

Feb 1

Program Verification Quiz 1

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1.

a) An interpretation is a mapping of propositional variables to truth values.

eg. $I = \{ p:1, q:0, r:1 \}$

b) Valuations take an interpretation and a propositional formula and assigns a truth value to the formula according to the interpretation.

eg. $A = p \vee (\neg q \wedge r)$

$Val(I, A) = 1$ (True)

c) Tautology is a formula that evaluates to true regardless of the interpretation.

eg. $p \vee \neg p$

d) Consider two formulas A_1, A_2

$A_1 \equiv A_2$ if and only if
 $Val(I, A_1) = Val(I, A_2)$
for all interpretations I .

eg. $\neg(\neg p \vee q) \equiv \neg\neg p \wedge \neg q$

2.

Proof:

1.

1.1 $(P \rightarrow q) \wedge (\neg r \rightarrow \neg q)$ Assumed1.1.1 P Assumed~~1.1.2 P~~ ~~1.1.1, $\wedge E$~~ 1.1.3 $P \rightarrow q$ 1.1, $\wedge E$ 1.1.4 q 1.1.1, 1.1.3, $\rightarrow E$ 1.1.5 $\neg r \rightarrow \neg q$ 1.1, $\wedge E_2$ ~~1.1.5.1 $\neg q$~~ Assumed1.1.5.2 \perp 1.1.4, 1.1.5.1, $\neg E$ 1.1.6 ~~$\neg q$~~ r 1.1.5.2, RAA1.2 $P \rightarrow r$ 1.1.1, 1.1.6 $\rightarrow I$ 2 $(P \rightarrow q) \wedge (\neg r \rightarrow \neg q) \rightarrow (P \rightarrow r)$, 1.1, 1.2 $\rightarrow I$

3.

rewriting $[(p \rightarrow q) \wedge (\neg r \rightarrow \neg q)] \rightarrow (p \rightarrow r)$ to a logically equivalent
formula using $a \rightarrow b \equiv \neg a \vee b$

$$\neg [(\neg p \vee q) \wedge (r \vee \neg q)] \vee (\neg p \vee r)$$