Mandatory assignment 1

Mek 4320/9320, autumn 2014

Formalities

You must turn your assignment in before 24.00 at October 2nd. It may be delivered in class or put in the shelf of Geir Pedersen in the "postal room" on 7th floor NHAbelshus).

The assignment can be typed electronically and printed or written by hand. In either case authentic printings of the requested snippets of code and figures must be included. Authentic printings means that code snippets are those which are actually run and that the figures are produced with the submitted code. Irregularities in this respect may lead to rejection of the whole assignment. You may also be requested to submit code and data electronically.

In solving the problems you may collaborate with the other students. However, the paper you turn in must be written by you personally and it should reflect your understanding of the material.

1 *Klein-Gordon's equation and stationary phase.* The equation of Klein-Gordon, expressed in some set of non-dimensional variables, reads:

$$\frac{\partial^2\eta}{\partial t^2} - \frac{\partial^2\eta}{\partial x^2} + q\eta = 0, \tag{1}$$

for $t > 0$ and $-\infty < x < \infty$. Waves are generated from the initial condition

$$\eta(x,0) = e^{-\left(\frac{x}{L}\right)^2}; \quad \frac{\partial\eta(x,0)}{\partial t} = 0, \tag{2}$$

a) Explain why the solution may depend on q and L only in the combination qL^2. However, keep the equation in the form (1). Subject the equation and initial conditions to the Fourier transform and show that the solution, for large positive x, may be obtained through the reverse transform

$$\eta(x,t) = \frac{1}{2\pi}\Re\int_0^\infty \tilde{\eta}_0(k)e^{i(kx-\omega(k)t)}dk. \tag{3}$$

b) Invert (3) by means of the stationary phase approximation. Discuss the solution: which waves (short or long) arrives first at a given point ? How does the wave heights evolve in time.

c) Set $q = 0.1$ and $L = 10$. On the web cite for notes on MEK4320 you will find numerical solutions for a few selected times. Invoke MatLab, or another suitable tool, to compare your stationary phase solution to the numerical one. Both the plots and authentic snippet must be included in your paper.