

# SENIOR HONOURS PROJECT



University of  
St Andrews

## Freeing Neural Training Through Surfing

*Georg Wölflein*  
*170011885*

*Supervisor: Dr. Mike Weir*

April 9, 2020

Word count: 1386 words

# Abstract

TODO

# Declaration

“I declare that the material submitted for assessment is my own work except where credit is explicitly given to others by citation or acknowledgement. This work was performed during the current academic year except where otherwise stated. The main text of this project report is 1386 words long, including project specification and plan. In submitting this project report to the University of St Andrews, I give permission for it to be made available for use in accordance with the regulations of the University Library. I also give permission for the title and abstract to be published and for copies of the report to be made and supplied at cost to any bona fide library or research worker, and to be made available on the World Wide Web. I retain the copyright in this work.”

*Georg Wölflein*

# Contents

<b>1</b>	<b>Introduction</b>	<b>5</b>
<b>2</b>	<b>Context survey</b>	<b>6</b>
<b>3</b>	<b>Requirements specification</b>	<b>7</b>
3.1	Ethics . . . . .	7
<b>I</b>	<b>Theory</b>	<b>8</b>
<b>4</b>	<b>Neural network theory</b>	<b>9</b>
4.1	Supervised learning . . . . .	9
4.2	Artificial neural networks . . . . .	10
4.2.1	Single-layer perceptron . . . . .	10
4.2.2	Multi-layer perceptron . . . . .	12
4.3	Activation functions . . . . .	12
<b>5</b>	<b>Neural network learning</b>	<b>14</b>
5.1	Gradient descent with mean squared error . . . . .	14
5.2	Local minimum problem . . . . .	14
5.3	Simulated annealing . . . . .	14
<b>6</b>	<b>Neural surfing theory</b>	<b>15</b>
6.1	Weight and output spaces . . . . .	15
<b>7</b>	<b>Problems</b>	<b>16</b>
7.1	Stripe problem . . . . .	16
<b>II</b>	<b>Framework</b>	<b>18</b>
<b>8</b>	<b>Design</b>	<b>19</b>
<b>9</b>	<b>Implementation</b>	<b>20</b>



# Chapter 1

## Introduction

*Describe the problem you set out to solve and the extent of your success in solving it. You should include the aims and objectives of the project in order of importance and try to outline key aspects of your project for the reader to look for in the rest of your report. **TODO***

## Chapter 2

# Context survey

*Surveying the context, the background literature and any recent work with similar aims. The context survey describes the work already done in this area, either as described in textbooks, research papers, or in publicly available software. You may also describe potentially useful tools and technologies here but do not go into project-specific decisions.*

- TensorFlow
- keras

**TODO**

## Chapter 3

# Requirements specification

Primary objectives:

1. Design a generic framework that can be used for various neural training algorithms with a clear set of inputs and outputs at each step. This framework should include benchmarking capabilities.
2. For a simple case of this framework (when the dimensionality of the control space and output space are suitably low), implement a visualisation tool that shows the algorithm's steps.
3. Implement a particular training algorithm for the framework that uses potential field techniques.
4. Evaluate the performance of this and other algorithms on tasks of differing complexity, especially with regard to the local minimum problem and similar issues.

Secondary objectives:

1. Investigate how this approach can be generalized to other numerical optimisation problems.

### 3.1 Ethics

There are no ethical considerations. All questions on the preliminary self-assessment form were answered with “NO” and hence no ethics form had to be completed.



# Part I

# Theory

## Chapter 4

# Neural network theory

### 4.1 Supervised learning

**Regression model** In machine learning, a regression model  $f$  is defined as a mathematical function of the form

$$f(\mathbf{x}) = \hat{y} = y + \epsilon \quad (4.1)$$

that models the relationship between a  $D$ -dimensional feature vector  $\mathbf{x} \in \mathbb{R}^D$  of independent (*input*) variables and the dependent (*output*) variable  $y \in \mathbb{R}$ . Given a particular  $\mathbf{x}$ , the model will produce a *prediction* for  $y$  which we denote  $\hat{y}$ . Here, the additive error term  $\epsilon$  represents the discrepancy between  $y$  and  $\hat{y}$ .

**Supervised learning** A supervised learning algorithm for a regression task infers the function  $f$  given in (4.1) from a set of *labelled training data*. This dataset consists of  $N$  tuples of the form  $\langle \mathbf{x}_i, y_i \rangle$  for  $i = 1, \dots, N$ . For each feature vector  $\mathbf{x}_i$ , the corresponding  $y_i$  represents the observed output, or *label* [Burkov, 2019]. We use the vector

$$\mathbf{y} = [y_1 \quad y_2 \quad \cdots \quad y_N]^\top \quad (4.2)$$

to denote all the labelled outputs in the dataset, and

$$\mathbf{X} = [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \cdots \quad \mathbf{x}_N]^\top \quad (4.3)$$

is the  $N \times D$  matrix representing the corresponding feature vectors.

Similarly,

$$\hat{\mathbf{y}} = [\hat{y}_1 \quad \hat{y}_2 \quad \cdots \quad \hat{y}_N]^\top \quad (4.4)$$

denotes a particular prediction for each training sample.

## 4.2 Artificial neural networks

Artificial neural networks (ANNs) take inspiration from the human brain and can be regarded as a set of interconnected neurons. More formally, an ANN is a directed graph of  $n$  neurons (referred to as *nodes* or *units*) with weighted edges (*links*). Each link connecting two units  $i$  and  $j$  is directed and associated with a real-valued weight  $w_{i,j}$ .

A particular unit  $i$ 's *excitation*, denoted  $ex_i$ , is calculated as the weighted sum

$$ex_i = \sum_{j=1}^n w_{j,i} a_j + b_i \quad (4.5)$$

where  $a_j \in \mathbb{R}$  is another unit  $j$ 's *activation* and  $b_i \in \mathbb{R}$  is the  $i$ th unit's *bias*. Notice that if there exists no link between unit  $i$  and a particular  $j$  then simply  $w_{i,j} = 0$  and therefore  $j$  will not contribute to  $i$ 's excitation.

The unit  $i$ 's activation is its excitation applied to a non-linear *activation function*,  $g$ . We have

$$a_i = g(ex_i) = g\left(\sum_{j=1}^n w_{j,i} a_j + b_i\right). \quad (4.6)$$

Activation functions will be explored in more detail in Section 4.3.

**ANNs as regression models** We can employ an ANN to model a regression problem of the form given in (4.1). To do so, we need at least  $D + 1$  neurons in the network. We consider the first  $D$  units to be the *input* neurons, and the last neuron,  $n$ , is the output unit. Furthermore, we require  $w_{j,k} = 0$  for  $j, k \in \mathbb{Z}^+$  where  $j \leq n$  and  $k \leq D$  to ensure that there are no links feeding into the input neurons.

To obtain the prediction  $\hat{y}$  given the  $D$ -dimensional feature vector  $\mathbf{x}$ , we set the activation of the  $i$ th unit to the value the  $i$ th element in  $\mathbf{x}$  for  $i = 1, \dots, D$ . Then, we propagate the activations using (4.6) until finally the prediction is the activation of the last neuron,  $\hat{y} = a_n$ . This process is often called *forward propagation* or *forward pass* [Russell and Norvig, 2010].

### 4.2.1 Single-layer perceptron

A single-layer perceptron (SLP) is a type of ANN that consists of two layers, an input and an output layer. Every input node is connected to every output node, but there are no intra-layer links (i.e. there are no links between any two input nodes or any two output nodes). This is what we call a *fully-connected feedforward* architecture. SLP architectures will always form a *directed acyclic graph* (DAG) because there are no intra-layer or backwards connections. Figure 4.1 shows an example of a DAG representing a SLP.

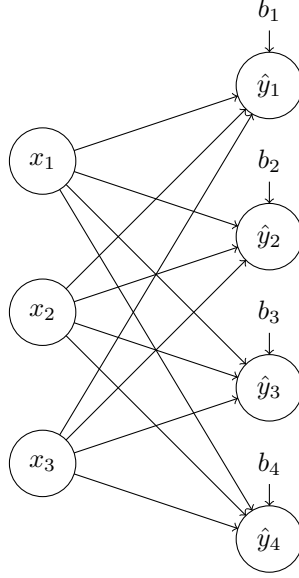


Figure 4.1: A single-layer perceptron with three input and four output neurons.

Let us consider a SLP with  $m$  inputs and  $n$  outputs. Since every output unit  $i$  only has connections from every input unit  $j$ , we can adapt (4.6) to give the activation of a particular output neuron  $i$  as

$$a_i = y_i = g(ex_i) = g\left(\sum_{j=1}^m w_{j,i}x_j + b_i\right) = g(\mathbf{w}_i\mathbf{x} + b_i) \quad (4.7)$$

where  $\mathbf{w}_i = [w_{1,i} \ w_{2,i} \ \cdots \ w_{m,i}]$  represents the weights of all the edges that connect to output unit  $i$ . If we use the  $n \times m$  matrix

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_n \end{bmatrix} = \begin{bmatrix} w_{1,1} & w_{2,1} & \cdots & w_{m,1} \\ w_{1,2} & w_{2,2} & \cdots & w_{m,2} \\ \vdots & \vdots & \ddots & \vdots \\ w_{1,n} & w_{2,n} & \cdots & w_{m,n} \end{bmatrix} \quad (4.8)$$

to capture all weights and the vector  $\mathbf{b} = [b_1 \ b_2 \ \cdots \ b_n]^\top$  for the biases, we can give a mathematical formula describing the relationship between the inputs and outputs as

$$\mathbf{f}_{\text{SLP}}(\mathbf{x}; \mathbf{W}, \mathbf{b}, \mathbf{g}) = \hat{\mathbf{y}} = \mathbf{g}(\mathbf{W}\mathbf{x} + \mathbf{b}). \quad (4.9)$$

Unlike the formula for a regression model, this is a vector-valued function, due to the fact that there are multiple outputs. Notice, however, that when  $n = 1$ , we reach the same form as in (4.1).

### 4.2.2 Multi-layer perceptron

A multi-layer perceptron (MLP) is a fully-connected feedforward ANN architecture with multiple layers which we will define in terms of multiple nested SLPs as in [Burkov, 2019]. A MLP with  $L$  layers is the mathematical function

$$f_{MLP}(\mathbf{x}) = \hat{y} = f_L(\mathbf{f}_{L-1}(\dots(\mathbf{f}_1(\mathbf{x})))) \quad (4.10)$$

where  $\mathbf{f}_l(\mathbf{x}) = \mathbf{f}_{SLP}(\mathbf{x}; \mathbf{W}_l, \mathbf{b}_l, \mathbf{g}_l)$  for  $l = 1, \dots, L - 1$ .

Therefore, to fully define a MLP, we need the three-tuple  $\langle \mathcal{W}, \mathcal{B}, \mathcal{G} \rangle$  where  $\mathcal{W} = \mathbf{W}_1, \mathbf{W}_2, \dots, \mathbf{W}_L$  are the weight matrices,  $\mathcal{B} = \mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_L$  the bias vectors, and  $\mathcal{G} = \mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_L$  the vector-valued activation functions. Notice that for every  $l < L$ ,  $\mathbf{W}_l$  is a  $n_l \times m_l$  matrix such that  $n_l = m_{l+1}$  to ensure that the number of outputs of layer  $l$  is the number of inputs to layer  $l + 1$ . This means that the MLP has  $m_1$  input neurons. The outermost function  $f_L$  is the scalar-valued function  $f_L(\mathbf{x}) = f_{SLP}(\mathbf{x}; \mathbf{W}_L, \mathbf{b}_L, \mathbf{g}_L)$  because it represents a SLP with only one output unit which also means that  $\|\mathbf{b}_L\| = 1$ ,  $\mathbf{W}_L$  has only one row, and finally  $n_L = 1$ .

The graph representing this type of network consists of connecting the outputs of the SLP representing layer  $l$  with the inputs of the SLP representing layer  $l + 1$ , as shown in Figure 4.2. The layers between the input and output layers are referred to as *hidden* layers.

Since MLPs are simply nested SLPs, it follows that MLPs retain the DAG property and are therefore *feedforward* networks as well. In the forward pass, the activations are propagated from layer to layer (i.e. nested function to nested function) as in (4.9).

## 4.3 Activation functions

Although units within a network can have different activation functions, this project solely employs networks where every unit uses the same  $g$ . Common activation functions include the sigmoid

$$S(x) = \frac{1}{1 + e^{-x}} \quad (4.11)$$

**TODO**

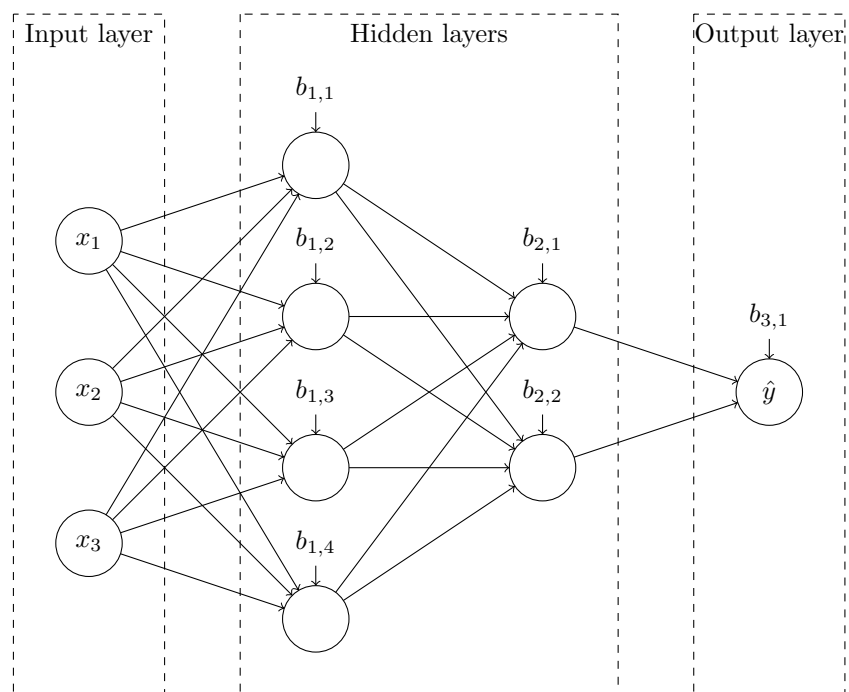


Figure 4.2: A multi-layer perceptron with three inputs and two hidden layers.

## Chapter 5

# Neural network learning

5.1 Gradient descent with mean squared error

5.2 Local minimum problem

5.3 Simulated annealing

## Chapter 6

# Neural surfing theory

### 6.1 Weight and output spaces

We define the weight space  $\mathcal{W}$  **TODO**

The output space  $\mathcal{O}$  spans the space of all possible output predictions on the training set,  $\hat{\mathbf{y}}$ , so  $\mathcal{O} = \mathbb{R}^N$  considering the fact that the training set has  $N$  samples.



## Chapter 7

# Problems

### 7.1 Stripe problem

# Ideas

Generalize to classification as regression with multiple output variables?

# **Part II**

# **Framework**

## Chapter 8

# Design

TODO

## Chapter 9

# Implementation

TODO

# Bibliography

- [Burkov, 2019] Burkov, A. (2019). *The Hundred-Page Machine Learning Book*. Andriy Burkov.
- [Russell and Norvig, 2010] Russell, S. and Norvig, P. (2010). *Artificial Intelligence: A Modern Approach*. Pearson, 3rd edition.