Trovare tutti gli invertibili in $\mathbb{Z}_n$
con $n =$ numero di lettere nel proprio nome

MICHELE $\Rightarrow n = 7$ sol $[1]_7\ [2]_7\ [3]_7\ [4]_7\ [5]_7\ [6]_7$

$\varphi(7) = 7\left(1 - \frac{1}{7}\right) = 6$

$\boxed{0} \equiv 1 \bmod 7 \longrightarrow$ mai invertibile ✗

$1 \equiv 1 \bmod 7 \quad MCD(1,7) = 1$ ✓

$2 \equiv 1 \bmod 7 \quad MCD(2,7) = 1$ ✓

$\vdots$

$6 \equiv 1 \bmod 7 \quad MCD(6,7) = 1$ ✓

Si risolva il sistema lineare $A(\alpha)\underline{x}=\underline{b}(\alpha)$ dove:

$$[A(\alpha)\mid \underline{b}(\alpha)] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & \alpha-1 & \alpha-1 & \alpha-1 \\ -1 & 0 & \alpha-2 & -1 \end{array}\right] \xrightarrow[E_{31}(1)]{} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & \alpha-1 & \alpha-1 & \alpha-1 \\ 0 & 0 & \alpha-1 & 0 \end{array}\right] = [B(\alpha)\mid \underline{c}(\alpha)]$$

CASO 1 : $\alpha-1=0 \Rightarrow \alpha=1$

$$[B\mid \underline{c}] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] = [C\mid \underline{e}] \Rightarrow$$

$\underline{e}$ non è dominante e C ha 2 colonne libere $\Rightarrow$ ci sono ∞^2 sol.

$$\underline{v} = \begin{bmatrix} x_1 = 1-h \\ x_2 = k \\ x_3 = h \end{bmatrix} \qquad sol = \left\{ \begin{bmatrix} 1-h \\ k \\ h \end{bmatrix} \;\middle|\; h,k \in \mathbb{C} \right\}$$

Caso 2: $\alpha-1\neq 0 \Rightarrow \alpha\neq 1$

$$[B(\alpha)\mid \underline{c}(\alpha)] \xrightarrow[E_2\left(\frac{1}{\alpha-1}\right)]{} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \alpha-1 & 0 \end{array}\right] \xrightarrow[E_3\left(\frac{1}{\alpha-1}\right)]{} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right] = [E\mid \underline{d}]$$

$\underline{d}$ non dominante
e E ha tutte le colonne
dominanti $\Rightarrow \exists$ un unica sol

$$\underline{v}' = \begin{bmatrix} x_1 = 1 \\ x_2 = 1 \\ x_3 = 0 \end{bmatrix} \qquad sol = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$