Homework 2 - Chapter 1

- 1. (1 point) The conditional statements below are:
- i) $(p \land q) \rightarrow p$
- ii) $p \rightarrow (p \ V \ q)$
- *a. Logically equivalent
- b. Not logically equivalent
- c. Do not have a relation
- d. Cannot be compared
- 2.(1 point) Let p and q be the propositions
- p = "You drive over 65 miles per hour."
- q = "You get a speeding ticket."

Write these propositions using p and q and logical connectives (including negations). If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

*a.
$$\neg p \rightarrow \neg q$$

d.
$$\neg p \rightarrow \neg q$$

- 3. (1 point) The conditional statement are:
- $(p \to q) \to r \text{ and } p \to (q \to r)$
- a. Logically equivalent
- *b. Not logically equivalent
- c. Do not have a relation
- d. Cannot be compared
- 4.(1 point) In the following proposition

- a. p is the conclusion and q is the premise
- b. not p is the consequent and not q is the conclusion
- c. p is an hypothesis and not q is the conclusion
- *d. not p is the antecedent and not q is the consequent
- 5. (1 points) How can the following English sentence be translated into a logical expression?

"Mary is nice; and it is Monday implies that Bob is tired."

Let p = It is Monday

```
q = Mary is nice
```

t = Bob is tired

a.
$$((q \land p) \rightarrow \neg t)$$

b.
$$((q \land p) \rightarrow t)$$

c.
$$(q \rightarrow t)$$

*d.
$$(q \land (p \rightarrow t))$$

- 6. (1 point) Which statement best captures in logic the English sentence, "Somewhere, somebody done somebody wrong?"
- a. $\forall x \exists y \forall z (W(x, y, z))$
- *b. $\exists x \exists y \exists z (W(x, y, z))$
- c. $\exists x \forall y \forall z (W(x, y, z))$
- d. $\forall x \forall y \forall z (W(x, y, z))$
- 7. (1 point) Which of the following is a logical equivalence?

a.
$$(p \rightarrow q) \equiv p \land \neg q$$

b.
$$\neg(p \rightarrow q) \equiv p \lor \neg q$$

c.
$$\neg(p \rightarrow q) \equiv p \land q$$

- *d. None of the above
- 8. (1 point) Which of the following is a tautology?

*a.
$$(\neg r \land (q \rightarrow r)) \rightarrow \neg q$$

b.
$$(r \land (q \rightarrow r)) \rightarrow q$$

c.
$$(r \land (q \rightarrow r)) \rightarrow \neg q$$

$$d.\; (\neg r\; \wedge\; (\neg q \to r)) \to \neg q$$

9. (1 point) Let W(x, z) mean that student x has visited website y, where the domain for x and y consists of all students in your

school and the domain for z consists of all websites.

Which of the following expresses that there are two different people who have visited exactly the same websites ?

a.
$$\forall x \exists y \exists z ((x = y) \land (W(x, z) \leftrightarrow W(y, z)))$$

b.
$$\forall x \forall y \forall z ((x != y) \land (W(x, z) \leftrightarrow W(y, z)))$$

c.
$$\exists x \exists y \forall z ((x = y) \land (W(x, z) \leftrightarrow W(y, z)))$$

*d.
$$\exists x \exists y \forall z((x != y) \land (W(x, z) \leftrightarrow W(y, z)))$$

10. (1 point) Let L(x, y) be the statement "x loves y," where the domain for both x and y consists of all people in the world.

Which of the following English statements expresses this formal logic?

$$\exists x(\forall y L(y, x) \land \forall z((\forall w L(w, z)) \rightarrow z = x))$$

- *a. There is exactly one person whom everybody loves.
- b. There is somebody whom no one loves.
- c. There are exactly two people whom everyone loves.
- d. Everyone loves himself or herself.
- 11. (1 point) A theorem is a:
- a. statement that is always true
- b. false statement
- *c. statement that can be proven to be true
- d. proof
- 12. (1 point) Indirect proofs make use of the following:
- $a. p \land p \Leftrightarrow p$
- $b.\ p \to q \Leftrightarrow {\sim}p\ \lor\ q$
- c. p V T ⇔ T
- *d. $p \rightarrow q \Leftrightarrow \sim q \rightarrow \sim p$
- 13. (1 point) A proof that proves a theorem by checking every possibility is called a/an:
- a. Proof by contradiction
- *b. Exhaustive proof
- c. Proof by contraposition
- d. Irrational proof
- 14. (1 point) A direct proof of a conditional statement $p \rightarrow q$ assumes:
- a. p and q both are true
- b. q is true so then p must be true
- *c. p is true so then q must be true
- d. p and q both are false