Graphic Era
HILL UNIVERSITY
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University under section 2(f) of UGC Act, 1956

GEHU/04M/9.1.3

End Term Examination, December 2025

Roll No.................

Name of the Course: B. Tech.
Semester: I
Name of the Paper: Engineering Mathematics- I
Paper Code: TMA - 101
Time: 3 Hour **Maximum Marks: 100**

Note: (a) Answer **all the questions** by choosing ***any two of the sub questions.***

(b) Each question contains three parts (a) (b) & (c). Attempt any two parts of choice from each question.

Q 1. **CO-1 (10×2 Marks)**

a. Verify Cayley Hamilton theorem for $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ and hence find A^{-1}.

b. Determine for what values of p and q the following equations have (i) no solution and (ii) a unique solution; $x + y + z = 6, x + 2y + 3z = 10, x + 2y + pz = q$.

c. Find all eigenvalues and eigenvectors of $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

Q2. **CO-2 (10×2 Marks)**

a. If $u = \sin^{-1}[\frac{x+y}{\sqrt{x}+\sqrt{y}}]$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$.

b. If $y = \sin(m\sin^{-1} x)$, prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

c. Expand $e^{\sin x}$ by Maclaurin's series upto the terms containing x^4.

Q3. **CO-3 (10×2 Marks)**

a. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$.

b. If $u = xy + yz + zx$, $v = x^2 + y^2 + z^2$ and $w = x + y + z$, determine whether there is a functional relationship between u, v, w and if so, find it.

c. Find the point on the plane $ax + by + cz = p$ at which the function $f = x^2 + y^2 + z^2$ has a minimum value and find this minimum f.

Q4. **CO- 4 (10×2 Marks)**

a. Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$.

b. Using Beta and Gamma functions, evaluate $\int_0^{\infty}\frac{dx}{1+x^4}$.

c. Prove that $B(m,m) = 2^{1-2m}\,B(m,\frac{1}{2})$.

Q5. **CO-5 (10×2 Marks)**

a. Using Green's theorem, evaluate $\int_c (x^2y\,dx + x^2dy)$, where c is the boundary described counter clockwise of the triangle with vertices $(0,0), (1,0), (1,1)$.

b. Apply Stoke's theorem to find the value of $\int_c (ydx + z\,dy + x\,dz)$, where c is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$.

c. If $u = x + y + z, v = x^2 + y^2 + z^2, w = xy + yz + zx$ prove that $grad\ u, grad\ v\ and\ grad\ w$ are coplanar vectors.