Assignment 1

Automata Theory Monsoon 2021, IIIT Hyderabad

October 13, 2021

Total Marks: 30 | Due date: October 23, 2021

General Instructions: All symbols have the usual meanings (example: $\mathbb{R}$ is the set of reals, $\mathbb{N}$ the set of natural numbers, and so on.) FSM stands for finite state machine. DFA stands for deterministic finite automata. NFA stands for non-deterministic finite automata. a^* is the Kleene Star operation. Big Endian Form of binary representation: The significance of the digit increases from left to right, for example, 001 in big endian binary form represents 4.

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1. [2 points] If a FSM is used as a memory, how much memory do we have, in terms of the number of states and transition function?

2. [2 points] Let $C_n = \{\langle x \rangle \mid \langle x \rangle$ is the binary encoding of an integer multiple of n$\}$. Show that $\forall n \in \mathbb{N}$, the language C_n is regular.

3. [4 points] Let

$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

   Define the kleene star operation over Σ_2^* such that concatenation of $u, v \in \Sigma_2 = \begin{bmatrix} u_1 v_1 \\ u_2 v_2 \end{bmatrix}$. Each row of $w \in \Sigma_2^*$ represents a binary number in big endian form.

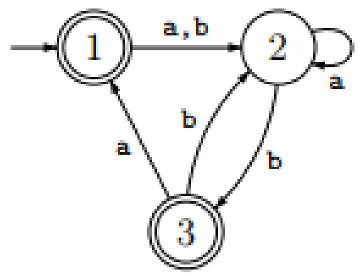
   We represent the numbers by $w(r_1)$ and $w(r_2)$, corresponding to the first and second rows of w respectively. Prove that the language $A = \{w \in \Sigma_2^* \mid w(r_1) = 2 \times w(r_2)\}$ is regular.

4. [5 points] Prove that the language A is not a regular language

$$A = \{\text{bits}(p) \mid p \text{ is a prime number } p \in \text{N}\}$$

   where bits(p) is the big endian binary representation of the number p.

5. [2 points] Find the regular expression for the language defined by the above automaton.



6. [5 points] Suppose that,

$$S = \{x \in \mathbb{N} \mid x \text{ has ones in all odd-numbered positions of its binary representation}\}$$

Call all numbers $\in S$ be known as cute numbers.

1. Draw the state diagram of the NFA for all numbers which are sum of three cute numbers.
2. Using the NFA constructed, prove that 333 and 420 are sums of three cute numbers.

7. [2 points] Prove that every NFA can be converted to an equivalent NFA that has a single accept/final state.

8. [2 points] Using the pumping lemma, show that the following languages are not regular:

1. $L = \{w \in \{\{,\}\}^* \mid w \text{ has balanced parentheses}\}$
2. $L = \{a^{n!} \mid n \in \{0, 1, 2 \ldots\}\}$ such that $a^x = aaa \ldots a$ repeated x times.

9. [2 points] Provide an algorithm for converting a right linear grammar to a left linear grammar.

10. [4 points] Consider the regular expression $R = (aa)^* + b^* + a^*b^*$.

1. Draw an NFA of the above regular expression with not more than 6 states.
2. Draw the equivalent DFA.
3. Find R' which recognizes the complement of the language recognized by R.