

# **On the Origin of Elements in the Milky Way**

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# Preface

These lecture notes for the course “On the origin of the elements in the Milky Way” are prepared as we go, so they will be in part influenced by you! If you find typos, errors, or other issues please let me know. The most recent copy of the L<sup>A</sup>T<sub>E</sub>X files and figures can also be found on [GitHub](#).

The lecture notes contain clickable links, e.g., all references are linked to the bibliography and all acronyms are linked to the acronyms’ definition page. These links are generally colored in [dark blue](#).

The end of the document contains a bibliography that contains details on all references. Note that references here are limited and often point to review articles that could be of interest to the reader. More detailed references to individual topics can be found in these articles. Each reference entry contains a digital object identifier ([doi](#))<sup>1</sup> that can be found online and accessed via a Brandeis library subscription. Alternatively, a link to ArXiV<sup>2</sup> is provided.

Specific boxes throughout the text discuss further information. They are defined as following:



**Background information** on topics that do not necessarily fit into the text but are important to keep in mind will be given in a box like this.



**Further information and reading** for the avid reader will be pointed out in a box like this. The scope of these boxes is generally slightly outside the realm of the class.



**Programming** is an integral part of scientific research, be it for running experiments, performing measurements, evaluating measurements, or modeling a system. A box like this will point you to more information with respect to coding. These boxes will often refer to coding in [python](#).

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<sup>1</sup><https://doi.org>

<sup>2</sup><https://arxiv.org>

# Acronyms

**r-process** rapid neutron capture process

**s-process** slow neutron capture process

**AGB** asymptotic giant branch

**AU** astronomical unit

**BBN** Big Bang nucleosynthesis

**CMB** cosmic microwave background

**dex** decimal exponent

**doi** digital object identifier

**E-ABG** early asymptotic giant branch

**ESA** European Space Agency

**EUV** extreme ultraviolet

**GCE** galactic chemical evolution

**HRD** Hertzsprung-Russell diagram

**HST** Hubble Space Telescope

**IMF** initial mass function

**IR** infrared radiation

**ISM** interstellar medium

**MESA** Modules for Experiments in Stellar Astrophysics

**pc** parsec

**pdf** portable document format

*Acronyms*

**PN** planetary nebula

**pp-chain** proton-proton-chain

**RGB** red giant branch

**SGB** subgiant branch

**SN** supernova

**SOHO** solar and heliospheric observatory

**TDU** third dredge up

**TP-AGB** thermally pulsing asymptotic giant branch

**UV** ultraviolet

**WD** white dwarf

**WMAP** Wilkinson microwave anisotropy probe

**ZAMS** zero age main sequence

# 1. Solar System Abundances

To understand the origin of the elements in the Milky Way, it is crucial to first understand what in fact needs explaining. One of the best studied points in the history of the Milky Way is the Solar System we inhabit, mainly since we actually live inside it and thus have direct access to its material.

The Solar System formed 4.567 Ga ago from a homogeneous molecular cloud. Since the Milky Way itself formed roughly 13 Ga ago, the molecular cloud from which the solar nebula formed evolved for roughly 8.4 Ga since the formation of the Milky Way and around 9 Ga since the Big Bang. The process that describes the formation and destruction of elements over this period of time is generally referred to as galactic chemical evolution ([GCE](#)).

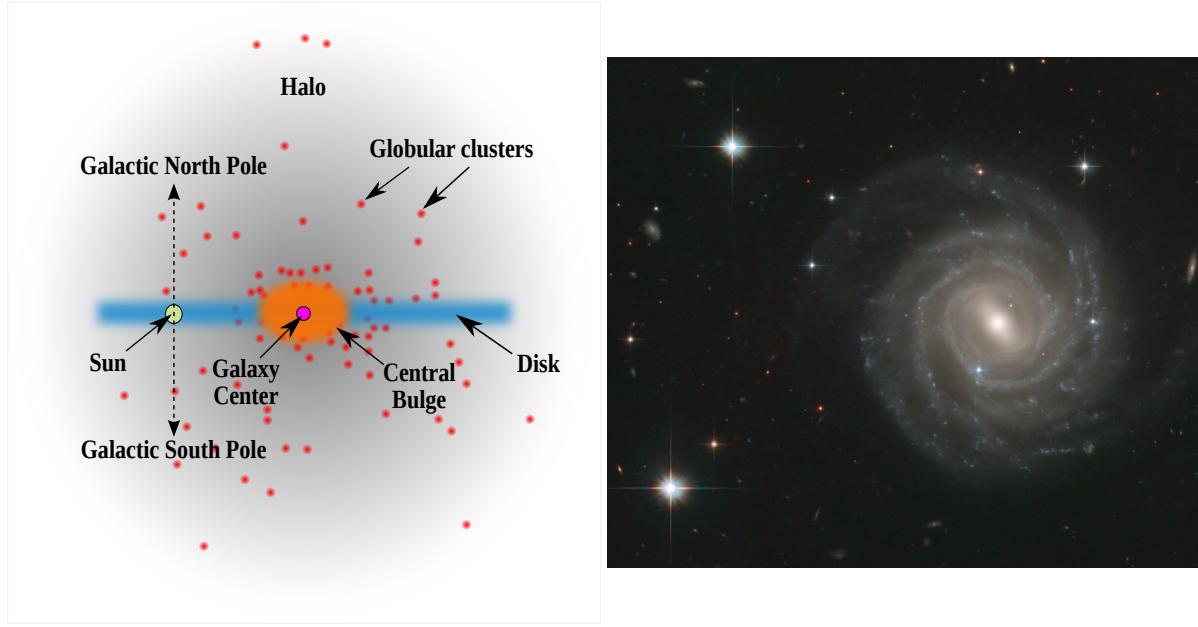
In order to put the Solar System abundances in perspective to the galaxy, Figure 1.1 shows a schematic of the Milky Way (left). The galactic center, which is likely a black hole, is surrounded by an area called the bulge. The disk of the Milky Way has a spiral structure. An analogue galaxy with a similar structure is UGC 12158, shown on the right in Figure 1.1. The densest regions of the Milky Way are close to the center (e.g., the bulge) while less dense areas are further out (e.g., the disk). The Solar System is located in the disk and about two-thirds out from the center in one of the spiral arms called the Orion-Cygnus arm. Disk and bulge are both surrounded by a low-density halo. This halo also contains globular clusters, which are gravitationally bound aggregations of stars. These clusters tend to be very old and are thus made of material that formed early on in the history of the Milky Way.

This chapter discusses the composition of the Solar System 4.567 Ga ago (the initial Solar System abundances) and today (abundances in the solar photosphere). While we can observe stars and determine their composition via spectroscopy, the Solar System is unique since we have meteorites available that represent unaltered solar nebula material. This material is especially useful since it can be studied in the laboratory using mass spectrometry techniques.



**Initial Solar System abundances in python** can easily be handled with the `iniabu` package. This package can be used make various databases of Solar System abundances available for interaction and is used here to create abundance plots using `matplotlib`. Full disclosure: I am the author of the package and am thus surely biased.

## 1. Solar System Abundances



**Figure 1.1.:** Left: Profile of the Milky Way with the current position of the Solar System indicated. Edited from Wikipedia. Right: Galaxy UGC 12158, which is thought to have a similar spiral structure as the Milky Way. Credit: ESA/Hubble & NASA

### 1.1. Definitions and Notations

#### 1.1.1. Terminology

**Elemental concentrations and abundances** are two terms that are often used interchangeably and their meaning, unfortunately, is often ambiguous. Let us for example look at a form of uranium oxide named yellowcake. The chemical formula for a common form of yellowcake is  $\text{U}_3\text{O}_8$ , i.e., the smallest unit consists of three uranium atoms and eight oxygen atoms. Expressing the stoichiometric elemental abundance of oxygen in yellowcake, i.e., the abundance by number of atoms, we would say that the oxygen abundance of yellowcake is  $8/(8 + 3) = 0.73$ . Alternatively and often (but not exclusively referred to as the concentration) we could determine the oxygen concentration by mass. Uranium (mean mass:  $\bar{m}_{\text{U}} = 237.3 \text{ u}$ ) is significantly heavier than oxygen (mean mass:  $\bar{m}_{\text{O}} = 16.0 \text{ u}$ ). The mean mass here is the mass averaged over all isotopes of the specific element. The average concentration of oxygen in yellowcake, by mass, can then be determined as:

$$[\text{O}] = \frac{8\bar{m}_{\text{O}}}{8\bar{m}_{\text{O}} + 3\bar{m}_{\text{U}}} = 0.15 \quad (1.1)$$

Note that when solar abundances with respect to meteorites, the stoichiometric / number abundances are generally used. Astronomers and astrophysicists on the other hand often

## 1. Solar System Abundances

report mass fractions (see below).

**Solar (elemental) mass fractions** are commonly used in astronomy. For astronomers, the universe consists, to first order, of three elements, namely hydrogen ( $X$ ), helium ( $Y$ ), and metals ( $Z$ ), which are all elements heavier than helium. This might sound strange since it implies that the air we breathe (78% nitrogen, 21% oxygen) consists of metals in the astronomical sense, however makes some sense when looking, e.g., at the composition of the Sun. The Sun's composition in this notation is  $X = 0.7389$ ,  $Y = 0.2463$ , and  $Z = 0.0148$ . These mass fractions are always given by weight.

**Solar System initial abundances**, often referred to as Solar System abundances, describe the original composition of the Solar System 4.567 Ga ago. To determine this composition, measurements of, e.g., meteorites must be decay corrected for radioactive nuclides and their products.

**Solar photospheric abundances** on the other hand refer to the present-day composition of the Sun. There are further important differences between initial and present-day abundances that are discussed in the subsequent sections.



**Radioactive decay** Unstable atomic nuclei undergo radioactive decay to so-called daughter nuclei. For example,  $^{26}\text{Al}$  is an unstable isotope and decays to the stable  $^{26}\text{Mg}$ . Its half-life ( $t_{1/2}$ ), which is the time after which only half the initial material is left, is  $7.17 \times 10^5$  a. Radioactive decay is an exponential process. Instead of the half-life the decay constant  $\lambda = \ln 2/t_{1/2}$  is often referred to. Radioactive decay can be calculated as:

$$N(t) = N_0 \exp(-\lambda t) \quad (1.2)$$

Here,  $N_0$  is the amount of radioactive material available at  $t = 0$  and  $N(t)$  is the amount of material after time  $t$  has elapsed.

### 1.1.2. Scales

As with definitions, several different scales are used. Here we define the three most commonly used ones. **Meteoritic abundances** are generally normalized to the number of silicon atoms, which is set to be equal to  $N_{\text{Si}} = 10^6$ . These abundances scale linearly.

**Spectroscopic abundances**, often used in astronomy, are frequently given with respect to hydrogen, the most abundant element in the universe. These abundances are usually reported as logarithmic abundances and the abundance of hydrogen is defined to be equal to 12. For example, the Solar System initial abundance of silicon in this logarithmic unit can be calculated as:

$$A(\text{Si}) = \log_{10} \left( \frac{N_{\text{Si}}}{N_{\text{H}}} \right) + 12 = 7.59 \quad (1.3)$$

Here,  $N_{\text{Si}}$  and  $N_{\text{H}}$  are the number abundances of silicon and hydrogen, respectively.

## 1. Solar System Abundances

Finally, **mass fractions** are also frequently used in astronomy and astrophysics. The mass fraction of an element or isotope  $x_i$  with number abundance  $N_i$  and mass  $m_i$  can be calculated as:

$$x_i = \frac{N_i m_i}{\sum_j N_j m_j} \quad (1.4)$$

Here, the sum over  $j$  adds up all elements or isotopes. If we were thus to sum up all mass fractions, we would get  $\sum_j x_j = 1$ .

### 1.1.3. Comparing measurements

In astronomy, chemical abundance ratios are generally expressed in “bracket”-notation. These chemical abundance ratios are then usually regarded with respect to a standard, which generally is the Sun. For example, to express the iron content of a given star in comparison to the Sun one would calculate:

$$[\text{Fe}/\text{H}] = \log_{10} \left( \frac{N_{\text{Fe}}}{N_{\text{H}}} \right)_{\text{star}} - \log_{10} \left( \frac{N_{\text{Fe}}}{N_{\text{H}}} \right)_{\odot} \quad (1.5)$$

Here,  $N_x$  represents the number abundance of a given element  $x$ . The symbol  $\odot$  is generally used for the Sun. Using this formulation, observations with higher iron content than the Sun would result in a positive number, and vice versa. While the  $[\text{Fe}/\text{H}]$  number is unitless, differences in this notation are commonly referred to as decimal exponent (**dex**). A star with a 0.3 dex enhancement in  $[\text{Fe}/\text{H}]$  would thus have twice as much iron than the Sun when normalized to hydrogen.

## 1.2. The Sun

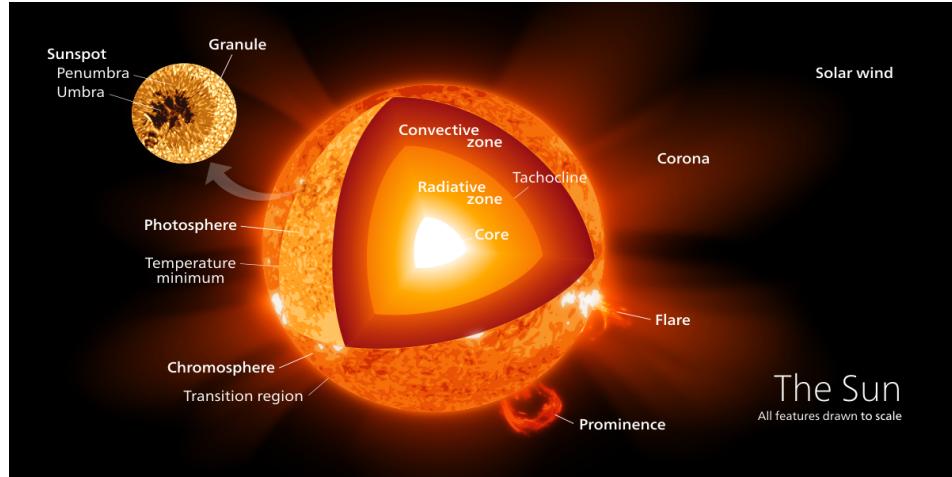
The Sun is the central body of the Solar System and contains with a mass of  $\sim 2 \times 10^{30}$  kg about 99.86% of the total mass in the Solar System. The total luminosity of the Sun is  $3.8 \times 10^{26}$  W and it's orbited by the Earth at a mean distance of  $\sim 1.5 \times 10^8$  km, which is equal to 1 astronomical unit (**AU**). The total solar irradiance at Earth is thus around  $1.3 \text{ kW/m}^2$ .

Since the Sun contains the majority of the Solar System's mass, it is reasonable to assume that this celestial body also represents well the Solar System's composition. Figure 1.2 shows an artistic rendering of the structure of the Sun. The Sun's core extends to about a quarter of its radius and is the main nuclear engine that fuses hydrogen to helium at a temperature of around  $16 \times 10^6$  K.<sup>1</sup> The solar core is not convectively connected to the outer layers, thus energy is mainly transported by radiative transfer.

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<sup>1</sup>Temperatures in astrophysics are often written as  $T_x = y$ . In this case, the actual temperature would be  $y \times 10^x$  K. For the core of the Sun we could write the temperature thus as  $T_{10} = 1.6$ .

## 1. Solar System Abundances



**Figure 1.2.:** The structure of the Sun. Credit: Wikipedia

The photosphere, which is the visible part of the Sun, is at a temperature of around 5800 K. The atmosphere of the Sun has a minimum temperature of around 4100 K about 500 km above the photosphere. It consists of the chromosphere which lays above the photosphere and the solar corona that extends from the chromosphere out into space. At higher altitudes the temperature of the corona increases and reaches temperatures in excess of  $10^6$  K. How such high temperatures can be reached in the solar corona is still unclear.



**Wien's displacement law** states that the radiation curve of a black-body has its peak at different wavelengths depending on the temperature. For a given temperature, the peak wavelength can be calculated as:

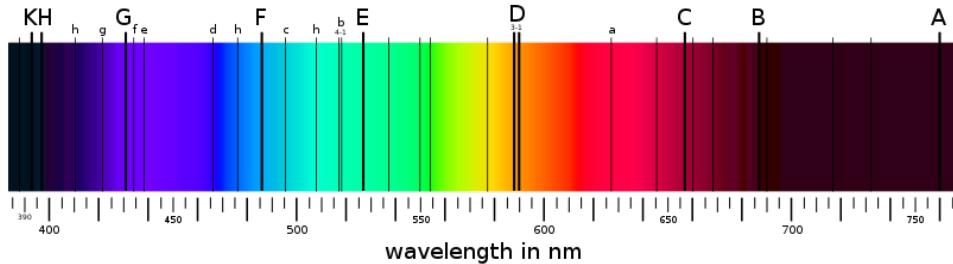
$$\lambda_{\max} = \frac{b}{T} \quad (1.6)$$

Here,  $T$  is the temperature in kelvin and  $b = 2.898 \times 10^{-3}$  m·K. Plugging in the photosphere temperature, we can calculate a peak wavelength at 500 nm, which is the color green in the visible part of the electromagnetic spectrum.

### 1.2.1. Spectroscopy and Absorption Spectra

In order to determine the composition of the Sun from the electromagnetic spectrum it radiates, astronomers use a technique called stellar spectroscopy. Using a prism or grating the light from the Sun is dispersed into its individual colors. Figure 1.3 shows the electromagnetic spectrum of the Sun between 380 nm and 710 nm. Clearly visible are dark spectral absorption lines, so-called Fraunhofer lines named after Joseph von

## 1. Solar System Abundances



**Figure 1.3.:** The solar spectrum broken up into its components. Credit: Wikipedia.

Fraunhofer who discovered, studied, and described them in 1814.

These absorption lines are fingerprints of the Sun's elemental composition. The dark lines form since photons coming out of the Sun are absorbed in the photosphere by atoms. The absorbed photons are subsequently re-emitted, however, this re-emission is not directed. Thus, the area of the spectrum that the original photon was absorbed in appears darker. For example, the double lines labeled "D" in Figure 1.3 are a feature of the sodium absorption lines.

### 1.2.2. Stellar Abundances

In order to interpret the Fraunhofer lines observed from the Sun with respect to its elemental composition, many physical parameters and models must be known. Here we give a brief overview of these modeling efforts. Further details can be found in two great reviews on determining the Sun's elemental composition by Asplund et al. (2009) and Allende Prieto (2016).

To determine the composition of the Sun two closely related conditions must be understood: (1) line formation and (2) the stellar atmosphere. These two fields include physics from several disciplines, namely fluid dynamics, statistical mechanics, and thermodynamics. To understand line formation the atomic parameters and stellar conditions, such as the opacity, need to be known. To model stellar atmospheres on the other hand we need to know the energy radiated through the atmosphere, the surface gravity, and the chemical composition. Note that the chemical composition is on one hand what we would like to determine from these observations, however it is also an integral part of the stellar atmospheric model. The chemical composition has thus to be determined in an iterative process. For a given set of interest, especially the amount of energy radiated through it, its surface gravity, and its chemical composition. Note that the chemical composition, which is what we would like to find, must be known in order to model the stellar atmosphere. Determining the composition of the Sun from observations is thus an iterative process. For a given model atmosphere and determined atomic and molecular lines and continuum opacities, so-called spectra synthesis codes can be used

## 1. Solar System Abundances

to predict the spectrum of the model star with all its absorption lines. These spectra are then compared to observations. If different, the parameters, especially the chemical composition is updated with better values and the models are run again. The chemical composition of the star is found when model and observations agree.

### Photospheric Abundances

The solar photosphere (see Figure 1.2) is the part of the Sun that we can actually see. Spectroscopy of the photosphere is thus generally used to model the solar abundances. With a temperature between 4500 K and 6000 K and an effective temperature of around 5800 K no molecules can form in the photosphere and thus only atoms are expected to contribute to its absorption lines. Furthermore, the densities of the chromosphere and corona above the photosphere are so low that they do not influence the absorption lines observed from the photosphere.

The present-day solar photosphere composition is slightly different from the initial Solar System abundance, i.e., the abundance that is representative of the homogeneous solar nebula. One difference of course is for radioactive nuclides that have decayed over the last 4.567 Ga since the beginning of the Solar System. Furthermore, thermal diffusion, gravitational settling, and radiative acceleration – the three of which are often collectively referred to as diffusion – also changed the photospheric composition over the past 4.567 Ga. Corrections for these effects must thus be applied before comparing Solar System initial abundances, as e.g., measured in meteorites (see Section 1.3) with photospheric measurements.

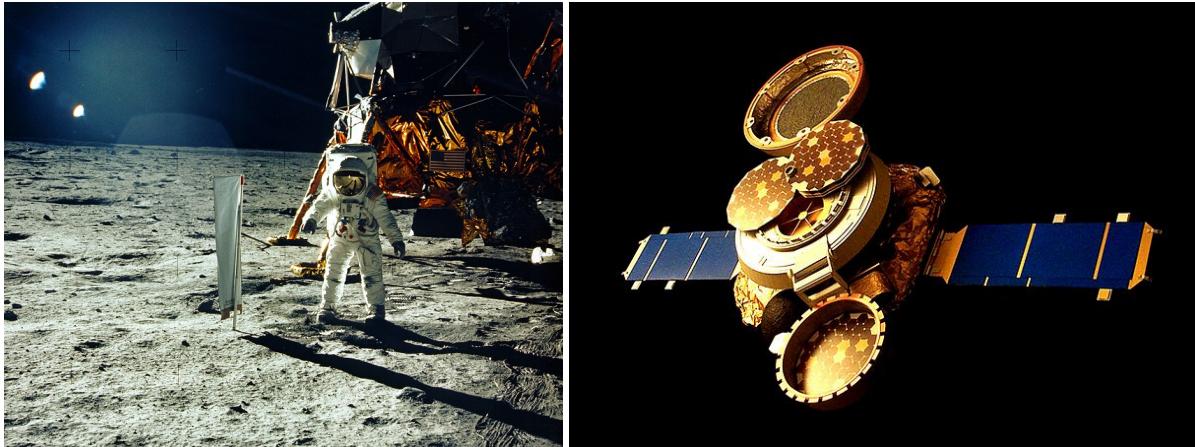
### Chromosphere & Corona Observations

The chromosphere and corona are the solar layers right above the photosphere. These layers are too thin for us to directly observe and can generally only be seen during a total solar eclipse. Since there is no homogeneous background irradiation for the chromosphere and corona in these cases, absorption spectra cannot be measured. Thus, the elemental composition of the solar atmosphere is determined by measuring emission spectra. Excited atoms fall back to their ground states at discreet energies. Such emission lines will show up as bright instead of dark lines when analyzed in a spectrograph.

Temperatures as low as 4100K have been measured in the chromosphere. This allows for the existence of simple molecules, which complicates the determination of the spectral abundances since many more atomic transitions are suddenly available. Furthermore, the chromosphere has a fairly large temperature gradient which also complicates the models.

Coronal abundances are difficult to determine. The extremely high temperatures of the coronal make it difficult to determine radiative transitions in the laboratory. A large part of the coronal emission lines are furthermore in the ultraviolet (**UV**) and extreme

## 1. Solar System Abundances



(a) Buzz Aldrin standing next to the solar sail during the Apollo 11 mission. (b) Artist conception of the Genesis spacecraft with unfolded collectors.

**Figure 1.4.:** The two types of solar wind catcher experiments that have been used to determine the composition of the Sun. Credit: NASA

ultraviolet (EUV), thus can not be detected from Earth due to the atmosphere. Spacecrafts such as solar and heliospheric observatory (SOHO) have been used to determine the elemental composition of the solar corona.

### Solar Wind

Mass ejections such as prominences and flares (see Figure 1.2) are the origin of the solar wind that penetrates the whole Solar System. The interaction of the solar wind with the Earth's magnetic field can, e.g., be seen as aurorae (borealis and australis), also known as the northern and southern lights. The solar wind consists to 98% of protons (charged hydrogen particles). The other 2% is mainly charged helium nuclei and a minute amount of heavier particles. This however means that the solar wind does carry some part of the Sun's composition out through the solar system.

While the solar wind cannot be measured on Earth, bodies without an atmosphere such as the Moon are constantly struck by solar wind. During the Apollo missions to the Moon, astronauts carried a solar sail, which was aluminum foil mounted on a pole in order to capture solar wind during their stay on the Moon. Figure 1.4a shows Apollo 11 astronaut Buzz Aldrin standing next to the solar sail. While valuable measurements were done using solar sails flown on Apollo missions, the total exposure time of the aluminum foil was very limited, i.e., only up to a few days each. Thus, only little material was captured which resulted in significant analytical uncertainties when measuring the captured content.

In 2001, NASA launched the Genesis spacecraft which was parked on a Lagrange

## 1. Solar System Abundances

point L<sub>1</sub>.<sup>2</sup> L<sub>1</sub> is a gravitationally semi-stable place in space in between the Sun and the Earth. Genesis exposed its collectors (Figure 1.4b) to the solar wind for a total of 850 d before returning to Earth. In order to avoid any contamination with terrestrial material, NASA's plan was for Genesis to re-enter the Earth's atmosphere, deploy its drogue parachute, and then capture the capsule mid-air using a helicopter with a long hook. The deployment mechanism for the parachute was connected to an accelerometer, which was unfortunately built backwards into the spacecraft. This resulted in the accelerometer measuring the acceleration with the wrong sign, thus never triggering the software to release the parachute. Needless to say that the "landing" of Genesis took place at a terminal velocity of around 86 m s<sup>-1</sup> (190 mph). This broke open the collector container and all collectors were significantly contaminated with material from the Utah dessert. Nevertheless, all main objectives of Genesis could still be achieved by meticulously cleaning and puzzling the collector array back together, a process that took several years.

### 1.3. Meteorites

Meteorites are rocks that originated in the Solar System and fell to Earth as meteors. They are either finds, meaning that they were found during searches, or falls, meaning that they were found after observing the meteor in the sky. Meteorites could have previously been part of another planet, e.g., Mars, part of the Moon, or, most often, part of an asteroid. Thousands of these rocks have been analyzed, and they are generally classified into various different groups. The two major groups these rocks belong to relate them to their origin; they were either part of a differentiated or an undifferentiated parent body. A differentiated Solar System body has at some point in its history gotten hot enough in order to be completely or partially molten. This separated the metal-loving (siderophile) from the silicate loving (lithophile) elements. The Earth for example is a differentiated body with an iron, nickel core at the center and a silicate mantel around it. Meteorites from undifferentiated bodies, called chondrites, still contain the metal and silicate in the same phases, i.e., they have never gotten hot enough to differentiate.

In order to determine the initial Solar System composition by analyzing meteorites, samples that have never been altered throughout its history are of special interest. These meteorites are called primitive. The most primitive subgroup of the chondrites are the carbonaceous chondrites, which have generally not even been heated enough in order to destroy the mineral phases that originally condensed from the solar nebula.

Alterations different from heat, e.g., chemical alterations, can also influence the composition and thus primitiveness of a meteorite. Meteorites that have been the least altered with respect to their chemical and isotopic composition are carbonaceous chondrites that are similar to the Ivuna meteorite, which is the type specimen for this group.

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<sup>2</sup>For more information, see [https://en.wikipedia.org/wiki/Lagrangian\\_point](https://en.wikipedia.org/wiki/Lagrangian_point)

## 1. Solar System Abundances

The name of this chondrite group is thus abbreviated CI chondrites. The Orgueil meteorite, a CI chondrite, represents the best studied sample for the Solar System initial abundance. This rock fell on May 14, 1864 just after 8pm local time near Orgueil, a town in southern France. Samples were recovered immediately, totaling 14 kg of material. Meteorite falls have the advantage that they did not spend any significant amounts of time exposed to the weather on Earth, thus they do not show any terrestrial alteration either.

To determine the composition of meteorites, their material is usually homogenized and then analyzed using a mass spectrometry. Depending on the element of interest sample material is atomized and ionized. These ions are then separated by mass-over-charge in a mass analyzer. This allows for precise determination of their elemental and isotopic composition.



**Stardust** Some primitive meteorites contain micrometer sized grains that in fact did not form in the Solar System at all but rather represent bona-fide stardust grains. These stardust grains formed in the outflow of dying stars, were transported through the interstellar medium (ISM), and then incorporated into meteorite parent bodies during their formation. We will discuss stardust grains in more detail later since these samples allow us to directly probe the processes in which elements are created.

## 1.4. The Composition of the Solar System

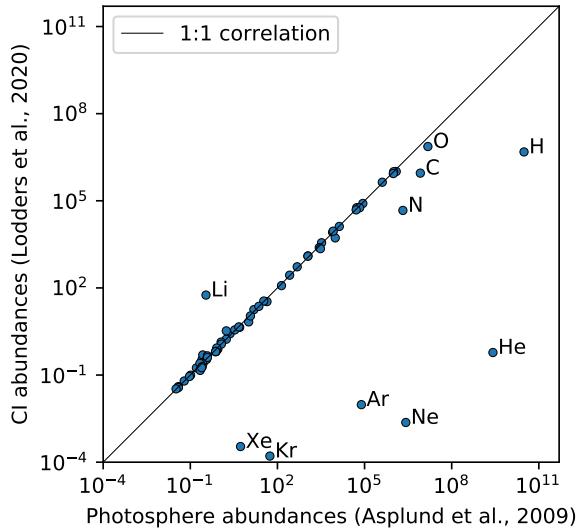
While CI chondrites seem to be primitive Solar System objects, their total mass is represents only a tiny fraction of the total mass of the Solar System. Photosphere observations on the other hand are associated with larger measurement uncertainties and require modeling of the physical processes underlying the formation of absorption spectra. The question thus arises on how the composition of the Solar System can best be determined.

### 1.4.1. Comparing CI chondrites and the Photosphere

Figure 1.5 shows the comparison of the photosphere measurements reported in Asplund et al. (2009) with the CI chondrite measurements reported in Lodders (2020). Most elements lie perfectly on the 1:1 correlation indicated in black in the figure, which spans a total of around 12 orders of magnitude. However, there are some very notable deviations that require further discussion.

**Lithium** is the only element that is more abundant in CI chondrites than in the Sun. In the Sun, lithium diffuses effectively. Since it is destroyed during the nuclear reactions

## 1. Solar System Abundances



**Figure 1.5.:** Comparison of solar photosphere measurements (Asplund et al., 2009) and the composition of CI meteorites (Lodders, 2020). Elements that do not lie close to the 1:1 correlation line are labeled.

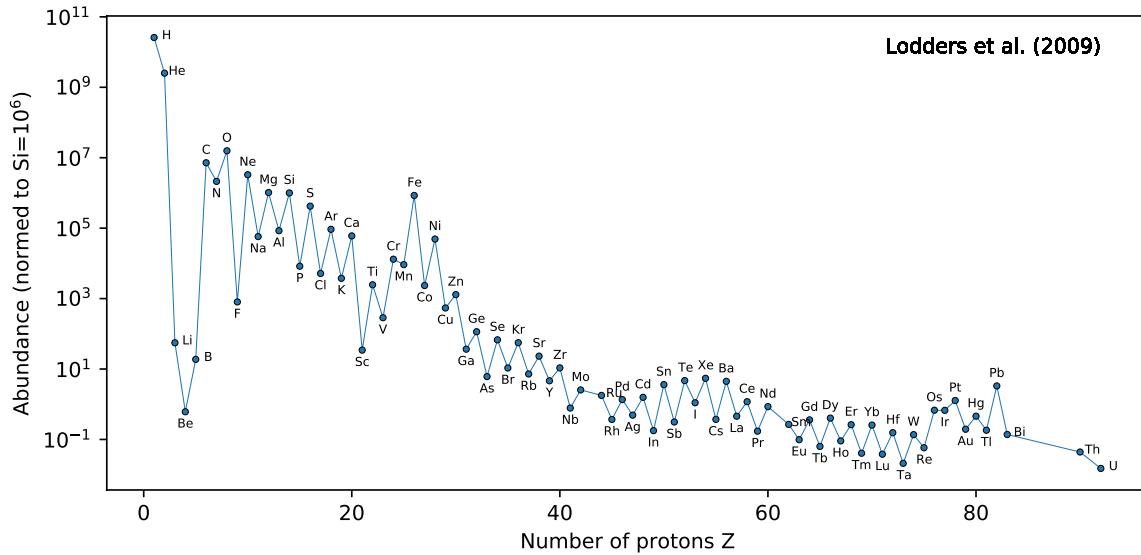
taking place in the stellar core, the overall abundance of lithium is expected to be depleted when compared to the bulk Solar System.

**The noble gases** helium, neon, argon, krypton, and xenon are all very volatile and are thus heavily depleted in meteorites, i.e., they never effectively condensed into the rocky material when meteorite parent bodies formed. The bulk Solar System helium abundance can fortunately be derived from helioseismology via observations of the Sun and is not highly model dependent. Various techniques have been used to determine the composition of the other noble gases. One of the most accurate methods to-date is the analysis of solar wind in the Genesis collectors, see page 11.

**Hydrogen** is also a highly volatile element and thus does not effectively condense into meteorite parent bodies. Hydrogen thus needs to be measured in the Sun by model comparison. The hydrogen mass is insensitive to the Z/X composition of the Sun, which results in an accurate determination of the hydrogen mass fraction X.

**Carbon, nitrogen, and oxygen** are prominent atoms that form volatile molecules, e.g., CO, CO<sub>2</sub>, N<sub>2</sub>, and O<sub>2</sub>. Thus, these elements are expected to be depleted in rocky materials. While these elements can all be measured in the solar photosphere, model improvements resulted in significant changes over time in their abundances.

## 1. Solar System Abundances



**Figure 1.6.:** Solar System initial abundances for all elements (Lodders et al., 2009).

### 1.4.2. Solar System Elemental Abundances

Figure 1.6 shows the elemental abundances of the solar nebula. Clearly, hydrogen and helium are the most abundant elements. Furthermore, we can see a clear peak around carbon, oxygen, and neon. These elements are made in most stars and thus reach such high abundances. There is also a clear peak at the iron isotopic composition. This is due to the fact that in this region, the binding energy per nucleon is the highest across all elements. That means that energy can be gained by fusing nuclides together up to iron. Past iron this does not happen anymore, thus the abundances of all heavier nuclides drop significantly.

Another striking feature of the abundance curve in Figure 1.6 is the clear zigzag pattern. This pattern is due to the fact that even nuclei, i.e., nuclei with an even number of nucleons (protons and neutrons), have a higher binding energy than odd nuclei and are thus more stable.

### 1.4.3. Solar System Isotopic Abundances

So far we have mostly discussed the abundance of the elements in the Solar System. Just as important is the abundance of the isotopes, which can be derived from CI chondrites with high precision. Atomic line differences of individual isotopes are generally too small in order to derive useful isotopic abundances from solar observations. Let us briefly discuss the origin of a few important isotopes cannot easily be derived from CI chondrites.

## 1. Solar System Abundances

**Deuterium and  ${}^3\text{He}$**  are both highly volatile, thus cannot be measured in CI chondrites, and have been significantly changed in the Sun. In its early stage the Sun underwent deuterium burning, essentially leaving it deuterium free at this point. This burning also produced  ${}^3\text{He}$  and thus changed its abundance in the Sun. The exact D/H ratio in the Solar System is still an active matter of research. Good analogues to determine this ratio are the atmosphere of Jupiter which consists mainly of the gaseous material that the solar nebula was made of, and in comets. Certain, so-called Kuiper-belt comets formed far out in the Solar System beyond the snow-line, which is the area of the solar nebula beyond which water only appears in solid form. These comets thus trapped the original D/H composition. For the early Solar System  ${}^3\text{He}/{}^4\text{He}$  ratio, the value for Jupiter's atmosphere is generally adopted.

**Carbon, nitrogen, and oxygen** are all very volatile. While the carbon isotopic composition in the Solar System is not very variable across different types of meteorites, the nitrogen and oxygen isotopic composition vary widely. To measure these isotopes in solar wind was one of the main objectives of the Genesis mission. After significant cleaning of these collectors, the solar oxygen and nitrogen isotopic composition have finally been determined by McKeegan et al. (2011) and Marty et al. (2011), respectively.

**Noble gases** are also volatile. As discussed before, their elemental and isotopic abundances were determined using the solar wind collectors on board the Genesis spacecraft. A detailed review of this work can be found in Heber et al. (2009).

### 1.4.4. Preview on the Origin of Elements and Isotopes

So far we have only briefly discussed the major features visible in the Solar System abundance pattern (Figure 1.6). We will look into a lot more details on how individual elements and groups formed later on. The production of elements and their isotopes takes place in a process called nucleosynthesis. A brief overview of these processes is given below.

All hydrogen and helium in universe formed right after the Big Bang in a process called Big Bang nucleosynthesis. The majority of all heavier elements were subsequently formed in stars. Elements up to the iron peak formed mainly in massive stars by nuclear fusion reactions. As discussed before, the binding energy per nucleon is the highest at iron and thus fusion reactions beyond this region are not energetically favorable. Thus, other processes must take over.

Most isotopes beyond iron are formed by neutron-capture reactions. Since neutrons are not electrically charged, they do not see the Coulomb barrier of a positively charged nucleus and can thus be more effectively added to it. Neutron addition takes place until the nucleus becomes unstable. Unstable, neutron-rich nuclei decay generally via

## 1. Solar System Abundances

$\beta^-$ -decay to the next stable isobar, which has one proton more and one neutron less. Proton-rich nuclei, e.g.,  $^{92}\text{Mo}$ , cannot form by neutron capture and various processes and locations for their formation are actively being discussed in the astrophysics community.

About half the elements beyond iron formed in the so called slow neutron capture process (*s*-process), in which the  $\beta^-$  decay to the stable isobar generally happens faster than another neutron can be captured. This way however, only elements up to bismuth can be made. Heavier elements, e.g., actinides such as thorium and uranium, cannot be formed in the *s*-process. Rapid capture of neutrons, the so-called rapid neutron capture process (*r*-process), must thus be invoked to explain their existence.

## 1.5. Other Stars

While we have Solar System samples, i.e., CI chondrites, to determine the composition of the solar nebula and thus the Sun precisely, representative rocks of other stars are not available in the Solar System. However, the light of other star still reaches us, i.e., we can see their photosphere and apply the same spectroscopic techniques to determine their elemental composition.

As discussed in Section 1.2.2, to determine the abundances of observed other stars, the stellar atmosphere must be modeled. For this, the size and mass of the star must be known, which is not always simple to determine from observations. Recent massive surveys such as the Sloan Digital Sky Survey<sup>3</sup> and the Gaia mission<sup>4</sup> have made it possible to determine the elemental composition of many more stars. An overview of existing database can, e.g., be found in Allende Prieto (2016). It turns out that the solar neighborhood is a diverse place (see, e.g., Bensby et al., 2014) and still leaves us for the time being with many open question on the origin of its elements and isotopes.

## 1.6. Reading

A great reading to further dive into the topic of solar abundances is the recent review by Lodders (2020).<sup>5</sup> Some questions and points of discussions for this paper are:

- Why are meteoritic measurements normalized to a silicon abundance of  $10^6$  while astronomical observations are generally given in dex noramlized such that the abundance of hydrogen is equal to 12?
- Why can elemental abundances with much higher precision in CI chondrites than in the Sun's photosphere?

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<sup>3</sup><https://www.sdss.org/>

<sup>4</sup><https://sci.esa.int/web/gaia/>

<sup>5</sup>At the time of this writing, the download of the document as a portable document format (pdf) file did not succeed. You can also find this article on ArXiV.

## *1. Solar System Abundances*

- Why does aqueous alteration not affect the chemical composition of CI chondrites?
- Explain from a nuclear physics point of view why there is no deuterium in the Sun.
- Why is it so difficult to determine certain elemental and isotopic abundances, e.g., noble gases and the D/H ratio of the Solar System?
- What are the difficulties that one encounters when analyzing meteorites by mass spectrometry?
- Why can carbon, oxygen, and nitrogen not be determined when analyzing meteorites?

# 2. Big Bang Nucleosynthesis

In order to understand primordial / Big Bang nucleosynthesis (BBN), we need to first look at some fundamental observations that define cosmology. In addition, we also need to introduce the standard model of cosmology. Note that this is only a very brief introduction into a topic that could be a whole course by itself.

## 2.1. Fundamental Cosmological Observations

### 2.1.1. Olbers' paradox

One of the most fundamental observations that goes back to Kepler (1610), Halley (1721), de Cheseaux (1744), and is today known as Olbers' (1823) paradox is the fact that the night sky is dark. If one assumes an infinite, homogeneous space filled with stars, this space would appear infinitely bright to the observer on Earth. The apparent luminosity of a star can be described as

$$l = \frac{L}{4\pi r^2} \propto r^{-2}, \quad (2.1)$$

where  $L$  is the star's luminosity and  $r$  its distance from Earth. Assuming a density of stars  $n$ , a spherical shell of thickness  $dr$  at distance  $r$  from Earth would contain

$$dN = 4\pi n r^2 dr \propto r^2 \quad (2.2)$$

stars. The total energy density of all stars for an observer can thus be calculated as

$$\varepsilon_s = \int_0^\infty \frac{L}{4\pi r^2} dN = nL \int_0^\infty dr. \quad (2.3)$$

In contrast to our experience this integral is divergent, thus the night sky should be infinitely bright.

This paradox cannot be simply solved by assuming absorption of light in the interstellar medium, e.g., by dust. The energy from all stars would over time heat up this dust until it radiates as bright as the stars themselves. One factor that slightly helps is that stars will overlap with each other. You can picture this scenario however like standing in a dense forest where, as far as you can see, every line of sites terminates in

## 2. Big Bang Nucleosynthesis

a tree trunk. For the universe this would mean that every line of sight will terminate at the surface of a star. Stars cover each other, so the night sky would not seem infinitely bright but homogeneously about as bright as the Sun. This still does not agree with our experience.

The solution to the paradox lies in the fact that the universe is expanding. The wavelength of the light that arrives expands along with the universe and is thus shifted to the red (Doppler shift). The result is that the relative luminosity, as given in equation (2.1), is in fact  $l \propto r^{-3}$ . Plugging this into equation (2.1) results in the integral converging. In addition, there the universe has a horizon at distance

$$R \simeq ct. \quad (2.4)$$

Here,  $c$  is the speed of light and  $t$  the approximate age of the universe. The visible part of space and thus the energy density become finite.



**Doppler effect** Let us assume that a source emits an electromagnetic wave with wavelength  $\lambda_s$ . An observer that moves with a velocity  $\vec{v}$  with respect to that source will register a shifted frequency  $\lambda_r$ . For light, and considering special relativity, this effect will result in a blueshift if the observer moves towards the source and in a redshift if the observer moves away from the source. For two objects that move directly towards or away from each other, the relativistic Doppler shift for light can be written as

$$\lambda_r = \sqrt{\frac{1 + \beta}{1 - \beta}} \lambda_s, \quad (2.5)$$

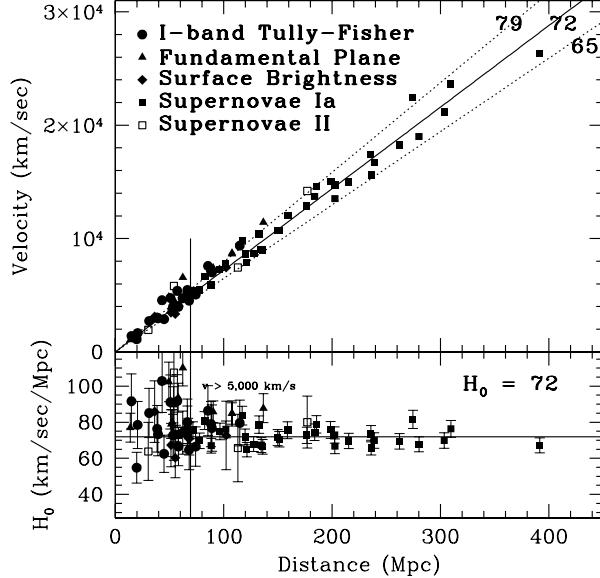
where  $\beta = v/c$ , i.e., the speed of the observer  $v$  with respect to the speed of light  $c$ . Sources moving away from each other hereby have a positive velocity ( $\beta > 0$ ), sources moving towards each other a negative one ( $\beta < 0$ ).

### 2.1.2. Hubble's Law

The expansion of the universe can in fact be observed. Figure 2.1 shows the Hubble diagram for many observations, taken from Freedman et al. (2001). The diagram on top shows the velocity of objects relative to Earth as a function of their distance. Using the Doppler shift one can see that the velocity, which is away from the observer in this case, is proportional to the redshift of the object. Here, the distance is given in megaparsec, where 1 parsec (pc) is approximately 3.26 light years. The linear relationship between velocity and distance ( $R$ ) can be written as

$$\left( \frac{\dot{R}}{R} \right)_0 = H_0 = \text{constant}. \quad (2.6)$$

## 2. Big Bang Nucleosynthesis



**Figure 2.1.:** Hubble diagram of velocity (top) and value of  $H_0$  (bottom) as a function of distance. Freedman et al. (2001), © 2001 The American Astronomical Society.

The subscript zero hereby describes the present-day value of the constant. The constant  $H_0$  is commonly called the Hubble constant and its value is about  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Note that the Hubble constant is given in units of one over seconds. The reciprocal value of the  $H_0$  thus defines a time, the so-called Hubble time. If the expansion of the universe is never accelerated, its age could be determined as

$$t_0 \leq \frac{1}{H_0} = 1.4 \times 10^{10} \text{ a.} \quad (2.7)$$

The distance of a galaxy is often expressed by its redshift. Let  $\lambda$  be the wavelength sent out from the galaxy in question and  $\lambda_0$  the wavelength received today. The redshift  $z$  can then be written as

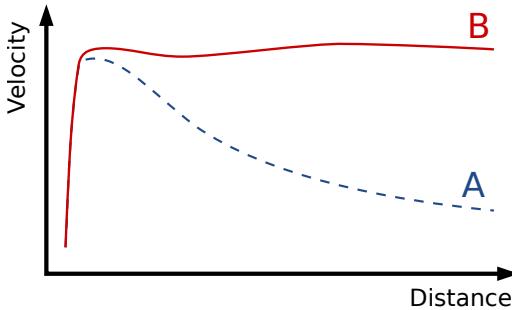
$$1 + z = \frac{\lambda_0}{\lambda} = \frac{R_0}{R}. \quad (2.8)$$

Here,  $R$  is introduced as the so-called scale factor of the universe.

Since the universe consists of mass that interacts gravitationally with each other, we can define a deceleration parameter  $q_0$  for the universe such that

$$q_0 \equiv - \left( \frac{\ddot{R}R}{\dot{R}^2} \right)_0 f = - \frac{\ddot{R}_0}{R_0 H_0^2}. \quad (2.9)$$

## 2. Big Bang Nucleosynthesis



**Figure 2.2.:** Galactic rotation: Case A shows the expected behavior, case B the observed one. Credit: Wikipedia.

### 2.1.3. Density of Matter

By density one generally refers to the total energy density of all components (radiation, matter, and dark energy) divided by  $c^2$ . While radiation dominated in the early universe, matter or even dark energy dominates today. The total density of matter in the universe can be measured by determining the total mass of individual galaxies. An estimated density can thus be given as

$$2 \times 10^{-31} \text{ g cm}^{-3} \leq \rho_0 \leq 2 \times 10^{-30} \text{ g cm}^{-3}. \quad (2.10)$$

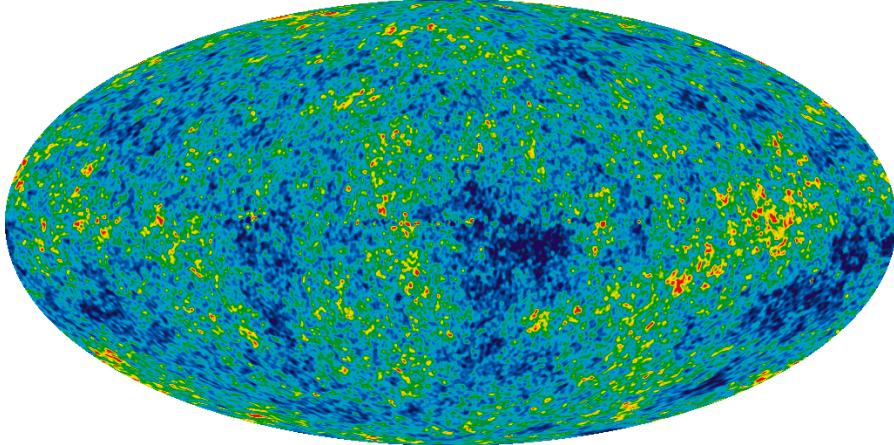


**Dark matter** In 1933, Fritz Zwicky noticed that galactic cluster do not rotate as expected. By constraining the observed mass in the cluster and observing its rotation, Zwicky noticed that some matter was missing. He called this dark matter. Figure 2.2 shows an example of the expected and observed rotation curves. From this and further observations we can derive that dark matter makes up around 85% of all matter in the universe. Only 15% of the matter in the universe consists of baryons, e.g., particles made up of quarks such as neutrons and protons.

### 2.1.4. Cosmic Microwave Background

In the Big Bang model, the universe started from a very hot and dense state. One expected remnant of this state is a relic radiation that in fact was detected in 1965 by Arno Penzias and Robert Wilson (Penzias & Wilson, 1965). The detection of this cosmic microwave background (CMB) is one of the pillars supporting the Big Bang model to describe the origin of the universe. The origin of the CMB will be discussed in further detail below.

## 2. Big Bang Nucleosynthesis



**Figure 2.3.:** WMAP's image of the CMB. Shown is averaged data from nine years of measurements. Credit: NASA.

The CMB is a thermal blackbody radiation with a measured temperature of  $2.72548 \pm 0.00057$  K. The peak of this radiation is in the microwave range and cannot be observed from Earth. Space missions such as Wilkinson microwave anisotropy probe (WMAP) and Planck measured the CMB in detail. A map of the CMB is shown in Figure 2.3.

## 2.2. The Standard Model of Cosmology

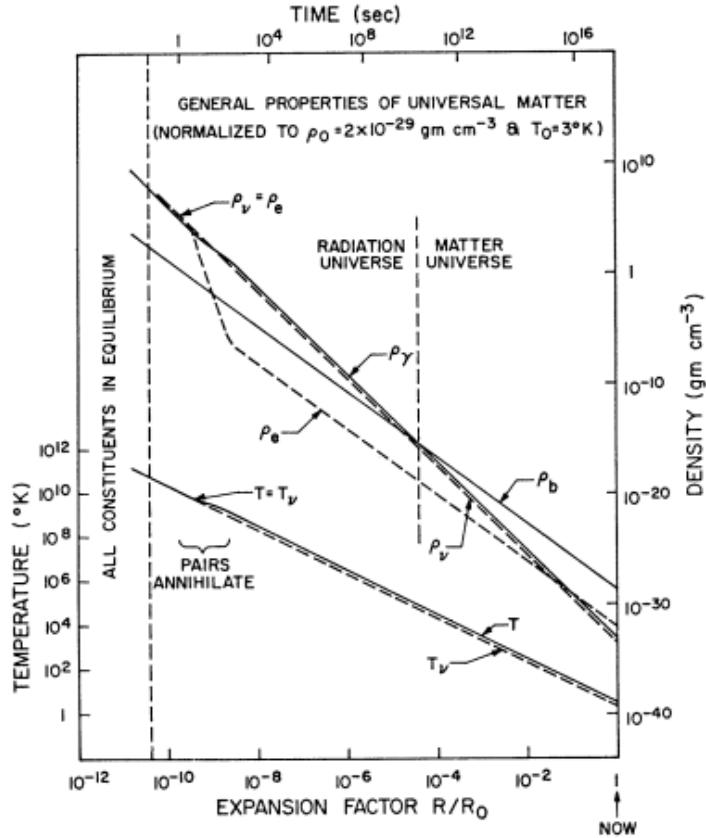
The standard model of cosmology describes the origin of the universe and its evolution based on one singular event: the Big Bang. With the Big Bang, space, time, and matter came into existence and the universe developed from its hot, dense state to the cold and transparent state we see today.

### 2.2.1. Assumptions

The standard model of cosmology is based on several key assumptions. Here we focus on the ones that are of importance to understand BBN.

1. The cosmological principle is valid. This states that the spatial distribution of matter in the universe is homogeneous and isotropic.
2. The overall charge in the universe is zero.
3. The universe is made of matter and not of antimatter. This assumption in fact directly contradicts the cosmological principle (assumption 1) since it requires that there was an overabundance of baryons compared to antibaryons at the start of

## 2. Big Bang Nucleosynthesis



**Figure 2.4.:** The temperature and density evolution of the universe in the framework of the standard model of cosmology. Taken from Wagoner et al. (1967). © American Astronomical Society.

the universe, making it thus inhomogeneous. This overabundance of matter is to this date not fully explained and remains under active investigation.

4. At temperature  $T < 10^{11}$  K (approximately 10 ms after the Big Bang), all heavy particles were annihilated and the density of the universe is defined by photons, neutrinos ( $\nu$ ), electron and positrons ( $e^\pm$ ), and the remaining baryons.

### 2.2.2. Temperature and Density Evolution

Figure 2.4 shows the temperature and density evolution of the universe in the framework of the standard model of cosmology (Wagoner et al., 1967). The horizontal axis show the expansion factor  $R/R_0$  as defined in equation 2.8 on the bottom and the time since the Big Bang on the top. The vertical axes show the temperature (left) and the density (right). The standard model description starts at around 10 ms with all constituents in

## 2. Big Bang Nucleosynthesis

equilibrium. Big Bang nucleosynthesis sets in at a temperature of around  $10^9$  K.

### 2.3. Big Bang Nucleosynthesis

#### 2.3.1. The Proton-to-Neutron Ratio

In the beginning at a temperature of around  $10^{11}$  K (see Figure 2.4), the protons and neutrons are in thermodynamic equilibrium. Statistical mechanics shows that the energy of such a system can be expressed using the Boltzmann distribution for the individual states. This can be used to express the ratio of the number of protons to the number of neutrons in equilibrium conditions as

$$\left(\frac{n_p}{n_n}\right)_{\text{eq}} = \frac{\lambda_{np}}{\lambda_{pn}} = \exp\left(\frac{(m_n - m_p)c^2}{kT}\right) = \exp\left(\frac{1.501}{T_{10}}\right). \quad (2.11)$$

Here,  $\lambda_{np}$  and  $\lambda_{pn}$  are the reaction rates to turn a neutron into proton and vice versa, respectively. For  $T \rightarrow \infty$  protons and neutrons will have equal abundances. At lower temperatures however, protons will become more abundant.

We can rewrite equation (2.11) to express the mass fractions of neutrons in equilibrium as

$$X_{n,\text{eq}} = \left(\frac{n_n}{n_p + n_n}\right)_{\text{eq}} = \frac{1}{(n_p/n_n)_{\text{eq}} + 1} = \frac{\lambda_{pn}}{\lambda_{pn} + \lambda_{np}}. \quad (2.12)$$

The nucleons react with each other via the weak force and the following reactions are possible:



Reaction (2.15) is the free decay of neutrons and has a half-life of 610 s. This reaction can thus be neglected at the beginning since it is very long compared to all other reactions. The neutron mass fraction over time follows thus the differential equation

$$\frac{d}{dt}X_n(t) = -\lambda_{np}(t)X_n(t) + \lambda_{pn}(t)[1 - X_n(t)]. \quad (2.16)$$

With decreasing temperature the reaction rates  $\lambda_{np}$  and  $\lambda_{pn}$  go rapidly towards zero such that after around 10 s the proton to neutron ratio is frozen in place. Calculating reaction rate values, e.g., as in Peebles (1966b), the neutron mass fraction at freezeout (when the reaction rates  $\lambda_{np}$  and  $\lambda_{pn}$  go to zero) is

$$X_{n,\text{freeze}} = 0.164. \quad (2.17)$$

## 2. Big Bang Nucleosynthesis

From this time on, the only reaction taking place is free neutron decay, see reaction (2.15). Since the half-life of neutrons is fairly short, they must rapidly after the freezeout be captured as part of atomic nuclei in order to not decay away.

### 2.3.2. Nucleosynthesis of Deuterium

Deuterium has a mass of  $m_D = 1875.612928 \text{ MeV}$ , while a proton and neutron have masses of  $m_p = 938.272088 \text{ MeV}$  and  $m_n = 939.565421 \text{ MeV}$  (see info box below for an explanation of measurements in eV). Deuterium, an isotope of hydrogen ( ${}^2\text{H}$ ), consists of one proton and one neutron. However, summing the mass of one proton and one neutron results in a mass that is  $\Delta m = 2.22 \text{ MeV}$  larger than the mass of deuterium. This so-called mass defect defines the binding energy of deuterium and is the reason why energy is released when a proton and neutron are fused together. In chemistry this would be called an exothermic reaction. It can be written as



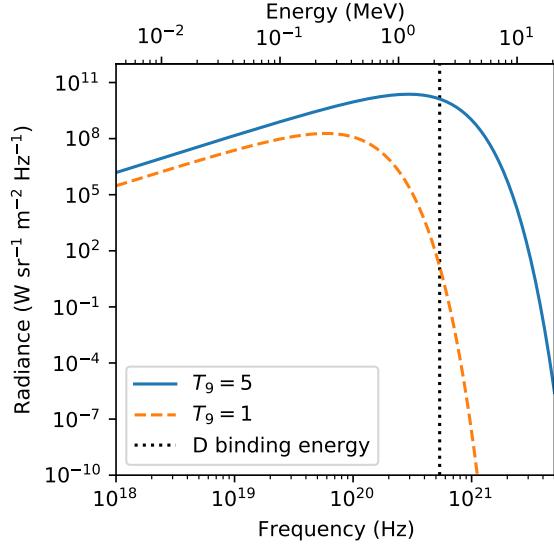
At the time when  $n_p/n_n$  is frozen, the temperature of the universe is still  $T \approx 5 \times 10^9 \text{ K}$ . Any deuterium that forms at this temperature is effectively destroyed right away again, since many photons have enough energy to dissociate the binding energy of 2.22 MeV. Figure 2.5 shows the spectral energy of a blackbody at this high temperature as a function of the frequency of the photons (bottom axis) and as function of their energy in MeV (top axis). The black, dashed line shows the deuterium binding energy. Note that both axes of the figure are logarithmic. Clearly, at  $T_9 = 5$  many photons still have high enough energies. The universe thus first needs to cool down below  $T_9 \approx 1$  before deuterium can effectively form.

The exact temperatures, as just described, do not only depend on the temperature but also on the abundance of photons. Furthermore, the temperature is directly related



**Measurements in eV** In nuclear physics, masses are often expressed in mega electronvolts or MeV, which is technically not a mass but an energy. The mass of the particle is related to its energy via Einstein's equation  $m = E/c^2$ . Thus, energy and mass are directly related with  $c$ , the speed of light, as a proportionality constant. Similarly we can write the momentum of a particle as  $\vec{p} = E\vec{c}^{-1}$  and the temperature of a system as  $T = Ek_B^{-1}$ . Here,  $k_B$  is the Boltzmann constant. Since mass, momentum, and temperature are related to energy via constants, these quantities are often expressed as energies.

## 2. Big Bang Nucleosynthesis



**Figure 2.5.:** Spectral energy of black body radiation at the temperature when neutrons and protons are in equilibrium ( $T_9 \approx 5$ ) and when deuterium fusion can start ( $T_9 \approx 1$ ). Prior to this temperature, the photon energies are too high and destroy newly formed deuterium effectively.

to the density of the universe. Figure 2.5 thus only gives a relative insight into the dissociation of deuterium. More detailed calculations can be found in the literature, e.g., in Wagoner et al. (1967).

The drop in temperature of the universe from when proton and neutron abundances are frozen until deuterium can effectively form without being destroyed takes about 220 s. During this time, the neutrons still undergo free decay. Using equation (1.2) we can calculate that by the time deuterium fusion becomes significant, another  $\sim 20\%$  of the neutrons decay. This leaves behind a neutron fraction of

$$X_n = 0.128 \quad (2.19)$$

at the start of BBN.

## 2. Big Bang Nucleosynthesis

### 2.3.3. Nucleosynthesis of Helium

Once deuterium has formed, more reactions can take place. Some of these reactions are:



Here, T is a tritium nucleus, another isotope of hydrogen with two neutrons ( ${}^3\text{H}$ ). The reaction rates for these individual paths can be calculated, however, let us first estimate the dominant product of big bang nucleosynthesis. For deuterium above we determined a binding energy of 2.22 MeV. Calculating the mass defect and thus the binding energy for  ${}^3\text{He}$  and  ${}^4\text{He}$  gives 6.70 MeV and 27.3 MeV, respectively. A better way to compare these binding energies is however to determine the binding energy per nucleon. For  ${}^3\text{He}$  and  ${}^4\text{He}$  these would be 2.23 MeV per nucleon and 6.81 MeV per nucleon. Thus,  ${}^4\text{He}$  is much favored to being produced.

If we assume that all neutrons will be bound into  ${}^4\text{He}$ , a fairly accurate assumption as we will see further down, we can now predict the mass fraction of helium ( $Y$ ) in the universe. After free decay of neutrons, we are left with  $X_n = 0.128$ . Since  ${}^4\text{He}$  is roughly four times heavier than hydrogen, we can estimate the mass fraction of  ${}^4\text{He}$  as

$$Y = \frac{4n_{\text{He}}}{4n_{\text{He}} + n_{\text{H}}} = \frac{2n_n}{n_p + n_n} = \frac{2(n_n/n_p)}{1 + (n_n/n_p)} = 0.23. \quad (2.28)$$

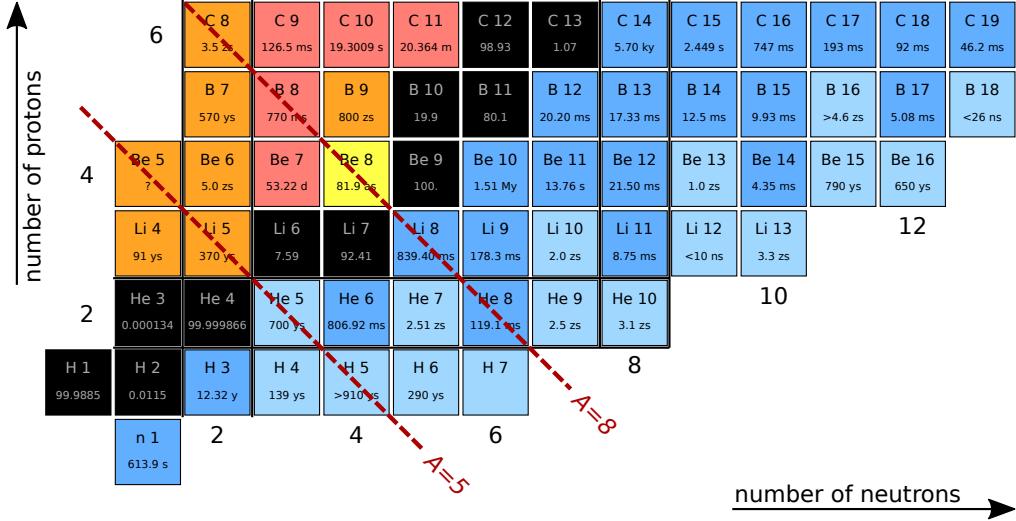
This estimated value is in excellent agreement with the observed abundance of helium in the universe of about 1/4 and is thus another robust pillar for the Big Bang model.

### 2.3.4. Nucleosynthesis of heavier elements

The highest binding energy per nucleon is found in the isotope  ${}^{62}\text{Ni}$ . However, several reasons contribute to the fact that BBN cannot synthesize nuclides heavier than  ${}^7\text{Li}$ .

Figure 2.6 shows an excerpt of the chart of the nuclides from hydrogen to carbon. Black isotopes are stable. The dashed, red lines indicate mass numbers  $A = 5$  and  $A = 8$ , at which no stable nuclides exist. Thus, reactions of two  ${}^4\text{He}$  nuclei or a  ${}^4\text{He}$  nucleus and a proton do not form stable nuclides, which prevents BBN from forming anything heavier than  ${}^7\text{Li}$ .

## 2. Big Bang Nucleosynthesis



**Figure 2.6.:** The low-mass section of the chart of the nuclides with red, dashed lines indicating  $A = 5$  and  $A = 8$ . These are the two mass regions that do not have any stable nuclides, thus effectively halting heavier element production in BBN. Chart generated with a python tool by Krzysztof Miernik.

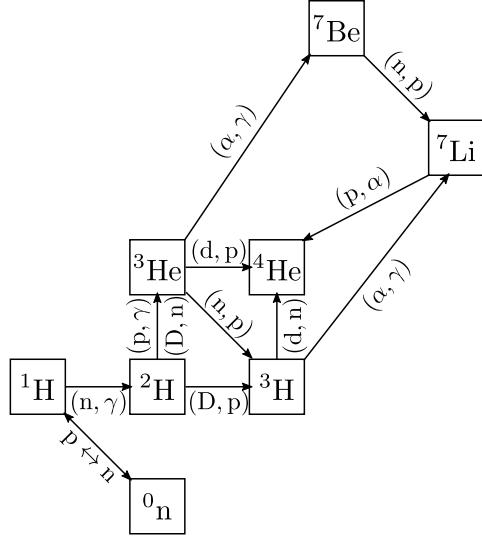
While nucleosynthesis takes place, the universe continues expanding, thus the temperature further decreases. This significantly limits the time in which new nuclides can form. Below a temperature of  $T_9 \approx 0.1$  BBN comes to a halt. This means that around 10-15 min after the Big Bang, all the hydrogen, helium, and other BBN products were formed.

Figure 2.7 shows the reaction network showing dominant reactions at work in BBN (after Nollett & Burles, 2000). As is common, we abbreviate the  $^4\text{He}$  nucleus with the symbol  $\alpha$ . Reactions are written in their abbreviated form, as is common in nuclear physics. For example  $^1\text{H}(\text{n},\gamma)^2\text{H}$  could also be written as  $^1\text{H} + \text{n} \longrightarrow ^2\text{H} + \gamma$ . Notably, there is no main reaction to produce  $^6\text{Li}$ . Lithium-7 can be produced in two ways: directly from tritium by capturing a  $\alpha$  particle (which is equivalent to  $^4\text{He}$  capturing a tritium nucleus) or by production of  $^7\text{Be}$  (from  $^3\text{He}$ ) and subsequent decay to  $^7\text{Li}$ .

### 2.3.5. Observational Constraints

So far, we have roughly derived the production of hydrogen and  $^4\text{He}$  expected to form during the Big Bang. We furthermore mentioned how D,  $^3\text{He}$ , and  $^7\text{Li}$  are produced. Our derivations were mainly based on the temperature and density evolution of the universe during BBN, two quantities that are coupled to each other. By observing the abundances of hydrogen, helium, and lithium in the universe and deriving the fractions of the species of interest that formed in the Big Bang, we can constrain the environment

## 2. Big Bang Nucleosynthesis



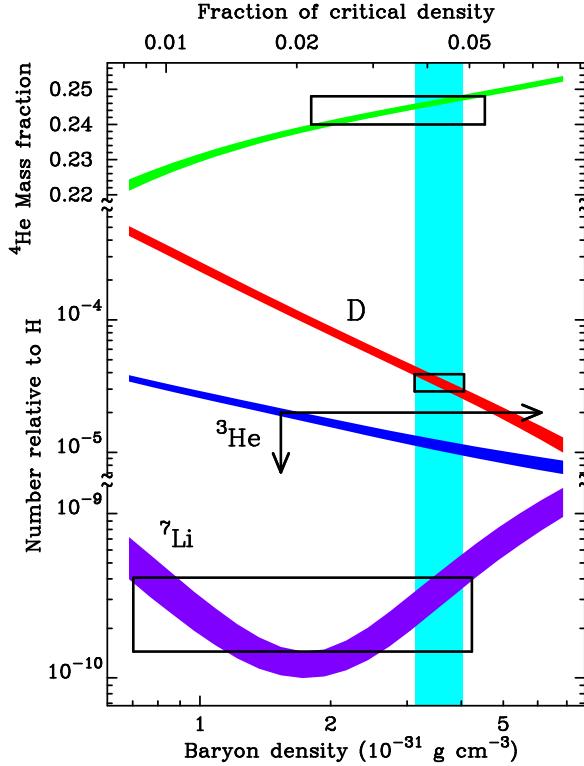
**Figure 2.7.:** BBN reaction rate network for dominant reactions, after Nollett & Burles (2000).

in which BBN took place.

Figure 2.8 shows the calculated abundances of D,  $^3\text{He}$ ,  $^4\text{He}$ , and  $^7\text{Li}$  when varying the present-day baryon density. The squares show the observed ratios, where available, and their uncertainties. The larger the baryon density in the universe, the larger is the baryon-to-photon ratio. As discussed before, this ratio and the temperature define when deuterium can form (see Figure 2.5). If the baryon-to-photon ratio goes up, fewer neutrons decay until they are captured and thus more  $^4\text{He}$  nuclei are ultimately formed. The conversion of D to  $^4\text{He}$  will also be more complete, thus less deuterium remains. The higher starting abundance of D and higher end abundance of  $^4\text{He}$  also results in burning out  $^3\text{He}$  more effectively, thus lowering its abundance with an increase in the baryon density. The curve in Figure 2.8 for  $^7\text{Li}$  shows the competition of two reactions. At low baryon densities,  $^7\text{Li}$  is mainly produced via  $^4\text{He} + \text{T} \longrightarrow ^7\text{Li} + \nu$ . At increased baryon density,  $^7\text{Li}$  however gets again efficiently destroyed by burning further. This destruction is compensated and overtaken at even higher baryon densities since  $^7\text{Be}$  is produced more efficiently, which then immediately decays to  $^7\text{Li}$ .

While difficult to observe, the abundance of BBN-produced hydrogen, helium, and lithium in the universe is a great way to determine crucial parameters of the Big Bang model. Similar insights can however also be gained by analyzing the CMB.

## 2. Big Bang Nucleosynthesis



**Figure 2.8.:** BBN predictions of the  $D$ ,  ${}^3He$ ,  ${}^4He$ , and  ${}^7Li$  abundance as a function of the present-day baryon density (Tytler et al., 2000). From arXiv:astro-ph/0001318.

## 2.4. Recombination

In Section 2.2.1 we laid out the assumptions of the standard model of cosmology, one of which is that the overall charge in the universe is zero. During BBN, the temperature too high to for electrons and nuclei to combine in order to produce a neutral gas. These components thus rather occur as a plasma. In order to create a neutral gas, the temperature has to sink to below 3000 K. At this temperature, the electrons combine with the atomic nuclei. Historically, this phase has been described as recombination, although the “re” part is slightly misleading since electrons and nuclei have never been combined previously.

During the plasma phase, i.e., prior to recombination, photons can scatter easily and the universe is thus opaque. After recombination the universe becomes transparent, and we transition into the matter dominated universe (see Figure 2.4). Since the Big Bang, roughly  $4 \times 10^5$  a have elapsed at this point.

The transition from the radiation dominated to the matter dominated universe can still be seen today as the horizon of the visible universe. It has significantly cooled down and is today seen as the CMB. An image of its fluctuations is shown in Figure 2.3.

## 2. Big Bang Nucleosynthesis

### 2.5. Reading

Since BBN took place fairly long ago at this point, it seems adequate to also look at the literature from a historic point of view. For the historical perspective one should read Alpher et al. (1948) and Peebles (1966a). Focus on comparing the two manuscripts: what are the differences between them and what are differences compared to what you have learned in class?

The present status of BBN was recently given by Cyburt et al. (2016). While this work contains fairly detailed methods on uncertainty calculations that are outside the scope of this lecture, the eager student might want to read sections III. Observations and V. The Lithium Problem in detail. You might also want to skim the introduction and the preliminaries.

For these readings it is important that you do not get hung up on details you do not understand, but rather try to follow the big picture. The following questions that can be discussed in class might help to focus on the big picture.

- What elements and isotopes are all formed in the Big Bang according to the work by Alpher et al. (1948)? What are the issues you see with this model, especially regarding it from the current state of knowledge?
- What nuclides were produced in the Big Bang according to Peebles (1966a) and how does this differ from the work by Alpher et al. (1948)?
- Note the neutron half-life that is used in Peebles (1966a). What is the issue here? This is also in detail discussed in the introduction by Cyburt et al. (2016).
- What importance does the nuclear reaction rate network have in Cyburt et al. (2016)? What reaction rates are to this day fairly uncertain?
- Discuss how the primordial abundance of H, D,  $^3\text{He}$ ,  $^4\text{He}$ , and  $^7\text{Li}$  can be derived from current observations. What are the problems?
- What is meant in Cyburt et al. (2016) by “regression to zero metallicity”?

# 3. Star Formation

In the last chapter we discussed the formation of the first elements in the Big Bang and derived that all the hydrogen and the vast majority of helium we see today in the universe formed in this event. In this chapter we will study how stars can form from hydrogen and helium gas and how they evolve throughout their lifetime. This topic is discussed generally for all stars, however, we will in the end focus on the very first and oldest star to form in the galaxy.

## 3.1. Molecular Cloud Collapse and Protostar Formation

To understand star formation, let us first discuss how molecular clouds collapse. Figure 3.1a shows a Hubble Space Telescope ([HST](#)) image of the famous horsehead nebula, a molecular cloud located in the constellation Orion. Molecular clouds are accumulations of gas and dust in the galaxy that prevent light from the stars behind it to come through. These structures thus look like clouds to the observer.

### 3.1.1. Jeans Criterium for Gravitational Collapse

For simplicity, let us assume a spherical, homogeneous sphere with a constant density  $\rho$  and temperature  $T$ . A schematic of such an idealized molecular cloud is shown in Figure 3.1b. The gravitational / potential that a given test mass  $m$  in the molecular cloud is in can be written as

$$E_{\text{pot}} = -\frac{GMm}{R}. \quad (3.1)$$

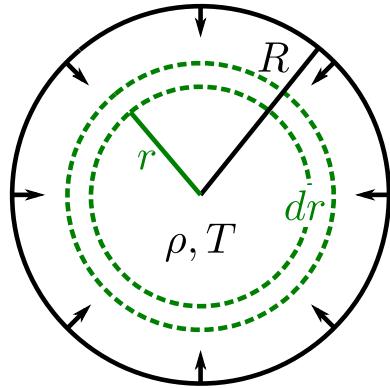
Here,  $R$  is the distance of the test mass from the center of the molecular cloud and  $M$  is the total mass of the cloud inside the radius  $R$ . Furthermore,  $G$  is the gravitational constant and is equal to  $G = 6.674 \times 10^{11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ . Infinitely far away from the molecular cloud, the gravitational potential is by definition zero. Thus, it becomes negative the closer to the center the test mass gets.

Instead of a test mass, let us now assume that we want to determine the gravitational energy of a mass shell with thickness  $dr$  at distance  $r$  from the center of the molecular cloud. A schematic of this setup is shown in Figure 3.1b. Assuming that the cloud is at

### 3. Star Formation



(a) The horsehead nebula. Credit: NASA/ESA/Hubble Heritage Team.



(b) A schematic representation of a molecular cloud collapsing.

**Figure 3.1.:** Molecular clouds are the structures stars form from in the universe.

a given temperature  $T$  and has a homogeneous density  $\rho$ , we can write the mass of the shell ( $m$ ) and the mass of the cloud inside the shell ( $M$ ) as

$$m = 4\pi\rho r^2 dr \quad (3.2)$$

$$M = \frac{4}{3}\pi\rho r^3. \quad (3.3)$$

We can now determine the total potential energy of the molecular cloud as

$$\begin{aligned} E_{\text{pot}} &= - \int_0^R \frac{G}{r} \left( \frac{4}{3}\pi\rho r^3 \right) (4\pi\rho r^2) dr \\ &= - \frac{16}{3}\pi^2\rho^2 G \int_0^R r^4 dr \\ &= - \frac{16}{15}\pi^2\rho^2 G R^5 \\ &= - \frac{3}{5}G \frac{M^2}{R}. \end{aligned} \quad (3.4)$$

In the last step we used the fact that the mass of the molecular cloud can be written as

$$M = \frac{4}{3}R^3\rho \quad (3.5)$$

to substitute it back into the equation.

### 3. Star Formation

During the collapse, gravitational energy is transformed into kinetic energy, which here is equal to a rise in temperature. The total kinetic energy of the system can be written as

$$E_{\text{kin}} = \frac{3}{2} N k T = \frac{3}{2} k T \frac{M}{\mu}, \quad (3.6)$$

where  $N$  is the number of molecules in the molecular cloud and  $\mu$  their average mass, i.e., to first approximation the mass of H<sub>2</sub>. For a cloud in hydrostatic equilibrium, i.e., a cloud that is neither expanding nor contracting, the virial theorem (which is derived in Appendix A) can be applied. This theorem states that the kinetic and potential / gravitational energy are related to each other as

$$2E_{\text{kin}} + E_{\text{pot}} = 0. \quad (3.7)$$

Assuming the molecular cloud is in equilibrium, we can plug equations (3.4) and (3.6) into (3.7) and solve for the mass.

$$\begin{aligned} 3kT \frac{M}{\mu} - \frac{3}{5} G \frac{M^2}{R} &= 0 \\ \frac{GM}{5R} &= \frac{kT}{\mu} \\ M &= \frac{5RkT}{G\mu} \equiv M_J \end{aligned} \quad (3.8)$$

Here,  $M_J$  is the so-called Jeans mass, named after Sir James Jeans who first described this derivation in 1904. It describes the mass  $M = M_J$  for which the molecular cloud is in balance, i.e., the kinetic and gravitational energy compensate each other. The Jeans mass thus also describes the minimum mass for star formation; a molecular cloud with  $M > M_J$  will collapse since the gravitational force dominates.

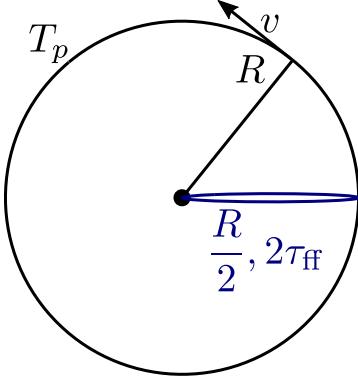
#### 3.1.2. Free Fall Time

To determine how long gravitational collapse of a molecular cloud takes, we can estimate the free fall time. The gravitational force of the system and its radius are known. Using Newton's law  $\vec{F} = m\vec{a}$ , we can thus write the slightly unwieldy differential equation

$$-G \frac{Mm}{R^2} = m\ddot{R}, \quad (3.9)$$

where  $m$  is a test mass at the edge of the cloud. However, the free fall time  $\tau_{\text{ff}}$  can also be derived more elegantly. Kepler's third law of planetary motion states that the cube of the semi-major axis of a planet divided by the square of its period is constant. This law in fact can directly be derived by comparing the gravitational and centrifugal force acting on a planet since these two forces, for a stable orbit, must compensate each

### 3. Star Formation



**Figure 3.2.:** Schematic to derive the free fall time  $\tau_{\text{ff}}$  using Kepler's third law of planetary motions. See text for details.

other. Figure 3.2 shows a schematic of the here discussed scenario. The molecular cloud (black circle), if not collapsing, would have to rotate at a given velocity  $v$  such that the gravitational force  $F_g$  and the centrifugal force  $F_c$  compensate each other. For a test mass at the outermost edge of the cloud, the gravitational force of the whole cloud is equivalent as if we pictured the mass of the cloud to be concentrated in the center. Disregarding vector quantities, we can derive the velocity a stable cloud would move around the center with as

$$\begin{aligned} F_g &= F_c \\ G \frac{Mm}{R^2} &= \frac{mv^2}{R} \\ \Rightarrow v &= \left( G \frac{M}{R} \right)^{1/2}. \end{aligned} \quad (3.10)$$

To determine the free fall scenario we can now halt the test particle in its motion, i.e., set  $v = 0$ . The “orbit” of the test particle would then look like a straight line into the gravitational center of the cloud. In Figure 3.2 this scenario is shown by drawing the line as a very skinny ellipse. The semi-major axis of this orbit would now be  $R/2$  and the orbital period  $2\tau_{\text{ff}}$ .

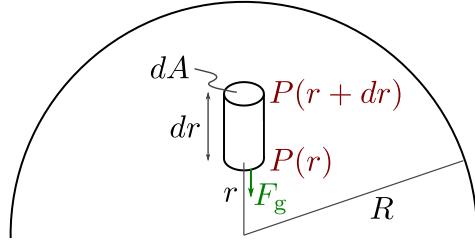
From Kepler's third law we know that the following statement must hold true.

$$\frac{R^3}{T_P^2} = \frac{\left(\frac{R}{2}\right)^3}{(2\tau_{\text{ff}})^2} \quad (3.11)$$

Here,  $T_P$  can be derived using the velocity as given in equation 3.10 and the length of the orbit ( $2\pi R$ ). This yields

$$T_p = 2\pi \left( \frac{R^3}{GM} \right)^{1/2}. \quad (3.12)$$

### 3. Star Formation



**Figure 3.3.:** Schematic drawing to derive the hydrostatic equilibrium equation. See text for details.

Plugging equation (3.12) into equation (3.11) and expressing the mass of the cloud using the density  $\rho$  as  $M = \frac{4}{3}R^3\rho\pi$ , we can finally derive the free fall time as

$$\tau_{\text{ff}} = \left( \frac{3\pi}{32G\rho} \right)^{1/2}. \quad (3.13)$$

#### 3.1.3. Protostar Birth

Fortunately, the molecular cloud collapse does not take place adiabatically. Heat is effectively radiated from the cloud as infrared radiation (IR), thus preventing the Jeans mass (which is proportional to the temperature) from exceeding the molecular cloud mass and thus bringing the cloud into hydrostatic equilibrium (see Section 3.2). The main cooling mechanisms producing IR radiation are molecular collisions and dust. Colliding molecules transfer energy into vibrational states which, when they decay, emit IR. Dust on the other hand will heat up and radiate as a blackbody with an effective temperature of less than around 1000 K, thus mostly radiating in the IR. The molecular cloud is mostly transparent to IR and can thus effectively cool thanks to these processes.

Ultimately, the rising heat will start dissociating molecules and evaporating dust, thus the effective cooling mechanism stops. At this point the temperature in the center rises and the Jeans mass will be exceeded. The center of the collapsing cloud defines the protostar in hydrostatic equilibrium (see below).

## 3.2. Hydrostatic equilibrium

Once a molecular cloud has reached a high enough temperature such that its mass is equal to the Jeans mass, the system has entered the so-called hydrostatic equilibrium. Understanding this equilibrium is crucial in terms of understanding stellar phases during the end of their lives and during nucleosynthesis events. The whole life of a star is dominated by hydrostatic equilibrium.

### 3. Star Formation

Figure 3.3 shows a cylinder of length  $dr$ , face area  $dA$ , and density  $\rho(r)$  at distance  $r$  from the center of the star. The gravitational force that acts on the cylinder can be written as

$$F_g = -G \frac{M_r dm}{r^2}, \quad (3.14)$$

where  $M_r$  is the mass of the star that is inside radius  $r$  and  $dm = \rho(r)drdA$  is the mass of the cylinder.

In addition, the cylinder is under pressure, i.e., the pressure  $P(r + dr)$  from above it pushing it down and the pressure  $P(r)$  from below it pushing it up. To force exerted by the pressure is

$$F_P = dPdA, \quad (3.15)$$

where  $dP = P(r + dr) - P(r)$ .

In order for the cylinder to be neutrally buoyant, i.e., to be in hydrostatic equilibrium, the gravitational force must exactly compensate the force exerted by the pressure. Setting equations (3.14) and (3.15) equal plus some simple algebra, we can derive the hydrostatic equilibrium condition as

$$\frac{dP}{dr} = -\rho(r) \frac{GM_r}{r^2}. \quad (3.16)$$

#### 3.2.1. Central Pressure

Using equation (3.16), we can estimate the central pressure of a star by integrating over  $r$ . For this estimate, let us replace the density  $\rho(r)$  with the mean density inside the star. This mean density can be expressed as

$$\bar{\rho} = \frac{M}{\frac{4}{3}\pi R^3}. \quad (3.17)$$

We can furthermore write  $M_r$  as a function of the distance  $r$  from the center of the star. Remember that  $M_r$  simply represents the mass of the star inside of radius  $r$ , thus

$$M_r = M(r) = \frac{4}{3}\pi r^3 \bar{\rho}. \quad (3.18)$$

Plugging these two quantities into equation (3.16) and integrating over  $r$ , the central pressure  $P_c$  can be estimated as

$$P_c = \frac{3}{8\pi} \frac{GM^2}{R^4}. \quad (3.19)$$

We assumed here that the pressure at the surface of the star is zero, which is surely true when compared to the central pressure. In the case of the Sun, using equation (3.19) we can calculate a central pressure of  $P_{c,\odot} \approx 10^{14}$  Pa.

### 3. Star Formation

Note that the presented estimate of the central pressure of a star necessarily represents a lower limit. The reason for this is that we assumed a constant density throughout the star, which surely is not correct. The density will be significantly higher in the center of the star compared to the outer parts. This always increases the pressure compared to our estimate. To visualize why, imagine all the mass of the star was concentrated within  $R/2$ . Integrating the pressure over the whole star using our estimate above would result in the same central pressure. However, from our visualization we know that the central pressure is concentrated in the inner half, thus integration of  $r$  only needs to be done up to  $R/2$  to determine the “real” central pressure. This value will necessarily be higher than what we estimated in equation (3.19).

For the Sun,  $\rho(r)$  needs to be modeled in order to exactly determine the actual pressure. Such models yield a central pressure for the Sun of  $2.5 \times 10^{16}$  Pa, which is two orders of magnitude higher than our lower limit estimate.

#### 3.2.2. Central Temperature

From the estimated central pressure, we can estimate the central temperature of a star assuming that the equation of state of an ideal gas holds in this scenario. This equation takes the form

$$pV = Nk_B T, \quad (3.20)$$

where  $p$  is the pressure,  $V$  the volume,  $N$  the number of particles,  $k_B$  Boltzmann’s constant, and  $T$  the temperature. The number of particles that are in the gas can be expressed as  $N = M/\mu$ , where  $\mu$  is the mean molecular weight of all particles. For a gas made of neutral hydrogen, the mean molecular mass per particle would be equal to the molecular mass of hydrogen  $\mu_H$ . However, we know that the temperature within the Sun is so high that the hydrogen gas is fully ionized. Thus, we have twice as many particles (protons and electrons) to consider. Since electrons have significantly less mass than protons, the mean molecular mass of the gas inside a star can be estimated as  $\mu = \mu_H/2$ . Using equation (3.19), the central temperature of a star can now be written as

$$T_c = \frac{P_c \mu}{\bar{\rho} k_B}. \quad (3.21)$$

Here, we replaced  $N$  with  $M/\mu$ , which results in  $V/M$  in this equation. This is of course equal to the reciprocal of the mean density  $\rho$ .

For the Sun, using the above determined pressure, we can estimate a temperature in the center of  $T_{c,\odot} \approx 10^7$  K. Considering all the approximation that we made up to here, this number agrees strikingly well with the central temperature of the Sun.

### 3. Star Formation

#### 3.2.3. Sustaining a Star via Gravity

Knowing the luminosity  $L$  of a star, e.g., the luminosity of the Sun, which is  $L_\odot = 3.828 \times 10^{26}$  W, we can calculate how long the star would live if its energy would solely originate from gravitational collapse. The total gravitational energy available in the system is already given in equation (3.4). Since the luminosity is simply energy per time, the total amount of time that a star's luminosity could be sustained by gravitational infall is

$$t = \frac{E_g}{L} = \frac{GM^2}{RL} \equiv t_{\text{KH}}. \quad (3.22)$$

This time is also known as the Kelvin-Helmoltz time ( $t_{\text{KH}}$ ).

For the Sun we can calculate that gravity alone would be able to sustain the current luminosity for a total of  $t_{\text{KH}} = 3 \times 10^7$  a. This time is more than two orders of magnitude too short. Looking at the Earth's fossil record, clear evidence has been found for life on Earth as far back as 3.5 Ga (Schopf et al., 2007). Dating meteorites furthermore yields a Solar System age of 4.567 Ga. Thus, the Sun must be at least be 100 times older than indicated by the Kelvin-Helmholtz time and another energy source is required to hold up the hydrostatic equilibrium.

#### 3.2.4. Nuclear Reactions

We estimated the temperature at the center of the Sun to be  $T_c \approx 10^7$  K, which is high enough to efficiently convert hydrogen to helium. Nuclear reactions are in fact the reason that stars can sustain the hydrostatic equilibrium. In Section 2.3.3 we established that converting four hydrogen atoms into one helium atom releases  $\Delta E = 28.3$  MeV of energy. If all the hydrogen mass ( $M_{\text{H}}$ ) of a star is available as fuel to sustain the hydrostatic equilibrium, the star's lifetime could be calculated as

$$t_{\text{nuc}} = \frac{M_{\text{H}} N_A \Delta E}{4 m_{\text{H}} L}. \quad (3.23)$$

Here,  $N_A$  is Avogadro's constant and  $m_{\text{H}} = 1.008 \text{ g mol}^{-1}$  the molecular mass of hydrogen.

For the Sun, considering that hydrogen makes up  $\sim 75\%$  of its mass, we can calculate  $t_{\text{nuc}} \approx 10^{11}$  a. However, not all the hydrogen is available as fuel since the Sun is not fully convective. Only around 10% of the hydrogen mass are accessible by the core and can thus be transformed to helium to produce energy. This puts the Sun's life while sustaining hydrostatic equilibrium via hydrogen burning at about 10 Ga, which means we have about 5 Ga left.

### 3.2.5. The Death of a Star

When stars run out of fuel in the center, nuclear reactions can no longer be sustained and thus the radiation pressure falls away. The star is thus no longer in hydrostatic equilibrium since  $dP/dr = 0$  in equation (3.16). Therefore, the system will start collapsing again due to gravitation. The temperature in the center of the star will rise further until the next burning stage can start, i.e., helium burning in the triple  $\alpha$  process producing  $^{12}\text{C}$ . These reactions produce again radiation pressure, thus reestablishing the hydrostatic equilibrium. How many burning stages can be accessed by the star depends on the star's initial mass and will be discussed later.



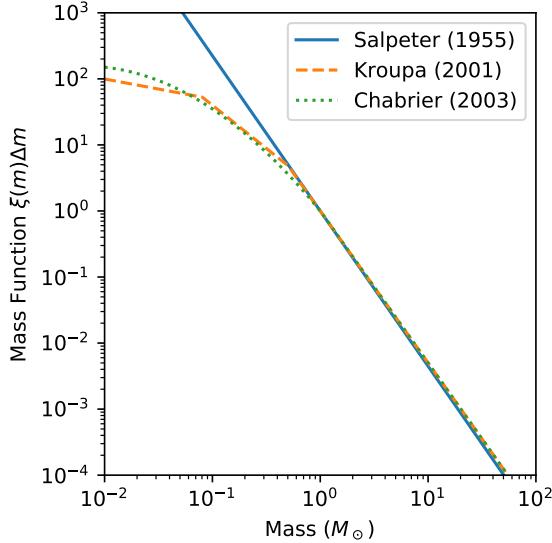
**Stellar evolution** The evolution of stars is generally modeled using elaborate stellar evolution codes. Many different codes exist, however, one open-source stellar evolution code has recently been widely adopted. The Modules for Experiments in Stellar Astrophysics ([MESA](#)) code, developed by Bill Paxton at UCSB, has enabled many new research groups to work on stellar evolution and its consequences. In fact, anybody can run and install [MESA](#), instructions can be found on the [MESA website](#). Note that while [MESA](#) is simple to use, users still must have some basic understanding of stellar evolution. The [garbage in, garbage out concept](#) applies to stellar evolution as well.

## 3.3. The Initial Mass Function

Star formation can only take place if a molecular cloud exceeds the Jeans mass. The likelihood of star formation in a given location of the galaxy depends on the density of material in that specific place. Depending on the total mass of the molecular cloud, stars of different masses can form. The so-called initial mass function ([IMF](#)) describes the number distribution of stars with different masses between  $0.1 M_{\odot}$  and  $100 M_{\odot}$  and has been derived from observations.

Figure 3.4 shows the IMF derived by three different researchers. The first IMF was published by Salpeter (1955). More detailed observations led to revisions of the IMF by Kroupa (2001) and Chabrier (2003). For stars heavier than  $1 M_{\odot}$ , all IMFs agree with each other. At low masses, however, the IMF by Salpeter (1955) significantly overestimates the abundance of stars compared to the predictions by Kroupa (2001) and Chabrier (2003). Today, the latter two IMFs are generally used for GCE models.

### 3. Star Formation



**Figure 3.4.:** Comparison of IMFs by Salpeter (1955), Kroupa (2001), and Chabrier (2003) normalized to  $m = 1 M_{\odot}$ .

## 3.4. The First Stars

We have seen above that temperature and cooling play an essential role in star formation. The temperature of a molecular cloud is proportional to the Jeans mass, see equation (3.8). This leads to two issues: (1) The universe at the beginning was hotter, thus more mass is required for a molecular cloud to undergo gravitational collapse. (2) The main cooling processes of molecular clouds is to radiate heat away to not fall too early into hydrostatic equilibrium. This requires molecules and dust to be present. In the early universe the metallicity was however practically zero, thus this cooling process, except the dissociation of H<sub>2</sub> molecules, cannot take place. As a result only very massive, metal-free stars are expected to form in the beginning.

These predicted, very massive stars with very low metallicity are called population III stars. Metal poor stars with  $10^{-4} Z_{\odot} < Z < 10^{-5} Z_{\odot}$ , which can mostly be found in old globular clusters, are called population II stars. All more metal-rich stars, mostly found in the galactic disk, are population I stars, e.g., the Sun.

Massive stars, as we will see in detail later on, have a much shorter lifespan than low-mass stars and will thus quickly enrich the early universe with freshly nucleosynthesized products, i.e., metals. Thus, second and later generation will start off with some metallicity. Possible nuclear reactions thus change and get more diverse. To study these earliest stars and nucleosynthesis events, spectroscopic observations of ultra metal-poor stars are an important part.

### 3.5. Reading

For the curious reader, Anna Frebel wrote an excellent book titled “Searching for the Oldest Stars” (Frebel, 2015). The book is available via the Brandeis Library online.

For the discussion section we will focus on astronomical observations of Reticulum II, an ultrafaint dwarf galaxy that orbits the Milky Way. Please read Croswell (2021), a news feature article in the proceedings of the national academy of sciences. This article should give you a very brief and interesting overview of the field of the first stars, *r*-process nuclei, and why they are important for understanding element formation in the universe. Furthermore, this brief article is written as a summary of recent events and discoveries. Second, please read the paper by Ji et al. (2016) on the original observations of r-process enhanced stars in Reticulum II. This is the main article that we will discuss in class. The following bullet points should serve as guidance on reading these two manuscripts.

- What are r-process nuclides and why are they important in this context?
- What are ultrafaint dwarf galaxies? Why are they so interesting when studying the oldest stars? What population of stars do these galaxies contain? What is special about Reticulum II?
- Why were neutron star mergers thought to be the main contributor to *r*-process nuclei today, but not in the early universe? Has our thinking changed due to the work by Ji et al. (2016)? Has our thinking since then changed again?
- What is the difference between neutron-capture elements and non-neutron-capture elements? Why is this important in this context?
- Why do Ji et al. (2016) argue that all *r*-process elements in Reticulum II were made by one single event?
- What is the neutron star merger to supernova (SN) rate ratio in the Milky Way? What factor do you think play into the determination of the rates?
- What alternative scenarios could explain the observations of Ji et al. (2016) aside from neutron star mergers?

# 4. The Life and Death of the Sun

In the last chapter we have seen how stars form from molecular clouds. We also derived the hydrostatic equilibrium, which is the conditions that stars spend most of their life in, and have analyzed some of its consequences. While we have already determined some key facts about the Sun, in this chapter we discuss the life of the Sun, its place among other stars in the Milky Way, and its ultimate fate in more detail. Understanding the Sun is useful; after all it is the energy source required for life on Earth. However, the Sun is also regularly used in comparison with other stars when discussing their lifetimes and fates.

## 4.1. The Sun's Place in the Milky Way

In Chapter 1 we have already discussed the place of the Sun in the Milky Way, see, e.g., Figure 1.1. Here, we will now also have a more detailed look at the Sun with respect to other stars – its neighbors – in the galaxy.

### 4.1.1. The Hertzsprung-Russell Diagram

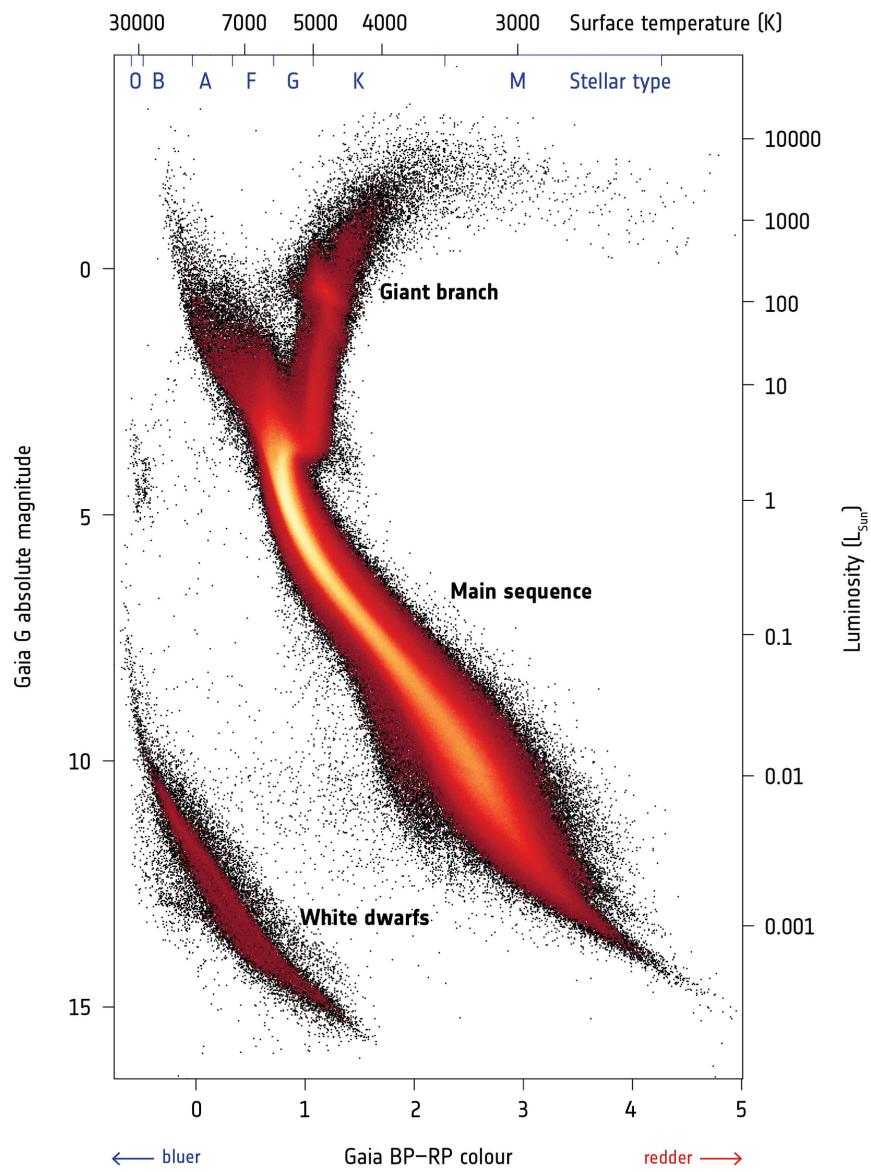
Large-scale photographic and spectroscopic observations of stars led in the early 19<sup>th</sup> century to the assembly of the first Hertzsprung-Russell diagram (**HRD**). An updated version of this diagram that includes around four million stars within 5000 ly of the Sun is shown in Figure 4.1. The observations for this **HRD** were done with **Gaia**, a space mission by the European Space Agency (**ESA**).

The **HRD** originally showed the magnitude of a star, i.e., its brightness, versus its stellar type. While the brightness is directly related to the luminosity, the stellar type is related to the peak in its spectrum and thus to the star's surface temperature. Wien's displacement law (see Section 1.2) allows us to directly correlate the stellar type with the surface temperature of a star. Thus, the **HRD** diagram plots the luminosity of a star, usually normalized to the solar luminosity  $L_{\odot}$ , with respect to its surface temperature.

Various notations exist for stellar types, see, e.g., [here](#). In Figure 4.1 the Harvard system is used, in which the stellar types from hottest to coldest stars are O, B, A, F, G, K, M (see the top axis of Figure 4.1).

The Sun has a surface temperature of around 5800 K and is thus a G-type star. Knowing its luminosity ( $1 L_{\odot}$ ), its position in the **HRD** can be identified. The Sun

## → GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM



**Figure 4.1.:** The Hertzsprung-Russell diagram put together from four million stars within 5000 ly of the Sun. Credit: [ESA/Gaia/DPAC](#), CC BY-SA 3.0 IGO.

#### 4. The Life and Death of the Sun

lays centered on a branch in the HRD that is labeled as main sequence. The main sequence is the location in the HRD where stars spend their lives, i.e., their quiescent burning phases. Once the nuclear fuel is exhausted, stars collapse further until the next burning stage can be activated. At this point, stars expand while keeping their surface temperature similar, thus they significantly increase their luminosity and wander up onto the giant branch. Finally, low mass stars end up as so-called white dwarfs (WDs). These objects, usually surrounded by planetary nebulae, are compact and hot. Due to their small size, their luminosity is fairly low, which places them in the bottom left corner of the HRD.

##### 4.1.2. Planetary Systems

With the discovery of the first exoplanet in 1995 by Michel Mayor and Didier Queloz of the Geneva Observatory, the Sun was suddenly not the sole star anymore that had planets orbiting it. The existence of other planets outside the Solar System has been long suspected; it is thought that Giordano Bruno (1548-1600), a Dominican monk, believed in a Copernican universe filled with an infinite number of inhabited worlds around other stars. While no evidence exists at this point for inhabited worlds, multiple exoplanets in the so-called Goldilocks or habitable zone have been discovered. These rocky planets are located at a distance from their parent star that allows for the existence of liquid water on the surface, like Earth. Note that while Mayor and Queloz received the 2019 Nobel Prize,<sup>1</sup> Bruno was executed for his beliefs.

According to NASA's exoplanet website,<sup>2</sup> the current number of confirmed exoplanets as of February 26, 2021 is 4352. Figure 4.2 shows a current figure for the size of confirmed exoplanets as a function of their orbital period. The size here is given in multiples of the size of Jupiter ( $\text{\textcircled{J}}$ ). Note that Earth ( $\text{\textcircled{E}}$ ) in this figure would plot at  $M_{\text{\textcircled{E}}} = 3.15 \times 10^{-3} M_{\text{\textcircled{J}}}$  and at a period of 365 d. Jupiter itself on the other hand, in comparison to other exoplanets, has an orbital period of 4333 d. This shows that the Solar System seems to be an outlier when compared to all discovered exoplanets. Many of these exoplanets are heavy and orbit their parent star at short distances. Some massive exoplanets have orbital periods of a day and shorter. To what extent these discrepancies are due to observational bias remains under investigation, see, e.g., Mulders et al. (2018) and Mulders et al. (2019).

##### 4.1.3. Is the Sun Special?

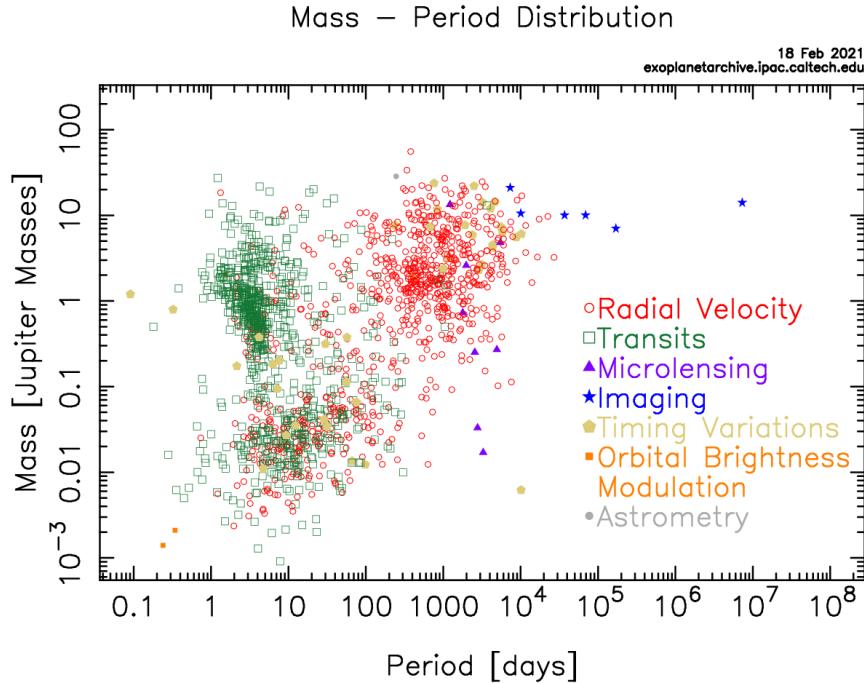
The Sun, overall, seems to be an average star like many others. While it plots in warmer areas of the HRD than most stars that are on the main sequence, see Figure 4.1, it is

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<sup>1</sup><https://www.nobelprize.org/prizes/physics/2019/summary/>

<sup>2</sup><https://exoplanets.nasa.gov/>

#### 4. The Life and Death of the Sun



**Figure 4.2.:** Size of discovered exoplanets as a function of their orbital periods. Source: Exoplanet Archive, IPAC, Caltech.

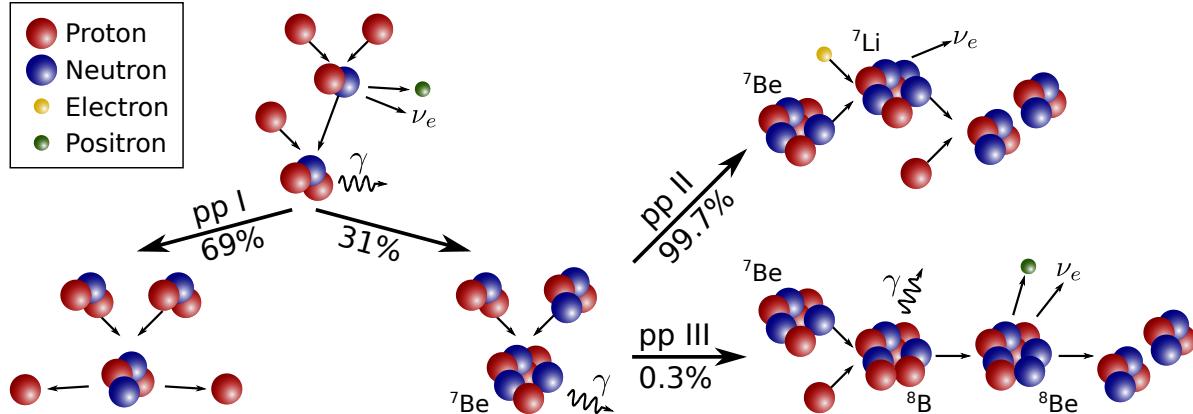
not outstandingly hot or cold. With respect to its planetary system it remains to be seen how common the Solar System as a whole is. Comparing the Solar System to exoplanets (Figure 4.2) it seems as if we are fairly unique, however, this could simply be the result of observation biases since it is currently difficult to detect Earth-sized planets that orbit their parent star at distances of 1 AU from their parent star. However, the habitable zone of a star of course does not just depend on the distance but also on the star's luminosity.

The Sun is special in one aspect that has not been discussed so far, namely, it does not have a companion star. More than half of all stars are part of multiple star systems.



**The Exoplanet Population Observer Simulator (EPOS)** is a python software package to simulate observations of exoplanet populations. This package allows to study observational biases in transit and radial velocity exoplanet surveys, e.g., when analyzing data from the Kepler mission. The software package is well documented on [Read the Docs](#) and its source is available on [GitHub](#).

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**Figure 4.3.:** The proton-proton-chain (pp-chain) and it's three branches. Percentages are with respect to hydrogen fusion in the Sun.

As we will see later, these systems can have a significant effect on the stellar evolution and nucleosynthesis, and in some cases even be crucial for the production of certain elements.

## 4.2. The Sun's Quiescent Burning Phase

After having formed from a collapsing molecular cloud, the Sun entered the main sequence of the [HRD](#) once nuclear fusion of hydrogen became the dominant heat source. Astronomers also refer to a star at the point when it enters the main sequence as a zero age main sequence ([ZAMS](#)) star. From here on, the quiescent burning phase starts, in which stars slowly fuses hydrogen to helium. The [HRD](#) in Figure 4.1 shows that most stars lay on the main sequence. The reason for this is that hydrogen burning is the longest phase a star goes through.

The four-body reaction  $4 \text{ } ^1\text{H} \rightarrow ^4\text{He} + \gamma$  is extremely unlikely to happen. Hydrogen fusion rather takes place in stages. Multiple processes are responsible as outlined below.

### 4.2.1. The Proton-Proton-Chain

The proton-proton-chain (pp-chain) is the main reaction chain that fuses hydrogen to helium in the Sun. Figure 4.3 shows a schematic of the pp-chain and its three main branches. All branches start out with the reactions



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Three protons are thus converted via deuterium ( $^2\text{H}$ ) into a  $^3\text{He}$  nucleus. At this point the pp-chain branches. The pp I chain, which is responsible for 69% of the energy produced in the Sun via the pp-chain, fuses two  $^3\text{He}$  nuclei and forms one  $^4\text{He}$  nucleus in the



reaction.

On the other branch in the pp-chain, which takes place 31% of the time in the Sun, a  $^3\text{He}$  and a  $^4\text{He}$  nuclei are first combined to form  $^7\text{Be}$ . The reaction is

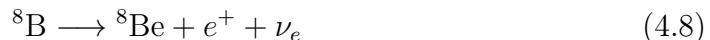


Two paths are possible from here. The pp II chain forms  $^4\text{He}$  via the following reactions from  $^7\text{Be}$ :



The  $^4\text{He}$  nucleus that is consumed in reaction (4.4) is gained back in the end. The pp II chain takes place 99.7% of the time after the first branching.

The rest of the time in this second branch, the pp III chain produces  $^4\text{He}$  from  $^7\text{Be}$ . The reactions that take place are as following:



The latter two reactions are here simply the decays of  $^8\text{B}$  and  $^8\text{Be}$  to form two  $^4\text{He}$ . Thus, the  $^4\text{He}$  consumed after the first branching is again gained back plus an additional  $^4\text{He}$  forms.

The nuclear energy generation rate in  $\text{W kg}^{-1}$ , simplified after equation (10.46) in Carroll & Ostlie (2017) (page 311), can be written as

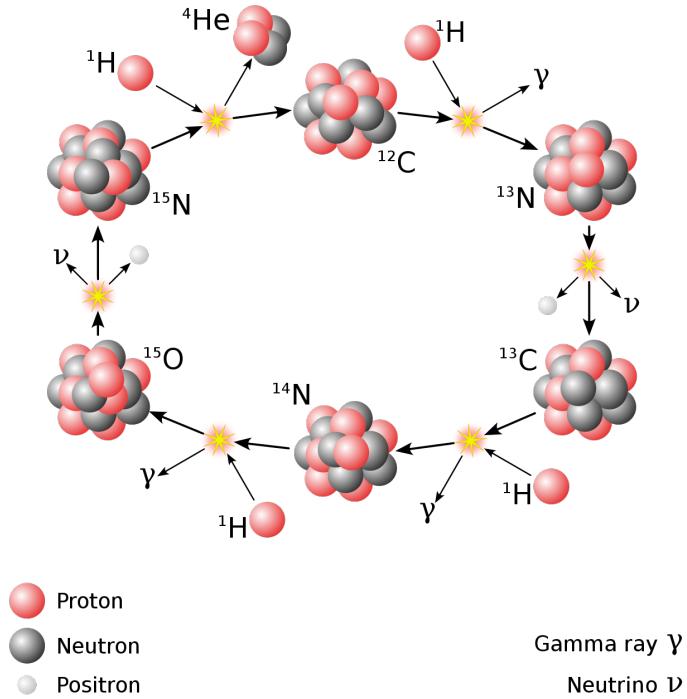
$$\epsilon_{\text{pp}} = 0.241\rho X^2 T_6^{-2/3} \exp\left(-33.8T_6^{-1/3}\right). \quad (4.10)$$

Here,  $\rho$  is the density of the star's center and  $T_6$  is the temperature expressed in multiples of  $10^6 \text{ K}$ . Assuming a density in the center of the Sun of  $\rho = 150 \text{ g cm}^{-3}$ , we can calculate the energy production of the pp-chain at  $\epsilon_{\text{pp}} = 3.4 \times 10^{-3} \text{ W kg}^{-1}$ .

#### 4.2.2. The CNO-Cycle

Compared to the pp-chain, the CNO cycle requires carbon, nitrogen, and oxygen as catalysts for the reaction in order to combine four hydrogen nuclei into  $^4\text{He}$ . Figure 4.4

#### 4. The Life and Death of the Sun



**Figure 4.4.:** Schematic drawing of the CNO cycle and the involved reactions. A  ${}^4\text{He}$  nucleus forms by combining four protons. The elements carbon, nitrogen, and oxygen act as catalysts for the reactions. Credit: Wikipedia.

shows a schematic of the CNO cycle. Starting at  ${}^{12}\text{C}$ , the involved reactions are as following:



As in the pp-chain, the CNO cycle is also branched. This branch occurs at the last reaction (4.16) and occurs only about 0.04% of the time. The branch starting at

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reaction (4.16), which is not shown in Figure 4.4, is



This branch thus does not drop back to  $^{12}\text{C}$  but rather produces  $^{14}\text{N}$ , which is also part of the CNO cycle and will be consumed further, see reaction (4.14).

The nuclear energy generation rate in  $\text{W kg}^{-1}$ , simplified after equation (10.58) in Carroll & Ostlie (2017) (page 312), can be written as

$$\epsilon_{\text{CNO}} = 8.67 \times 10^{20} \rho X X_{\text{CNO}} T_6^{-2/3} \exp(-152.28 T_6^{-1/3}). \quad (4.21)$$

Here,  $X_{\text{CNO}}$  is the mass fraction of CNO elements, i.e., the elements that are used as catalysts. In order for the CNO cycle to take place in a star, some metals must be present. For the Sun, the energy generation rate via the CNO cycle can be estimated as  $\epsilon_{\text{CNO}} = 2.3 \times 10^{-4} \text{ W kg}^{-1}$ .

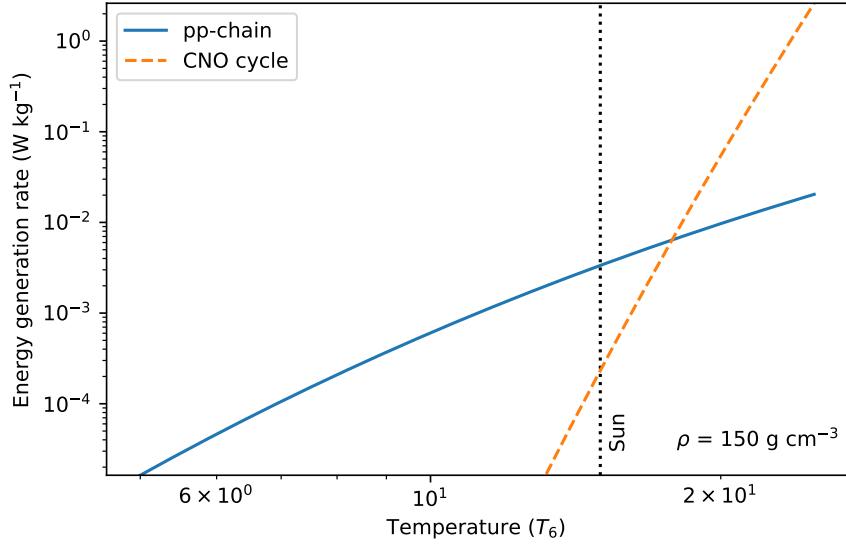
### 4.2.3. Energy Production in the Sun

Figure 4.5 shows the energy generation rates of the pp-chain and the CNO cycle (equations (4.10) and (4.21), respectively) as a function of the central temperature. A dotted line is shown at the Sun's central temperature. While the figure is scaled to the Sun's core density of  $\rho \approx 150 \text{ g cm}^{-3}$ , both energy generation rates for the pp-chain and the CNO cycle are proportional to  $\rho$ , thus the relative position of the two curves will stay the same for different stars and only depend on the temperature.

Figure 4.5 clearly shows that the pp-chain is the dominant energy source in the Sun. The CNO cycle's energy generation rate is more than an order of magnitude lower. However, at slightly higher temperatures, the CNO cycle will be the dominant source of nuclear energy that keeps a star in hydrostatic equilibrium.

In Section 3.4 we discussed star formation at zero metallicity and concluded that these stars must have been very massive since cooling rates for molecular clouds are minimal at zero metallicity. If a star starts with  $Z = 0$ , it can also only produce energy in its core via the pp-chain. In Figure 4.5 it can be seen that the energy generation rate increases slower for the pp-chain as a function of temperature compared to the CNO cycle. Thus, stars that start off with truly no metals must have gravitationally collapsed further until the temperature was high enough for the pp-chain to compensate this infall and go into hydrostatic equilibrium.

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**Figure 4.5.:** Energy generation rate due to the pp-chain and CNO cycle as a function of temperature. The core density here is fixed to  $150 \text{ g cm}^{-3}$ , which is approximately the density in the Sun's core.

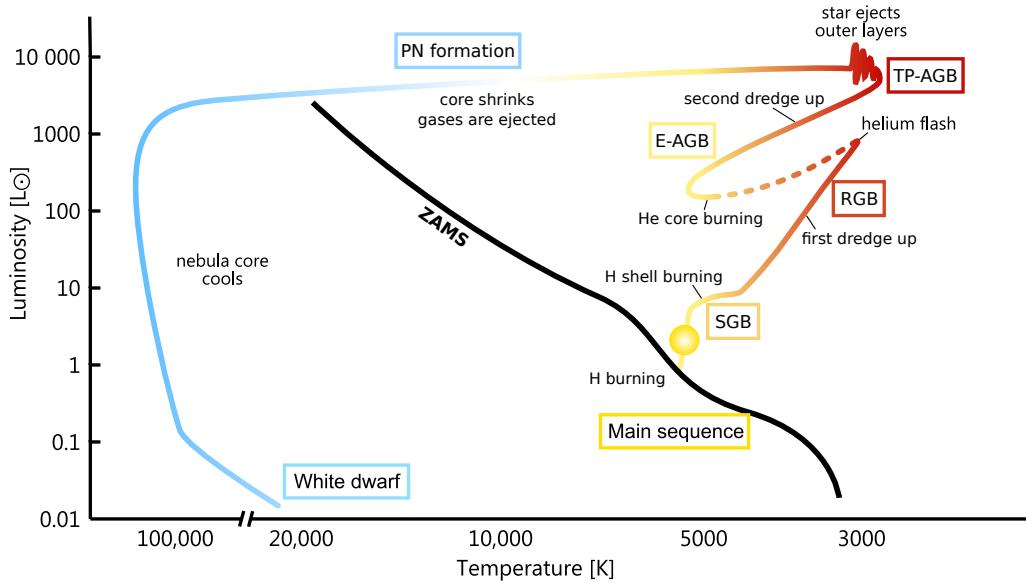
### 4.3. The Dying Sun

Figure 4.6 shows the evolution of the Sun in the HRD. The black line in the background represents the ZAMS and here is where the Sun starts its life. Currently, the Sun is still on the main sequence quiescently burning hydrogen, as discussed above. While on the main sequence, the Sun is slowly moving upwards in the HRD since continuous hydrogen burning slowly changes the average composition of the core. At present, about half of the nuclear fuel in the core has been consumed. Since the Sun has currently only at the surface a convective region, further hydrogen from the envelope is not accessible as fuel for hydrogen fusion.

Note that the discussion here follows the details described in Iliadis (2015). Since this is not a completely settled topic, some details might differ in other work, e.g., Schröder & Smith (2008).

**Turning off the main sequence** Once the Sun runs out of hydrogen in the core it starts to turn off the main sequence. There is still enough hydrogen left to burn in a shell around the helium core, but the nuclear reactions cannot sustain the hydrostatic equilibrium anymore and thus the Sun starts to contract. The Sun at this point has not developed a fully convective envelope yet and it is called a subgiant branch (SGB) star.

#### 4. The Life and Death of the Sun



**Figure 4.6.:** Schematic of the evolution of the Sun in the HRD. Figure adopted after Szczurek via Wikipedia Commons.

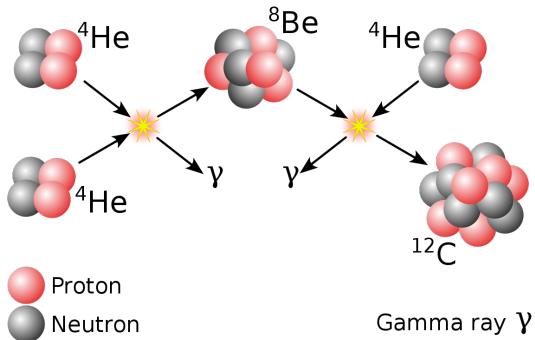
**The red giant branch** Eventually, the envelope becomes fully convective, which makes more fuel available to the hydrogen burning shell. This results in a dramatic expansion of the Sun out to around the orbit of Mercury. The Sun thus climbs the red giant branch (RGB). While the hydrogen burns in the shell, the core of the Sun continuously contracts. Its density and temperature therefore increases. The core density will become so high that matter becomes electron degenerate. The convectiveness of the envelope during the RGB phase also deepens and ultimately dredges up the products of hydrogen burning from the outer core (first dredge up).



**Electron degenerate matter** Most of the electron energy levels in ordinary, fermionic matter are unfilled. The electrons are thus free to move among these states. With increased density, the lower energy levels become more and more populated until all electrons are in the lowest possible states. Due to the Pauli exclusion principle, no two fermions can be in the same quantum state, thus, pressure and temperature become de-coupled. An increase in temperature at this point cannot anymore result in an increase of pressure. See Wikipedia for more details.

**The helium flash** When the temperature in the Sun's core reaches  $T_6 \approx 100$ , helium can start fusing via the triple- $\alpha$  process to carbon and on to oxygen, see Figure 4.7.

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**Figure 4.7.:** Schematic of the triple- $\alpha$  process. Credit: Borb via English Wikipedia.

In the triple- $\alpha$  process, also known as the Salpeter process, helium fusion to  ${}^{12}\text{C}$  takes place in the following way:



The nuclear energy generation rate in  $\text{W kg}^{-1}$  of the triple- $\alpha$  process, as described in Carroll & Ostlie (2017) (equation (10.62) on page 312) is given as

$$\epsilon_{3\alpha} = 50.9\rho^2 Y^3 T_8^{-3} f_{3\alpha} \exp(-44.027T_8^{-1}). \quad (4.24)$$

Here,  $f_{3\alpha}$  is the screening factor for the triple- $\alpha$  process. Once enough carbon is produced, it becomes possible to capture another  ${}^4\text{He}$  on  ${}^{12}\text{C}$ , thus producing  ${}^{16}\text{O}$  in the reaction



The energy release from fusing three  ${}^4\text{He}$  to  ${}^{12}\text{C}$  is 7.367 MeV. While regularly, this additional energy would result in an expansion of the star, which would thus cool down and go into hydrostatic equilibrium again, the pressure does not rise in a degenerate gas. Thus, the temperature keeps rising, resulting in a vast acceleration of the triple- $\alpha$  process. This positive feedback loop results in a thermonuclear runaway, which ultimately ends in the so-called helium flash. The energy released during this helium flash does however not result in a stellar explosion but is rather used up in order to lift the degeneracy of the matter in the core, which subsequently yields a rapid expansion of the core. This expansion leads to a cooling in the hydrogen burning shell around the core, which has up to now been the sole supporter of the luminosity of the star. Thus, the Sun's luminosity drops significantly and it enters the horizontal branch on which it quietly burns helium in the core and hydrogen in a shell surrounding the core via the CNO cycle. This burning stage lasts for about 0.1 Ga.

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**The asymptotic giant branch** Once the helium in the core is exhausted, the Sun will have CO core in the center. The Sun will contract again until helium burning in a shell around the core starts. At the same time, hydrogen will be burning further out. The two areas are separated by an intershell region consisting mainly of helium. During this phase the Sun is on the early asymptotic giant branch (E-ABG). The AGB is named in this way since it asymptotically almost merges with the RGB. The expanding and cooling star leads the convective envelope to again penetrate deeper, thus initiating the second dredge up.

**The thermally pulsing asymptotic giant branch** During the Sun's ascent of the AGB, helium burning becomes thermally unstable and the hydrogen and helium shells start burning alternatively. About 90% of the time the hydrogen shell is burning, creating helium which subsequently increases the mass of helium and thus the temperature and pressure at the bottom of the intershell. At some point the temperature is high enough again to suddenly ignite helium burning at the bottom of the intershell, resulting in a thermal pulse. The Sun has now reached the TP-AGB phase. The radius of the Sun varies by a factor of around four during this time and at its maximum radius can reach the Earth's orbit. Several of these thermal pulses will repeat at time intervals of around  $10^5$  a. Strong stellar winds during this period result in the Sun loosing a significant amount of mass. After about 20 Ma in total, the Sun leaves the AGB branch.

**Planetary nebula and white dwarf formation** As strong stellar winds continue, the Sun will lose enough mass to start loosing its hydrogen envelope. This exposes deeper and hotter layers, thus moving the star to the left in the HRD. Once this surface temperature is high enough it will ionize the surrounding ejecta material via UV radiation, making the material fluoresce brightly. Figure 4.8 shows a Hubble image of the Catseye Nebula as an example of a planetary nebula (PN).

With ultimately all the hydrogen gone, the Sun itself will further contract until an electron degenerate CO WD is left behind. At this point it weighs about half of its original mass. The gravitational force will be opposed by the degeneracy pressure. The WD will continue to radiate heat away into space and thus slowly cool.

### 4.4. The Death of More Massive Stars

Intermediate mass stars ( $2 M_{\odot} < M \lesssim 8 M_{\odot}$ ) will undergo a similar evolution to the one discussed for the Sun. During the TP-AGB phase though, these stars will undergo multiple, so-called third dredge up (TDU) events. These events mix material from the intershell with the envelope. As we will see later, the *s*-process takes place in the helium intershell. Thus, these TDU events mix freshly nucleosynthesized material from the helium intershell into the convective envelope. Since the envelope gets ultimately

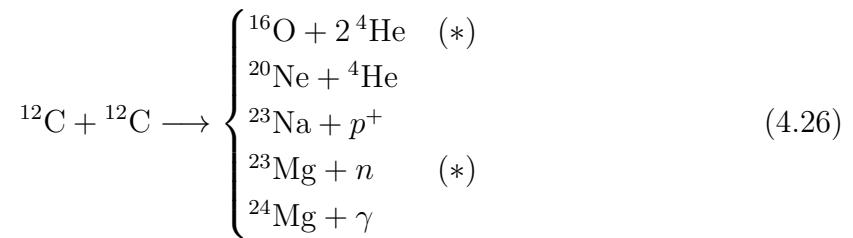
#### 4. The Life and Death of the Sun



**Figure 4.8.:** The Catseye Nebula – a planetary nebula – photographed with the Hubble space telescope. Credit: NASA, ESA, HEIC, and The Hubble Heritage Team (STScI / AURA).

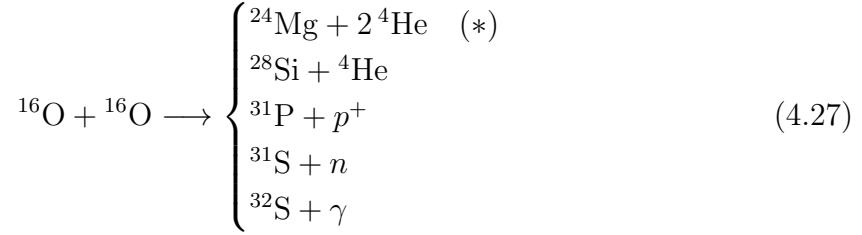
ejected into space by stellar winds, *s*-process material is effectively recycled into the Milky Way.

Stars more massive than  $5 M_{\odot} \lesssim M \lesssim 8 M_{\odot}$  furthermore can successfully burn the carbon and oxygen further. The reactions that take place for carbon burning are the following:



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For oxygen burning, the reactions are:



Note that reactions labeled with (\*) in fact consume energy and do not produce any. This behavior is however enabled by the high temperatures present during CO burning. Instead of a CO WD, stars in this mass range leave behind a ONe WD

Stars more massive than  $8 M_\odot$  undergo further burning reactions and end up in their final state as either neutron stars or black holes. These massive stars – at least some – are also expected to explode as SNe. We will discuss their fate further in the next chapter.

## 4.5. Stellar Lifetimes

While the quiescent burning of the Sun takes place over around 10 Ga, subsequent burning phases are exponentially faster. The total life of the Sun is thus defined by the quiescent burning phase. This can be generalized to other stars. If the mass of a star increases, the central pressure and temperature will necessarily go up as can be seen in equations (3.19) and (3.21), respectively. Thus, the nuclear fuel will burn faster resulting in a higher luminosity of the star. From observations of main sequence stars, an empirical mass-luminosity relation can be determined approximately as

$$L = L_\odot \left( \frac{M}{M_\odot} \right)^{3.5}. \quad (4.28)$$

The lifetime of a star is proportional to its luminosity divided by its mass since the mass is proportional to the amount of fuel. We can therefore roughly estimate the lifetime dependency in comparison to the solar lifetime with respect to a star's mass. The dependency yields

$$t_{\text{nuc}}(M) \approx t_{\odot,\text{total}} \left( \frac{M_\odot}{M} \right)^{2.5}. \quad (4.29)$$

This equation should solely be used for back-of-the-envelope-type calculations, however, it helps to understand the fact that more massive stars die faster. Detailed determinations of the stellar lifetimes require adequate models that include the energy generation equations, e.g., equations (4.10) and (4.21), as well as an accurate thermodynamic model of the inside of a star.

## 4.6. Reading

In this chapter we have in detail discussed the inner workings of the Sun. The reading chosen to supplement our theoretical understanding of the Sun discusses the experimental work that went into actually observing the quiescent burning using neutrino detectors.

Please read Davis (1955) and Bahcall & Davis (1976). We will briefly discuss Davis (1955) and these pioneering measurements and then focus on Bahcall & Davis (1976). The following points might help to guide the reading:

- What is the general understanding of neutrino research in 1955? How does this influence the hypothesis of Davis (1955)?
- Discuss the experimental setup of Davis (1955). It might help to make a drawing of the setup to the best of your understanding. Split the setup up into two parts, (1) how to get  $^{37}\text{Ar}$  out of the tank and (2) how it is subsequently detected.
- What are the (stunning) conclusions of Davis (1955)?
- How has the knowledge of neutrinos changed in between Davis (1955) and Bahcall & Davis (1976)?
- What neutrinos can be detected with the experiment of Bahcall & Davis (1976)? Which reactions do these neutrinos originate from? Put your findings into context of Figure 4.3.
- What is different in the experimental setup discussed in Bahcall & Davis (1976) compared to Davis (1955)?
- Discuss why the experiment described in Bahcall & Davis (1976) was performed underground? Compare the Homestake mine to other underground laboratories. Was the shielding of the mountain enough to achieve the required sensitivity?
- How did the experimentalists ensure that their result is correct?
- What solar processes can be excluded from these measurements? Where lies the solar neutrino problem?
- What alternative explanations are given by Bahcall & Davis (1976) that might solve the solar neutrino problem?
- Who won in the end, the astronomers or the physicists? What implication did this finding have – for the world and for Ray Davis?

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# Appendices

# A. Deriving the Virial Theorem

We can use the hydrostatic equilibrium equation (3.16) to derive the virial theorem that has been used extensively in Chapter 3. To start we can multiply the hydrostatic equilibrium equation (3.16) on both sites by  $4\pi r^3 dr$  and integrate over  $r$ . Writing this out gives us

$$\int_0^R 4\pi r^3 \frac{dP}{dr} dr = - \int_0^R \rho G \frac{M(r)}{r^2} 4\pi r^3 dr. \quad (\text{A.1})$$

Let us first discuss the right-hand side of the equation. Minor algebra of this side allows us to rewrite it since we know that  $4\pi r^2 \rho dr$  is equal to the mass of a mass shell in the star with thickness  $dr$  at distance  $r$  from the center. We can thus write this whole part as  $dM$ . We then have to integrate over the mass instead of the radius. The right-hand side now looks like

$$- \int_0^R \left( \frac{GM(r)}{r} \right) 4\pi r^2 \rho dr = - \int_0^M \frac{GM(r)}{r} dM. \quad (\text{A.2})$$

This part is of course equal to the gravitational or potential energy  $E_{\text{pot}}$ , see equation (3.1).

To evaluate the left-hand side of equation (A.1) we can use integration by parts for the integration over  $dr$ . Integration by parts states that

$$\int_a^b u(x)v'(x)dx = u(x)v(x)|_a^b - \int_a^b u'(x)v(x)dx. \quad (\text{A.3})$$

We can set  $u = 4\pi r^3$  and  $v = P$  and write

$$\begin{aligned} \int_0^R 4\pi r^3 \frac{dP}{dr} dr &= 4\pi P r^3|_0^R - \int_0^R 12\pi P r^2 dr \\ &= -3 \int_0^R 4\pi P r^2 dr. \end{aligned} \quad (\text{A.4})$$

The first term  $4\pi P r^3|_0^R$  is zero because at the inner limit  $r = 0$  we know that  $r(0) = 0$  and at the upper limit  $r = R$  we can assume that  $P(R) = 0$ . Using the ideal gas equation

$$PV = Nk_B T \quad (\text{A.5})$$

### A. Deriving the Virial Theorem

and the kinetic energy of an ideal gas, which is given as

$$E_{\text{kin}} = \frac{3}{2} N k_B T, \quad (\text{A.6})$$

we can write the pressure  $P$  as

$$P = \frac{2}{3} \frac{E_{\text{kin}}}{V}. \quad (\text{A.7})$$

Plugging this expression for the pressure into equation (A.4), we can write for the left-hand side of equation A.1

$$E_{\text{kin}} \frac{1}{V} \int_0^R 4\pi r^2 dr. \quad (\text{A.8})$$

For a sphere,  $\int_0^R 4\pi r^2 dr$  is of course nothing else than its volume,  $V$  thus crosses out, and we have determined that the left-hand side of equation (A.1) is equal to  $-2E_{\text{kin}}$ . Thus, we have derived the virial theorem, which states that

$$-2E_{\text{kin}} = E_{\text{pot}}. \quad (\text{A.9})$$