Algebra e matematica discreta, a.a. 2020/2021,

Scuola di Scienze - Corso di laurea: Informatica

ESERCIZIO TIPO 10

Si consideri l'applicazione lineare $T : \mathbb{C}^2 \to \mathbb{C}^3$ definita da

$$T(\begin{pmatrix} a \\ b \end{pmatrix}) = \begin{pmatrix} 4a + b \\ 3a \\ a - 2b \end{pmatrix}.$$

Si determini la matrice $\mathbf{A}$ associata ad T rispetto alle basi ordinate

$$\boldsymbol{\mathcal{B}} = \left\{ \begin{pmatrix} 2 \\ 6 \end{pmatrix}; \begin{pmatrix} 2 \\ -4 \end{pmatrix} \right\} \quad \text{e} \quad \boldsymbol{\mathcal{D}} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}; \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}$$

su dominio e codominio rispettivamente.

La matrice che cerchiamo è

$$\mathbf{A} = \begin{pmatrix} C_{\boldsymbol{\mathcal{D}}}(T(\begin{pmatrix} 2 \\ 6 \end{pmatrix})) & C_{\boldsymbol{\mathcal{D}}}(T(\begin{pmatrix} 2 \\ -4 \end{pmatrix})) \end{pmatrix}.$$

Poichè

$$T(\begin{pmatrix} 2 \\ 6 \end{pmatrix}) = \begin{pmatrix} 14 \\ 6 \\ -10 \end{pmatrix} \quad \text{e} \quad T(\begin{pmatrix} 2 \\ -4 \end{pmatrix}) = \begin{pmatrix} 4 \\ 6 \\ 10 \end{pmatrix},$$

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$$\mathbf{A} = \begin{pmatrix} C_{\boldsymbol{\mathcal{D}}}(\begin{pmatrix} 14 \\ 6 \\ -10 \end{pmatrix}) & C_{\boldsymbol{\mathcal{D}}}(\begin{pmatrix} 4 \\ 6 \\ 10 \end{pmatrix}) \end{pmatrix}.$$

Piuttosto che calcolare separatamente $C_{\boldsymbol{\mathcal{D}}}(\begin{pmatrix} 8 \\ 3 \\ -13 \end{pmatrix})$ e $C_{\boldsymbol{\mathcal{D}}}(\begin{pmatrix} -1 \\ 6 \\ 8 \end{pmatrix})$, e calcoliamo $C_{\boldsymbol{\mathcal{D}}}(\begin{pmatrix} a \\ b \\ c \end{pmatrix})$ per un generico vettore $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \in \mathbb{R}^3$, e specializziamo la formula ottenuta ai due diversi vettori $\begin{pmatrix} 14 \\ 6 \\ -10 \end{pmatrix}$ e $\begin{pmatrix} 4 \\ 6 \\ 10 \end{pmatrix}$. Poichè

$$C_{\boldsymbol{\mathcal{D}}}(\begin{pmatrix} a \\ b \\ c \end{pmatrix}) = \begin{pmatrix} \alpha \\ \beta \\ \delta \end{pmatrix} \Bigg| \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} + \delta \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} \alpha + \delta \\ 3\beta \\ \alpha - \delta \end{pmatrix}$$

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$$\begin{cases} \alpha + \delta = a \\ 3\beta = b \\ \alpha - \delta = c \end{cases} \quad \Longrightarrow \quad \begin{cases} \alpha = (a+c)/2 \\ \beta = b/3 \\ \delta = (a-c)/2 \end{cases} \quad \Longrightarrow \quad C_{\boldsymbol{\mathcal{D}}}\left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}\right) = \begin{pmatrix} (a+c)/2 \\ b/3 \\ (a-c)/2 \end{pmatrix}.$$

Ponendo $a = 14$, $b = 6$ e $c = -10$ otteniamo $C_{\boldsymbol{\mathcal{D}}}\left(\begin{pmatrix} 14 \\ 6 \\ -10 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 2 \\ 12 \end{pmatrix}$; ponendo $a = 4$, $b = 6$ e $c = 10$ otteniamo $C_{\boldsymbol{\mathcal{D}}}\left(\begin{pmatrix} 4 \\ 6 \\ 10 \end{pmatrix}\right) = \begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix}$. Quindi

$$\mathbf{A} = \begin{pmatrix} 2 & 7 \\ 2 & 2 \\ 12 & -3 \end{pmatrix}.$$