classmate

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Intro to Quantum Computing
Assignment 1

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2. $\rightarrow \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \sigma_x^* = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\therefore \sigma_x$ is hermitian

$\sigma_x \sigma_x^* = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\therefore \sigma_x$ is unitary

$\rightarrow \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = \sigma_y^* = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\therefore \sigma_y$ is hermitian

$\sigma_y \sigma_y^* = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \therefore \sigma_y$ is unitary

$\rightarrow \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \sigma_z^* = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ $\therefore \sigma_y$ is hermitian

$\sigma_z \sigma_z^* = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \therefore \sigma_z$ is unitary tensor products of the Pauli matrices:

$$\sigma_x \otimes \sigma_y = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \qquad \sigma_x \otimes \sigma_z = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\sigma_y \otimes \sigma_x = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \qquad \sigma_y \otimes \sigma_z = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$\sigma_z \otimes \sigma_x = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & \cancel{0}-1 & 0 \end{pmatrix} \qquad \sigma_z \otimes \sigma_{\cancel{x}y} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{pmatrix}$$

Consider any two unitary operators A, B and their tensor product $A \otimes B$, ~~the~~ ~~any~~ ~~then~~ take the product with the adjoint,

$$(A \otimes B)(A \otimes B)^*$$
$$= (A \otimes B)(A^* \otimes B^*)$$
$$= (AA^* \otimes BB^*)$$
$$= I \otimes I = I$$

$\therefore$ tensor product of unitary ~~u~~ matrices are the unitary.

hence, all $\sigma_i \otimes \sigma_j$ are unitary.

When taking the adjoints of all the $\sigma_i \otimes \sigma_j$ they remain the same.
i.e.

$$(\sigma_i \otimes \sigma_j) = (\sigma_i \otimes \sigma_j)^*$$

$\therefore$ they are all hermitian.

~~3. a) Since P is a projector,~~

$$\cancel{P = PP = P^2}$$
$$\cancel{P^{-1}P = P^{-1}PP}$$
$$\cancel{I = P}$$

3.a) ~~Since~~ Let a be an eigenvector associated with λ, an eigenvalue of P.

Consider $\quad Pa = \lambda a \quad \rightarrow (1)$

$P^2 a = \lambda (Pa) \quad \because P$ is a projector

$P^2 a = \lambda(\lambda a) \quad$ Using (1)

$Pa = \lambda^2 a \quad \because P$ is a projector $\rightarrow (2)$

(1) = (2), $\quad \lambda a = \lambda^2 a$

$(\lambda - \lambda^2) a = 0$

$\Rightarrow \lambda = \lambda^2$ i.e. $\lambda = 1$ or $\lambda = 0$

6 ~~b~~) Consider ~~Hermitian~~ Unitary operators A and B. and their tensor product $A \otimes B$.

~~Hath~~ taking its product with its adjoint.

$$(A \otimes B)(A \otimes B)^*$$
$$= (A \otimes B)(A^* \otimes B^*)$$
$$= (AA^* \otimes BB^*)$$
$$= (I \otimes I) = I$$

$\therefore (A \otimes B)$ is ~~herm~~ unitary.

~~a b) Consider Hermitian operators~~

1. Consider the Bell states as columns in a matrix.

$$A = \frac{1}{\sqrt{2}} \begin{array}{c} \begin{array}{cccc} \phi^+ & \phi^- & \psi^+ & \psi^- \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \end{array}$$

$$A^* = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} = A$$

$AA^* = I$ $\therefore$ the columns of A i.e. Bell States are an orthonormal basis.