Graphic Era HILL UNIVERSITY

Established by an Act of the State Legislature of Uttarakhand (Adhiniyam Sankhya 12 of 2011)
University under section 2(f) of UGC Act, 1956

End Term (Odd) Semester Examination November 2025

Roll no............................

Name of the Course and semester: Bachelor of Technology, 3rd semester
Name of the Paper: Discrete Structures and Combinatorics
Paper Code: TMA 316
Time: 3 hours | Maximum Marks: 100

Note:

(i) All the questions are compulsory.
(ii) Answer any two sub questions from a, b and c in each main question.
(iii) Total marks for each question is 20 (twenty).
(iv) Each sub-question carries 10 marks.

Q1. (2x10=20 Marks) (CO1)

a. Let $X=\{a,b,c\}$ and $P(X)$ denotes the power set of set X. Prove that $(P(X),\subseteq)$ is a poset and hence construct its Hasse diagram. Find
(i) Greatest element
(ii) Least element
(iii) Greatest lower bound of $A=\{\{a\},\{c\}\}$
(iv) Upper bound(s) of $B=\{\{a\},\{b,c\}\}$

b. Determine whether the function $f:\mathbb{R}\to\mathbb{R}$ defined as $f(x)=4(x-3)^5+21$ is invertible. If yes, find its inverse.

c. Define an equivalence relation. Test whether $R=\{(x,y):|x-y| \text{ is an even number}\}$, where R is a relation defined on the set of integers, is an equivalence relation.

Q2. (2x10=20 Marks) (CO2)

a. A random variable X has the following probability distribution:

X	0	2	4	6	8	10	12	14
$P(X=x)$	$8a$	$3a$	$5a$	$6a$	$3a$	$6a$	$4a$	$5a$

(i) Find the value of a.
(ii) Find the mean and variance of X.
(iii) Find $P(X<10)$, $P(X>6)$ and $P(2<X<10)$.

b. The length of life of a product produced by a machine has a normal distribution with a mean of 30 months and a standard deviation of 5 months. Find the probability that a product which is produced by this machine will last
(i) less than 30 months (ii) between 26 and 40 months (iii) more than 45 months.
Given $\phi(Z=2)=0.9772$, $\phi(Z=0.8)=0.7881$, $\phi(Z=3)=0.9987$, $\phi(Z=0)=0.5$, where $\phi(Z=z)=P(Z\le z)$.

c. If X is a Poisson variate such that $P(X=2)=9\times P(X=4)+90\times P(X=6)$, find the value of the parameter λ.

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Q3. (2x10=20 Marks) (CO3)

a. Define a valid argument. Determine whether the following argument is valid:
"If I study, then I will pass in examination. If I do not go to cinema, then I will study. But I failed in examination. Therefore, I went to cinema."

b. Using the principle of mathematical induction, prove that $n^3 - 4n + 6$ is divisible by 3 for all positive integers n.

c. Determine whether the given propositions are tautologies:
(i) $\left[(p \to r) \wedge (\neg q \to p) \wedge \neg r\right] \to q$ (ii) $\left[(p \vee q) \wedge (p \to r) \wedge (q \to r)\right] \to r$,
where $\neg q$ denotes negation of q.

Q4. (2x10=20 Marks) (CO4)

a. Show that the set of all positive rational numbers Q^{+} forms an abelian group under the composition defined by $a * b = \dfrac{ab}{2}$.

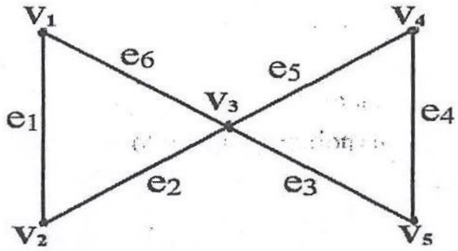
b. Solve the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2$ with initial conditions $a_0 = 1$ and $a_1 = -1$.

c. Find the number of ways in which a string of 4 letters (without repetition) can be formed using the set of letters $\{$A, B, C, D, E, F$\}$, if the following conditions are to be satisfied:
(i) The string starts with A.
(ii) The string starts with A and terminates in B.
(iii) The string starts either with A or with B.
(iv) The string starts either with A or with B and terminates either in A or in B such that the starting and terminating letters are different.

Q5. (2x10=20 Marks) (CO5)

a. (i) State and prove the Handshaking theorem.
(ii) A graph G has three vertices of degree 2, two vertices of degree 4, and the remaining vertices of degree 3. If there are 10 edges in the graph, then find the number of vertices in the graph.

b. Consider the graph $G(V, E)$ given below and answer the following:



(i) Find adjacency matrix of $G(V, E)$.
(ii) Is $G(V, E)$ Eulerian? If yes, find any one Eulerian circuit.
(iii) Is $G(V, E)$ Hamiltonian? If yes, find any one Hamiltonian cycle.
(iv) What is the order and size of $G(V, E)$?

c. (i) Define tree, rooted tree and binary tree with examples.
(ii) If T is a tree with n vertices, prove that T has exactly $(n-1)$ edges.