SIGNAL PROCESSING FUNDAMENTALS Through NCERT

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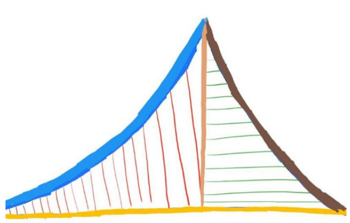

Contents

Introduction

This book introduces some concepts in signal processing through maths and physics problems in NCERT textbooks.

Chapter 1

Analog

1.1. Harmonics

1.1.1 A charged particle oscillates about its mean equilibrium position with a frequency of $10^9 Hz$. What is the frequency of the electromagnetic waves produced by the oscillator?

Solution:

Symbol	Value	Description
$y(t)$	$\cos(2\pi f_c t)$	Wave equation of electro-magnetic wave
f_c	10^9	Frequency of electromagnetic wave
t	seconds	Time

Table 1.1: Variable description

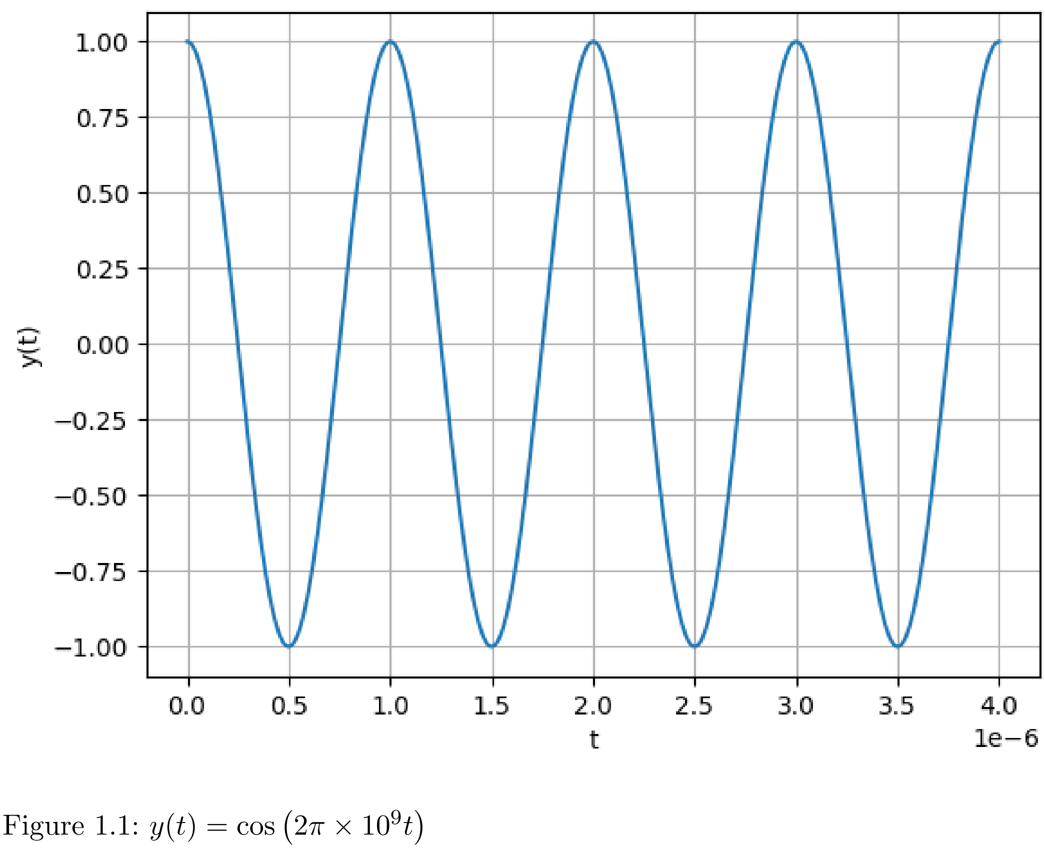

Figure 1.1: $y(t) = \cos\left(2\pi \times 10^9 t\right)$

Chapter 2

Discrete

Appendix A

Axioms

Appendix B

Z-transform

B.1 The Z-transform of $p(n)$ is defined as

$$P(z) = \sum_{n=-\infty}^{\infty} p(n) z^{-n} \tag{B.1.1}$$

B.2 If

$$p(n) = p_1(n) * p_2(n), \tag{B.2.1}$$

$$P(z) = P_1(z) P_2(z) \tag{B.2.2}$$

The above property follows from Fourier analysis and is fundamental to signal processing.

B.3 For a Geometric progression defined as follows

$$x\left(n\right) = x\left(0\right) r^{n} u\left(n\right) \tag{B.3.1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \tag{B.3.2}$$

$$= \sum_{n=0}^{\infty} x(0)\, r^n z^{-n} \tag{B.3.3}$$

$$= \sum_{n=0}^{\infty} x(0) \left(rz^{-1}\right)^n \tag{B.3.4}$$

$$= \frac{x(0)}{1 - rz^{-1}} \qquad \left|rz^{-1}\right| < 1 \tag{B.3.5}$$

$$ROC \implies |z| > |r| \tag{B.3.6}$$

B.4 For a given Arithmetic progression defined as follows

$$x(n) = [x(0) + nd]\, u(n) \tag{B.4.1}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{B.4.2}$$

$$= \sum_{n=-\infty}^{\infty} [x(0) + nd]\, u(n) z^{-n} \tag{B.4.3}$$

$$= x(0) \sum_{n=-\infty}^{\infty} u(n) z^{-n} + d \sum_{n=-\infty}^{\infty} n u(n) z^{-n} \tag{B.4.4}$$

Let us consider

$$U(z) = \sum_{n=-\infty}^{\infty} u(n) z^{-n} = \frac{1}{1 - z^{-1}} \qquad \text{For } |z| > 1 \tag{B.4.5}$$

$$\frac{dU(z)}{dz} = \frac{-1}{z} \sum_{n=-\infty}^{\infty} n u(n) z^{-n} \tag{B.4.6}$$

$$\implies z^{-1} \frac{1}{(1 - z^{-1})^2} = \sum_{n=-\infty}^{\infty} n u(n) z^{-n} \tag{B.4.7}$$

$$\implies X(z) = \frac{x(0)}{1 - z^{-1}} + \frac{dz^{-1}}{(1 - z^{-1})^2} \qquad \text{For } |z| > 1 \tag{B.4.8}$$