ESE 505 & MEAM 513 ONLY - SPRING 2012
HOMEWORK #13
DUE 23-Apr-2012 (Wednesday, 25-Apr-2012 @3PM with late pass)

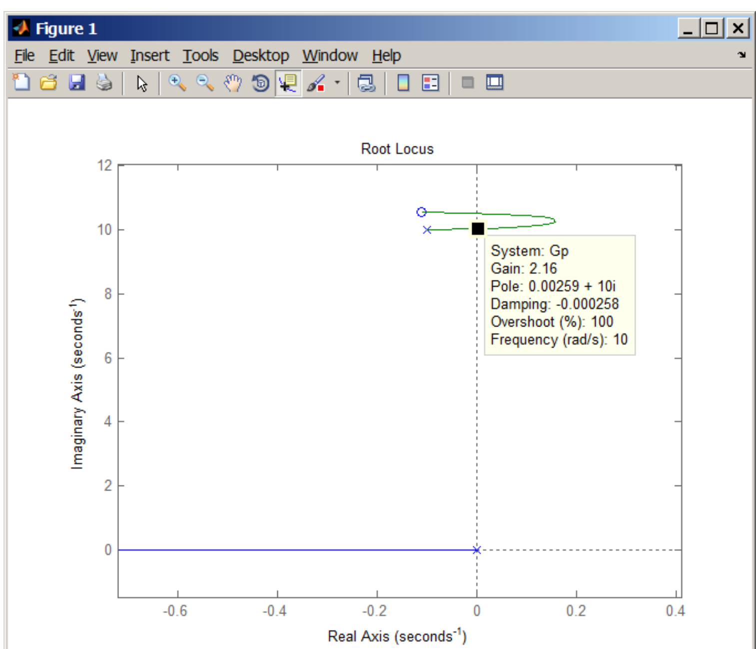
Let's do a modern control design for a problem we know how to do with classical methods. Consider a system described by the following state-space model:

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -100 & -0.2 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 0 \\ 0.1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \underline{x}$$

The corresponding transfer function is

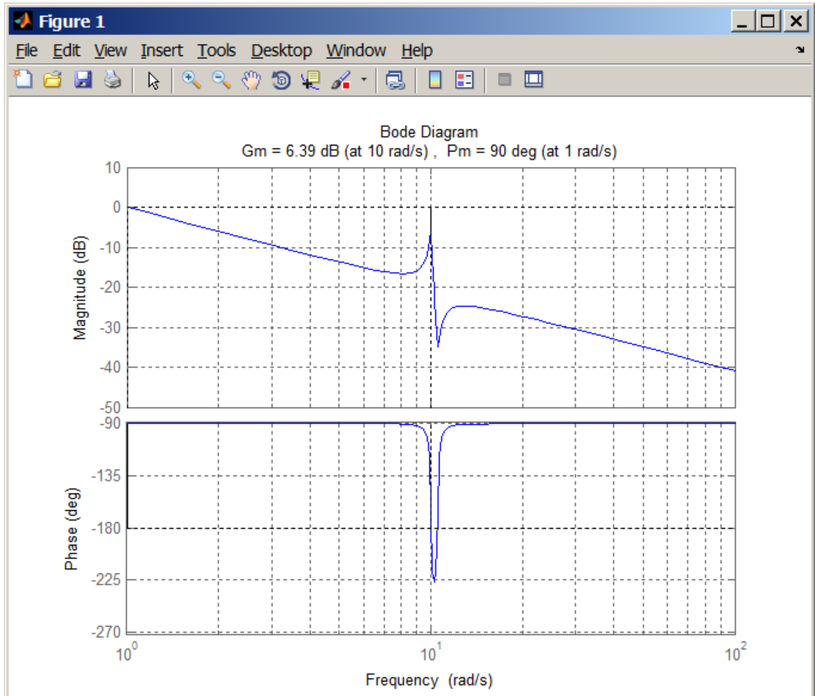
$$\frac{Y}{U} = G_P(s) = \frac{0.9s^2 + 0.2s + 100}{s\left(s^2 + 0.2s + 100\right)}$$

Suppose we would like to move the closed-loop pole at the origin to -5. At low frequency, the transfer function looks like $G_P(s) \approx \frac{1}{s}$, so we would just use proportional feedback with $K = 5$. The problem is that this will destabilize the lightly damped mode. We can easily see this root locus on G_P:



The gain for neutral stability is found to be slightly larger than 2.

We could also understand the situation with a bode plot of G_P:

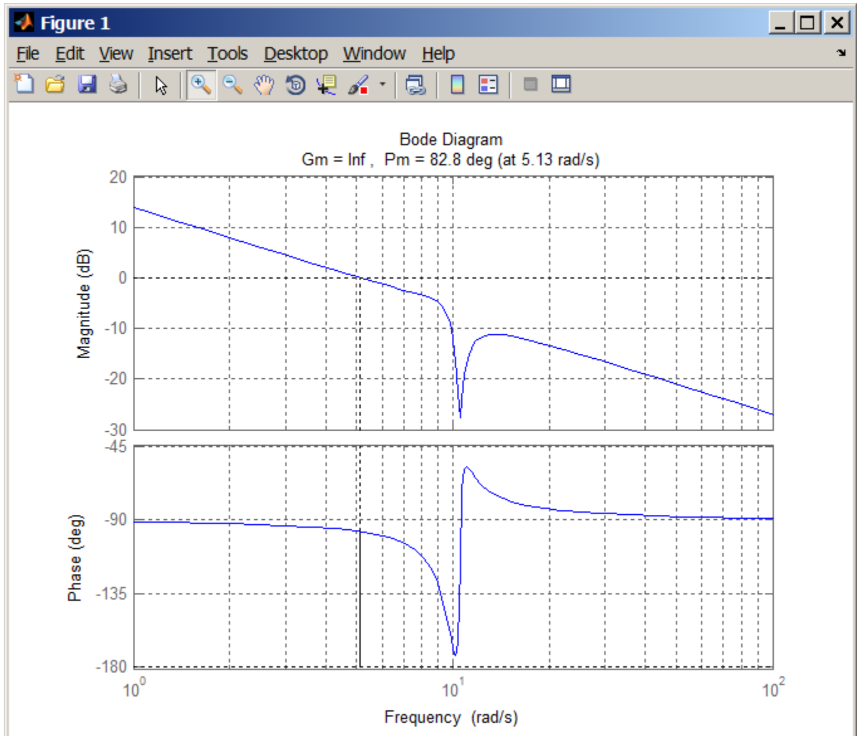


And of course the gain margin is slightly larger than 6dB (factor of 2).

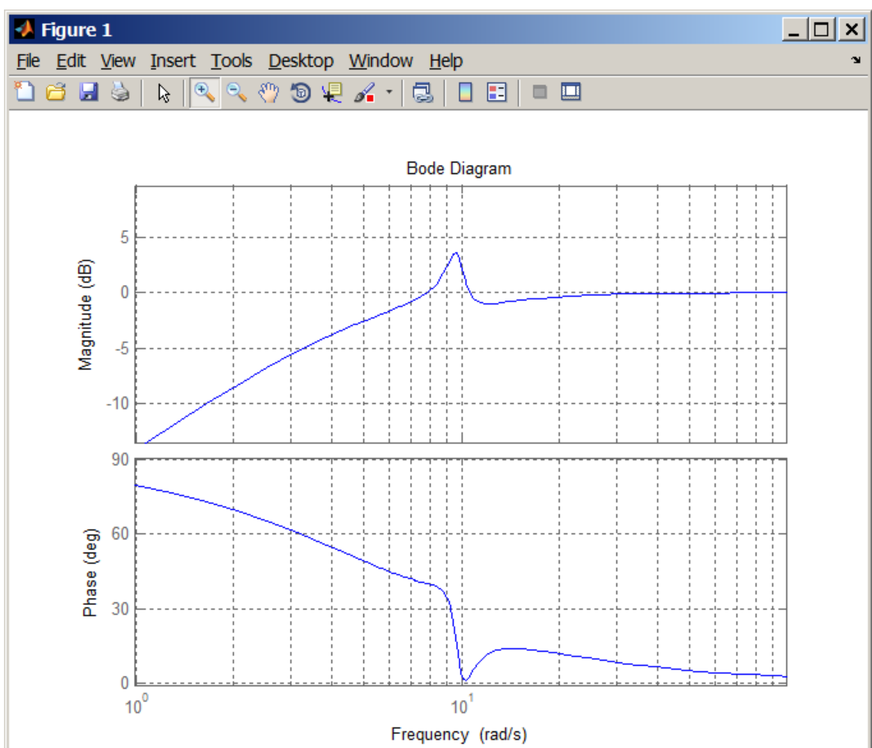
We know how to fix this: we should us a notch filter. Our knowledge of the system might tell us how accurately we know the frequency of the lightly damped mode and inform our choice of how wide to make the notch. The nominal damping ratio of the mode is about 1%; and we will set the depth on the basis of this estimate. To get our desired gain of 5 (about 14dB) with a gain margin of at least 6dB, we need a notch that is at least 14dB deep. Maybe we would go for 20dB, just to be safe and go with this:

$$G_C(s) = K\frac{s^2 + 0.2s + 100}{\left(s^2 + 2s + 100\right)}$$

The corresponding loop bode with K=5 is shown below. Notice that the phase change caused by the notch has actually raised the phase above -180 where we had the low gain margin in the plant. But we know that small changes in open-loop delay might change this, so we aren't too comfortable with our "infinite" gain margin here. In fact, we can look at the sensitivity function, $S(s) = \dfrac{1}{1 + G_C(s)G_P(s)}$ to get a sense of our maximum disturbance amplification. As shown below, we still have a notable, if reasonable, peak in S at the critical frequency.



Loop Bode



Sensitivity Function

Okay, now let's try modern control...

1. First, let's check controllability. After entering the state matrices shown above, we generate the controllability matrix. Check that these two commands give the same result:

   ```
   C1 = [B A*B A*A*B];
   C2 = ctrb(A,B);
   ```

   You can verify that the matrix is non-singular (has rank 3):

   ```
   rank(C1);
   ```

2. Since we have a controllable system, we know we can put the closed-loop poles where we like using proportional state feedback. MATLAB's "place" function will tell us the required gain matrix:

   ```
   K = place(A,B,p)
   ```

   where `p` is a vector containing the closed-loop poles we want, for example:

   ```
   p = [-5   -0.1+10*j   -0.1-10*j];
   ```

   This leaves the lightly damped open-loop poles where they were and moves the pole at the origin to the left, as desired. If you like, you can come back later and experiment with trying to stabilize the lightly damped modes. But for now, let's see what happens if we stick with the more modest objective.

   You can check that the given K works as intended by computing the eigenvalues of the closed-loop state matrix:

   ```
   eig(A-B*K)
   ```

3. Okay, now we need to design an estimator. We could confirm observability with `obsv(A,C)`. Or we could use duality: `ctrb(A',C')'`. Since we have an observable system, we can use feedback to place the observer poles wherever we want them. Let's do something simple and see how it works out. Since the observer is completely under our control (it would be implemented in the digital computer), we don't have to worry that the "open loop" estimator poles are different than we think. We don't want our error dynamics to be lightly damped. So, let's put the closed-loop observer poles on the negative real axis, at a point significantly left of where the desired closed-loop plant pole will be.

   Due to a limitation with how the `place` command works, we can't put all 3 of the observer poles at the same place, so let's choose `p=[-19   -20   -21]`. Now we use duality to design the observer gain matrix:

   ```
   L = place(A',C',p)'
   ```

Again, we can verify that we got the desired observer poles with `eig(A-L*C)`.

4. We now have a design that looks like Figure 7.35 in the textbook[1]. Equation 7.177 in the book expresses the effective compensator transfer function that results from our combine state-feedback + estimator. Let's form that transfer function in MATLAB.

```
Ac = A - B*K - L*C
Bc = L
Cc = K
Dc = 0
[numc, denc]=ss2tf(Ac,Bc,Cc,Dc)
Gc = tf(numc,denc)
```

5. Let's compare the compensator from our modern design with the compensator from our classical design:

```
Gcclassical = 5*tf([1 0.2 100],[1 2.0 100]);
bode(Gcclassical); grid on; hold on; bode(Gc);
set(gcf,'Color','w');
```

The modern control came up with some sort of notch filter automatically! That is pretty cool! On the other hand, we don't have insight into how wide the notch is, nor how deep it is; both of these were design choices we made very deliberately in the classical approach. And there is the puzzling fact that the low-frequency gain of the compensator is much lower than our classical compensator, but somehow, the closed-loop poles are in very similar locations? Evidently, the lead at high frequency (the modern controller gain is higher between 20 and 50 rps) allows the modern controller to move the pole further to the left than we might expect. Is this lead a good idea? What unmodeled dynamics might be lurking out there to cause problems? Such questions are intended to remind you of the great value of the methods you have spent so much time learning this semester.

6. Finally, make a loop bode plot comparing the modern and classical designs. Submit this plot.

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[1] The textbook uses "west coast" notation (F, G, H) instead of our "east coast" notation (A, B, C). I think most of the literature has converted to the east coast notation by now, but the conversion is not difficult.