CS 302.1 - Automata Theory

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Quick Recap

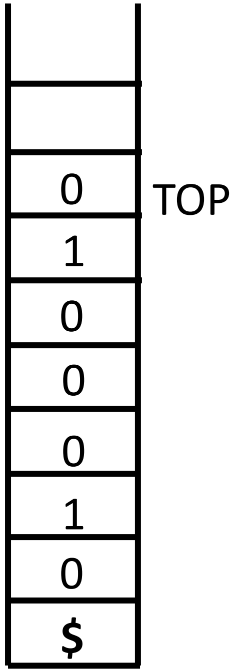
Chomsky Normal Form: If every *rule* of the CFG is of the form

$$\boldsymbol{A \to BC} \qquad [B, C \text{ are not start variables}]$$
$$\boldsymbol{A \to a} \qquad [a \text{ is a terminal}]$$
$$\boldsymbol{S \to \epsilon} \qquad [S \text{ is the Start Variable}]$$

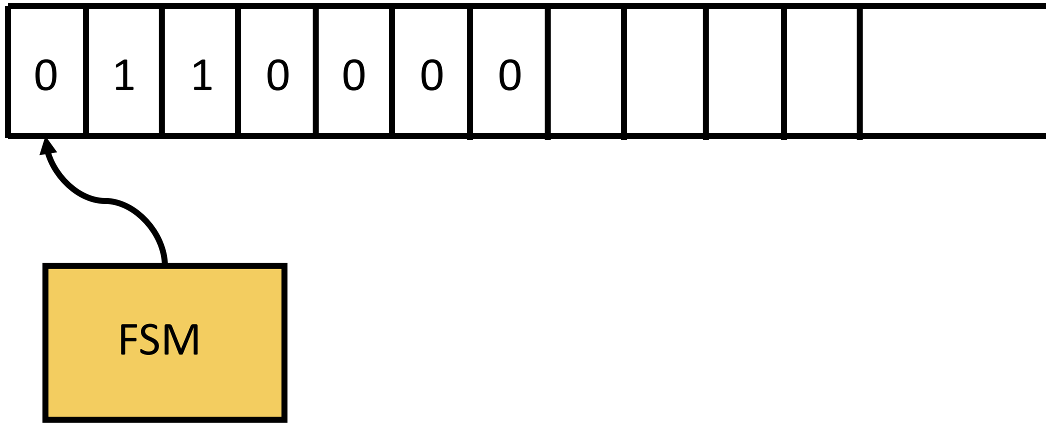
- Any CFG can be converted to a grammar in CNF that generates the same language.
- The number of steps required to derive a string $w = 2|w| - 1$.
- Is crucial in deciding whether w is generated by a CFG G.

Pushdown Automata

- Automata that recognizes CFLs
- FSM + stack
- FSM transitions by reading an input symbol and by interacting with the stack



One-way infinite tape holding the input string



Pushdown Automata

PDAs are **non-deterministic**. (Multiple transitions/input symbol possible)

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- If FSM is at Q_0, and a 1 is read, pop a 0 off the Stack and transition to Q_1.

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- If FSM is at Q_1, and a 1 is read, pop a 0 off the Stack and remain at Q_1.
- If FSM is at Q_1, and a 1 is read, pop a 0 off the Stack, push 1 on to the stack and transition to Q_2

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- If FSM is at Q_1, and a 1 is read, pop a 0 off the Stack, push 1 on to the stack and transition to Q_2
- If the input is finished exactly when the stack is empty (TOP = $), ACCEPT the input.

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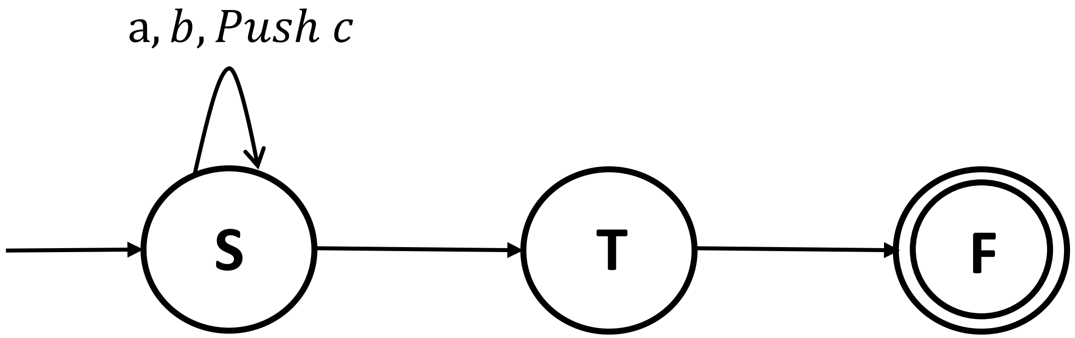
- If FSM is at Q_1, and a 1 is read, pop a 0 off the Stack, push 1 on to the stack and transition to Q_2
- If the input is finished exactly when the stack is empty (TOP = $), ACCEPT the input.
- REJECT otherwise (Stack becomes empty before all the inputs are read/non-empty after the entire input is read)

Pushdown Automata

- How to represent a transition in a PDA?



If input symbol = a, Stack top = b, then Pop b and Push c onto the Stack

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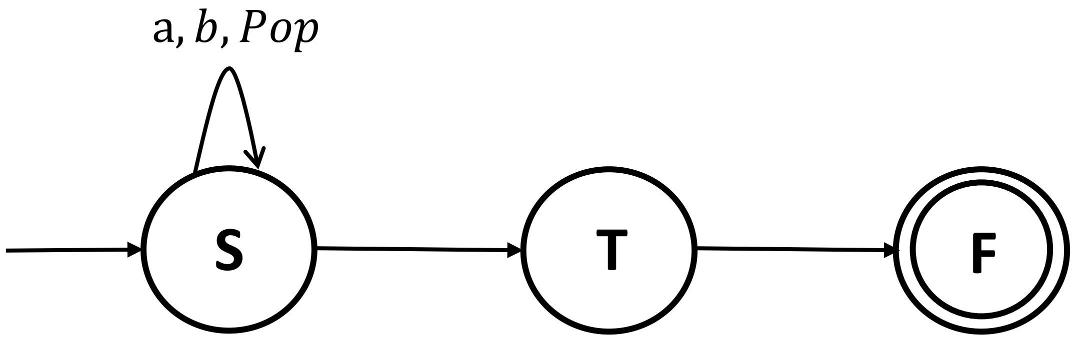

If input symbol = a and Stack top = b, then Pop b

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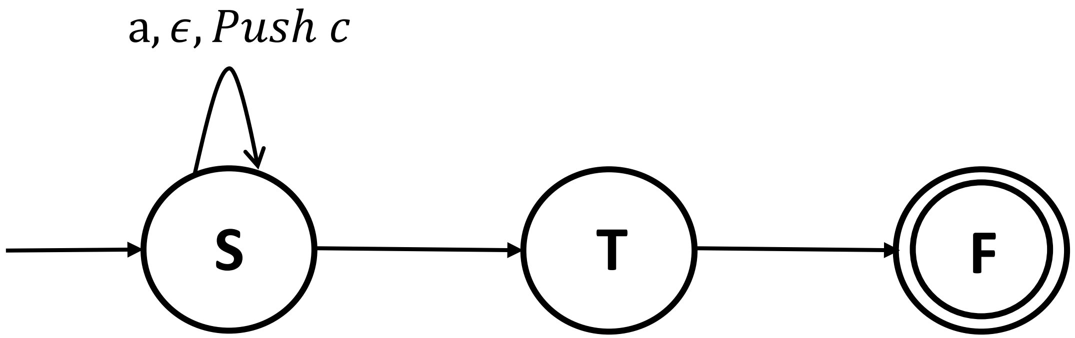

If input symbol = a, then Push c

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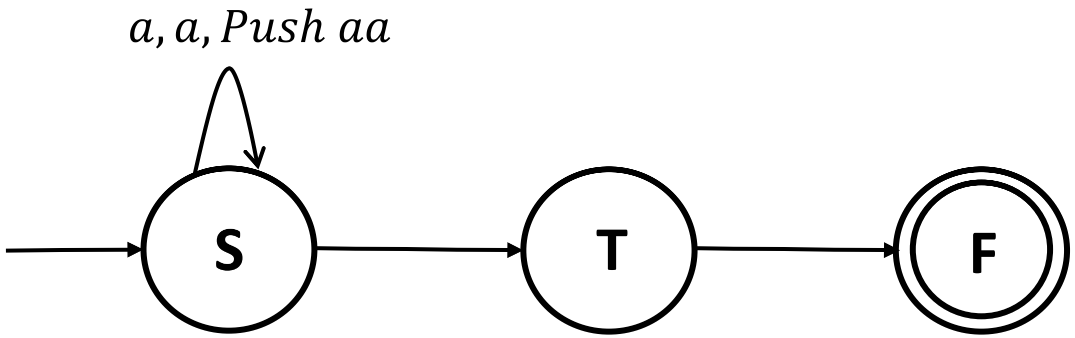

If input symbol = a, then Pop a and Push aa.

So effectively, the PDA pushes a onto the stack if it reads a on the input tape and the stack top = a.

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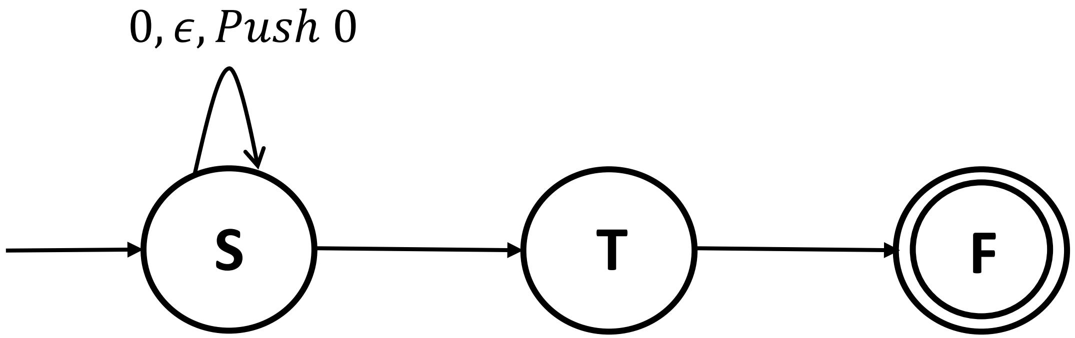

If input symbol = 0, Push 0 onto the Stack irrespective of the element at the top of the stack

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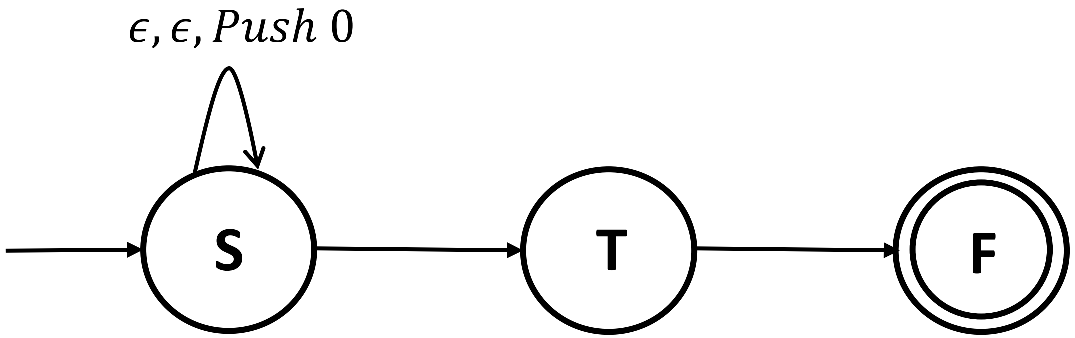

Without reading the input symbol and the Stack top, Push 0 onto the Stack

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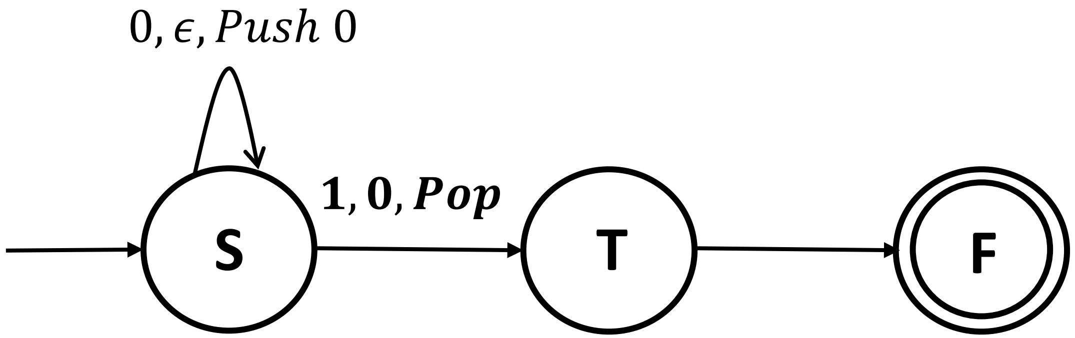

If the input symbol is 1, and the element at the top of the stack is 0, pop it **(Pop 0)**.

Pushdown Automata

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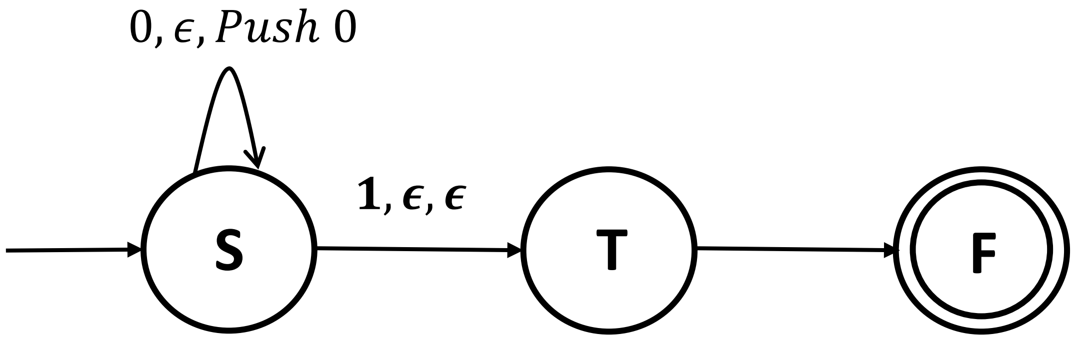

If the input symbol is 1, transition to T by ignoring the stack top completely.

If this happens at every step of the execution of the PDA, then it is as powerful as an NFA.

Pushdown Automata

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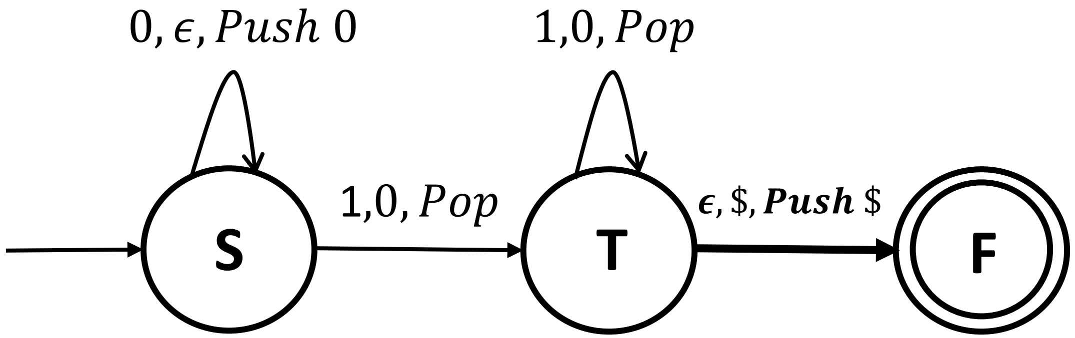

If the Stack is empty, i.e. TOP = $, transition to F from T, without reading the input

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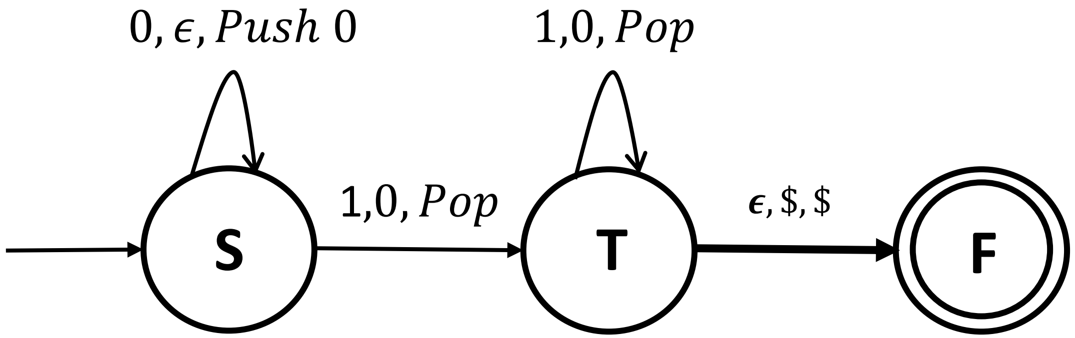

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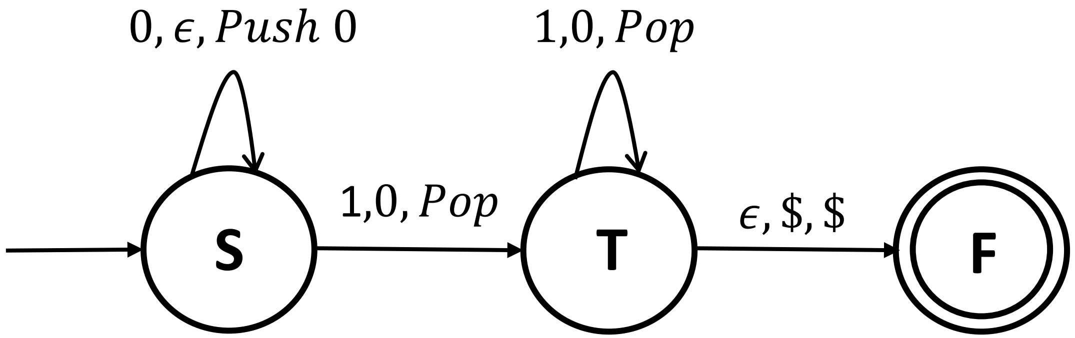

What is the language accepted by this PDA?

Pushdown Automata

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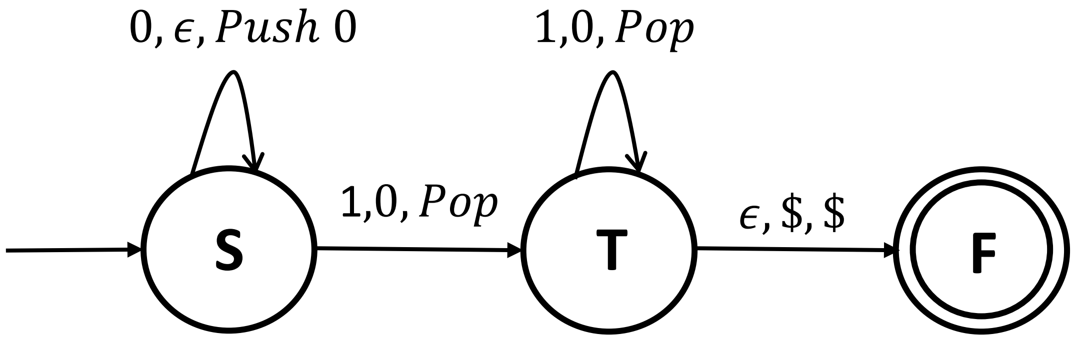

What is the language recognized by this PDA?

Verify that it is $\boldsymbol{L = \{0^n 1^n, n \geq 1\}}$

Pushdown Automata

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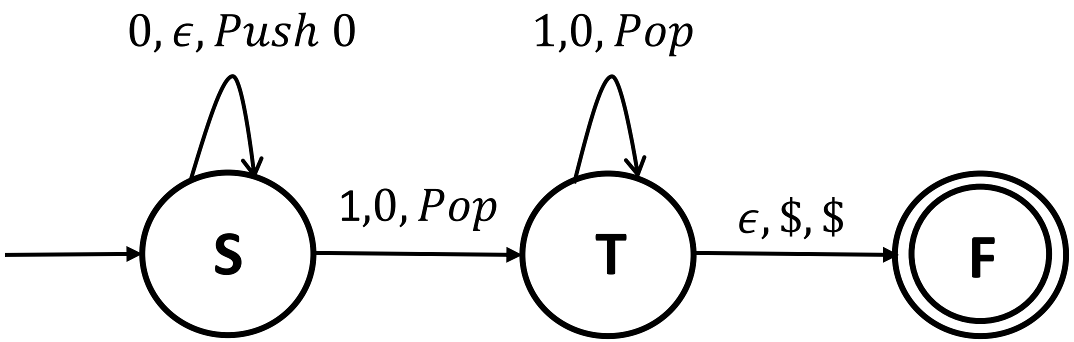

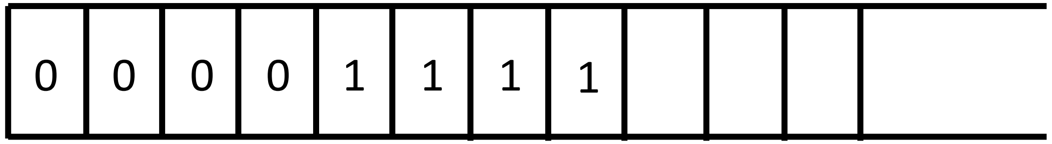

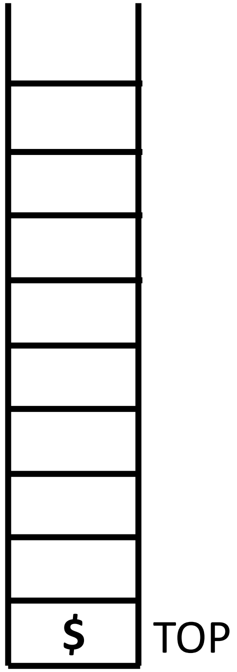

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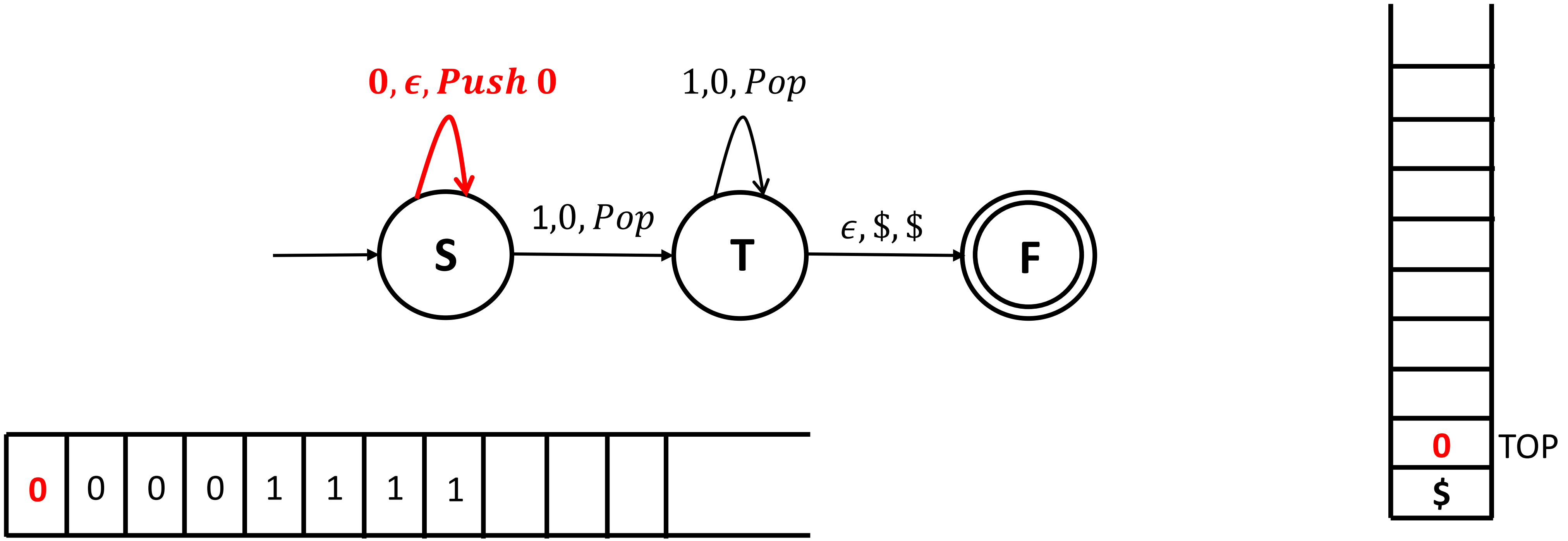

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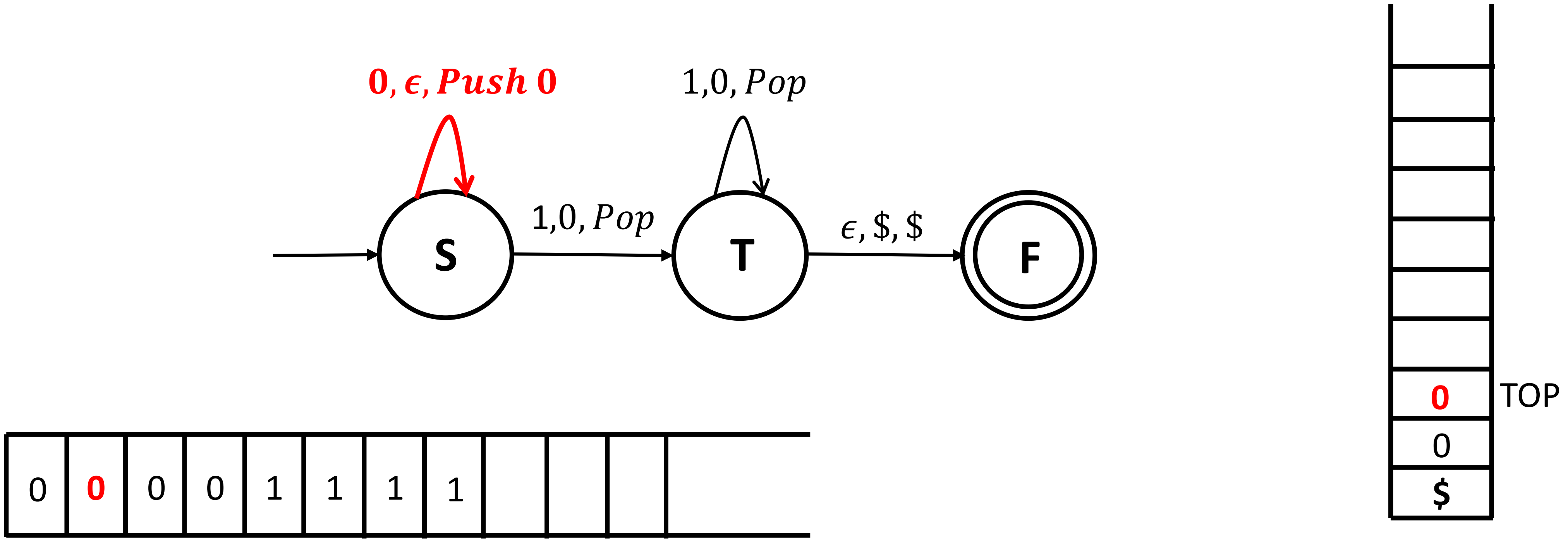

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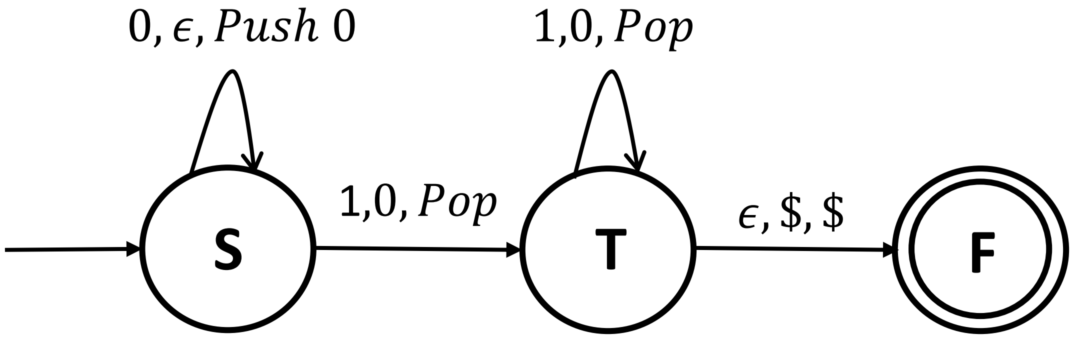

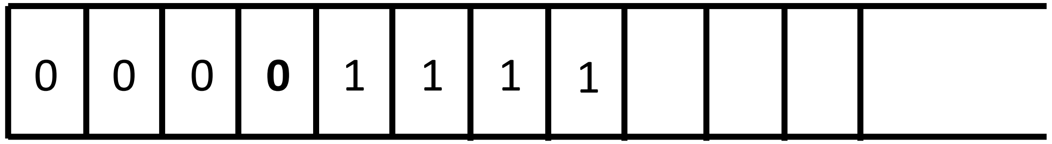

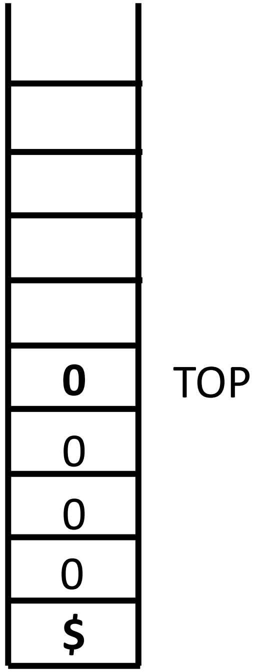

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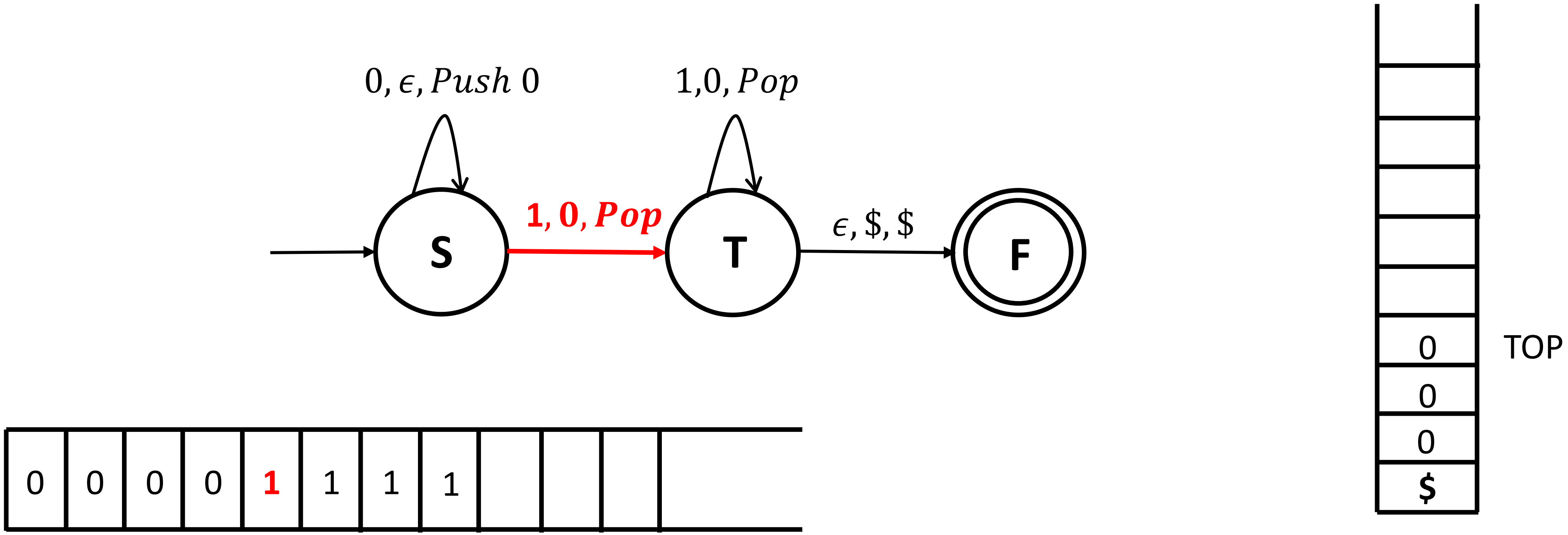

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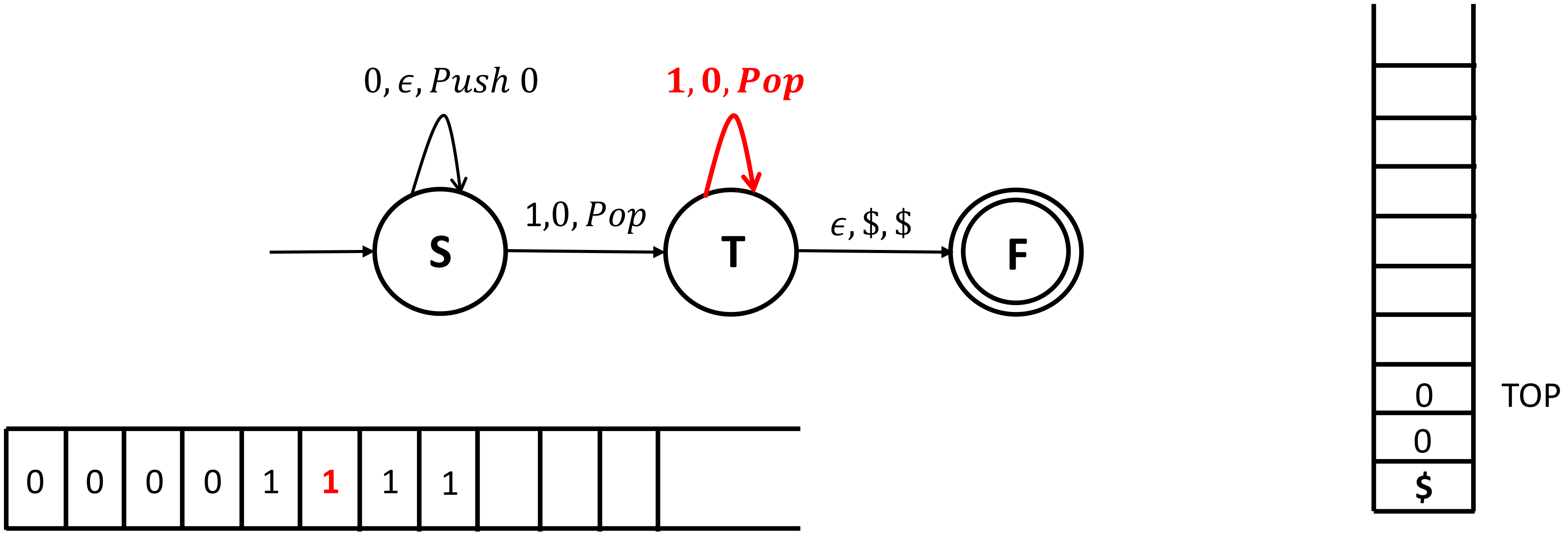

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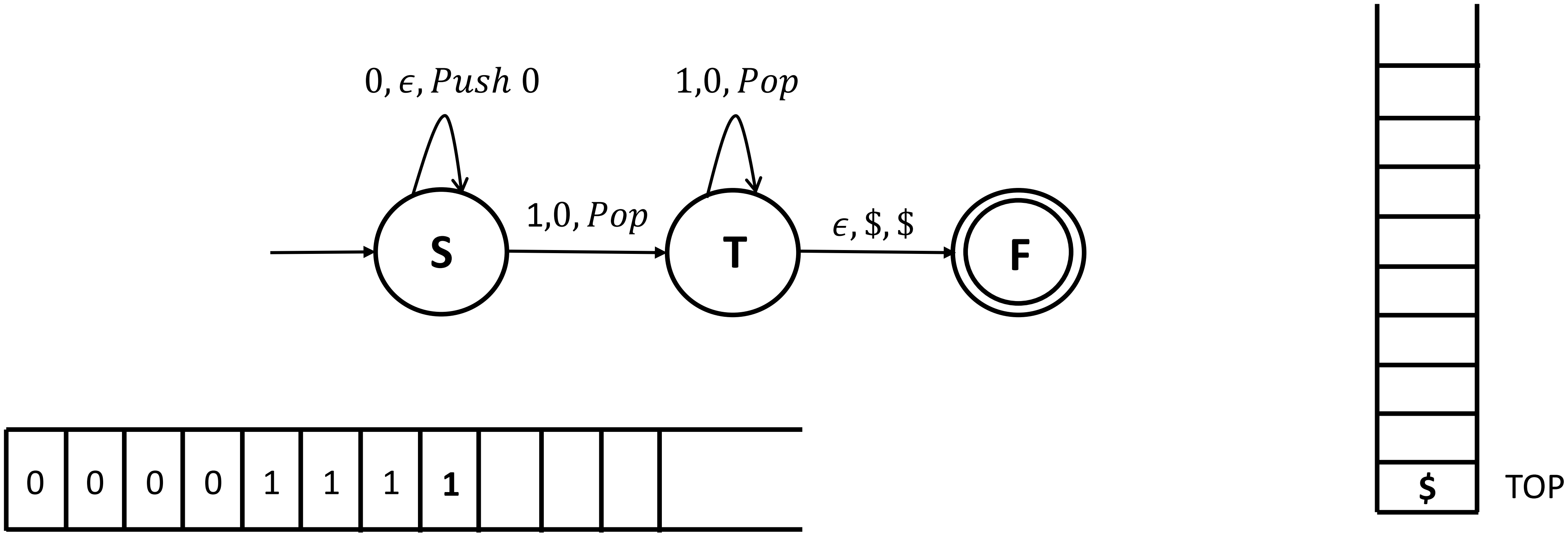

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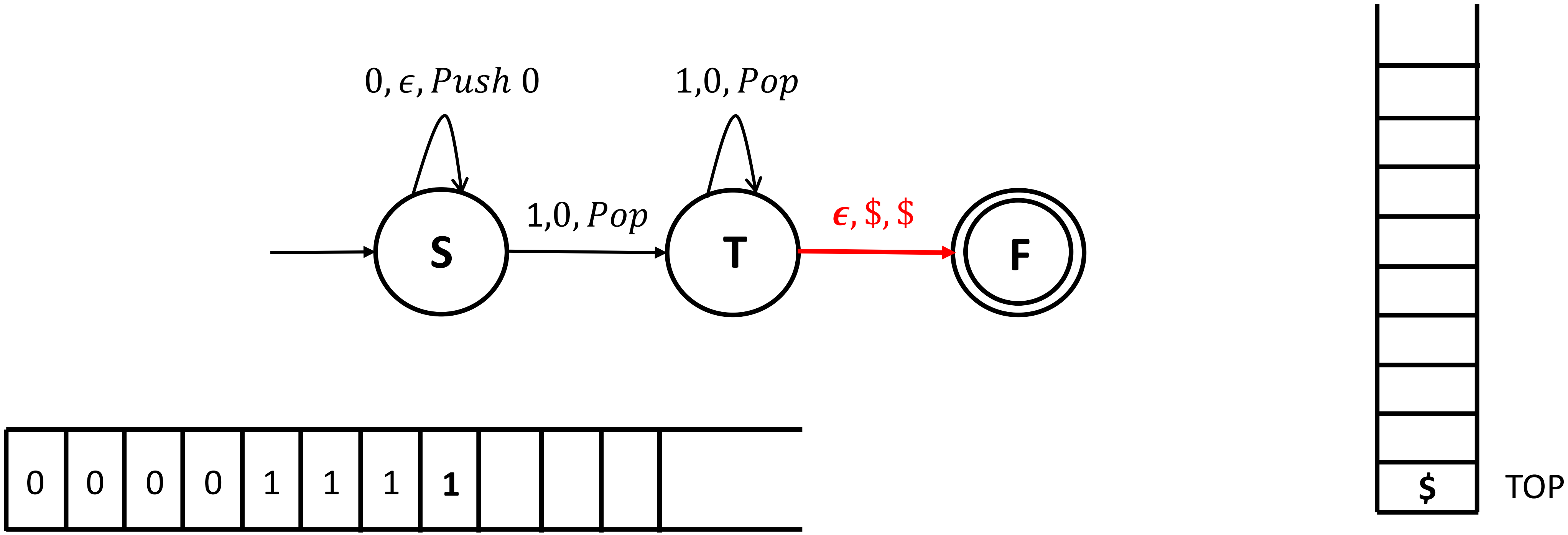

The language recognized by the PDA: $\boldsymbol{L = \{0^n 1^n, n \geq 1\}}$

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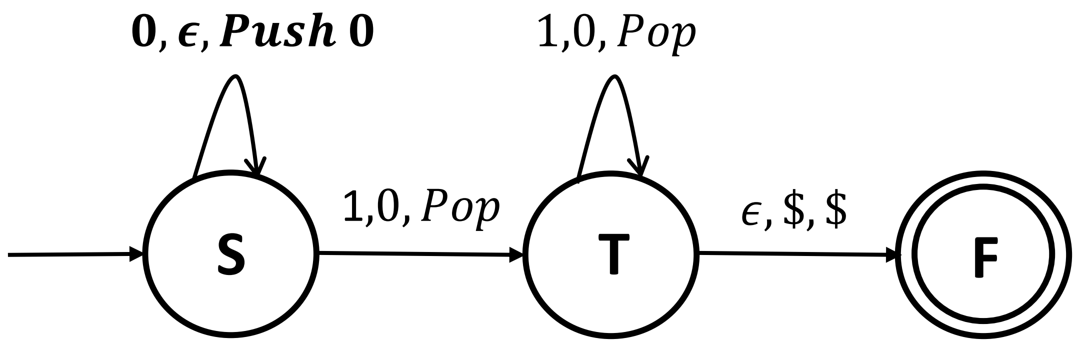

In some references (such as Sipser):

- The transitions of the PDA are labelled as "$\boldsymbol{a}, \boldsymbol{b} \rightarrow \boldsymbol{c}$", implying: If the input symbol read is $\boldsymbol{a}$, the element at the top of the stack is $\boldsymbol{b}$, then pop $\boldsymbol{b}$ and push $\boldsymbol{c}$ on to the Stack.

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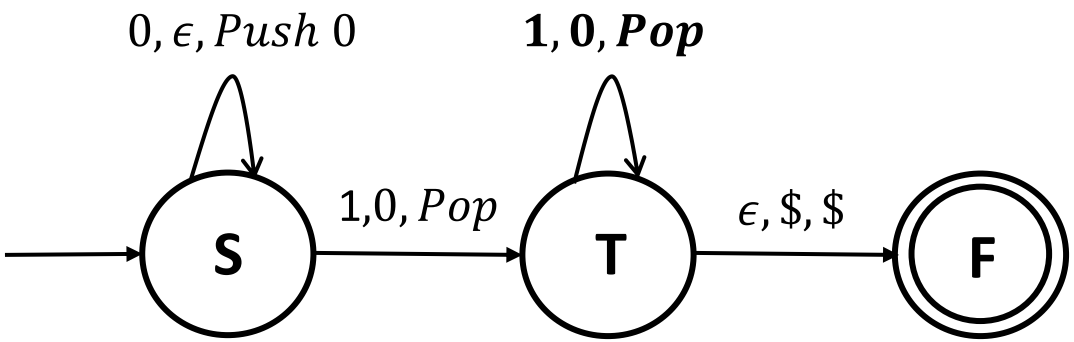

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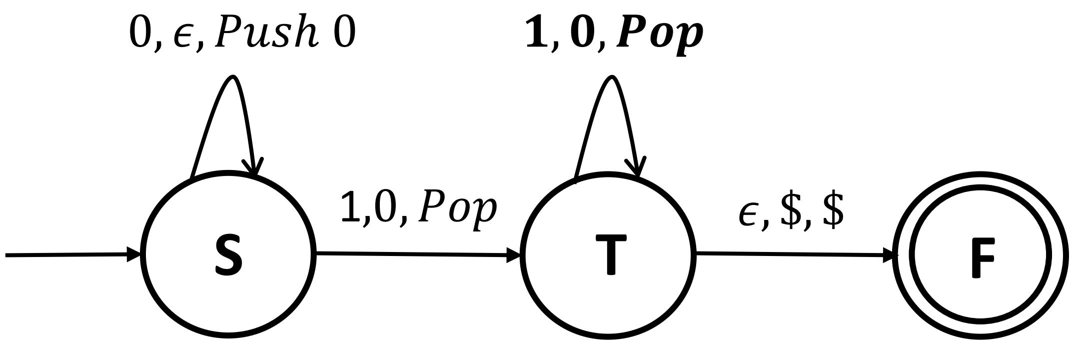

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- The symbol signifying the bottom of the Stack \$ is pushed at the very beginning.

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Formally, a PDA M is a 6-tuple $(\mathrm{Q}, \Sigma, \Gamma, \delta, q_0, F)$ where

- Q is a finite set called the ***states.***
- Σ is the set of input ***alphabets.***
- Γ is the ***Stack alphabet***
- $\delta: Q \times \Sigma_\epsilon \times \Gamma_\epsilon \mapsto \mathcal{P}(Q \times \Gamma_\epsilon)$ is the ***transition function*** [$\Sigma_\epsilon = \Sigma \cup \{\epsilon\}$ and $\Gamma_\epsilon = \Gamma \cup \{\epsilon\}$]
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- $\boldsymbol{\delta(q_i, \epsilon, \$) = (q_j, \$)}$: Transition from q_i to q_j if the stack is empty.

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- If the input symbol read is a and the stack top = a, then Push a and remain at q_i : ?

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- Q is a finite set called the ***states.***
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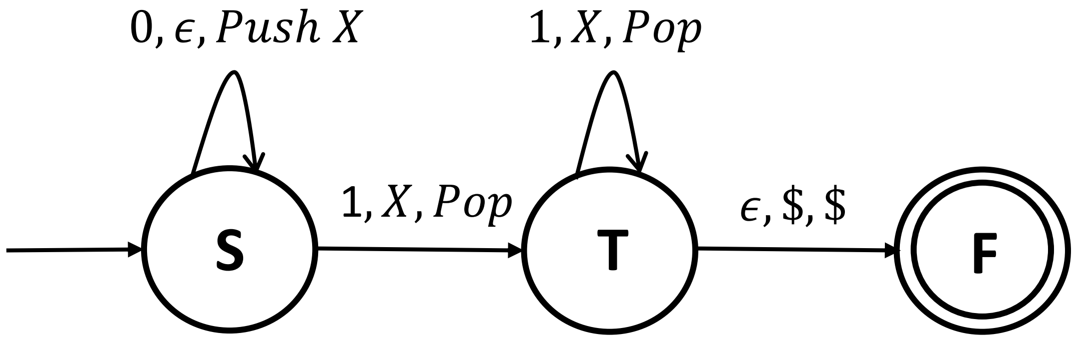

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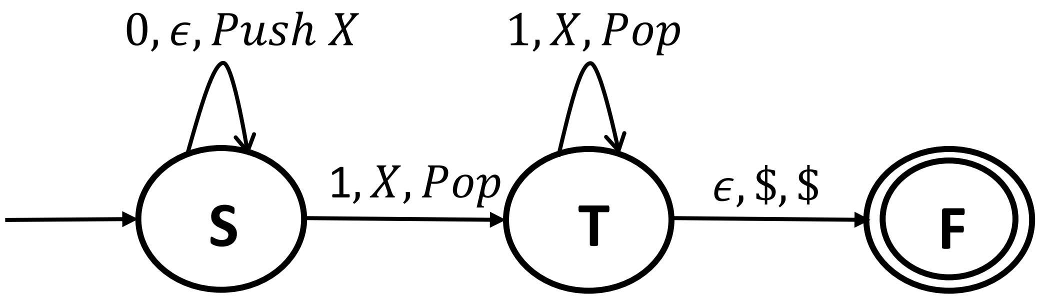

$$\delta(S, 0, \epsilon) = (S, X)$$

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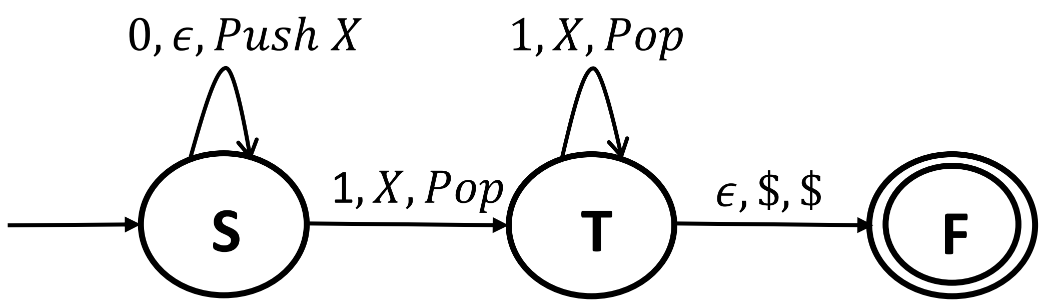

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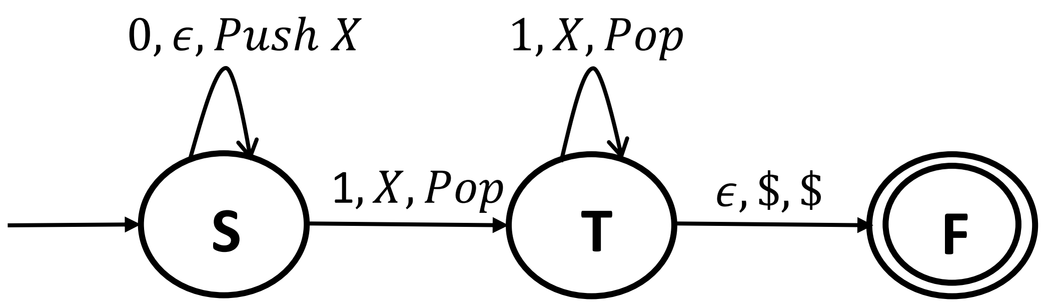

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- The above intuition is applicable for even length palindromes of the form ww^R.

- What about odd length palindromes?
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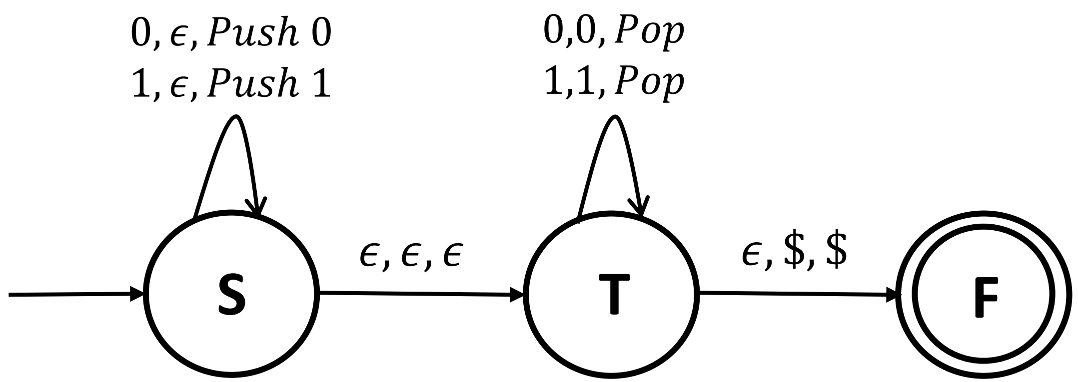

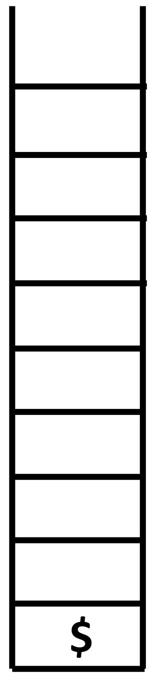

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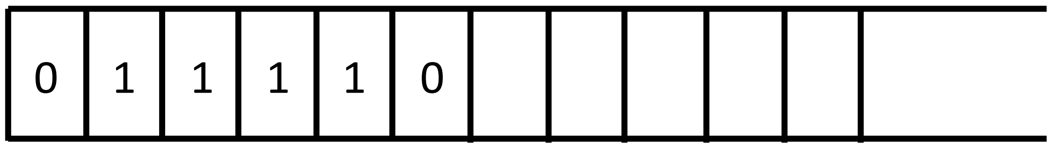

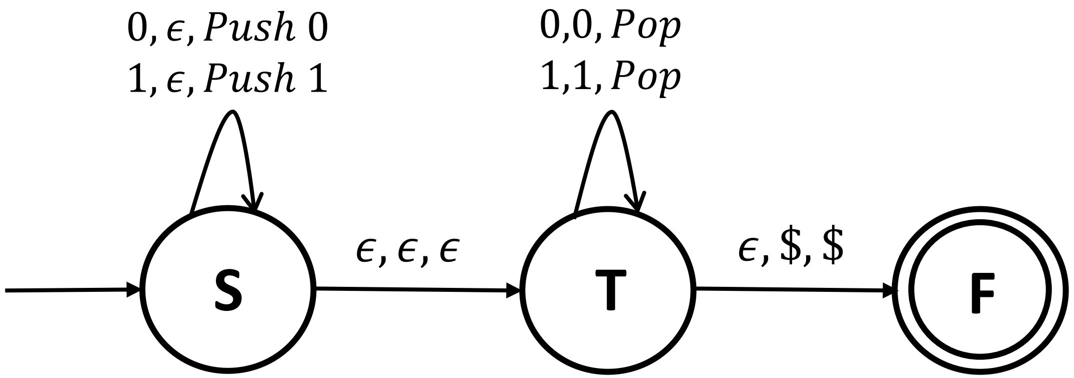

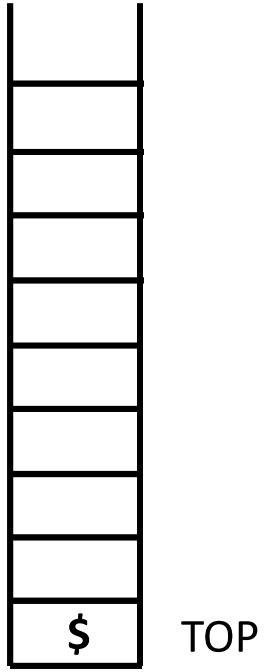

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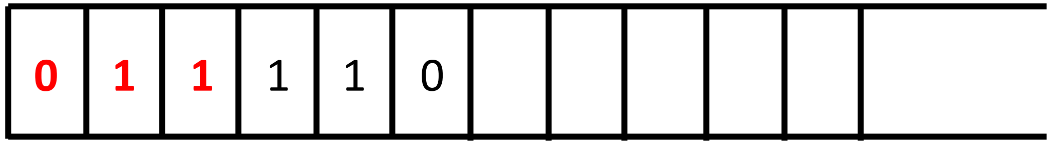

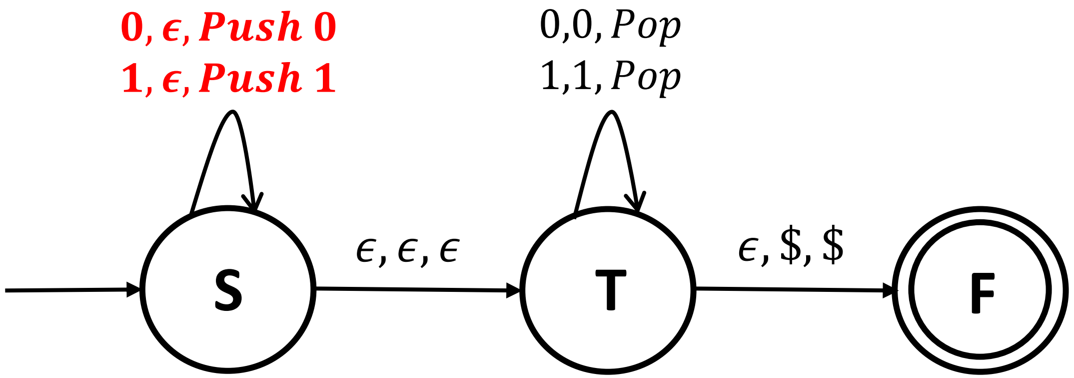

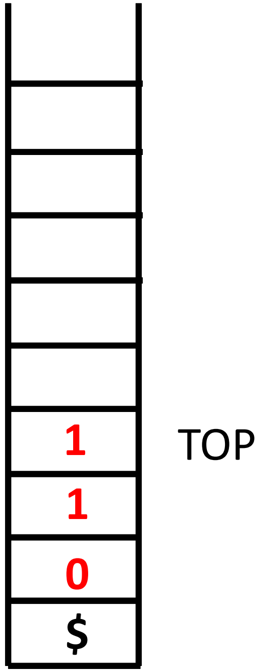

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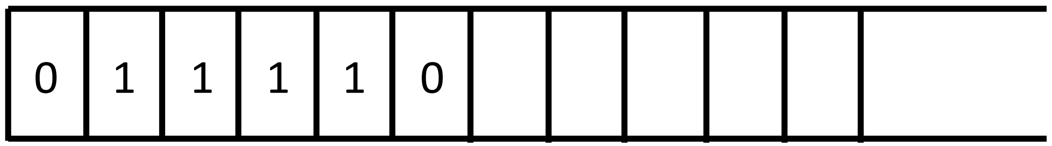

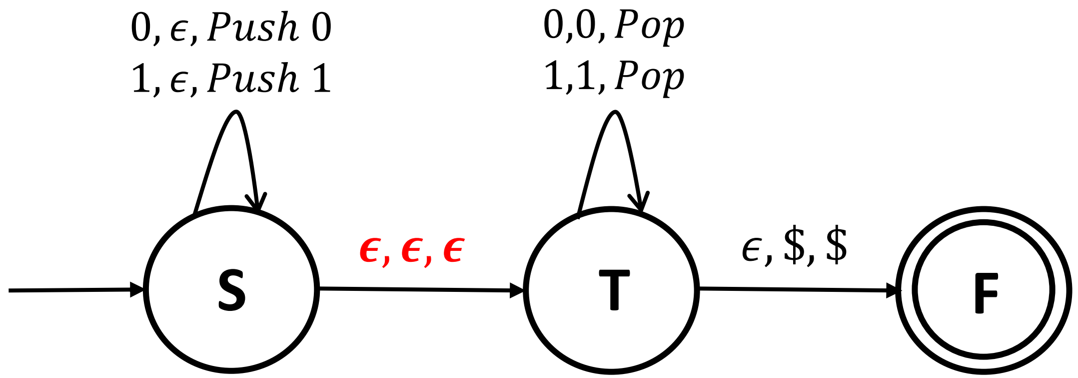

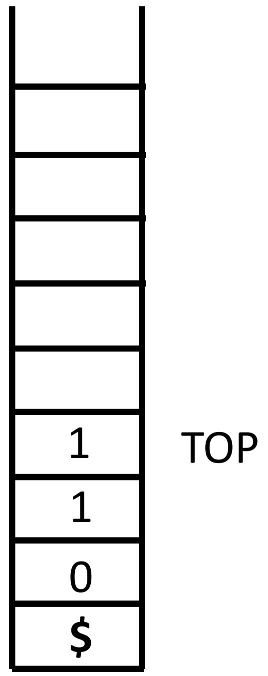

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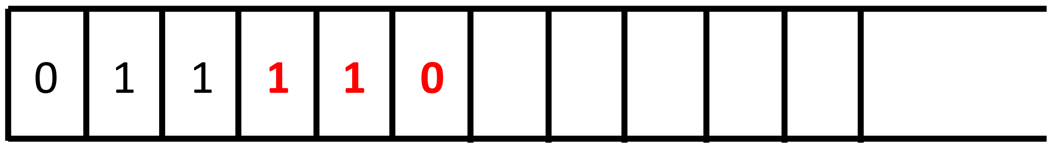

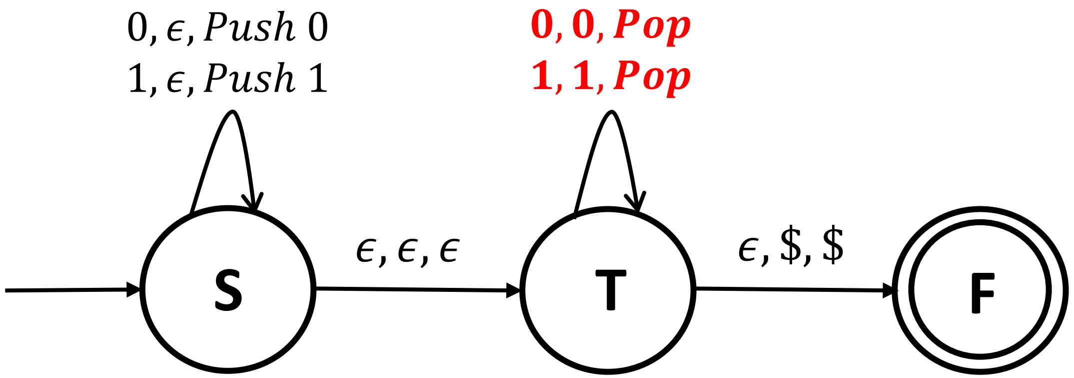

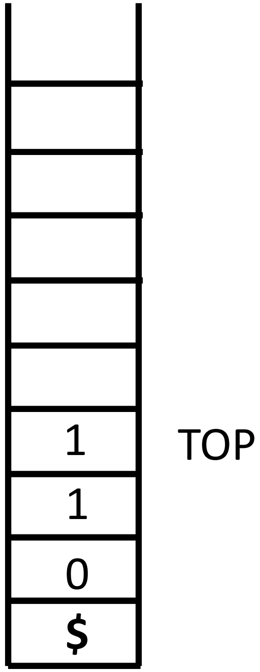

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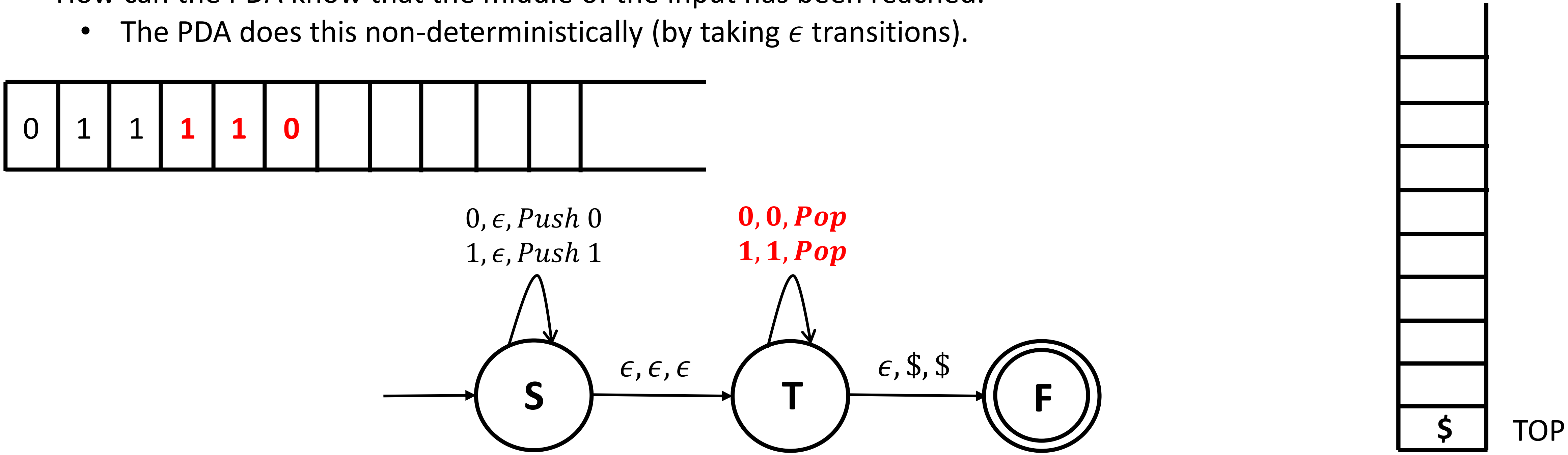

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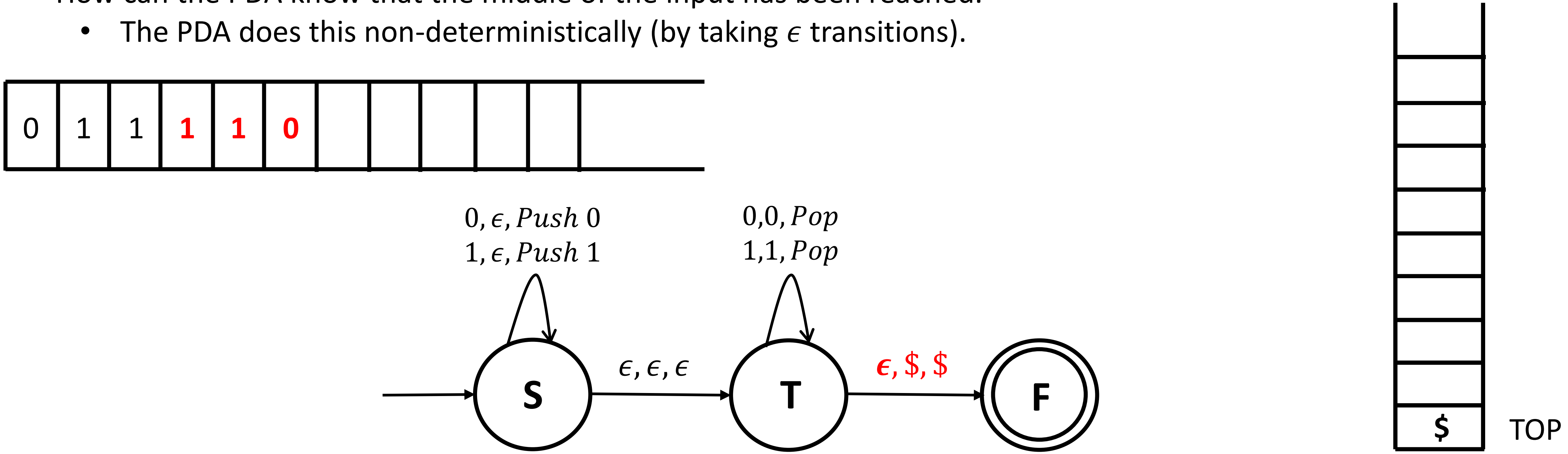

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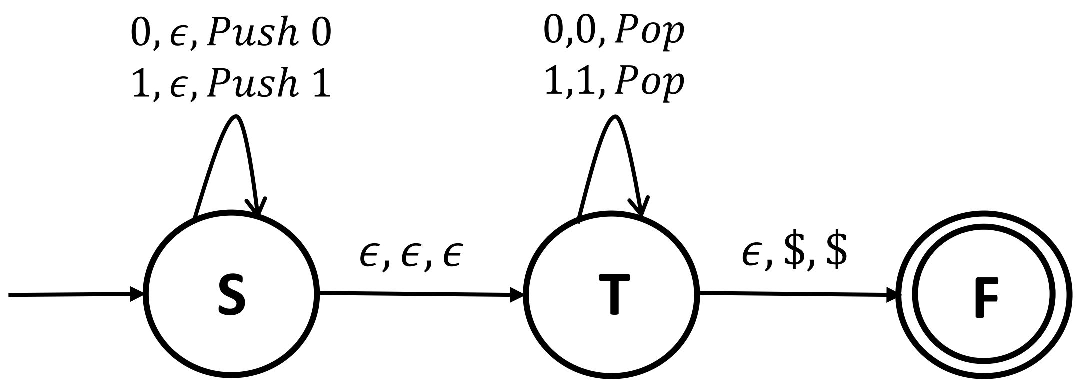

Recognizes even length palindromes of the form: ww^R

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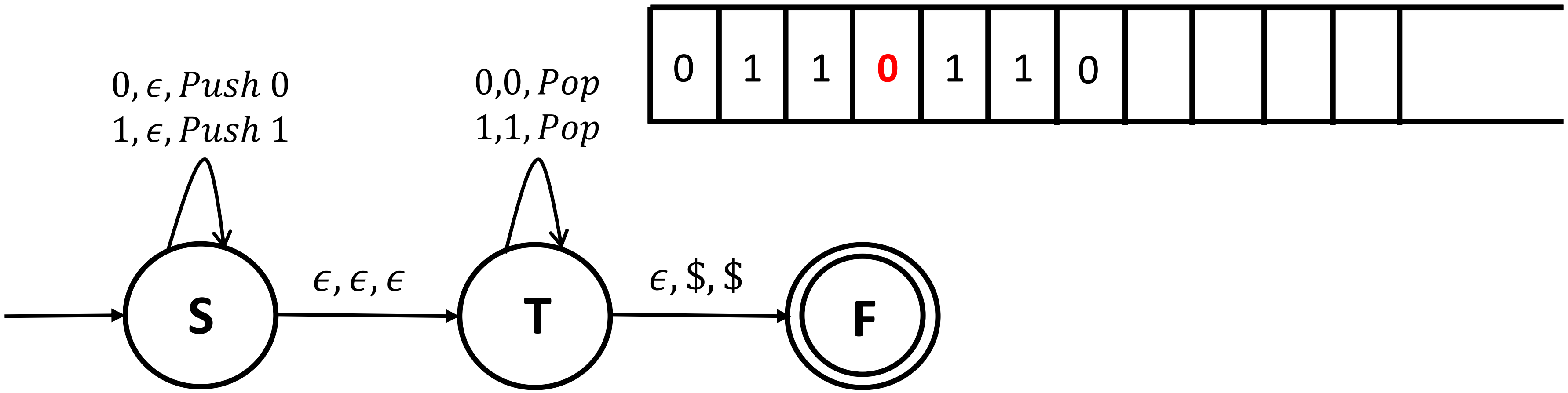

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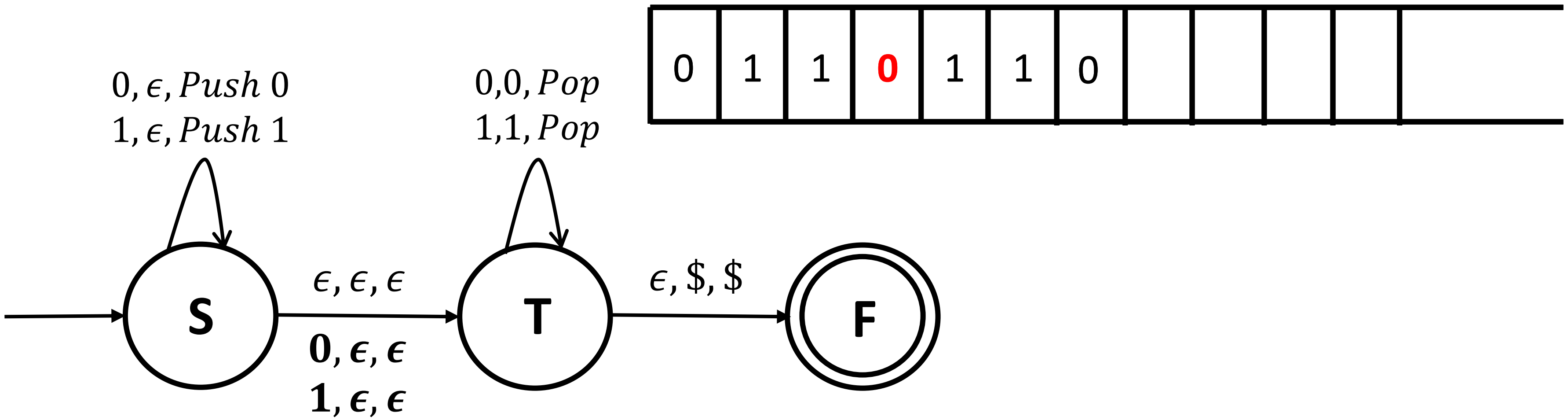

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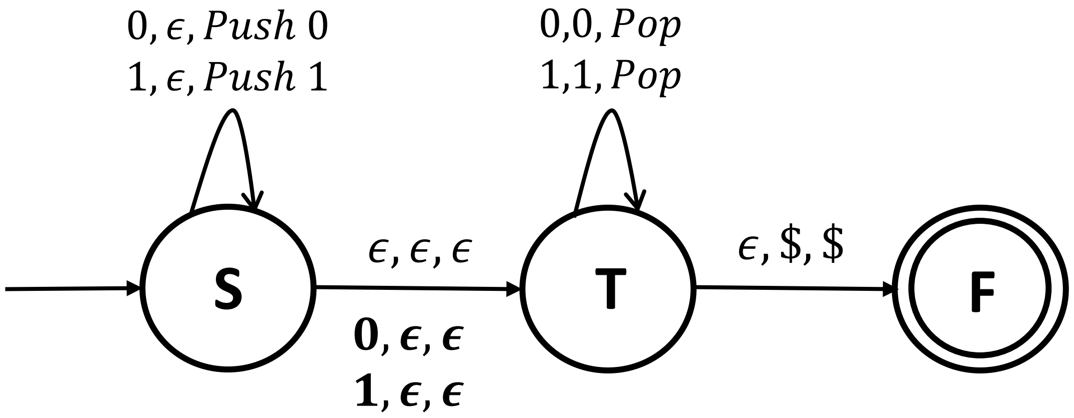

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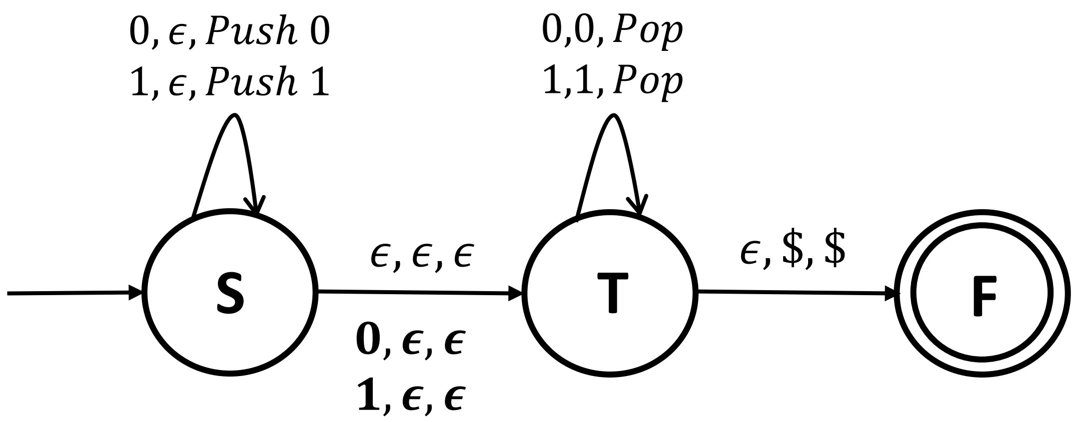

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This allows the PDA to **recognize strings of the form:** $\boldsymbol{\omega c w^R}$, where the aforementioned transitions non-deterministically guessed $c \in \{0,1\}$

Thank You!