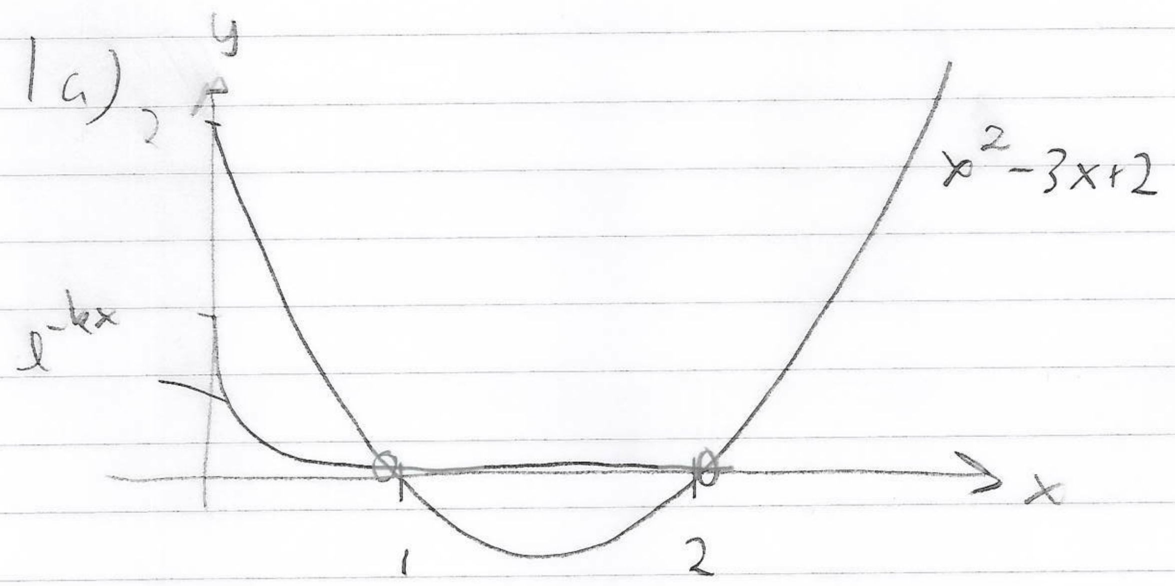
1a)



k - stor $\quad e^{-kx}$ liten ved $x = 1, 2$

$x^2 - 3x + 2 \approx 0 \Rightarrow x \simeq 1$ eller $x \simeq 2$

$X = 1 + x_1 + x_2 \qquad 1 \gg x_1 \gg x_2$

$1 + 2x_1 + 2x_2 + x_1^2 + 2x_1x_2 + \ldots - 3 - 3x_1 - 3x_2 + 2 + \ldots$

$= e^{-k - kx_1 - kx_2}$

Mulige dominante ledd

$2x_1 - 3x_1 = e^{-k - kx_1}$

Hvis $kx_1 \ll 1$ ($k \to \infty$, $x_1 \to 0$ må sjekkes)

$\underline{x_1 = -e^{-k}}$

Da $|kx_1| = k e^{-k} << 1$ når $k \to \infty$ OK

b) Kandidater:

$$2x_2 + x_1^2 + \underbrace{\cancel{2x_1x_2}}_{<< x_1} - 3x_2 + e^{-k} = e^{-k-kx_1}$$

Fordi $kx_1 << 1$: $e^{-k-kx_1} = e^{-k} e^{-kx_1} = e^{-k}(1 - kx_1 \dots)$

$$-x_2 = -x_1^2 - e^{-k} k x_1 = (k-1) e^{-2k}$$

$$\underline{x_2 = (1-k) e^{-2k}}$$

2

a) $$\left.\begin{array}{cccccc} x & x_0 & \alpha & g & m & t \\ L & L & \frac{1}{L} & \frac{L}{T^2} & M & T \end{array}\right\} 6-3\ \Pi'\text{er}$$

$$\Pi_1 = \frac{x}{x_0}, \quad \Pi_2 = \alpha x_0, \quad \Pi_3 = \alpha g t^2$$

$$\Pi_1 = F(\Pi_2, \Pi_3)$$

$$\Rightarrow \quad x = x_0 F(\alpha x_0, \alpha g t^2)$$

b)

$$\vec{r} = x\vec{i} + y\vec{j} = x\vec{i} + \tfrac{1}{2}\alpha x^2 \vec{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{x}\vec{i} + \alpha x \dot{x}\vec{j}$$

$$T = \tfrac{1}{2} m (1 + \alpha^2 x^2)\dot{x}^2$$

$$V = mgy = \tfrac{1}{2} m g \alpha x^2$$

$$L = T - V$$

$$0 = \frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = -mg\alpha x - \frac{d}{dt}\left(m(1+\alpha^2 x^2)\dot{x}\right)$$

$$\frac{\partial \bar{L}}{\partial t} = 0 \Rightarrow \text{const. } H = \dot{x}\frac{\partial \bar{L}}{\partial \dot{x}} - \bar{L} = \frac{1}{2}m(1+\alpha^2x^2)\dot{x}^2 + \bar{V}$$
$$= T + V = E$$

c)

$$\frac{d}{dt}\left((1+\alpha^2x^2)\dot{x}\right) + g\alpha x = 0$$

$$x(0) = x_0 \quad , \quad \dot{x}(0) = 0$$

Initial $\Rightarrow$ $x = x_0 z$, $t = q\tau$

q ná finas

$$\frac{1}{q^2}\frac{d}{d\tau}\left((1+(\alpha x_0)^2)\frac{dz}{d\tau}\right) + g\alpha z = 0$$

$$q^2 g\alpha \Rightarrow 1 \qquad \tau = \sqrt{g\alpha}\, t = \sqrt{\Pi_3}$$

$$\varepsilon = \alpha^2 x_0^2 = \Pi_2^2$$

d) Poincaré - Lindstedt

$\sigma = \omega \tau$, period 2π in σ (*)

$$\omega^2 \frac{d}{d\sigma}\left((1+\varepsilon z^2) \frac{dz}{d\sigma} \right) + z = 0$$

$$z = z_0 + \varepsilon z_1 + \dots \qquad \omega = \omega_0 + \varepsilon \omega_1 + \dots$$

ε^0:

$$\omega_0^2 \frac{d^2 z_0}{d\sigma^2} + z_0 = 0 \qquad z_0(0) = 1,\ \frac{dz_0}{d\sigma}(0) = 0$$

$(*) \Rightarrow \omega_0 = 1$

$$z_0 = \frac{1}{2}\left(e^{i\sigma} + e^{-i\sigma}\right) = \cos\sigma$$

ε^1:

$$\frac{d^2 z_1}{d\sigma^2} + z_1 = -2\omega_1 \frac{d^2 z_0}{d\sigma^2} - \frac{d}{d\sigma}\left(z_0^2 \frac{dz_0}{d\sigma}\right)$$

$$z_0^2 \frac{dz_0}{d\sigma} = \frac{i}{8}\left(e^{i\sigma} + e^{-i\sigma}\right)\left(e^{2i\sigma} - e^{-2i\sigma}\right)$$

$$= \frac{i}{8}\left(e^{3i\sigma} + e^{i\sigma} - e^{-i\sigma} - e^{-3i\sigma}\right)$$

$$-\frac{d}{d\sigma}\left(z_0^2\,\frac{dz_0}{d\sigma}\right) = \frac{1}{8}\left(3e^{3i\sigma}+e^{i\sigma}+e^{-i\sigma}+3e^{-3i\sigma}\right)$$

$$\frac{d^2 z_1}{d\sigma^2}+z_1 = \left(\omega_1+\frac{1}{8}\right)e^{i\sigma}+\frac{3}{8}e^{3i\sigma}+c.c.$$

Unngå sek. ledd (*) $\quad \omega_1 = -\frac{1}{8}$

$$z_1 = -\frac{3}{64}\left(e^{3i\sigma}+e^{-3i\sigma}\right)+Ae^{i\sigma}+\bar{A}e^{-i\sigma}$$

$$\frac{dz_1}{d\sigma}(0)=0 \;,\quad z_1(0)=0 \implies A=\frac{3}{64}=\bar{A}$$

$$z_1 = \frac{3}{32}\left(\cos\sigma-\cos 3\sigma\right)$$

$$z = \cos\sigma+\frac{3\varepsilon}{32}\left(\cos\sigma-\cos 3\sigma\right)+O(\varepsilon^2)$$

$$\omega = 1-\frac{\varepsilon}{8}+O(\varepsilon^2)$$

3a)

$$y_0 = + \frac{g(x)}{f(x)}$$

Oppfyller generelt ikke $y(0)=1$ eller $y(1)=?$.
$\Rightarrow$ randlag begge sider.

b)

$x \to 0$ $\quad \xi = \frac{x}{\delta}$ $\quad , f(x) = f(\delta\xi)$
$= f(0) + \delta\xi f'(0) + \dots$

$$\frac{\varepsilon}{\delta^2} y_{\xi\xi} - f(0) y = g(0)$$

$\delta = \varepsilon^{\frac{1}{2}}$, to-ledds balanse

$$Y_{\xi\xi} - f(0) Y = g(0)$$

$$Y = Y_H + Y_p = A e^{\sqrt{f(0)}\xi} + B e^{-\sqrt{f(0)}\xi} + \frac{g(0)}{f(0)}$$

$\xi \to \infty \quad Y < \infty \Rightarrow A = 0$

$$Y(0) = 1 \Rightarrow B = 1 - \frac{g(0)}{f(0)}$$

$$Y = \left(1 - \frac{g(0)}{f(0)}\right) e^{-\xi} + \frac{g(0)}{f(0)}$$

3c)

$\varepsilon y'' - f y = 0$

$x \neq 1$: $y_0 = 0$

$x \to 1$: $\cancel{\xi} = \frac{x}{\delta}$, $f(x) = f(\delta\xi) = \cancel{f(0)}$
$+ f'(0)\,\delta\xi + \cdots = \delta\xi\, f'(0)$

$\frac{\varepsilon}{\delta^2} \bar{Y}_{\xi\xi} - f'(0)\,\delta\xi\,\bar{Y} = 0$

$\delta = \varepsilon^{\frac{1}{3}}$

$\bar{Y}_{\xi\xi} - f'(0)\,\xi\,\bar{Y} = 0$

$\sigma = \alpha\xi$

$\alpha^2 \bar{Y}_{\sigma\sigma} - f'(0)\,\alpha\,\sigma\,\bar{Y} = 0$

$\alpha = f'(0) \Rightarrow$ Vi får likn 3 på nh

$\bar{Y} = a\,Ai(f'(0)\xi) + b\,Bi(f'(0)\xi)$

$x \to 1$, $\xi = \frac{1-x}{\delta}$, $x = 1 - \delta\xi$

$f(x) = f(1) - f'(1)\delta\xi + \dots$

dom.

$$\frac{\varepsilon}{\delta^2} \bar{Y}_{R\xi\xi} - f(1)\bar{Y}_R = g(1)$$

$\delta = \varepsilon^{\frac{1}{2}}$ ahhhmt ridue ron fr x>0

$$\bar{Y}_R = \left(2 - \frac{g(1)}{f(1)}\right) e^{-\xi\sqrt{f(1)}} + \frac{g(1)}{f(1)}$$

Match Vento

$$\left.\begin{aligned} y_0 &\to \frac{g(0)}{f(0)} = y_{m L} \\ \bar{Y} &\to \frac{g(0)}{f(0)} = y_{m L} \end{aligned}\right\} \text{ok}$$

Match hau

$$\left.\begin{aligned} y_0 &\to \frac{g(1)}{f(1)} = y_{m R} \\ \bar{Y}_R &\to y_{m R} \end{aligned}\right\} \text{OK}$$

$$y_{unif} = y_0 + \bar{Y}_R + \bar{Y}_L - y_{mL} - y_{mR}$$
$$= \frac{g(x)}{f(x)} + \left(1 - \frac{g(0)}{f(0)}\right) e^{-\frac{x}{\varepsilon^{1/2}}} + \left(2 - \frac{g(1)}{f(1)}\right) e^{-\sqrt{f(1)}\frac{(1-x)}{\varepsilon^{1/2}}}$$