

Lognormal Equations for Anderson-Darling MDE

Conventions

Using $z = y[n-1-i]$

Using $y = y[i]$

Primary Optimization

$d\{\log(F(y)) + \log(1-F(z))\}/du$

$$\frac{\partial}{\partial u} \left(\log \left(\frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) \right) \right) + \log \left(1 - \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) \right) \right) \right) =$$

$$\frac{e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{\sqrt{2 \pi} s \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)} - \frac{\sqrt{\frac{2}{\pi}} e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{s \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)}$$

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$d\{\log(F(y)) + \log(1-F(z))\}/ds$

$$\frac{\partial}{\partial s} \left(\log \left(\frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) \right) \right) + \log \left(1 - \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) \right) \right) \right) =$$

$$\frac{(\log(z)-u) e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{\sqrt{2 \pi} s^2 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)} - \frac{\sqrt{\frac{2}{\pi}} (\log(y)-u) e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{s^2 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)}$$

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Jacobian Matrix

$$J[0,0] = d^2\{\log(F(y)) + \log(1-F(z))\}/du^2$$

$$\begin{aligned} & \frac{\partial}{\partial u} \left(\frac{\partial}{\partial u} \left(\log \left(\frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) \right) \right) + \log \left(1 - \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) \right) \right) \right) \right) = \\ & - \frac{\frac{(\log(y)-u)^2}{2 e^{\frac{(\log(y)-u)^2}{2 s^2}}}}{\pi s^2 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)^2} - \frac{\frac{(\log(z)-u)^2}{e^{\frac{(\log(z)-u)^2}{2 s^2}}}}{2 \pi s^2 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)^2} - \\ & \frac{\sqrt{\frac{2}{\pi}} (\log(y)-u) e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{s^3 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)} + \frac{(\log(z)-u) e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{\sqrt{2} \pi s^3 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)} \end{aligned}$$

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$$J[0,1] = d^2\{\log(F(y)) + \log(1-F(z))\}/ds du$$

$$\begin{aligned} & \frac{\partial}{\partial s} \left(\frac{\partial}{\partial u} \left(\log \left(\frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) \right) \right) + \log \left(1 - \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) \right) \right) \right) \right) = \\ & \frac{\sqrt{\frac{2}{\pi}} e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{s^2 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)} - \frac{\frac{(\log(z)-u)^2}{e^{\frac{(\log(z)-u)^2}{2 s^2}}}}{\sqrt{2} \pi s^2 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)} - \\ & \frac{\sqrt{\frac{2}{\pi}} (\log(y)-u)^2 e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{s^4 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)} + \frac{(\log(z)-u)^2 e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{\sqrt{2} \pi s^4 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)} - \\ & \frac{2 (\log(y)-u) e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{\pi s^3 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)^2} - \frac{(\log(z)-u) e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{2 \pi s^3 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)^2} \end{aligned}$$

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$$J[1,0] = d^2\{\log(F(y)) + \log(1-F(z))\}/du ds$$

$$\begin{aligned} & \frac{\partial}{\partial u} \left(\frac{\partial}{\partial s} \left(\log \left(\frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) \right) \right) + \log \left(1 - \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) \right) \right) \right) \right) = \\ & \frac{\sqrt{\frac{2}{\pi}} e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{s^2 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)} - \frac{e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{\sqrt{2 \pi} s^2 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)} - \\ & \frac{\sqrt{\frac{2}{\pi}} (\log(y)-u)^2 e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{s^4 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)} + \frac{(\log(z)-u)^2 e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{\sqrt{2 \pi} s^4 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)} - \\ & \frac{2 (\log(y)-u) e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{\pi s^3 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)^2} - \frac{(\log(z)-u) e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{2 \pi s^3 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)^2} \end{aligned}$$

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$$J[1,1] = d^2\{\log(F(y)) + \log(1-F(z))\}/ds^2$$

$$\begin{aligned} & \frac{\partial}{\partial s} \left(\frac{\partial}{\partial s} \left(\log \left(\frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) \right) \right) + \log \left(1 - \frac{1}{2} \left(1 + \operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) \right) \right) \right) \right) = \\ & - \frac{\sqrt{\frac{2}{\pi}} (\log(y)-u)^3 e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{s^5 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)} + \frac{(\log(z)-u)^3 e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{\sqrt{2 \pi} s^5 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)} - \\ & \frac{2 (\log(y)-u)^2 e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{\pi s^4 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)^2} - \frac{(\log(z)-u)^2 e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{2 \pi s^4 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)^2} + \\ & \frac{2 \sqrt{\frac{2}{\pi}} (\log(y)-u) e^{-\frac{(\log(y)-u)^2}{2 s^2}}}{s^3 \left(\operatorname{erf} \left(\frac{\log(y)-u}{\sqrt{2} s} \right) + 1 \right)} - \frac{\sqrt{\frac{2}{\pi}} (\log(z)-u) e^{-\frac{(\log(z)-u)^2}{2 s^2}}}{s^3 \left(\frac{1}{2} \left(-\operatorname{erf} \left(\frac{\log(z)-u}{\sqrt{2} s} \right) - 1 \right) + 1 \right)} \end{aligned}$$

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