Michele Nester nr Matricola 2000559

Ax

1 $\neg \exists x\, C(p,x)$

2 $\neg \exists x\, C(x,x)$

3 $\forall x \forall y (C(x,y) \to C(y,x))$

4 $\forall x\, (x = m \to C(v,m))$

5 $\neg \exists x\, \neg C(\ell,x)$

Tax-dx1

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\vdash \mathrm{Ax}\ 5 \qquad \cfrac{\cfrac{\vdash \mathrm{Ax}\ 1 \qquad \cfrac{\cfrac{\vdash \mathrm{Ax}\ 3 \qquad \cfrac{\cfrac{\cfrac{\cfrac{\cfrac{\vdash C(\ell,p), \neg C(\ell,p), C(p,\ell), \bot \quad \pi}{C(\ell,p) \to C(p,\ell) \vdash \neg C(\ell,p), C(p,\ell), \bot}\ \to S}{\forall y (C(\ell,y) \to C(y,\ell)) \vdash \neg C(\ell,p), C(p,\ell), \bot}\ \forall\text{-}S}{\forall x \forall y (C(x,y) \to C(y,x)) \vdash \neg C(\ell,p), C(p,\ell), \bot}\ \forall\text{-}S}{\forall x \forall y (C(x,y) \to C(y,x)) \vdash \exists x \neg C(\ell,x), C(p,\ell), \bot}\ \exists\text{-}D}{\forall x \forall y (C(x,y) \to C(y,x)) \vdash C(p,\ell), \exists x \neg C(\ell,x), \bot}\ \text{sc}_{dx}}{}}{}}{}}{}}{}}{}}{}}{}}{}$$

$\forall x \forall y (C(x,y) \to C(y,x)) \vdash \exists x\, C(p,x), \exists x \neg C(\ell,x), \bot$ — $\exists$-D

$\vdash \mathrm{Ax}\ 3$ — comp

$\vdash \exists x\, C(p,x), \exists x \neg C(\ell,x), \bot$ — $\neg$-S

$\neg \exists x\, C(p,x) \vdash \exists x \neg C(\ell,x), \bot$

$\vdash \mathrm{Ax}\ 1$ — comp

$\vdash \exists x \neg C(\ell,x), \bot$ — $\neg$-S

$\neg \exists x \neg C(\ell,x) \vdash \bot$

$\vdash \mathrm{Ax}\ 5$ — comp

$\vdash \bot$

Michele Nesler   nr Matricola 2000559

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$$\frac{\overset{\text{ax-id}}{\nearrow}}{\begin{array}{c} , C(p,l) \vdash \neg C(l,p), C(p,l), \perp \end{array}} \qquad \frac{, C(p,l) \vdash \neg C(l,p), C(p,l), \perp}{\pi}$$

avendo dimostrato il falso posso dimostrare Tutti i seguenti teoremi ponendoli al posto di Tm nella procedura sottostante :

T

① $\neg C(p,l)$

② $C(l,p)$

③ $C(m,v)$

④ $m \neq v$

dove sopra

$$\frac{\vdash \perp \qquad \overset{\text{ax-}\perp}{\nearrow}\ \perp \vdash Tm}{\vdash Tm}\ \text{comp}$$

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3

IP	$I \to \neg\forall x\, S(x)$	☺
A	$\neg\exists x\, S(x) \to \neg I$	☺
B	$I \to \exists x \neg S(x) \vee \exists x\, S(x)$	☺
C	$\forall x\, S(x) \to \neg I$	☺

$$\dfrac{\dfrac{\dfrac{\overset{\neg\text{-ax-dx}_2}{\forall x\, S(x) \vdash \neg I, I} \qquad \overset{\neg\text{-ax-sx}_1}{\forall x\, S(x), \neg\forall x\, S(x) \vdash \neg I}}{\forall x\, S(x), I \to \neg\forall x\, S(x) \vdash \neg I}\ \to S}{I \to \neg\forall x\, S(x), \forall x\, S(x) \vdash \neg I}\ sc\text{-}sx}{I \to \neg\forall x\, S(x) \vdash \forall x\, S(x) \to \neg I}\ \to D$$

☹ manca la risposta

Michele [illegible] matricola [illegible]

π_3

$$\dfrac{\dfrac{\exists x A(xx) \vdash}{\vdash \neg \exists x A(xx)}\ \neg\text{-D} \qquad \forall x A(xx) \vdash}{\neg \exists x A(xx) \to \forall x A(xx) \vdash}\ \to\text{-S}$$

Contromodello: $D = \{ali\}$ pongo $A(xx)^{D}(ali) = 1$

e quindi $(\exists x A(xx))^{D} = 1$ e $(\forall x A(xx))^{D} = 1$

quindi la foglia π_3 va a 0 ed anche la radice

di conseguenza perchè: $(\exists x A(xx) \to 0)^{D} = 1 \to 0 = 0$

Modello che va a 1: $D^m = \{ali\}$ pongo $A(xx)^{D^m}(ali) = 0$

e quindi siccome ali è l'unico elemento del domino

$(\exists x A(xx))^{D^m} = 0$ e $\forall x A(xx)^{D^m} = 0$ quindi il sequente

radice va a 1, il che vuol dire che è opinione siccome esiste

anche un Contromodello oltre ad un Modello che va a 1, perchè:

$((\neg \exists A(xx) \to \forall x A(xx)) \to 0)^{D^m} = (\neg 0 \to 0) \to 0 = (1 \to 0) \to 0 =$

$0 \to 0 = 1$

Michele Nesler nr matricola 2000559

Manca risposta!

$$
\dfrac{
  \dfrac{
    \vdash \exists x\, S(x), \neg I, I \;\; (\uparrow \neg\text{ax-dx-2})
    \qquad
    \dfrac{\vdash \forall x\, S(x), \exists x\, S(x), \neg I \;\; (\text{Loop})}{\neg \forall x\, S(x) \vdash \exists x\, S(x), \neg I}\;\neg\text{-S}
  }{
    \dfrac{I \to \neg \forall x\, S(x) \vdash \exists x\, S(x), \neg I}{I \to \neg\forall x\, S(x), \neg \exists x\, S(x) \vdash \neg I}\;\neg\text{-S}
  }\;\to\text{S}
}{
  I \to \neg \forall x\, S(x) \vdash \neg \exists x\, S(x) \to \neg I
}\;\to\text{D}
$$

Modello che va a 1: $\mathcal{D} = \{ali\}$ pongo $S(x)^{\mathcal{D}}(ali) = 1$ quindi $(\exists x\, S(x))^{\mathcal{D}} = 1$ quindi l'unica foglia che non è assioma, e che quindi determina se la radice vada a 0 o vada a 1, va a 1 perché $(1 \to \forall x\, S(x) \vee \exists x\, S(x) \vee \neg I)^{\mathcal{D}} = 1 \to ? \vee 1 \vee ? = 1$ e quindi anche il seguente radice.

Contromodello: $\mathcal{D}^c = \{ali\}$ pongo $S(x)^{\mathcal{D}^c}(ali) = 0$ e $I = 1$ quindi, visto che ali è l'unico elemento di $\mathcal{D}^c$: $(\exists x\, S(x))^{\mathcal{D}^c} = 0$ e $(\forall x\, S(x))^{\mathcal{D}^c} = 0$ quindi sempre la stessa foglia va a 1, ed essendo l'unica non assioma anche la radice, infatti: $(1 \to \forall x\, S(x) \vee \exists x\, S(x) \vee \neg I)^{\mathcal{D}^c} =$

$1 \to 1 \vee 1 \vee \neg 0 = 1 \to 1 \vee 1 \vee 1 = 1$

Michele Nesler 2000559 matricola nr

le scansioni sono fatte in ordine inverso, leggo dal basso verso l'alto

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ax-id

$$b=w,\ b=c \vdash b=c \qquad w=b$$

$$\frac{b=w,\ b=c \vdash b=c}{b=w,\ b=c \vdash w=c}\ =\!-S$$

$$\frac{b=w,\ b=c \vdash w=c}{\forall x(\dots),\ b=c \vdash w=c}\ \forall\text{-}S_v \quad +5\ C5\ x$$

$$\frac{\forall x(\dots),\ b=c \vdash w=c}{\forall x\ b=x \vdash w=c}\ \forall\text{-}S$$

$$\frac{\forall x\ b=x \vdash w=c}{\forall x\ b=x \vdash \forall y\ y=c}\ \forall\text{-}D\ \ w \notin VL$$

¬ax-dx-2 ¬ax-dx-2

$$\frac{\neg T(w) \vdash \neg C(w), C(w) \qquad \neg T(w), T(w) \vdash \neg C(w)}{C(w) \to T(w), \neg T(w) \vdash \neg C(w)}\ \to\!-S$$

$$\frac{C(w) \to T(w), \neg T(w) \vdash \neg C(w)}{C(w) \to T(w) \vdash \neg T(w) \to \neg C(w)}\ \to\!-D \quad +5\ C5\ x$$

$$\frac{C(w) \to T(w) \vdash \neg T(w) \to \neg C(w)}{\forall x(C(x) \to T(x)) \vdash \neg T(w) \to \neg C(w)}\ \forall\text{-}S_v$$

$$\frac{\forall x(C(x) \to T(x)) \vdash \neg T(w) \to \neg C(w)}{\forall x(C(x) \to T(x)) \vdash \forall y(\neg T(x) \to \neg C(x))}\ \forall\text{-}D\ \ w \notin VL$$