Algebra e matematica discreta, a.a. 2020/2021,

Scuola di Scienze - Corso di laurea: Informatica

ESERCIZIO TIPO 4

Sia $\mathbf{A}(\alpha) = \begin{pmatrix} \alpha-1 & 1 & \alpha-1 \\ \alpha-1 & 1 & -1 \\ 0 & 1 & 0 \end{pmatrix}$, dove $\alpha \in \mathbb{R}$. Per quegli $\alpha \in \mathbb{R}$ per cui $\mathbf{A}(\alpha)$ è non singolare, si calcoli $\mathbf{A}(\alpha)^{-1}$.

$$\big(\mathbf{A}(\alpha) \mid \mathbf{I}_3\big) = \left(\begin{array}{ccc|ccc} \alpha-1 & 1 & \alpha-1 & 1 & 0 & 0 \\ \alpha-1 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{\boxed{\alpha \neq 1 : \mathbf{A}(1) \text{ non ha inversa}}}$$

$$\xrightarrow{E_{21}(-\alpha+1)E_1(\frac{1}{\alpha-1})} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{\alpha-1} & 1 & \frac{1}{\alpha-1} & 0 & 0 \\ 0 & 0 & -\alpha & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \xrightarrow{E_{23}}$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & \frac{1}{\alpha-1} & 1 & \frac{1}{\alpha-1} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\alpha & -1 & 1 & 0 \end{array}\right) \xrightarrow{\boxed{\alpha \neq 0 : \mathbf{A}(0) \text{ non ha inversa}} \quad E_3(-\frac{1}{\alpha})}$$

$$\rightarrow \left(\begin{array}{ccc|ccc} 1 & \frac{1}{\alpha-1} & 1 & \frac{1}{\alpha-1} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{\alpha} & -\frac{1}{\alpha} & 0 \end{array}\right) \xrightarrow{E_{13}(-1)} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{\alpha-1} & 0 & \frac{1}{\alpha(\alpha-1)} & \frac{1}{\alpha} & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{\alpha} & -\frac{1}{\alpha} & 0 \end{array}\right) \rightarrow$$

$$\xrightarrow{E_{12}(-\frac{1}{\alpha-1})} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{\alpha(\alpha-1)} & \frac{1}{\alpha} & -\frac{1}{\alpha-1} \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{\alpha} & -\frac{1}{\alpha} & 0 \end{array}\right) = \big(\mathbf{I}_3 | \mathbf{A}(\alpha)^{-1}\big).$$

Se $\boxed{\alpha \notin \{0,1\}}$ $\quad \mathbf{A}(\alpha)^{-1} = \begin{pmatrix} \frac{1}{\alpha(\alpha-1)} & \frac{1}{\alpha} & -\frac{1}{\alpha-1} \\ 0 & 0 & 1 \\ \frac{1}{\alpha} & -\frac{1}{\alpha} & 0 \end{pmatrix}.$