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Automata Theory
Final Exam

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A.1. Pushdown automata are decidable with respect to the halting problem.

A PDA can be represented by a CFG. If that context free grammar is converted to Chomsky Normal Form, then the PDA is guaranteed to end in $2n-1$ steps.

We can simulate a PDA using a 2-tape turing machine with the stack maintained on one tape and the input maintained on the other with one space reserved to count the number of steps on the stack tape.

A.2 A) $RE_1 \cup RE_2$ = RE language
since RE languages are closed under union

C) $R_1 - R_2$ is recursively enumerable
$\therefore A - B$ is RE if A is RE and B is recursive.

A.3 Consider A to be the NFA that accepts L.

Make every state in the NFA an accepting state. This Automata A will accept $Pre(L)$ since only crash runs are those strings that aren't prefixes of L. Hence $Pre(L)$ is regular.

A.4. It is known that DFAs can be minimised using Turing machines.

To recognise EQ_{DFA}, we minimise M_1 and M_2 such that the minimised encodings are as follows on the tape:

$\langle$ Min. encoding of $M_1 \rangle \langle$ Min encoding of $M_2 \rangle$

Next we check whether these two are the same. i.e.

$M_1 = (Q_1, \Sigma_1, \delta_1, q_1, F_1)$ is the same as $M_2 = (Q_2, \Sigma_2, \delta_2, q_2, F_2)$.

A.5. (A) is the correct order since we know of languages that are context free and not regular, recursive but not context free and recursively enumerable but not recursive.

A.6 Assume L is context free.
Then a pumping length p exists.

Consider $K \in L$ such that $K = w_1 w_2 w_3$ where $|w| = p$. $\Rightarrow K$ is of at least pumping length p and $w_1 = w_2 = w_3$

Consider the following cases for $uvxyz$:

Case 1: v or y contains symbols only from w_1 or w_2 or w_3 exclusively.

$uv^2xy^2z \notin L$ since only w_1, w_2 or w_2, w_3 or w_1, w_3 will be pumped leaving the remaining w unequal to the others.

Case 2: v or y contains symbols from two adjacent w.

$uv^2xy^2z \notin L$ since symbols from a pair of w will be pumped unequally and the $w_1 = w_2 = w_3$ equality will not maintain.

There are no more cases such as one where v or y contains symbols from w_1, w_2 and w_3 since $|vxy| \leq p$.

$\Rightarrow$ our assumption is incorrect and that L is not a CFL.

A.7 Consider the non-CFL,

$$L = \{ a^m b^m c^m d^m \mid m \in \mathbb{N} \}$$

a prefix of any string in L is $a^m b^m c^m$ which is not context free either.

A.8 (b) denotes the same set as $0^*(10^*)^*$

B) An NFA for that set is: