Presentation problem 5

Problem 5 ***Bores.***
Again we start from rest and use (8) as initial elevation with $x_0 = 0$. However, this time we employ a long initial condition and a long wave tank according to $L = 1000$ and $\lambda = 100$. In the two first subproblems we have constant depth $h = 1$.

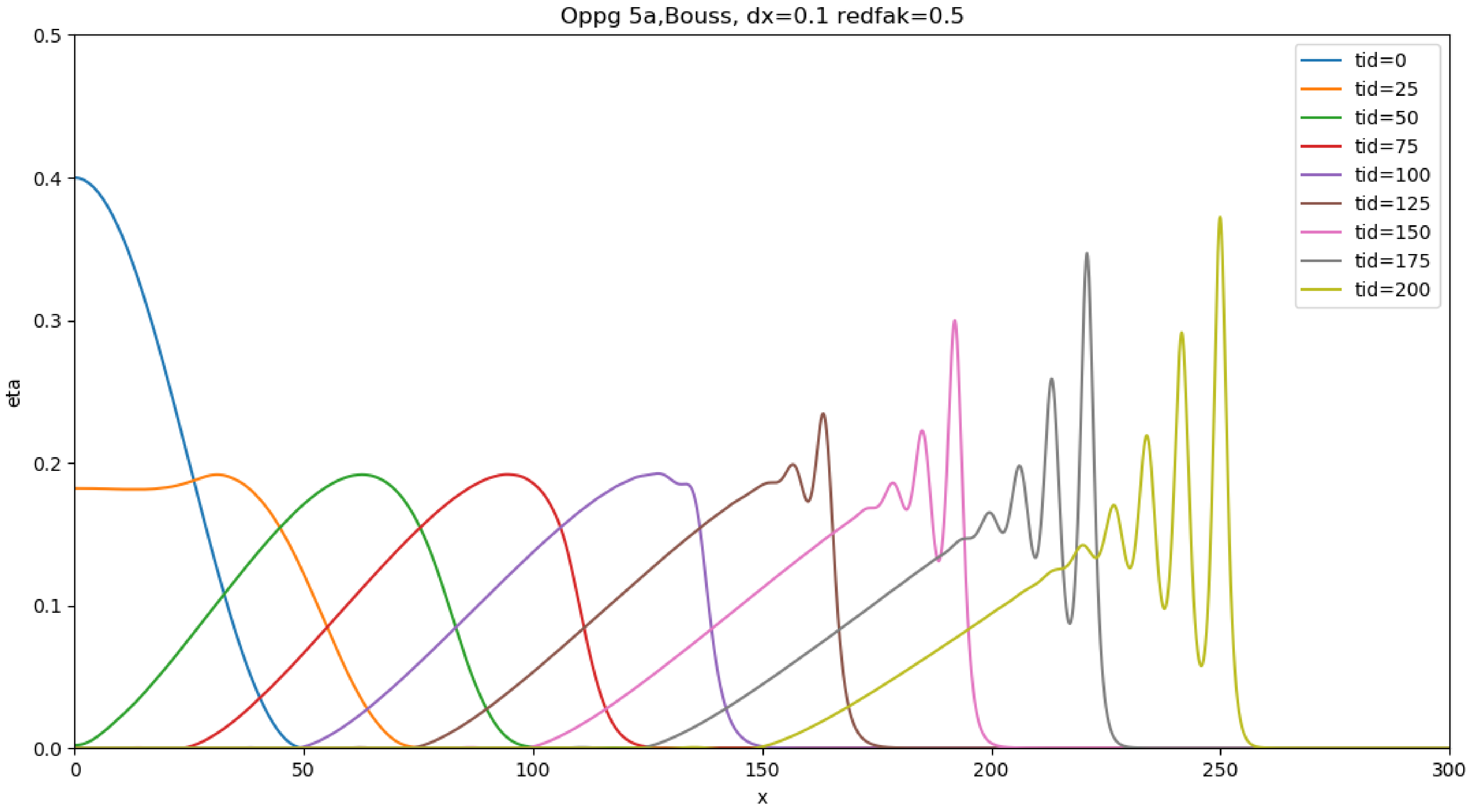
a) We focus on $t < 200$. Do simulations with NLSW and Boussinesq equations. How long are the results similar ? Illustrate with graphs.

b) Do the Boussinesq simulation until $t = 800$. What happens? Use Matlab to compare the second crest from the front to the solitary wave solution in the syllabus (slides or in: Lecture Notes Mek 4320: Hydrodynamic Wave theory.) Can you expect perfect agreement ? How can the sbouss program be used to make a more accurate comparison ?

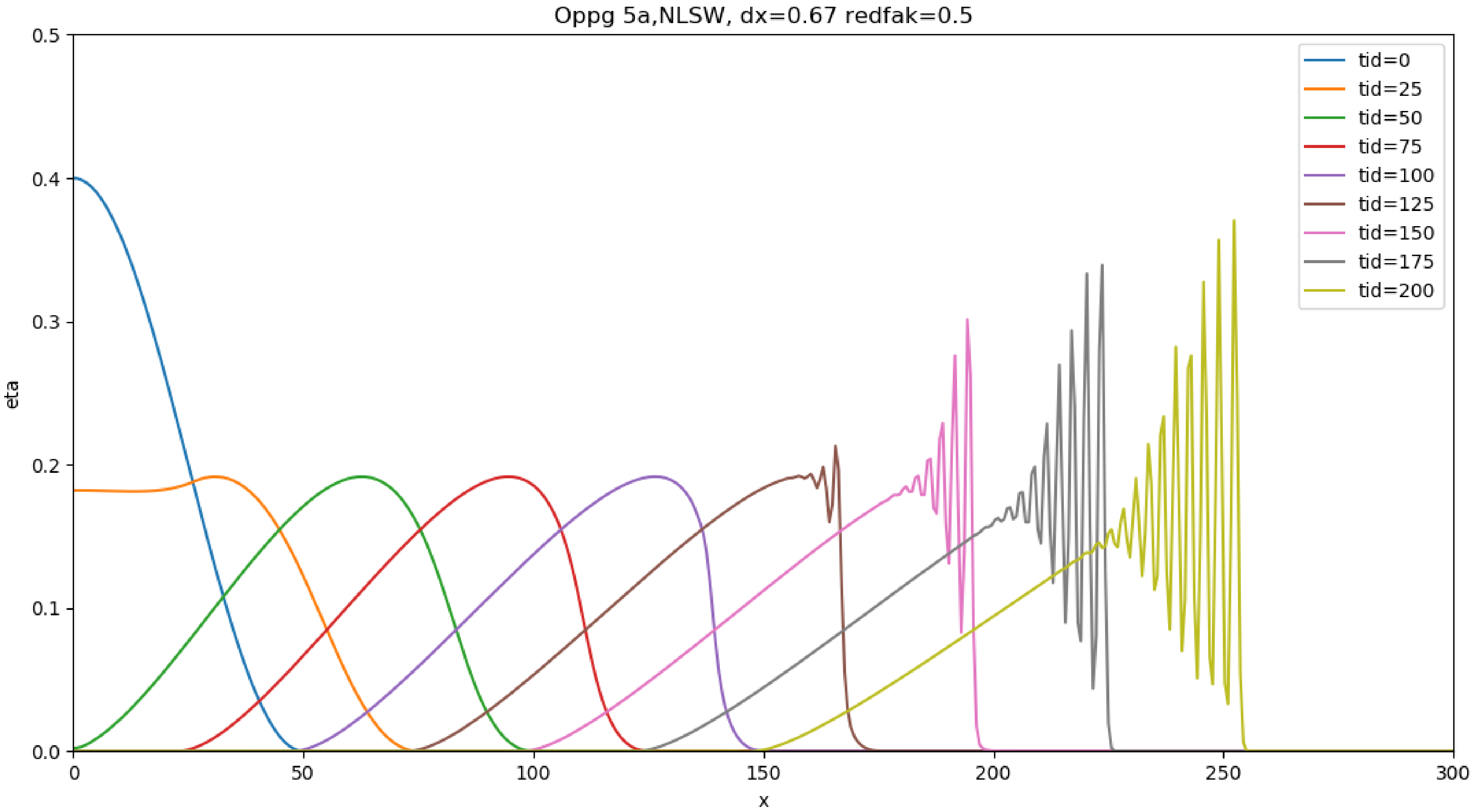
c) A sequence of crests like the ones in the previous sub-problem is called an undular bore. It may also be generated due to shoaling. Make a depth file to sbouss which corresponds to $h = 1$ for $x < 40$, $h = 0.2$ for $150 > x > 50$ and a linear slope in between. Run a solitary wave with amplitude $A = 0.05$ from the deep to the shallow region (you need a fine grid due to the shallow shelf). Show that an undular bore is generated. Explain the relation to the previous sub problem.

Problem 5a

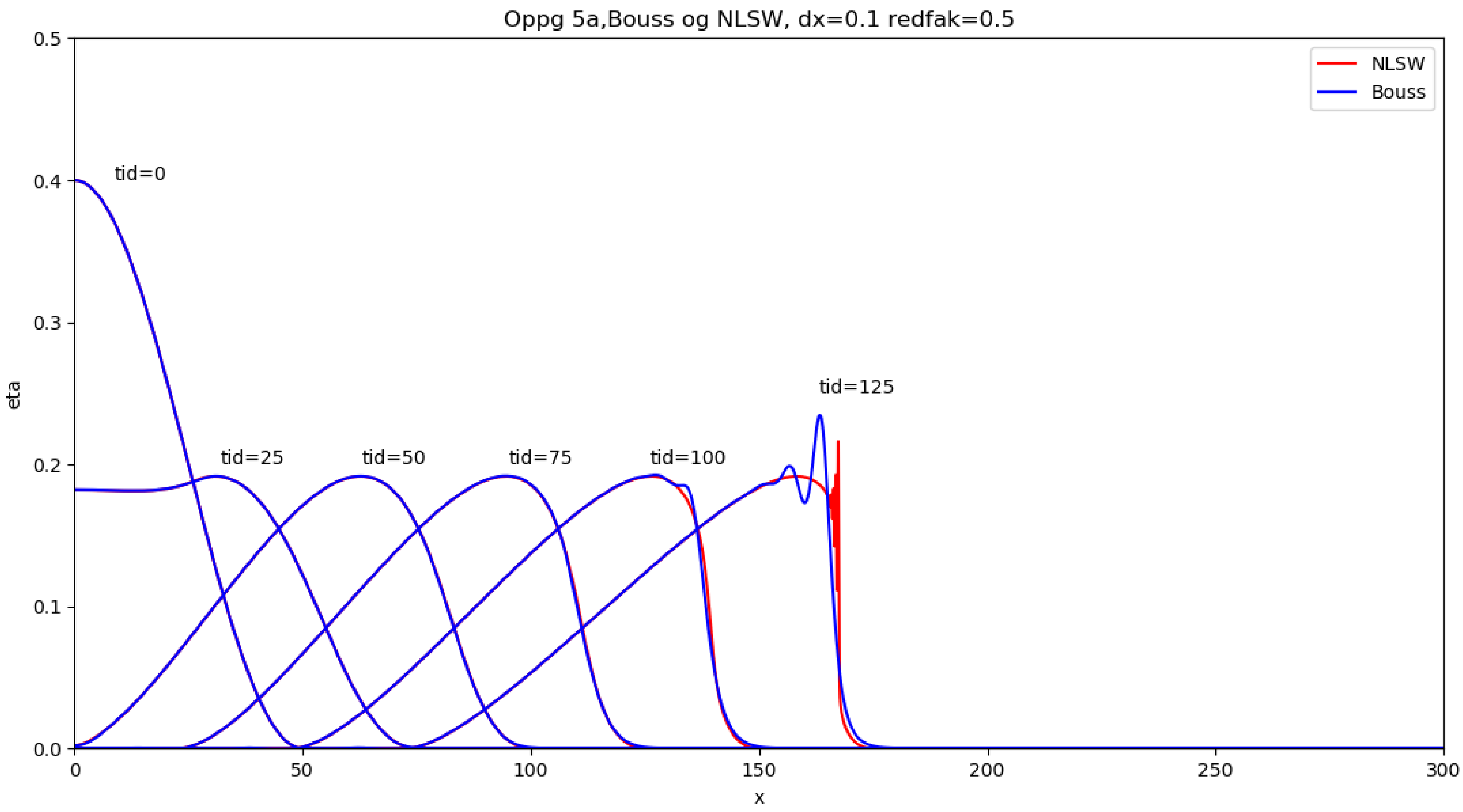
a) We focus on $t < 200$. Do simulations with NLSW and Boussinesq equations. How long are the results similar ? Illustrate with graphs.

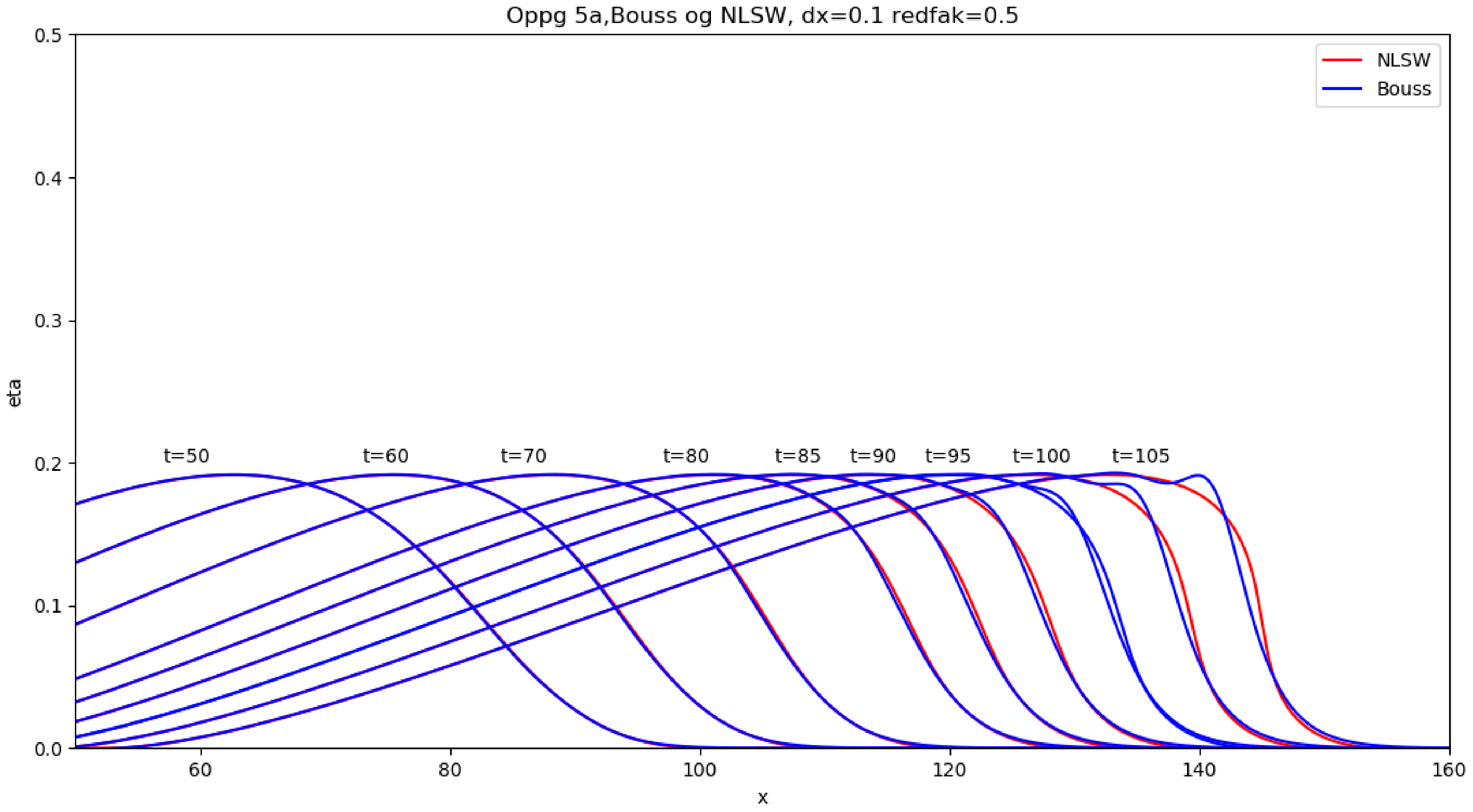


Oppg 5a,Bouss, dx=0.1 redfak=0.5
eta
x
0.0
0.1
0.2
0.3
0.4
0.5
0
50
100
150
200
250
300
tid=0
tid=25
tid=50
tid=75
tid=100
tid=125
tid=150
tid=175
tid=200



Oppg 5a,NLSW, dx=0.67 redfak=0.5
tid=0
tid=25
tid=50
tid=75
tid=100
tid=125
tid=150
tid=175
tid=200
eta
x
0.0
0.1
0.2
0.3
0.4
0.5
0
50
100
150
200
250
300

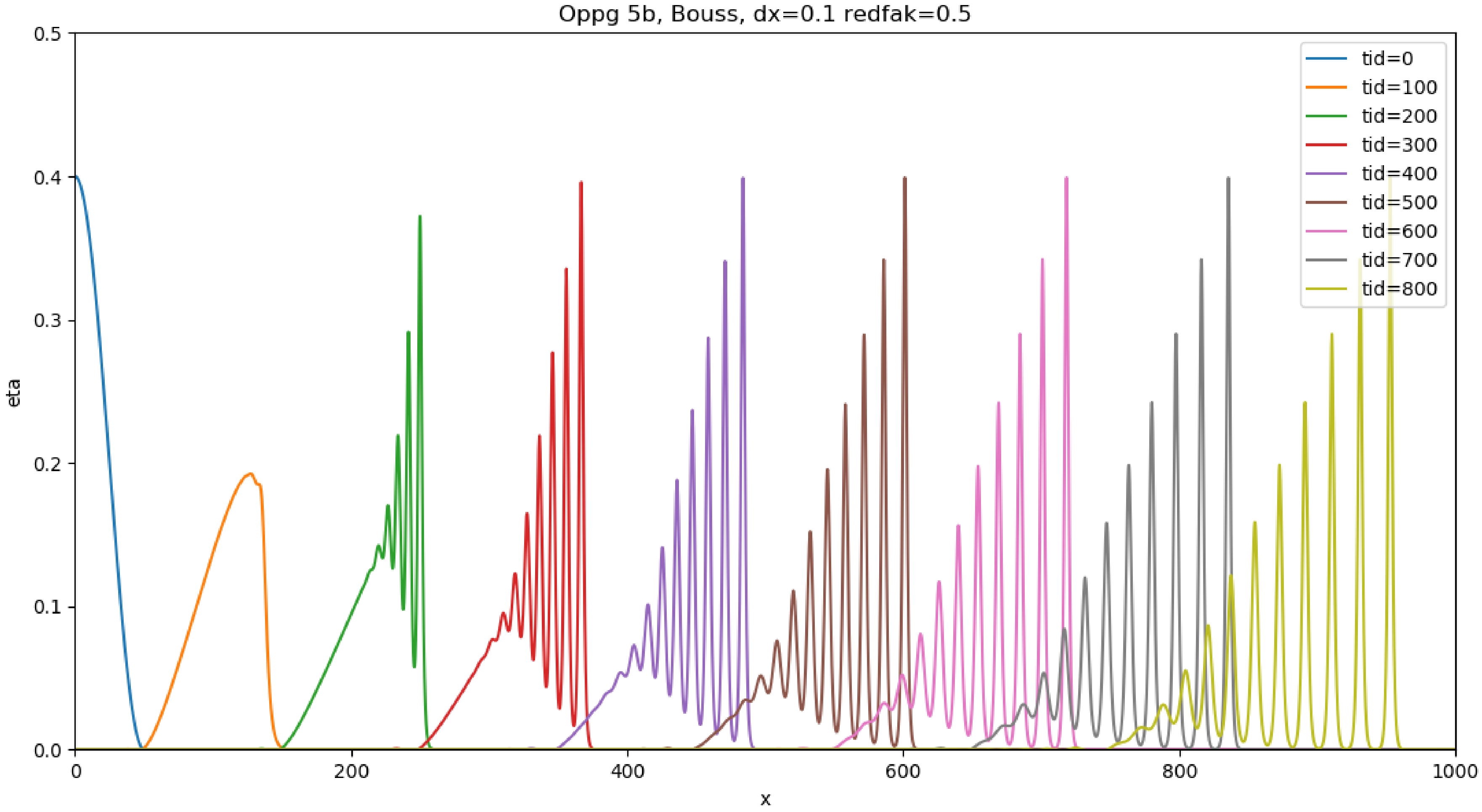

Oppg 5a,Bouss og NLSW, dx=0.1 redfak=0.5
NLSW
Bouss
tid=0
tid=25
tid=50
tid=75
tid=100
tid=125
eta
x
0.0
0.1
0.2
0.3
0.4
0.5
0
50
100
150
200
250
300



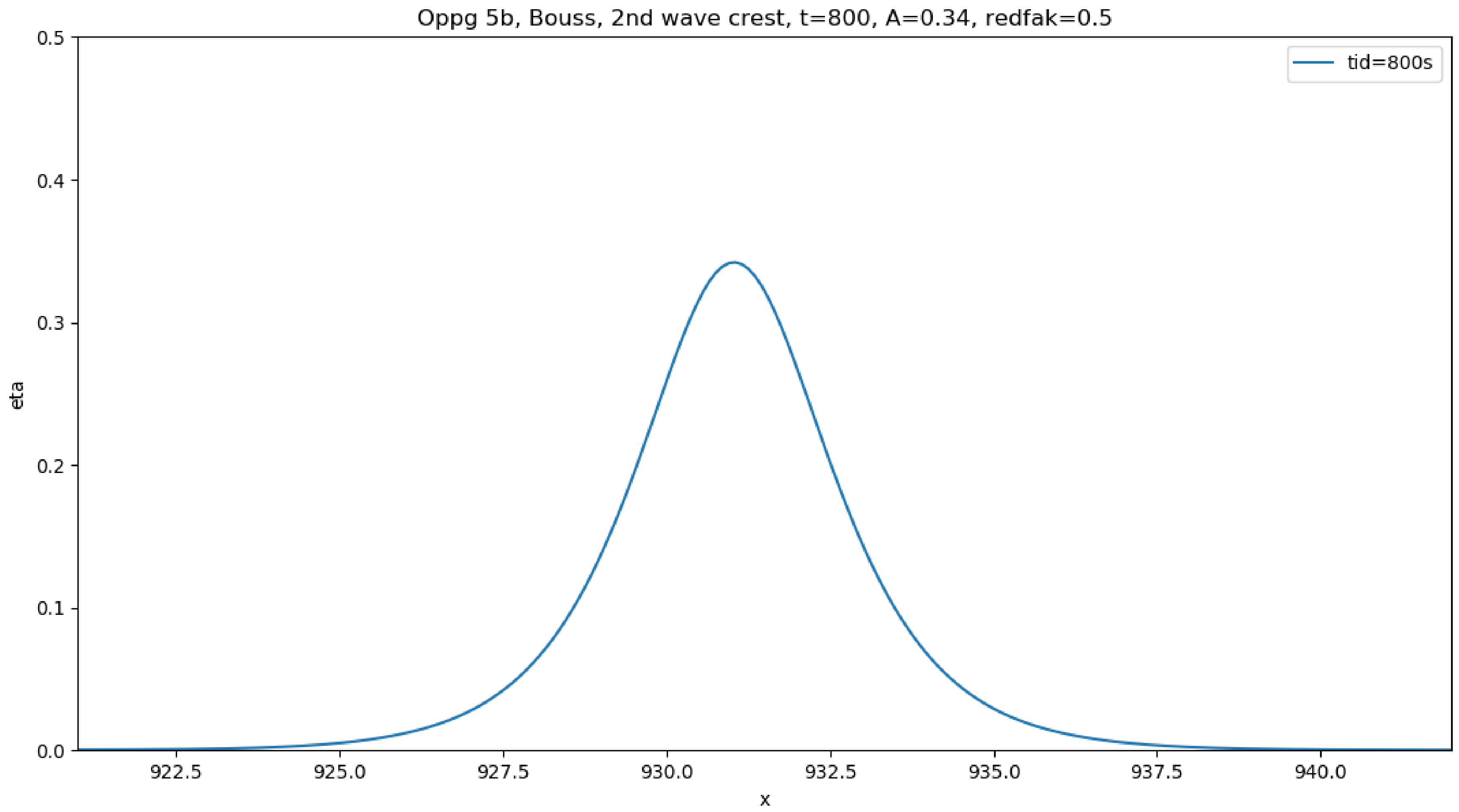
Oppg 5a,Bouss og NLSW, dx=0.1 redfak=0.5
NLSW
Bouss
eta
x
t=50
t=60
t=70
t=80
t=85
t=90
t=95
t=100
t=105
0.0
0.1
0.2
0.3
0.4
0.5
60
80
100
120
140
160

Problem 5b

b) Do the Boussinesq simulation until $t = 800$. What happens? Use Matlab to compare the second crest from the front to the solitary wave solution in the syllabus (slides or in: Lecture Notes Mek 4320: Hydrodynamic Wave theory.) Can you expect perfect agreement ? How can the sbouss program be used to make a more accurate comparison ?



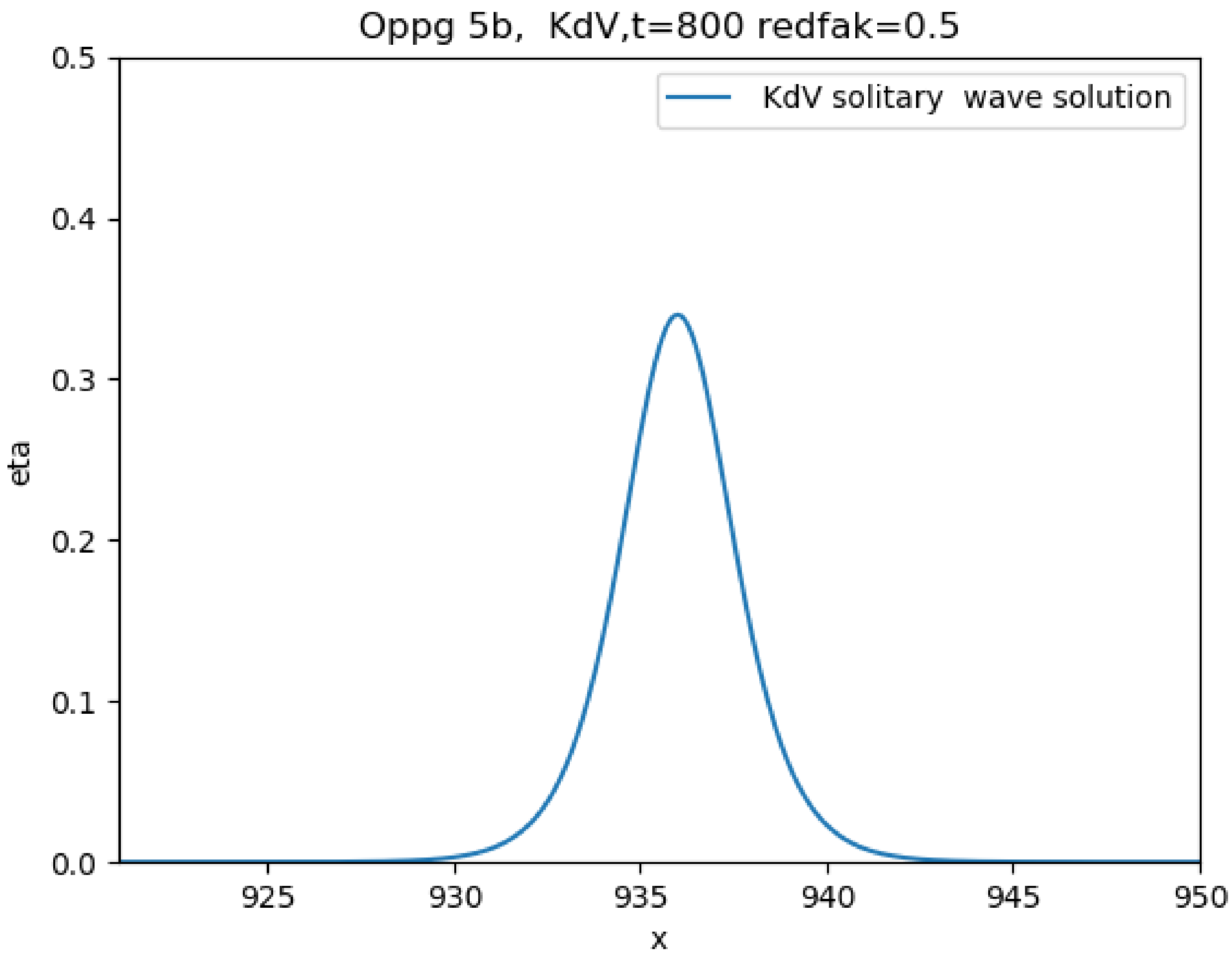
Oppg 5b, Bouss, dx=0.1 redfak=0.5
eta
x
0.0
0.1
0.2
0.3
0.4
0.5
0
200
400
600
800
1000
tid=0
tid=100
tid=200
tid=300
tid=400
tid=500
tid=600
tid=700
tid=800



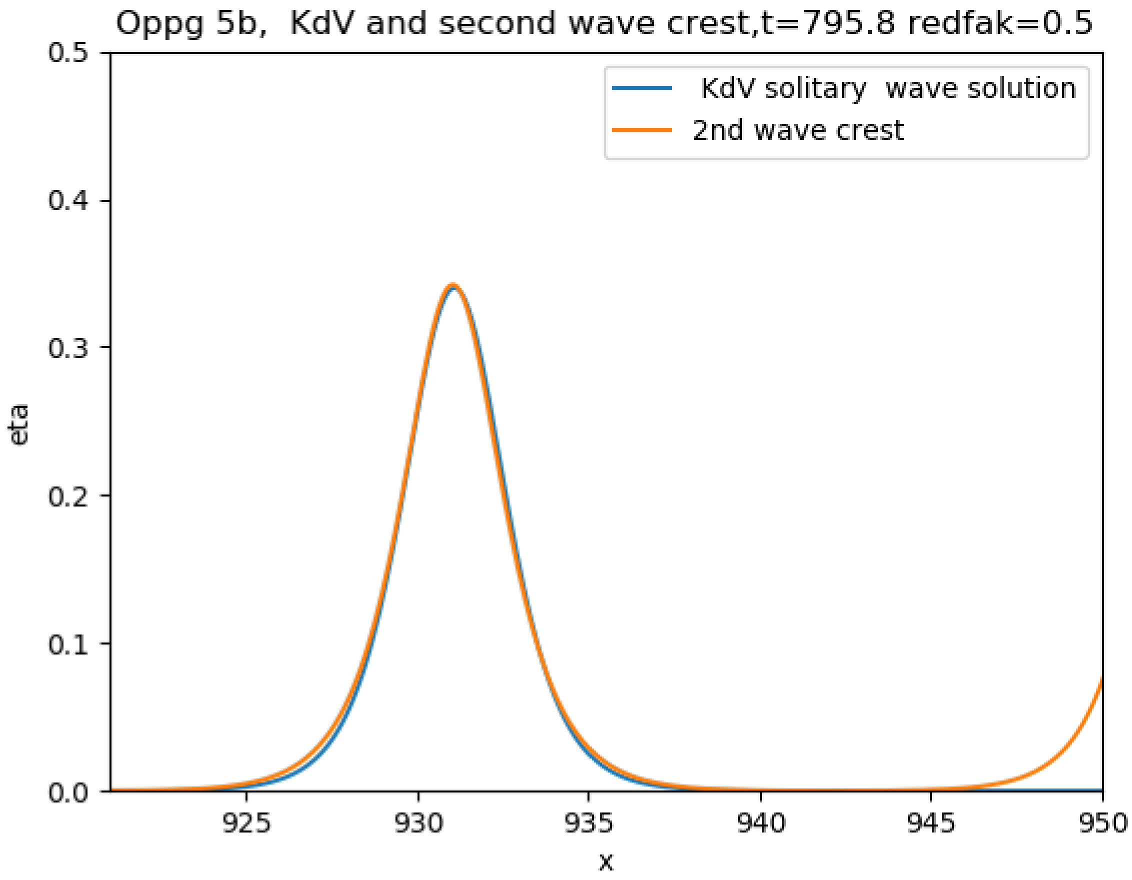
Oppg 5b, Bouss, 2nd wave crest, t=800, A=0.34, redfak=0.5
tid=800s
eta
x
0.0
0.1
0.2
0.3
0.4
0.5
922.5
925.0
927.5
930.0
932.5
935.0
937.5
940.0

The KdV solitary wave solution

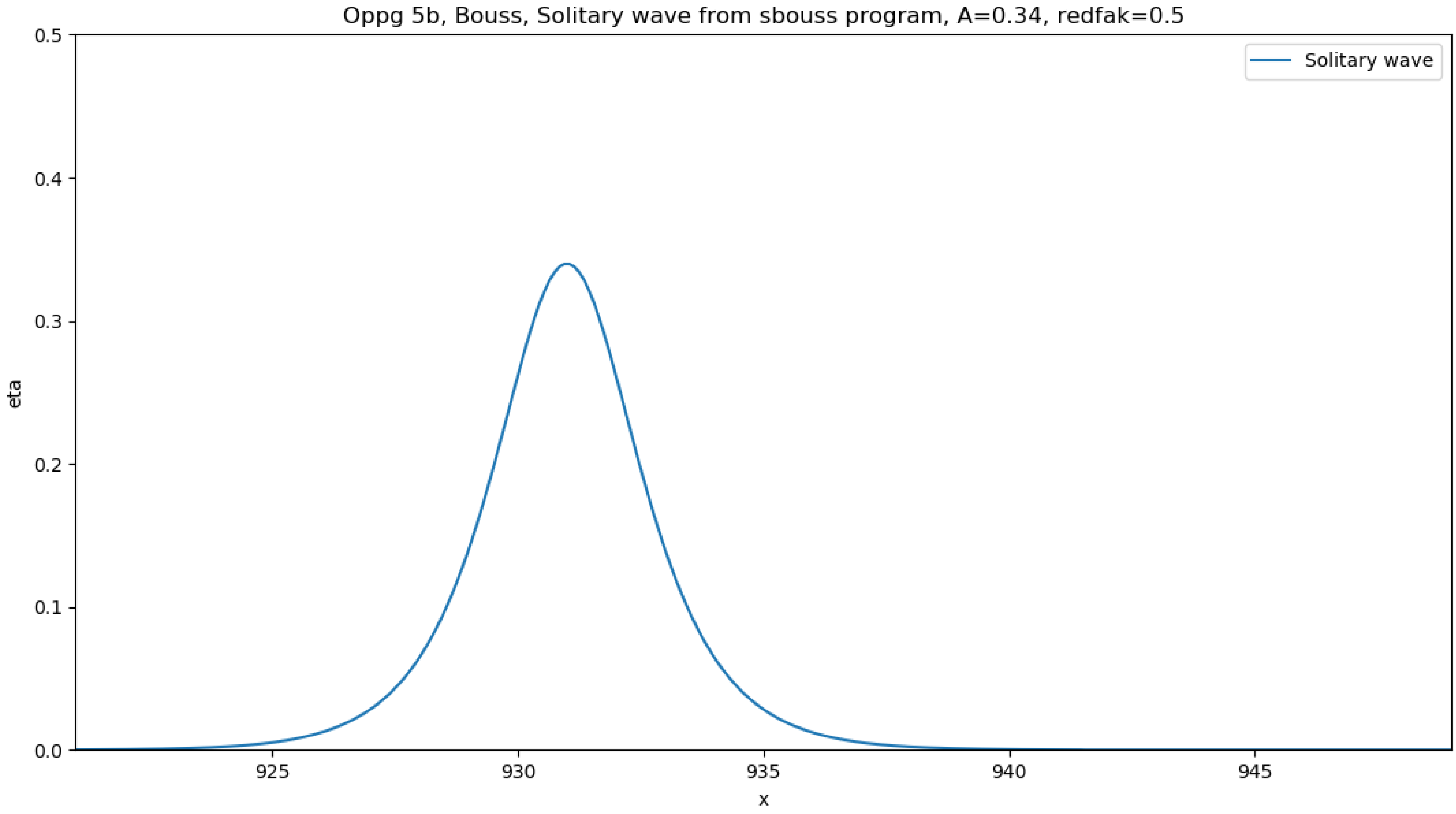
$$\eta = \alpha\,\mathrm{sech}^2\left(\frac{1}{2}\sqrt{3\alpha}(x - ct)\right), \quad c = (1 + \frac{1}{2}\alpha). \qquad (2)$$

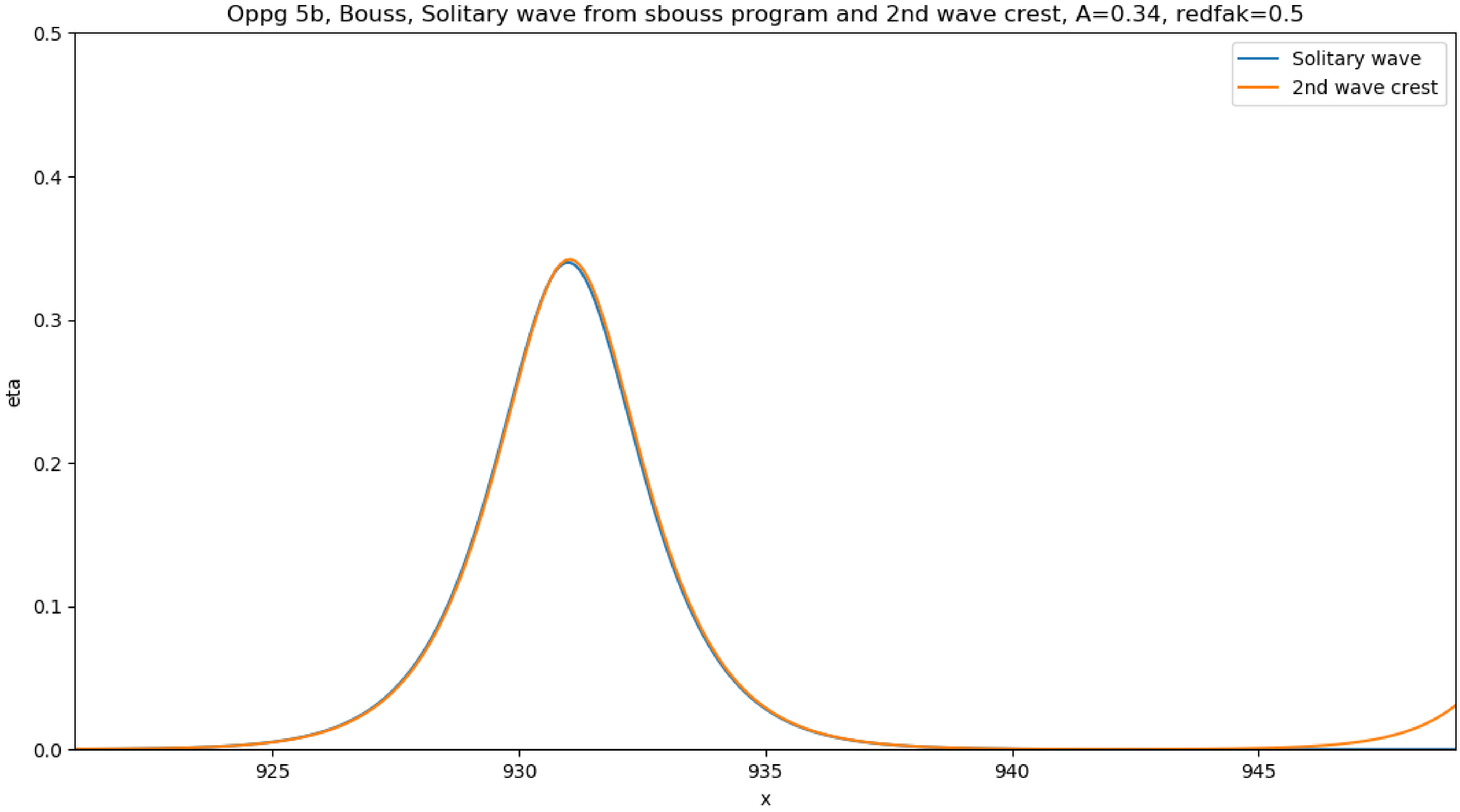


Oppg 5b, KdV,t=800 redfak=0.5
KdV solitary wave solution
eta
x
0.0
0.1
0.2
0.3
0.4
0.5
925
930
935
940
945
950



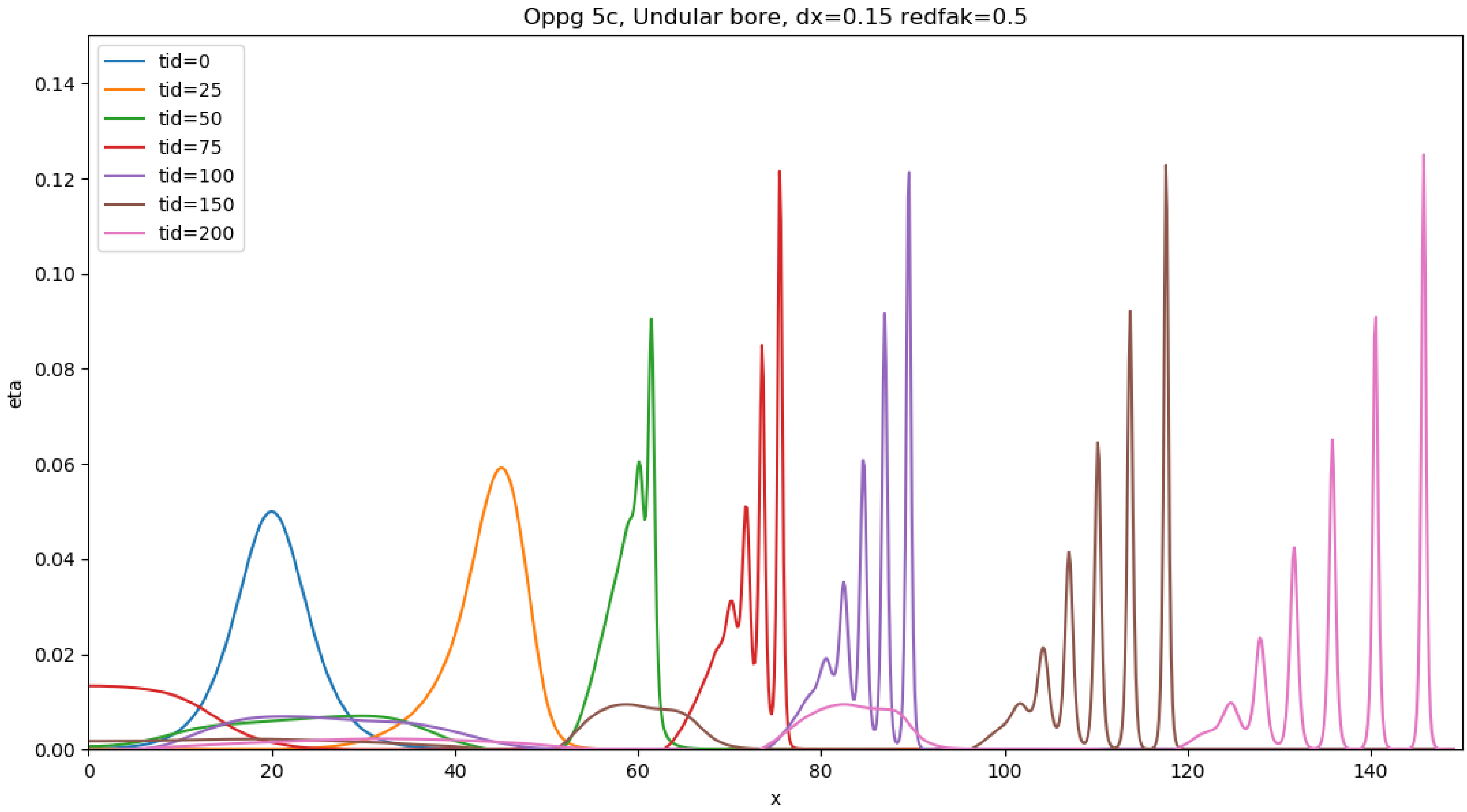
Oppg 5b, KdV and second wave crest,t=795.8 redfak=0.5
KdV solitary wave solution
2nd wave crest
eta
x
0.0
0.1
0.2
0.3
0.4
0.5
925
930
935
940
945
950

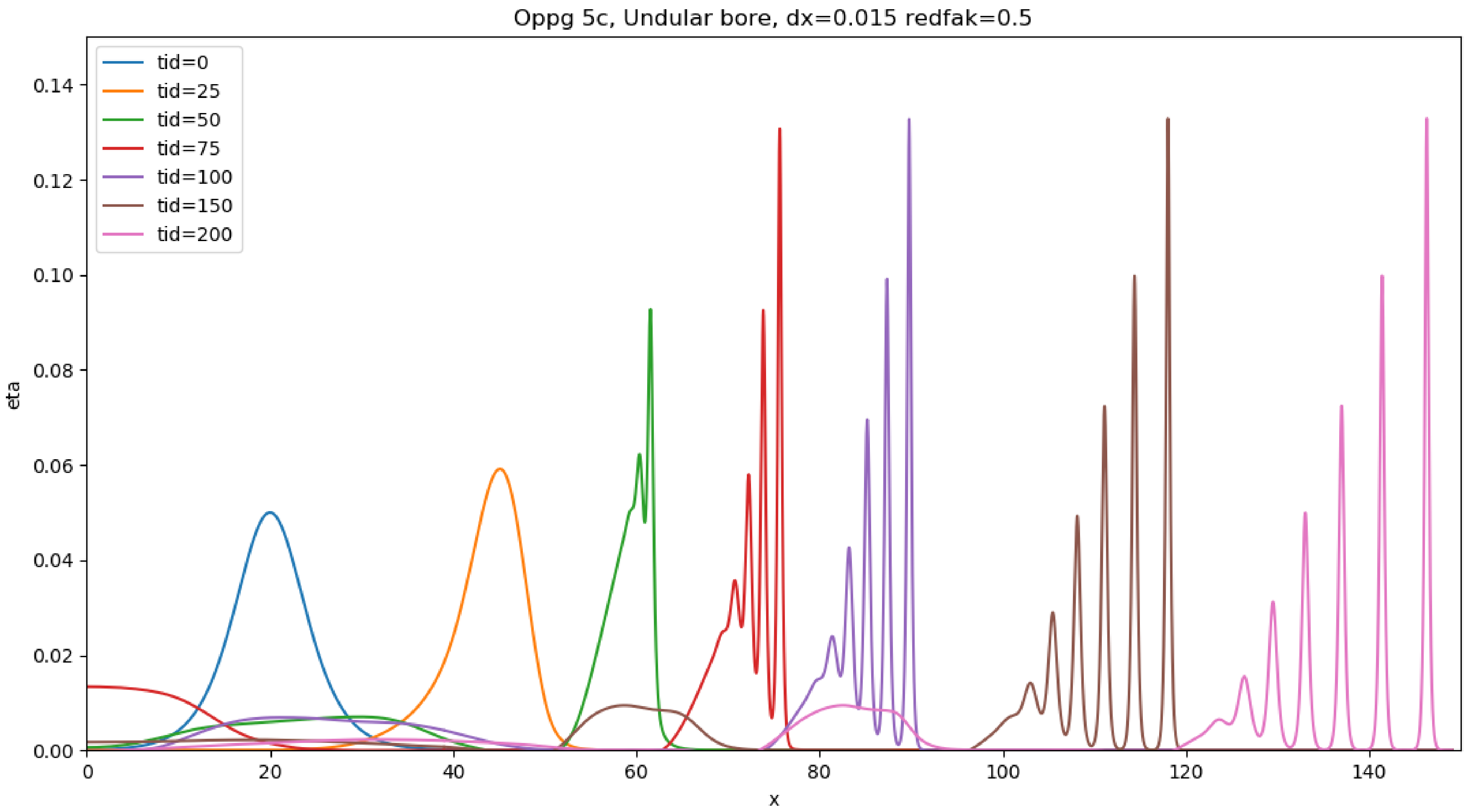

Oppg 5b, Bouss, Solitary wave from sbouss program, A=0.34, redfak=0.5
Solitary wave
eta
x
0.0
0.1
0.2
0.3
0.4
0.5
925
930
935
940
945


Oppg 5b, Bouss, Solitary wave from sbouss program and 2nd wave crest, A=0.34, redfak=0.5
Solitary wave
2nd wave crest
eta
x
0.0
0.1
0.2
0.3
0.4
0.5
925
930
935
940
945

Problem 5c

c) A sequence of crests like the ones in the previous sub-problem is called an undular bore. It may also be generated due to shoaling. Make a depth file to sbouss which corresponds to $h = 1$ for $x < 40$, $h = 0.2$ for $150 > x > 50$ and a linear slope in between. Run a solitary wave with amplitude $A = 0.05$ from the deep to the shallow region (you need a fine grid due to the shallow shelf). Show that an undular bore is generated. Explain the relation to the previous sub problem.


Oppg 5c, Undular bore, dx=0.15 redfak=0.5
tid=0
tid=25
tid=50
tid=75
tid=100
tid=150
tid=200
eta
x
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0
20
40
60
80
100
120
140



Oppg 5c, Undular bore, dx=0.015 redfak=0.5
tid=0
tid=25
tid=50
tid=75
tid=100
tid=150
tid=200
eta
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
x
0
20
40
60
80
100
120
140