Ex. 88 ***On the relation between the WKB, two-scale and boundary layer techniques.***

a) Boundary layer problem A boundary layer problem is defined by

$$\epsilon^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - W(x)y = 0; \quad y(x_a) = a, \quad y(x_b) = b, \tag{115}$$

where $W > 0$. Show that the boundary layer technique gives

$$y \approx a e^{-\sqrt{W(x_a)}\frac{(x-x_a)}{\epsilon}} + b e^{\sqrt{W(x_b)}\frac{(x-x_b)}{\epsilon}}. \tag{116}$$

b) Exact solution of simplified boundary layer problem Assume now that W is constant. Find the exact solution solution of (115). Discuss the errors inherent in (116) in this case. Relate this to the steps from the WKB solution to the boundary layer solution in electronic presentation *Mek4100; The WKB method*, slides 20 and 21.

c) Failed application of the boundary layer technique Now the problem is changed to

$$\epsilon^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + W(x)y = 0; \quad y(x_a) = a, \quad y(x_b) = b,$$

where $W > 0$. Try to apply the boundary layer technique and demonstrate how it fails.

d) Two scale expansion Use two-scale expansion to find two independent solutions of

$$\epsilon^2 \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + W(x)y = 0,$$

where $W > 0$. We have now waived the boundary conditions. Compare to the results in *Mek4100; The WKB method.*