

# Student Solutions For Honors Algebra (MATH10069) Past Papers

April 19, 2022

## Contents

|                       |          |
|-----------------------|----------|
| <b>Exam 2014-2015</b> | <b>2</b> |
| Question 1 . . . . .  | 2        |
| Q1a . . . . .         | 2        |
| Q1b . . . . .         | 2        |
| Q1c . . . . .         | 2        |
| Question 2 . . . . .  | 2        |
| Q2a . . . . .         | 2        |
| Q2b . . . . .         | 2        |
| Question 3 . . . . .  | 2        |
| Question 4 . . . . .  | 2        |
| Question 5 . . . . .  | 2        |
| <b>Exam 2015-2016</b> | <b>3</b> |
| Question 1 . . . . .  | 3        |
| Question 2 . . . . .  | 3        |
| Question 3 . . . . .  | 3        |
| Question 4 . . . . .  | 3        |
| Question 5 . . . . .  | 3        |
| <b>Exam 2016-2017</b> | <b>4</b> |
| Question 1 . . . . .  | 4        |
| Question 2 . . . . .  | 4        |
| Question 3 . . . . .  | 4        |
| Question 4 . . . . .  | 4        |
| Question 5 . . . . .  | 4        |
| <b>Exam 2017-2018</b> | <b>5</b> |
| Question 1 . . . . .  | 5        |
| Question 2 . . . . .  | 5        |
| Question 3 . . . . .  | 5        |
| Question 4 . . . . .  | 5        |
| Question 5 . . . . .  | 5        |
| <b>Exam 2018-2019</b> | <b>6</b> |
| Question 1 . . . . .  | 6        |
| Question 2 . . . . .  | 6        |
| Question 3 . . . . .  | 6        |
| Question 4 . . . . .  | 6        |

|                           |              |
|---------------------------|--------------|
| <b>Exam 2019-2020</b>     | <b>7</b>     |
| Question 1 . . . . .      | 7            |
| Question 2 . . . . .      | 7            |
| Question 3 . . . . .      | 7            |
| Q3a . . . . .             | 7            |
| Question 4 . . . . .      | 7            |
| <br><b>Exam 2020-2021</b> | <br><b>8</b> |
| Question 1 . . . . .      | 8            |
| Question 2 . . . . .      | 8            |
| Question 3 . . . . .      | 8            |

**Exam 2014-2015****Question 1****Q1a**

An example of infinite dimensional vector space over a field is  $\mathbb{R}[x]$ , the set of polynomials with coefficients in  $\mathbb{R}$ .

**Q1b**

An vector space with exactly 16 elements is  $\mathbb{Z}/16\mathbb{Z} = \{0, 1, 2, \dots, 15\}$

**Q1c****Question 2****Q2a**

To show that  $\mathcal{B}$  forms a basis, consider the matrix that represents  $\mathcal{B}$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

using gaussian elimination we find that

$$\text{rref}(B) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

or that  $\dim B = 3$  so  $\mathcal{B}$  spans  $V = \mathbb{R}^3$

**Q2b**

i)

Denote the equivalence class  $[v]$  for  $v \in V$  by

$$[v] = \{v + u : u \in U\}$$

and addition and multiplication is defined as follows

$$k[n] = [kn]$$

for all  $k \in \mathbb{R}$ , and

$$[v_1] + [v_2] = [v_1 + v_2]$$

thus the canonical mapping is simply

$$\text{can}(v) : V \rightarrow V/U = [v]$$

and therefore  $\ker(\text{can}) = 0$  as

$$[0] = \{0 + u : u \in U\}$$

**Question 3****Question 4****Question 5**

**Exam 2015-2016****Question 1****Question 2****Question 3****Question 4****Question 5**

**Exam 2016-2017****Question 1****Question 2****Question 3****Question 4****Question 5**

**Exam 2017-2018****Question 1****Question 2****Question 3****Question 4****Question 5**

**Exam 2018-2019****Question 1****Question 2****Question 3****Question 4**

**Exam 2019-2020****Question 1****Question 2****Question 3****Q3a**

i) Recall that the complex inner product has the following properties:

$$(\lambda \vec{x} + \mu \vec{y}, \vec{z}) = \lambda(\vec{x}, \vec{z}) + \mu(\vec{y}, \vec{z})$$

$$(\vec{x}, \vec{y}) = \overline{(\vec{y}, \vec{x})}$$

$$(\vec{x}, \vec{x}) \geq 0$$

**Question 4**



## Exam 2020-2021

Question 1

Question 2

Question 3