1 Exercise III.1

Compute the probability of measuring $|0\rangle$ and $|1\rangle$ for each of the following quantum states:

1. $0.6|0\rangle + 0.8|1\rangle$.
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2. $\frac{1}{\sqrt{3}}|0\rangle + \sqrt{2/3}|1\rangle$.
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3. $\frac{\sqrt{3}}{2}|0\rangle - \frac{1}{2}|1\rangle$.
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4. $-\frac{1}{25}(24|0\rangle - 7|1\rangle)$.
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5. $-\frac{1}{\sqrt{2}}|0\rangle + \frac{e^{i}\pi/6}{\sqrt{2}}|1\rangle$.
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2 Exercise III.2

Compute the probability of the four states if the following are measured in the computational basis:

1. $(e^{i}|00\rangle + \sqrt{2}|01\rangle + \sqrt{3}|10\rangle + 2e^{2}i|11\rangle)/\sqrt{10}$.
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2. $\frac{1}{2}(-|0\rangle + |1\rangle) \otimes (e^{\pi i}|0\rangle + e^{-\pi i}|1\rangle)$.
    -
3. $(\sqrt{1/3}|0\rangle - \sqrt{2/3}|1\rangle) \otimes \sqrt{2}(\frac{e^{\pi i/4}}{2}|0\rangle + \frac{e^{\pi i/2}}{2}|1\rangle)$.
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3 Exercise III.3

Suppose that a two-quibit register is in the state

$$|\psi\rangle = \frac{3}{5}|00\rangle - \frac{\sqrt{7}}{5}|01\rangle + \frac{e^{i\pi/2}}{\sqrt{5}}|10\rangle - \frac{2}{5}|11\rangle$$

1. Suppose we measure just the first qubit. Compute the probability of measuring a $|0\rangle$ or a $|1\rangle$ and the resulting register state in each case.

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2. Do the same, but supposing instead that we measure just the second qubit.
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4 Exercise III.12

Show that the four Bell states are orthonormal (i.e., both orthogonal and normalized).

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5 Exercise III.13

Prove that $|\beta_{11}\rangle$ is entangled.

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6 Exercise III.14

Prove that $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$ is entangled.

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7 Exercise III.30

Use a single Toffoli gate to implement each of NOT, NAND, and XOR.

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8 Exercise III.34

Prove:

$$|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle),$$

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$$|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle).$$

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9 Exercise III.35

What are the possible outcomes (probabilities and resulting states) of measuring $a|+\rangle + b|-\rangle$ in the *computational basis* (of course, $|a|^2 + |b|^2 = 1$)?

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