Mechatronics System Design

EC4.404 - S2023

Lecture - 9

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Recap...

Revolute – 1

prismatic – 1

universal – 2

Cylindrical – 2

helical – 2

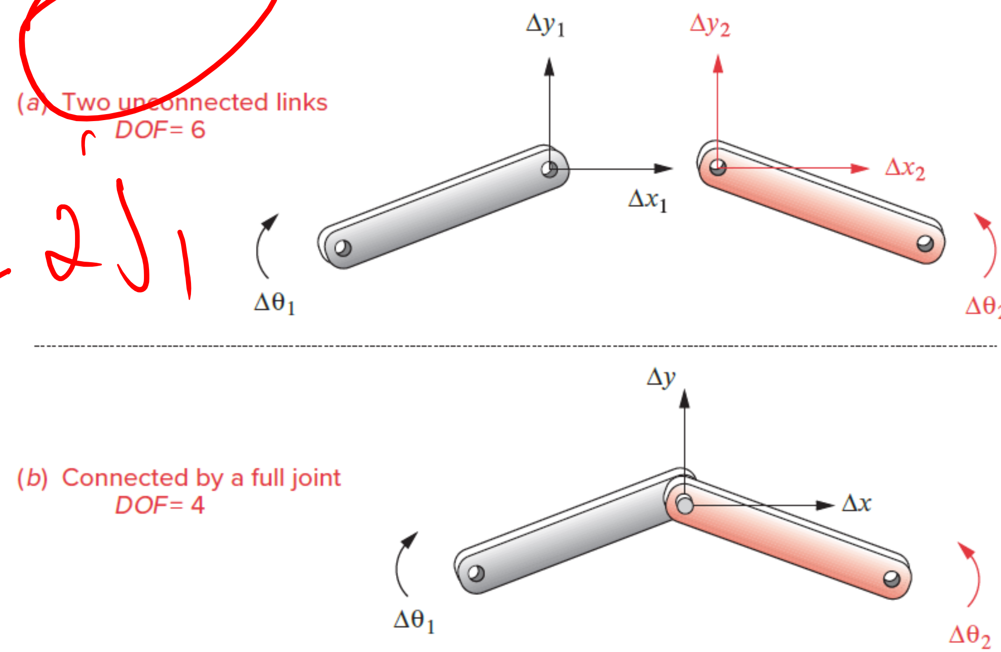
Spherical – 3

planar – 3

Degree of Freedom in Planar Mechanisms

- **Gruebler condition:** to investigate the DoF of any assembly of linkages.

$M = (3n-1) - 2J_1$



$$M = 3(N-1) - 2J_1$$

1 DOF

- Kutzbach's modification of Gruebler's equation is

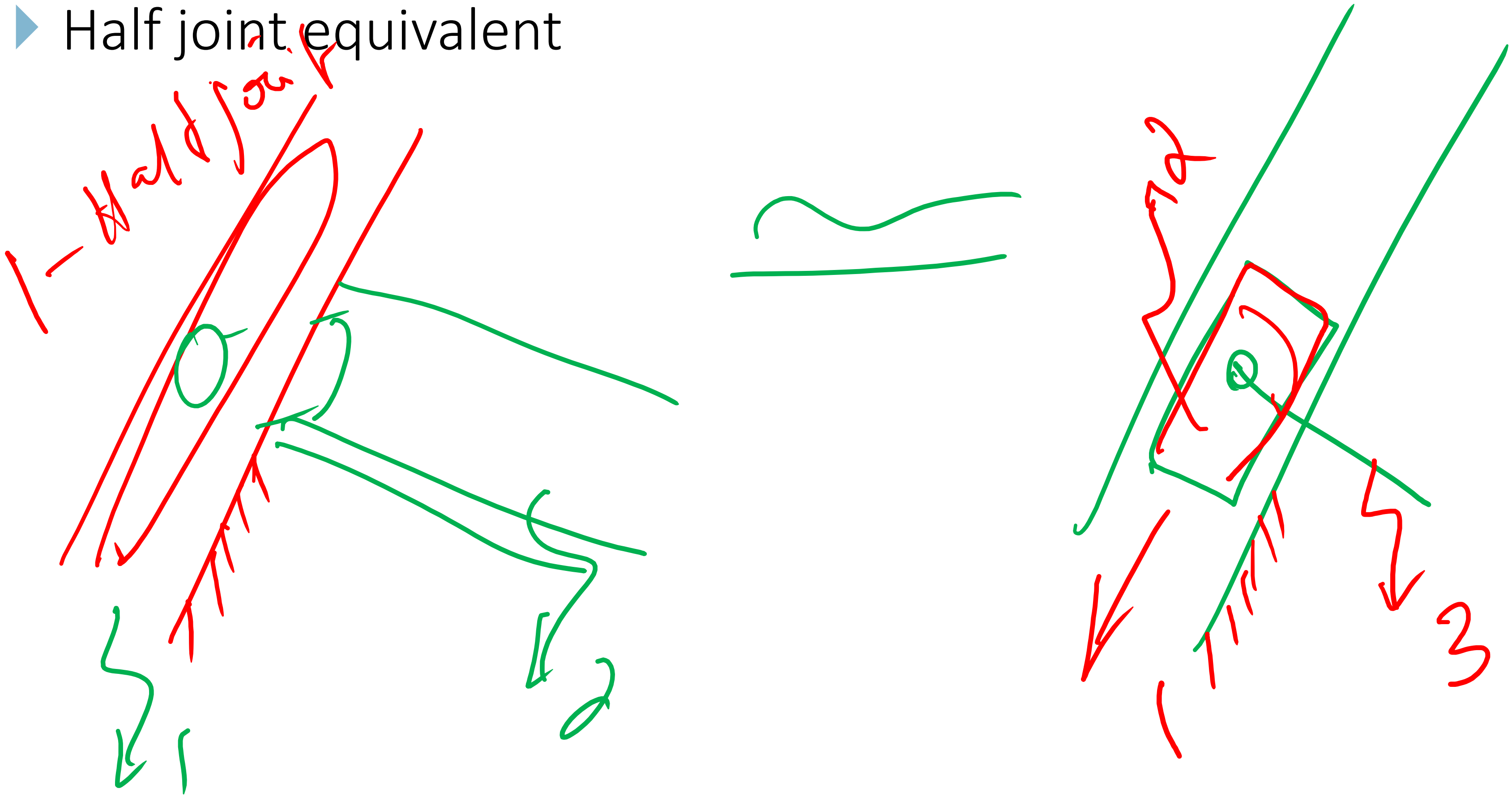
$$M = 3(N-1) - 2J_1 - J_2$$

Half joint

$$J_1 = \text{no. of 1DOF joints}, J_2 = \text{no. of half joints}$$

Degree of Freedom in Planar Mechanisms

- Half joint equivalent



Degree of Freedom (Mobility) in Spatial Mechanisms

$x, y, z = 0$
$\theta_x, \theta_y = 0$

$6(N-1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5$

A one-freedom joint removes 5 DOF

no. of joints with 1 DoF

2 DoF

The Kutzbach mobility equation for spatial linkages

$$\boxed{M = 6(N-1) - 5J_1 - 4J_2 - 3J_3 - 4J_4 - J_5}$$

Joints

3 - 2 = 1

x = 0, y = 0

x

Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1		
Prismatic (P)	1		
Helical (H)	1		
Cylindrical (C)	2		
Universal (U)	2		
Spherical (S)	3		

6 - 4 = 2

The number of degrees of freedom f and constraints c provided by common joints.

Gruebler's Condition

the number of constraint equations imposed by j joints

$$DoF = K(N-1) - \boxed{\overset{3-1}{\sum_{i=1}^{j}(K - f_i)}}$$

$3(N-1) - 2j_1$

3 or 6

if Planar

spatial

K = No. of parameters required to specify a single link 3 or 6

N = no. of links

j = no. of joints

f_i = is the freedom of the jth joint

i

$$\boxed{DoF = K(N-1-j) + \sum_{i=1}^{j} f_i}$$

2

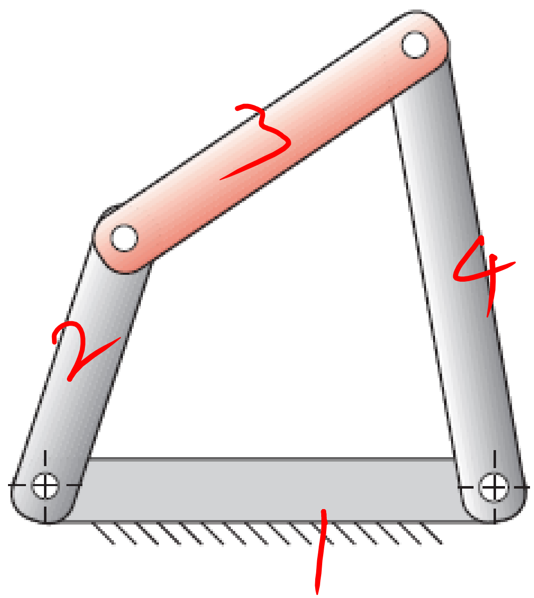
Gruebler's Condition

$$M = k(N-1-j)+\sum_{i=1}^{j} f_i$$

$$= 3(4-1-4)+4$$

$$= 1$$

$k=3$
$N=4$
$j=4$

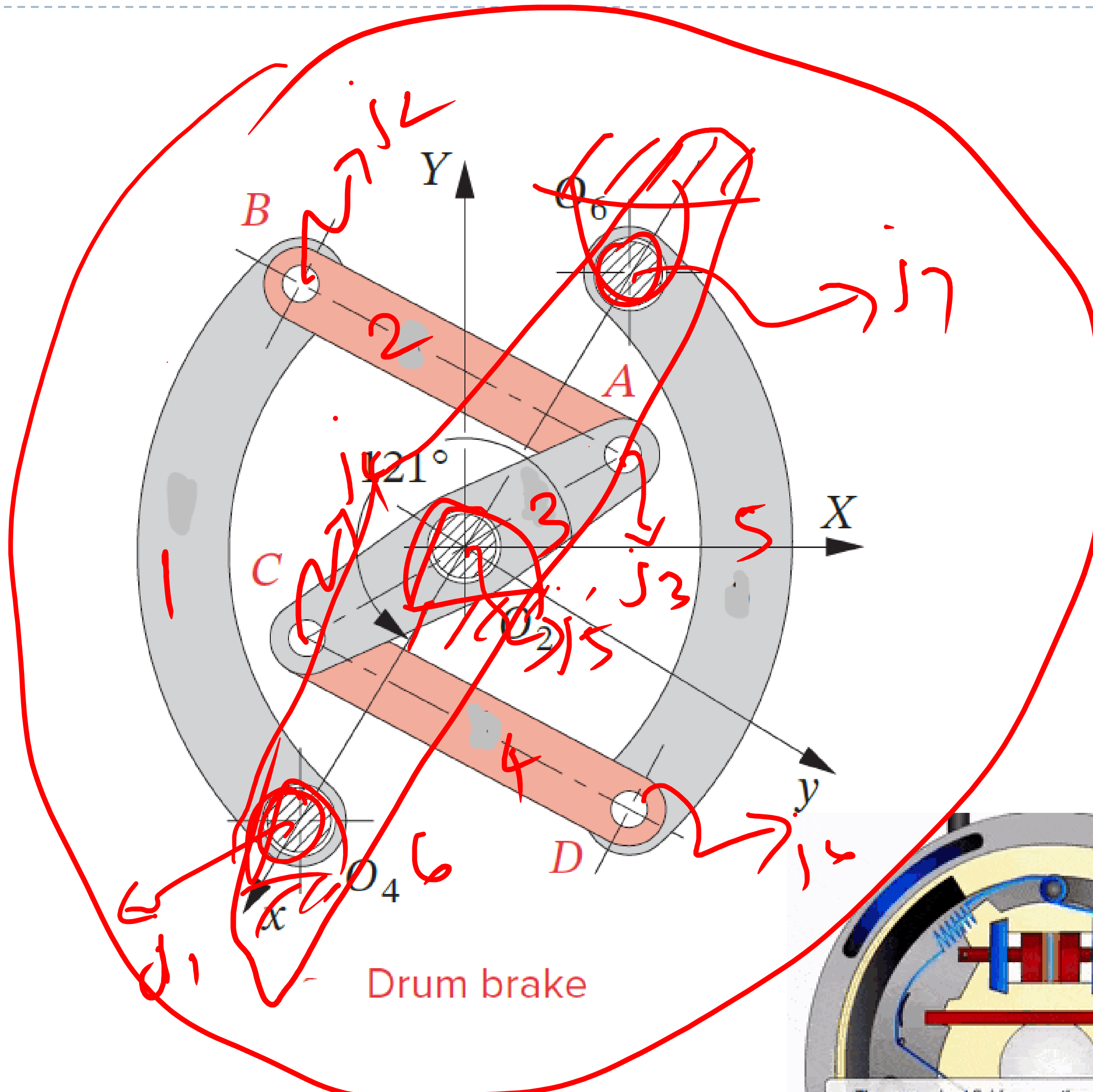


$M=1$

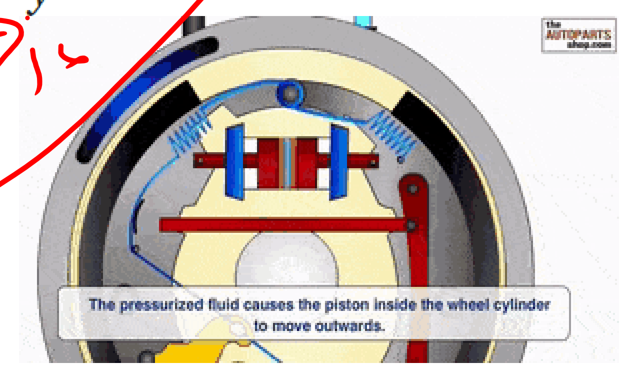
Gruebler's Condition

$K = 3$

$N = 6$



Drum brake

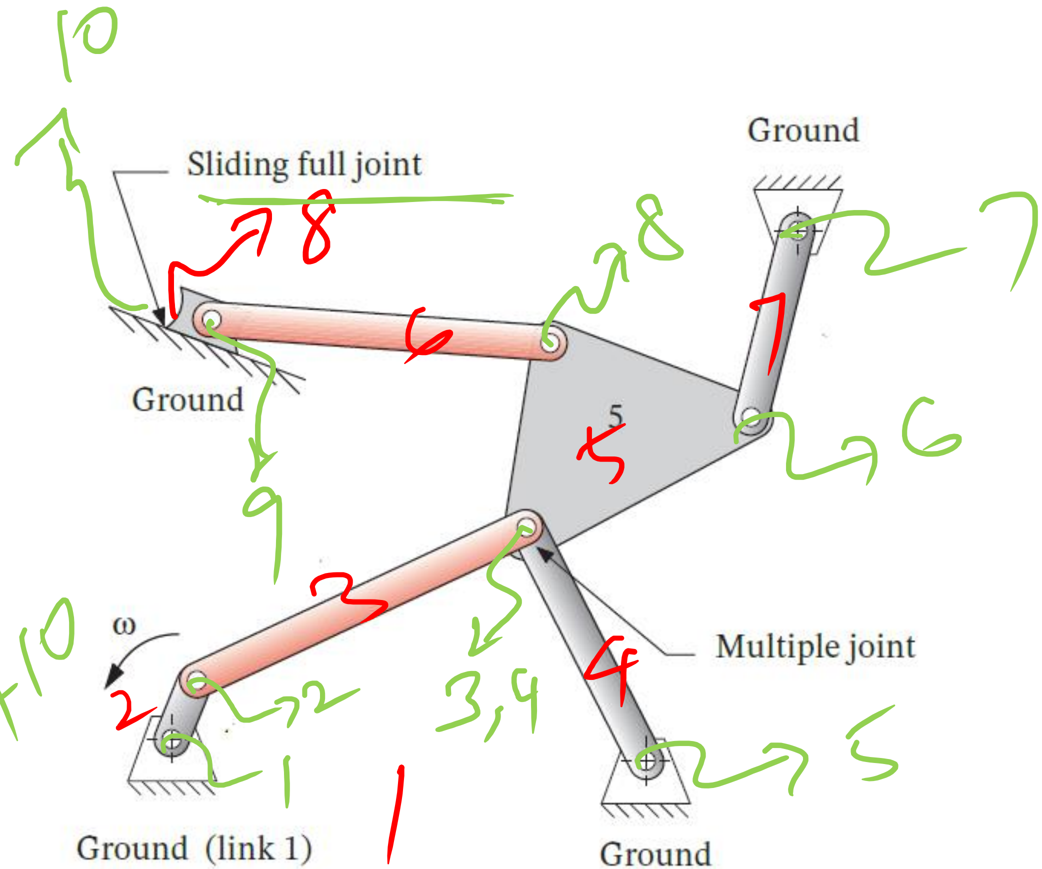


Gruebler's Condition

$N =$

$J = 10$

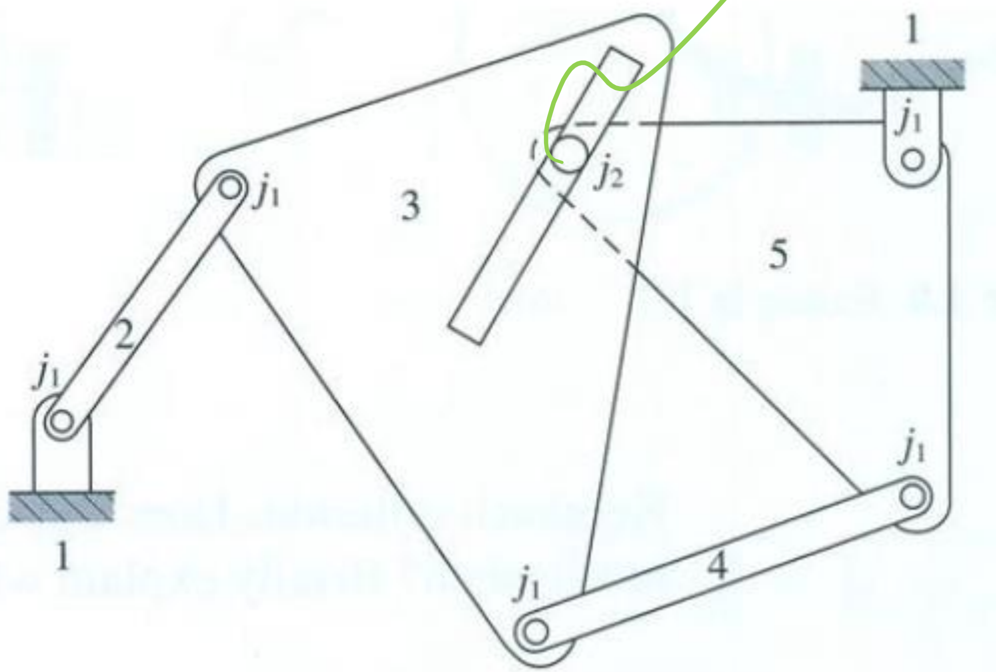
$3(8-1-10)+10$



(a) Linkage with full and multiple joints

Gruebler's Condition

$3(n-1)-2j_1-j_2$



0

MECHANISMS AND STRUCTURES

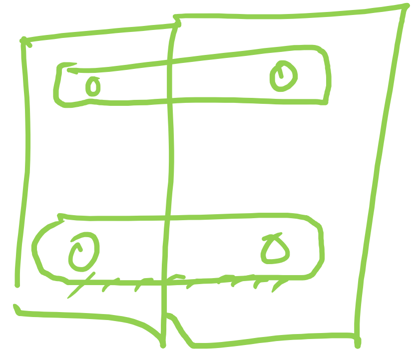
- If the DOF is positive, it will be a **mechanism**, and the links will have relative motion

- If the DOF is exactly zero, then it will be a **structure**, and no motion is possible.

- If the DOF is negative, then it is a **preloaded structure**, which means that no motion is possible and some stresses may also be present at the time of assembly.

Failure of Grubler's Equation

- Fails when
  - Special dimensions
  - Special geometry

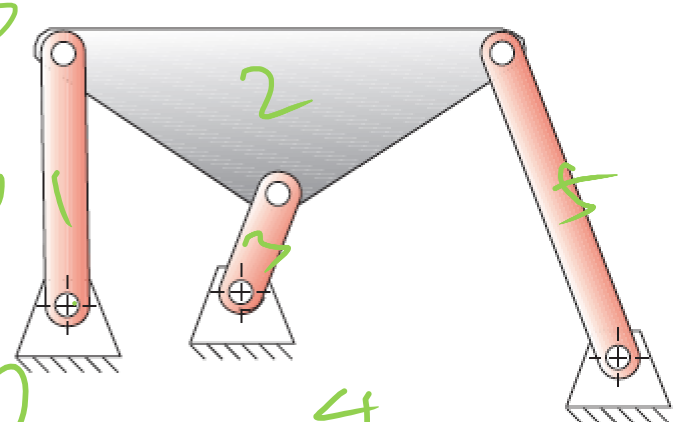


redundant link

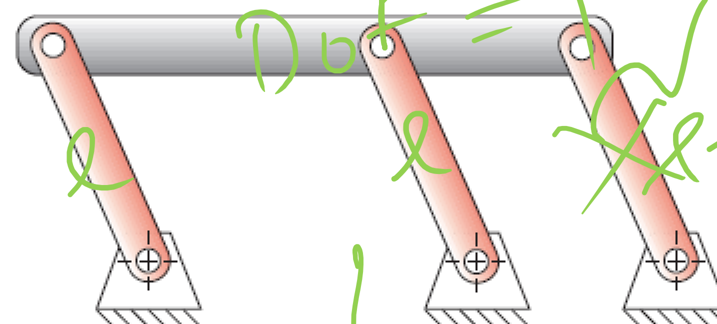
$N = 5$

$j = 6$

$M = 0$



DoF = 1



Due to parallelogram configuration, the linkage can move. However, this is a overconstrained linkage with redundant constraint.

DoF = 0

DoF = 1

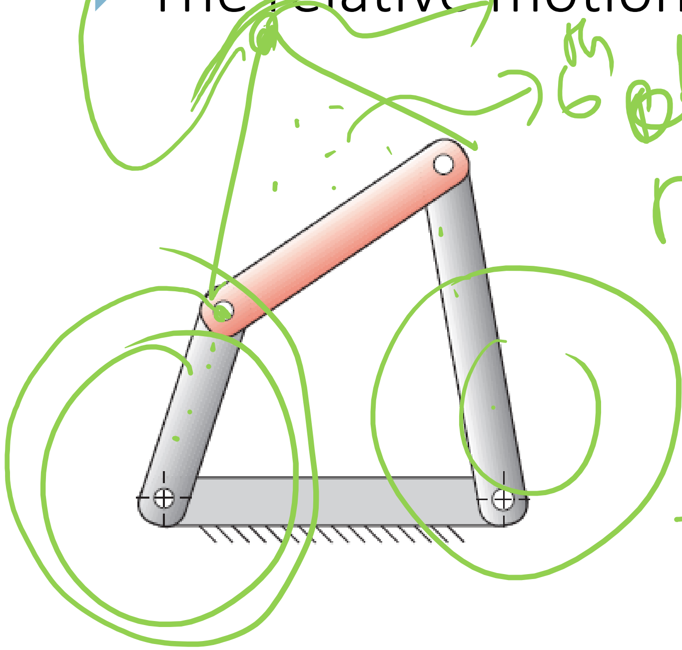
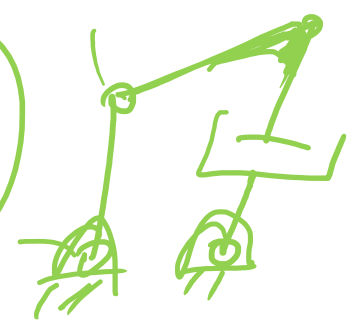
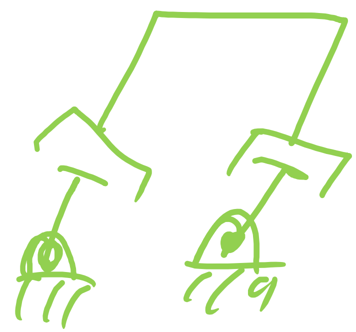
It may jam if there are any manufacturing errors.

Constrained Mechanism

- A mechanism with exactly 1 DOF
- The relative motion between any two links remain fixed.

coupler

6th order polynom

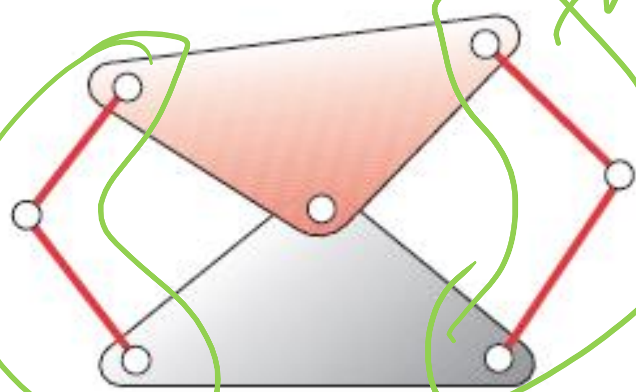





- Example 4R 3R1P 2R2P RPRP

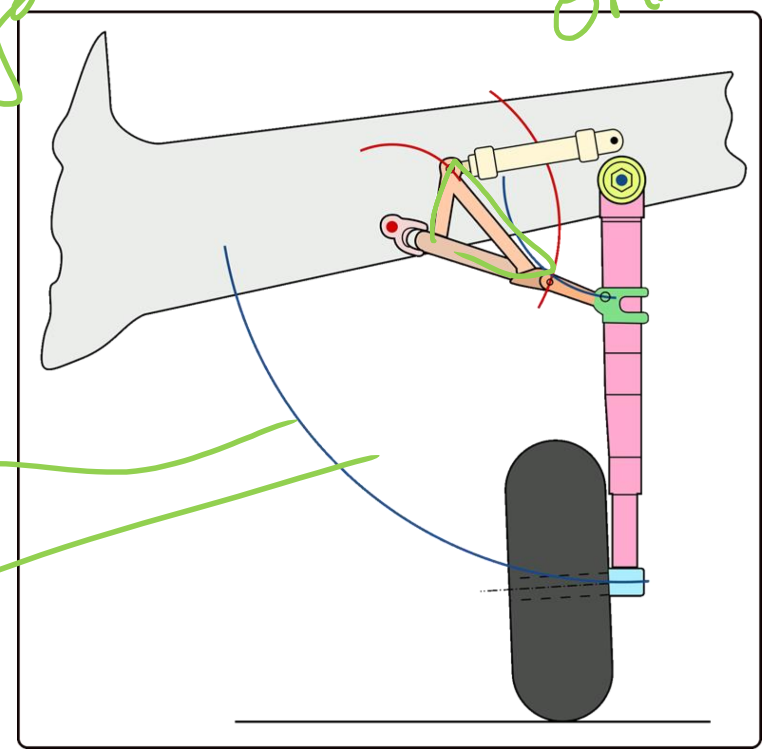
Constrained Mechanism

4B − 2T

n-dyad ⇒ 2 binary link with one revlt

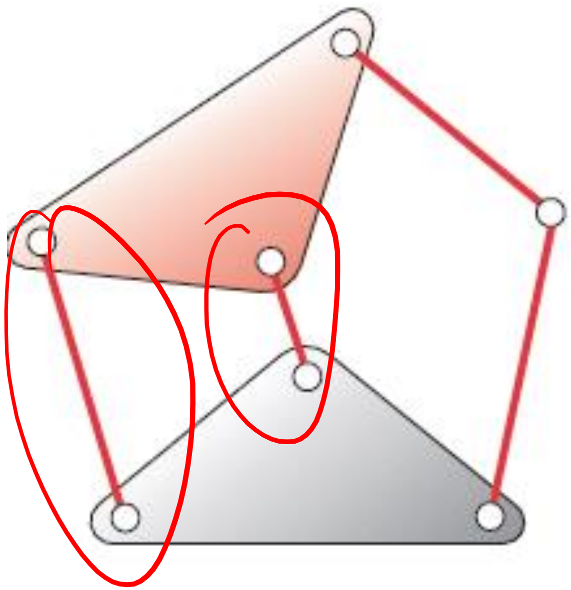


Watt's sixbar isomer

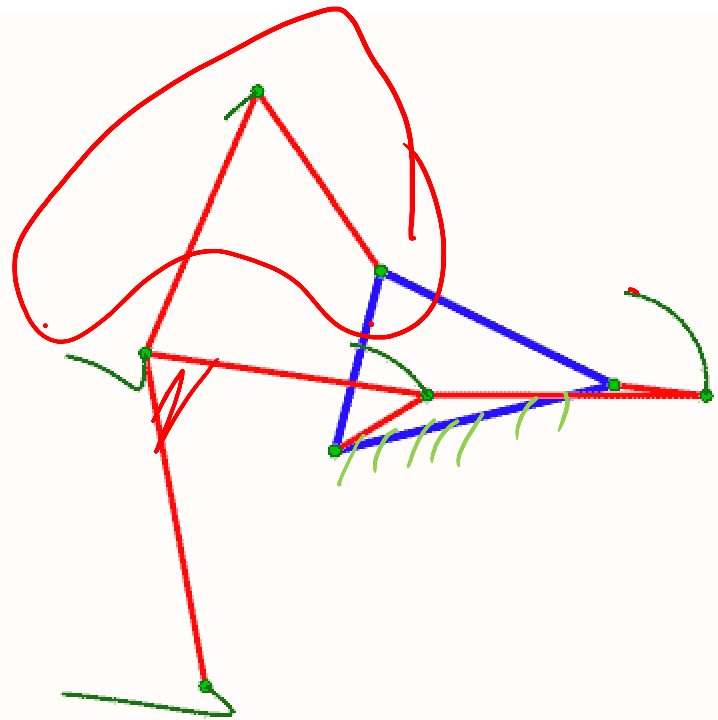


Constrained Mechanism

4B-2T

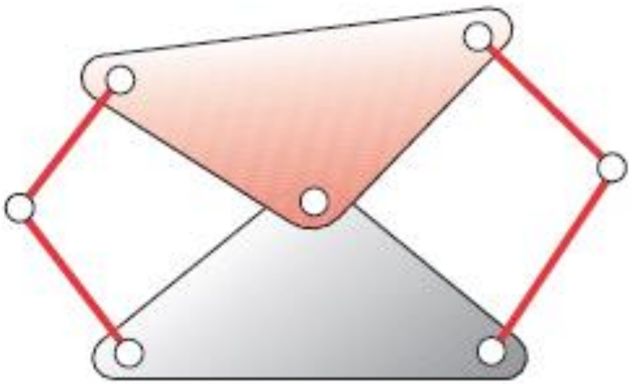


Stephenson's sixbar isomer

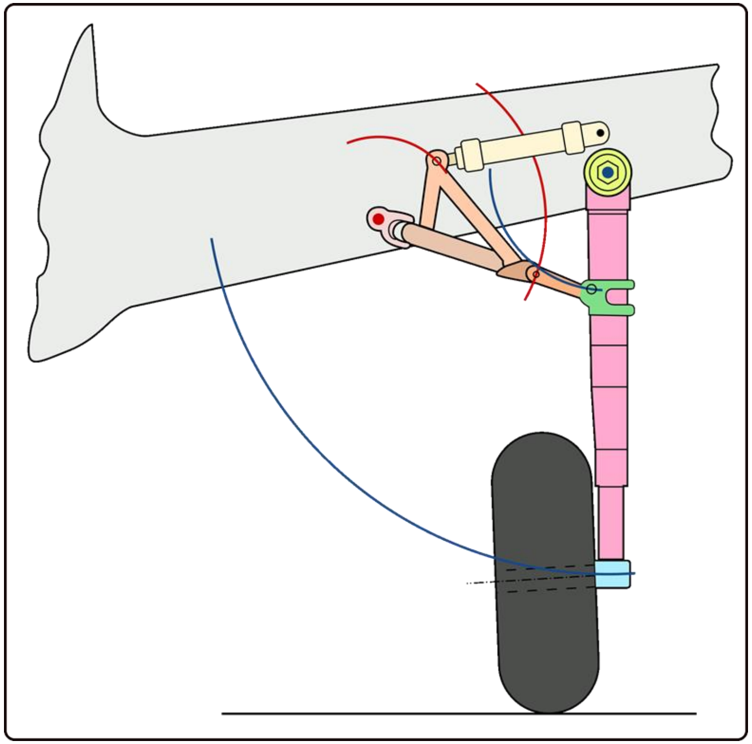


Klann walking mechanism

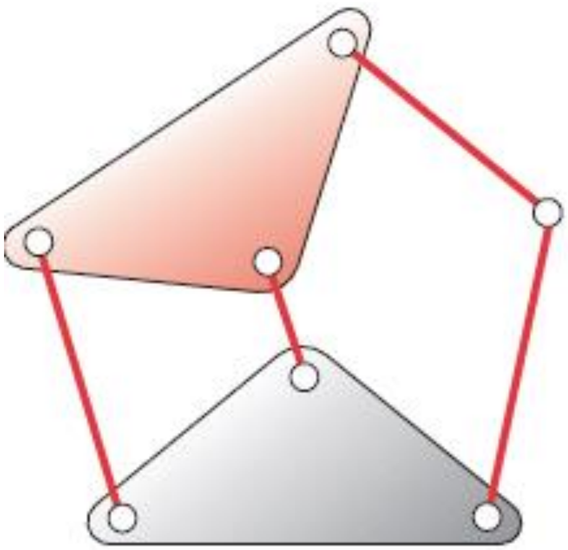
Constrained Mechanism



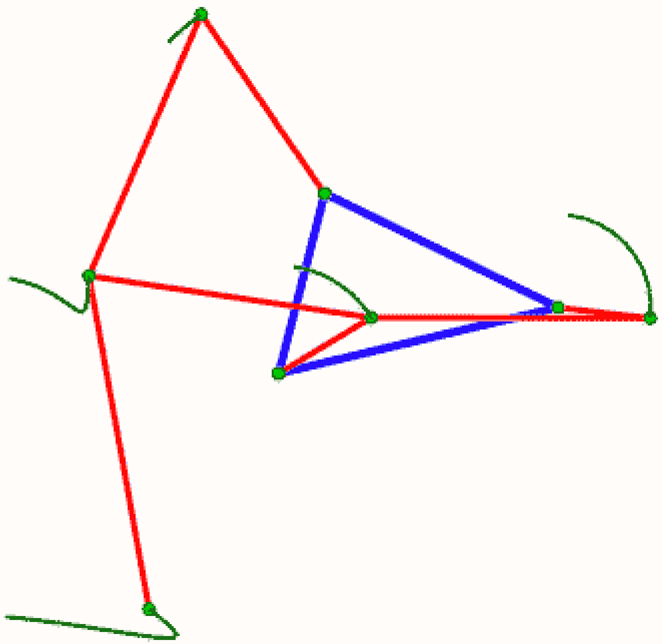
Watt's sixbar isomer



Constrained Mechanism

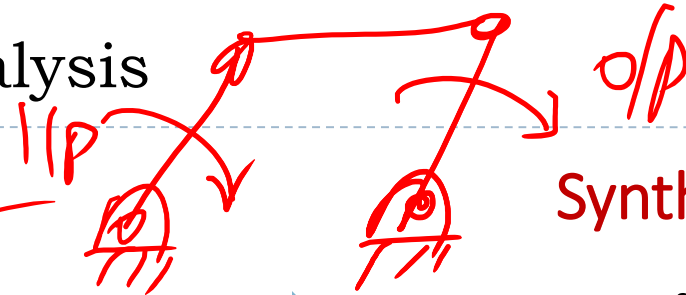


Stephenson's sixbar isomer



Klann walking mechanism

Synthesis and Analysis

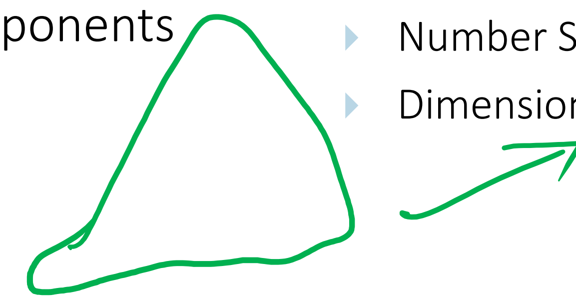


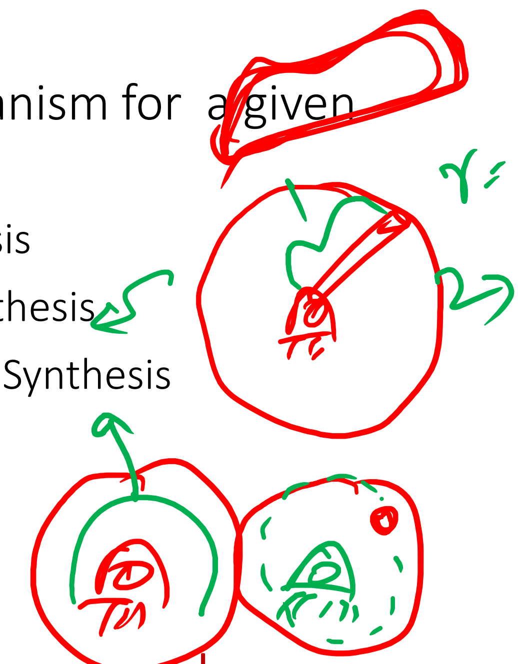
Analysis

- A particular given mechanism is investigated based on the mechanism geometry plus other characteristics (i/p angular velocity, acceleration etc)
- Given the mechanism, the motion characteristics of its components will be determined.
  - Displacement analysis
  - Velocity analysis
  - Acceleration analysis

Synthesis

- It is a process of designing a mechanism to accomplish the desired task.
- Creating mechanism for a given motion
  - Type Synthesis
  - Number Synthesis
  - Dimensional Synthesis



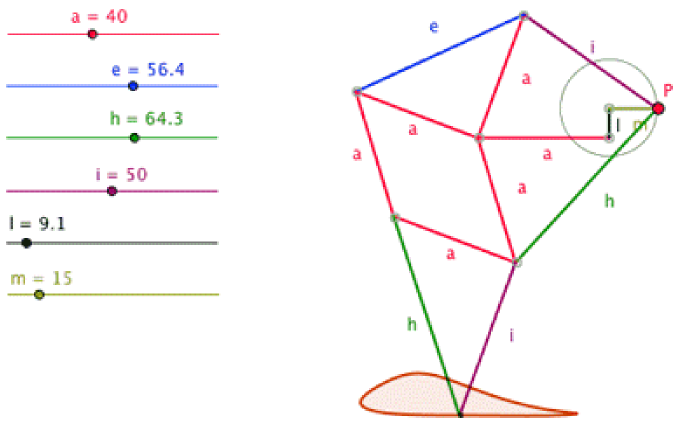


- The process of drawing kinematic diagrams and determining DOF of more complex mechanisms are the first steps in both kinematic analysis and synthesis


Analysis
6
cusp

Mechanism design

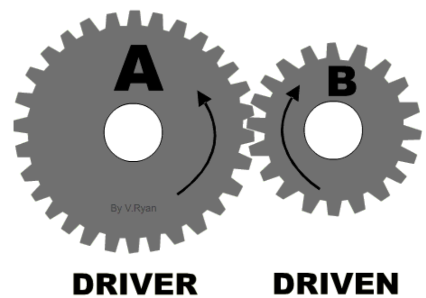
- Mechanism design involves finding a mechanism which carries out a user specified task.
- The process involves selection of joint types and link dimensions.
- Example – Eight-bar **Theo-Jansen** linkage enables robotic walking.

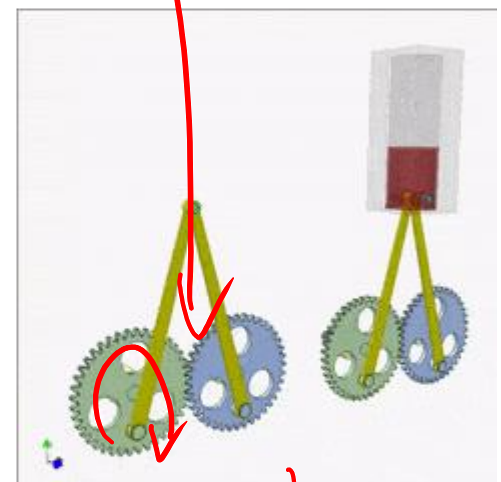


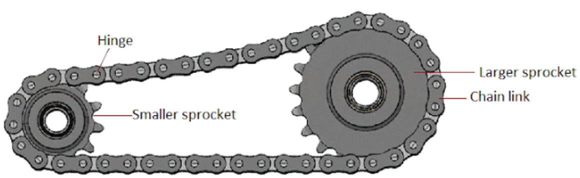
Type Synthesis:

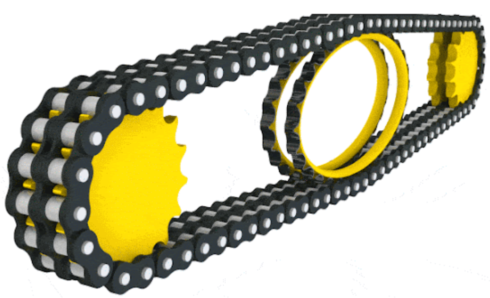
Given a task to be produced by a mechanism, what type of mechanism will be suitable?

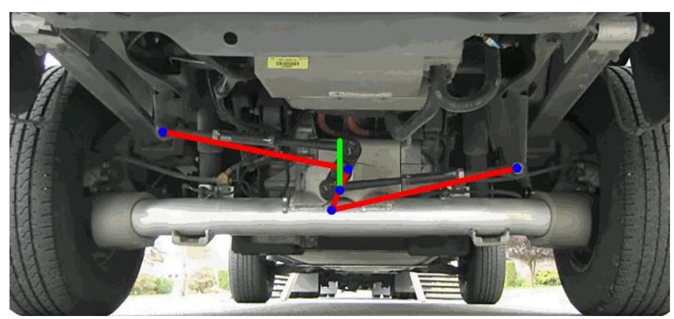
- Linkages
- Gear trains
- Belt drives etc











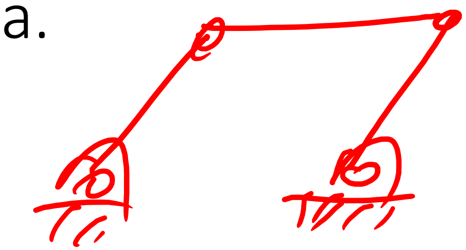
Number synthesis

- The determination of the number and order of links and joints necessary to produce motion of a particular DOF
- Link order - refers to the number of nodes per link, i.e., binary, ternary, quaternary, etc.
- exhaustive determination of all possible combinations of links that will yield any chosen DOF

Number synthesis - Example

- all the possible link combinations for one DOF, including sets of up to eight links

Hypothesis: If all joints have 1 DOF, an odd number of DOF requires an even number of links and vice-versa.



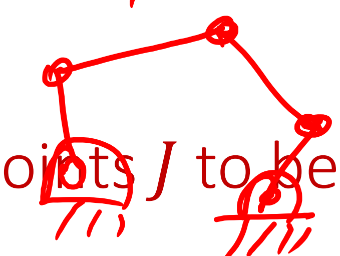
1-DOF
4-links

Proof:

$$M = 3(N-1) - 2J$$

$$J = \frac{3}{2}(N-1) - \frac{1}{2}$$

N must be **2,4,6,8, …** links to ensure the no. of joints J to be a positive integer.



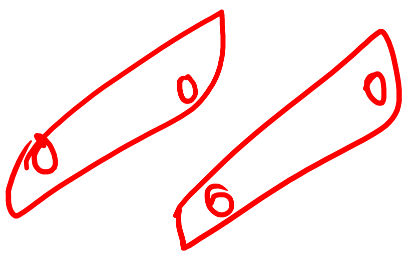
DOF = 2
N = 5

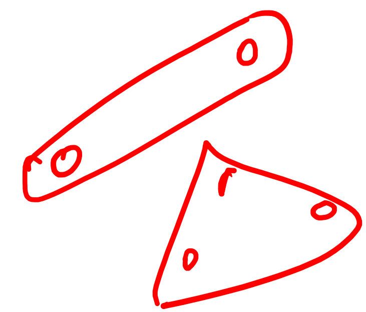
Number synthesis - Contd.

- Total number of links

$$N = B + T + Q + P + \cdots$$

$$B = \text{no. of binary links}$$
$$T = \text{no. of ternary links}$$
$$Q = \text{no. of quaternary links} \ldots$$





Two link nodes are needed to make one joint

$$J = \frac{nodes}{2}$$

$$\text{no. of nodes} = \text{order of links} * \text{no. of links of that order}$$

Number synthesis - Contd.

$$J = \frac{nodes}{2} = 2B + 3T + 4Q$$

$$J = \frac{(2B + 3T + 4Q + 5P + 6H)}{2}$$

$j = 3/2$

Substitute in Gruebler's equ.

$$\frac{(2B + 3T + 4Q + 5P + 6H \ldots)}{2} = \frac{3}{2}(N - 1) - \frac{M}{2}$$

$$2B + 3T + 4Q + 5P + 6H = 3N - 3 - M$$

$N = B + T + Q$

$= 3B + 3T + 3Q + 3P + 3H - 4$

$$\boxed{B + Q + 2P + 3H = -4}$$

Number synthesis - Contd.

Sub $\boxed{B = N - T - Q - P - H}$ in

$N = B + T + Q + (P + 1)$

$$2B + 3T + 4Q + 5P + 6H = 3N - 3 - M$$

$$\boxed{T + 2Q + 3P + 4H = N - 4}$$

$$T + 2Q + 3P + 4H = N - 4 \rightarrow 1$$
$$B = N - T - Q - P - H \rightarrow 2$$

N must be even for odd DOF

- Case 1 ; $N = 2$

$$T + 2Q + 3P + 4H = -2$$
$$\cancel{B = N - T - Q - P - H}$$

Requires $N = -2$ links - impossible

- Case 2 ; $N = 4$

$$T + 2Q + 3P + 4H = 0$$
$$B = 4 - T - Q - P - H$$

The simplest 1DoF requires $N = 4$ binary links

$$T + 2Q + 3P + 4H = N - 4$$

$$B = N - T - Q - P - H$$

- Case 3 ; $N = 6$

$$T + 2Q + 3P + 4H = 2 \qquad P = H = 0$$

$T\ may\ be\ 0, 1, or\ 2$ $\qquad$ $Q\ may\ be\ 0\ or\ 1$

$$If\ Q = 0$$
$$then\ T = 2,$$
$$B = 4$$

$$If\ Q = 1$$
$$then\ T = 0,$$
$$B = 5$$

$$B = 6 - T - Q - P - H$$

- Case 3 ; $N = 6$

$$T + 2Q + 3P + 4H = 2 \qquad P = H = 0$$

$$T\ may\ be\ 0,1, or\ 2 \qquad Q\ may\ be\ 0\ or\ 1$$

$$If\ Q = 0$$
$$then\ T = 2,$$
$$B = 4$$

$$If\ Q = 1$$
$$then\ T = 0,$$
$$B = 5$$

$$B = 6 - T - Q - P - H$$

Number synthesis - Contd.

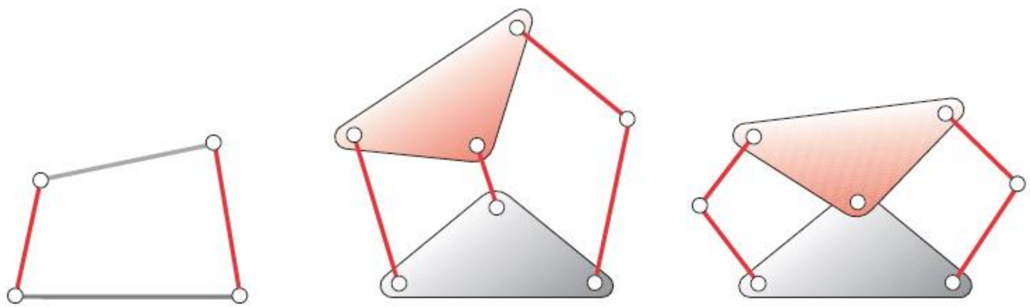
TABLE 2-2 1-*DOF* Planar Mechanisms with Revolute Joints and Up to 8 Links

N	Link Sets				
	Binary	Ternary	Quaternary	Pentagonal	Hexagonal
4	4	0	0	0	0
6	4	2	0	0	0
6	5	0	1	0	0
8	7	0	0	0	1
8	4	4	0	0	0
8	5	2	1	0	0
8	6	0	2	0	0
8	6	1	0	1	0

ISOMERS

- Like Isomers in chemistry - compounds that have the same number and type of atoms but which are interconnected differently and thus have different physical properties.

Valid Isomers



The only fourbar isomer — Stephenson's sixbar isomer — Watt's sixbar isomer

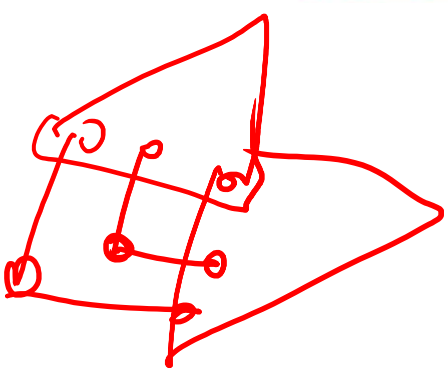
(*b*) All valid isomers of the fourbar and sixbar linkages

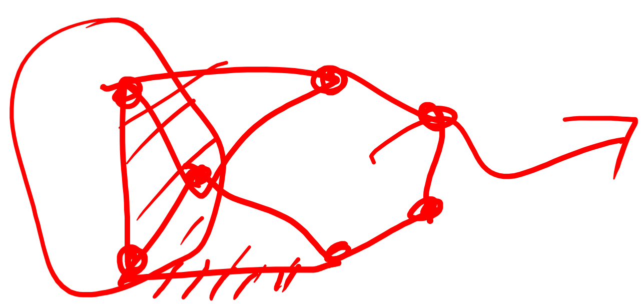
Number of Valid Isomers

Links	Valid Isomers
4	1
6	2
8	16
10	230
12	6856

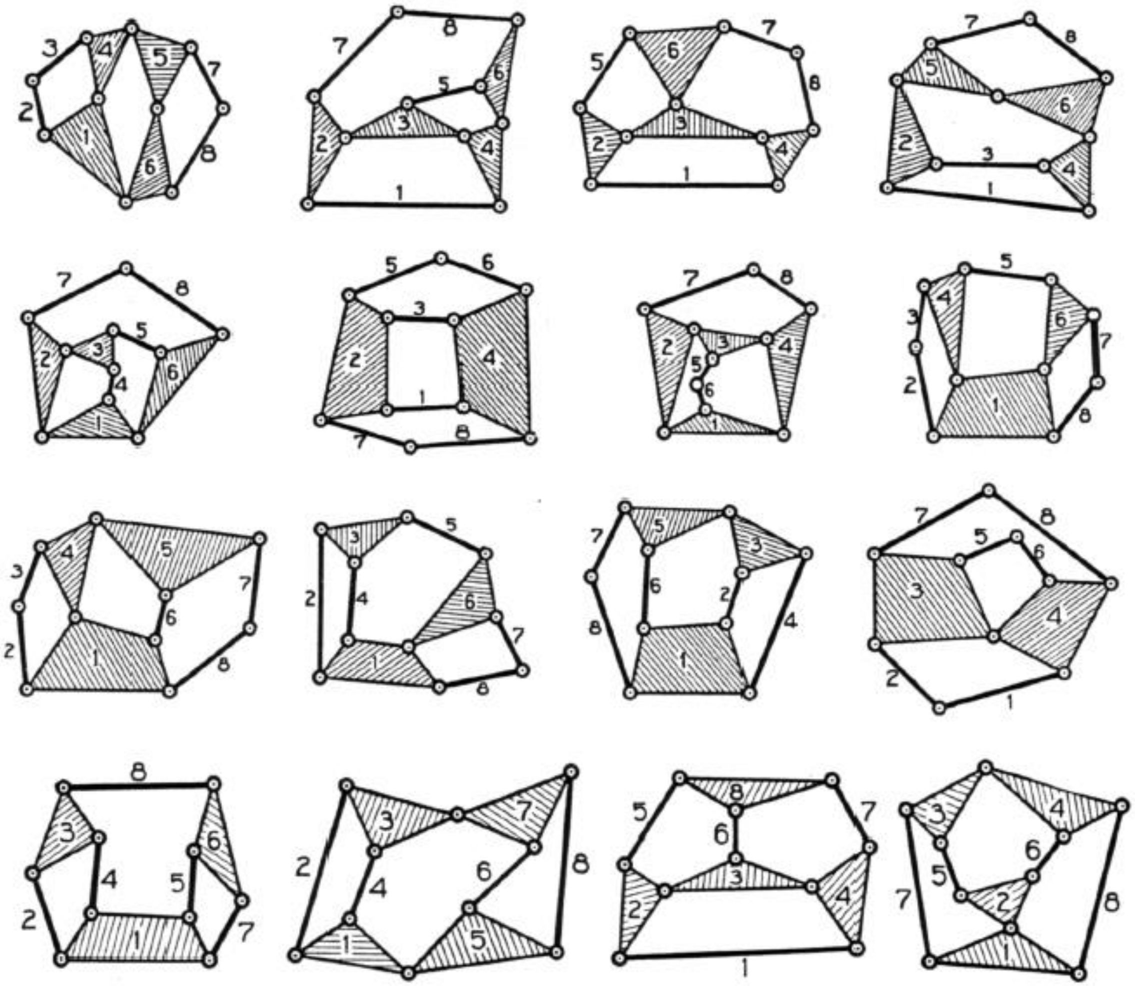
4 bar

Invalid Isomers





Eightbar 1-DOF ISOMERS



(*d*) All the valid eightbar 1-*DOF* isomers