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Street Network Representation and Nonplanarity in Urban Form Studies

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Abstract

Models of street networks underlie research in urban travel behavior, accessibility, design patterns, and morphology. These models are commonly defined as "planar," meaning that they can be represented in two dimensions without any underpasses or overpasses. However, real-world urban street networks exist in three-dimensional space and frequently feature grade separation such as bridges and tunnels. How well do planar models represent real-world street networks? What does a city's extent of nonplanarity reveal about its infrastructure and urban development? This study measures the nonplanarity of drivable and walkable street networks in the centers of 50 cities worldwide, then examines the variation of nonplanarity across a single city. It develops new measures of a network's nonplanarity as indicators of infrastructure and urbanization. While some street networks are approximately planar, many are poorly-modeled by planar simplifications that inconsistently but drastically misrepresent routing, connectivity, intersection counts/densities, and block lengths.

Keywords

street network, GIS, urban form, transportation, urban design

Introduction

In urban planning and transportation research, street networks are routinely used to calculate accessibility between origins and destinations or to compute indicators of the urban form, such as block sizes or intersection density and connectivity. Mathematical models of street networks, called *graphs*, have grown ubiquitous in the urban studies literature in recent years as they have been used to model household travel patterns, access and equity, pedestrian volume, urban design patterns, spatial morphology, and location centrality and polycentricity (Marshall and Garrick 2010; Porta et al. 2014; Marshall et al. 2014; Hajrasouliha and Yin 2015; Parthasarathi et al. 2015; Knight and Marshall 2015; Gil 2016; Zhong et al. 2017).

In the urban studies literature, street networks are often referred to as "planar" or "approximately planar," meaning that they can be well-represented by a two-dimensional model that inherently precludes overpasses or underpasses. However, this claim has not been well studied or defined empirically. In the real world, urban street networks are embedded in three-dimensional space and often feature grade separation, bridges, and tunnels. This leads to two intertwined questions. First, how well do these planar graphs model urban street networks? Second, what does the extent to which a network is nonplanar tell us about a city's infrastructure and development?

This study tests the planarity of street networks around the world. It also presents two new measures of the extent of nonplanarity that can be generalized to other types of spatial networks. These indicators help describe the nature of the urban form and transportation infrastructure. Despite common claims in the literature that urban street networks are planar graphs, this study finds that they generally are nonplanar and that planar graphs poorly model the street networks of many urban centers. Further, the magnitude of this bias varies substantially across cities and urbanization types.

This paper first introduces the basics of graph theory relevant to urban studies, focusing on discussions in the research literature about street network planarity. Next it discusses the methods used to acquire and analyze the street networks in this study. Then it present the results of this analysis before concluding with a discussion of ramifications for street network research and urban form studies.

Background and Motivation

Graph theory is the mathematical study of networks (Newman 2010). Graphs can model real-world networks such as friendships, the world wide web, or spatial networks such as urban street networks (Barthelemy 2011). A graph G consists of a set of nodes N connected to one another by a set of edges E. An edge uv in a directed graph points in one direction from some node u to some node v, but an undirected graph's edges all point mutually in both directions. In a street network, the nodes represent intersections and dead-ends, and the (directed) edges represent the street segments that connect them. How a graph's nodes and edges connect to one another defines its *topology*. For example, a node's *degree* is a topological trait that represents how many edges connect to that node.

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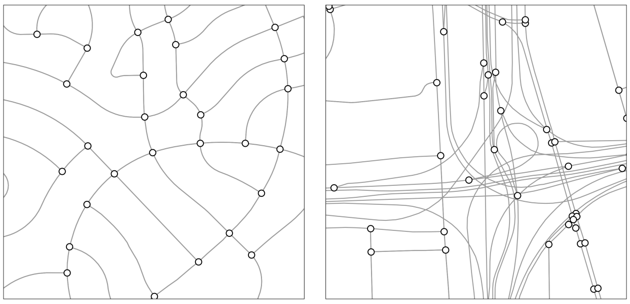

Figure 1. Formally and spatially planar (left) and nonplanar (right) road networks.

A *planar* graph can be drawn on a two-dimensional plane without any of its edges crossing each other, except where they intersect at nodes. If the graph cannot be drawn — or redrawn — to meet this criterion, then the graph is nonplanar (Trudeau 1994). Street networks are embedded in space, which provides them with geometry — such as geographical coordinates, lengths, areas, shapes, and angles — along with their topology.

This creates a minor wrinkle when we consider planarity: we must distinguish between a graph's mathematical/topological planarity, which we refer to as *formal planarity*, and the planarity of its particular spatial embedding, which we refer to as *spatial planarity*. For example, a street network might be spatially nonplanar due to its embedding in space (i.e., it contains overpasses or underpasses in the real world), but it could still be formally planar. If we "redraw" the graph by moving its nodes and edges around in space without changing how they are connected to one another (i.e., altering its geometry without altering its topology), there may exist some spatial embedding that prevents edges crossing anywhere but at nodes (Barthelemy 2017b, p. 6). In such a case, the street network is formally planar from a topological perspective, but its real-world embedding is spatially nonplanar.

Consider the examples in Figure 1. The left network comprises a set of paths and streets without any bridges or tunnels. In two-dimensions, its edges intersect each other only at nodes so it is by definition planar. The right network comprises surface streets and a grade-separated freeway interchange with many overpasses and underpasses. In two-dimensions, its edges sometimes cross each other at non-nodes (i.e., overpasses), so it is spatially nonplanar. Furthermore it is impossible to redraw the graph by moving its nodes and edges around in space such that its edges only intersect at nodes. Therefore it is also formally nonplanar. At a city- or region-wide spatial scale, these nonplanar edge crossings may be relatively uncommon: we might call such a network *approximately* planar. Approximate planarity constrains nonplanar spatial networks such that they do not exhibit certain characteristics found among nonplanar aspatial graphs, such as small-world effects or power-law distributed node degrees (Crucitti et al. 2006; O'Sullivan 2014).

In the urban studies literature, street networks are commonly referred to as planar graphs. Table 1 presents a survey of statements and reasoning around this claim. Some authors prefer to hedge slightly, arguing that street networks are *approximately* or *essentially* planar graphs that are close enough to be well-modeled as such.

If street networks can be sufficiently well-modeled by planar graphs, there are certain methodological benefits to doing so. Planar graphs offer computational simplicity and tractability. They enable simple polygonal analysis of city blocks and form (Fohl et al. 1996; Barthelemy 2017a) as well as the α and γ indices popular in road network analysis (Eppstein and Goodrich 2008). In mathematics, a bijection exists planar graphs and trees, and classifying planar graphs presents a trivial problem (Louf and Barthelemy 2014). Planar graphs are easier to visualize and can be faster to run algorithms on (Liebers 2001). Accordingly, Barthelemy (2011, p. 3) argues that "planar spatial networks are the most important and most studies have focused on these examples". But in contrast, Masucci et al. (2009) and Masucci et al. (2013) argue that planar graphs remain a compelling research domain for urban scholars because they were understudied until recently for two reasons: they appear topologically trivial and planarity does not lend itself to certain popular graph-theoretic analyses. Discussing the open research area around street networks as planar graphs, Viana et al. (2013, p. 1) state, "there is still a lack of global, high-level metrics allowing to characterize their structure and geometrical patterns."

Despite the computational advantages of simple planar models, street networks are often nonplanar in reality (Levinson 2012, p. 7): many include at least one overpass or underpass that results in the failure of formal proofs of their planarity, such as the Kuratowski (1930) theorem or the Hopcroft and Tarjan (1974) algorithm (cf. Gastner and Newman 2006). Mandloi and Thill (2010, p. 199) note that "quite often the transportation network has overpasses and underpasses that require a non-planar network representation." O'Sullivan (2014, p. 1258) explains that "for many infrastructure networks, [planarity] is approximately true, although bridges and tunnels in ground-transport networks are an obvious (but generally minor) exception." However, the presence of such nonplanar elements can vex models. "The planar network data model has received widespread acceptance and use. Despite its popularity, the model has limitations for some areas of transportation analysis, especially where complex network structures are involved. One major problem is caused by the planar embedding requirement... intersections at grade cannot be distinguished from intersections with an overpass or underpass that do not cross at grade" (Fischer et al. 2004, p. 395).

If a planar graph models a street network poorly, it could do so in multiple ways. Imposing planarity on a nonplanar street network produces artificial nodes in the graph at bridges and tunnels, which breaks routing. For example, this misrepresentation would allow a routing engine to recommend a left turn at an "intersection" that is actually an overpass (Kwan et al. 1996, p. 6). Intersection counts and densities would be overestimated in the presence of these false nodes. Consequently, edge lengths would be underestimated due to these artificial breakpoints splitting up street segments. Finally, this bias might behave inconsistently across different kinds of cities and street network types based on the extents to which they are planar.

Table 1. Survey of recent statements in the research literature regarding the representation of street networks as planar graphs.

"In a planar graph, no links intersect, except by nodes. This feature represents a transportation network well." (Dill 2004, p. 6)
"Street networks are planar graphs composed of junctions and street segments..." (Batty 2005, p. 18)
"Any of these street networks (SNS) can be described by an embedded planar graph... Street networks are planar graphs and such planarity strongly constrains their heterogeneity..." (Buhl et al. 2006, pp. 514 & 521)
"Planar graphs are those graphs forming vertices whenever two edges cross, whereas nonplanar graphs can have edge crossings that do not form vertices. The graphs representing urban street patterns are, by construction, planar..." (Cardillo et al. 2006, p. 3)
"The connection and arrangement of a road network is usually abstracted in network analysis as a directed planar graph..." (Xie and Levinson 2007, p. 340)
"Urban street patterns form planar networks... The simplest description of the street network consists of a graph whose links represent roads and whose vertices represent road intersections and end points. For these graphs, links intersect essentially only at vertices and are thus planar." (Barthelemy and Flammini 2008, p. 1)
"Urban street networks as spatial networks are embedded in planar space, which give many constraints." (Hu et al. 2008, p. 1)
"...a street network is a strange network when compared to other social or biological networks in the sense that it is embedded in the Euclidian [sic] space and the edges do not cross each other. In graph theory, such a network is called a planar graph." (Masucci et al. 2009, p. 259)
"...street networks are embedded in space and are planar in nature..." (Porta et al. 2010, p. 114)
"Roads, rail, and other transportation networks are spatial and to a good accuracy planar networks. For many applications, planar spatial networks are the most important..." (Barthelemy 2011, p. 3)
"...urban road systems can be (in good approximation) considered as planar networks, i.e., links cannot 'cross' each other without forming a physical intersection (node) as long as there are no tunnels or bridges... The meaningful definition of link angles requires the presence of a planar network, which is assumed to be the case in urban road systems." (Chan et al. 2011, pp. 563 & 567)
"Road networks are planar graphs consisting of a series of land cells surrounded by street segments." (Strano et al. 2012, p. 3)
"Planar graphs are basic tools for understanding transportation systems embedded in two-dimensional space, in particular urban street networks... As these graphs are embedded in a two-dimensional surface, the planarity criteria requires that the links do not cross each other." (Masucci et al. 2013, p. 1)
"...street networks are essentially planar; in the absence of tunnels and bridges, the streets (the links) cannot cross without generating an intersection or a junction, that is, a node." (Gudmundsson and Mohajeri 2013, p. 1).
"Networks of street patterns belong to a particular class of graphs called planar graphs, that is, graphs whose links cross only at nodes." (Strano et al. 2013, p. 1074)
"In city science, planar networks are extensively used to represent, to a good approximation, various infrastructure networks... in particular, transportation networks and more recently streets patterns..." (Viana et al. 2013, p. 1)
"...finding a typology of street patterns essentially amounts to classifying planar graphs..." (Louf and Barthelemy 2014, p. 2)
"...we are dealing with spatial graphs, which tend to be planar..." (Zhong et al. 2014, p. 2191)
"Urban transport systems as networks can be represented as planar graphs..." (Wang 2015, p. 2)
"Modeling a road network as a planar graph seems very natural." (Aldous 2016, p. 42)
"In city science, planar networks are extensively used to represent various infrastructure networks. In particular, transportation networks and street patterns..." (Barthelemy 2017a, p. 257)
"In graph theory, a spatial street network is a type of planar graph embedded in Euclidean space." (Law 2017, p. 168)

Given these issues, there are some unanswered questions in the theoretical and empirical literature. What do *approximately planar* and *well-modeled* mean for street network research? How close is "close enough" for a planar graph to competently model a formally nonplanar street network? Do the biases of planar models behave consistently across geographies and development eras or do they misrepresent different cities to different extents? And if street networks are at least generally approximately planar, how can we measure just how planar or nonplanar a given street network is?

The graph theory literature offers some measures of how "far off" a nonplanar graph G is from being planar, including its *crossing number* — the minimum number of edge crossings of any drawing of G — and its *skewness* — the minimum number of edges that must be removed from G to produce a planar graph (Liebers 2001; Chimani and Gutwenger 2009). However, these measures are imperfect and hard to compute (Székely 2004; Chimani et al. 2012). They also fail to adjust for the size, density, or real-world embedding of a spatial network. In his discussion of road networks and approximate spatial planarity, Newman (2010, p. 133) argues that due to these drawbacks no widely-accepted measure of the extent of nonplanarity has emerged and calls for the development of better indicators.

Such measures would be particularly useful for street networks, as the extent to which a network is (or is not) planar can characterize the nature of its circulation infrastructure and urban form. For instance, late 20th-century freeway-oriented American cities might exhibit lower planarity than older walkable European cities. What about informal settlements in developing countries or rapidly urbanizing Chinese cities? Beyond the question of graph model goodness-of-fit, such indicators could provide useful information about urban development, civil infrastructure, and transportation system character.

Methods

This study develops two new measures of the extent to which a spatial network is planar. Then it analyzes various city centers worldwide to better understand how well planar graphs model urban street networks and the extent to which bias (i.e., model misrepresentation) varies across places and types of urbanization. Finally, it considers what these indicators suggest about the urban form and transportation infrastructure in different cities.

Data

Following Jacobs (1995) and Cardillo et al. (2006), we analyze a consistently sized, square-mile network at the centers of 50 cities worldwide. This allows us to consistently examine central urban street networks without being swamped by metropolitan-scale variation or the idiosyncrasies of individual municipalities' spatial extents. The 50 sampled cities span Africa, Asia, Australia, Europe, and North and South America. We look separately at each city's drivable and walkable street networks. For cities such as Moscow, with a newer commercial central business district (CBD) that lies apart from an "old town" center, we take the modern CBD as the city center.

To acquire these street networks, we use OSMnx to download the data for each city and network type from OpenStreetMap. OpenStreetMap is a collaborative online mapping platform commonly used by researchers because of its good worldwide coverage (Haklay 2010; Jokar Arsanjani et al. 2015). OSMnx is a Python-based software tool that allows us to automatically download a street network from OpenStreetMap for any study site in the world, and automatically process it into a length-weighted nonplanar directed graph (Boeing 2017). It differentiates between walkable and drivable routes in the circulation network based on individual elements' metadata that describe how the route may be used. Thus the walkable network may contain surface streets, paths through parks, pedestrian passageways between buildings or under roads, and other walkable paths. The drivable network may contain surface streets, grade-separated freeways, and other drivable routes.

OpenStreetMap's raw data contain many interstitial nodes in the middle of street segments (forming an expansion graph) to allow streets to curve through space via a series of straight-line approximations. OSMnx automatically simplifies each graph's topology to retain nodes only at intersections and dead-ends, while faithfully retaining each edge's true spatial geometry. This provides an accurate count of intersections and an accurate measure of edge lengths for comparison between the planar and nonplanar representations of these networks.

Analysis

Once we have acquired and prepared the networks, we calculate three measures of planarity. The first is an aspatial test of formal planarity using the algorithm described by Boyer (2012). This assesses if it is possible to rearrange the graph's nodes and edges in space, while preserving its topology, so that edges cross only at nodes. This binary true/false indicator tells us if the graph is formally planar, ignoring its real-world spatial embedding. However, street networks *are* spatially embedded: every spatially planar network is formally planar, but not every formally planar network is spatially planar. Accordingly, for this study, we have developed a second and third indicator to assess the "extent" to which they are planar.

The second measure is the Spatial Planarity Ratio ϕ. It represents the ratio of nonplanar intersections i_n (i.e., non-dead-end nodes in the nonplanar, three-dimensional, spatially-embedded graph) to planar intersections i_p (i.e., edge crossings in the two-dimensional spatially-embedded graph):

$$\phi = \frac{i_n}{i_p} \tag{1}$$

Thus, ϕ represents what proportion of the two-dimensional edge crossings in the planar graph are true intersections in the nonplanar graph. This indicates the extent to which planarity overstates intersections and connectivity in a street network. A spatially planar network with no bridges or tunnels will have an ϕ score of 1.0, while lower values indicate the percentage extent to which the network is planar. From this indicator, we can further calculate by what percentage the planar graph overstates intersection counts as:

$$1 - \frac{1}{\phi} \tag{2}$$

The third measure is the Edge Length Ratio (λ). It represents the ratio of the mean edge length in the planar graph l_p to the mean edge length in the nonplanar graph l_n:

$$\lambda = \frac{l_p}{l_n} \tag{3}$$

This indicates the extent to which planarity understates edge lengths in each street network by fragmenting street segments at overpasses or underpasses. A spatially planar street network will thus have a λ score of 1.0, while lower values indicate the percentage extent to which the network is planar. From the λ, we can further calculate by what percentage the planar graph understates the average edge length as:

$$1 - \lambda \tag{4}$$

Finally, we explore how these three indicators vary throughout a single city. To do so we analyze the drivable street network of Oakland, California as a case study. Oakland is a reasonably representative midsized American city with a variety of urban form types from gridded street patterns in its flatlands, to winding culs-de-sac in its hills, to freeways and dense blocks around its downtown. First we analyze the entire city of Oakland. Then we recreate the aforementioned methodology by randomly sampling 100 points within the city limits and analyzing the square-mile street networks centered on each. The resulting statistical dispersion of planarity demonstrates the extent to which analyzing an entire city's neighborhoods as a single graph may obscure neighborhood-scale infrastructure characteristics.

Results

Table 2 lists the planarity indicators for these 50 city centers. Among the drivable street networks, only 20% are formally planar. On average, they are 88.5% spatially planar by the ϕ measure and 90% by the λ measure. The individual ϕ values indicate that spatial planarity ranges from a high of 100% in seven of these cities to a low of 57% in Moscow. The λ values indicate that spatial planarity ranges from a high of 100% in seven of these cities to a low of 63.5% in Los Angeles. On average across these networks, planar representations overcount intersections by 16% and underestimate street segment lengths by 10% (Equations 2 and 4).

Among walkable street networks, only 10% are formally planar. On average, they are 92% spatially planar by the ϕ measure and 91.5% by the λ measure. The individual ϕ values indicate that spatial planarity ranges from a high of 100% in two cities to a low of 67% in Shanghai. The λ values indicate that spatial planarity ranges from a high of 100% in two cities to a low of 66% in Shanghai. On average across these networks, planar representations overcount intersections by 9% and underestimate street segment lengths by 8.5%. Fewer walkable than drivable networks are formally planar, but on average these walking networks are slightly more spatially planar than the driving networks.

Not all formally planar street networks are spatially planar. For example, Toronto's drivable network is formally planar but only 93% (ϕ) and 96% (λ) spatially planar. In total, three drivable networks (Toronto, Jakarta, and Copenhagen) and three walkable networks (Dallas, Delhi, and Bologna) are formally planar but spatially nonplanar to various extents. About a third of the city centers studied demonstrate ϕ spatial planarity of 97% or higher, suggesting that they are "approximately" planar. However, another third of the city centers are less than 87% planar. Dallas, Los Angeles, and Moscow have ϕ values below 60%, suggesting planar graphs poorly model these city centers. Moreover, planar graphs overstate the intersection counts in these three networks by 67%, 72%, and 74% respectively.

Mogadishu is the only city studied that demonstrates perfect planarity across all three indicators for both network types. All three Italian cities demonstrate perfect planarity in their centers' drivable networks, but not in their walkable networks. The extent of planarity is not consistent across network types: Dallas's walkable ϕ is 61% greater than its drivable ϕ, while Geneva's drivable ϕ is 19% greater than its walkable ϕ. Figure 2 maps the distribution of ϕ values around the world. While nearly every European city is in the highest tercile, indicating their networks are more planar, most American cities are in the lowest tercile, indicating their networks are more nonplanar.

Figure 3 depicts the relationship between the ϕ and λ indicators across all 50 cities for both network types, as a log-log plot. The indicators' log-log relationship is linear, positive, and very strong (drivable $r^2 = 0.98$ and walkable $r^2 = 0.99$). The coefficients of determination tell us that these two indicators unsurprisingly provide redundant statistical information about the extent to which a network is spatially planar. However, each assesses different implications of this bias for measuring the urban form.

Finally, we examine how these measures behave across an entire city. We find that Oakland's city-wide street network is formally nonplanar. The city has a ϕ score of 91.8% and a λ score of 93.6%. This suggests that the planar representation of Oakland's drivable street network overstates the number of intersections — and thus, the network's connectivity — by 8.9% city-wide and understates the average edge length by 6.4% city-wide.

However, these indicators' values vary across the city. To explore this statistical variation, Table 3 presents summary statistics of these planarity indicators across 100 square-mile samples of Oakland's drivable street network. Our samples' mean ϕ and λ scores are reasonably close to the city-wide values. However, the samples range from spatial planarity lows of 56.9% (ϕ) and 63.7% (λ) up to highs of 100%. 67% of the samples pass the formal planarity test; however, 63% of the samples are at least somewhat spatially nonplanar (i.e., with $\phi < 1$).

Discussion

Are street networks planar graphs?

These findings suggest that the street networks at the centers of most major cities are formally and spatially nonplanar. However, this depends on the scale of measurement: across an entire city there is likely to be at least one overpass or underpass somewhere, while individual neighborhoods or small towns may be entirely planar. The type and era of urbanization represent another factor. Medieval European towns or informal settlements in the global south may contain fewer grade-separated roads — and thus would be more planar — than 20th-century American or 21st-century Chinese metropolises. This results from the prevailing transportation and engineering technologies when the urban form developed, as well as local terrain, wealth, culture, and politics (Southworth and Ben-Joseph 1995).

Street networks are frequently spatially nonplanar because they are embedded in three dimensions, not two: they have a (less-extensive) z-axis along with their x- and y-axes. But because they are usually "mostly" planar (average drivable ϕ of 88.5% and average walkable ϕ of 92%), typically with only a few overpasses or underpasses, they could often be described as *approximately* planar. However, claiming that urban street networks universally are "planar," or modeling them as such, misrepresents them in several ways:

1. Intersection counts are overestimated due to false nodes where grade-separated edges cross
2. Average edge lengths are underestimated
3. Connectivity is misrepresented for routing, accessibility analysis, and other topological studies

Our results demonstrate how this is a bigger problem in Los Angeles than in Florence. But even in Florence the

Table 2. Planarity measures for central street networks in 50 cities worldwide (Planar = whether street network passed the formal test of planarity; ϕ = Spatial Planarity Ratio; λ = Edge Length Ratio).

		Drive			Walk		
Country	City	Planar	ϕ	λ	Planar	ϕ	λ
Argentina	Buenos Aires	Yes	1.000	1.000	No	0.946	0.947
Australia	Sydney	No	0.741	0.749	No	0.909	0.901
Brazil	Sao Paulo	No	0.791	0.790	No	0.852	0.831
Canada	Toronto	Yes	0.930	0.958	No	0.858	0.848
	Vancouver	No	0.929	0.948	No	0.929	0.926
Chile	Santiago	No	0.875	0.887	No	0.972	0.971
China	Beijing	No	0.818	0.872	No	0.842	0.848
	Hong Kong	No	0.846	0.835	No	0.840	0.818
	Shanghai	No	0.682	0.717	No	0.670	0.659
Denmark	Copenhagen	Yes	0.992	0.988	No	0.991	0.987
Egypt	Cairo	No	0.900	0.916	No	0.918	0.906
France	Lyon	No	0.991	0.989	No	0.960	0.957
	Paris	No	0.988	0.993	No	0.920	0.917
Germany	Berlin	No	0.939	0.950	No	0.943	0.936
India	Delhi	Yes	1.000	1.000	Yes	0.993	0.992
Indonesia	Jakarta	Yes	0.983	0.986	No	0.962	0.960
Iran	Tehran	No	0.962	0.973	No	0.957	0.956
Italy	Bologna	Yes	1.000	1.000	Yes	0.996	0.996
	Florence	Yes	1.000	1.000	No	0.980	0.978
	Milan	Yes	1.000	1.000	No	0.875	0.860
Japan	Osaka	No	0.868	0.871	No	0.951	0.949
	Tokyo	No	0.927	0.923	No	0.922	0.912
Kenya	Nairobi	No	0.974	0.974	No	0.949	0.943
Mexico	Mexico City	No	0.940	0.952	No	0.913	0.917
Nigeria	Lagos	No	0.952	0.967	No	0.988	0.987
Peru	Lima	No	0.939	0.941	No	0.932	0.931
Philippines	Manila	No	0.946	0.953	No	0.906	0.895
Russia	Moscow	No	0.574	0.680	No	0.856	0.858
Singapore	Singapore	No	0.868	0.874	No	0.899	0.890
Somalia	Mogadishu	Yes	1.000	1.000	Yes	1.000	1.000
South Africa	Johannesburg	No	0.851	0.883	No	0.997	0.997
Spain	Barcelona	Yes	1.000	1.000	No	0.904	0.900
Switzerland	Geneva	No	0.985	0.982	No	0.828	0.813
Thailand	Bangkok	No	0.988	0.988	No	0.993	0.989
Turkey	Istanbul	No	0.975	0.982	No	0.980	0.978
UAE	Dubai	No	0.685	0.722	No	0.860	0.850
UK	Edinburgh	No	0.973	0.968	No	0.988	0.988
	London	No	0.979	0.981	No	0.865	0.847
USA	Atlanta	No	0.736	0.777	No	0.738	0.724
	Chicago	No	0.776	0.814	No	0.807	0.804
	Cincinnati	No	0.730	0.757	No	0.931	0.927
	Dallas	No	0.598	0.650	Yes	0.963	0.959
	Los Angeles	No	0.583	0.635	No	0.793	0.799
	Miami	No	0.647	0.662	No	0.964	0.961
	New York	No	0.881	0.901	No	0.942	0.941
	Phoenix	No	0.955	0.962	No	0.979	0.977
	San Francisco	No	0.935	0.941	No	0.948	0.944
	Seattle	No	0.732	0.779	No	0.933	0.926
	Washington DC	No	0.948	0.956	No	0.967	0.967
Venezuela	Caracas	No	0.953	0.957	Yes	1.000	1.000

walking network is both spatially and formally nonplanar due to the *sottopassaggio* (pedestrian subway) near Stazione di Santa Maria Novella, its central train station. The results also suggest that spatial planarity is inconsistent both across cities as well as across different neighborhoods within individual cities.

The problem with planar models is particularly pronounced around the downtowns of North American cities,

Figure 2. Map of world cities from Table 2 grouped by ϕ terciles (lower values mean less planar).

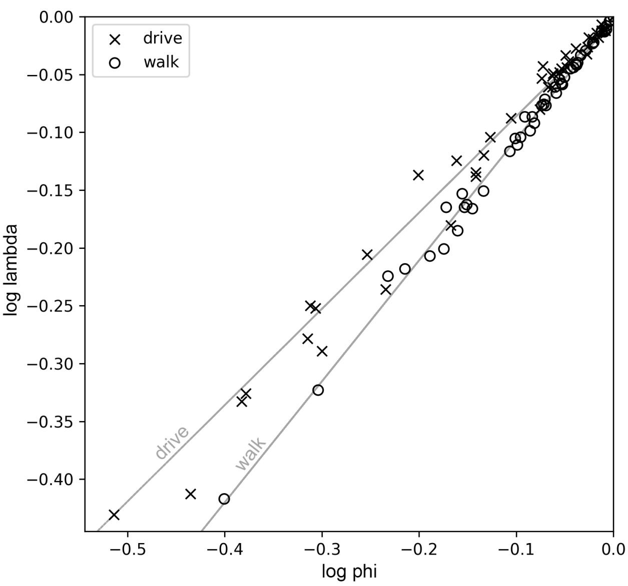

Figure 3. Log-log plot of λ vs ϕ by network type with simple regression lines.

Table 3. Summary statistics of planarity indicators across 100 square-mile samples of Oakland, California's drivable network.

	ϕ	λ
count	100	100
mean	0.930	0.947
σ	0.101	0.082
min	0.569	0.637
max	1.000	1.000

due to the prevalence of freeways, bridges, and underpasses. Drivable networks are affected by these in particular, but even walkable networks are affected by pedestrian flyovers and subways, as in Florence. Furthermore, even networks of non-freeway, non-pedestrian-only surface streets could easily be nonplanar due to tunnels in hilly neighborhoods or bridges over rivers.

A graph is not planar because its edges *usually* intersect only at nodes: by definition it is planar because its edges *exclusively* intersect at nodes. Contrary to some of the statements in the urban studies literature, our results suggest that it cannot be universally claimed that urban street networks are planar graphs — formally or spatially. However, as Newman (2010) points out, debating the semantics of formal planarity may be missing the point – more interesting is the *extent* to which a network is planar and its implications for modeling.

Are street networks well-modeled by planar graphs?

As George Box famously said, "All models are wrong but some are useful." Even if they are not formally planar, can street networks be simplified to planar graphs and still be usefully well-modeled? Our results suggest that the answer depends on the study site and on the type of analysis. In limited circumstances — where the circulation network exhibits few (or ideally zero) underpasses, overpasses, or grade-separation — then perhaps yes, especially if the study focuses on polygonal spatial analysis. But universally we cannot answer yes, especially for topological studies. Most egregiously, imposing planarity on a nonplanar street network forces false nodes at underpasses and overpasses, breaking routing and network-based accessibility modeling. For this reason, nonplanar graphs have been the standard for decades in transportation engineering, real-world traffic assignment models, and routing engines.

But planar graphs are often used in the literature to characterize urban form and morphology. Aside from routing, do planar graphs offer useful models for this type of research? Again, only in limited circumstances, such as the drivable networks in the three Italian cities we analyzed. Elsewhere, the results in Table 2 showed how common urban form measures such as intersection counts are overstated by planar models (16% on average in the drivable networks), while average street segment lengths are consequently understated (10% on average in the drivable networks). Moreover, this misrepresentation behaves inconsistently from place to place: Figure 2 and Table 3 demonstrated how the magnitude of bias varies across cities and modes of urbanization.

We might shoehorn real-world data into a chosen model and be able to perform a desired computation, but we lose the underlying ability to reason when the real world no longer obeys the rules and constraints of the chosen model. Although planar graphs offer computational tractability and allow for the polygonal analysis of urban blocks, they often model these street networks poorly when compared to nonplanar graphs.

What can nonplanarity tell us?

We might repurpose this discrepancy between planar and nonplanar representations of a street network to reveal something useful about urban form. The ϕ and λ scores of the major US city centers indicate the greater three-dimensionality of their transportation infrastructure compared to that of the European city centers. These US city centers feature more automobile-oriented roadways and are often bounded by grade-separated freeways. By contrast, these European city centers feature less three-dimensionality as their streets and paths tend to be at-grade. Examining how ϕ and λ change over time in different cities could reveal the type and pace of urban development. Future research can further explore how these indicators correlate with other measures of urbanization, development, and era.

Most street networks are formally nonplanar while demonstrating ϕ and λ values that are only slightly nonplanar. Approximate planarity is common across cities because of costs, technology, and politics: it is expensive to engineer a three-dimensional network with z-axis extents (nearly) as extensive as the x- and y-axes. Furthermore, a city fully consumed by overpasses and tunnels would degrade quality of life, making it politically infeasible.

Conclusion

Street networks are often defined and modeled as planar graphs in the urban studies literature despite typically being nonplanar in reality. This study tested the formal planarity of central urban street networks and developed two new measures of their spatial planarity: the Spatial Planarity Ratio ϕ and the Edge Length Ratio λ. Although planar models may be useful in limited circumstances, they behave inconsistently within and across cities by misrepresenting connectivity, accessibility, routing, intersection counts and densities, and street segment lengths. Nonplanar graphs usually provide more faithful models, and ϕ and λ can characterize urbanization in different cities, particularly through the transportation infrastructure's three-dimensionality.

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