Fourier Series

G V V Sharma*

CONTENTS

Abstract—**This manual provides a simple introduction to Fourier Series**

1 PERIODIC FUNCTION

Let

$$x(t) = A_0 \left| \sin\left(2\pi f_0 t\right) \right| \quad (1.1)$$

1.1 Plot $x(t)$.

1.2 Show that $x(t)$ is periodic and find its period.

2 FOURIER SERIES

Consider $A_0 = 12$ and $f_0 = 50$ for all numerical calculations.

2.1 If

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f_0 t} \quad (2.1)$$

show that

$$c_k = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} x(t) e^{-j2\pi k f_0 t}\, dt \quad (2.2)$$

2.2 Find c_k for (1.1)

2.3 Verify (1.1) using python.

2.4 Show that

$$x(t) = \sum_{k=0}^{\infty} \left(a_k \cos \jmath 2\pi k f_0 t + b_k \sin \jmath 2\pi k f_0 t\right)$$

$$(2.3)$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

and obtain the formulae for a_k and b_k.

2.5 Find a_k and b_k for (1.1)

2.6 Verify (2.3) using python.

3 FOURIER TRANSFORM

3.1

$$\delta(t) = 0, \quad t \neq 0 \quad (3.1)$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \quad (3.2)$$

3.2 The Fourier Transform of $g(t)$ is

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t}\, dt \quad (3.3)$$

3.3 Show that

$$g(t - t_0) \overset{\mathcal{F}}{\longleftrightarrow} G(f) e^{-j2\pi f t_0} \quad (3.4)$$

$$(3.5)$$

3.4 Show that

$$G(t) \overset{\mathcal{F}}{\longleftrightarrow} g(-f) \quad (3.6)$$

3.5 $\delta(t) \overset{\mathcal{F}}{\longleftrightarrow}?$

3.6 $e^{-j2\pi f_0 t} \overset{\mathcal{F}}{\longleftrightarrow}?$

3.7 $\cos(2\pi f_0 t) \overset{\mathcal{F}}{\longleftrightarrow}?$

3.8 Find the Fourier Transform of $x(t)$ and plot it. Verify using python.

3.9 Show that

$$\operatorname{rect}(t) \overset{\mathcal{F}}{\longleftrightarrow} \operatorname{sinc}(t) \quad (3.7)$$

Verify using python.

3.10 $\operatorname{sinc}(t) \overset{\mathcal{F}}{\longleftrightarrow}?$. Verify using python.

4 FILTER

4.1 Find $H(f)$ which transforms $x(t)$ to DC 5V.

4.2 Find $h(t)$.

4.3 Verify your result using through convolution.

5 FILTER DESIGN

5.1 Design a Butterworth filter for $H(f)$.

5.2 Design a Chebyschev filter for $H(f)$.

5.3 Design a circuit for your Butterworth filter.
5.4 Design a circuit for your Chebyschev filter.