Exercise on stationary phase.

Mek 4320/9320, autumn 2015

1 ***Klein-Gordon's equation and stationary phase.*** The equation of Klein-Gordon, expressed in some set of non-dimensional variables, reads:

$$\frac{\partial^2 \eta}{\partial t^2} - \frac{\partial^2 \eta}{\partial x^2} + q\eta = 0, \tag{1}$$

for $t > 0$ and $-\infty < x < \infty$. Waves are generated from the initial condition

$$\eta(x, 0) = e^{-\left(\frac{x}{L}\right)^2}; \quad \frac{\partial \eta(x, 0)}{\partial t} = 0, \tag{2}$$

a) Explain why the solution may depend on q and L only in the combination qL^2. However, keep the equation in the form (1). Subject the equation and initial conditions to the Fourier transform and show that the solution, for large positive x, may be obtained through the reverse transform

$$\eta(x, t) = \frac{1}{2\pi} \Re \int_0^\infty \tilde{\eta_0}(k) e^{i(kx - \omega(k)t)} dk. \tag{3}$$

b) Invert (3) by means of the stationary phase approximation. Discuss the solution: which waves (short or long) arrives first at a given point ? How does the wave heights evolve in time.

c) (Optional) Set $q = 0.1$ and $L = 10$. On the web cite for notes on MEK4320 you will find numerical solutions for a few selected times. Invoke MatLab, or another suitable tool, to compare your stationary phase solution to the numerical one. Both the plots and authentic snippet must be included in your paper.