Discrete Mathematics Quiz 12

Name: ______________________________

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Each question carries one point. Please do all 5 questions on both sides of the paper. Total will be best 4 out of 5.

1.1) How many binary strings of length 10 have at least 2 adjacent bits that are the same ("00" or "11") somewhere in the string?
a. 2^{10}
b. $2^{10} - 1$
*c. $2^{10} - 2$
d. $2^{10}/2$

1.2) How many strings of length 10 over the alphabet {a, b, c, d} have at least one b somewhere in the string?
a. 3^{10}
*b. $4^{10} - 3^{10}$
c. $10 \cdot 4^9$
d. $10 \cdot 3^9$

1.3) How many binary strings of length 10 are there in which the number of 0's in the string is not equal to the number of 1's in the string?
a. 2^{10}
b. 2^9
c. $2^{10} - 2^9$
*d. $2^{10} - \binom{10}{5}$

1.4) A class of 30 students with 14 boys and 16 girls must select 4 leaders. How many ways are there to select the 4 leaders so that at least one girl is selected?
a. $16 \cdot \binom{29}{3}$

b. $16 \cdot \binom{30}{3}$
c. $\binom{30}{4} - \binom{16}{4}$
*d. $\binom{30}{4} - \binom{14}{4}$

2.1) A store sells 6 varieties of donuts. Chocolate is one of the varieties sold. How many ways are there to select 14 donuts if at most 4 Chocolate donuts are selected?
a. $\binom{19}{5} - \binom{15}{5}$
b. $\binom{19}{5} - \binom{15}{4}$
*c. $\binom{19}{5} - \binom{14}{5}$
d. $\binom{19}{5}$

2.2) A store sells 6 varieties of donuts. Chocolate is one of the varieties sold. How many ways are there to select 14 donuts if at least 4 Chocolate donuts are selected?
*a. $\binom{15}{5}$
b. $\binom{19}{5} - \binom{15}{4}$
c. $\binom{19}{5} - \binom{14}{5}$
d. $\binom{14}{5}$

2.3) A store sells 57 varieties of juice. Juice is sold in cans and two cans of the same variety are identical. How many ways are there to select 6 cans of juice if there is no restriction on the number of each variety selected?
a. $\binom{57}{5}$
b. $\binom{57}{6}$
c. $\binom{62}{5}$
*d. $\binom{62}{56}$

2.4) How many solutions are there to the equation: $x_1 + x_2 + x_3 + x_4 + x_5 = 39$,
where each x_i is an integer that satisfies $x_i \geq 3$?
a. $\binom{24}{5}$
*b. $\binom{28}{4}$
c. $\binom{36}{5}$
d. $\binom{40}{4}$

3.1) How many strings of length 14 over the alphabet {a, b, c, d} have exactly three a's or exactly three b's or both?

a. $2 \cdot \binom{14}{3} \cdot 3^{11} - \binom{14}{6} \cdot 2^{8}$

b. $2 \cdot \binom{14}{3} \cdot 3^{11} - \binom{14}{6} \cdot 3^{8}$

*c. $2 \cdot \binom{14}{3} \cdot 3^{11} - \binom{14}{3} \cdot \binom{11}{3} \cdot 2^{8}$

d. $2 \cdot \binom{14}{3} \cdot 3^{11} - \binom{14}{3} \cdot \binom{11}{3} \cdot 3^{8}$

3.2) How many strings of length 12 over the alphabet {a, b, c, d} start with "aa" or end with "aa" or both?

a. $2 \cdot 4^{10} + 4^{8}$

*b. $2 \cdot 4^{10} - 4^{8}$

c. $2 \cdot \binom{12}{2} \cdot 4^{10} + \binom{12}{4} \cdot 4^{8}$

d. $2 \cdot \binom{12}{2} \cdot 4^{10} - \binom{12}{4} \cdot 4^{8}$

4.1) How large must a group of people be in order to guarantee that there are at least two people in the group whose birthdays fall in the same month?

a. 1 people

b. 2 people

c. 12 people

*d. 13 people

4.2) Each person in a group weighs at least 100 pounds and at most 130 pounds. How large must the group be in order to guarantee that there are at least 2 people whose weights differ by at most 9 pounds?

*a. 5 people

b. 6 people

c. 30 people

d. 31 people

4.3) A basket holds a set of balls. Each ball is red, green, or blue. How many balls must there be in the basket in order to guarantee that there are at least 5 balls of the same color?
a. 12 balls
*b. 13 balls
c. 14 balls
d. 15 balls

5.1)

Given the binomial Theorem: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$ Prove: $2^n = \sum_{k=0}^{n} \binom{n}{k}$

Let a = 1 and b = 1, $(1+1)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} 1^k$, $(2)^n = \sum_{k=0}^{n} \binom{n}{k} 1^n$, $2^n = \sum_{k=0}^{n} \binom{n}{k}$

5.2)

Given the binomial Theorem: $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$ Prove: $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$

Let a = 1 and b = -1, $(1-1)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} (-1)^k$, $(0)^n = \sum_{k=0}^{n} \binom{n}{k} (-1)^k$,
$0 = \sum_{k=0}^{n} \binom{n}{k} (-1)^k$, $\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$