Graphic Era HILL UNIVERSITY

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University under section 2(f) of UGC Act, 1956

GEHU/04M/9.1.3

Term Evaluation (Odd) Semester Examination : September 2025

Roll No...........................

Name of the Course : **B.Tech. (Mechanical Engineering)**
Semester : **Third**
Name of the Paper : **Engineering Mathematics -III**
Paper Code : **TMA-303**

Time : 1.5 Hour **Maximum Marks : 50**

Note : (i) Answer all the questions by choosing any one of the sub-questions.
(ii) Each question carries 10 marks.

1. (a) Solve the partial differential equation $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$. (CO1) **(10 Marks)**

OR

(b) Solve $(mz-ny)\dfrac{\partial z}{\partial x} + (nx-lz)\dfrac{\partial z}{\partial y} = (ly-mx)$. (CO1) **(10 Marks)**

2. (a) Solve $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x \partial y} - 6\dfrac{\partial^2 z}{\partial y^2} = \sqrt{x+y}$. (CO1) **(10 Marks)**

OR

(b) Solve $\dfrac{\partial^3 z}{\partial x^2 \partial y} - 2\dfrac{\partial^3 z}{\partial x \partial y^2} + \dfrac{\partial^3 z}{\partial y^3} = \dfrac{1}{x^2}$. (CO1) **(10 Marks)**

3. (a) The vibrations of a elastic string is generated by the partial differential equation- (CO1) **(10 Marks)**

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}.$$

The lenght of the string is π and the ends are fixed. The initial velocity is zero and the initial deflection is $u(x,0) = 2(\sin x + \sin 3x)$. Find the deflection $u(x,t)$ of the vibrating string for $t > 0$.

OR

(b) Find the solution to the following IBVP - (CO2) **(10 Marks)**

$$u_t = 3u_{xx},\ 0 \le x \le \pi,\ 0 < t < \infty,$$
$$u(0,t) = u(\pi,t) = 0,\ 0 < t < \infty,$$
$$u(x,0) = 3\sin 2x - 6\sin 5x,\ 0 \le x \le \pi.$$

4. (a) Find a positive root of the equation $xe^x = 1$, using Fixed-Point Iteration method. (CO2) **(10 Marks)**

OR

(b) Using Euler's method, solve the initial value problem (CO2) **(10 Marks)**

$$\frac{dy}{dx} = x + y + xy, \quad y(0) = 1.$$

Hence, compute the value of y at $x = 0.1$ using a step size of $h = 0.05$.

5. (a) Use the Runge-Kutta method with step sizes $h = 0.1$, $h = 0.05$, and $h = 0.025$ to find approximate values of the solution to the initial value problem - (CO2)

$$y' + 2xy = 3x^3 + 1, \quad y(1) = 1$$

(10 Marks)

at

$$x = 1.0,\ 1.1,\ 1.2,\ 1.3,\ \ldots,\ 2.0.$$

Hence, compare these approximate values with the values of the exact solution

$$y = \frac{1}{3}x^2\left(9\ln x + x^3 + 2\right).$$

OR

(b) Find the approximate value of - (CO2)

(10 Marks)

$$I = \int_1^2 \frac{dx}{5 + 3x},$$

using Simpson's $\frac{1}{3}$ rule with 4 and 8 equal subintervals. Using the exact solution, find the absolute errors.