Mandatory assignment 1 for MEK4320

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In the present assignment all enumeration refers to the English version of the compendium. Throughout this assignment we ignore capillary effects.

Ex. 1 ***Deep and shallow water.***

a) Assume that $kH \to \infty$. Show (in detail) how (2.14) and (2.15) simplifies to

$$\phi = -ace^{kz}\cos(kx - \omega t), \quad u = a\omega e^{kz}\sin(kx - \omega t), \quad w = -a\omega e^{kz}\cos(kx - \omega t).$$

b) For the solution of the preceding point: as we move downwards from the surface; at which value of z/λ is the velocity reduced to 1% of the value at the surface?

c) We now assume that kH, and hence $|kz|$ are small. Expand (2.14) and (2.15) in kH and kz, keeping quadratic terms and lower, to obtain an expression

$$\phi = -\frac{ac_0}{kH}\left(1 + k^2 q(z, H) + O(k^4)\right)\cos\chi, \quad \omega = c_0 k(1 - \frac{1}{6}(kH)^2 + O(k^4)), \tag{1}$$

where $c_0 = \sqrt{gH}$, $\chi = kx - \omega t$ and q is a second order polynomial of z and H. Find also the corresponding velocities.

d) Employ (1) to obtain an expression for the pressure. Show that the pressure is hydrostatic, in the sense that $p = \rho g(\eta - z)$, when $kH \to 0$. An important quantity is the horizontal pressure gradient, because this relates to the horizontal acceleration. Find a measure of the relative deviation in $\partial p/\partial x$ from that of the hydrostatic pressure when $\lambda = 4\pi H$ by taking the $k^2 q$ term in (1) into consideration.

e) Correspondingly, show that u is independent of z when $kH \to 0$ and find the total variation of u in a vertical column. Does the result relate to that of the preceding point ?

f) Show that $w \to 0$ as $kH \to 0$. What is the consequence for the particle trajectories?

g) The velocity potential should fulfill

$$\nabla^2\phi = 0.$$

In what sense does the approximation (1) satisfy this equation ?

Ex. 2 ***Nonlinear terms.***

We want to investigate the omission of nonlinear terms in the equations used for the harmonic wave mode given by (2.14) and (2.15). To this end we insert these expressions into the nonlinear versions of the equations and assess the magnitude of the terms we omit. Even though this procedure neither yields qualitative error estimates or a rigorous proof for the consistency of linearization it does give an indication of validity.

a) Assume infinite depth $kH \to \infty$. Insert the harmonic mode in the kinematic boundary condition (2.5) and expand it to find all quadratic terms in a. Compare the maximum size of the nonlinear terms to the maximum linear ones. Identify a parameter that measures the significance of nonlinear terms.

b) Apply the same analysis to (2.7) versus (2.9).

c) We now assume long waves $kH \to 0$. Look at the size of the nonlinear terms of (2.5), as compared to the linear ones, for this case. What is the parameter that measures significance of nonlinearity now ?