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IQC Assignment 2

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Section - B

1.

a) $\psi_1 = |00\rangle$

$\rho_1 = \psi_1 \psi_1^{+} = |00\rangle\langle 00|$

$\psi_3 = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$\rho_3 = \frac{1}{2}(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|)$

$\boxed{\rho_1}$

b) $\rho_1 = |00\rangle\langle 00|$

eigenvalues are 1, 0, 0, 0

$S(\rho_1) = -(1\log 1 + 0 + 0 + 0) = 0$

$\rho_1^A = \mathrm{tr}_B(\rho_1) = |0\rangle\langle 0|$

$\rho_1^B = |0\rangle\langle 0|$

$S(\rho_1^A) = -(1\log 1 + 0) = 0$

$S(\rho_1^B) = S(\rho_1^A) = 0$

c) $S_{A|B}(\rho) = S_{A,B}(\rho_1) - S_B(\rho_1)$

$= S(\rho_1) - S(\rho_1^B)$

$= 0$

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$\boxed{\rho_2}$

b) $= \rho_2 = \frac{1}{4}\left(|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|\right)$

$\rho_2^A = tr_B(\rho_2) = \frac{1}{4}\left(|0\rangle\langle 0| + |0\rangle\langle 0| + |1\rangle\langle 1| + |1\rangle\langle 1|\right)$

$= \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$

$\rho_2^B = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$

~~ρ_2 = the~~ eigenvalue of $\rho_2 = \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$

$S(\rho_2) = -4 \times \frac{1}{4} \log \frac{1}{4} = 2$

$S(\rho_2^A) = -2 \times \frac{1}{2} \log \frac{1}{2} = 1$

$S(\rho_2^B) = 1$

c) $S_{A|B}(\rho) = S_{A,B}(\rho_2) - S_B(\rho_2)$

$= 2 - 1 = 1$

$\boxed{\rho_3}$

$\rho_3 = \frac{1}{2}\left(|00\rangle\langle 00| + |00\rangle\langle 11| + |11\rangle\langle 00| + |11\rangle\langle 11|\right)$

$\rho_3^A = tr_B(\rho_3) = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) = I/2$

$\rho_3^B = tr_A(\rho_3) = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) = I/2$

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$$\rho_3 = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$ eigenvalues are 1, 0, 0, 0

$$S(\rho_3) = -(1 \log 1 + 0) = 0$$

$$S(\rho_3^A) = -2 \times \frac{1}{2} \log \frac{1}{2} = \underline{1}$$

$$S(\rho_3^B) = \underline{1}$$

c) $$S_{A|B}(\rho) = S_{AB}(\rho) - S_B(\rho)$$
$$= S(\rho_3) - S(\rho_3^B)$$
$$= \underline{\underline{-1}}$$

$\boxed{\rho_4}$

$$\rho_4 = 0.7 \langle \psi_3 | \psi_3 \rangle + 0.3 (\rho_2)$$

$$\langle \psi_3 | \psi_3 \rangle = \tfrac{1}{2}(1+1) = 1$$

$$\rho_2 = \tfrac{1}{4} I_4$$

$$\rho_4 = 0.7 I_4 + 0.3/4 \, I_4$$
$$= \frac{3.1}{4} I_4 = \frac{3.1}{4} (|00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|)$$

$$\rho_4^A = \rho_4^B = \frac{3.1}{4}\left(|0\rangle\langle 0| + |0\rangle\langle 0| + |1\rangle\langle 1| + |1\rangle\langle 1|\right)$$

$$= \frac{3.1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

b) $S(\rho_4) = -\frac{\cancel{4\times}3.1}{4}\left(\log\frac{3.1}{4}\right) = -3.1\left(2-\log(3.1)\right)$

$$S(\rho_4^A) = S(\rho_4^B) = -2\times\frac{3.1}{2}\left(\log\frac{3.1}{2}\right)$$

$$= 3.1\left(1-\log 3.1\right)$$

c) $S_{A|B}(\rho_4) = S(\rho_4) - S(\rho_4^B)$

$$= 3.1\left(2-\log 3.1\right)$$

$$= 3.1(2-1) = \underline{3.1}$$

2. $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|000\rangle + |111\rangle\right)$

density matrix of $|\psi\rangle$

$$= \frac{1}{2}\left(|000\rangle\langle 000| + |000\rangle\langle 111| + |111\rangle\langle 000| + |111\rangle\langle 111|\right)$$

$$\rho_{BC} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

$$= \rho_{AB} = \underline{\rho_{AC}}$$

$$\rho_B = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)$$

$$= \rho_C = \rho_A$$

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3.

a) $\rho = 1/4 (I \otimes I)$

$I = 1/2 (|+\rangle\langle +| + |-\rangle\langle -| + |+\rangle\langle +| + |-\rangle\langle -|)$
$= |+\rangle\langle +| + |-\rangle\langle -|$

$\rho = 1/4 (|+\rangle\langle +| + |-\rangle\langle -|) \otimes (|+\rangle\langle +| + |-\rangle\langle -|)$
$\rho_B = tr_A(\rho) = 1/4 (|+\rangle\langle +| + |-\rangle\langle -| + |+\rangle\langle +| + |-\rangle\langle -|)$
$= 1/2 (|+\rangle\langle +| + |-\rangle\langle -|) = I/2$

b) $\rho = 1/2 (|0\rangle\langle 0| + |+\rangle\langle +|) \otimes I$

$= 1/4 [3(|++\rangle\langle ++| + |+-\rangle\langle +-|) + |++\rangle\langle -+|$
$+ |+-\rangle\langle --| + |-+\rangle\langle ++| + |--\rangle\langle +-|$
$+ |-+\rangle\langle -+| + |--\rangle\langle --|]$
$\rho_B = 1/4 [3(|+\rangle\langle +| + |-\rangle\langle -|) + |+\rangle\langle +| + |-\rangle\langle -|)$
$= |+\rangle\langle +| + |-\rangle\langle -| = I$

c) $\rho = \frac{p}{4} I + (1-p) |\psi\rangle\langle\psi|$

$I_{4\times 4} = I \otimes I$

$|\psi\rangle = \frac{|00\rangle + x|11\rangle}{\sqrt{1+|x|^2}}$

$a = \frac{1}{\sqrt{1+|x|^2}}$

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$$|\psi\rangle\langle\psi| = a^2 \left[|00\rangle\langle 00| + x^* |00\rangle\langle 11| + x|11\rangle\langle 00| + |x|^2 |11\rangle\langle 11| \right]$$

$$= a^2 \left[|0\rangle\langle 0|_A \otimes |0\rangle\langle 0|_B + x^* |0\rangle\langle 1|_A \otimes |0\rangle\langle 1|_B + x|1\rangle\langle 0|_A \otimes |1\rangle\langle 0|_B + |x|^2 |1\rangle\langle 1|_A \otimes |1\rangle\langle 1|_B \right)$$

$\cancel{tr_A}$

$$tr_A\left[(1-p)|\psi\rangle\langle\psi|\right]$$

$$= (1-p)a^2 tr_A \Big[\tfrac{1}{2}\left(|+\rangle\langle +| + |+\rangle\langle -| + |-\rangle\langle +| + |-\rangle\langle -|\right) \otimes |0\rangle\langle 0|_B$$

$$+ \frac{x^*}{2}\left(|+\rangle\langle +| - |+\rangle\langle -| + |-\rangle\langle +| - |-\rangle\langle -|\right) \otimes |0\rangle\langle 1|$$

$$+ \frac{x}{2}\left(|+\rangle\langle +| + |+\rangle\langle -| - |-\rangle\langle +| - |-\rangle\langle -|\right) \otimes |1\rangle\langle 0|$$

$$+ \frac{|x|^2}{2}\left(|+\rangle\langle +| - |+\rangle\langle -| - |-\rangle\langle +| + |-\rangle\langle -|\right) \otimes |1\rangle\langle 1|$$

$$= (1-p)a^2 \left[|0\rangle\langle 0| + |x|^2 |1\rangle\langle 1| \right]$$

$$\text{output} = (p/2)I + (1-p)a^2\left(|0\rangle\langle 0| + |x|^2 |1\rangle\langle 1|\right)$$

$$= \frac{p + |x|^2 p + 2 - 2p}{2(1+|x|^2)} |0\rangle\langle 0| + \frac{p + |x|^2 p + 2 - 2|x|^2 p}{2(1+|x|^2)} |1\rangle\langle 1|$$

$$= \left(|x|^2 p - p + 2\right)|0\rangle\langle 0| + \left(p + 2 - |x|^2 p\right)|1\rangle\langle 1| \times \frac{1}{2(1+|x|^2)}$$

Section A

2. $|\psi\rangle_{12} = \dfrac{|00\rangle + x|11\rangle}{\sqrt{1+|x|^2}}$ $\quad |\psi\rangle_{34} = \dfrac{|00\rangle + m|11\rangle}{\sqrt{1+|x|^2}}$

$$|\psi\rangle_{12} \otimes |\psi\rangle_{34} = \frac{1}{\sqrt{1+|x|^2}\sqrt{1+|m|^2}} \Big[|0\rangle_1 |00\rangle_{23} |0\rangle_4 + m|0\rangle_1 |01\rangle_{23} |1\rangle_4 + x|1\rangle_1 |10\rangle_{23} |0\rangle_4 + xm|1\rangle|11\rangle|1\rangle \Big]$$

$$= \frac{1}{\sqrt{2(1+|x|^2)(1+|m|^2)}} \Big[\phi^+_{23}\left(|00\rangle_{14} + xm|11\rangle_{14}\right) + \psi^+_{23}\left(m|01\rangle_{14} + x|10\rangle_{14}\right) + \phi^-_{23}\left(|00\rangle_{14} - xm|11\rangle_{14}\right) + \psi^-_{23}\left(m|01\rangle_{14} - x|01\rangle_{14}\right) \Big]$$

these are the entangled states $\alpha^+, \beta^+, \alpha^-, \beta^-$

$$= \frac{1}{\sqrt{2(1+|x|^2)(1+|m|^2)}} \left[\frac{\phi^+_{23}\,\alpha^+}{\sqrt{1+x^2m^2}} + \frac{\psi^+_{23}\,\beta^+}{\sqrt{x^2+m^2}} + \frac{\phi^-_{23}\,\alpha^-}{\sqrt{1+x^2m^2}} + \frac{\psi^-_{23}\,\beta^-}{\sqrt{x^2+m^2}} \right]$$