Graphic Era HILL UNIVERSITY
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End Term (Odd) Semester Examination November 2025

Roll no..............................

Name of the Course and semester: **B.Tech. ECE V**
Name of the Paper: **Probability Theory and Stochastic Processes**
Paper Code: **TEC 505**
Time: 3 hour Maximum Marks: 100

Note:
- *(i)* All the questions are compulsory.
- *(ii)* Answer any two sub questions from a, b and c in each main question.
- *(iii)* Total marks for each question is 20 (twenty).
- *(iv)* Each sub-question carries 10 marks.

Q1. (2X10=20 Marks)

a. Prove that $(A \cup B)' = A' \cap B'$. CO1

b. In a competitive examination 30 candidates are to be selected. In all 600 candidates appear in written test, and 100 will be called for interview. What is the probability that a person will be called for interview? Find the probability of a person getting selected, if he has been called for interview. CO1

c. In a test, an examinee either guesses or copies or knows the answer to a multiple-choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it is $\frac{1}{8}$. Find the probability that he knows the answer to the question, given that he correctly answered it. CO1

Q2. (2X10=20 Marks)

a. If X is random variable for probability mass function is $p(x) = \begin{cases} \frac{x}{15}, & x = 1,2,3,\ldots \\ 0, & elsewhere \end{cases}$. CO2

Find (i) $P(X = 1 \text{ } or \text{ } 2)$ and (ii) $P\left(\left(\frac{1}{2} < X < \frac{5}{2}\right)/(X > 1)\right)$.

b. Four coins are tossed what is the expectation of a number of head? CO2

c. A random variable X has the following probability function:

X	-2	-1	0	1	2	3
$P\ (X = x)$	0.1	k	0.2	$2k$	0.3	k

Find the value of k and calculate mean and variance. CO2

Q3. (2X10=20 Marks)

a. A random variable gives measurements X between 0 and 1 with a probability function: CO3

$$f(x) = \begin{cases} 12x^3 + 21x^2 + 10x, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}$$

(i) Find $P\left(x \le \frac{1}{2}\right)$ and $P\left(x > \frac{1}{2}\right)$.

(ii) Find a number K such that $P(X \le K) = \frac{1}{2}$.

b. The frequency function of a continuous random variable is given by $f(x) = y_0\, x(2 - x),\ 0 \le x \le 2$. Find the value of y_0, mean and variance of X. CO3

c. The length of a telephone call made to a company is denoted by the continuous random variable T. It is modelled by the probability density function $f(t) = \begin{cases} kt, & 0 \le t \le 10 \\ 0, & otherwise \end{cases}$ CO3

(i) Find the value of k (ii) Find $P(T > 6)$ (iii) Calculate an exact value for $E(T)$ and for $Var(T)$

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Q4. (2X10=20 Marks)

a. The joint probability distribution of X and Y is given by the following table: CO4

$X \backslash Y$	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

(i) Find the marginal probability distribution of Y.
(ii) Find the conditional distribution of Y given that $X = 2$.

b. Toss 3 coins. Let X denotes the number of heads on the first two and Y denotes the number of heads on the last two. CO4

(i) Find the joint distribution of X and Y.
(ii) Find $E(Y/X = 1)$
(iii) Find correlation coefficient $\rho\ (X, Y)$.

c. If X and Y are two random variables having joint density function CO4

$$f(x,y) = \begin{cases} \frac{1}{8}(6 - x - y), & 0 < x < 2,\ 2 < y < 4 \\ 0, & otherwise \end{cases}$$

Find (i) $\boldsymbol{P(X < 1 \cap Y < 3)}$, (ii) $\boldsymbol{P(X + Y < 3)}$, (iii) $\boldsymbol{P(X < 1 | Y < 3)}$.

Q5. (2X10=20 Marks)

a. Let $\{X(t), t \in \mathbb{R}\}$ be a continuous time random process, defined as

$$X(t) = A\,cos(2t + \Phi),$$

Where $A \sim U(0,1)$ and $\Phi \sim U(0, 2\pi)$ are two independent random variables. CO5

(i) Find the mean function, $\mu_X(t)$.
(ii) Find the correlation function $R_X(t_1, t_2)$.
(iii) Is $X(t)$ a WSS process?

b. Let $X(t)$ be a zero-mean WSS Gaussian random process with $R_X(\tau) = e^{-\pi\tau^2}$. Suppose that $X(t)$ is input to an LTI system with transfer function $|H(f)| = e^{-\frac{3}{2}\pi f^2}$. Let $Y(t)$ be the output. CO5

(i) Find μ_Y, $R_Y(\tau)$ and $Var(Y(t))$.
(ii) Find $E(Y(3)|Y(1) = -1)$ and $Var(Y(3)|Y(1) = -1)$.

c. Given the power spectral density of a continuous process as $S_{XX}(\omega) = \frac{\omega^2+9}{\omega^4+5\omega^2+4}$. Find the mean square value of the process. CO5