

$$> A := \begin{bmatrix} 0.5 & 0.4 & 0.2 \\ 0.1 & 0.4 & 0.3 \\ 0.4 & 0.2 & 0.5 \end{bmatrix}$$

$$A := \begin{bmatrix} 0.500 & 0.400 & 0.200 \\ 0.100 & 0.400 & 0.300 \\ 0.400 & 0.200 & 0.500 \end{bmatrix} \quad (1)$$

> with(LinearAlgebra)
 [&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]

$$> \text{CharacteristicPolynomial}(A, a) \\ -0.070 + 0.470 a - 1.400 a^2 + a^3 \quad (3)$$

$$> \lambda, x := \text{Eigenvectors}(A) \\ \lambda, x := \begin{bmatrix} 1.000 + 0.000 I \\ 0.200 + 0.173 I \\ 0.200 - 0.173 I \end{bmatrix}, \begin{bmatrix} 0.611 + 0.000 I & 0.577 + 0.000 I & 0.577 - 0.000 I \\ 0.433 + 0.000 I & -0.289 + 0.500 I & -0.289 - 0.500 I \\ 0.662 + 0.000 I & -0.289 - 0.500 I & -0.289 + 0.500 I \end{bmatrix} \quad (4)$$

$$> \lambda R := \max(\text{Re}(\lambda)) \\ \lambda R := 1.000 \quad (5)$$

-----Matrix A is not productive!-----

> $xR := \text{abs}(\text{Column}(x, 1))$

$$xR := \begin{bmatrix} 0.611 \\ 0.433 \\ 0.662 \end{bmatrix} \quad (6)$$

> $At := \text{Transpose}(A)$

$$At := \begin{bmatrix} 0.500 & 0.100 & 0.400 \\ 0.400 & 0.400 & 0.200 \\ 0.200 & 0.300 & 0.500 \end{bmatrix} \quad (7)$$

> $\lambda t, xt := \text{Eigenvectors}(At)$

$$\lambda t, xt := \begin{bmatrix} 1.000 + 0.000 I \\ 0.200 + 0.173 I \\ 0.200 - 0.173 I \end{bmatrix}, \begin{bmatrix} 0.577 + 0.000 I & -0.206 + 0.516 I & -0.206 - 0.516 I \\ 0.577 + 0.000 I & 0.642 + 0.000 I & 0.642 - 0.000 I \\ 0.577 + 0.000 I & -0.229 - 0.476 I & -0.229 + 0.476 I \end{bmatrix} \quad (8)$$

> $xL := \text{abs}(\text{Column}(xt, 1))$

$$xL := \begin{bmatrix} 0.577 \\ 0.577 \\ 0.577 \end{bmatrix} \quad (9)$$

$$> E3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

> $B := \text{MatrixInverse}(E3 - A)$

Error, (in LinearAlgebra:-LA Main:-MatrixInverse) singular matrix

-----Matrix A is not productive, so B does not exist-----

> $Balt := \text{add}(\text{MatrixPower}(A, k), k = 1..10)$

$$Balt := \begin{bmatrix} 3.727 & 3.682 & 3.383 \\ 2.334 & 2.678 & 2.633 \\ 3.939 & 3.640 & 3.984 \end{bmatrix} \quad (11)$$

> $Balt := \text{add}(\text{MatrixPower}(A, k), k = 1..20)$

$$Balt := \begin{bmatrix} 7.309 & 7.264 & 6.965 \\ 4.872 & 5.215 & 5.170 \\ 7.820 & 7.521 & 7.864 \end{bmatrix} \quad (12)$$

Diagram illustrating the relationship between the price vector and the growth of B :

- Top line: $(B \text{ grows to infinity})$
- Middle line: Price vector does not exist
- Bottom line: (unlabeled)