Probability and Statistics

UG2, Core course, IIIT,H

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Outline

1. 123456 T2ab2le:C
2. ons:id2db53
3. r2b3d x:d:aCl:ap x2a245n
4. rs:ib2e mbCdabltdb53C
5. on26se:C 5w mbCdabltdb53C
6. ons:id2db53C 5w r56: mbCdabltdb53C

Define Random Variable

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$E \rightarrow$ id

Definition of Random Variable

A random variable X is a function from the sample space to the real numbers.

$$X : S \rightarrow \mathbb{R}$$

Sample space

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Examples of Random Variables...

Find the range of the following random variables:

- I toss a coin 10 times. Let X be the number of heads I observe

$\{0, 1, 2, \ldots, 10\}$

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Examples of Random Variables...

$\{1, 2, \ldots\}$

Find the range of the following random variables:

- I toss a coin 10 times. Let X be the number of heads I observe
- I toss a coin until the first tail appears. Let Y be the total number of coin tosses

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Consider an Experiment: 3 coins are flipped. Let X be the number of tails. Answer the following:

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  - (H, H, H) $X = 0$
  - (T, T, H) $X = 2$

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Consider an Experiment: 3 coins are flipped. Let X be the number of tails. Answer the following:

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  - (T, T, H)
- What is the event when $X = 2$? ← event: no of tails is 2

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Consider an Experiment: 3 coins are flipped. Let X be the number of tails. Answer the following:

- What is the value of X for the following outcomes?
  - (H, H, H)
  - (T, T, H)
- What is the event when $X = 2$?
- What is $P(X = 2)$?

$\{(H, H, H), (H, H, T), \ldots (T, T, T)\}$

$2 \times 2 \times 2 = 8$

$X = 2 = \{(TTH), (HTT), (THT)\}$

$P(X=2) = \frac{3}{8}$

Random Variables are Not Events!

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$X = 2$

Remarks on Random variables

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Remarks on Random variables

- random variables are not events!
- when a random variable is assigned a value, then it becomes event

$X = x$	Set of Outcomes	$P(X = k)$
$X = 0$	$\{(T, T, T)\}$	$1/8$
$X = 1$	$\{(H, T, T), (T, H, T), (T, T, H)\}$	$3/8$
$X = 2$	$\{(H, H, T), (H, T, H), (T, H, T)\}$	$3/8$
$X = 3$	$\{(H, H, H)\}$	$1/8$
$X \geq 4$	$\{\ \}$	0

Table: Consider an experiment where 3 coins are flipped, and X denotes number of heads

Discrete Random Variables...

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$X =$

Recall: countable sets

A set A is countable if either it is a finite set, or it can be put in 1-1 correspondence with set of natural numbers.

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1. discrete random variables
2. continuous random variables

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2. I toss a coin until the first heads appears. Let Y be the total number of coin tosses
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4. Let X be the height of students in a class

Probability Mass Function...

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Definition of probability mass function and some remarks...

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$$\underbrace{P_X(x_k)} = \underbrace{P(X = x_k)}, \text{ for } k = 1, 2, 3, \ldots$$

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$P_Y(y_k)$

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- The subscript in $P_X(x_k)$ indicates that it is the PMF of random variable X
- For discrete random variable, PMF is also called probability distribution
- The term probability distribution function is almost always reserved for cumulative distribution(to be introduced)

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Answer

- Sample space $S = \{HH, HT, TH, TT\}$. No. of heads: 0,1,2. Hence, $R_X = \{0, 1, 2\}$

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- We now find the PMF of X : $P_X(k) = P(X = k)$ for $k = 0, 1, 2$

Example of PMF...

$\{X=0\} \cup \{X=1\} \cup \{X=2\} = S$

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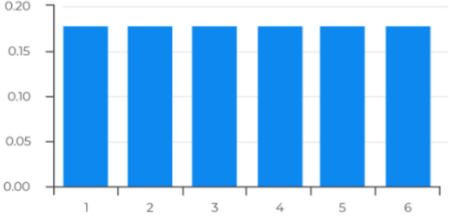
$\{HT, TH\} \subseteq S$

$$
\begin{aligned}
P_X(0) &= P(X = 0) = P(TT) = 1/4 \\
P_X(1) &= P(X = 1) = P(\boxed{\{HT, TH\}}) = 1/4 + 1/4 = 1/2 \\
P_X(2) &= P(X = 2) = P(HH) = 1/4
\end{aligned}
$$


0.20
0.15
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1
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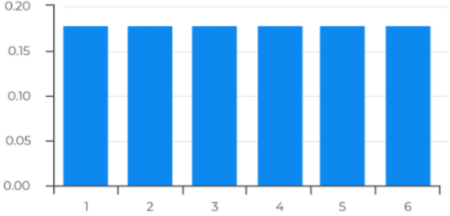
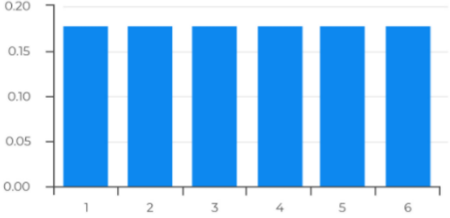


- Consider an experiment of rolling a six-sided die



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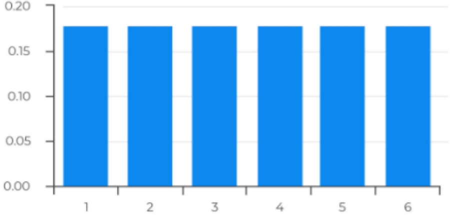

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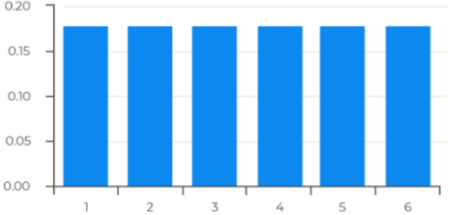

fair

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$$P_X(x) = \begin{cases} 1/6, & x \in \{1, 2, 3, 4, 5, 6\} \\ 0, & \text{otherwise} \end{cases}$$

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Problem on PMF

Consider an unfair coin for which $P(H) = p$.

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Consider an unfair coin for which $P(H) = p$. We toss the coin repeatedly until we observe a head for the first time.

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Consider an unfair coin for which $P(H) = p$. We toss the coin repeatedly until we observe a head for the first time. Let Y be the total number of times the coin was tossed.

Example of PMF...

$R_Y = \{1, 2, \ldots\}$
countable

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Consider an unfair coin for which $P(H) = p$. We toss the coin repeatedly until we observe a head for the first time. Let Y be the total number of times the coin was tossed.

1. Is Y a discrete random variable? Yes

Example of PMF...

Problem on PMF

Consider an unfair coin for which $P(H) = p$. We toss the coin repeatedly until we observe a head for the first time. Let Y be the total number of times the coin was tossed.

1. Is Y a discrete random variable?
2. Find PMF of the random variable Y

$R_Y = \{1, 2, 3, \ldots\}$

Answer to the problem

We have

PMF

$$P_Y(1) = P(Y = 1) = P(H) = p$$

$$P_Y(2) = P(Y = 2) = P(TH) = (1 - p)p$$

$$\vdots$$

$$P_Y(k) = P(Y = k) = P(TT \cdots TH) = (1 - p)^{k-1}p$$

$\boxed{P_Y(y)}$

$S = \{Y=1\} \cup \{Y=2\} \cup \ldots \cup \{Y=k\} \ldots$

Properties of PMF...

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1. $0 \leq P_X(x) \leq 1$ for all x
2. $\sum_{x \in R_X} P_X(x) = 1$
3. For any set $A \subset R_X$, $P(X \in A) = \sum_{x \in A} P_X(x)$

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1. Check $\sum_{y \in R_Y} P_Y(y) = 1$, here R_Y is the range of random variable Y

Check Properties of PMF...

$P_Y(k) = P_Y(Y=k)$

Problem on PMF

Consider an unfair coin for which $P(H) = p$. We toss the coin repeatedly until we observe a head for the first time. Let Y be the total number of times the coin was tossed.

1. Check $\sum_{y \in R_Y} P_Y(y) = 1$, here R_Y is the range of random variable Y
2. If $p = 1/2$, find $P(2 \leq Y < 5)$

PMF $P_Y(k)$

$$P(2 \leq Y < 5) = \sum_{k=2}^{4} P_Y(k) = (1-p)p + (1-p)^2 p + (1-p)^3 p$$

$$= \frac{1}{2}\cdot\frac{1}{2} + \left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^3 \cdot \frac{1}{2}$$

$$= \frac{1}{4} + \frac{1}{2}\left(\frac{1}{4}\right) + \frac{1}{4}\cdot\frac{1}{4}$$

$$= \frac{4+2+1}{16} = 7/16$$

$P_Y(2) = P\{TH\} = (1-p)p$

$P_Y(3) = P\{TTH\} = (1-p)^2 p$

$P_Y(4) = P\{TTTH\} = (1-p)^3 p$

Independent Random Variables...

Recall: $P(A \cap B) = P(A, B)$

$= P(A) \cdot P(B) \Rightarrow A \,\&\, B$ are ind.

$\{X = k\}$

$\{X = m\}$ $=$

$= P_X(k) \cdot P_X(m)$

$P(\ldots) = P_X(k, m) =$

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We consider two random variables independent if

$$P(X = x, Y = y) = P(X = x)P(Y = y), \quad \text{for all } x, y$$

Independent Random Variables...

$P(A|B) = P(A)$

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In general, if two random variables are independent, then

$$P(X \in A, X \in B) = P(X \in A)P(Y \in B), \quad \text{for all sets } A \text{ and } B$$

Also, we have

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Also, we have

$$\overset{A}{\underbrace{P(Y = y}} \overset{|}{|} \overset{B}{\underbrace{X = x)}} = \overset{A}{\underbrace{P(Y = y)}}, \quad \text{for all } x, y$$

Example: independent random variables, PMF

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Problem

We toss a coin twice, and let X denote the number of heads we observe.

Example: independent random variables, PMF

$$\underbrace{\_\ \_}_{X} \;\Big|\; \underbrace{\_\ \_}_{Y}$$

Problem

We toss a coin twice, and let X denote the number of heads we observe. After this, we then toss the coin two more times and define Y to be the number of heads we observe.

Example: independent random variables, PMF

$\{HH, HT, TH, TT\}$

$\{HH, HT, TH, TT\}$

1 head case

Problem

We toss a coin twice, and let X denote the number of heads we observe. After this, we then toss the coin two more times and define Y to be the number of heads we observe. What is $P((X < 2)$ and $(Y > 1))$?

$$P((X<2)\cap(Y>1)) = P(X<2)P(Y>1)$$

[Since X, Y independent]

$$= (P_X(0)+P_X(1))\,P_Y(2) = \left(\frac{1}{4}+\frac{2}{4}\right)\cdot\frac{1}{4} = \frac{3}{4}\cdot\frac{1}{4} = \frac{3}{16}$$

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1. 123456 T2ab2le:C
2. ons:id2db53
3. r2b3d x:d:aCl:ap x2a245n
4. rs:ib2e mbCdabltdb53C
5. on26se:C 5w mbCdabltdb53C
6. ons:id2db53C 5w r56: mbCdabltdb53C

Definition of Expectation..

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The expectation of a discrete random variable X is defined as

$P_X(x) \geq 0$

$$E[X] = \sum_{x: p(x) > 0} p(x) \cdot x$$

removed

sub X

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$$E[X] = \sum_{x:p(x)>0} p(x) \cdot x$$

- Note we sum over all values of X that have non-zero probability
- Other names of expectations: mean, expected value, weighted average, center of mass, first moment

Example of Expectation...

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Example of Expectation...

What is the expected value of 6-sided die roll?

$$R_x = \{1, 2, 3, 4, 5, 6\}$$

$$\frac{\cancel{6}^{3} \cdot 7}{\cancel{2}} = 21$$

$$P_x(1) = 1/6$$
$$P_x(2) = 1/6$$
$$P_x(3) = 1/6$$
$$P_x(4) = 1/6$$
$$P_x(5) = 1/6$$
$$P_x(6) = 1/6$$

$$E[X] = \sum_{i=1}^{6} P_x(x_i)\, x_i$$

$$= \frac{1}{6}\cdot 1 + \frac{1}{6}\cdot 2 + \frac{1}{6}\cdot 3 + \frac{1}{6}\cdot 4 + \frac{1}{6}\cdot 5 + \frac{1}{6}\cdot 6$$

$$= \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.$$

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Answer to the question

- Define a random variable: $X =$ RV for value of a roll

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- Calculate Expectation:

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- Define a random variable: $X =$ RV for value of a roll
- Find PMF:

$$P(X = x) = \begin{cases} 1/6, & x \in \{1, \ldots, 6\} \\ 0, & \text{otherwise} \end{cases}$$

- Calculate Expectation:

$$E[X] = 1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) = \left(\frac{7}{2}\right) = 3.5$$

Recall: Linear Functions...

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Definition of linear function

Let A, B be two vector spaces.

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Let A, B be two vector spaces. A function $f : A \rightarrow B$ is called linear function if the following holds:

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- $f(cu) = cf(u), c$ is scalar

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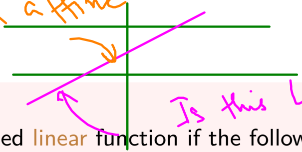
Equivalent definition is:

Recall: Linear Functions...

called affine



Is this linear fn ?

Definition of linear function

Let A, B be two vector spaces. A function $f : A \to B$ is called linear function if the following holds:

- $f(u+v) = f(u) + f(v)$
- $f(cu) = cf(u)$, c is scalar

$\exists$ fn passes through origin ?

i.e., $f(0) = 0$?

Equivalent definition is:

$$f(\alpha u + \beta v) = \alpha f(u) + \beta f(v), \quad \forall u, v \in A, \quad \alpha, \beta \text{ scalars}$$

Q Is $f(x) = 10$ a linear fn ?

Ans: No

$$f(0) = f(x - x) = f(x) - f(x) = 0$$

$$f(0) = f(0 \cdot 0) = 0 \cdot f(0) = 0$$

vector

scalar

Quiz on linear functions...

Quiz on linear functions...

Problem on linear function

Which of the following functions are linear?

Quiz on linear functions...

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1. $f(x) = x$

Quiz on linear functions...

Problem on linear function

Which of the following functions are linear?

1. $f(x) = x$
2. $f(x) = ax + b$

Quiz on linear functions...

Problem on linear function

Which of the following functions are linear?

1. $f(x) = x$
2. $f(x) = ax + b$
3. $f(x) = 10$

Quiz on linear functions...

Problem on linear function

Which of the following functions are linear?

1. $f(x) = x$
2. $f(x) = ax + b$
3. $f(x) = 10$
4. $f(x) = Ax + b$

matrix

vector

Quiz on linear functions...

Problem on linear function

Which of the following functions are linear?

1. $f(x) = x$
2. $f(x) = ax + b$
3. $f(x) = 10$
4. $f(x) = Ax + b$
5. $f(x) = 3\sin x$

Quiz on linear functions...

$f:\mathbb{R}^2 \to \mathbb{R}$

$$f\Big(\underbrace{\begin{bmatrix}x\\y\end{bmatrix}}_{u}\Big) = 3x+4y$$

Problem on linear function

Which of the following functions are linear?

1. $f(x) = x$
2. $f(x) = ax + b$
3. $f(x) = 10$
4. $f(x) = Ax + b$
5. $f(x) = 3\sin x$
6. $f(x, y) = 3x + 4y$

$$f\left(\begin{bmatrix}a\\b\end{bmatrix}+\begin{bmatrix}c\\d\end{bmatrix}\right) = f\left(\begin{bmatrix}a\\b\end{bmatrix}\right)+f\left(\begin{bmatrix}c\\d\end{bmatrix}\right)$$

$$f\left(c\begin{bmatrix}a\\b\end{bmatrix}\right) = c\,f\left(\begin{bmatrix}a\\b\end{bmatrix}\right)$$

Quiz on linear functions...

Problem on linear function

Which of the following functions are linear?

1. $f(x) = x$
2. $f(x) = ax + b$
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4. $f(x) = Ax + b$
5. $f(x) = 3\sin x$
6. $f(x, y) = 3x + 4y$
7. $f(x, y, z) = 3 + x + y + z$

because of this / f does not pass through origin.

Quiz on linear functions...

$$f:\mathbb{R}^3 \to \mathbb{R}^3$$

Problem on linear function

Which of the following functions are linear?

1. $f(x) = x$
2. $f(x) = ax + b$
3. $f(x) = 10$
4. $f(x) = Ax + b$
5. $f(x) = 3\sin x$
6. $f(x, y) = 3x + 4y$
7. $f(x, y, z) = 3 + x + y + z$
8. $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + y \\ y - z \\ z \end{pmatrix}$

$$\to\ f\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix}\right) = f\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) + f\left(\begin{bmatrix} d \\ e \\ f \end{bmatrix}\right)$$

$$\to\ f\left(c\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = c\,f\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right)$$

Properties of Expectation...

Properties of Expectation...

$E[\alpha X + \beta Y] = \alpha E[X] + \beta E[Y]$

Linearity of Expectation

$$E[aX + b] = aE[X] + b$$

Properties of Expectation...

$2E[X] - 1 \cdot E[1]$

$E[c] = c$ constant

Linearity of Expectation

$$E[aX + b] = aE[X] + b$$

Let X = 6 sided dice roll. Let $Y = 2X - 1$. If $E[X] = 3.5$, then what is $E[Y] = ?$

$E[Y] = E[2X-1] = 2E[X] - 1$

$= 2 \cdot 3.5 - 1 = 7 - 1$

$= 6$

$E[X]$