

Formulas for Mek3100/4100

Trigonometric identities

$$\begin{aligned}
 \cos^2 \theta &= \frac{1}{2}(1 + \cos 2\theta), & \sin^2 \theta &= \frac{1}{2}(1 - \cos 2\theta), \\
 \sin \theta \cos \theta &= \frac{1}{2} \sin 2\theta, & \sin \theta \cos 2\theta &= \frac{1}{2}(\sin 3\theta - \sin \theta), \\
 \cos \theta \cos 2\theta &= \frac{1}{2}(\cos \theta + \cos 3\theta), & \sin \theta \sin 2\theta &= \frac{1}{2}(\cos \theta - \cos 3\theta), \\
 \cos \theta \sin 2\theta &= \frac{1}{2}(\sin 3\theta + \sin \theta), & \sin^3 \theta &= \frac{3}{4} \sin \theta - \frac{1}{4} \sin 3\theta, \\
 \cos^3 \theta &= \frac{3}{4} \cos \theta + \frac{1}{4} \cos 3\theta, & \cos \theta \sin^2 \theta &= \frac{1}{4} \cos \theta - \frac{1}{4} \cos 3\theta, \\
 \cos^2 \theta \sin \theta &= \frac{1}{4} \sin \theta + \frac{1}{4} \sin 3\theta, & \cosh(x) &= \frac{1}{2}(e^x + e^{-x}), \\
 e^{i\theta} &= \cos \theta + i \sin \theta, & \sinh(x) &= \frac{1}{2}(e^x - e^{-x}), \\
 \sin \theta &= \theta - \frac{1}{6}\theta^3 + \dots = \sum_{j=0}^{\infty} \frac{(-1)^j}{(2j+1)!} \theta^{(2j+1)}, & \cos \theta &= \theta - \frac{1}{2}\theta^2 + \dots = \sum_{j=0}^{\infty} \frac{(-1)^j}{(2j)!} \theta^{2j}.
 \end{aligned}$$

First order differential equations

The equation set

$$\frac{\partial y}{\partial x} + f(x)y = g(x), \quad y(0) = a, \quad (1)$$

has the solution

$$y(x) = e^{-\int_0^x f(t)dt} \left(a + \int_0^x g(t) e^{\int_0^t f(s)ds} dt \right).$$

Particular solutions

Equation

$$\frac{d^2 y}{dt^2} + \omega^2 y = F(t) \quad (2)$$

where ω is a nonzero constant. Selected inhomogeneous solutions

$F(t)$	Part. løsning
$\cos \sigma t, \sigma \neq \pm \omega$	$(\omega^2 - \sigma^2)^{-1} \cos \sigma t$
$\sin \sigma t, \sigma \neq \pm \omega$	$(\omega^2 - \sigma^2)^{-1} \sin \sigma t$
$\cos \omega t$	$\frac{1}{2\omega} t \sin \omega t$
$\sin \omega t$	$-\frac{1}{2\omega} t \cos \omega t$