(6) Risolvere le seguenti relazioni di ricorrenza:

- $a_n = 3a_{n-1} + 4a_{n-2}$, $a_0 = a_1 = 1$;
- $a_n = a_{n-2}$, $a_0 = a_1 = 1$;
- $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$, $a_0 = a_1 = 1$, $a_2 = 2$;

- $a_n = 3a_{n-1} - 2$, $a_0 = 0$;
- $a_n = 2a_{n-1} + 2n^2$, $a_0 = 3$;

1) $a_n = 3a_{n-1} + 4a_{n-2}$ $\qquad a_0 = a_1 = 1 \quad r = 2$

$$x^n = 3x^{n-1} + 4x^{n-2}$$

$$\frac{x^n}{x^{n-2}} = \frac{3x^{n-1}}{x^{n-2}} + \frac{4\cancel{x^{n-2}}}{\cancel{x^{n-2}}} \Rightarrow x^2 = 3x + 4 \Rightarrow x^2 - 3x - 4 = 0$$

$$\alpha_{1,2} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} \Rightarrow \alpha_1 = -1 \quad \alpha_2 = 4$$
$$m_1 = 1 \quad m_2 = 1$$

$$a_n = A_1(-1)^n + A_2 \cdot 4^n$$

$$\begin{cases} A_1(-1)^0 + A_2 \cdot 4^0 = 1 \\ A_1(-1)^1 + A_2 \cdot 4^1 = 1 \end{cases} \Rightarrow \begin{cases} A_1 + A_2 = 1 \\ -A_1 + 4A_2 = 1 \end{cases}$$

$$\begin{cases} A_1 = 1 - A_2 \\ -1 + A_2 + 4A_2 = 1 \end{cases} \Rightarrow \begin{cases} A_1 = 3/5 \\ A_2 = 3/5 \end{cases} \Rightarrow \boxed{a_n = \frac{3}{5} \cdot (-1)^n + \frac{2}{5} \cdot 4^n}$$

2) $a_n = a_{n-2}$ $\qquad a_0 = a_1 = 1 \qquad r = 2$

$$x^n = 0 \cdot x^{n-1} + x^{n-2}$$

$$\frac{x^n}{x^{n-2}} = \left(0 \cdot \frac{x^{n-1}}{x^{n-2}}\right) + \cancel{\frac{x^{n-2}}{x^{n-2}}} \Rightarrow x^2 = (0 \cdot x) + 1$$

$x^2 - 1 = 0$ $\qquad \alpha_1 = -1 \quad \alpha_2 = 1$
$\qquad m_1 = 1 \quad m_2 = 1$

$$a_n = A_1(-1)^n + A_2(1)^n$$

$$\begin{cases} A_1(-1)^0 + A_2(1)^0 = 1 \\ A_1(-1)^1 + A_2(1)^1 = 1 \end{cases} \Rightarrow \begin{cases} A_1 + A_2 = 1 \\ -A_1 + A_2 = 1 \end{cases}$$

$$\begin{cases} A_1 = 1 - A_2 \\ -1 + A_2 + A_2 = 1 \end{cases} \Rightarrow \begin{cases} A_1 = 0 \\ A_2 = 1 \end{cases} \Rightarrow \boxed{a_n = 1}$$

3) $a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$ $\quad a_0 = a_1 = 1 \quad a_2 = 2$

$r = 3$

$$x^n = 3x^{n-1} - 3x^{n-2} + x^{n-3}$$

$$x^3 = 3x^2 - 3x + 1 \Rightarrow x^3 - 3x^2 + 3x - 1 = 0$$

$$-3x(x-1) + (x^3 - 1) = 0 \Rightarrow -3x(x-1) + (x-1)(x^2+x+1) = 0$$

$$(x-1)(-3x + x^2 + x + 1) = 0 \Rightarrow (x-1)(x^2 - 2x + 1) = 0$$

$$(x-1)^3 = 0 \qquad \alpha_1 = 1 \quad m_1 = 3$$

$$a_n = A_1(1)^n + A_2 n(1)^n + A_3 n^2 (1)^n$$

$$\begin{cases} A_1(1)^0 + \cancel{A_2 \cdot 0 \cdot (1)^0} + \cancel{A_3 \cdot 0^2 \cdot (1)^0} = 1 \\ A_1(1)^1 + A_2 \cdot 1 \cdot (1)^1 + A_3 \cdot 1^2 (1)^1 = 1 \\ A_1(1)^2 + A_2 \cdot 2 \cdot (1)^2 + A_3 \cdot 2^2 \cdot (1)^2 = 2 \end{cases}$$

$$\Rightarrow \begin{cases} A_1 = 1 \\ \cancel{1} + A_2 + A_3 = \cancel{1} \\ 1 + 2A_2 + 4A_3 = 2 \end{cases} \Rightarrow \begin{cases} A_1 = 1 \\ A_2 = -A_3 \\ 1 - 2A_3 + 4A_3 = 2 \end{cases} \Rightarrow \begin{cases} A_1 = 1 \\ A_2 = -\frac{1}{2} \\ A_3 = \frac{1}{2} \end{cases}$$

$$\boxed{a_n = \frac{1}{2}n^2 - \frac{1}{2}n + 1}$$

4) $a_n = 3a_{n-1} - 2 \qquad a_0 = 0$

SOLUZ. GENERALE DI $a_n = 3a_{n-1}$

$a_n = A\,3^n$

$d = -2 \Rightarrow p(n) = B$

$a_n = A \cdot 3^n + B$

$a_1 = 3 \cdot a_0 - 2 = -2$

$$\begin{cases} A \cdot 3^0 + B = 0 \\ A \cdot 3^1 + B = -2 \end{cases} \Rightarrow \begin{cases} A = -B \\ -3B + B = -2 \end{cases} \Rightarrow \begin{cases} A = -1 \\ B = 1 \end{cases}$$

$a_n = -3^n + 1$