Graphic Era HILL UNIVERSITY
Established by an Act of the State Legislature of Uttarakhand (Adhiniyam Sankhya 12 of 2011)
University under section 2(f) of UGC Act, 1956

End Term (Odd) Semester Examination, December 2025

Roll No............................

Name of the Course and Semester : **BCA (AI&DS) - I Semester**
Name of the Paper : **Mathematical Foundations for AI**
Paper Code : **TBD-103**
Time : 3 Hours

Maximum Marks: 100

Note : (i) All the questions are compulsory.
(ii) Answer any two sub questions from a, b and c in each main question.
(iii) Total marks for each question is 20 (twenty).
(iv) Each sub-question carries 10 marks.

1. (a) Explain disjoint set, equal set, difference of two sets and union of two sets with suitable examples. (CO1)

(b) Prove the Demorgan's Law on basis of the set theory. (CO1)

(c) Out of 80 students in a school, 60 play cricket, 53 play hockey, and 35 play both the games. How many students:
(i) do not play these games,
(ii) play only hockey but not cricket. (CO1)

(20 Marks)

2. (a) Define the following terms with an example:
(i) Cartesian product of two sets,
(ii) Bijective function,
(iii) Inverse function. (CO2)

(b) Define composition of the functions. Let f, g and h be functions from N to N, where N is the set of natural numbers, such that $f(x) = x^2 + 2$, $g(x) = 3x + 5$ and $h(x) = 2x$. Then determine:
(i) fog (ii) gof (iii) foh (CO2)

(c) Show that the relation $\leq$ (less than or equal to) defined on the set of positive integers is a partial order relation. (CO2)

(20 Marks)

3. (a) Define the following terms with an example:
(i) Proposition (or Statement),
(ii) Contingency,
(iii) Truth tables. (CO3)

(b) With the help of truth tables, prove that (note that, $\neg$ denotes the negation, $\wedge$ denotes the conjunction and $\vee$ dentoes the disjunction):
(i) $\neg p \vee \neg q = \neg(p \wedge q)$,
(ii) $p \vee q = \neg(\neg p \wedge \neg q)$. (CO3)

(c) Define the three basic logical operations, namely, conjunction, disjunction and negation with truth tables. (CO3)

(20 Marks)

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4. (a) Define the following terms with an example:
   (i) Random experiment,
   (ii) Mutually exclusive events,
   (iii) Dependent event,
   (iv) Independent event. (CO4)

   (b) Using the principle of mathematical induction, prove that
   $P(n): 1^2 + 2^2 + 3^2 + 4^2 + \cdots + n^2 = \frac{n(n+1)(n+2)}{6}$. (CO4)

   (c) Using the principle of mathematical induction, prove that
   $P(n): 1.2 + 2.2^2 + 3.2^3 + \cdots + n.2^n = (n-1)2^{n+1} + 2$. (CO4)

   (20 Marks)

5. (a) Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}.$$

(CO5)

   (b) If

$$A = \begin{bmatrix} 3 & -1 \\ -3 & -5 \end{bmatrix},$$

   then find $A^3 - 3A + 3I$. (CO5)

   (c) Find the determinant of the following matrices by using rule of Sarrus:

$$A = \begin{bmatrix} 1 & -1 & 3 \\ -2 & 0 & 1 \\ 6 & 3 & 1 \end{bmatrix}, \quad and \quad B = \begin{bmatrix} 1 & 0 & 8 \\ -3 & 4 & 3 \\ -1 & -2 & 3 \end{bmatrix}.$$

(CO5)

   (20 Marks)