Probability and Statistics

UG2, Core course, IIIT,H

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IIIT, Hyderabad

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Outline

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Motivation for Uniform Distribution: Distribution of a Die Roll...

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Example: Motivation for Uniform Distribution

Consider rolling a fair die. The possible outcomes are $\{1, 2, 3, 4, 5, 6\}$. Then the PMF is given by

$$p(x) = \begin{cases} \frac{1}{6}, & x \in \{1, 2, 3, 4, 5, 6\} \\ 0, & \text{otherwise} \end{cases}$$

We note here that $\sum_{x \in \mathbb{Z}} p(x) = 1$. We note here that PMF takes uniform values for all values of $X = x$.

Uniform Distribution...

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Definition: Uniform Distribution

Motivated from the previous example, we now define uniform distribution on $\{1, 2, \ldots, n\}$ by

$$p(x) = \begin{cases} \frac{1}{n}, & x \in \{1, 2, \ldots, n\} \\ 0, & \textit{otherwise} \end{cases}$$

We verify here that $\sum_{x \in \mathbb{Z}} p(x) = 1$.

Bernoulli Distribution

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A random variable X is called a Bernoulli random variable with parameter p, denoted by $X \sim Bernoulli(p)$, if its PMF is given by

$$P_X(x) = \begin{cases} p & \text{for } x = 1, \\ 1 - p & \text{for } x = 0, \\ 0 & \text{otherwise,} \end{cases}$$

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- This models random experiments that have two possible outcomes
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- Example: A coin is tossed, the outcome is either heads or tails

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- If $X_1, X_2, \ldots, X_n$ are independent Bernoulli(p) random variables, then $X = X_1 + \cdots + X_n$ has Bernoulli(n, p) distribution.
- We verify that $\sum_{x \in \mathbb{Z}} P_X(x) = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = 1$

Example

Example

Let $X \sim Binomial(n, p)$ and $Y \sim Binomial(m, p)$ be two independent random variables. We define a random variable $Z = X + Y$. What is the PMF of Z?

Scratch Space...

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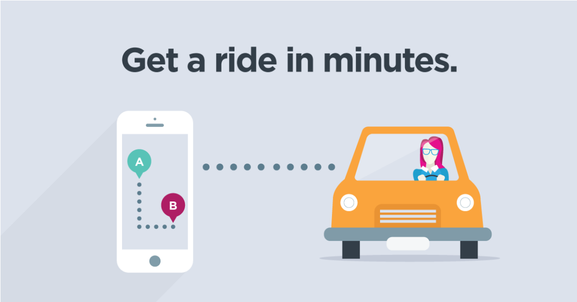

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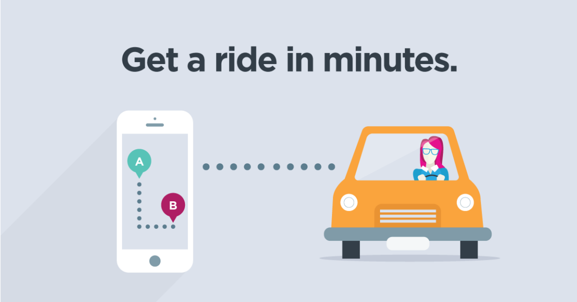

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Binomial in the Limit is Poisson Distribution...

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Derivation:

Definition of Poisson Distribution...

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Definition of Poisson

A random variable X is said to be a Poisson random variable with parameter λ, shown as $X \sim Poisson(\lambda)$, if its range is $R_X = \{0, 1, 2, \ldots, \}$, and its PMF is given by

$$P_X(k) = \begin{cases} \frac{e^{-\lambda}\lambda^k}{k!}, & k \in R_X \\ 0 & \text{otherwise} \end{cases}$$

- Simeon-Denis Poisson, was a French mathematician (1781-1840)
- He published his first paper at 18, became professot at 21
- He published over 300 papers

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- With $X \sim \text{Poisson}(0.01)$ be the number of corrupted bits

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- When n large, and p small: can use Poisson!

Another Example of Poisson Distribution...

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$$\begin{aligned} P(X > 1) &= 1 - P(X = 0) - P(X = 1) \\ &= 1 - e^{-2.8}\frac{2.8^0}{0!} - e^{-2.8}\frac{2.8^1}{1!} \\ &= 1 - e^{-2.8} - 2.8e^{-2.8} \\ &\approx 1 - 0.06 - 0.17 = 0.77 \end{aligned}$$

Outline

Expectation of Poisson Distribution...

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Expectation of Poisson

Let $X \sim Poisson(\lambda)$, with PMF given by

$$P_X(k) = \begin{cases} \frac{e^{-\lambda}\lambda^k}{k!}, & k \in R_X \\ 0 & \text{otherwise} \end{cases}$$

Show that the expectation $E[X] = \lambda$.

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Proof

$$E[X] = \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} = \sum_{x=1}^{\infty} x \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$

$$= e^{-\lambda} \sum_{z=0}^{\infty} \frac{\lambda^{z+1}}{z!} = \lambda e^{-\lambda} \sum_{z=0}^{\infty} \frac{\lambda^z}{z!} = \lambda.$$

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$$E[X] = \sum_{x=0}^{n} x \binom{n}{x} p^x q^{n-x} = \sum_{x=1}^{n} x \binom{n}{x} p^x q^{n-x}.$$

$$\text{For } 0 < x \leq n, \quad x \binom{n}{x} = x \frac{n!}{(n-x)!x!} = \frac{n!}{(n-x)!(x-1)!} = n \binom{n-1}{x-1},$$

$$\implies \quad E[X] = \sum_{x=1}^{n} n \binom{n-1}{x-1} p^x q^{n-x} = \sum_{z=0}^{n-1} n \binom{n-1}{z} p^{z+1} q^{n-1-z} = np.$$

Outline

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Motivation for Variance

- The expectation value $E(X)$ (also called the mean) of a discrete random variable is a rather coarse measure of how X is distributed. For example, consider the following three situations:

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- The three cases differ in the spread of PMF. This can be measured by Variance.
- Recall Saint Petersburg Paradox! High Risk High Reward!

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$$\sigma(X) = \sqrt{\sum_x (x-\mu)^2 f_X(x)} = \sqrt{\mathrm{Var}(X)}.$$

Example of Computing Variance...

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Example

Let X be the value on one roll of a 6-sided die. Recall that $E[X] = 7/2$. What is $Var(X)$?

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- Third case: $\text{Var}(X) = 10^{-3}(10^6 - 10^3)^2 + 999 \times 10^{-3}(0 - 10^3)^2 \approx 10^9$

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For example, the data sets 199, 200, 201 and 0, 200, 400 both have the same average (200) yet they have very different standard deviations.

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Theorem (Another Expression for Variance)

If X is a discrete random variable with mean μ, then

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$$\begin{aligned}
\mathrm{Var}(X) &= \sum_x (x-\mu)^2 p_X(x) = \sum_x (x^2 - 2\mu x + \mu^2) p_X(x) \\
&= \sum_x x^2 p_X(x) - 2\mu \sum_x x\, p_X(x) + \mu^2 \sum_x p_X(x) \\
&= E[X^2] - 2\mu E[X] + \mu^2 = E[X^2] - \mu^2
\end{aligned}$$

Properties of Variance...

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Theorem

Let X be a discrete random variable and α a constant. Then

$$\mathrm{Var}(\alpha X) = \alpha^2 \mathrm{Var}(X) \quad \text{and} \quad \mathrm{Var}(X + \alpha) = \mathrm{Var}(X)$$

Computing Variance: Binomial

Variance of Binomial Distribution

Let $X \sim \text{Binomial}(n, p)$. Then the variance $\text{Var}(X) = np(1 - p)$.

Variance of Binomial Distribution...

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Outline

Higher Order Moments...

Define nth moment

The nth moment about the mean or nth central moment of a real valued random variable X is defined as follows

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Generating Moments...

Is there a quick way to generate moments?

Moment Generating Function...

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The moment generating function $M_X(t)$ is the expectation value

$$M_X(t) = E[e^{tX}] = \sum_x e^{tx} p_X(x)$$

Lemma

- $M_X(0) = 1$
- $E[X] = M'_X(0)$, where ' is the derivative w.r.t. t

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M_X(t) &= \sum_{x=0}^{n} \binom{n}{x} p^{x} (1-p)^{n-x} e^{tx} \\
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- Differentiating w.r.t. to t,

$$M'_X(t) = e^{\lambda(e^t-1)} \lambda e^t$$

- Setting $t = 0, \quad M'_X(0) = \lambda$

Variance Using Moment Generating Function...

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$$\text{Var}(X) = M_X''(0) - M_X'(0)^2$$

Computing Variance Using Moment Generating Function...

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Find the Variance using $M_X(t)$.

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Find the Variance using $M_X(t)$.