Student Solutions For Honors Algebra (MATH10069) Past Papers

April 19, 2022

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Exam 2014-2015

Question 1

Q1a

An example of infinite dimensional vector space over a field is $\mathbb{R}[x]$, the set of polynomials with coefficients in $\mathbb{R}$.

Q1b

An vector space with exactly 16 elements is $\mathbb{Z}/16\mathbb{Z} = \{0, 1, 2, \ldots, 15\}$

Q1c

Question 2

Q2a

To show that $\mathcal{B}$ forms a bsis, consider the matrix that represents $\mathcal{B}$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

using gaussian elimination we find that

$$rref(B) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

or that $\dim B = 3$ so $\mathcal{B}$ spans $V = \mathbb{R}^3$

Q2b

i)

Denote the equivalence class $[v]$ for $v \in V$ by

$$[v] = \{v + u : u \in U\}$$

and addition and multiplication is defined as follows

$$k[n] = [kn]$$

for all $k \in \mathbb{R}$, and

$$[v_1] + [v_2] = [v_1 + v_2]$$

thus the canonical mapping is simply

$$can(v) : V \to V/U = [v]$$

and therefore $\ker(can) = 0$ as

$$[0] = \{0 + u : u \in U\}$$

Question 3

Question 4

Question 5

Exam 2015-2016

Question 1

Question 2

Question 3

Question 4

Question 5

Exam 2016-2017

Question 1

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Question 3

Question 4

Question 5

Exam 2017-2018

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Exam 2018-2019

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Exam 2019-2020

Question 1

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Q3a

(i) Recall that the complex inner product has the following properties:

$$(\lambda\vec{x}+\mu\vec{y},\vec{z}) = \lambda(\vec{x},\vec{z})+\mu(\vec{y},\vec{z}) \tag{1}$$
$$(\vec{x},\vec{y}) = \overline{(\vec{y},\vec{x})} \tag{2}$$
$$(\vec{x},\vec{x}) \geq 0 \tag{3}$$

and T being self adjoint

$$(T\vec{x},y) = (\vec{x},T\vec{y})$$

Thus, let v be an eigenvector with eigenvalue λ.

$$\begin{aligned}
\lambda(x,x) &= (x,\lambda x) && \textit{(property 1)}\\
&= (x,Tx) && \textit{(eigenvec property)}\\
&= (Tx,x) && \textit{(self adjoint prop)}\\
&= (\lambda x,x) && \textit{(eigenvec propety)}\\
&= \overline{(x,\lambda x)} && \textit{(property 2)}\\
&= (x,\overline{\lambda}x) && \textit{(Not sure why)}\\
&= \overline{\lambda}(x,x) && \textit{(property 1+2)}
\end{aligned}$$

implying that $\lambda = \overline{\lambda}$, which can only be true is $\lambda \in \mathbb{R}$.

(ii)

Question 4

Exam 2020-2021

Question 1

Question 2

Question 3