Formulas for 4100

Trigonometric formulas

$$
\begin{aligned}
&\cos^2\theta = \tfrac{1}{2}(1+\cos 2\theta), && \sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta),\\
&\sin\theta\cos\theta = \tfrac{1}{2}\sin 2\theta && \\
&\cos^3\theta = \tfrac{3}{4}\cos\theta + \tfrac{1}{4}\cos 3\theta, && \sin^3\theta = \tfrac{3}{4}\sin\theta - \tfrac{1}{4}\sin 3\theta,\\
&\cos^2\theta\sin\theta = \tfrac{1}{4}\sin\theta + \tfrac{1}{4}\sin 3\theta, && \cos\theta\sin^2\theta = \tfrac{1}{4}\cos\theta - \tfrac{1}{4}\cos 3\theta,\\
&\cos\theta\cos 2\theta = \tfrac{1}{2}(\cos\theta + cos3\theta), && \sin\theta\sin 2\theta = \tfrac{1}{2}(\cos\theta - \cos 3\theta),\\
&\cos\theta\sin 2\theta = \tfrac{1}{2}(\sin\theta + \sin 3\theta), && \sin\theta\cos 2\theta = \tfrac{1}{2}(-\sin\theta + \sin 3\theta),\\
&\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta, && \\
&\sinh(x) = \tfrac{1}{2}(\mathrm{e}^{x} - \mathrm{e}^{-x}), && \cosh(x) = \tfrac{1}{2}(\mathrm{e}^{x} + \mathrm{e}^{-x}),\\
&\sin\theta = \theta - \tfrac{1}{6}\theta^3 + \ldots = \sum_{j=0}^{\infty}\tfrac{(-1)^j}{(2j+1)!}\theta^{(2j+1)}, && \cos\theta = 1 - \tfrac{1}{2}\theta^2 + \ldots = \sum_{j=0}^{\infty}\tfrac{(-1)^j}{(2j)!}\theta^{2j}.
\end{aligned}
$$

Taylors formula

$$
f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \ldots + \frac{1}{n!}f^{(n)}(a)(x-a)^n + R_n,
$$

where the residual is $R_n = \frac{1}{(n+1)!}f^{(n+1)}(c)(x-a)^{(n+1)}$ for some c between a and x.

First order differential equations

The equation set

$$
\frac{\mathrm{d}y}{\mathrm{d}x} + f(x)y = g(x), \quad y(0) = a, \tag{1}
$$

has the solution

$$
y(x) = e^{-\int_0^x f(t)\mathrm{d}t}\left(a + \int_0^x g(t)e^{\int_0^t f(s)\mathrm{d}s}\,\mathrm{d}t\right).
$$

Particular solutions

Equation

$$
\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + \omega^2 y = F(t) \tag{2}
$$

where ω is a nonzero constant. Selected inhomogeneous solutions

$F(t)$	Part. løsning
$\cos\sigma t$, $\sigma \neq \pm\omega$	$(\omega^2-\sigma^2)^{-1}\cos\sigma t$
$\sin\sigma t$, $\sigma \neq \pm\omega$	$(\omega^2-\sigma^2)^{-1}\sin\sigma t$
$\cos\omega t$	$\frac{1}{2\omega}t\sin\omega t$
$\sin\omega t$	-$\frac{1}{2\omega}t\cos\omega t$