Calcolo dei sequenti $\mathbf{LC_{=}}$ della Logica classica predicativa con uguaglianza

Il calcolo $LC_{=}$ è composto dai seguenti schemi di assiomi e regole:
ove si ricorda che usiamo lettere greche maiuscole del tipo

$$\boldsymbol{\Gamma}, \qquad \boldsymbol{\Delta}, \qquad \boldsymbol{\Sigma}\ldots$$

come META-VARIABILI per indicare una generica **LISTA** di **formule** anche vuota.

ax-id	**ax-**$\perp$	ax-$\top$
$\boldsymbol{\Gamma}, \mathtt{pr}_1, \boldsymbol{\Gamma}' \vdash \boldsymbol{\Delta}, \mathtt{pr}_1, \boldsymbol{\Delta}'$	$\boldsymbol{\Gamma}, \perp, \boldsymbol{\Gamma}' \vdash \nabla$	$\boldsymbol{\Gamma} \vdash \nabla, \top, \nabla'$

$$\frac{\boldsymbol{\Sigma}, \boldsymbol{\Gamma}, \boldsymbol{\Theta}, \boldsymbol{\Gamma}', \boldsymbol{\Delta} \vdash \boldsymbol{\Sigma}'}{\boldsymbol{\Sigma}, \boldsymbol{\Gamma}', \boldsymbol{\Theta}, \boldsymbol{\Gamma}, \boldsymbol{\Delta} \vdash \boldsymbol{\Sigma}'}\ \mathrm{sc_{sx}} \qquad \frac{\boldsymbol{\Gamma} \vdash \boldsymbol{\Sigma}, \boldsymbol{\Delta}, \boldsymbol{\Theta}, \boldsymbol{\Delta}', \nabla}{\boldsymbol{\Gamma} \vdash \boldsymbol{\Sigma}, \boldsymbol{\Delta}', \boldsymbol{\Theta}, \boldsymbol{\Delta}, \nabla}\ \mathrm{sc_{dx}}$$

$$\frac{\boldsymbol{\Gamma} \vdash \mathtt{pr}_1, \boldsymbol{\Delta} \quad \boldsymbol{\Gamma} \vdash \mathtt{pr}_2, \boldsymbol{\Delta}}{\boldsymbol{\Gamma} \vdash (\mathtt{pr}_1)\&(\mathtt{pr}_2), \boldsymbol{\Delta}}\ \&-\mathrm{D} \qquad \frac{\boldsymbol{\Gamma}, \mathtt{pr}_1, \mathtt{pr}_2 \vdash \boldsymbol{\Delta}}{\boldsymbol{\Gamma}, (\mathtt{pr}_1)\&(\mathtt{pr}_2) \vdash \boldsymbol{\Delta}}\ \&-\mathrm{S}$$

$$\frac{\boldsymbol{\Gamma} \vdash \mathtt{pr}_1, \mathtt{pr}_2, \boldsymbol{\Delta}}{\boldsymbol{\Gamma} \vdash (\mathtt{pr}_1)\vee(\mathtt{pr}_2), \boldsymbol{\Delta}}\ \vee-\mathrm{D} \qquad \frac{\boldsymbol{\Gamma}, \mathtt{pr}_1 \vdash \boldsymbol{\Delta} \quad \boldsymbol{\Gamma}, \mathtt{pr}_2 \vdash \boldsymbol{\Delta}}{\boldsymbol{\Gamma}, (\mathtt{pr}_1)\vee(\mathtt{pr}_2) \vdash \boldsymbol{\Delta}}\ \vee-\mathrm{S}$$

$$\frac{\boldsymbol{\Gamma}, \mathtt{pr}_1 \vdash \boldsymbol{\Delta}}{\boldsymbol{\Gamma} \vdash \neg(\mathtt{pr}_1), \boldsymbol{\Delta}}\ \neg-\mathrm{D} \qquad \frac{\boldsymbol{\Gamma} \vdash \mathtt{pr}_1, \boldsymbol{\Delta}}{\boldsymbol{\Gamma}, \neg(\mathtt{pr}_1) \vdash \boldsymbol{\Delta}}\ \neg-\mathrm{S}$$

$$\frac{\boldsymbol{\Gamma}, \mathtt{pr}_1 \vdash \mathtt{pr}_2, \boldsymbol{\Delta}}{\boldsymbol{\Gamma} \vdash (\mathtt{pr}_1)\to(\mathtt{pr}_2), \boldsymbol{\Delta}}\ \to-\mathrm{D} \qquad \frac{\boldsymbol{\Gamma} \vdash \mathtt{pr}_1, \boldsymbol{\Delta} \quad \boldsymbol{\Gamma}, \mathtt{pr}_2 \vdash \boldsymbol{\Delta}}{\boldsymbol{\Gamma}, (\mathtt{pr}_1)\to(\mathtt{pr}_2) \vdash \boldsymbol{\Delta}}\ \to-\mathrm{S}$$

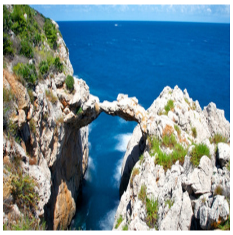
$$\frac{\boldsymbol{\Gamma},\ \forall \mathbf{x}\ \mathtt{fr}\ ,\ \mathtt{fr}[\mathbf{x}/\mathbf{t_{ter}}] \vdash \nabla}{\boldsymbol{\Gamma}\ ,\ \forall \mathbf{x}\ \mathtt{fr} \vdash \nabla}\ \forall-\mathrm{S} \qquad \frac{\boldsymbol{\Gamma} \vdash \mathtt{fr}[\mathbf{x}/\mathbf{w}]\ ,\ \nabla}{\boldsymbol{\Gamma} \vdash\ \forall \mathbf{x}\ \mathtt{fr}\ ,\ \nabla}\ \forall-\mathrm{D}\ (\mathbf{w} \notin \mathbf{VL}(\boldsymbol{\Gamma},\ \forall \mathbf{x}\ \mathtt{fr}\ ,\ \nabla))$$

$$\frac{\boldsymbol{\Gamma},\ \mathtt{fr}[\mathbf{x}/\mathbf{w}] \vdash \nabla}{\boldsymbol{\Gamma},\ \exists \mathbf{x}\ \mathtt{fr} \vdash \nabla}\ \exists-\mathrm{S}\ (\mathbf{w} \notin \mathbf{VL}(\boldsymbol{\Gamma}, \exists \mathbf{x}\ \mathtt{fr}, \boldsymbol{\Delta})) \qquad \frac{\boldsymbol{\Gamma} \vdash \mathtt{fr}[\mathbf{x}/\mathbf{t_{ter}}]\ ,\ \exists \mathbf{x}\ \mathtt{fr}\ , \nabla}{\boldsymbol{\Gamma} \vdash \exists \mathbf{x}\ \mathtt{fr}\ ,\ \nabla}\ \exists-\mathrm{D}$$

$$\frac{\boldsymbol{\Sigma}\ ,\ \mathbf{t_{ter}} = \mathbf{s_{ter}}\ ,\ \boldsymbol{\Gamma}(\mathbf{t_{ter}})\ \vdash\ \boldsymbol{\Delta}(\mathbf{t_{ter}})\ ,\ \nabla}{\boldsymbol{\Sigma}\ ,\ \boldsymbol{\Gamma}(\mathbf{s_{ter}})\ ,\ \mathbf{t_{ter}} = \mathbf{s_{ter}}\ \vdash\ \boldsymbol{\Delta}(\mathbf{s_{ter}})\ ,\ \nabla}\ = -\mathrm{S} \qquad \begin{array}{c} = -\mathrm{ax} \\ \boldsymbol{\Gamma} \vdash \mathbf{t_{ter}} = \mathbf{t_{ter}}\ ,\ \boldsymbol{\Delta} \end{array}$$

Calcolo dei sequenti $LC_=$ della Logica classica predicativa con uguaglianza (presentazione con predicati atomici)

Il calcolo $LC_=$ è composto dai seguenti schemi di assiomi e regole
+ TUTTE le regole ottenibili da loro SOSTITUENDO i predicati atomici $\mathbf{A}$, $\mathbf{B}$, $\mathbf{A}(\mathbf{x})$ e $\mathbf{B}(\mathbf{x})$ con formule qualsiasi:

ax-id	ax-$\bot$	ax-tt
$\mathbf{\Gamma}, \mathbf{A}, \mathbf{\Gamma}' \vdash \mathbf{\Delta}, \mathbf{A}, \mathbf{\Delta}'$	$\mathbf{\Gamma}, \bot, \mathbf{\Gamma}' \vdash \nabla$	$\mathbf{\Gamma} \vdash \mathbf{\Delta}, \mathtt{tt}, \nabla$

$$\frac{\mathbf{\Sigma}, \mathbf{\Gamma}, \mathbf{\Theta}, \mathbf{\Gamma}', \mathbf{\Delta} \vdash \mathbf{\Sigma}'}{\mathbf{\Sigma}, \mathbf{\Gamma}', \mathbf{\Theta}, \mathbf{\Gamma}, \mathbf{\Delta} \vdash \mathbf{\Sigma}'}\ \mathrm{sc_{sx}} \qquad \frac{\mathbf{\Gamma} \vdash \mathbf{\Sigma}, \mathbf{\Delta}, \mathbf{\Theta}, \mathbf{\Delta}', \nabla}{\mathbf{\Gamma} \vdash \mathbf{\Sigma}, \mathbf{\Delta}', \mathbf{\Theta}, \mathbf{\Delta}, \nabla}\ \mathrm{sc_{dx}}$$

$$\frac{\mathbf{\Gamma}, \mathbf{A}, \mathbf{B} \vdash \mathbf{\Delta}}{\mathbf{\Gamma}, \mathbf{A}\&\mathbf{B} \vdash \mathbf{\Delta}}\ \&-\mathrm{S} \qquad \frac{\mathbf{\Gamma} \vdash \mathbf{A}, \mathbf{\Delta} \quad \mathbf{\Gamma} \vdash \mathbf{B}, \mathbf{\Delta}}{\mathbf{\Gamma} \vdash \mathbf{A}\&\mathbf{B}, \mathbf{\Delta}}\ \&-\mathrm{D}$$

$$\frac{\mathbf{\Gamma}, \mathbf{A} \vdash \mathbf{\Delta} \quad \mathbf{\Gamma}, \mathbf{B} \vdash \mathbf{\Delta}}{\mathbf{\Gamma}, \mathbf{A} \vee \mathbf{B} \vdash \mathbf{\Delta}}\ \vee-\mathrm{S} \qquad \frac{\mathbf{\Gamma} \vdash \mathbf{A}, \mathbf{B}, \mathbf{\Delta}}{\mathbf{\Gamma} \vdash \mathbf{A} \vee \mathbf{B}, \mathbf{\Delta}}\ \vee-\mathrm{D}$$

$$\frac{\mathbf{\Gamma} \vdash \mathbf{A}, \mathbf{\Delta}}{\mathbf{\Gamma}, \neg\mathbf{A} \vdash \mathbf{\Delta}}\ \neg-\mathrm{S} \qquad \frac{\mathbf{\Gamma}, \mathbf{A} \vdash \mathbf{\Delta}}{\mathbf{\Gamma} \vdash \neg\mathbf{A}, \mathbf{\Delta}}\ \neg-\mathrm{D}$$

$$\frac{\mathbf{\Gamma} \vdash \mathbf{A}, \mathbf{\Delta} \quad \mathbf{\Gamma}, \mathbf{B} \vdash \mathbf{\Delta}}{\mathbf{\Gamma}, \mathbf{A} \rightarrow \mathbf{B} \vdash \mathbf{\Delta}}\ \rightarrow-\mathrm{S} \qquad \frac{\mathbf{\Gamma}, \mathbf{A} \vdash \mathbf{B}, \mathbf{\Delta}}{\mathbf{\Gamma} \vdash \mathbf{A} \rightarrow \mathbf{B}, \mathbf{\Delta}}\ \rightarrow-\mathrm{D}$$

$$\frac{\mathbf{\Gamma}, \forall \mathbf{x}\ \mathbf{A}(\mathbf{x}), \mathbf{A}(\mathbf{t_{ter}}) \vdash \nabla}{\mathbf{\Gamma}, \forall \mathbf{x}\ \mathbf{A}(\mathbf{x}) \vdash \nabla}\ \forall-\mathrm{S} \qquad \frac{\mathbf{\Gamma} \vdash \mathbf{A}(\mathbf{w}), \nabla}{\mathbf{\Gamma} \vdash \forall \mathbf{x}\ \mathbf{A}(\mathbf{x}), \nabla}\ \forall-\mathrm{D}\ (w \notin VL(\mathbf{\Gamma}, \forall \mathbf{x}\ \mathbf{A}(\mathbf{x}), \nabla))$$

$$\frac{\mathbf{\Gamma}, \mathbf{A}(\mathbf{w}) \vdash \nabla}{\mathbf{\Gamma}, \exists \mathbf{x}\ \mathbf{A}(\mathbf{x}) \vdash \nabla}\ \exists-\mathrm{S}\ (w \notin VL(\mathbf{\Gamma}, \exists \mathbf{x}\ \mathbf{A}(\mathbf{x}), \nabla)) \qquad \frac{\mathbf{\Gamma} \vdash \mathbf{A}(\mathbf{t_{ter}}), \exists \mathbf{x}\ \mathbf{A}(\mathbf{x}), \nabla}{\mathbf{\Gamma} \vdash \exists \mathbf{x}\ \mathbf{A}(\mathbf{x}), \nabla}\ \exists-\mathrm{D}$$

$$\frac{\mathbf{\Sigma}\ ,\ \mathbf{t_{ter}} = \mathbf{s_{ter}}\ ,\ \mathbf{\Gamma}(\mathbf{t_{ter}})\ \vdash\ \mathbf{\Delta}(\mathbf{t_{ter}})\ ,\ \nabla}{\mathbf{\Sigma}\ ,\ \mathbf{\Gamma}(\mathbf{s_{ter}})\ ,\ \mathbf{t_{ter}} = \mathbf{s_{ter}}\ \vdash\ \mathbf{\Delta}(\mathbf{s_{ter}})\ ,\ \nabla}\ =-\mathrm{S} \qquad \begin{array}{c} =-\mathrm{ax} \\ \mathbf{\Gamma} \vdash \mathbf{t_{ter}} = \mathbf{t_{ter}}\ ,\ \mathbf{\Delta} \end{array}$$