ASYMPTOTISK BØLGEFRONT

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Fourier integral

$$\eta(x,t) = \frac{1}{2\pi}\Re \int_0^\infty \tilde{\eta}_0 \mathrm{e}^{\imath\chi}\mathrm{d}k$$

$$\chi \equiv kx - \omega(k)t$$

Stasjonært punkt

$$\frac{\mathrm{d}\chi(k_s)}{\mathrm{d}k} = 0$$

som gir

$$c_g(k_s) = \frac{x}{t}$$

Dominant bidrag rundt k_s

Begrensning stasj. fase

$c_g \leq c_0 \equiv \sqrt{gH} \Rightarrow$ Stasjonær fase for $\frac{x}{t} \leq c_0$

$\frac{x}{t} \sim c_0 \Rightarrow$ bølgefront

Nær bølgefront

Bidrag fra $k \to 0$.

$\chi(0) = \chi''(0) = 0 \Rightarrow$

$$\chi \approx kx - c_0 \left(k - \frac{H^2}{6} k^3 \right) t$$

og

$$\eta(x,t) \sim \frac{1}{2\pi} \Re \int_0^\infty \tilde{\eta}_0(0) \mathrm{e}^{\imath \left(kx - (c_0 k - \frac{H^2}{6} c_0 k^3) t \right)} \mathrm{d}k \qquad (1)$$

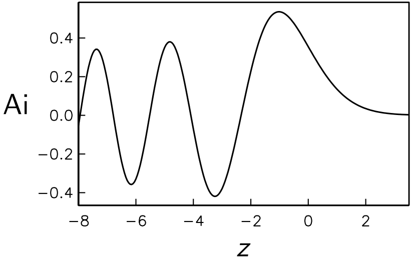
AIRY FUNKSJONEN

Integral i (1) uttrykkes ved:

$$I(z) = \frac{1}{\pi} \int_0^\infty \cos\left(\frac{1}{3}s^3 + zs\right) \mathrm{d}s$$

Integralformel for Airy funksjonen Ai som løser

$$\frac{\mathrm{d}^2 F}{\mathrm{d}z^2} - zF = 0 \tag{2}$$



BØLGEFRONT

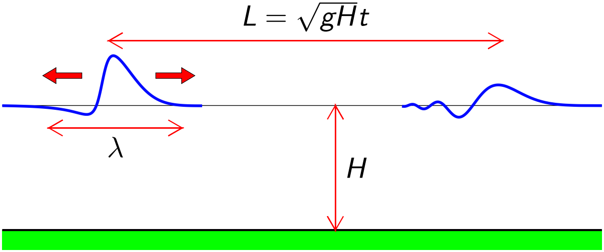
Nær $x = c_0 t$

$$\eta \sim \frac{\frac{1}{2}V}{(\frac{1}{2}c_0 H^2 t)^{\frac{1}{3}}} \mathrm{Ai}\left(\frac{x - c_0 t}{(\frac{1}{2}c_0 H^2 t)^{\frac{1}{3}}}\right)$$

der $V = \tilde{\eta}(0)$ er volumet under $\eta(x, 0)$

- Første topp nær $x = c_0 t$
- Form uavhengig av $\eta(x, 0)$
- Bølgehøyde $\sim t^{-\frac{1}{3}}$
- Lengde øker $\sim t^{\frac{1}{3}}$

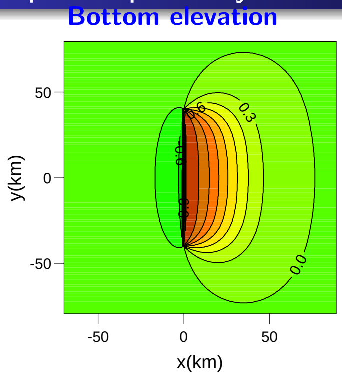
Wave front and dispersion in tsunami propagation

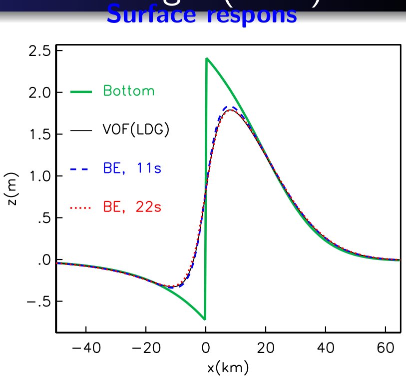


Dispersion often neglected for tsunamis due to large wavelength.
Its significance depends on:

1. Extent of source relative to depth.
2. Propagation distance.

Exampel inspired by earthquake off Portugal (1969)





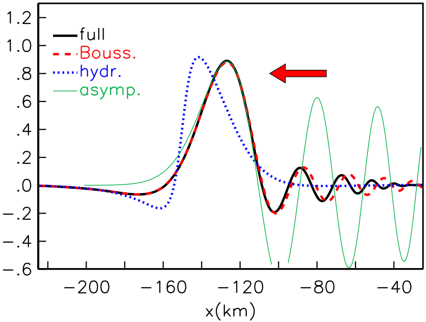
Magnitude: $M_s = 7.9$, $H = 5000\,\mathrm{m}$, inverse thrust fault, large dip angle $\approx 50^\circ$, fault length $\approx 70\,\mathrm{km} \Rightarrow$ rather confined bottom uplift

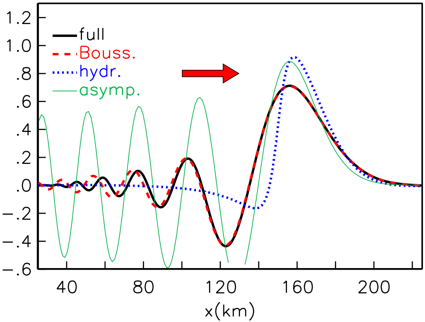
Left panel: co-seismic bottom-uplift from Okada's formula

Right panel: hydrodynamic response for centerline from 2D theories

2D response used as initial condition.

After $t = 11.3 \Rightarrow L = c_0 t = 150\,\mathrm{km}$





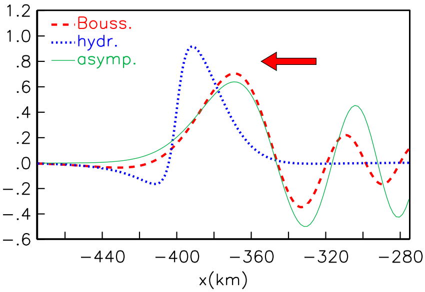
Curves:

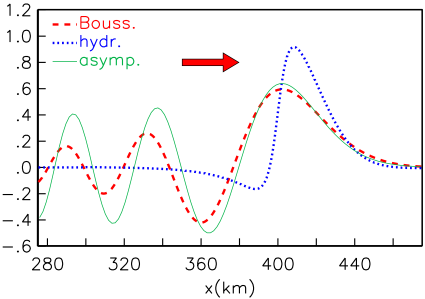
'full' and Boussinesq: numerical solutions with dispersion

'hydr': is half the initial elevation

'asymp': Asymptotic solution for wave front

After $t = 30$ min $\Rightarrow L = 400\mathrm{k}m$





Now asymptote and numerical solution are becoming close.