Mechatronics System Design

EC4.404 - M2023

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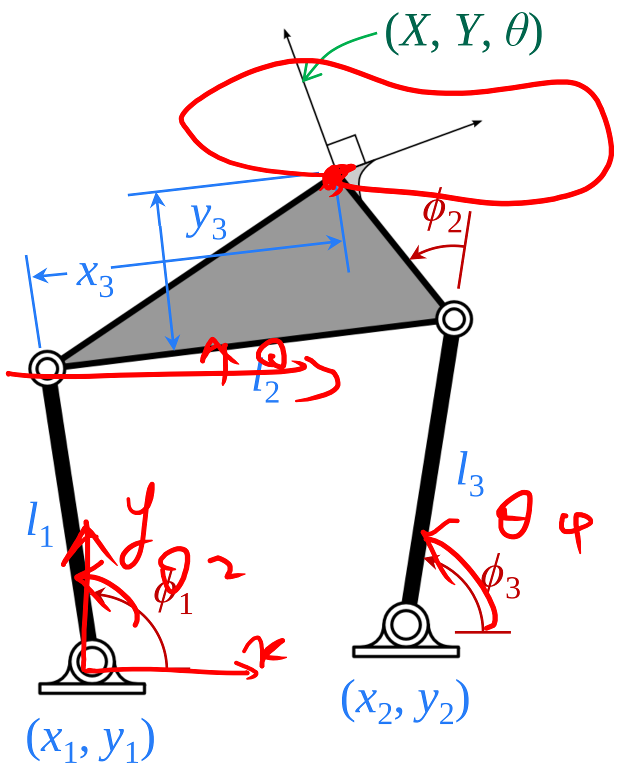
Problems in Kinematics

Dimensions

Joint Parameters

End Effector Coordinates

Analysis



Forward Kinematics
Known: Dimensions, Joint Parameters
Solve for: End Effector Coordinates

Inverse Kinematics
Known: Dimensions, End Effector Coordinates
Solve for: Joint Parameters

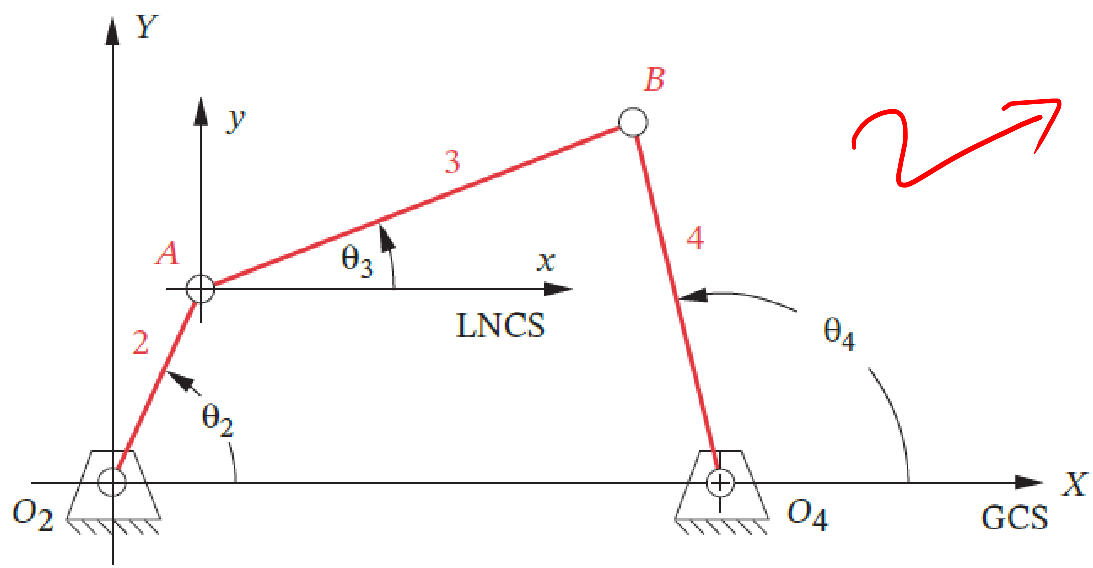
Synthesis
Known: End Effector Coordinates
Solve for: Dimensions, Joint Parameters

Graphical and Analytical

- Graphical
  - Have limitations of accuracy
  - Not suitable for computer simulation
  - Parameters are not easily manipulated to create new solutions
- Analytical
  - Graphical techniques are essential at the initial phases of kinematic synthesis.
  - Suitable for computer simulation

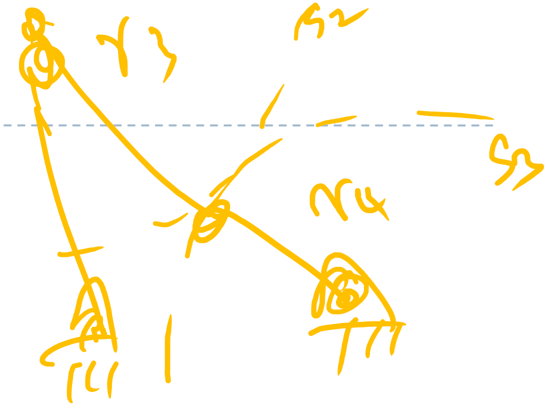
Graphical Position Analysis

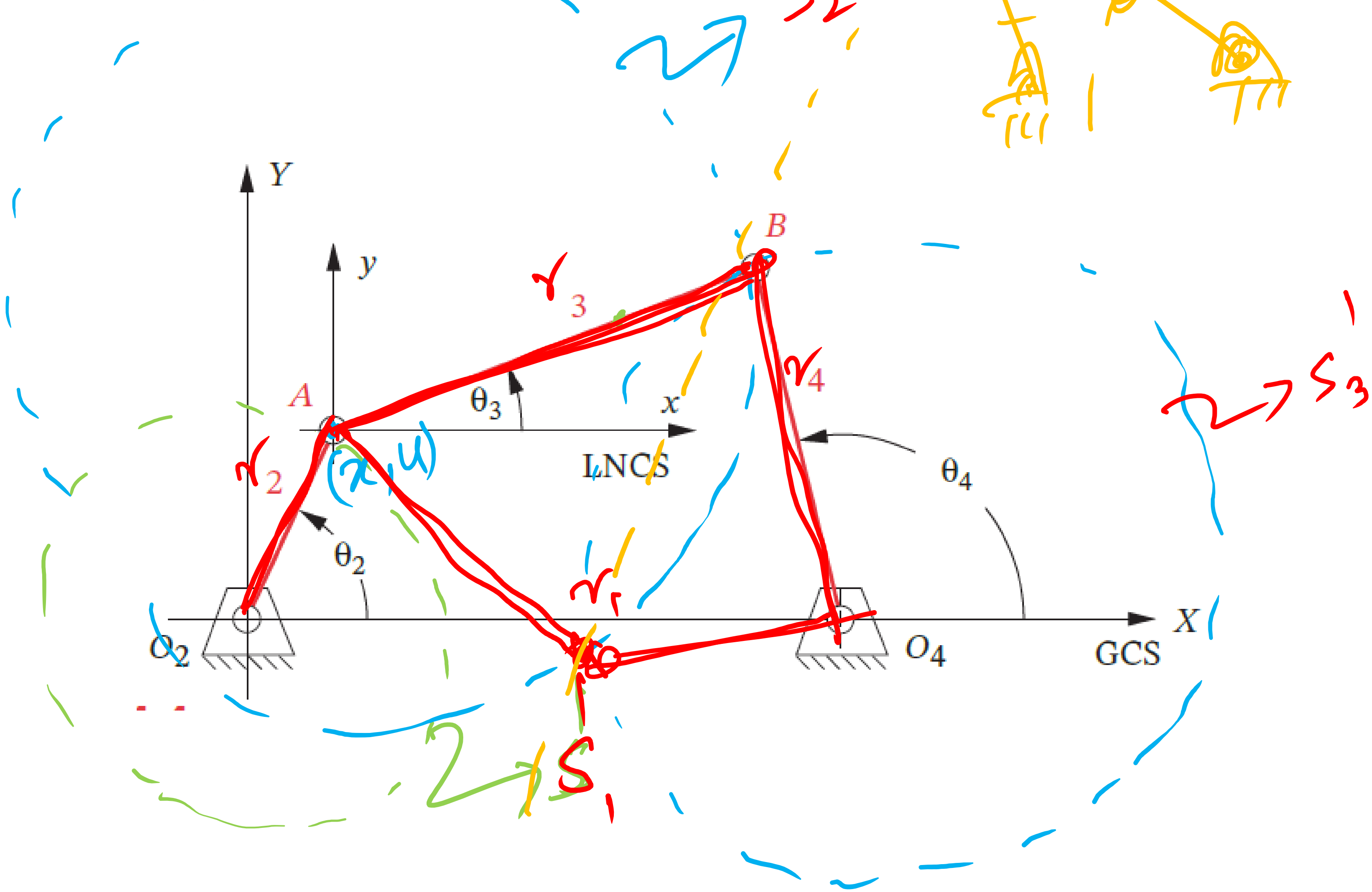
- For fourbar – one parameter is required to completely specify all the links
- The typical parameter is the crank angle θ_2
- Given the link lengths, find θ_3 and θ_4



2
1

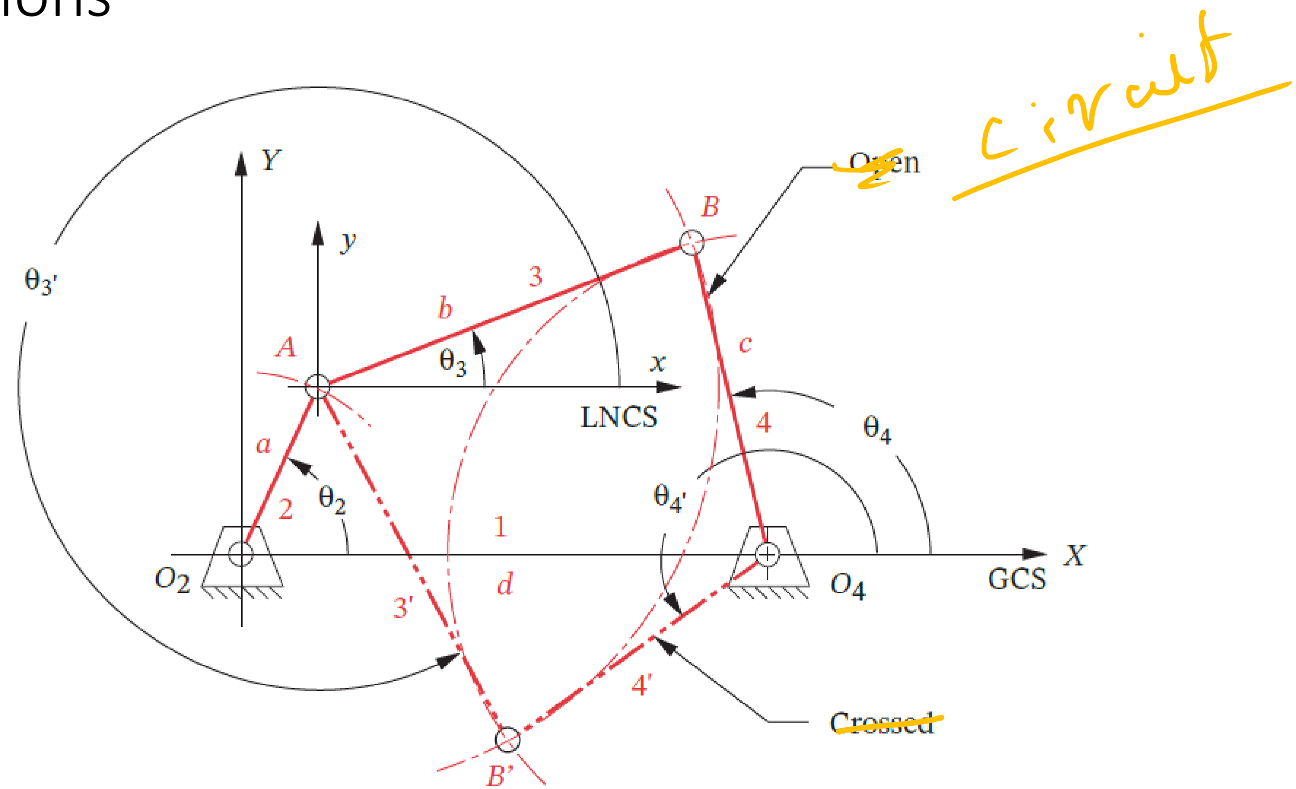
Graphical Position Analysis





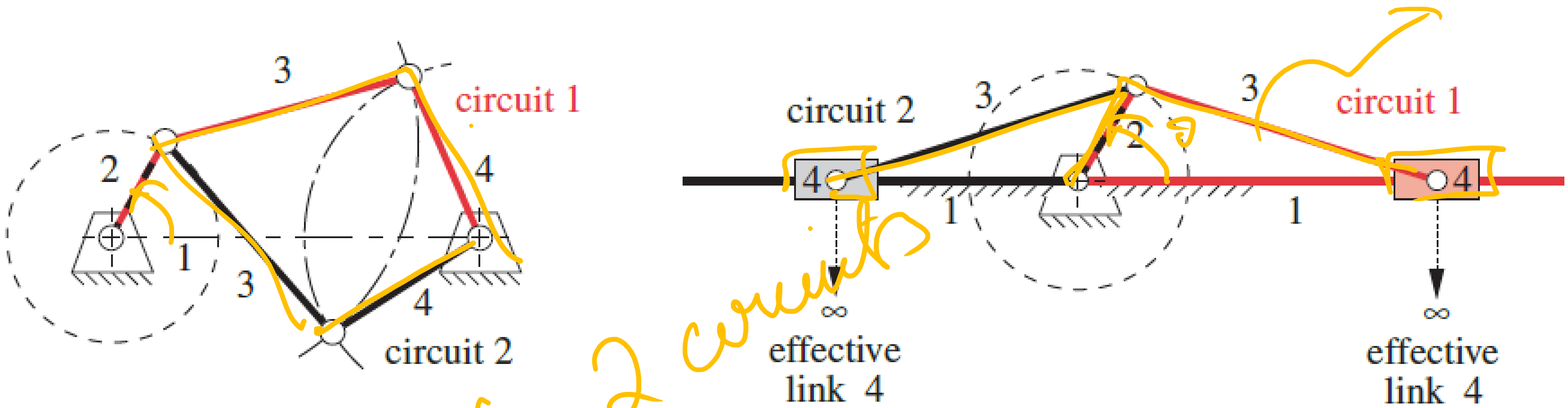
Graphical Position Analysis

- These two arcs will have two intersections at B and B' that define the two solutions to the position problem for a fourbar linkage which can be assembled in two configurations



CIRCUITS In Linkages

- all possible orientations of the links that can be realized without disconnecting any of the joints



Grashof = 2 circuits
non = 1 cir.

Position Analysis – Fourbar mechanism

$$\vec{r_2}+\vec{r_3}-\vec{r_4}-\vec{r_1}=0$$

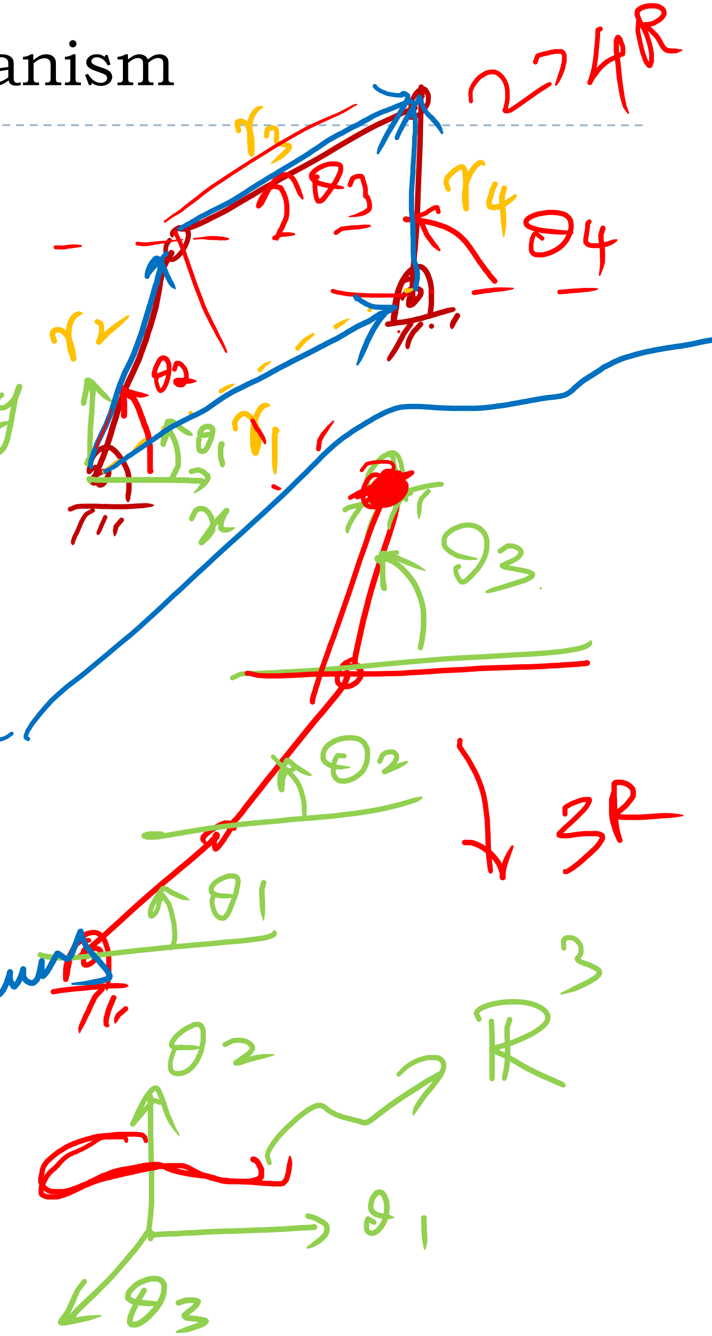
$$r_2 e^{i\theta_2}+r_3 e^{i\theta_3}=r_4 e^{i\theta_4}+r_1 e^{i\theta_1}$$

$$\Downarrow$$

$$C\theta + iS\theta$$

r_2, r_3, r_4, r_1 are given

$\cancel{\theta_1}, \theta_2, \theta_3, \theta_4$ are unknown



Position Analysis – Fourbar mechanism

$$r_2 c\theta_2 + r_3 c\theta_3 = r_4 c\theta_4 + r_1$$

$$r_2 s\theta_2 + r_3 s\theta_3 = r_4 s\theta_4$$

$$1 \leftarrow \quad r_3 c\theta_3 = r_4 c\theta_4 + r_1 - r_2 c\theta_2$$

$$2 \leftarrow \quad r_3 s\theta_3 = r_4 s\theta_4 - r_2 s\theta_2$$

$$1^2 + 2^2 \Rightarrow \quad r_3^2 = r_1^2 + r_4^2 + r_2^2 - 2r_1 r_2 c\theta_2 - 2r_2 r_4 s\theta_2 s\theta_4 + 2r_1 r_4 c\theta_4 - 2r_2 r_4 c\theta_2 c\theta_4$$

$$r_3^2 = r_1^2 + r_4^2 + r_2^2 - \underbrace{2r_1 r_2}_{K_1} c\theta_2 + \underbrace{2r_1 r_4}_{K_2} c\theta_4 - \overbrace{2r_2 r_4}^{K_3} c(\theta_2 - \theta_4)$$

Position Analysis – Fourbar mechanism

$$c(\theta_2-\theta_4) = \underbrace{\frac{r_1^2+r_2^2-r_3^2+r_4^2}{2r_2r_4}}_{k_3} - \underbrace{\frac{r_1}{r_4}}_{k_2}c\theta_2 + \underbrace{\frac{r_1}{r_2}}_{k_1}c\theta_4 .$$

$$\boxed{c(\theta_2-\theta_4) = k_1 c\theta_4 + k_2 c\theta_2 + k_3}$$

Freudenstein eqn

$$\boxed{s\theta_2 s\theta_4 + \cancel{c\theta_2 c\theta_4} = k_1 c\theta_4 + k_2 c\theta_2 + k_3} \rightarrow (3)$$

$$\Downarrow$$

Position Analysis – Fourbar mechanism

$$A c\theta + B s\theta + C = 0$$

$$s\theta = \frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)} \qquad c\theta = \frac{1-\tan^2(\theta/2)}{1+\tan^2(\theta/2)}$$

$$t = \tan(\theta/2)$$

Sub

$$\boxed{s\theta_4 = \frac{2t}{1+t^2} \qquad c\theta_4 = \frac{1-t^2}{1+t^2}}$$

in 3

$$t^2(\overbrace{K_2 C\theta_2 + K_3 + C\theta_2 - K_1}^{A}) +$$

$$(\underbrace{-2S\theta_2}_{B})t$$

$$(\underbrace{K_2 C\theta_2 + K_3 - C\theta_2 + K_1}_{C}) = 0$$

$$At^2 + Bt + C = 0$$

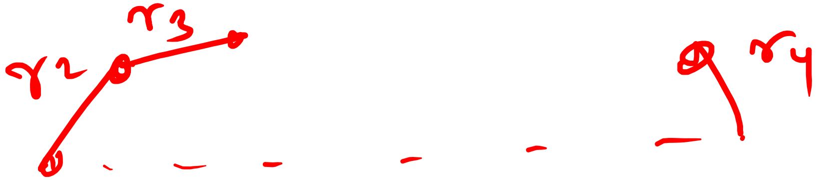
$$t_{1,2} = \frac{-B \pm \sqrt{\overbrace{B^2 - 4AC}^{D}}}{2A}$$

$\Delta > 0$, real distinct roots

$\Delta = 0$, real & repeated roots

$\Delta < 0$, Complex conjugate pairs.

$t = \tan\left(\frac{\theta_4}{2}\right)$ $\theta_4 = 2\tan^{-1}(t)$



Types of solution

If the discriminant under the radical is negative:

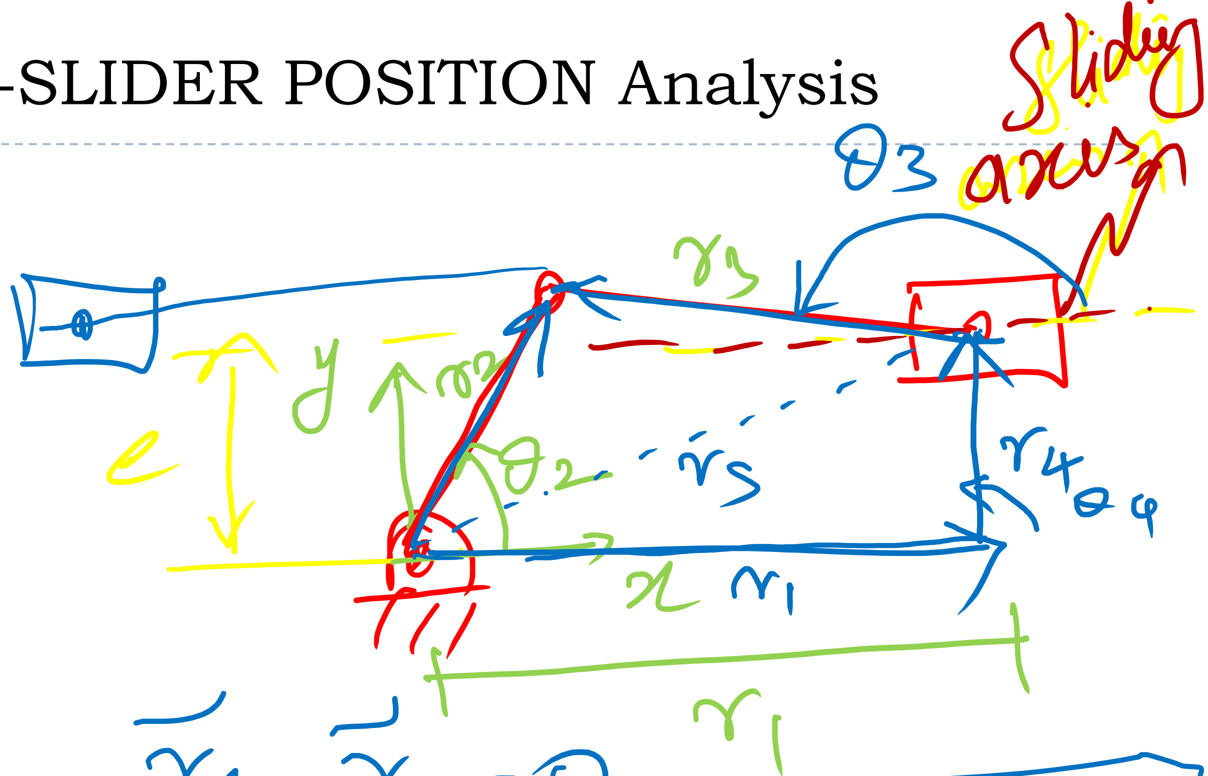
- link lengths chosen are not capable of connection for the chosen value of the input angle θ_2
- non-Grashof linkage, when the input angle is beyond a toggle limit position.

Otherwise, the solution will usually be real and unequal:

- There are two values of θ_4 corresponding to any one value of θ_2
- These are referred to as the **crossed** and **open** configurations of the linkage and also as the two **circuits** of the linkage.

FOURBAR CRANK-SLIDER POSITION Analysis

$\vec{r_5} = \vec{r_2} + \vec{r_3}$



$\vec{r_2} - \vec{r_3} - \vec{r_4} - \vec{r_1} = 0$

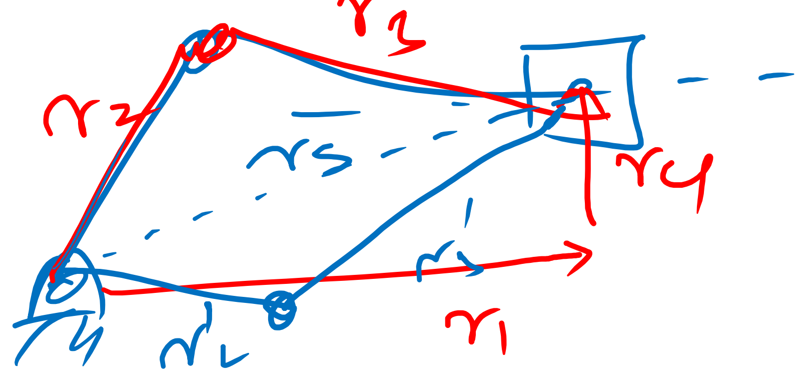
$\theta_3 = ?$

$r_1 = ?$

$$\boxed{\begin{gathered}\theta_3 = \arcsin\left(\frac{-r_4 + r_2 S\theta_2}{r_3}\right) \\ r_1 = r_2 C\theta_2 - r_3 C\theta_3\end{gathered}}$$

FOURBAR ~~CRANK-SLIDER~~ POSITION Analysis

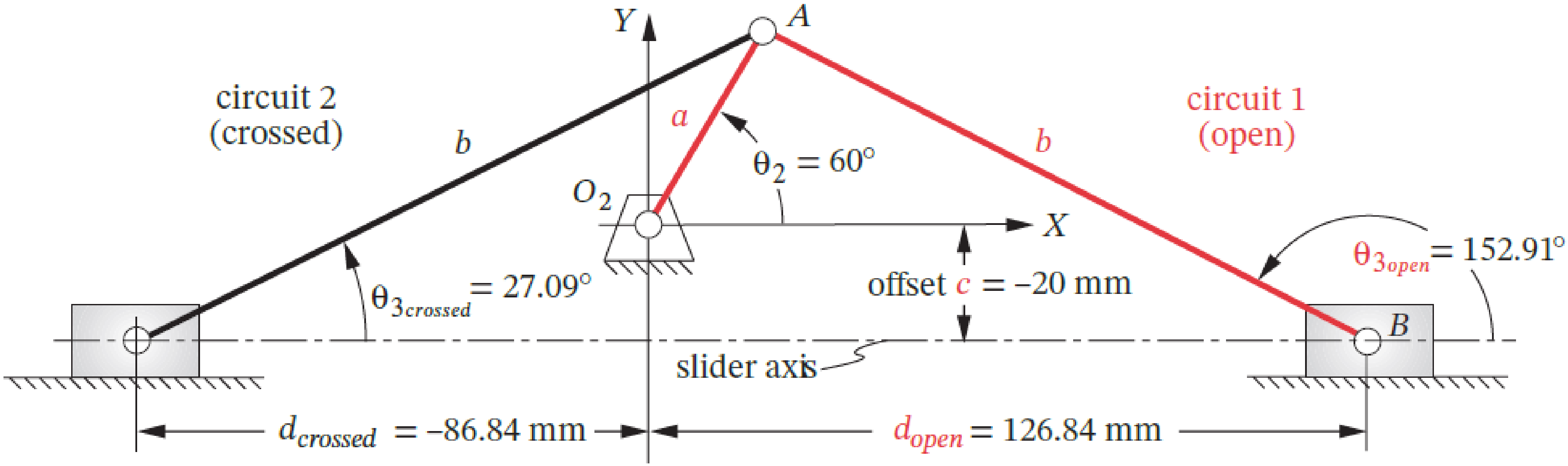
slider-crank



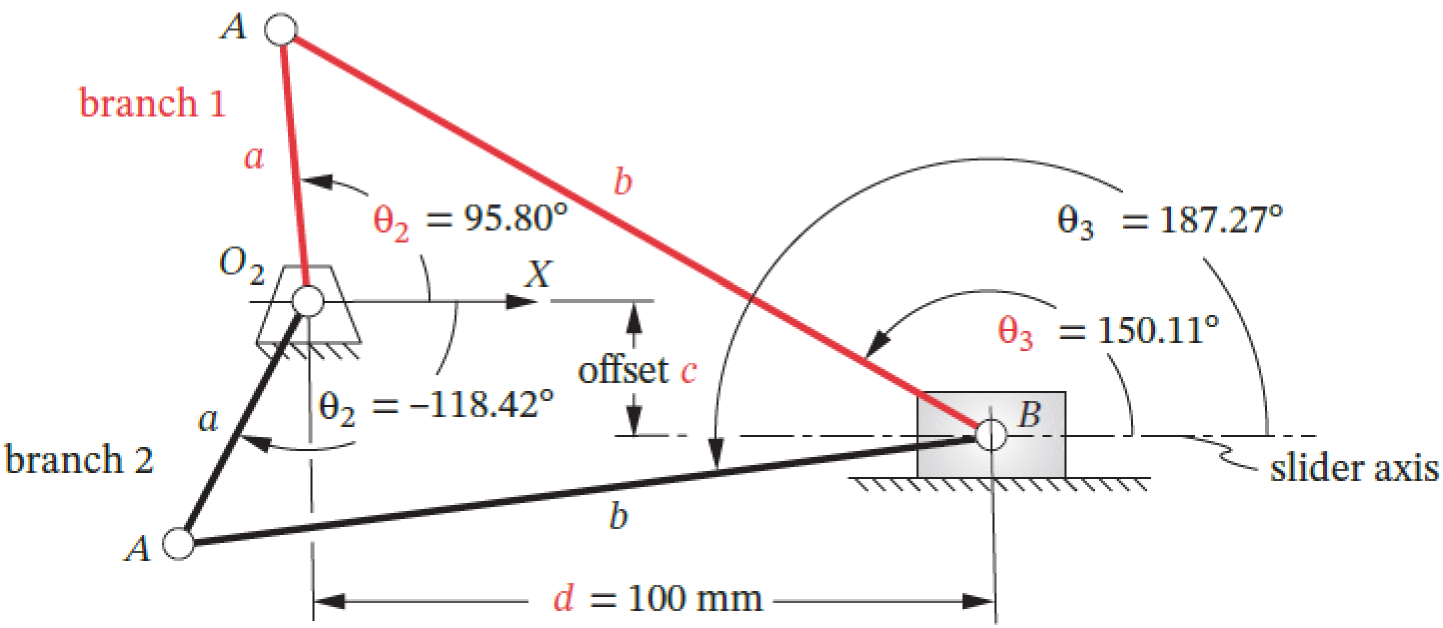
given r_1

find θ_2 & θ_3

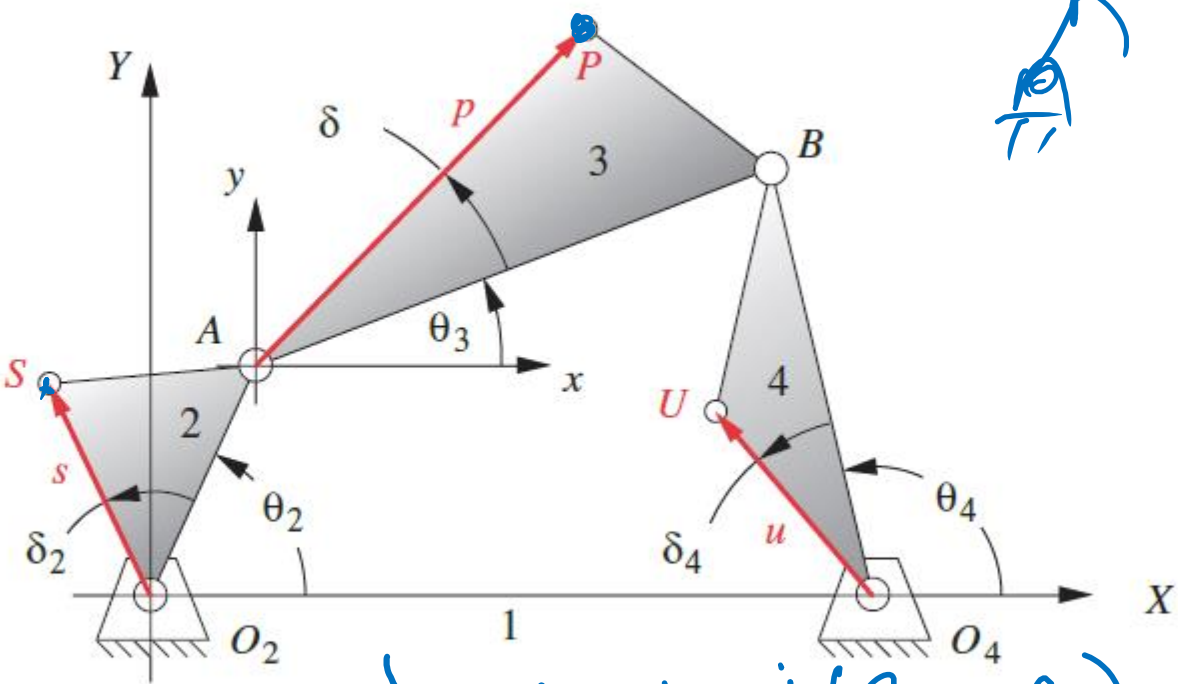
FOURBAR CRANK-SLIDER POSITION SOLUTION

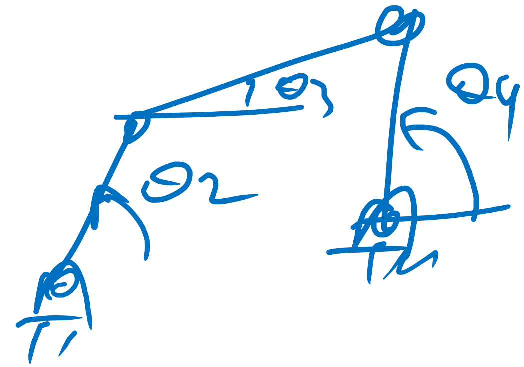


SLIDER-CRANK POSITION SOLUTION



POSITION OF ANY POINT ON A LINKAGE

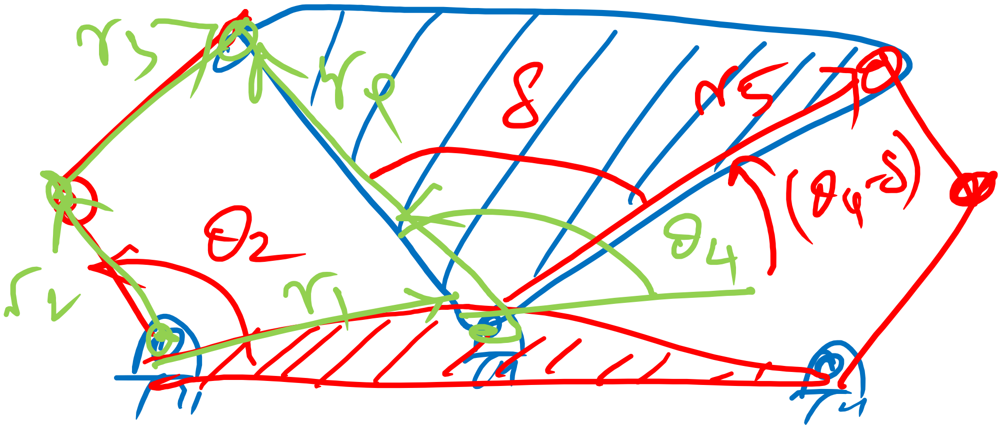




$$\vec{AP} = |AP|\, e^{i(\theta_3 + \delta)}$$

$$\vec{O_2 S} = |O_2 S|\, e^{i(\theta_2 + \delta_2)}$$

Watt's Six bar mechanism



$$(\theta_4 - \delta) = \lambda$$