L. PARMEGGIANI

- LGEBRA E MATEMATICA DISCRETA

Corso di Laurea: Informatica

SVOLGIMENTO DEGLI ESERCIZI PER CASA 1 (2ª PARTE)

4 In tutti i casi considerati nell'Esercizio 2, ricordando che d il massimo comun divisore positivo di a e b, si trovino $m, n \in \mathbb{Z}$ tali che $d = ma + nb$.

1) $a = 126$, $b = 56$, $d = 14$

$126 = 56 \cdot 2 + 14 \Rightarrow 14 = 126 - 56 \cdot 2$

(126: a, 56: b, 14: d)

$\Rightarrow \begin{cases} m = 1 \\ n = -2 \end{cases}$

2) $a = 234$, $b = 273$, $d = 39$

$273 = 234 \cdot 1 + 39 \Rightarrow 39 = 273 - 234$

(273: b, 234: a, 39: d)

$\Rightarrow \begin{cases} m = -1 \\ n = 1 \end{cases}$

3) $a = -168$, $b = 180$, $d = 12$

$180 = 168 \cdot 2 + 12$

$\Rightarrow 180 = (-168) \cdot (-1) + 12 \Rightarrow 12 = 180 + (-168)$

(180: b, -168: a, 12: d)

$\Rightarrow \begin{cases} m = 1 \\ n = 1 \end{cases}$

4) $a = 231$, $b = 165$, $d = 33$

$231 = 165 \cdot 1 + 66 \Rightarrow \boxed{66 = 231 - 165}$

$165 = 66 \cdot 2 + 33 \Rightarrow 33 = 165 - 66 \cdot 2 =$

(33: d)

$= 165 - (231 - 165) \cdot 2 =$

$= 165 - 231 \cdot 2 + 165 \cdot 2 =$

$= 165 \cdot 3 - 231 \cdot 2$

$$\Rightarrow \underbrace{33}_{d} = \underbrace{165}_{b} \cdot 3 + \underbrace{231}_{a} \cdot (-2)$$

$$\Rightarrow \begin{cases} m = -2 \\ n = 3 \end{cases}$$

5) $a = -136, \quad b = 48, \quad d = 8$

$$136 = 48 \cdot 2 + 40 \Longrightarrow \boxed{40 = 136 - 48 \cdot 2}$$

$$48 = 40 \cdot 1 + \underbrace{8}_{d} \Rightarrow 8 = 48 - 40 =$$

$$= 48 - (136 - 48 \cdot 2) =$$

$$= 48 - 136 + 48 \cdot 2 =$$

$$= 48 \cdot 3 - 136$$

$$\Rightarrow \underbrace{8}_{d} = \underbrace{48}_{b} \cdot 3 + \underbrace{(-136)}_{a}$$

$$\Rightarrow \begin{cases} m = 1 \\ n = 3 \end{cases}$$

6) $a = -208, \quad b = 286, \quad d = 26$

$$286 = 208 \cdot 1 + 78 \Rightarrow \boxed{78 = 286 - 208}$$

$$208 = 78 \cdot 2 + 52 \Rightarrow \boxed{52 = 208 - 78 \cdot 2}$$

$$78 = 52 \cdot 1 + \underbrace{26}_{d} \Rightarrow 26 = 78 - 52 =$$

$$= 78 - (208 - 78 \cdot 2) =$$

$$= 78 - 208 + 78 \cdot 2 =$$

$$= 78 \cdot 3 - 208 =$$

$$= (286 - 208) \cdot 3 - 208 =$$

$$= 286 \cdot 3 - 208 \cdot 3 - 208 =$$

$$= 286 \cdot 3 - 208 \cdot 4$$

$$\Rightarrow \begin{cases} m = 4 \\ n = 3 \end{cases}$$

7) $a = 132,\ b = 180,\ d = 12$

$180 = 132 + 48 \Rightarrow \boxed{48 = 180 - 132}$

$132 = 48 \cdot 2 + 36 \Rightarrow \boxed{36 = 132 - 48 \cdot 2}$

$48 = 36 \cdot 1 + \underbrace{12}_{d} \Rightarrow 12 = 48 - 36 =$

$= 48 - (132 - 48 \cdot 2) =$

$= 48 - 132 + 48 \cdot 2 =$

$= 48 \cdot 3 - 132 =$

$= (180 - 132) \cdot 3 - 132 =$

$= 180 \cdot 3 - 132 \cdot 3 - 132 =$

$= 180 \cdot 3 - 132 \cdot 4$

$\Rightarrow \underbrace{12}_{d} = \underbrace{180}_{b} \cdot 3 - \underbrace{132}_{a} \cdot 4$

$\Rightarrow \begin{cases} m = -4 \\ n = 3 \end{cases}$

5 Si dica quali delle seguenti congruenze sono vere e quali false:

1) $132 \equiv 8 \mod 9$ FALSA : $132 = 9 \cdot 14 + 6$

$\Rightarrow 132 \equiv 6 \mod 9$

2) $132 \equiv 1 \mod 11$ FALSA : $132 = 11 \cdot 12 + 0$

$\Rightarrow 132 \equiv 0 \mod 11$

3) $132 \equiv 0 \mod 12$ **VERA** $132 = 12 \cdot 11 + 0$

$\Rightarrow 132 \equiv 0 \mod 12$

4) $132 \equiv 4 \mod 13$ FALSA : $132 = 13 \cdot 10 + 2$

$\Rightarrow 132 \equiv 2 \mod 13$