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Automata Theory
Theory Assignment - 2

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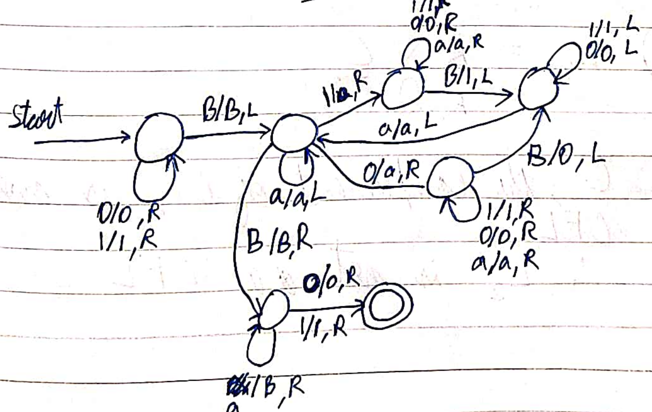
(1) Assuming the Turing ~~a~~ Machine is space bounded ~~by~~ by N,
for a turing macahine $T = (Q, \Gamma, b, \Sigma, \delta, q_0, F)$
A single configuration of a Turing Machine consists of ~~a~~ (S, q, K)

where $S \in \Gamma^*$, is the string on the tape
$q \in Q$ is the current state
$K \in \mathbb{N}$ is the position on the tape.

Now
$|\Gamma^*| = |\Gamma|^N$
$|Q| = |Q|$
$|\mathbb{N}| = N$, since ~~the~~ T is space bound.

$\therefore$ cardinality of the configuration space
$= |\Gamma|^N \times |Q| \times N$

(2)



③ 1) $L_1 = C \setminus R$
$= C \cap \overline{R}$

R is regular $\therefore \overline{R}$ is regular and since the intersection of a regular language and a CFL ~~a~~ is a CFL,

L_1 is a CFL

2) $L_2 = R \setminus C$
$= R \cap \overline{C}$

$\overline{C}$, the complement of C, is not a CFL
$\therefore L_2$ is not ~~a~~ a CFL

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(4) $L = \{ a^i b^j c^k$; where i, j, k are different $\}$

satisfies the pumping lemma but is not context free.

(5) Begin by ~~xxxx~~ encoding the entire input in the two stacks. Push the ~~xx~~ entire input to a stack. We treat the two stacks as the content of the tape to the right and left of the head.

Start by pushing every symbol in the input tho the left stack until blank symbol.

Replace left moving instructions in the Turing machine,

$x / y \; L$ with

x, pop x, push ϵ, pop ϵ, push y

Replace the right moving instructions ~~with~~

$x / y, \; R$ with

x, pop ϵ, push y, pop x, push ϵ

∴ the 2-stack PDA will first have a ~~xxxx~~ part that pushes the full input to the stack, and once replaced as mentioned, the ~~a~~ PDA is now equivalent to a Turing Machine.

(6) Suppose each cell in the 2D grid is assigned some coordinates. (0,0), (1,-1)...

Consider the ~~new~~ Γ',

~~Even~~ $\Gamma' = \{(a,b,x);\ a,b \in \mathbb{N},\ x \in \Gamma\}$

In this way we can encode the entire 2D grid on the 1D tape and similarly operation can be performed.

(7) Consider a machine T that addresses the halting problem for (N, y) i.e. for the machine ~~L~~ N ~~is~~ which is L(N) and on input y.

The input to T is x i.e. T runs N on input y for x steps.

In this way we can reduce the problem to ~~a~~ a form of the halting problem which is known to be undecidable

hence L is undecidable.

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(8) 1) for $L = \{ a^n b^j c^k ; k = jn \}$, let L be be context free.
Let p be a pumping length,

Consider a string
$a^p b^p c^{p^2}$

Choosing $uvwxy$ for the pumping lemma can be done in the following ways:

- v and y has either only a's, b's or c's - pumping this case will definitely yield a string $\notin L$ since $k \neq jn$
- v has a's or b's and y with c's - pumping the same amount of a's or b's and c's will give a string in L only if j = 1 or n = 1
- v has a's, y has b's - pumping a's and b's without pumping c's guarantees that $k \neq jn$.
- Any other case with combination v and/or y gives inconsistencies such as a's after b's etc.

$\therefore$ L is not context free, since it contradicts.

2) Let $L = \{ a^{n!} ; n \geq 0 \}$ be context free. with pumping length p.

Consider $a^{p!}$
Choosing any $uvwxy$,
if $|v| = i$ and $|x| = j$
~~then~~

then pumping k times gives a ~~b~~ string of total length

~~Rx~~ $p! + k(i+j) \neq x!$ for any $x \in \mathbb{N}$

~~there~~ $\therefore$ L is not context free, since it contradicts.