Assignment 1

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1 Solve the following questions :

1. In the product Hilbert Space $C^2 \otimes C^2$, the Bell states are given by :

$$\left|\phi^+\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad \left|\phi^-\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$\left|\psi^+\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad \left|\psi^-\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Show that these form the orthonormal basis for C^4. Here, $\{|0\rangle, |1\rangle\}$ is an arbitrary orthonormal basis in the Hilbert space C^2. Let :

$$|0\rangle = \begin{pmatrix} e^{i\phi}\cos\theta \\ \sin\theta \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} -e^{i\phi}\sin\theta \\ \cos\theta \end{pmatrix}$$

   (a) Find $\left|\phi^+\right\rangle, \left|\phi^-\right\rangle, \left|\psi^+\right\rangle, \left|\psi^-\right\rangle$ for this basis
   (b) Find it for the special case of when $\phi = 0$, and $\theta = 0$.

2. Show that all the Pauli Matrices σ_i are Unitary and Hermitian. Find all $\sigma_i \otimes \sigma_j$. Find out whether $\sigma_i \otimes \sigma_j$ are Hermitian and Unitary or not?

3. a) Prove that the eigen values of the projector P are all either 0 or 1. b) Show that for any operator A, A^+A is positive.c) Prove that two eigen vectors of an Hermitian operator with different eigenvalues are necessarily orthogonal.

4. Suppose Alice and Bob shares an entangled state $|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, where the first qubit is with Alice and the second qubit is with Bob. Alice measures her qubit in the basis $[|+\rangle, |-\rangle]$. Find the combined state of Alice and Bob after measurement.

5. Consider the following settings of sequential Stern-Gerlachs, tell what will be the final outputs with proper reasons

   (a) $S_x \to S_y \to S_x \to S_y$ ($-$, $-$, $-$ rays are blocked sequentially).
   (b) $S_z \to S_z \to S_x$ ($-$, $+$ rays are blocked sequentially)
   (c) $S_x \to S_y \to S_z \to S_y$ ($+$, $+$, $+$ rays are blocked sequentially).

6. a) Show that tensor product of two Hermitian operators is Hermitian b) Show that tensor product of two Unitary operators is Unitary c) Find out all possible tensor products of the states

$$\left|\phi^+\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad \left|\phi^-\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$\left|\psi^+\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad \left|\psi^-\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$