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Nonplanarity and Street Network Representation in Urban Form Studies

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Abstract

Mathematical models of street networks, called graphs, are widely used throughout the urban studies literature to examine travel behavior, accessibility, urban design patterns, and morphology. These networks are commonly called planar graphs, meaning that they can be represented in two dimensions without any underpasses or overpasses. However, real urban street networks exist in three-dimensional space and frequently include grade separation such as bridges and tunnels. How well do planar graphs model real-world street networks? What does the extent of nonplanarity tell us about a place's infrastructure and urban development? We measure the nonplanarity of drivable and walkable street networks in the centers of 50 cities worldwide, then examine the variation of planarity across a single city. We find that while some street networks are approximately planar, many are poorly-modeled by planar graphs. In particular, planarity can drastically misrepresent routing, connectivity, intersection counts/densities, and block lengths.

Keywords

street network, GIS, urban form, transportation, urban design

Introduction

In urban planning and design research, street networks are routinely used to calculate accessibility between origins and destinations or to compute indicators of the urban form, such as block sizes or intersection density and connectivity. Mathematical models of street networks, called graphs, have grown ubiquitous in the urban studies literature in recent years as they have been used to model household travel patterns, access and equity, pedestrian volume, urban design patterns and spatial morphology, and location centrality and polycentricity (Southworth and Ben-Joseph 1995; Marshall and Garrick 2010; Porta et al. 2014; Marshall et al. 2014; Hajrasouliha and Yin 2015; Parthasarathi et al. 2015; Knight and Marshall 2015; Gil 2016; Zhong et al. 2017).

In the urban studies literature, street networks are typically referred to as "planar" or "approximately planar" (see Table 1), meaning that they can be well-modeled by a flat two-dimensional model that inherently precludes overpasses or underpasses. Of course in the real world, urban street networks are embedded in three-dimensional space and often feature grade separation, bridges, and tunnels. This leads to two intertwined questions. First, how well do these planar graphs model urban street networks? Second, what does the extent to which a network is nonplanar tell us about a city's infrastructure and development?

This study tests the planarity of street networks around the world. It also presents two new measures of the degree of nonplanarity that can be generalized to other types of spatial networks. These indicators help describe the nature of the urban form and transportation infrastructure. Despite common claims in the literature that street networks are planar graphs, this study finds that they generally are nonplanar and that planar graphs poorly model the street networks of many cities. Further, the magnitude of this bias varies substantially across cities and urbanization types.

This article is organized as follows. The following section introduces the basics of graph theory relevant to urban studies, focusing on discussions in the research literature about street network planarity. The next section discusses the methods used to acquire and analyze the street networks in this study. Then we present the results of this analysis before concluding with a discussion of their ramifications for street network research and urban form studies.

Background and Motivation

Graph theory is the mathematical study of networks (Newman 2010). Graphs can model real-world networks such as friendships, the world wide web, or spatial networks such as urban street networks (Barthelemy 2011). A graph G consists of a set of nodes N connected to one another by a set of edges E. An edge uv in a directed graph points in one direction from some node u to some node v, but an undirected graph's edges all point mutually in both directions. In a street network, the nodes represent intersections and dead-ends, and the (directed) edges represent the street segments that connect them. How a graph's nodes and edges connect to one another defines its *topology*. For example, a node's *degree* is a topological trait that represents how many edges connect to that node. A *planar* graph can be drawn on a two-dimensional plane without any of its edges crossing each other, except where

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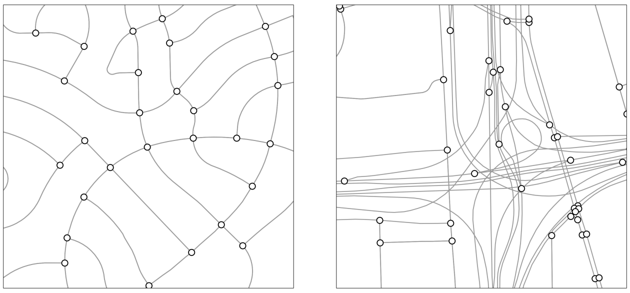

Figure 1. Formally and spatially planar (left) and nonplanar (right) road networks.

they intersect at nodes. If the graph cannot be drawn — or redrawn — to meet this criterion, then the graph is nonplanar (Trudeau 1994). Street networks are embedded in space, which provides them with geometry — such as geographical coordinates, lengths, areas, and angles — along with their topology.

This creates a minor wrinkle when we consider planarity: we must distinguish between a graph's mathematical/topological planarity, which we refer to as *formal planarity*, and the planarity of its particular spatial embedding, which we refer to as *spatial planarity*. For example, a street network might be spatially nonplanar due to its embedding in space (i.e., it contains overpasses or underpasses in the real world), but it could still be formally planar. If we "redraw" the graph by moving its nodes and edges around in space without changing how they are connected to one another (i.e., altering its geometry without altering its topology), there may exist some spatial embedding that prevents edges crossing anywhere but at nodes (Barthelemy 2017b, p. 6). In such a case, the street network is formally planar from a topological perspective, but its real-world embedding is spatially nonplanar.

Consider the examples in Figure 1. The left network comprises a set of paths and streets without any bridges or tunnels. In two-dimensions, its edges intersect each other only at nodes so it is by definition planar. The right network comprises surface streets and a grade-separated freeway interchange with many overpasses and underpasses. In two-dimensions, its edges sometimes cross each other at non-nodes (i.e., overpasses), so it is spatially nonplanar. Furthermore it is impossible to redraw the graph by moving its nodes and edges around in space such that its edges only intersect at nodes. Therefore it is also formally nonplanar. However, at a city- or region-wide spatial scale, these nonplanar edge crossings may be relatively uncommon; we might call such a network *approximately* planar. Approximate planarity constrains nonplanar spatial networks such that they do not exhibit certain characteristics found among nonplanar aspatial graphs, such as small-world effects or power-law distributed node degrees (O'Sullivan 2014).

In the urban studies literature, street networks are commonly referred to as planar graphs. Table 1 presents a survey of statements and reasoning around this claim. Some authors prefer to hedge slightly, arguing that street networks are *approximately* or *essentially* planar graphs that are close enough to be well-modeled as such.

If street networks can be sufficiently well-modeled by planar graphs, there are certain methodological benefits to doing so. Planar graphs offer computational simplicity and tractability. They enable easy polygonal spatial analysis of city blocks and form (Fohl et al. 1996; Barthelemy 2017a) as well as the Euler characteristic. In mathematics, there is a bijection between planar graphs and trees, and classifying planar graphs presents a trivial problem (Louf and Barthelemy 2014). Planar graphs are easier to visualize and can be faster to run algorithms on (Liebers 2001). Accordingly, Barthelemy (2011, p. 3) argues that "planar spatial networks are the most important and most studies have focused on these examples". But in contrast, Masucci et al. (2009) and Masucci et al. (2013) argue that planar graphs remain a compelling research domain for urban scholars because they were understudied until recently for two reasons: they appear topologically trivial and planarity does not lend itself to certain popular graph-theoretic analyses. Discussing the open research area around street networks as planar graphs, Viana et al. (2013, p. 1) state, "there is still a lack of global, high-level metrics allowing to characterize their structure and geometrical patterns."

Despite the computational and mathematical advantages of simple planar models, street networks are often nonplanar in reality: many include at least one overpass or underpass that results in the failure of formal proofs of their planarity, such as the Kuratowski (1930) theorem or the Hopcroft and Tarjan (1974) algorithm (cf. Gastner and Newman 2006). As Levinson (2012, p. 7) points out, "Real networks are neither perfect, nor planar, nor grids, though they may approximate them."

Other authors have commented on this characteristic of street networks. Mandloi and Thill (2010, p. 199) note that "quite often the transportation network has overpasses and underpasses that require a non-planar network representation." O'Sullivan (2014, p. 1258) explains that "for many infrastructure networks, [planarity] is approximately true, although bridges and tunnels in ground-transport networks are an obvious (but generally minor) exception." However, the presence of such nonplanar elements can vex models. "The planar network data model has received widespread acceptance and use. Despite its popularity, the model has limitations for some areas of transportation analysis, especially where complex network structures are involved. One major problem is caused by the planar embedding requirement... intersections at grade cannot be distinguished from intersections with an overpass or underpass that do not cross at grade" (Fischer et al. 2004, p. 395). Twenty years ago, Fohl et al. (1996, p. 18) called for a nonplanar model to better represent truly nonplanar spatial networks.

If a planar graph models a street network poorly, it could do so in multiple ways. Forcing planarity on a nonplanar street network creates artificial nodes in the graph at bridges and tunnels, which breaks routing. As Kwan et al. (1996, p. 6) describe it, "the difficulty in accurately representing overpasses or underpasses may lead to problems when running various routing algorithms (e.g. recommending that a traveler make a left-turn at an intersection that proves to be an overpass)". Intersection counts and densities will be overestimated in the presence of these false nodes.

Table 1. Recent statements in the research literature regarding the representation of street networks as planar graphs.

"In a planar graph, no links intersect, except by nodes. This feature represents a transportation network well." (Dill 2004, p. 6)
"Street networks are planar graphs composed of junctions and street segments..." (Batty 2005, p. 18)
"The number of long-range connections and the number of edges that can be connected to a single node are limited by the spatial embedding. This is particularly evident in planar networks e.g., those networks forming vertices whenever two edges cross, as urban streets or ant networks of galleries..." (Crucitti et al. 2006, p. 1)
"Any of these street networks (SNS) can be described by an embedded planar graph... Street networks are planar graphs and such planarity strongly constrains their heterogeneity..." (Buhl et al. 2006, pp. 514 & 521)
"Planar graphs are those graphs forming vertices whenever two edges cross, whereas nonplanar graphs can have edge crossings that do not form vertices. The graphs representing urban street patterns are, by construction, planar..." (Cardillo et al. 2006, p. 3)
"The connection and arrangement of a road network is usually abstracted in network analysis as a directed planar graph..." (Xie and Levinson 2007, p. 340)
"Urban street patterns form planar networks... The simplest description of the street network consists of a graph whose links represent roads and whose vertices represent road intersections and end points. For these graphs, links intersect essentially only at vertices and are thus planar." (Barthelemy and Flammini 2008, p. 1)
"Urban street networks as spatial networks are embedded in planar space, which give many constraints." (Hu et al. 2008, p. 1)
"...a street network is a strange network when compared to other social or biological networks in the sense that it is embedded in the Euclidian [sic] space and the edges do not cross each other. In graph theory, such a network is called a planar graph." (Masucci et al. 2009, p. 259)
"...street networks are embedded in space and are planar in nature..." (Porta et al. 2010, p. 114)
"Roads, rail, and other transportation networks are spatial and to a good accuracy planar networks. For many applications, planar spatial networks are the most important..." (Barthelemy 2011, p. 3)
"...urban road systems can be (in good approximation) considered as planar networks, i.e., links cannot 'cross' each other without forming a physical intersection (node) as long as there are no tunnels or bridges... The meaningful definition of link angles requires the presence of a planar network, which is assumed to be the case in urban road systems." (Chan et al. 2011, pp. 563 & 567)
"Road networks are planar graphs consisting of a series of land cells surrounded by street segments." (Strano et al. 2012, p. 3)
"Planar graphs are basic tools for understanding transportation systems embedded in two-dimensional space, in particular urban street networks... As these graphs are embedded in a two-dimensional surface, the planarity criteria requires that the links do not cross each other." (Masucci et al. 2013, p. 1)
"...street networks are essentially planar; in the absence of tunnels and bridges, the streets (the links) cannot cross without generating an intersection or a junction, that is, a node." (Gudmundsson and Mohajeri 2013, p. 1).
"Networks of street patterns belong to a particular class of graphs called planar graphs, that is, graphs whose links cross only at nodes." (Strano et al. 2013, p. 1074)
"In city science, planar networks are extensively used to represent, to a good approximation, various infrastructure networks... in particular, transportation networks and more recently streets patterns..." (Viana et al. 2013, p. 1)
"...finding a typology of street patterns essentially amounts to classifying planar graphs..." (Louf and Barthelemy 2014, p. 2)
"...we are dealing with spatial graphs, which tend to be planar..." (Zhong et al. 2014, p. 2191)
"Urban transport systems as networks can be represented as planar graphs..." (Wang 2015, p. 2)
"Modeling a road network as a planar graph seems very natural." (Aldous 2016, p. 42)
"In city science, planar networks are extensively used to represent various infrastructure networks. In particular, transportation networks and street patterns..." (Barthelemy 2017a, p. 257)
"In graph theory, a spatial street network is a type of planar graph embedded in Euclidean space." (Law 2017, p. 168)

Consequently, edge lengths will be underestimated due to these artificial breakpoints splitting up street segments. Finally, this bias would likely behave inconsistently across different kinds of cities and street network types based on the extents to which they are planar.

Given these issues, what do *approximately planar* and *well-modeled* mean for street network research? How close is "close enough" for a planar graph to competently model a formally nonplanar street network? Do the biases of planar models behave consistently across geographies and development eras or do they misrepresent different cities to different degrees? And if street networks are at least generally approximately planar, how can we measure just how planar or nonplanar a given street network is?

The graph theory literature offers some measures of how "far off" a nonplanar graph G is from being planar, including its *crossing number* — the minimum number of edge crossings of any drawing of G — and its *skewness* — the minimum number of edges that must be removed from G to produce a planar graph (Liebers 2001; Chimani and Gutwenger 2009). However, these measures are imperfect and hard to compute (Székely 2004; Chimani et al. 2012).

They also fail to correct for the size, density, or real-world embedding of a spatial network. In his discussion of road networks and approximate spatial planarity, Newman (2010, p. 133) argues that due to these drawbacks "no widely accepted metric for degree of planarity has emerged," and calls for the development of better indicators.

Such measures would be particularly useful for street networks, as the extent to which a network is (or is not) planar can characterize the nature of its circulation infrastructure and urban form. For instance, late 20th-century freeway-oriented American cities might exhibit lower planarity than older walkable European cities. What about informal settlements in developing countries or rapidly urbanizing Chinese cities? Beyond the question of graph model goodness-of-fit, such indicators could provide useful information about urban development, civil infrastructure, and transportation system character.

Methods

In this study, we develop two new measures of the extent to which a spatial network is planar. We then analyze various world cities to better understand how well planar graphs model their street networks as well as the extent to which bias (i.e., model misrepresentation) varies across different places and types of urbanization. Finally, we consider what these indicators suggest about the nature of the urban form and transportation infrastructure in different cities.

Data

Following Jacobs (1995) and Cardillo et al. (2006), we analyze a consistently sized, square-mile network at the centers of 50 cities worldwide. This allows us to consistently examine central urban street networks without being swamped by metropolitan-scale variation or the idiosyncrasies of individual municipalities' spatial extents. The 50 sampled cities span Africa, Asia, Australia, Europe, and North and South America. We look separately at each city's drivable and walkable street networks. For cities such as Moscow, with a newer commercial central business district (CBD) that lies apart from an "old town" center, we take the modern CBD as the city center.

To acquire these street networks, we use OSMnx to download the data for each city and network type from OpenStreetMap. OpenStreetMap is a collaborative online mapping platform commonly used by researchers because of its good worldwide coverage (Haklay 2010; Jokar Arsanjani et al. 2015). OSMnx is a Python-based software tool that allows us to automatically download a street network from OpenStreetMap for any study site in the world, and automatically process it into a length-weighted nonplanar directed graph (Boeing 2017). It differentiates between walkable and drivable routes in the circulation network based on individual elements' metadata that describe how the route may be used. Thus the walkable network may contain surface streets, paths through parks, pedestrian passageways between buildings or under roads, and other walkable paths. The drivable network may contain surface streets, grade-separated freeways, and other drivable routes.

OpenStreetMap's raw data contain many interstitial nodes in the middle of street segments (forming an expansion graph) to allow streets to curve through space via a series of straight-line approximations. OSMnx automatically simplifies each graph's topology to retain nodes only at intersections and dead-ends, while faithfully retaining each edge's true spatial geometry. This provides us with an accurate count of intersections and an accurate measure of edge lengths for comparison between the planar and nonplanar representations of our networks.

Analysis

Once we have acquired and prepared our networks, we calculate three measures of planarity. The first is an aspatial test of formal planarity using the algorithm described by Boyer (2012). This assesses if it is possible to rearrange the graph's nodes and edges in space, while preserving its topology, so that edges cross only at nodes. This binary true/false indicator tells us if the graph is formally planar, ignoring its real-world spatial embedding. However, street networks *are* spatially embedded. Accordingly, for this study, we have developed a second and third indicator to assess the "extent" to which they are planar.

The second measure is the spatial planarity ratio ϕ. It represents the ratio of nonplanar intersections i_n (i.e., non-dead-end nodes in the nonplanar, three-dimensional, spatially-embedded graph) to planar intersections i_p (i.e., edge crossings in the planar, two-dimensional, spatially-embedded graph):

$$\phi = \frac{i_n}{i_p} \tag{1}$$

This indicates the extent to which planarity overstates intersections and connectivity in each street network. In other words, ϕ tells us what proportion of the two-dimensional edge crossings in the planar graph are true intersections in the nonplanar graph. A truly planar network with no bridges or tunnels will thus have an ϕ score of 1.0, while lower values indicate the percentage extent to which the network is planar. From the ϕ, we can further calculate by what percentage the planar graph overstates intersection counts as:

$$1 - \frac{1}{\phi} \tag{2}$$

The third measure is the edge length ratio (λ). It represents the ratio of the mean edge length in the planar graph l_p to the mean edge length in the nonplanar graph l_n:

$$\lambda = \frac{l_p}{l_n} \tag{3}$$

This indicates the extent to which planarity understates edge lengths in each street network by fragmenting street segments at overpasses or underpasses. A truly planar street network with no bridges or tunnels will thus have an λ score of 1.0, while lower values indicate the percentage extent to which the network is planar. From the λ, we can further calculate by what percentage the planar graph understates the average edge length as:

$$1 - \lambda \tag{4}$$

Finally, we explore how these three indicators vary across a single city. To do so we analyze the drivable street network of Oakland, California, a reasonably representative midsized American city with a variety of urban form types from gridded street patterns in its flatlands, to winding culs-de-sac in its hills, to freeways and dense blocks around its downtown. First we analyze the entire city of Oakland. Then we recreate the aforementioned methodology by randomly sampling 100 points within the city limits and analyzing the square-mile street networks centered on each. The resulting statistical dispersion of planarity demonstrates the extent to which analyzing an entire city's neighborhoods as a single graph may obscure neighborhood-scale planarity characteristics.

Results

Table 2 shows ϕ, λ, and results of the formal planarity test for the walkable and the drivable networks in these 50 city centers. Among the drivable street networks, only 20% are formally planar. On average, they are 88.5% planar by the ϕ measure and 90% planar by the λ measure. The individual ϕ values indicate that spatial planarity ranges from a high of 100% in seven of these cities to a low of 57% in Moscow. The λ values indicate that spatial planarity ranges from a high of 100% in seven of these cities to a low of 63.5% in Los Angeles. On average across these networks, planar representations overcount intersections by 16% and underestimate street segment lengths by 10%.

Among walkable street networks, only 10% are formally planar. On average, they are 92% planar by the ϕ measure and 91.5% planar by the λ measure. The individual ϕ values indicate that spatial planarity ranges from a high of 100% in two cities to a low of 67% in Shanghai. The λ values indicate that spatial planarity ranges from a high of 100% in two cities to a low of 66% in Shanghai. On average across these networks, planar representations overcount intersections by 9% and underestimate street segment lengths by 8.5%. Fewer walkable than drivable networks are formally planar, but on average these walking networks are slightly more spatially planar than the driving networks.

Not all formally planar street networks are spatially planar. For example, Toronto's drivable network is formally planar but only 93% and 96% spatially planar according to its ϕ and λ measures. In total, three drivable networks (Toronto, Jakarta, and Copenhagen) and three walkable networks (Dallas, Delhi, and Bologna) are formally planar but spatially nonplanar to various extents. About a third of the cities studied demonstrate spatial planarity of 97% or higher according to the ϕ, suggesting that they are "approximately" planar. However, another third of the cities studied are less than 87% planar. Dallas, Los Angeles, and Moscow have ϕ values below 60%, suggesting planar graphs poorly model these city centers. Moreover, planar graphs overstate the intersection counts in these three networks by 67%, 72%, and 74% respectively (Equation 2).

Mogadishu is the only city studied that demonstrates perfect planarity across all three indicators for both network types. All three Italian cities demonstrate perfect planarity in their centers' drivable networks, but not in their walkable networks. The extent of planarity is not consistent across network types: Dallas's walkable ϕ is 61% greater than its drivable ϕ, while Geneva's drivable ϕ is 19% greater than its walkable ϕ. Figure 2 shows the distribution of ϕ values around the world. While nearly every European city is in the highest tercile, indicating their networks are more planar, most American cities are in the lowest tercile, indicating their networks are more nonplanar.

Figure 3 shows the relationship between the ϕ and λ indicators across all 50 cities for both network types, as a log-log plot. The indicators' log-log relationship is linear, positive, and very strong (drivable $r^2 = 0.98$ and walkable $r^2 = 0.99$). This strong correlation tells us that these two indicators unsurprisingly provide redundant statistical information about the extent to which a network is planar. However, each assesses different implications of this bias for measuring the urban form.

Finally, we examine how these measures behave across an entire city. We find that Oakland's city-wide street network is formally nonplanar. The city has a ϕ score of 91.8% and an λ score of 93.6%. This suggests that the planar representation of Oakland's drivable street network overstates the number of intersections — and thus, the network's connectivity — by 8.9% city-wide (Equation 2) and understates the average edge length by 6.4% city-wide (Equation 4).

However, these indicators' values vary across the city. To explore this statistical variation, Table 3 presents summary statistics of these planarity indicators across 100 square-mile samples of Oakland's drivable street network. Our samples' mean ϕ and λ scores are reasonably close to the city-wide values. However, the samples range from spatial planarity lows of 56.9% (ϕ) and 63.7% (λ) up to highs of 100%. 67% of the samples pass the formal planarity test. However, 63% of the samples are at least somewhat spatially nonplanar (i.e., with ϕ less than 1).

Discussion

Are street networks planar graphs?

Our findings suggest that the street networks at the centers of most major cities are formally nonplanar. However, this depends on the scale of measurement: across an entire city there is likely to be at least one overpass or underpass somewhere, while individual neighborhoods or small towns might be formally planar in their entirety. The type and era of urbanization represent another factor. Medieval European towns or informal settlements in the global south may contain fewer grade-separated roads — and thus are more planar — than 20th-century American or 21st-century Chinese metropolises. This is a result of the prevailing transportation technologies when the urban form was developed, as well as the local terrain, wealth, culture, and politics.

Street networks are frequently spatially nonplanar because they are embedded in three dimensions, not two: they have a z-coordinate (elevation) along with their x- and y-coordinates. But because they are usually "mostly" planar (average drivable ϕ of 88.5% and average walkable ϕ of 92%), typically with only a few overpasses or underpasses, they could often be described as *approximately planar*. However, claiming that urban street networks universally are "planar" misrepresents them in several ways:

Table 2. Planarity measures for the central street networks of 50 cities worldwide (Planar = whether street network passed the formal test of planarity; ϕ = spatial planarity ratio; λ = edge length ratio).

		Drive			Walk		
Country	City	Planar	ϕ	λ	Planar	ϕ	λ
Argentina	Buenos Aires	Yes	1.000	1.000	No	0.946	0.947
Australia	Sydney	No	0.741	0.749	No	0.909	0.901
Brazil	Sao Paulo	No	0.791	0.790	No	0.852	0.831
Canada	Toronto	Yes	0.930	0.958	No	0.858	0.848
	Vancouver	No	0.929	0.948	No	0.929	0.926
Chile	Santiago	No	0.875	0.887	No	0.972	0.971
China	Beijing	No	0.818	0.872	No	0.842	0.848
	Hong Kong	No	0.846	0.835	No	0.840	0.818
	Shanghai	No	0.682	0.717	No	0.670	0.659
Denmark	Copenhagen	Yes	0.992	0.988	No	0.991	0.987
Egypt	Cairo	No	0.900	0.916	No	0.918	0.906
France	Lyon	No	0.991	0.989	No	0.960	0.957
	Paris	No	0.988	0.993	No	0.920	0.917
Germany	Berlin	No	0.939	0.950	No	0.943	0.936
India	Delhi	Yes	1.000	1.000	Yes	0.993	0.992
Indonesia	Jakarta	Yes	0.983	0.986	No	0.962	0.960
Iran	Tehran	No	0.962	0.973	No	0.957	0.956
Italy	Bologna	Yes	1.000	1.000	Yes	0.996	0.996
	Florence	Yes	1.000	1.000	No	0.980	0.978
	Milan	Yes	1.000	1.000	No	0.875	0.860
Japan	Osaka	No	0.868	0.871	No	0.951	0.949
	Tokyo	No	0.927	0.923	No	0.922	0.912
Kenya	Nairobi	No	0.974	0.974	No	0.949	0.943
Mexico	Mexico City	No	0.940	0.952	No	0.913	0.917
Nigeria	Lagos	No	0.952	0.967	No	0.988	0.987
Peru	Lima	No	0.939	0.941	No	0.932	0.931
Philippines	Manila	No	0.946	0.953	No	0.906	0.895
Russia	Moscow	No	0.574	0.680	No	0.856	0.858
Singapore	Singapore	No	0.868	0.874	No	0.899	0.890
Somalia	Mogadishu	Yes	1.000	1.000	Yes	1.000	1.000
South Africa	Johannesburg	No	0.851	0.883	No	0.997	0.997
Spain	Barcelona	Yes	1.000	1.000	No	0.904	0.900
Switzerland	Geneva	No	0.985	0.982	No	0.828	0.813
Thailand	Bangkok	No	0.988	0.988	No	0.993	0.989
Turkey	Istanbul	No	0.975	0.982	No	0.980	0.978
UAE	Dubai	No	0.685	0.722	No	0.860	0.850
UK	Edinburgh	No	0.973	0.968	No	0.988	0.988
	London	No	0.979	0.981	No	0.865	0.847
USA	Atlanta	No	0.736	0.777	No	0.738	0.724
	Chicago	No	0.776	0.814	No	0.807	0.804
	Cincinnati	No	0.730	0.757	No	0.931	0.927
	Dallas	No	0.598	0.650	Yes	0.963	0.959
	Los Angeles	No	0.583	0.635	No	0.793	0.799
	Miami	No	0.647	0.662	No	0.964	0.961
	New York	No	0.881	0.901	No	0.942	0.941
	Phoenix	No	0.955	0.962	No	0.979	0.977
	San Francisco	No	0.935	0.941	No	0.948	0.944
	Seattle	No	0.732	0.779	No	0.933	0.926
	Washington DC	No	0.948	0.956	No	0.967	0.967
Venezuela	Caracas	No	0.953	0.957	Yes	1.000	1.000

1. Intersection counts are overestimated due to false nodes where grade-separated edges cross
2. Average edge lengths are underestimated
3. Connectivity is misrepresented for routing, accessibility analysis, and other topological studies

Figure 2. Map of world cities from Table 2 grouped by ϕ terciles (lower values mean less planar).

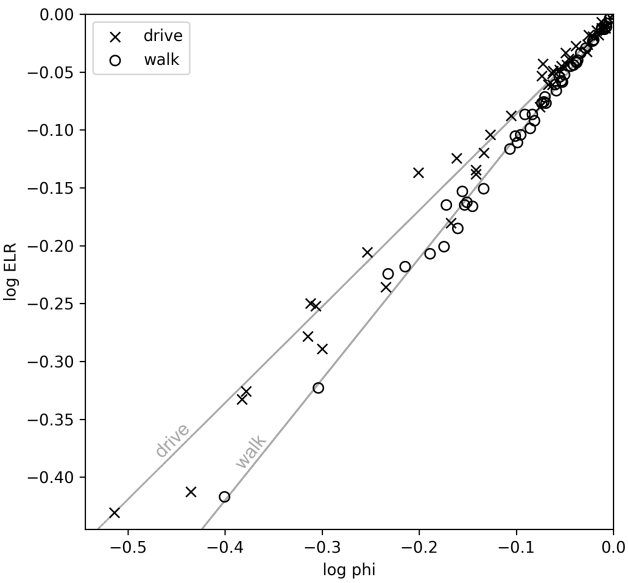

Figure 3. Log-log plot of λ vs ϕ, by network type, with simple regression lines.

Table 3. Summary statistics of planarity indicators across 100 random samples of Oakland, California's drivable network.

	ϕ	λ
count	100	100
mean	0.930	0.947
σ	0.101	0.082
min	0.569	0.637
max	1.000	1.000

Our results demonstrate how this is a bigger problem in Los Angeles than in Florence, but even in Florence the walking network is spatially and formally nonplanar due to the *sottopassaggio* (pedestrian subway) near Stazione di Santa Maria Novella, its central train station. Our results suggest that spatial planarity is inconsistent both across cities as well as across different neighborhoods within individual cities.

The problem with planar models is particularly pronounced around the downtowns of North American cities, due to the prevalence of freeways, bridges, and underpasses. Drivable networks are affected by these in particular. Walkable networks are more affected by pedestrian flyovers and subways, as in Florence. However, even networks of non-freeway, non-pedestrian-only surface streets could easily be nonplanar due to bridges or tunnels in hilly neighborhoods or over rivers.

Contrary to some of the statements in the urban studies literature, our results suggest that it cannot be universally claimed that urban street networks are planar graphs. But perhaps some of it comes down to vocabulary. If road is not synonymous with street, then a road network and a street network may not be synonymous. A road network, including freeways and boulevards, may frequently be nonplanar, but a street network, focusing on municipal streets lined by land parcels, may be at-grade and planar. But this ignores the fact that even residential streets sometimes include bridges and tunnels in hilly neighborhoods, and the fact that our analysis earlier showed that walkable circulation networks in city centers often include pedestrian tunnels and footbridges. A graph is not planar because its edges *usually* intersect only at nodes: by definition it is planar because its edges *exclusively* intersect at nodes. Thus, street networks are not universally planar graphs.

However, as Newman (2010) points out, debating the semantics of formal planarity may be missing the point – more interesting is the *extent* to which a network is planar.

Are street networks well-modeled by planar graphs?

As George Box once said, "All models are wrong but some are useful." Even if they are not formally planar, can street networks be simplified to planar graphs and still be usefully well-modeled? Our results suggest that the answer depends on the study site and on the type of analysis. In limited circumstances, where the circulation network and built form exhibit few (or ideally zero) underpasses, overpasses, or grade-separation, then perhaps yes.

But universally, we cannot answer yes. Most egregiously, imposing planarity on a nonplanar street network forces false nodes at underpasses and overpasses, breaking routing and network-based accessibility modeling. For this reason, nonplanar graphs have been the standard for decades in transportation engineering, real-world traffic assignment models, and routing engines.

But planar graphs are often used in the literature to characterize urban form and morphology. So, aside from routing, do planar graphs offer *useful* models for this type of research? Again, only in limited circumstances, such as the drivable networks in the three Italian cities we analyzed. Elsewhere, the results in Table 2 showed how common urban form measures such as intersection counts are overstated by planar models, while average street segment lengths are consequently understated. Moreover, this misrepresentation behaves inconsistently from place to place: Figure 2 demonstrated how the magnitude of bias varies across cities and modes of urbanization. Although planar graphs offer computationally tractable models and allow for the polygonal spatial analysis of urban blocks, they often modeled these street networks poorly compared to nonplanar graphs.

What can nonplanarity tell us?

We might repurpose this discrepancy between planar and nonplanar representations of a street network to reveal something useful about urban form. The ϕ and λ scores of the major US city centers indicate the greater three-dimensionality of their transportation infrastructure compared to that of the European city centers. These US city centers feature more automobile-oriented roadways and are often bounded by grade-separated freeways. By contrast, these European city centers feature less three-dimensionality as their streets and paths tend to be at-grade. Examining how ϕ and λ change over time in different cities could tell us something about the type and pace of urban development.

It is also interesting that most street networks are formally nonplanar while demonstrating ϕ and λ scores that are only slightly nonplanar. Approximate planarity is common across cities because of costs, technology, and politics. Why? It is expensive to engineer a three-dimensional network with z-axis extents (nearly) as extensive as the x- and y-axes. Furthermore, a city fully consumed by overpasses and tunnels would present degraded livability and walkability, making it politically infeasible.

Conclusion

Urban researchers often use planar graphs to model street networks. This study demonstrated that, although in limited circumstances these models may be accurate, they behave inconsistently across different kinds of cities by misrepresenting connectivity, accessibility, routing, intersection counts and densities, and street segment lengths. In most circumstances, nonplanar graphs provide better models. It also demonstrated how these indicators can be used to characterize urbanization in different cities, in particular through the transportation infrastructure's three-dimensionality.

Future research can further explore this latter finding, as it likely correlates with other measures of urbanization, development, and era. Finally, future research might examine how nonplanar intersection counts represent true intersections if multiple adjacent edges form multiple graph intersections at a point where only one true intersection exists from an urban design perspective.

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