Mek4100
A first order boundary value problem

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A first order singular problem

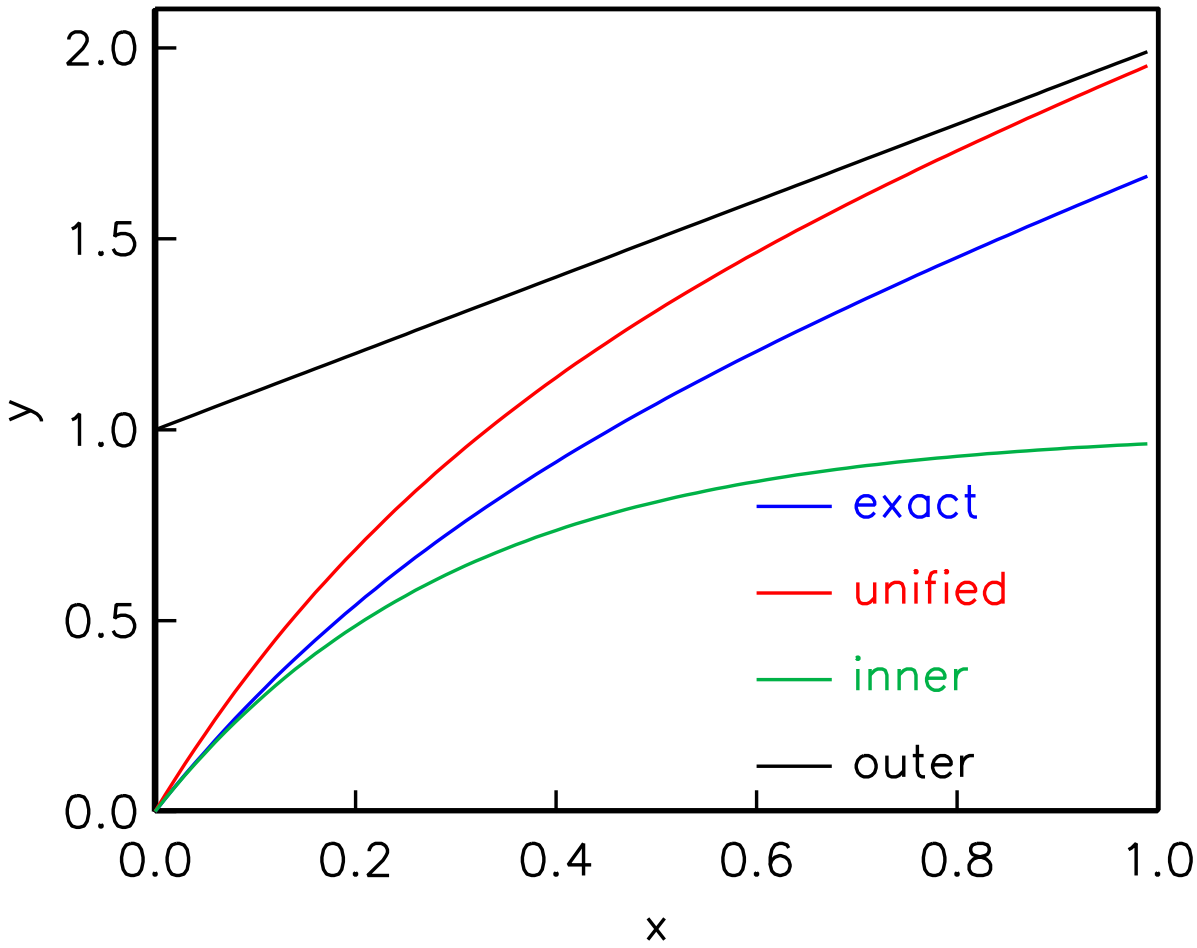
Equation set

$$\epsilon y' + y = x + 1, \quad y(0) = 0.$$

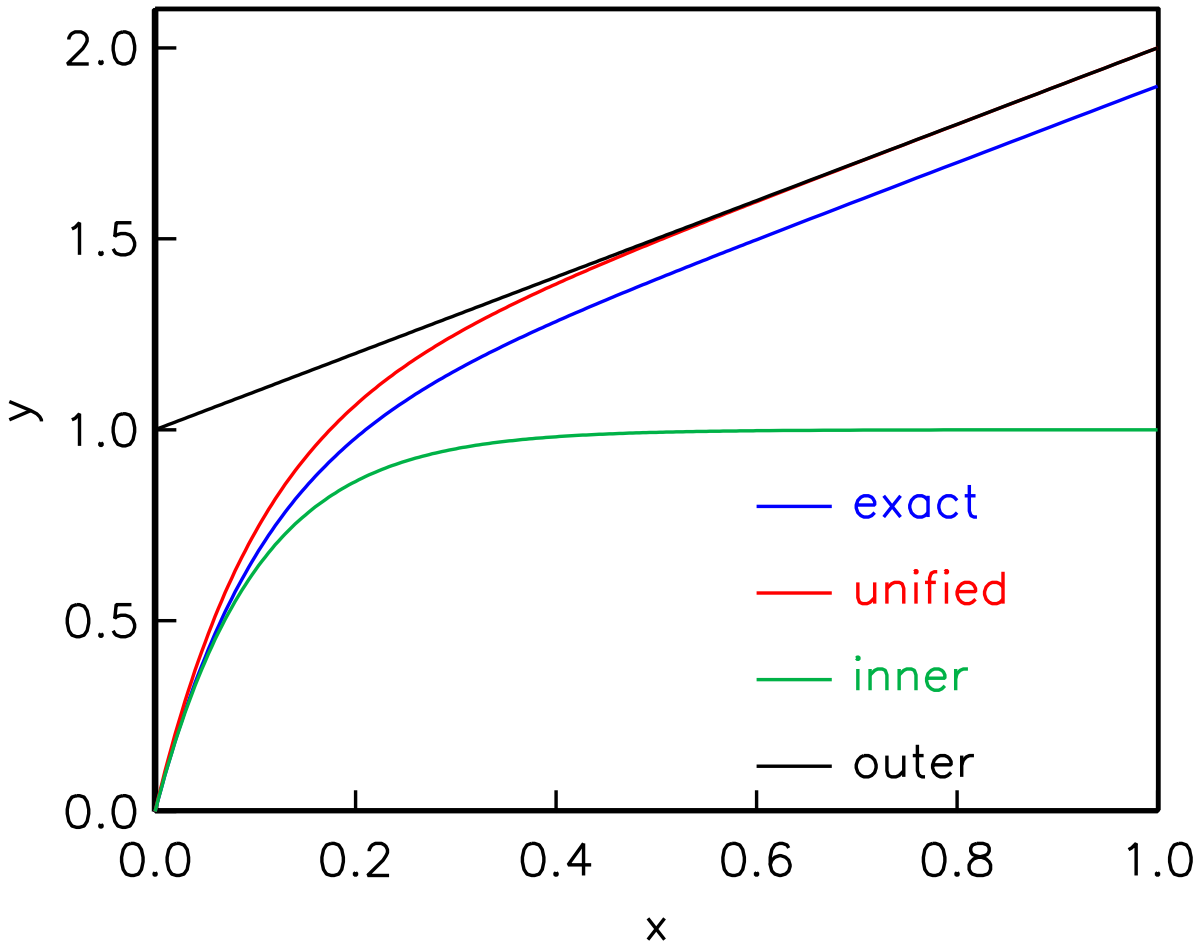
Solutions

Exact solution	$y = x + (1-\epsilon)(1 - e^{-\frac{x}{\epsilon}})$
Inner solution	$y_i = 1 - e^{-\frac{x}{\epsilon}}$
Outer solution	$y_o = x + 1$
Matching solution	$y_{\text{match}} = 1$
Unified solution	$y_{\text{unif}} = y_i + y_o - y_{\text{match}} = x + 1 - e^{-\frac{x}{\epsilon}}$
Error	$y - y_{\text{unif}} = -\epsilon(1 - e^{-\frac{x}{\epsilon}})$

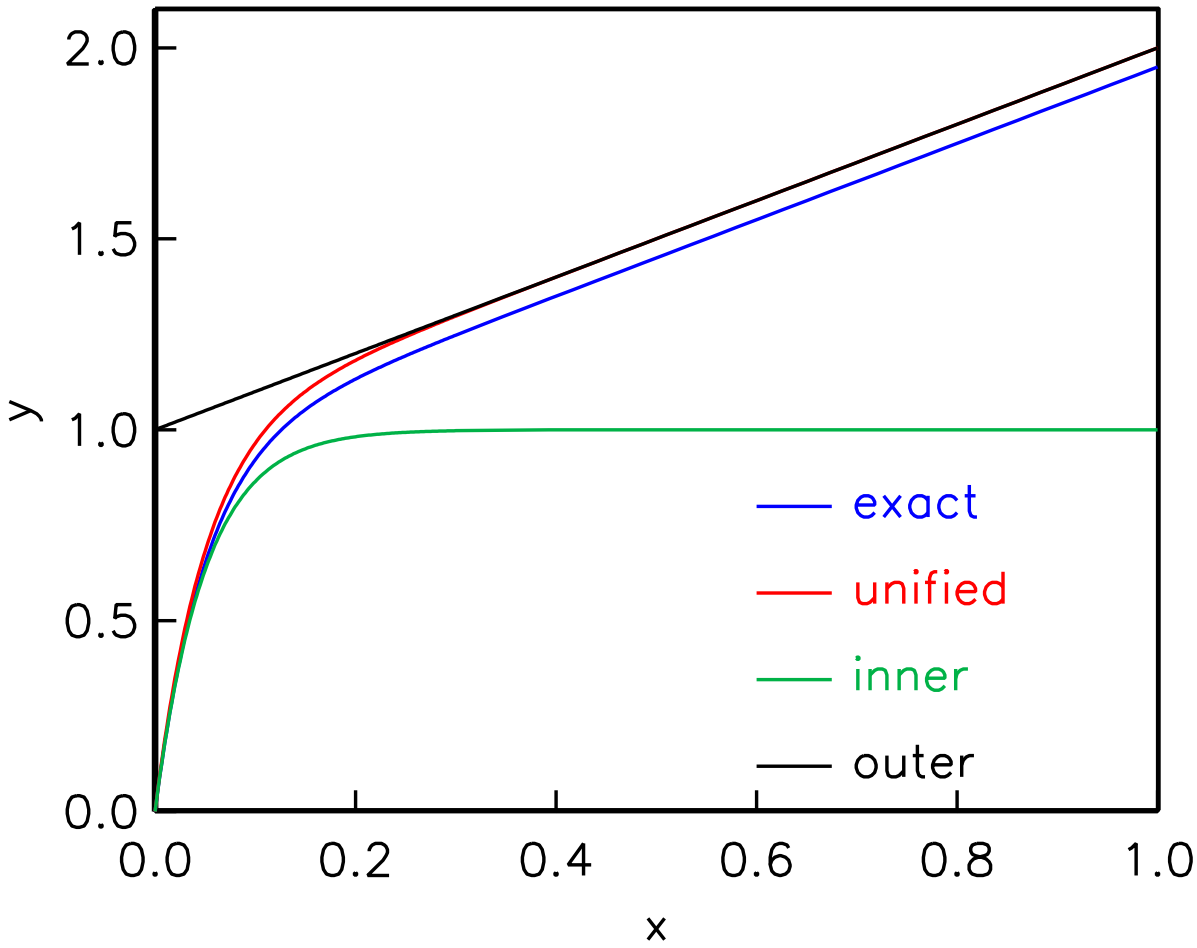
$\epsilon = 0.3$



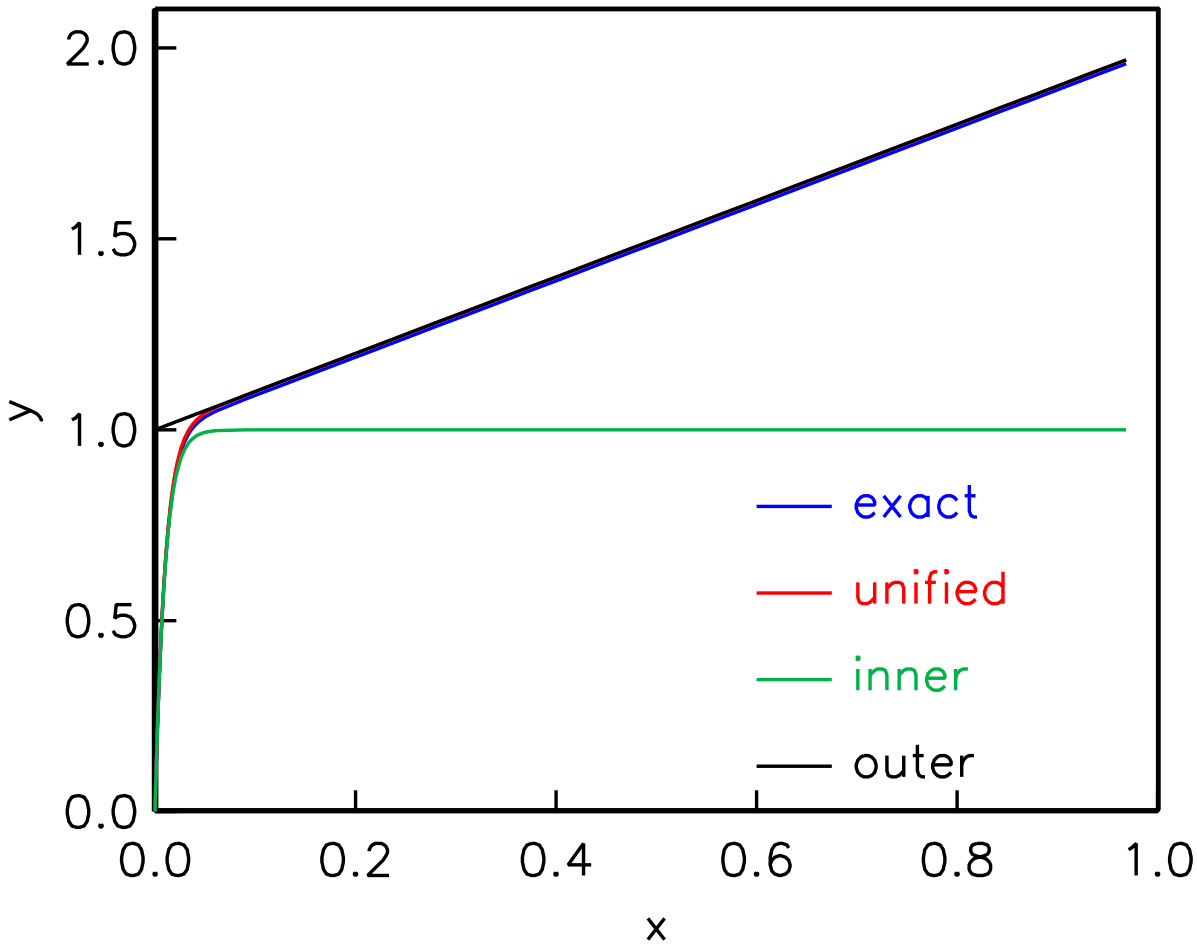
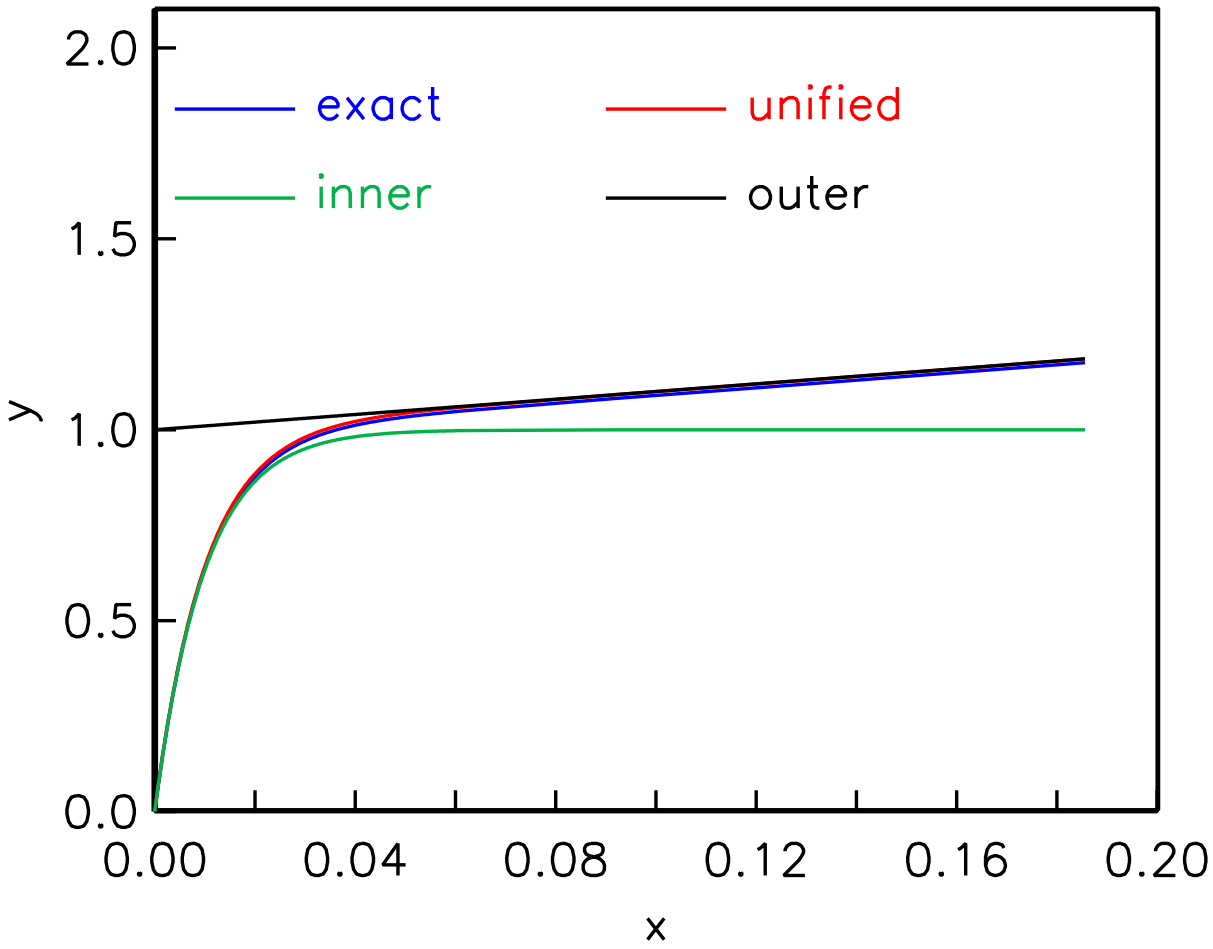
$\epsilon = 0.1$



$\epsilon = 0.05$



$\epsilon = 0.01$



Blow-up $\epsilon = 0.01$