Probability and Statistics

UG2, Core course, IIIT,H

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IIIT, Hyderabad

November 5, 2021

Outline

Normal (Gaussian) Distribution...

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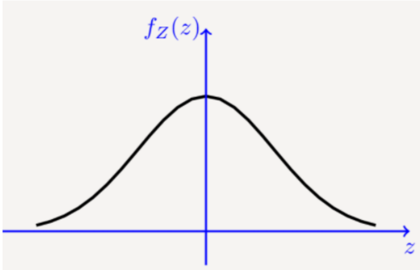

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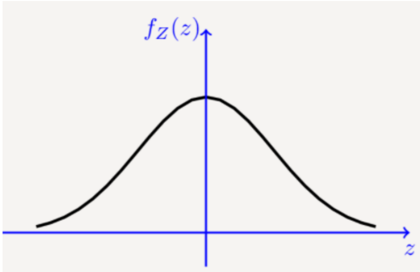

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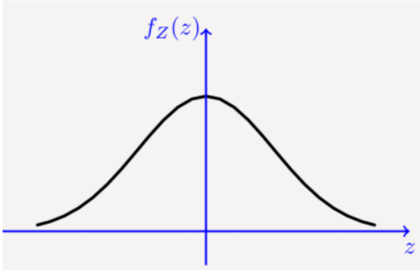

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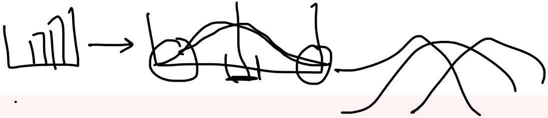
- Central Limit Theorem (TODO):

Normal (Gaussian) Distribution...



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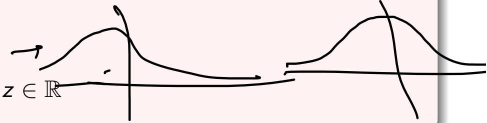
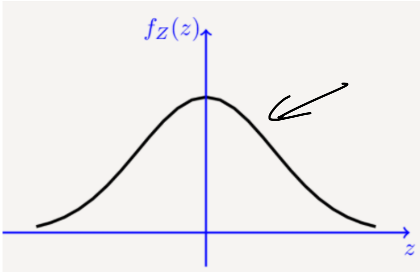


→ Mixed Gaussian

→ Probability Machine Learning



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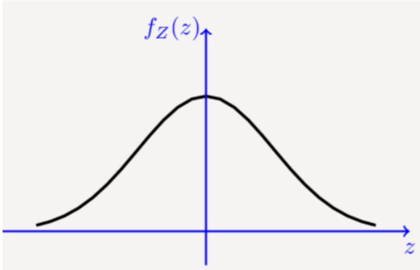

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Normal (Gaussian) Distribution...

Ex Check that $\mathrm{Var}[Z] = 1$

$$\int_{-\infty}^{\infty} e^{-z^2/2} = \sqrt{2\pi}$$

check

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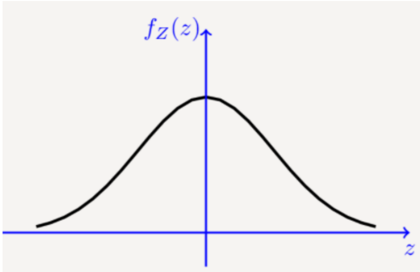
$$E[Z] = \int_{-\infty}^{\infty} z\, f(z)\, dz = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z\, e^{-z^2/2}\, dz = 0\ ?$$

odd fn



- Most important Probability Distribution!
- Central Limit Theorem (TODO):
  - If we add large number of random variable, then the distribution of the sum is normal (proof later)
- Here $1/\sqrt{2\pi}$ is there to make area under curve 1

+ve area

-ve area

→ cancels.

Mean and Variance of Standard Normal Distribution...

Mean and Variance of Standard Normal Distribution...

Mean and Variance of Standard Normal Distribution

Let Z be a normal distribution, i.e., $Z \sim N(0, 1)$, then $E[Z] = 0$ and $\text{Var}(Z) = 1$.

Recall

If $g(u) : \mathbb{R} \to \mathbb{R}$. If $g(u)$ is an odd function, i.e., $g(-u) = -g(u)$, and

$$\left| \int_0^{\infty} g(u)\, du \right| < \infty,$$

then

$$\int_{-\infty}^{\infty} g(u)\, du = 0.$$

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Answer to previous problem...

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CDF of Standard Normal Distribution...

CDF of Standard Normal Distribution... $N(0,1)$

Definition of CDF of Standard Normal Distribution

The CDF of the standard normal distribution is denoted by Φ

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The CDF of the standard normal distribution is denoted by Φ

cumulative

Can we integrate this to have closed form

$$\Phi(x) = P(Z \leq x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}}\, du \;=$$

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- The integral does not have a closed form solution!

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- The integral does not have a closed form solution!
- However, values of $\cancel{P(Z)}$ have been tabulated

$\phi(x)$

CDF of Standard Normal Distribution...

GATE:

NET-JRF:

UPSC:

IAS

Definition of CDF of Standard Normal Distribution

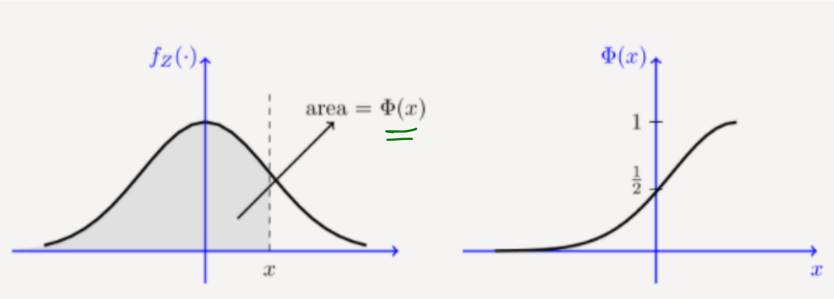
The CDF of the standard normal distribution is denoted by Φ

$$\Phi(x) = P(Z \leq x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{u^2}{2}}\, du$$

- The integral does not have a closed form solution!
- However, values of $F(Z)$ have been tabulated
- The CDF of any normal distribution can be written in terms of Φ function

CDF of Standard Normal Distribution...

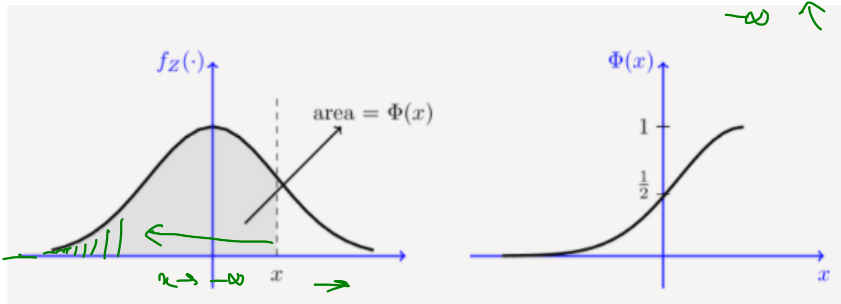
CDF of Standard Normal Distribution...



The Φ function satisfies the following properties:

CDF of Standard Normal Distribution...

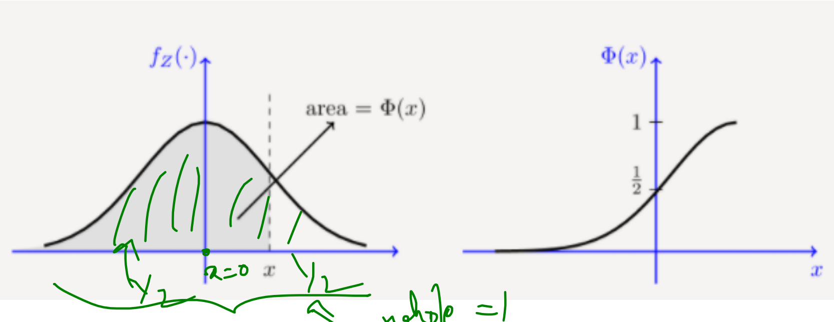
$\int_{-\infty}^{x} f(x)\,dx$ PDF



The Φ function satisfies the following properties:

- $\lim_{x\to\infty} \Phi(x) = 1$, $\quad \lim_{x\to-\infty} \Phi(x) = 0$

CDF of Standard Normal Distribution...

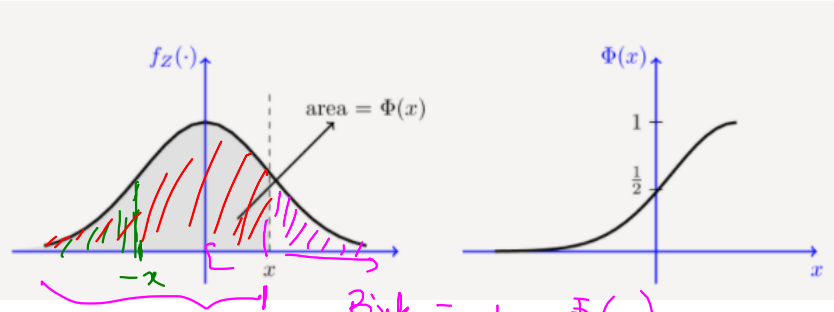


whole $= 1$

The Φ function satisfies the following properties:

- $\lim_{x\to\infty} = 1, \quad \lim_{x\to-\infty} = 0$
- $\Phi(0) = \frac{1}{2}$

CDF of Standard Normal Distribution...



Pink $= 1 - \Phi(x)$

Pink $=$ green (by symmetry)

The Φ function satisfies the following properties:

- $\lim_{x\to\infty} = 1, \quad \lim_{x\to-\infty} = 0$
- $\Phi(0) = \dfrac{1}{2}$
- $\Phi(-x) = 1 - \Phi(x)$, for all $x \in \mathbb{R}$

$\Phi(-x) =$ green $=$ pink $= 1 - \Phi(x)$

Bound for Φ Function...

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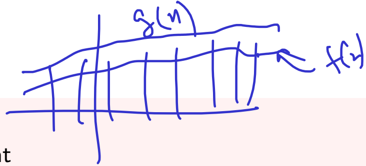

If $f(x) \le g(x)$

$$\int_a^b f(x) \le \int_a^b g(x)$$

Bound for Φ Function

Let $Z \sim N(0,1)$. We recall that

$$\Phi(x) = P(Z \le x).$$

For all $x \geq 0$, the Φ−function satisfies the following bound

$$\frac{1}{\sqrt{2\pi}}\frac{x}{x^2+1}e^{-x^2/2} \le 1-\Phi(x) \le \frac{1}{\sqrt{2\pi}}\frac{1}{x}e^{-x^2/2}$$

$$1-\Phi(x) = P(Z > x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \underbrace{e^{-u^2/2}}_{f(x)}\,du \le \frac{1}{\sqrt{2\pi}}\int_x^\infty \underbrace{\frac{u}{x}e^{-u^2/2}}_{g(x)}\,du$$

Note that $\frac{u}{x} \geq 1$ $[u \geq x]$

Can I write this?

Answer to previous problem...

$$\frac{1}{\sqrt{2\pi}} \int_x^\infty \frac{u}{x} e^{-u^2/2}\, du$$

$$= \frac{1}{x\sqrt{2\pi}} \int_x^\infty u e^{-u^2/2}\, du$$

By subst. $u^2 = t \Rightarrow 2u\,du = dt$

$u = x \Rightarrow t = x^2, \quad u = \infty \Rightarrow t = \infty$

$$= \frac{1}{2}\frac{1}{x\sqrt{2\pi}} \int_{x^2}^\infty e^{-t/2}\, dt$$

$$= \frac{1}{x\,\cancel{2}\sqrt{2\pi}}\,\cancel{-2}\left[e^{-t/2}\right]_{x^2}^\infty$$

$$= \frac{-1}{x\sqrt{2\pi}}\left[\cancel{e^{-\infty}}\;0 - e^{-x^2/2}\right]$$

$$= \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} \quad //$$

upper bound.

Ex Prove the lower bound as assignment.

Answer to previous problem...

Normal Random Variable...

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Definition of Normal Random Variables

- Have seen the standard normal RV, can obtain any normal RV by

Normal Random Variable...

$X \sim N(\mu, \sigma)$

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$E[Z] = 0$,

$$E[X] = \sigma E[Z] + \mu = \mu,$$
$$\mathrm{Var}(X) = \sigma^2 \mathrm{Var}(Z) = \sigma^2$$

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- In this case, we write $X \sim N(\mu, \sigma^2)$
- Conversely, if $X \sim N(\mu, \sigma^2)$, then $Z = \dfrac{X - \mu}{\sigma}$ is standard RV, i.e., $Z \sim N(0, 1)$

CDF and PDF of Normal Random Variable...

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$\boxed{\Phi(x)}$

CDF and PDf of Normal Random Variable

- To find the CDF of $X \sim N(\mu, \sigma^2)$, we have the following

Φ

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$$f_X(x) = \frac{d}{dx} F_X(x)$$

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ϕ is CDF of std. normal

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Summary: PDF, CDF, Computing Probabilities for Normal RV...

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$$P(a < X \le b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

$$F(b) - F(a) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$$

Solved Example...

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Let $X \sim N(-5, 4)$

Solved Example...

not std.

Solved Example

Let $X \sim N(-5, 4)$

- Find $P(X < 0)$

Solved Example...

Solved Example

Let $X \sim N(-5, 4)$

- Find $P(X < 0)$
- Find $P(-7 < X < -3)$

Solved Example...

$P(X < a) = P(X \leq a)$ In continuous!

(see table

Solved Example

Let $X \sim N(-5, 4)$

- Find $P(X < 0)$ $= P(X \leq 0) = F_X(0) = \phi\left(\frac{0-(-5)}{2}\right) = \Phi(2.5)$
- Find $P(-7 < X < -3)$ $= F_X(-3) - F_X(-7) = \phi\left(\frac{-3+5}{2}\right) - \phi\left(\frac{-7+5}{2}\right)$
- Find $P(X > -3 \mid X > -5)$

$$\frac{P(X > -3)}{P(X > -5)} =$$

[Cont. case]

$$= F_X(b) - F_X(a)$$

$$\phi(-x) = 1 - \phi(x)$$

$$= \phi(1) - \phi(-1)$$

$$= \phi(1) - [1 - \phi(1)] = 2\phi(1) - 1$$

Answer to previous problem...

Answer to previous problem...

Linear Transformation of a Normal RV is a Normal RV...

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Theorem

If $X \sim N(\mu_X, \sigma_X^2)$, and $Y = aX + b$, where $a, b \in \mathbb{R}$, then $Y \sim N(\mu_Y, \sigma_Y^2)$ where

$$\mu_Y = a\mu_X + b, \quad \sigma_Y^2 = a^2\sigma_X^2.$$

$$Y = a(\sigma_X Z + \mu_X) + b$$

$$= \underbrace{(a\sigma_X)}_{\sigma_Y} Z + \underbrace{(a\mu_X + b)}_{\mu_Y} \longrightarrow N(\mu_Y, \sigma_Y^2)$$

Gamma Distribution...

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Gamma Function: Extension of Factorial Function

The Gamma function denoted by $\Gamma(x)$ is an extension of the factorial function to real numbers.