Program Verification
Final exam

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1 a) In short, if a sequent is valid, then it can be proved using the sequent calculus.
And if a proof is done using the sequent calculus then the proved sequent is valid.

b) a state ~~m~~ at which all ~~points~~ inputs in a discrete flow eventually converge at is the convergence point.
i.e for $D = \langle X, F \rangle$, $\forall x \in X$, $F(x) =$ a convergent pt.

c) The largest A is (7, 7)

i.e $(u = 7, v = 7)\{F\}B$

2 a) Consider $x, y \in \mathbb{N}^+$, $i = 0$, $x \geq y$

~~x, y is~~

$Div(x, y) =$ ~~...~~ (r, q)
where q is the quotient and r is the remainder.

~~q is computed by~~

it is true that $x = q \times y + r$.
The problem is to write an algorithm that calculates Div.

$M = \langle P, F, \pi \rangle$

b) $P : N \times N^{+} \rightarrow N \times N^{+} \times N$

$X = N \times N^{+} \times N$

$P(x, y) = (x, y, 0)$

$F : X \rightarrow X$

$$F(x, y, i) = \begin{cases} (x-y, \ y, \ i+1) & \text{if } x \geq y \\ (x, \ y, \ i) & \text{otherwise} \end{cases}$$

$\pi : X \rightarrow N \times N$

$\pi(x, y, i) = (x, i)$

c) consider a function

$\lambda(x, y, i) = x$

$\lambda(F(x, y, i)) = x - y$

$\lambda(x, y, i) < \lambda(F(x, y, i))$

it follows that $\lambda(x, y, i) < \lambda(F^{i}(x, y, i))$
this ~~[illegible]~~ chain eventually

ends when $x < y$.
$x \in \mathbb{N}$ and
$(\mathbb{N}, <)$ is a well founded relation
$\therefore$ M is an algorithm.

d) Consider the invariant function

$\theta(x, y, i) = y \times i + x \longrightarrow ①$

$\theta(F(x, y, i)) = y \times (i+1) + (x-y)$

$= y \times i + y + x - y$
$= y \times i + x$
$=$ eq ①

$\therefore$ θ is an invariant function and hence M is partially correct.

1. c) An invariant function is one such that for a mapcode machine $M = \langle \rho, F, \pi \rangle$

$\theta(\rho(i)) = \theta(F^j(\rho(i)))$

for any input i in I (input space)
and for any $j \in [1, \infty)$

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3. Pseudocode for the repeated subtraction division algorithm is

1. $(x, y, i) := (x, y, 0)$
2. while $(x \geq y)$ do:
   2.1 $(x, y, i) := (x-y, y, i+1)$

required specification is :

$$\text{TRUE} \; \{P\} \; x' = i * y + x$$

where x' is the initial x.