Q Metric Variation – Minimization & Jacobian

$$y = x_i$$

 $k_i = sqrt(2)*erf_inverse(2*a_i - 1)$
 $a_i = i/n+1$
 $F = (1/2)(1 + erf[(log[y] - u)/(sqrt[2] s)])/exp[u + s k]$

$$\frac{\partial \frac{\text{erf}\left(\frac{\log\left(y\right)-u}{\sqrt{2}\text{ s}}\right)+1}{2\text{ exp}\left(k\text{ s}+u\right)}}{\partial u} = -\frac{1}{2}\text{ e}^{-k\text{ s}-u}\left(\text{erf}\left(\frac{\log\left(y\right)-u}{\sqrt{2}\text{ s}}\right)+1\right) - \frac{\text{e}^{-k\text{ s}-\frac{\left(\log\left(y\right)-u\right)^{2}}{2\text{ s}^{2}}-u}}{\sqrt{2}\pi\text{ s}}\right)$$

$$\begin{split} &\frac{\partial \frac{\text{erf}\left(\frac{log\left(y\right)-u}{\sqrt{2} \text{ s}}\right)+1}{2 \text{ exp}\left(k \text{ s+u}\right)}}{\partial \text{ s}} = \\ &-\frac{1}{2} \text{ k } \text{ e}^{-k \text{ s-u}} \left(\text{erf}\left(\frac{log\left(y\right)-u}{\sqrt{2} \text{ s}}\right)+1\right) - \frac{\left(log\left(y\right)-u\right) \text{ e}^{-k \text{ s}-\frac{\left(log\left(y\right)-u\right)^{2}}{2 \text{ s}^{2}}-u}}{\sqrt{2 \, \pi} \text{ s}^{2}} \end{split}$$

D[D[F,s],u] = D[D[F,u],s]

$$\frac{\partial \frac{\operatorname{erf}\left(\frac{\log\left(y\right)-u}{\sqrt{2}\ s}\right)+1}{2\exp\left(k\ s+u\right)}}{\partial u} = \frac{1}{2}\ k\ e^{-k\ s-u}\ \left(\operatorname{erf}\left(\frac{\log\left(y\right)-u}{\sqrt{2}\ s}\right)+1\right) + \frac{k\ e^{-k\ s-\frac{\left(\log\left(y\right)-u\right)^{2}}{2\ s^{2}}-u}}{\sqrt{2\ \pi}\ s} - \frac{1}{2}\left(\frac{\log\left(y\right)-u}{\sqrt{2}\ s}\right) + \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\ s}\right)}{\sqrt{2}\left(\frac{\log\left(y\right)-u}{\sqrt{2}\ s}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\ s}\right) + \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\ s}\right)}{\sqrt{2}\left(\frac{\log\left(y\right)-u}{\sqrt{2}\ s}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\ s}\right) + \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\ s}\right)}{\sqrt{2}\left(\log\left(y\right)-u}\right) + \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)}{\sqrt{2}\left(\log\left(y\right)-u}\right) + \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}\right)} - \frac{\log\left(y\right)-u}{\sqrt{2}\left(\log\left(y\right)-u}$$

$$\frac{\left(\log \left(y\right)-u\right) \; \left(\frac{\log \left(y\right)-u}{s^{2}}-1\right) \; e^{-k \; s-\frac{\left(\log \left(y\right)-u\right)^{\; 2}}{2 \; s^{2}}-u}}{\sqrt{2 \; \pi} \; s^{2}} \; + \; \frac{e^{-k \; s-\frac{\left(\log \left(y\right)-u\right)^{\; 2}}{2 \; s^{2}}-u}}{\sqrt{2 \; \pi} \; s^{2}}$$

D[D[F,u],u]

$$\frac{\partial}{\partial u} \left(\frac{\partial}{\partial u} \frac{1 + \operatorname{erf}\left(\frac{\log(y) - u}{\sqrt{2} s}\right)}{2 \exp(u + s k)} \right) =$$

$$\frac{1}{2} e^{-k s - u} \left(\operatorname{erf} \left(\frac{\log(y) - u}{\sqrt{2} s} \right) + 1 \right) - \frac{\left(\frac{\log(y) - u}{s^2} - 1 \right) e^{-k s - \frac{(\log(y) - u)^2}{2 s^2} - u}}{\sqrt{2 \pi} s} + \frac{e^{-k s - \frac{(\log(y) - u)^2}{2 s^2} - u}}{\sqrt{2 \pi} s}$$

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D[D[F,s],s]

$$\frac{\partial}{\partial s} \left(\frac{\partial}{\partial s} \frac{1 + \operatorname{erf}\left(\frac{\log(y) - u}{\sqrt{2} \, s}\right)}{2 \, \exp(u + s \, k)} \right) = \frac{1}{2} \, k^2 \, e^{-k \, s - u} \left(\operatorname{erf}\left(\frac{\log(y) - u}{\sqrt{2} \, s}\right) + 1 \right) + \frac{k \, (\log(y) - u) \, e^{-k \, s - \frac{(\log(y) - u)^2}{2 \, s^2} - u}}{\sqrt{2 \, \pi} \, s^2} + \frac{\sqrt{\frac{2}{\pi}} \, (\log(y) - u) \, e^{-k \, s - \frac{(\log(y) - u)^2}{2 \, s^2} - u}}{s^3} - \frac{(\log(y) - u) \, e^{-k \, s - \frac{(\log(y) - u)^2}{2 \, s^2} - u} \left(\frac{(\log(y) - u)^2}{s^3} - k\right)}{\sqrt{2 \, \pi} \, s^2} + \frac{(\log(y) - u) \, e^{-k \, s - \frac{(\log(y) - u)^2}{2 \, s^2} - u}}{s^3} \right) + \frac{1}{\sqrt{2 \, \pi} \, s^2} + \frac{1}{\sqrt{2 \,$$

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