HILL UNIVERSITY
Established by an Act of the State Legislature of Uttarakhand (Adhiniyam Sankhya 12 of 2011)
University under section 2(f) of UGC Act, 1956

Term Evaluation (Odd) Semester Examination September 2025

Roll No.............................

Name of the Course: B.Tech
Semester: I
Name of the Paper: Engineering Mathematics-I
Paper Code: TMA-101
Time: 1.5 Hour   **Maximum Marks: 50**

Note:
 (i)   Answer all the questions by choosing any one of the sub-questions.
 (ii) Each question carries 10 marks.

Q1.   (10 Marks, CO1)
a.   Find the rank of the matrix A, by reducing it to Normal form.

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}.$$

OR

b. Find for what values of λ and μ, the system of linear equations:

$$x + y + z = 6,$$
$$x + 2y + 5z = 10,$$
$$2x + 3y + \lambda z = \mu,$$

has (i) a unique solution (ii) no solution (iii) infinite solutions.

Q2.   (10 Marks, CO1)

a. Using Cayley-Hamilton theorem, find the inverse of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}.$

OR

b.   Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$

Q3.   (10 Marks, CO1&CO2)
a. Show that the following matrix is Skew-Hermitian matrix

$$A = \begin{bmatrix} -i & 3+3i & -2-i \\ -3+3i & 0 & 3-4i \\ 2-i & -3-4i & -2i \end{bmatrix}.$$

OR

b. Evaluate $\lim\limits_{x \to 0}\left(\dfrac{\tan x}{x}\right)^{1/x}.$

Graphic Era
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Q4. (10 Marks, CO2)

a. Prove that $\dfrac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \dfrac{b-a}{1+a^2}, \; 0 < a < b.$ Hence, show that $\dfrac{\pi}{4} + \dfrac{3}{25} < \tan^{-1}\dfrac{4}{5} < \dfrac{\pi}{4} + \dfrac{1}{6}.$

OR

b. Find the maximum and minimum value of the function
$y = \sin x(1 + \cos x)$ in $(0, \pi)$.

Q5. (10 Marks, CO2)

a. If $y = (\sin^{-1} x)^2$, prove that

$$y_n(0) \;=\; 0, \qquad\qquad \text{for } n \text{ odd and}$$

$$y_n(0) \;=\; 2.2^2.4^2.6^2 \ldots\ldots\ldots (n-2)^2, \quad n \neq 2 \text{ for } n \text{ even.}$$

OR

b. Find the Maclaurin's series for the function $f(x) = (1+x)^m$ where m is not necessarily an integer and

hence, use your answer to find the expansion of $\dfrac{1}{\sqrt{1-x^2}}$ up to the term in x^6.