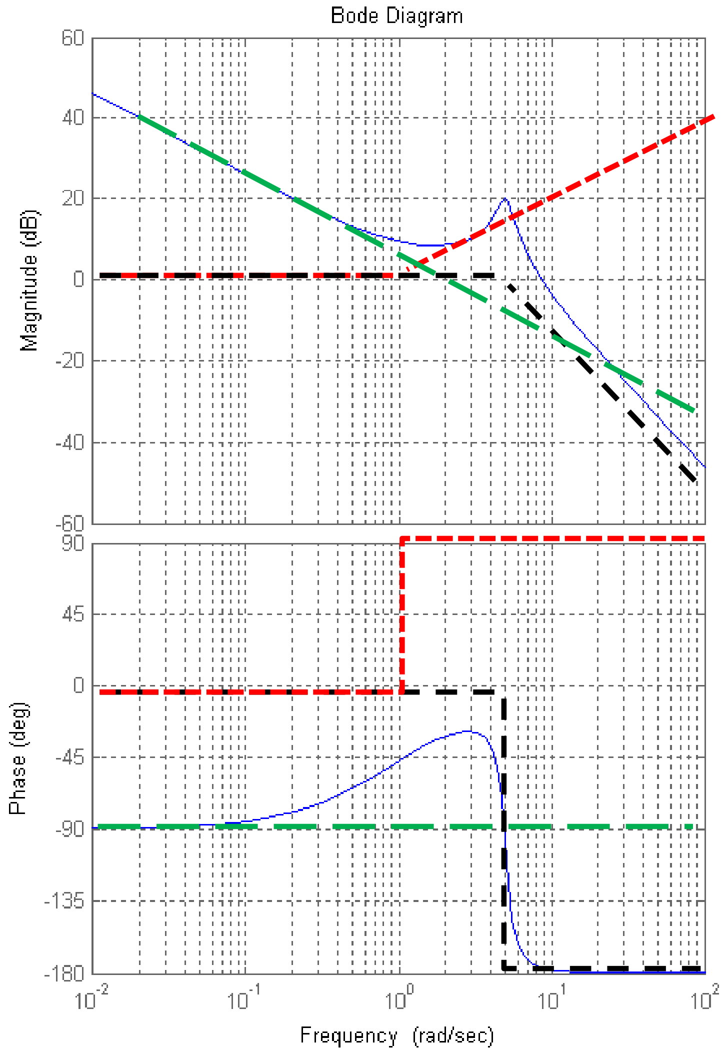
$$G(s)=\frac{50s+50}{s\left(s^2+s+25\right)}$$

$$G(s)=\frac{2}{s}(s+1)\frac{1}{\left(\frac{s^2}{5^2}+2(0.1)\frac{s}{5}+1\right)}$$

Slope of -20db/dec
Passing Through
Zero db @ 2 rps

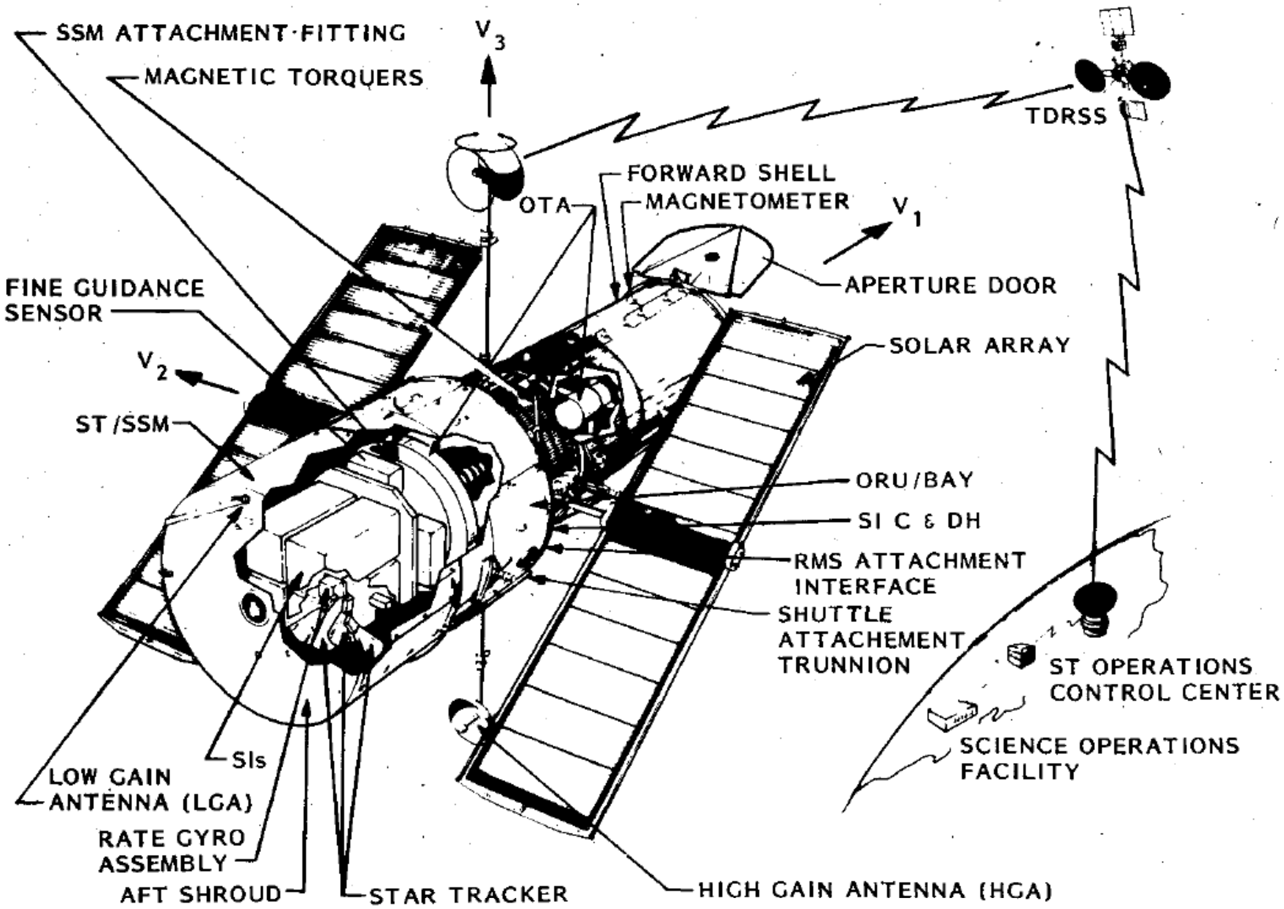
First-Order
Zero @ 1rps

Lightly Damped
Second-Order Pole
@ 5rps



Space Telescope

THE Space Telescope, shown in Fig. 1, has a 2.4-m telescope designed to allow scientists to observe the universe with a clarity and to distances never before achieved.



http://www.gsfc.nasa.gov/gsfc/service/gallery/fact_sheets/spacesci/hst3-01/hubble_space_telescope_systems.htm

Space Telescope Pointing Controller

VOL. 5, NO. 4, JULY-AUGUST 1982 J. GUIDANCE

AIAA 80-1784R

Space Telescope Pointing Control System

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Bendix Guidance Systems Division, Teterboro, N.J.
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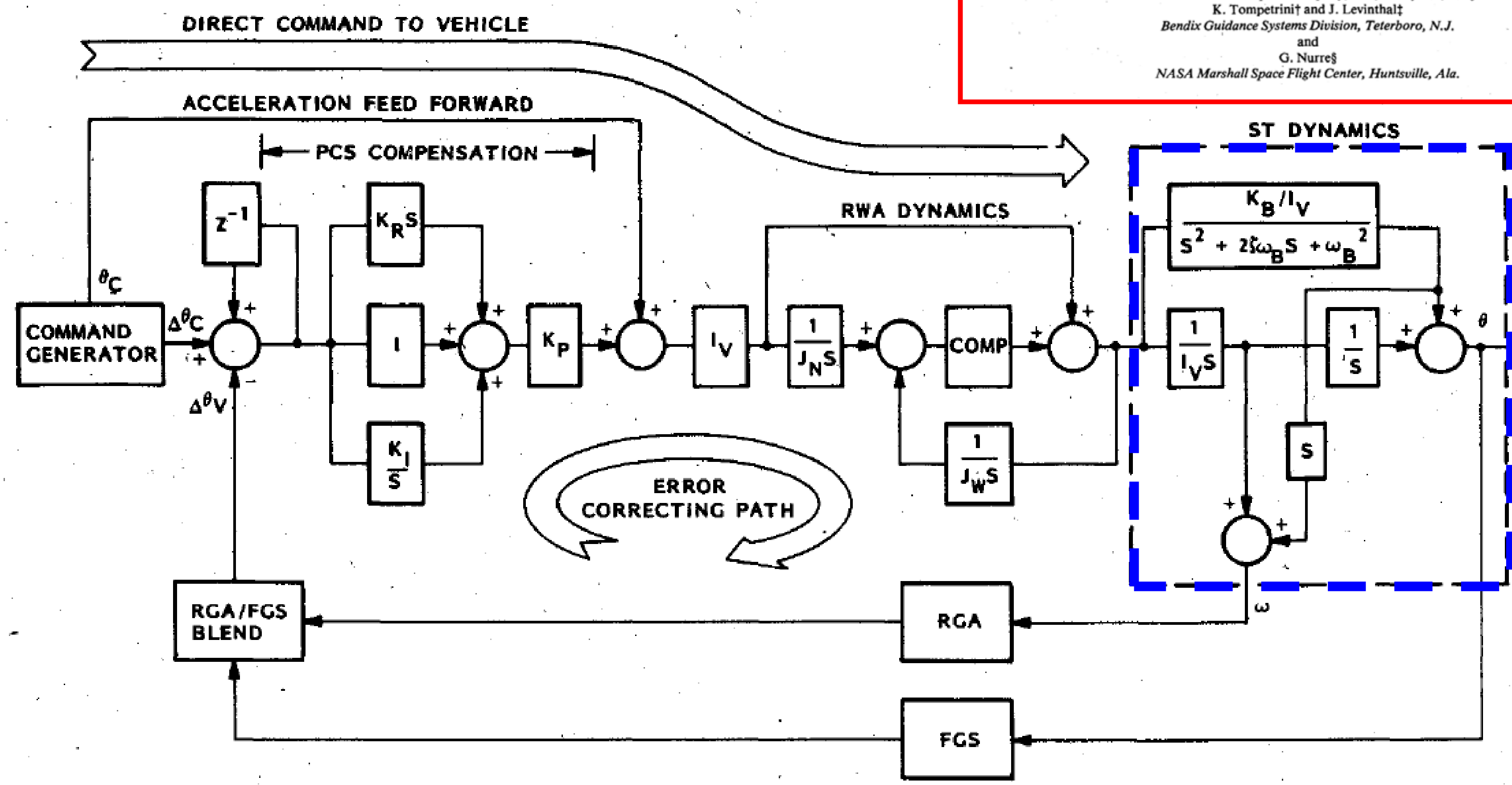
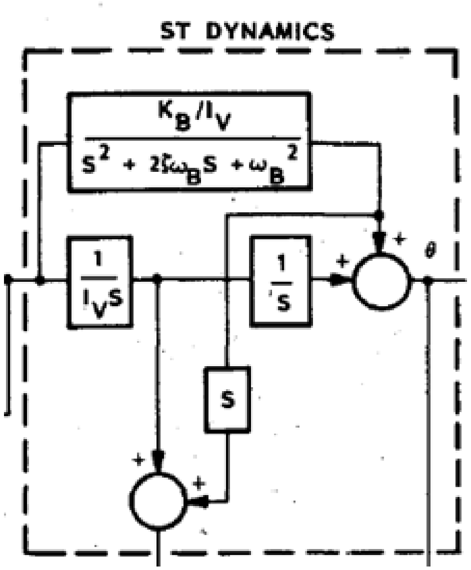
NASA Marshall Space Flight Center, Huntsville, Ala.

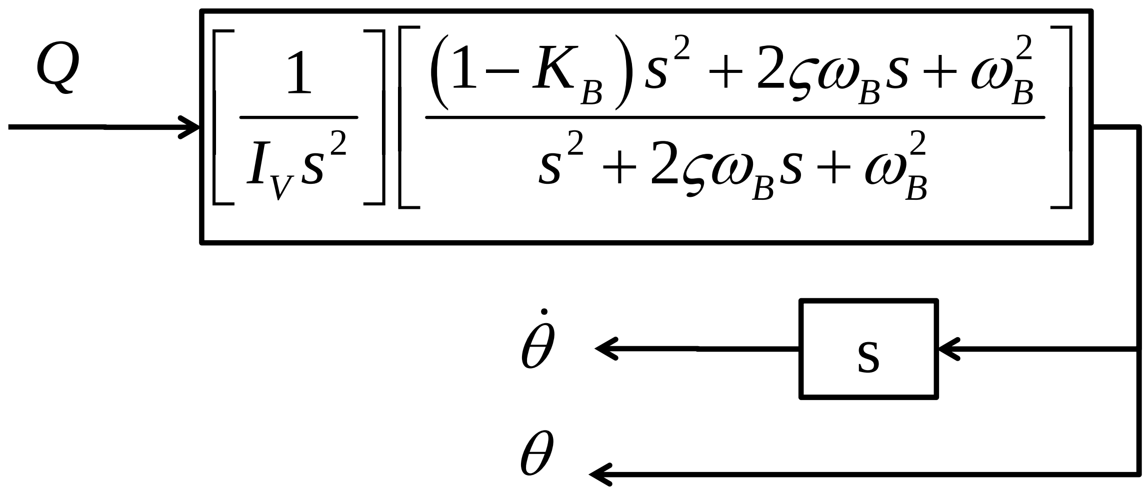


Space Telescope Dynamics

1) *Structural modes:* The solar array and optical telescope assembly modes have large modal coefficients. For example, the value of the solar array inertia about the Space Telescope center of mass is almost one-half that of the Space Telescope centerbody, which comprises the support systems module and optical telescope assembly. The control system sample rate and compensation are chosen to stabilize the modes. The command generator shapes maneuvers to limit structural excitations during maneuvers.

Modal Coefficient = K_B



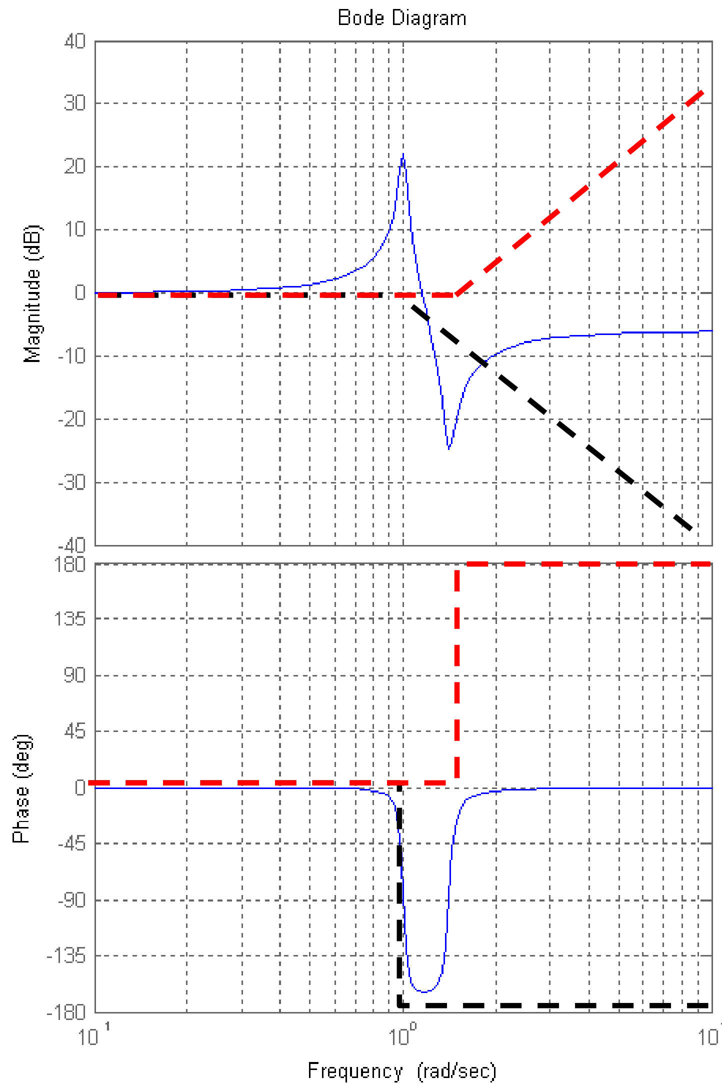


Using Bode Plot to Understand Dynamics

$$\frac{\theta}{Q} = \underbrace{\left[\frac{1}{I_V s^2}\right]}_{\text{Usual Rigid-Body Response}} \underbrace{\left[\frac{(1-K_B)s^2 + 2\varsigma\omega_B s + \omega_B^2}{s^2 + 2\varsigma\omega_B s + \omega_B^2}\right]}_{\text{“Filter” To Account for Structural Vibration}}$$

Usual Rigid-Body Response

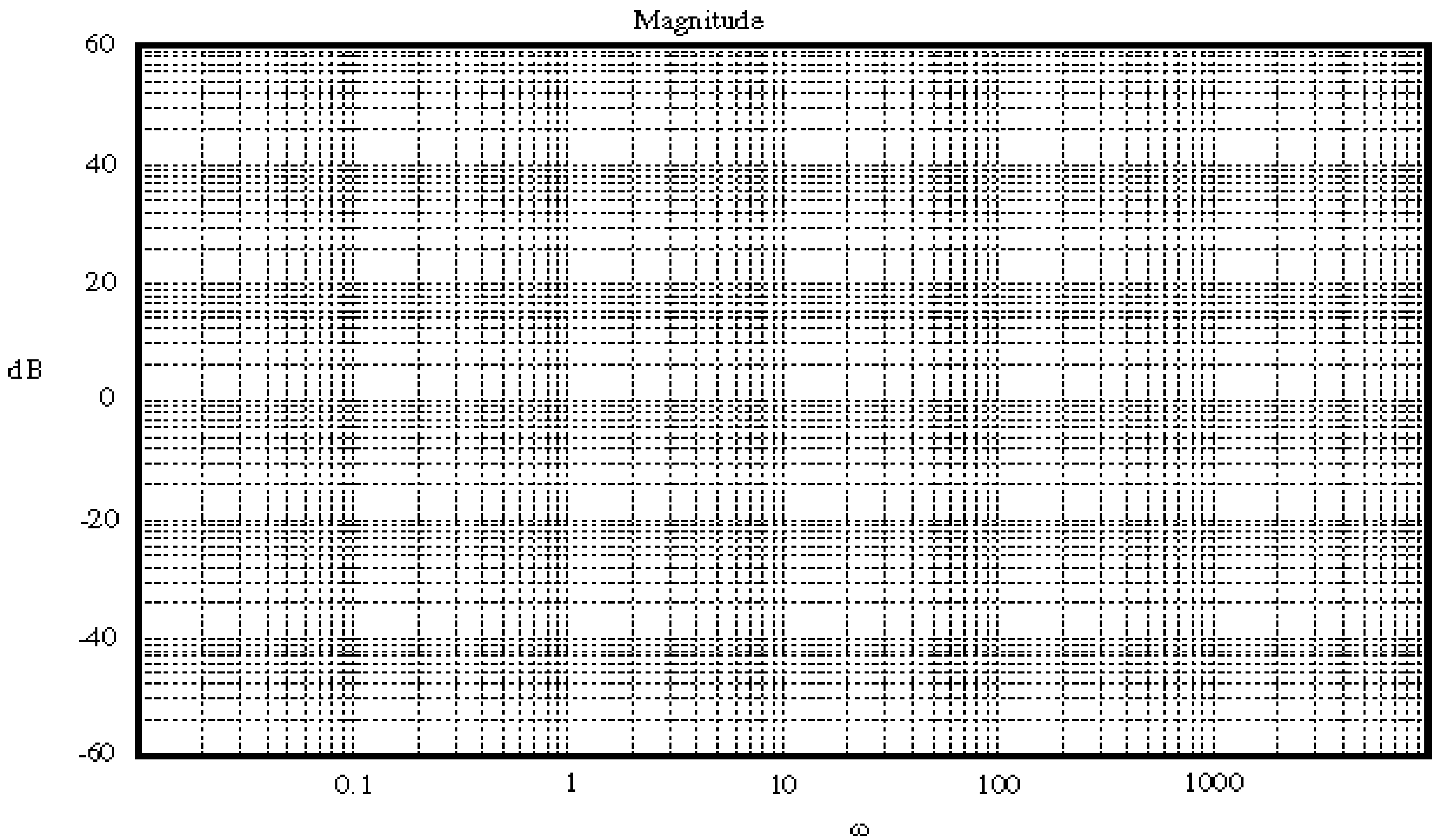
“Filter” To Account for Structural Vibration



K_B ~ 0.5 is an Easily Excited Mode
ζ~ 0.02 (or Less) is Typical!

(Plot has ω_n=1; Actual Space Station Modes Much Lower Frequency)

Bode Plot Paper (Magnitude)



Bode Plot Paper (Phase)