

# Student Solutions For Honors Algebra (MATH10069) Past Papers

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**Exam 2014-2015****Question 1****Q1a**

An example of infinite dimensional vector space over a field is  $\mathbb{R}[x]$ , the set of polynomials with coefficients in  $\mathbb{R}$ .

**Q1b**

An vector space with exactly 16 elements is  $\mathbb{Z}/16\mathbb{Z} = \{0, 1, 2, \dots, 15\}$

**Q1c****Question 2****Q2a**

To show that  $\mathcal{B}$  forms a basis, consider the matrix that represents  $\mathcal{B}$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

using gaussian elimination we find that

$$\text{rref}(B) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

or that  $\dim B = 3$  so  $\mathcal{B}$  spans  $V = \mathbb{R}^3$

**Q2b**

i)

Denote the equivalence class  $[v]$  for  $v \in V$  by

$$[v] = \{v + u : u \in U\}$$

and addition and multiplication is defined as follows

$$k[n] = [kn]$$

for all  $k \in \mathbb{R}$ , and

$$[v_1] + [v_2] = [v_1 + v_2]$$

thus the canonical mapping is simply

$$\text{can}(v) : V \rightarrow V/U = [v]$$

and therefore  $\ker(\text{can}) = 0$  as

$$[0] = \{0 + u : u \in U\}$$

**Question 3****Question 4****Question 5**

**Exam 2015-2016****Question 1****Question 2****Question 3****Question 4****Question 5**

**Exam 2016-2017****Question 1****Question 2****Question 3****Question 4****Question 5**

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**Exam 2019-2020****Question 1****Question 2****Question 3****Q3a**

(i) Recall that the complex inner product has the following properties:

$$(\lambda \vec{x} + \mu \vec{y}, \vec{z}) = \lambda(\vec{x}, \vec{z}) + \mu(\vec{y}, \vec{z}) \quad (1)$$

$$(\vec{x}, \vec{y}) = \overline{(\vec{y}, \vec{x})} \quad (2)$$

$$(\vec{x}, \vec{x}) \geq 0 \quad (3)$$

and  $T$  being self adjoint

$$(T\vec{x}, \vec{y}) = (\vec{x}, T\vec{y})$$

Thus, let  $v$  be an eigenvector with eigenvalue  $\lambda$ .

$$\begin{aligned} \lambda(x, x) &= (x, \lambda x) && \text{(property 1)} \\ &= (x, Tx) && \text{(eigenvec property)} \\ &= (Tx, x) && \text{(self adjoint prop)} \\ &= (\lambda x, x) && \text{(eigenvec property)} \\ &= \overline{(x, \lambda x)} && \text{(property 2)} \\ &= (x, \overline{\lambda}x) && \text{(Not sure why)} \\ &= \overline{\lambda}(x, x) && \text{(property 1+2)} \end{aligned}$$

implying that  $\lambda = \overline{\lambda}$ , which can only be true is  $\lambda \in \mathbb{R}$ .

(ii)

**Question 4**



**Exam 2020-2021****Question 1****Question 2****Question 3**