Discrete Mathematics Final, Spring 2018

Professor Callahan

Name: _____

Net ID: _____ Section A or B: ___

40 multiple choice, 3 points each:

1. Consider the following function: what is its Θ run time (assuming the loop variable can be changed inside the loop)?

```
f(n):
   for i from 1 to n:
        print i;
        i = floor( i / 2)
```

a. n
b. $n \log n$
c. $\log n$
d. $n / 2$

2. A customer can choose one of 5 monitors, one of 3 keyboards, one of 2 CPUs and one of 4 printers, to buy a computer system. Determine the number of possible systems that the customer can choose from.
a. 14
b. 15
c. 100
d. 120

4. If Helen has a bag with many red, blue, yellow and green shoes and if she decides to blindly select single shoes from the bag, how many shoes must she pull out to guarantee that she has two shoes with matching colors?
a. 4
b. 5
c. 3
d. Cannot be determined

5. A coin is tossed 7 times. What's the probability of getting 7 tails?
a. 1/64
b. 1/7
c. 1/128
d. 1/32

6. Disha draws two cards out of a deck of 52 cards. What is the probability of drawing a queen and then a king without replacement? (A deck of 52 cards has 4 queens and 4 kings.)
a. 2/663
b. 4/663
c. 1/310
d. 2/371

7. In how many different ways can the letters of the word 'REAPING' be arranged, such that the vowels always come together?
a. 360
b. 480
c. 720
d. 5040

8. Which of the following functions grows the most slowly?
a. $f(x) = x \log x$
b. $f(x) = 7x$
c. $f(x) = \log x$
d. $f(x) = 7x + 2x^2$

9. Euclid's algorithm for finding the greatest common divisor works because:
a. He was a real smart guy.
b. $ax + by = 1$.
c. The number of primes is infinite.
d. Whatever divides a and b also divides b and $a - bx$, for any integer x.

10. If $A \cap B = B \cap A$, then:
a. $A = B$
b. B is a subset of A
c. A is a subset of B

d. This is always true, so it tells us nothing about *A* or *B*.

11. Which of these functions is one-to-one from R to R.
a. $f(x) = x + 5$
b. $f(x) = -3x^2 + 7$
c. $f(x) = -x^2 + 1$
d. Both a and c

12. Which of the following is a tautology?
a. $(\neg r \wedge (q \rightarrow r)) \rightarrow \neg q$
b. $(r \wedge (q \rightarrow r)) \rightarrow q$
c. $(r \wedge (q \rightarrow r)) \rightarrow \neg q$
d. $(\neg r \wedge (\neg q \rightarrow r)) \rightarrow \neg q$

13. Mathematically, a graph is defined as
a. a non-empty set of edges and a set of vertices.
b. a non-empty set of vertices and a set of edges.
c. a set of edges and a set of vertices.
d. a collection of lines and dots on paper or a computer screen.

14. How many different 8 letter words are possible if the first and third letters are both 'P'?

a. 26^4
b. 26*25*24*23*22*21*20*19
c. 26^6
d. 26^2

15. Mathematically speaking, a function
a. takes an input value in its domain and maps it to any output.
b. takes an input value in its domain and maps it to a single output.
c. takes an integer and maps it to a real number.
d. accepts some input, but may or may not produce any output.

16. What integers can be "reached" from the equation $22x + 63y$ by choosing appropriate integer values of *x* and *y*?
a. all integers
b. all multiples of 2

c. all multiples of 3
d. all multiples of 7

17. What integers can be "reached" from the equation $21x + 60y$ by choosing appropriate integer values of x and y?
a. all integers
b. all multiples of 2
c. all multiples of 3
d. all multiples of 8

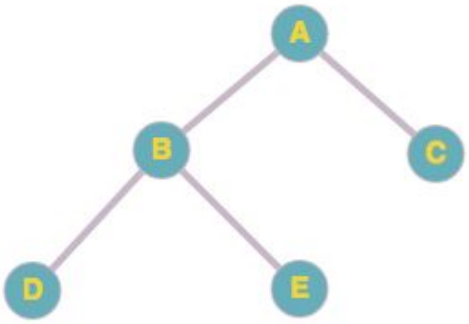
18. Is the number 9 relatively prime?
a. No, it has a prime factor of 3.
b. The question makes no sense: it is like asking "Is Joe shorter?"
c. Yes.

19. The unique prime factorization of 96 is:
a. 2 * 2 * 2 * 2 * 2 * 3
b. 2 * 48
c. 2 * 3
d. 3 * 32

20. A solution for the system $x \equiv 3 \pmod 4$ and $x \equiv 5 \pmod 6$ is
a. 36
b. 23
c. 15
d. There is no solution.

21. To find the maximum value in a binary search tree, we...
a. can just follow the rightmost path in the tree to a leaf.
b. can just follow the leftmost path in the tree to a leaf.
c. find the successor of the root node.
d. find the successor of the rightmost leaf.

22. Consider the following tree:



Which of the following traversals yield DEBCA?
a. Inorder
b. Preorder
c. Postorder
d. None of the above

23. If we are part way through building a minimum spanning tree, and we see that we have a forest containing several trees, that means we are running
a. Prim's algorithm
b. Kruskal's algorithm
c. Either Prim's algorithm or Kruskal's algorithm
d. Can't be Kruskal's or Prim's

24. Which of the following is a valid Huffman coding for an alphabet with letters having the these frequencies:
a: .01, b: .03, c: .09, d: .16, e: .24, f: .47

a. a: 00000, b: 0000, c: 000, d: 001, e: 01, f: 1
b. a: 1, b: 01, c: 001, d: 0001, e: 00001, f: 00000
c. a: 11111, b: 1111, c: 111, d: 11, e: 1, f: 0
d. a: 00000, b: 00001, c: 0001, d: 001, e: 01, f: 1

25. What sort of traversal does the following code make?

```
XTraversal(tree_root):
                if tree_root == nil:
                    return
                else:
                    XTraversal(tree_root.left)
```

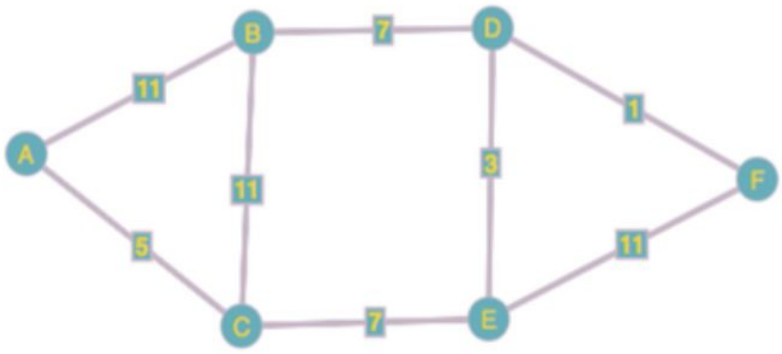
```
print(tree_root)
XTraversal(tree_root.right)
```

a. Inorder traversal
b. Postorder traversal
c. Preorder traversal
d. Prim's traversal

26. (1 point) What is the value of following prefix expression?
↑ − ∗ 2 4 ∗ 3 2 6
a. 32
b. 8
c. 4
d. 64

27. A graph has 21 vertices and 20 edges. Might it be a tree?
a. It definitely is.
b. Could be.
c. No way!
d. Can't say.

28. In finding the shortest path from A to F, what is the first time a path distance is updated by finding a second, shorter path to a vertex?



a. When we discover getting from C to B.
b. When we discover getting from B to D.
c. When we discover getting from E to D.
d. When we discover getting from E to F.

29. Consider the multigraph with following adjacency matrix:

	a	**b**	**c**	**d**	**e**
a	0	1	1	0	1
b	1	0	0	1	1
c	1	0	0	1	1
d	0	1	1	0	2
e	1	1	1	2	0

Which of the following is an Eulerian circuit in the graph?
a. d-e-c-d-e-b-a-e-b-d
b. a-b-e-d-e-d-c-e-d-b-a
c. d-e-d-c-e-d-b-a-b-e-d
d. e-b-a-e-d-c-e

30. Consider the graph described by the adjacency list:
{a: (b, c, d, g), b: (a, c), c: (a, b, d), d: (a, c), e: (g), f: (g), g: (e, f, a)}
Does it contain an Eulerian path?
a. Yes
b. No
c. Not enough information

31. A connected planar graph having 8 vertices and 12 edges contains _____ faces.
a. 20
b. 4
c. 5
d. 6

32. There are 21 people at a party. Is it possible for each person to exchange business cards with exactly 5 other people?
a. Yes
b. No
c. Not enough Information

33. K_9 is the name for:
a. Tandon's dog patrol unit.
b. The complete graph with 9 vertices.

c. The complete graph with 9 edges.
d. The cycle graph with 9 vertices.

34. A graph is called a _____ if it consists of disconnected trees.
a. worthless graph
b. directed graph
c. multigraph
d. forest

35. For which of the following combinations of the degrees of vertices would a connected graph have an Eulerian circuit?
a. 1, 2, 0
b. 2, 1, 1
c. 2, 2, 2
d. All of the above

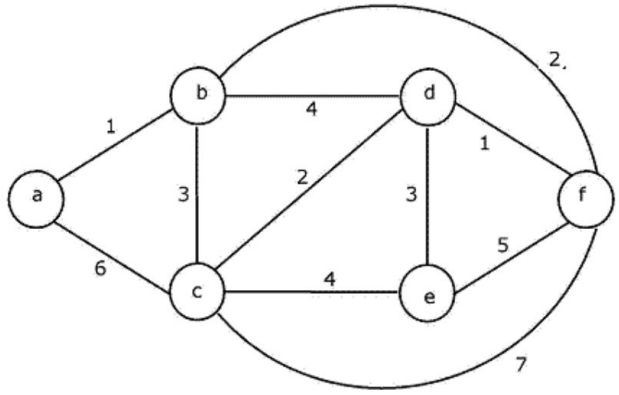
36. Using the master theorem, the runtime complexity of the recurrence $T(n) = 8T(n/2) + n^3$ is:
a. $\Theta(n^2 \log n)$
b. $\Theta(n^2)$
c. $\Theta(n^3 \log n)$
d. Can't be solved using the master theorem.

37. Using the master theorem, the runtime complexity of the recurrence $T(n) = 5T(2n) + n \log n$ is:
a. $\Theta(n^{2.32})$
b. $\Theta(n)$
c. $\Theta(n^5)$
d. Can't be solved using the master theorem.

38. All integers greater than 1 are either
a. prime or pseudo-prime
b. prime or composite
c. composite or perfect numbers
d. even or prime
39. (18, 24) =
a. 6
b. 2

c. 3
d. 1

40. Which one of the following is a possible sequence of edges added, in order, to a minimum spanning tree using Prim's algorithm?



a. (a—b), (b—f), (f—d), (d—c), (d—e)
b. (a—b), (d—f), (d—c), (b—c), (d—e)
c. (a—b), (b—d), (d—c), (b—f), (d—e)
d. (a—b), (d—f), (b—f), (d—e), (d—c)

5 long answer questions, 6 points each:

1) How does discrete mathematics differ from a branch of math like calculus?

2) How does discrete probability differ from the distributions studied in a statistics class, such as a Gaussian (bell curve) distribution?

3) Prove Euler's formula (vertices - edges + faces = 2) for a planar graph using induction.

4) Prove that Kruskal's algorithm, in making the greedy choice, must find a minimum spanning tree.

5) Prove that if d | (divides) ab, and d and a are relatively prime, then d | b.