Student Solutions For Honors Algebra (MATH10069) Past Papers

April 19, 2022

Contents

Exam 2014-2015

Question 1

Q1a

An example of infinite dimensional vector space over a field is $\mathbb{R}[x]$, the set of polynomials with coefficients in $\mathbb{R}$.

Q1b

An vector space with exactly 16 elements is $\mathbb{Z}/16\mathbb{Z} = \{0, 1, 2, \ldots, 15\}$

Q1c

Question 2

Q2a

To show that $\mathcal{B}$ forms a bsis, consider the matrix that represents $\mathcal{B}$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

using gaussian elimination we find that

$$rref(B) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

or that $\dim B = 3$ so $\mathcal{B}$ spans $V = \mathbb{R}^3$

Q2b

i)

Denote the equivalence class $[v]$ for $v \in V$ by

$$[v] = \{v + u : u \in U\}$$

and addition and multiplication is defined as follows

$$k[n] = [kn]$$

for all $k \in \mathbb{R}$, and

$$[v_1] + [v_2] = [v_1 + v_2]$$

thus the canonical mapping is simply

$$can(v) : V \to V/U = [v]$$

and therefore $\ker(can) = 0$ as

$$[0] = \{0 + u : u \in U\}$$

Question 3

Question 4

Question 5

Exam 2015-2016

Question 1

Question 2

Question 3

Question 4

Question 5

Exam 2016-2017

Question 1

Question 2

Question 3

Question 4

Question 5

Exam 2017-2018

Question 1

Question 2

Question 3

Question 4

Question 5

Exam 2018-2019

Question 1

Question 2

Question 3

Question 4

Exam 2019-2020

Question 1

Question 2

Question 3

Q3a

i) Recall that the complex inner product has the following properties:

$$\begin{aligned}(\lambda\vec{x} + \mu\vec{y}, \vec{z}) &= \lambda(\vec{x}, \vec{z}) + \mu(\vec{y}, \vec{z}) \\ (\vec{x}, \vec{y}) &= \overline{(\vec{y}, \vec{x})} \\ (\vec{x}, \vec{x}) &\geq 0\end{aligned}$$

Question 4

Exam 2020-2021

Question 1

Question 2

Question 3