Formulas for 4100

Trigonometric formulas

$$
\begin{array}{ll}
\cos^2\theta = \frac{1}{2}(1+\cos 2\theta), & \sin^2\theta = \frac{1}{2}(1-\cos 2\theta),\\
\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta & \\
\cos^3\theta = \frac{3}{4}\cos\theta + \frac{1}{4}\cos 3\theta, & \sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta,\\
\cos^2\theta\sin\theta = \frac{1}{4}\sin\theta + \frac{1}{4}\sin 3\theta, & \cos\theta\sin^2\theta = \frac{1}{4}\cos\theta - \frac{1}{4}\cos 3\theta,\\
\cos\theta\cos 2\theta = \frac{1}{2}(\cos\theta + cos 3\theta), & \sin\theta\sin 2\theta = \frac{1}{2}(\cos\theta - \cos 3\theta),\\
\cos\theta\sin 2\theta = \frac{1}{2}(\sin\theta + \sin 3\theta), & \sin\theta\cos 2\theta = \frac{1}{2}(-\sin\theta + 3\sin\theta),\\
\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta, & \\
\sinh(x) = \frac{1}{2}(\mathrm{e}^{x} - \mathrm{e}^{-x}), & \cosh(x) = \frac{1}{2}(\mathrm{e}^{x} + \mathrm{e}^{-x}),\\
\sin\theta = \theta - \frac{1}{6}\theta^3 + \ldots = \sum_{j=0}^{\infty} \frac{(-1)^j}{(2j+1)!}\theta^{(2j+1)}, & \cos\theta = 1 - \frac{1}{2}\theta^2 + \ldots = \sum_{j=0}^{\infty} \frac{(-1)^j}{(2j)!}\theta^{2j}.
\end{array}
$$

Taylors formula

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \ldots + \frac{1}{n!}f^{(n)}(a)(x-a)^n + R_n,$$

where the residual is $R_n = \frac{1}{(n+1)!}f^{(n+1)}(c)(x-a)^{(n+1)}$ for some c between a and x.

First order differential equations

The equation set

$$\frac{\mathrm{d}y}{\mathrm{d}x} + f(x)y = g(x), \quad y(0) = a, \tag{1}$$

has the solution

$$y(x) = e^{-\int_0^x f(t)\mathrm{d}t}\left(a + \int_0^x g(t)e^{\int_0^t f(s)\mathrm{d}s}\mathrm{d}t\right).$$

Particular solutions

Equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \omega^2 y = F(t) \tag{2}$$

where ω is a nonzero constant. Selected inhomogeneous solutions

$F(t)$	Part. løsning
$\cos\sigma t$, $\sigma \neq \pm\omega$	$(\omega^2-\sigma^2)^{-1}\cos\sigma t$
$\sin\sigma t$, $\sigma \neq \pm\omega$	$(\omega^2-\sigma^2)^{-1}\sin\sigma t$
$\cos\omega t$	$\frac{1}{2\omega}t\sin\omega t$
$\sin\omega t$	-$\frac{1}{2\omega}t\cos\omega t$