Quantum Computing
~~Tests~~ Final exam

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Section A

Q1.

$$|\psi\rangle = \frac{|00\rangle + n|11\rangle}{\sqrt{1+|n|^2}}$$

$$|\psi\rangle\langle\psi| = \frac{1}{1+|n|^2}\Big(|00\rangle\langle 00| + |00\rangle n^*\langle 11| + n|11\rangle\langle 00| + n|11\rangle n^* \cancel{}\langle 11|\Big)$$

$$\rho_{AB} = P/4 \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix} + \frac{(1-P)}{1+|n|^2}\left(\begin{bmatrix} 1&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix} + n^*\begin{bmatrix} 0&0&0&1\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0 \end{bmatrix} + n\begin{bmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 1&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0\\ 0&0&0&0\\ 0&0&0&0\\ 0&0&0&1 \end{bmatrix}\right)$$

$$= P/4 \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix} + \frac{1-P}{1+|n|^2}\begin{bmatrix} 1&0&0&n^*\\ 0&0&0&0\\ 0&0&0&0\\ n&0&0&1 \end{bmatrix}$$

$$\rho_A = tr_A(\rho_{AB}) = P/4 \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix} + \frac{1-P}{1+|n|^2}\left(|0\rangle\langle 0| + |n|^2|1\rangle\langle 1|\right)$$

$$\rho_B = tr_B(\rho_{AB}) = P/4 \begin{bmatrix} 1&0&0&0\\ 0&1&0&0\\ 0&0&1&0\\ 0&0&0&1 \end{bmatrix} + \frac{1-P}{1+|n|^2}\left(|0\rangle\langle 0| + |n|^2|1\rangle\langle 1|\right)$$

~~AB~~ eigenvalues of $\rho_{AB} = \left(P/4,\ P/4,\ \frac{P}{4}\left(\frac{1-P}{1+|n|^2}\right) - |n|,\ \frac{P}{4}\left(\frac{1-P}{1+|n|^2}\right) + |n|\right)$

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$$S(\rho_{AB}) = \cancel{-\frac{p}{4}\log(p/4)}$$

$$= -\left[\frac{p}{4}\log\left(\frac{p}{4}\right) + \frac{p}{4}\log\left(\frac{p}{4}\right) + \left[\frac{p}{4}\left(\frac{1-p}{1+|n|^2}\right) - |n|\right]\log\left[\frac{p}{4}\left(\frac{1-p}{1+|n|^2}\right) - |n|\right]\right.$$

$$\left. + \left[\frac{p}{4}\left(\frac{1-p}{1+|n|^2}\right) + |n|\right]\log\left[\frac{p}{4}\left(\frac{1-p}{1+|n|^2}\right) + |n|\right]\right]$$

~~$S(\rho_A)$~~ eigenvalues of ρ_A are

$$\left(\frac{p}{4} + \frac{1-p}{1+|n|^2},\ \frac{p}{4} + \frac{|n|^2(1-p)}{1+|n|^2}\right)$$

eigenvalues of ρ_B are

$$\left(\frac{p}{4} + \frac{1-p}{1+|n|^2},\ \frac{p}{4} + \frac{|n|^2(1-p)}{1+|n|^2}\right)$$

$$S(\rho_A) = -\left(\left[\frac{p}{4} + \frac{1-p}{1+|n|^2}\right]\log\left(\frac{p}{4} + \frac{1-p}{1+|n|^2}\right) + \left[\frac{p}{4} + \frac{|n|^2(1-p)}{1+|n|^2}\right] \times \log\left[\frac{p}{4} + \frac{|n|^2(1-p)}{1+|n|^2}\right]\right)$$

$$= S(\rho_B)$$

$$S(\rho_{A|B}) = S(\rho) - S(\rho_B)$$

$$S(\rho_{B|A}) = S(\rho) - S(\rho_A)$$

2. Alice wants to send two bits. These can be: 00, 01, 10, 11.

Apply $I \otimes I$ on $|x\rangle$

$$\psi = (I_A \otimes I_B)|x\rangle_{AB} = \frac{1}{\sqrt{1+|n|^2}}(|00\rangle + n|11\rangle)$$

An unchanged state $\therefore$ 00 was sent.

Apply $\sigma_z \otimes I$

$$|x_1'\rangle = (\sigma_{zA} \otimes I_B)|x\rangle_{AB}$$

$$= \frac{1}{\sqrt{1+|n|^2}}(|00\rangle - n|11\rangle)_{AB}$$

$\langle \cancel{x} x | x_1' \rangle \cancel{AB} = 0$ and now $|x'\rangle$ is entangled.

measuring $|x_1'\rangle$ will mean 01 was sent.

Applying $\sigma_x \otimes I$,

$$|x_2\rangle = (\sigma_{xA} \otimes I_B)|x\rangle_{AB} = \frac{1}{\sqrt{1+|n|^2}}(|10\rangle + n|01\rangle)_{AB}$$

$\langle x_1 | x_2 \rangle = 0$ so if x_2 is measured 10 was sent.

Apply $(i\sigma_y \otimes I)$,

$$|\chi_3\rangle = \frac{1}{\sqrt{1+|n|^2}}\left(|10\rangle - n|01\rangle\right)$$

$\langle \chi_2 | \chi_3 \rangle = 0$

A measurement of $|\chi^3\rangle$ means $|1$ was sent.