Q Metric Variation – Minimization & Jacobian

y = x_i

k_i = sqrt(2)*erf_inverse(2*a_i – 1)

a_i = i/n+1

F = (1/2)(1 + erf[(log[y] - u)/(sqrt[2] s)])/exp[u + s k]

D[F,u]
$$\frac{\partial \frac{\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)+1}{2\exp(k\,s+u)}}{\partial u} = -\frac{1}{2} e^{-k\,s-u} \left(\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)+1\right) - \frac{e^{-k\,s-\frac{(\log(y)-u)^2}{2 s^2}-u}}{\sqrt{2\pi}\, s}$$

D[F,s]
$$\frac{\partial \frac{\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)+1}{2\exp(k\,s+u)}}{\partial s} = -\frac{1}{2} k\, e^{-k\,s-u} \left(\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)+1\right) - \frac{(\log(y)-u)\, e^{-k\,s-\frac{(\log(y)-u)^2}{2 s^2}-u}}{\sqrt{2\pi}\, s^2}$$

D[D[F,s],u] = D[D[F,u],s]

$$\frac{\partial \dfrac{\partial \dfrac{\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)+1}{2\exp(k\, s+u)}}{\partial s}}{\partial u} = \frac{1}{2}\, k\, e^{-k\, s-u} \left(\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)+1\right) + \frac{k\, e^{-k\, s-\frac{(\log(y)-u)^2}{2\, s^2}-u}}{\sqrt{2\,\pi}\, s} - \frac{(\log(y)-u)\left(\frac{\log(y)-u}{s^2}-1\right) e^{-k\, s-\frac{(\log(y)-u)^2}{2\, s^2}-u}}{\sqrt{2\,\pi}\, s^2} + \frac{e^{-k\, s-\frac{(\log(y)-u)^2}{2\, s^2}-u}}{\sqrt{2\,\pi}\, s^2}$$

D[D[F,u],u]

$$\frac{\partial}{\partial u}\left(\frac{\partial}{\partial u}\, \frac{1+\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)}{2\exp(u+s\, k)}\right) =$$

$$\frac{1}{2}\, e^{-k\, s-u} \left(\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)+1\right) - \frac{\left(\frac{\log(y)-u}{s^2}-1\right) e^{-k\, s-\frac{(\log(y)-u)^2}{2\, s^2}-u}}{\sqrt{2\,\pi}\, s} + \frac{e^{-k\, s-\frac{(\log(y)-u)^2}{2\, s^2}-u}}{\sqrt{2\,\pi}\, s}$$

D[D[F,s],s]

$$\frac{\partial}{\partial s}\left(\frac{\partial}{\partial s}\, \frac{1+\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)}{2\exp(u+s\, k)}\right) = \frac{1}{2}\, k^2\, e^{-k\, s-u} \left(\operatorname{erf}\left(\frac{\log(y)-u}{\sqrt{2}\, s}\right)+1\right) + \frac{k\,(\log(y)-u)\, e^{-k\, s-\frac{(\log(y)-u)^2}{2\, s^2}-u}}{\sqrt{2\,\pi}\, s^2} + \frac{\sqrt{\frac{2}{\pi}}\,(\log(y)-u)\, e^{-k\, s-\frac{(\log(y)-u)^2}{2\, s^2}-u}}{s^3} - \frac{(\log(y)-u)\, e^{-k\, s-\frac{(\log(y)-u)^2}{2\, s^2}-u}\left(\frac{(\log(y)-u)^2}{s^3}-k\right)}{\sqrt{2\,\pi}\, s^2}$$