

Ampliación de Matemáticas 2: Formulario*

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1. Cambio de Variable

Jacobiano

$$J(T) = \text{abs} \left(\det \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{pmatrix} \right)$$

Coordenadas Esféricas

$$T : \begin{cases} x = r \cdot \text{sen}(\alpha) \cdot \cos(\beta) \\ y = r \cdot \text{sen}(\alpha) \cdot \text{sen}(\beta) \\ z = r \cdot \cos(\alpha) \end{cases}$$
$$J(T) = r^2 \cdot \text{sen}(\alpha)$$

$$r > 0 \quad \alpha \in [0, \pi] \quad \beta \in [0, 2\pi]$$

Coordenadas Cilíndricas

$$T : \begin{cases} x = r \cdot \cos(\alpha) \\ y = r \cdot \sin(\alpha) \\ z = z \end{cases}$$
$$J(T) = r$$

$$r \geq 0 \quad \alpha \in [0, 2\pi] \quad z \in \mathbb{R}$$

2. Integral Impropia

Criterios de Comparación

B := Interior de bola de radio 1 centrada en 0

B^* := Exterior de bola de radio 1 centrada en 0

$$r = \sqrt{x_1^2 + \dots + x_n^2}$$

$$\int_B r^{-\alpha} dx_1 \dots dx_n \begin{cases} \alpha < n & \text{convergente} \\ \alpha \geq n & \text{divergente} \end{cases}$$

$$\int_{B^*} r^{-\alpha} dx_1 \dots dx_n \begin{cases} \alpha > n & \text{convergente} \\ \alpha \leq n & \text{divergente} \end{cases}$$

3. Ecuación en Diferencias

Solución Particular del problema homogéneo

$$\{k^n r^k\} = \{k^n \rho^k \cos(\theta k)\}$$
$$\{k^n \tilde{r}^k\} = \{k^n \rho^k \sin(\theta k)\}$$

Solución Particular del problema no homogéneo

$$b_k = \rho^k (P_p(k) \cos(\theta k) + Q_q(k) \sin(\theta k))$$
$$z_k = k^s \rho^k (P_m^*(k) \cos(\theta k) + Q_m^*(k) \sin(\theta k))$$

b_k	z_k
C	A
k^t	$A_t k^t + \dots + A_1 k^1 + A_0$
C^k	AC^k
$k^t C^k$	$C^k (A_t k^t + \dots + A_1 k^1 + A_0)$
$\sin(\theta k)$	$A \cos(\theta k) + B \sin(\theta k)$
$\cos(\theta k)$	$A \cos(\theta k) + B \sin(\theta k)$

4. Métodos Numéricos

Integración Numérica

$$I(f) = \sum_{k=0}^n \left(\int_a^b l_k(x) dx \right) f(x_k)$$
$$+ \int_a^b \frac{1}{(n+1)!} f^{(n+1)}(\xi) \omega(x)$$

$$I_{pm}^c(f) = 2h \sum_{k=0}^{n/2} f(x_{2k})$$
$$+ \frac{1}{6} (b-a) h^2 f^{(2)}(\xi)$$

$$I_{tr}^c(f) = \frac{1}{2} h (f(x_0) + 2 \sum_{k=1}^{2n-1} f(x_k) + f(x_{2n}))$$
$$+ \frac{1}{12} (b-a) h^2 f^{(2)}(\xi)$$

$$I_{si}^c(f) = \frac{1}{3} (f(x_0) + 4 \sum_{k=1}^n f(x_{2k-1}) + 2 \sum_{k=1}^{2n-1} f(x_{2k}))$$
$$+ \frac{1}{180} (b-a) h^4 f^{(4)}(\xi)$$

A. Trigonometría

Valores de Referencia

$$\sin(0) = 0 \quad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \quad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} \quad \sin\left(\frac{\pi}{2}\right) = 1$$

$$\cos(0) = 1 \quad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \quad \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \quad \cos\left(\frac{\pi}{2}\right) = 0$$

Igualdades Trigonómicas

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$$
$$\cos(a+b) = \cos(a)\cos(b) - \sin(b)\sin(a)$$

*<https://github.com/garciparedes/am2-cheatsheet>