

BLG435E

Artificial Intelligence

İTÜ



Practice Session 3: Logic



Logical Representation

- Write down logical representations for the following sentences.
 - Some students took both Communications and AI courses.
 - There is a student who does not take any art courses.
 - No person likes an expensive and low quality appliance.
 - Only one student registered in AI took Project course.
 - All students take the same exam.

Logical Representation



- Some students took both Communications and AI courses.



Logical Representation

- Some students took both Communications and AI courses.

$$\exists x \text{ Student}(x) \wedge \text{Take}(x, \text{Communications}) \wedge \text{Take}(x, \text{AI})$$

- There is a student who does not take any art courses.

Logical Representation

- There is a student who does not take any art courses.

$$\exists x \forall y \text{ Student}(x) \wedge \text{ArtCourse}(y) \Rightarrow \neg \text{Take}(x, y)$$

- No person likes an expensive and low quality appliance.

Logical Representation

- No person likes an expensive and low quality appliance.

$$\forall x, y \text{ Person}(x) \wedge \text{Appliance}(y) \wedge \text{Expensive}(y) \wedge \text{LowQuality}(y) \Rightarrow \neg \text{Like}(x, y)$$

Logical Representation



- Only one student registered in AI took Project course.



Logical Representation

- Only one student registered in AI took Project course.

$$\exists x \forall y \text{ Student}(x) \wedge \text{Take}(x, \text{AI}) \wedge \text{Take}(x, \text{Project}) \\ \wedge \text{Student}(y) \wedge \text{Take}(y, \text{AI}) \wedge \text{Take}(y, \text{Project}) \Rightarrow x = y$$

Logical Representation

- All students take the same exam.



- All students take the same exam.

$$\forall x, y, z \text{ Student}(x) \wedge \text{Student}(y) \wedge \text{Exam}(z) \wedge \text{Take}(x, z) \\ \Rightarrow \text{Take}(y, z)$$

- Write down logical representations for the following sentences, suitable for use with Generalized Modus Ponens.
 - Horses, cows, and pigs are mammals.
 - An offspring of a horse is a horse.
 - Bluebeard is a horse.
 - Bluebeard is Charlie's parent.
 - Offspring and parent are inverse relations.
 - Every mammal has a parent.

- Horses, cows, and pigs are mammals.

- Horses, cows, and pigs are mammals.

$$\forall x \text{ Horse}(x) \Rightarrow \text{Mammal}(x)$$

$$\forall x \text{ Cow}(x) \Rightarrow \text{Mammal}(x)$$

$$\forall x \text{ Pig}(x) \Rightarrow \text{Mammal}(x)$$

- An offspring of a horse is a horse.

- An offspring of a horse is a horse.

$$\forall x, y \text{ Offspring}(x, y) \wedge \text{Horse}(y) \Rightarrow \text{Horse}(x)$$

- Bluebeard is a horse.

- Bluebeard is a horse.

Horse(Bluebeard)

- Bluebeard is Charlie's parent.

- Bluebeard is Charlie's parent.

Parent(Bluebeard, Charlie)

- Offspring and parent are inverse relations.

- Offspring and parent are inverse relations.

$$\forall x, y \text{ Offspring}(x, y) \Rightarrow \text{Parent}(y, x)$$

$$\forall x, y \text{ Parent}(x, y) \Rightarrow \text{Offspring}(y, x)$$

- Every mammal has a parent.

- Every mammal has a parent.

$$\forall x \exists y \text{ Mammal}(x) \Rightarrow \text{Parent}(y, x)$$

- Use resolution to prove the query: *Horse(Charlie)* by using the KB built in the first part of the question.
- First, we need to convert the KB into CNF.
- Then, we will apply resolution by showing $KB \wedge \neg\alpha$ is unsatisfiable.

$$\forall x \text{ Horse}(x) \Rightarrow \text{Mammal}(x)$$

$$\forall x \text{ Cow}(x) \Rightarrow \text{Mammal}(x)$$

$$\forall x \text{ Pig}(x) \Rightarrow \text{Mammal}(x)$$

$$\forall x \text{ Horse}(x) \Rightarrow \text{Mammal}(x)$$

$$\forall x \text{ Cow}(x) \Rightarrow \text{Mammal}(x)$$

$$\forall x \text{ Pig}(x) \Rightarrow \text{Mammal}(x)$$

- **In CNF:**

$$\neg \text{Horse}(x) \vee \text{Mammal}(x)$$

$$\neg \text{Cow}(x) \vee \text{Mammal}(x)$$

$$\neg \text{Pig}(x) \vee \text{Mammal}(x)$$

$$\forall x, y \text{ Offspring}(x, y) \wedge \text{Horse}(y) \Rightarrow \text{Horse}(x)$$

$$\forall x, y \text{ Offspring}(x, y) \wedge \text{Horse}(y) \Rightarrow \text{Horse}(x)$$

- **In CNF:**

$$\neg(\text{Offspring}(x, y) \wedge \text{Horse}(y)) \vee \text{Horse}(x)$$

$$\neg\text{Offspring}(x, y) \vee \neg\text{Horse}(y) \vee \text{Horse}(x)$$

Horse(Bluebeard)
Parent(Bluebeard, Charlie)

- **Already in CNF:**

Horse(Bluebeard)

Parent(Bluebeard, Charlie)



$\forall x, y \text{ Offspring}(x, y) \Rightarrow \text{Parent}(y, x)$

$\forall x, y \text{ Parent}(x, y) \Rightarrow \text{Offspring}(y, x)$

$$\forall x, y \text{ Offspring}(x, y) \Rightarrow \text{Parent}(y, x)$$

$$\forall x, y \text{ Parent}(x, y) \Rightarrow \text{Offspring}(y, x)$$

- **In CNF:**

$$\neg \text{Offspring}(x, y) \vee \text{Parent}(y, x)$$

$$\neg \text{Parent}(x, y) \vee \text{Offspring}(y, x)$$

$$\forall x \exists y \text{ Mammal}(x) \Rightarrow \text{Parent}(y, x)$$

$$\forall x \exists y \text{ Mammal}(x) \Rightarrow \text{Parent}(y, x)$$

- **In CNF:**

$\neg \text{Mammal}(x) \vee \text{Parent}(G(x), x)$ where $G(x)$ is a skolem constant

- **KB in CNF:**

- 1) $\neg \text{Horse}(x) \vee \text{Mammal}(x)$
- 2) $\neg \text{Cow}(x) \vee \text{Mammal}(x)$
- 3) $\neg \text{Pig}(x) \vee \text{Mammal}$
- 4) $\neg \text{Offspring}(x, y) \vee \neg \text{Horse}(y) \vee \text{Horse}(x)$
- 5) $\text{Horse}(\text{Bluebeard})$
- 6) $\text{Parent}(\text{Bluebeard}, \text{Charlie})$
- 7) $\neg \text{Offspring}(x, y) \vee \text{Parent}(y, x)$
- 8) $\neg \text{Parent}(x, y) \vee \text{Offspring}(y, x)$
- 9) $\neg \text{Mammal}(x) \vee \text{Parent}(G(x), x)$

Resolution

- 1) $\neg \text{Horse}(x) \vee \text{Mammal}(x)$
- 2) $\neg \text{Cow}(x) \vee \text{Mammal}(x)$
- 3) $\neg \text{Pig}(x) \vee \text{Mammal}(x)$
- 4) $\neg \text{Offspring}(x, y) \vee \neg \text{Horse}(y) \vee \text{Horse}(x)$
- 5) $\text{Horse}(\text{Bluebeard})$
- 6) $\text{Parent}(\text{Bluebeard}, \text{Charlie})$
- 7) $\neg \text{Offspring}(x, y) \vee \text{Parent}(y, x)$
- 8) $\neg \text{Parent}(x, y) \vee \text{Offspring}(y, x)$
- 9) $\neg \text{Mammal}(x) \vee \text{Parent}(G(x), x)$
- 10) $\neg \alpha: \neg \text{Horse}(\text{Charlie})$

Resolution

- 1) $\neg \text{Horse}(x) \vee \text{Mammal}(x)$
- 2) $\neg \text{Cow}(x) \vee \text{Mammal}(x)$
- 3) $\neg \text{Pig}(x) \vee \text{Mammal}(x)$
- 4) $\neg \text{Offspring}(x, y) \vee \neg \text{Horse}(y) \vee \text{Horse}(x)$
- 5) $\text{Horse}(\text{Bluebeard})$
- 6) $\text{Parent}(\text{Bluebeard}, \text{Charlie})$
- 7) $\neg \text{Offspring}(x, y) \vee \text{Parent}(y, x)$
- 8) $\neg \text{Parent}(x, y) \vee \text{Offspring}(y, x)$
- 9) $\neg \text{Mammal}(x) \vee \text{Parent}(G(x), x)$
- 10) $\neg \alpha: \neg \text{Horse}(\text{Charlie})$
- 11) **$\text{Offspring}(\text{Charlie}, \text{Bluebird})$ by using 6 and 8 (x:Bluebeard, y:Charlie)**

Resolution

- 1) $\neg \text{Horse}(x) \vee \text{Mammal}(x)$
- 2) $\neg \text{Cow}(x) \vee \text{Mammal}(x)$
- 3) $\neg \text{Pig}(x) \vee \text{Mammal}(x)$
- 4) $\neg \text{Offspring}(x, y) \vee \neg \text{Horse}(y) \vee \text{Horse}(x)$
- 5) $\text{Horse}(\text{Bluebeard})$
- 6) $\text{Parent}(\text{Bluebeard}, \text{Charlie})$
- 7) $\neg \text{Offspring}(x, y) \vee \text{Parent}(y, x)$
- 8) $\neg \text{Parent}(x, y) \vee \text{Offspring}(y, x)$
- 9) $\neg \text{Mammal}(x) \vee \text{Parent}(G(x), x)$
- 10) $\neg \alpha: \neg \text{Horse}(\text{Charlie})$
- 11) **$\text{Offspring}(\text{Charlie}, \text{Bluebird})$ by using 6 and 8 (x:Bluebeard, y:Charlie)**
- 12) **$\neg \text{Horse}(\text{Bluebeard}) \vee \text{Horse}(\text{Charlie})$ by using 4 and 11 (x:Charlie, y:Bluebeard)**

Resolution

- 1) $\neg \text{Horse}(x) \vee \text{Mammal}(x)$
- 2) $\neg \text{Cow}(x) \vee \text{Mammal}(x)$
- 3) $\neg \text{Pig}(x) \vee \text{Mammal}$
- 4) $\neg \text{Offspring}(x, y) \vee \neg \text{Horse}(y) \vee \text{Horse}(x)$
- 5) $\text{Horse}(\text{Bluebeard})$
- 6) $\text{Parent}(\text{Bluebeard}, \text{Charlie})$
- 7) $\neg \text{Offspring}(x, y) \vee \text{Parent}(y, x)$
- 8) $\neg \text{Parent}(x, y) \vee \text{Offspring}(y, x)$
- 9) $\neg \text{Mammal}(x) \vee \text{Parent}(G(x), x)$
- 10) $\neg \alpha: \neg \text{Horse}(\text{Charlie})$
- 11) **$\text{Offspring}(\text{Charlie}, \text{Bluebird})$ by using 6 and 8 (x:Bluebeard, y:Charlie)**
- 12) **$\neg \text{Horse}(\text{Bluebeard}) \vee \text{Horse}(\text{Charlie})$ by using 4 and 11 (x:Charlie, y:Bluebeard)**
- 13) **$\text{Horse}(\text{Charlie})$ by using 5 and 12**

Resolution

- 1) $\neg \text{Horse}(x) \vee \text{Mammal}(x)$
- 2) $\neg \text{Cow}(x) \vee \text{Mammal}(x)$
- 3) $\neg \text{Pig}(x) \vee \text{Mammal}$
- 4) $\neg \text{Offspring}(x, y) \vee \neg \text{Horse}(y) \vee \text{Horse}(x)$
- 5) $\text{Horse}(\text{Bluebeard})$
- 6) $\text{Parent}(\text{Bluebeard}, \text{Charlie})$
- 7) $\neg \text{Offspring}(x, y) \vee \text{Parent}(y, x)$
- 8) $\neg \text{Parent}(x, y) \vee \text{Offspring}(y, x)$
- 9) $\neg \text{Mammal}(x) \vee \text{Parent}(G(x), x)$
- 10) $\neg \alpha: \neg \text{Horse}(\text{Charlie})$
- 11) **$\text{Offspring}(\text{Charlie}, \text{Bluebird})$ by using 6 and 8 (x:Bluebeard, y:Charlie)**
- 12) **$\neg \text{Horse}(\text{Bluebeard}) \vee \text{Horse}(\text{Charlie})$ by using 4 and 11 (x:Charlie, y:Bluebeard)**
- 13) **$\text{Horse}(\text{Charlie})$ by using 5 and 12**
- 14) **$\{\}$ empty clause by using 10 and 13 $\rightarrow KB \wedge \neg \alpha$ is unsatisfiable**