

10.5.4-3

EE23BTECH11023-ABHIGNYA GOGULA

Question:

A ladder has rungs 25cm apart. The rungs decrease uniformly in length from 45cm at the bottom to 25cm at the top. If the top and bottom rungs are 2 and 1/2 meter apart, what is the length of wood required for the rungs?

The length of wood required for the rungs is 385 cm.

parameter	value
n	10
$x(0)$	45
$x(10)$	25

TABLE 0

DESCRIPTION OF PARAMETERS

SOLUTION

Total number of rungs:

$$\frac{\langle \frac{5}{2} \rangle 100}{25} + 1 = 11 \quad (1)$$

As the length of rungs decreases uniformly, it is in A.P:

$$x(n) = \langle x(0) + nd \rangle u(n) \quad (2)$$

$$25 = 45 + 10d \quad (3)$$

$$d = -2 \quad (4)$$

The sum of the lengths of all rungs gives the total length of wood required. So, finding the sum of A.P using Z-transform:

$$X(z) = \frac{45}{1 - z^{-1}} + \frac{-2z^{-1}}{\langle 1 - z^{-1} \rangle^2} \quad (5)$$

$$y(n) = x(n) * u(n) \quad (6)$$

$$Y(z) = X(z)U(z) \quad (7)$$

$$Y(z) = \frac{45}{\langle 1 - z^{-1} \rangle^2} + \frac{-2z^{-1}}{\langle 1 - z^{-1} \rangle^3} \quad (8)$$

Taking the inverse of the Z-transform using counter-integration:

$$y(n) = x(0)\langle n+1 \rangle u(n) + \frac{d}{2}\langle n \langle n+1 \rangle u(n) \rangle \quad (9)$$

$$y(n) = \frac{n+1}{2}\langle 2x(0) + nd \rangle u(n) \quad (10)$$

$$y(10) = 385 \quad (11)$$