14 SEPTEMBER 2021, PROBABILITY AND STATISTICS LECTURE

SOME PROBLEMS IN PROBABILITY

Problem: Boy/Girl Problem

On the morning of January 1, a hospital nursery has 3 boys and some number of girls. That night, a woman gives birth to a child, and the child is placed in the nursery. On January 2, a statistician conducts a survey and selects a child at random from the nursery (including the newborn and every child from January 1). The selected child is a boy. What is the probability the child born on January 1 was a boy?

Answer

On Jan 1: 3 boys + k girls + 1(born on Jan 1)

On Jan 2: A boy is selected from $3+k+1$

Consider $k=3$, If the born baby is a boy then $(4 \text{ boys}, 3 \text{ girls})$ and if it is a girl then $(3 \text{ boys}, 4 \text{ girls})$

Let $P(B)$ be the event a boy is selected. And $P(N)$ be the event that a boy is born.

so then $P(B) = 1/2 * 4/7 + 1/2 * 3/7$

So

$$P(N|B) = \frac{P(N \cap B)}{P(B)} = \frac{1/2 * 4/7}{1/2} = \frac{4}{7}$$

Now suppose there are g girls,

If the born baby is a boy then $(4 \text{ boys}, g \text{ girls})$ and if it is a girl then $(3 \text{ boys}, g+1 \text{ girls})$

Now $P(B) = \frac{7}{2(g+4)}$

$$P(N|B) = \frac{1/2 * 4/(g+4)}{\frac{7}{2(g+4)}} = 4/7$$

Problem: Probability of seeing a car

If the probability of seeing a car on the highway in 30 minutes is 0.95, what is the probability of seeing a car on the highway in 10 minutes? (assume a constant default probability)

Answer

$$p = P(\text{no car in 10 min})$$
$$P(\text{no car in 20 min}) = p^2$$
$$P(\text{no car in 30 min}) = p^3$$
$$P(\text{car in 20 min}) = 1 - p^3$$
$$\text{solving,}$$
$$p =$$

Problem: Skewed Die Problem

A standard dice has the 6 showing with a probability of 1/6, which is the same as every other number. A loaded dice has the 6 showing with 1/2 the probability of the other numbers. What is the probability of rolling a 6?

Answer

$P(6) = P(\text{other numbers})/2 = p/2$

$p/2 + 5p = 1$

Then solve for p

Problem

It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?

$$P(A) = \text{email detected as spam}$$
$$P(B) = \text{email is spam}$$
$$P(B^c) = \text{email isn't spam}$$
$$P(B) = 1/2$$
$$P(A|B) = 0.99$$
$$P(A|B^c) = 0.05$$
$$\text{required probability,}$$
$$P(B^c|A) = \frac{P(A|B^c)P(B^c)}{P(A|B)P(B) + P(A|B^c)(B^c)}$$

INDEPENDENCE

Two events E, F are independent if

$P(E \cap F) = P(E)P(F)$

Also, $P(E|F) = P(E)$

Examples

- $E : D_1 = 1$
- $F : D_2 = 6$
- $G : D_1 + D_2 = 5$
- That is, $G = \{(1,4), (2,3), (3,2), (4,1)\}$

1. Are E and F independent?
2. Are E and G independent?

$$P(E \cap F) = 1/36$$
$$P(E) = 1/6$$
$$P(F) = 1/6$$
$$P(E \cap F) = P(E)P(F)$$

Therefore E and F are independent

$$P(E) = 1/6$$
$$P(G) = 1/9$$
$$P(E)P(G) = 1/54 \neq P(E \cap G)$$

N independent events

General Definition for many events

The n events $E_1, E_2, \ldots, E_n$ are independent if

for $r = \cancel{1}\,2, \ldots, n$:

for every subset $E_1, E_2, \ldots, E_r$:

$$P(E_1 \cap E_2 \cap \cdots \cap E_r) = P(E_1)P(E_2)\cdots P(E_r)$$

Independent trials