UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Examination in: MEK4100 — Mathematical Methods in Mechanics

Day of examination: Thursday 11. June 2015

Examination hours: 9.00 – 13.00

This problem set consists of 2 pages.

Appendices: Formula sheet

Permitted aids: Mathematical handbook, by K. Rottmann.
Approved calculator

Please make sure that your copy of the problem set is complete before you attempt to answer anything.

Problem 1 (weight 25%)

Nonlinear oscillations are governed by the equations

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + x - \epsilon x^3 = 0, \quad x(0) = 1, \quad \frac{\mathrm{d}x(0)}{\mathrm{d}t} = 0, \tag{1}$$

where ϵ is a small parameter. We seek strictly periodic solutions of the set (1).

1a (weight 12.5%)

Apply a straightforward perturbation scheme $x(t) = x_0(t) + \epsilon x_1(t) + ..$ and explain why this scheme breaks down to order ϵ.

1b (weight 12.5%)

Modify the pertubation scheme to obtain a uniform, well behaved solution through order ϵ. Hint: you should be able to re-use most of the arithmetics from subproblem a.

Problem 2 (weight 25%)

An equation is specified according to

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \epsilon\gamma(\epsilon t)\frac{\mathrm{d}y}{\mathrm{d}t} + y = \epsilon f(\epsilon t)\cos(t), \tag{2}$$

where ϵ is a small parameter and γ and f are functions. The initial conditions are

$$y(0) = 1, \quad \frac{\mathrm{d}y(0)}{\mathrm{d}t} = 0. \tag{3}$$

2a (weight 20%)

Apply the multiple scale technique to (2) and (3). Find equations for how the two amplitudes of the leading order approximations evolve with time, but do not solve them in the general form.

2b (weight 5%)

Find the explicit solution for $\gamma = 0$ and $f(\tau) = e^{-\alpha\tau}$.

Problem 3 (weight 25%)

A particle of mass m moves freely in three dimensions in a force field with potential $V = \alpha_x x^2 + \alpha_y y^2$, where x and y are two Cartesian coordinates, with z as the third one.

3a (weight 12.5%)

Work out the Lagranges equations in Cartesian coordinates.

3b (weight 12.5%)

Show that the Lagranges equations are equivalent to Newtons second law in this case. Are there first intgrals in this case ? If so, what do they correspond to physically?

Problem 4 (weight 25%)

A second order equation is written

$$\epsilon y'' + W(x)y = 0, \tag{4}$$

where $\epsilon \to 0$, x is of order 1, and W is positive everywhere and of order 1. The prime denotes differentiaton with respect to x.

4a (weight 12.5%)

Substitute

$$y = e^{\int P(x)dx},$$

and show that P must fullfill the equation

$$\epsilon(P' + P^2) + W(x) = 0. \tag{5}$$

Apply the method of dominant balance to find the leading approximation, P_0, for P.

4b (weight 12.5%)

Write $P = P_0 + P_1$, where $P_1 \ll P_0$ and find an approximation for P_1 by a secondary dominant balance. Find y corresponding to $P_0 + P_1$.

THE END