HOMEWORK 1

1. Determine in which sets, the first is a subset of the second.
a. the set of people who play football and the set of people who play rugby
b. the set of people who speak English and the set of people
who speak Mandarin
*c. the set of prime numbers completely divisible by a coprime and the set of real numbers
d. the set of integers and the set of integers with a square root among the real numbers

2. Determine whether each of these pairs of sets are equal.
a. $\{\{1\}, \{\varnothing\}\}, \{1\}$
b. $\{\{1\}\}, \{1\}$
c. Both of the above
*d. Neither of the above

3.Determine which of these statements is true.
1) $x \in \{x\}$
2) $\{x\} \subseteq \{x\}$
3) $\{x\} \in \{x\}$
4) $\{x\} \in \{\{x\}\}$
5) $\varnothing \subseteq \{x\}$
6) $\varnothing \in \{x\}$

a. 1, 2
b. 1, 2, 3, 6
*c. 1, 2, 4, 5
d. 1, 2, 3, 4, 5

4. What is the cardinality of each of these sets?
1) $\varnothing$
2) $\{\varnothing\}$
3) $\{\varnothing, \{\varnothing\}\}$
4) $\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\}$

*a. 0, 1, 2, 3
b. 1, 1, 2, 4
c. 0, 2, 3, 4
d. 0, 1, 2, 4

5. Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. What is $A \times B \times C$

*a. {(a, x, 0), (a, x, 1), (b, x, 0), (c, x, 0), (c, x, 1), (b, x, 1), (a, y, 0), (a, y, 1), (b, y, 0), (b, y, 1), (c, y, 0), (c, y, 1)}
b. {(a, x, 0), (a, x, 1), (b, x, 0), (c, x, 0), (c, x, 1), (b, x, 1), (a, y, 0), (a, y, 1), (b, y, 0), (b, y, 1)}
c. {(a, x, 0), (a, x, 1), (b, x, 0), (c, x, 0), (c, x, 1), (b, x, 0), (b, y, 1), (a, y, 0), (a, y, 1), (b, y, 0), (b, y, 1), (c, y, 0), (c, y, 1)}
d. {(a, x, 0), (a, x, 1), (b, x, 0), (c, x, 0), (c, x, 1), (b, x, 0), (b, y, 1), (a, y, 0), (a, y, 1), (b, y, 0), (b, y, 1), (c, y, 0), (c, y, 1)}

6.Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find
i) $A \cup B$.
ii) $A \cap B$.
iii) $A - B$.
iv) $B - A$

a. **i)** {1, 2, 3, 4, 5, 0, 3, 6}, **ii)** {1, 2, 4, 5}, **iii)** {3}, {0, 6}, **iv)** {3}
b. **i)** {1, 2, 3, 4, 5, 0, 3, 6}, **ii)** {3}, {0, 6}, **iii)** {1, 2, 4, 5}, **iv)** {3}
c. **i)** {1, 2, 3, 4, 5, 0, 3, 6}, **ii)** {3}, **iii)** {0, 6}, **iv)** {3}
*d. **i)** {0, 1, 2, 3, 4, 5, 3, 6}, **ii)** {3}, **iii)** {1, 2, 4, 5}, **iv)** {0, 6}

7. If $A = \{3, 7, 9\}$, $B = \{4, 6, 12, 16, 24\}$ and $C = \{1, 3, 12, 24, 36\}$, then $(A \cup B) \cap C$:
a. {1, 3, 6, 7, 9, 12, 16, 24, 36}
b. {1, 3, 12, 24, 36}
c. {3, 7, 9, 24, 36}
*d. {3, 12, 24}

8. If $A \cup B = A$, then:
a. $B = \varnothing$
*b. B is a subset of A
c. B is the powerset of A
d. A is a subset of B

9.If $A \cap B = A$, then:
*a. A is a subset of B
b. B is a subset of A
c. $A = \varnothing$
d. $A = B$

10. If $A \cap B = B \cap A$, then:
a. $A = B$
b. B is a subset of A
c. A is a subset of B
*d. This is always true, so it tells us nothing about A or B

11.Determine whether f is a function from Z to R if

(i) $f(n) = \pm(2n + 3)$.
(ii) $f(n) = 0$
(iii) $f(n) = 1 / (n^2 - 1)$.
(iv) $f(n) = n / (n^2 + 1)$

a. Yes, Yes, No, Yes
*b. No, Yes, No, Yes
c. No, Yes, Yes, Yes
d. Yes, Yes, Yes, Yes

12. Determine whether each of these functions from Z to Z
is one-to-one.
(i) $f(n) = \sqrt{n+1}$
(ii) $f(n) = n^2 + 3n + 4$
(iii) $f(n) = n^5 + 5$
(iv) $f(n) = \lceil n/4 \rceil$

a. Yes , Yes, No, Yes
b. No , No, No, Yes
c. No , Yes, Yes, Yes
*d. Yes , No, Yes, No

13. Determine which of these functions is a bijection from R to R.
a. $f(x) = 3x + 4$
b. $f(x) = -3x^2 + 7$
c. $f(x) = x^2 + 1$
*d. Both a and c

14. The Fibonacci sequence is defined as follows:
$f_0 = 0$
$f_1 = 1$
$f_2 = f_1 + f_0 = 1 + 0 = 1$
$f_3 = f_2 + f_1 = 1 + 1 = 2$
Generally, for any term in position n in the sequence: $f_n = f_{n-1} + f_{n-2}$
The Fibonacci numbers f_4, f_5, f_6, f_7 are:
a. 3, 4, 6, 8
*b. 3, 5, 8, 13
c. 3, 5, 7, 12
d. 3, 5, 8, 12

15. Given the following terms $a_1 = 9$ and $a_2 = 14$, what is a_3 in an arithmetic sequence?

a. 19
b. 64
*c. 19
d. 40