Numerical investigations on nonlinear and dispersive wave propagation.

MEK4320

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Problem 2:

Solitary wave propagation and grid effects. We will test the performance of the model by investigating how well the shape of a solitary wave is preserved etc. What model (LSW, NLSW, linear dispersive or Boussinesq) is the appropriate choice for this task? Use the sbouss program to simulate the propagation of a solitary wave, with amplitude $\alpha = 0.1$, starting at $x = 20$ until $t = 70$ on constant depth $h = 1$. Make certain that the numerical wave tank is long enough. Use Matlab, or some other tool, to find the maximum η at $t = 70$ when you choose $\Delta x = 2$, $\Delta x = 1$ and $\Delta x = 0.5$, while the time step reduction factor is set to 0.5 and the discrete correction term is turned off. Do also depict all three solutions in the same diagram. Comment on what resolution that seems relevant.

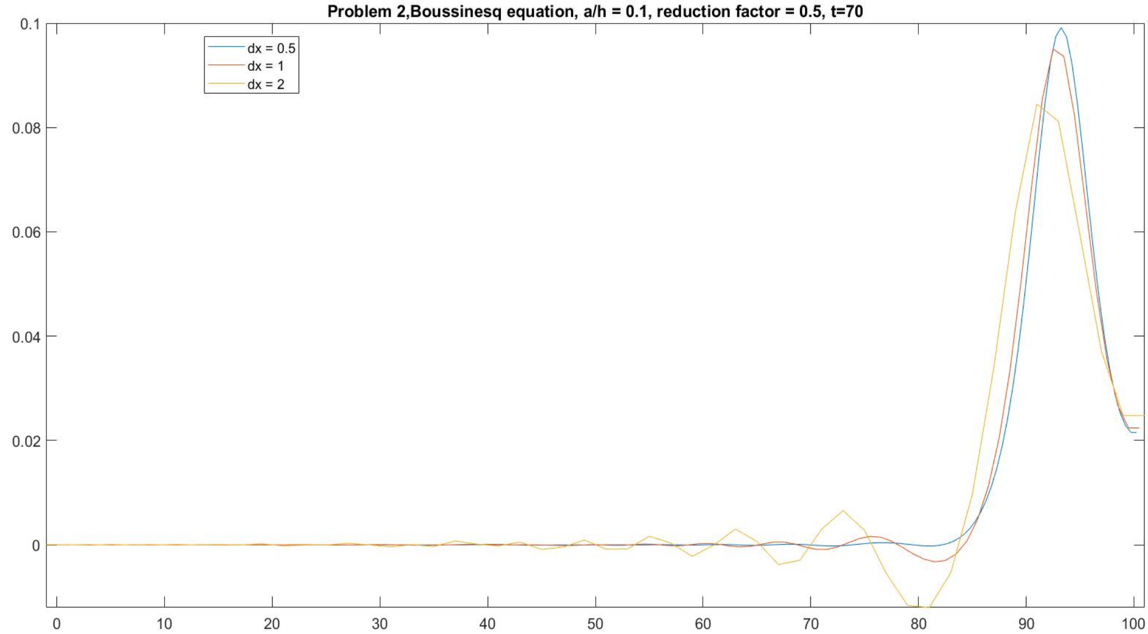


Figure 1 The three solution with different number of grid points using Boussinesq model

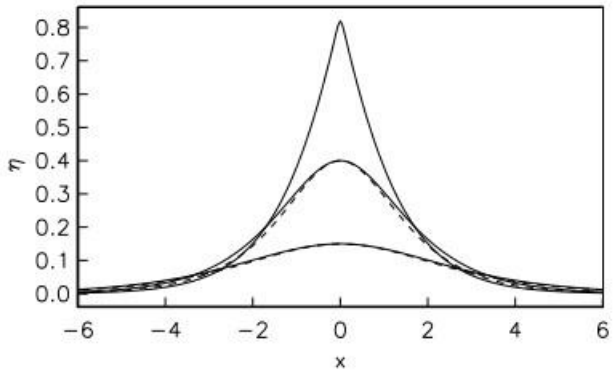
Length of the tank=100

Number of cycles=7

Time interval=10

Total time =70

Boussinesq is an appropriate model for this task. Boussinesq and the full potential theory have a good agreement for small values of alpha as we have seen in the lecture notes (Solitary waves; MEK4320). Also, Boussinesq have the same shape and size over time, but when we decrease the resolution, we start to get oscillation. This is due to numerical dispersion effects.



Boussinesq solitary waves (dashed) and full potential theory, numerical, (solid line) for $\alpha = 0.15, 0.4$. Potential solution for $\alpha = 0.82$ near maximum; crest close to cusp with 120° opening.

Figure 2 Boussinesq soliton vs. full potential theory.

The maximum values of eta are shown in the table below:

Table 1 The maximum value of eta for different values of grid points

Number of grid points	Δx	Maximum value of eta
200	0.5	0.0991
100	1	0.0950
50	2	0.0845

The amplitude is decreasing with the resolution.

To conclude, the highest resolution is the most relevant since it gives the most accurate numbers. In other words, as the number of grid points increases the accuracy of the solution will increase.

Problem 4:

Wave dispersion.

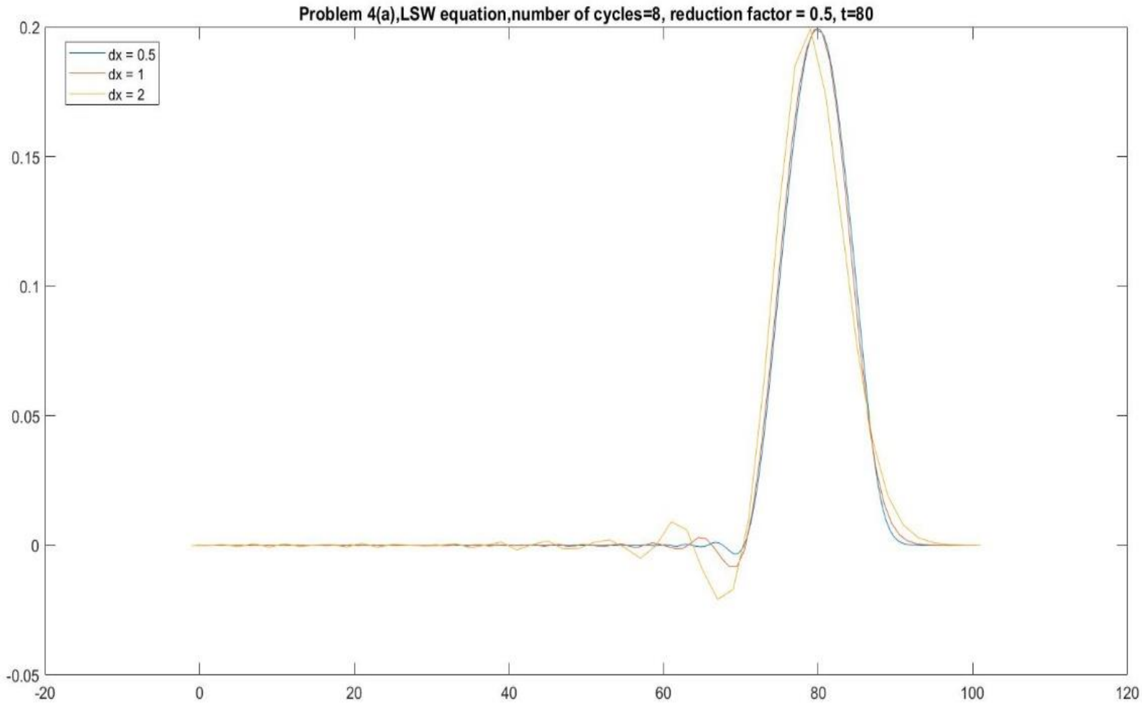
Use Matlab, for instance, to produce an initial elevation

η(x,0) =(2Acos^2(π(x−x0)/ λ) if − 1 2λ < x−x0 < 1 2λ

otherwise

η(x,0) =0

which is combined with u(x,0) = 0. Explain why this initial condition has continuous first derivative and why it to leading order yields a wave in each direction with amplitude A and length λ. Choose λ = 20, x0 = 0, a tank length L = 100 and simulation time t = 80. Remember



that the wave tank starts at x = 0. We still have constant depth h = 1.

Figure 3 The three solution with different number of grid points using LSW model

a) Solve the LSW equations. What should the result be? Adjust the resolution to obtain this result well enough (explain what you mean).

   The result is a soliton with an oscillatory tail staying behind. For higher resolution we will get only a soliton.