Graphic Era HILL UNIVERSITY
Established by an Act of the State Legislature of Uttarakhand (Adhiniyam Sankhya 12 of 2011)
University under section 2(f) of UGC Act. 1956

End Term (Odd) Semester Examination November 2025

Roll no..............................

Name of the Course and Semester: Bachelor of Technology Semester: I
Name of the Paper: Engineering Mathematics for Artificial Intelligence-I
Paper Code: TMA-102
Time: 3:00 hours **Maximum Marks: 100**

Note:
- (i) All the questions are compulsory.
- (ii) Answer any two sub questions from a, b and c in each main question.
- (iii) Total marks for each question is 20 (twenty).
- (iv) Each sub-question carries 10 marks.

Q1. $(10\times 2=20)$ CO:1

a. Using elementary transformations find the inverse of $A=\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

b. Is the system of vector $X_1=(2,2,1)^T$, $X_2=(1,3,1)^T$, $X_3=(1,2,2)^T$ linearly dependent?

c. Find all eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

Q2. $(10\times 2=20)$ CO:2

a. With the help of truth tables prove that:
$(i)\ p\vee q = \sim(\sim p \wedge \sim q)$ $(ii)\ \sim(p\wedge q) = \sim p \vee \sim q$.

b. Consider the functions $f,g: R\to R$ defined by $f(x)=x^2+3x+1$, $g(x)=2x-3$. Find the composition functions (i) fof and (ii) fog.

c. What is equivalence relation?
Let $A=\{1,2,3,4\}$ and $R=\{(1,1),(1,3),(2,2),(2,4),(3,1),(3,3),(4,2),(4,4)\}$.
Show that R is an equivalence relation.

Q3. $(10\times 2=20)$ CO:3

a. If $u=e^{xyz}$, find the value of $\dfrac{\partial^3 u}{\partial x\partial y\partial z}$.

b. Evaluate $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{1/x^2}$.

c. If $y=\sin ax+\cos ax$, prove that $y_n = a^n\left[1+(-1)^n \sin 2ax\right]^{1/2}$.

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Q4. $(10\times 2=20)$ CO:4

a. Examine $f(x,y)=x^3+y^3-3axy$ for maximum and minimum values.

b. If $y_1=\dfrac{x_2x_3}{x_1},\ y_2=\dfrac{x_3x_1}{x_2},\ y_3=\dfrac{x_1x_2}{x_3}$, show that the *Jacobian* of y_1, y_2, y_3 with respect x_1, x_2, x_3 is 4.

c. If $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, show that: (i) $grad\, r=\dfrac{\vec{r}}{r}$ and (ii) $grad\left(\dfrac{1}{r}\right)=-\dfrac{\vec{r}}{r^3}$.

Q5. $(10\times 2=20)$ CO:5

a. Evaluate: (i) $\int_0^\infty \sqrt{x}\, e^{-\sqrt[3]{x}}\,dx$ (ii) $\int_0^1 x^4\left(1-\sqrt{x}\right)^5 dx$.

b. Change the order of integration and evaluate $\displaystyle\int_0^a\int_y^a \frac{x}{x^2+y^2}\,dx\,dy$.

c. Evaluate: $\displaystyle\int_0^{\log 2}\int_0^x\int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$