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Feb 2 Intro to quantum info and Computation Mid Quiz

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Section A

2. Consider two unitary operators A, B

Consider the adjoint of their tensor product.

$(A \otimes B)^{+}$

$= A^{+} \otimes B^{+}$

Now, ~~multiplying~~ taking the product of the adjoint and $(A \otimes B)$

$(A \otimes B)^{\cancel{*}} (A \cancel{*} \otimes B)^{+}$

$= (A \otimes B)(A^{+} \otimes B^{+})$

$= (AA^{+} \otimes BB^{+})$ (By property of unitary ~~me~~ operators)

$= I \cancel{*} \otimes I = I$

$\therefore (A \otimes B)(A \cancel{*} \otimes B)^{+} = I$

we can say that the tensor product of $A \otimes B$ is also unitary.

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3. $[\sigma_x, \sigma_y]$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} - \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} - \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 2i & 0 \\ 0 & -2i \end{bmatrix}$$

$[\sigma_y, \sigma_z]$

$$= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} - \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2i \\ 2i & 0 \end{bmatrix}$$

$[\sigma_z, \sigma_x]$

$$= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$