EC4.404: Mechatronics System Design

Harikumar Kandath

Robotic Research Center,
IIIT Hyderabad

harikumar.k@iiit.ac.in

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General Information

Mechatronics: Study of the integration of mechanical hardware, electrical/electronic hardware with computer hardware and software. Named by Tetsuro Mori from Japan when working with Yaskawa Electric Coorporation.
Applications: Robotics, Aerospace industry, automotive industry, process industry etc.

Course Objective: To introduce the design and development of a mechatronic system.
Instructors: Harikumar Kandath and Nagamanikandan Govindan.

Course Contents

UNIT 1 ♢ Sensors - structure of measurement systems, static characteristics, dynamic characteristics. ♢ Sensors in robotics - position, speed, acceleration, orientation, range. ♢ Actuators - general characteristics, motors, control valves.

UNIT 2 ♢ Computer based feedback control: Sampled data control, sampling and hold, PID control implementation, stability, bilinear transformation.

Instructor: Harikumar Kandath

Course Contents

UNIT 3 $\Diamond$: Introduction to mechanical elements and transformations, basic concepts of kinematics and dynamics.

UNIT 4 $\Diamond$ Design and analysis of mechanisms.

UNIT 5 $\Diamond$ Programming and hardware experiments.

Instructor: Nagamanikandan Govindan

Structure of Measurement System

Measurement system: Contains four basic modules.

- Sensing element.
- Signal conditioning element.
- Signal processing element.
- Data presentation element.

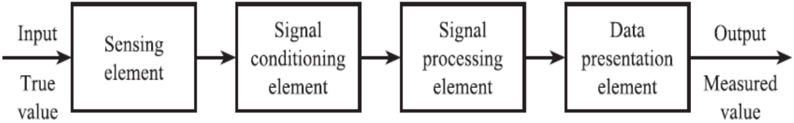


Figure: Structure of measurement system

Static Characteristics of Measurement System

- Range
- Span
- Non-linearity
- Sensitivity
- Environmental effects
- Hysteresis
- Resolution
- Error

Dynamic Characteristics of a Measurement System

When the input I changes suddenly, the output O will take some time to respond to it. For example, when a thermocouple is exposed to a sudden change in temperature from 25^{o}C (e.g. room temperature) to 100^{o}C (e.g. boiling water), the output will change from 1 mV to 4 mV after a while.

- Time response: time constant
- Frequency response: bandwidth

Time response of first and second order system

- First order system:

$$\frac{d\Delta O(t)}{dt} + k_1 \Delta O(t) = k_2 \Delta I(t) \tag{1}$$

Example: For thermocouple $\Delta I(t) = 100 - 25 = 75^o C$ and $\Delta O(t) = 4 - 1 = 3\, mV$.
Time constant (τ),

$$\tau = \frac{1}{k_1} \tag{2}$$

Step Response of first order system

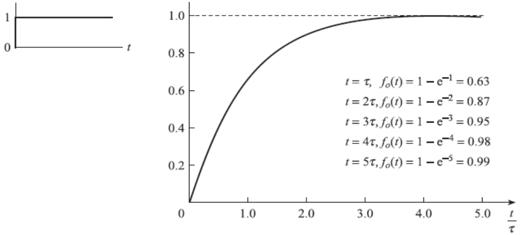


Figure: Unit step input and response

For input $\Delta I(t)$, $0 \implies 25^o C$ and $1 \implies 100^o C$ and for output $\Delta O(t)$, $0 \implies 1$ mV and $1 \implies 4$ mV.

$$f_o(t) = \Delta O(t) = \frac{k_2}{k_1}(1 - e^{-k_1 t}) = \frac{k_2}{k_1}(1 - e^{\frac{-t}{\tau}}) \tag{3}$$

Response to Sinusoidal Input

$$\Delta I(t) = A\, sin\omega t \tag{4}$$

$$\Delta O(t) = \frac{k_2}{k_1}\left(\frac{A\omega\tau}{1+\tau^2\omega^2} e^{\frac{-t}{\tau}} + \frac{A}{\sqrt{1+\tau^2\omega^2}} sin(\omega t + \phi_d)\right) \tag{5}$$

$$\phi_d = -tan^{-1}(\omega\tau) \tag{6}$$

Transfer Function Approach

Definition: Ratio of the Laplace transform of the output to the Laplace transform of the input with all the initial conditions set to zero.

$$\mathcal{L}(f(t)) = F(s) = \int_0^\infty f(t)e^{-st}dt \tag{7}$$

Transfer function $G(s)$

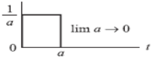
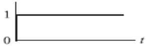
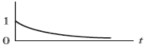
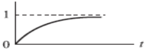
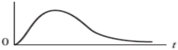
$$G(s) = \frac{\Delta O(s)}{\Delta I(s)} = \frac{k_2}{s+k_1} = \frac{k_2/k_1}{\tau s+1} \tag{8}$$

For $\Delta I(s) = \frac{1}{s}$,

$$\Delta O(s) = \frac{k_2}{k_1}\left(\frac{1}{s} - \frac{1}{s+1/\tau}\right) \tag{9}$$

$$\Delta O(t) = \mathcal{L}^{-1}\Delta O(s) = \frac{k_2}{k_1}(1 - e^{\frac{-t}{\tau}}) \tag{10}$$

Table of Laplace Transform

Function	Symbol	Graph	Transform
1st derivative	$\frac{d}{dt}f(t)$		$s\bar{f}(s) - f(0-)$
2nd derivative	$\frac{d^2}{dt^2}f(t)$		$s^2\bar{f}(s) - sf(0-) - \dot{f}(0-)$
Unit impulse	$\delta(t)$		1
Unit step	$\mu(t)$		$\frac{1}{s}$
Exponential decay	$\exp(-\alpha t)$		$\frac{1}{s+\alpha}$
Exponential growth	$1 - \exp(-\alpha t)$		$\frac{\alpha}{s(s+\alpha)}$
Sine wave	$\sin \omega t$		$\frac{\omega}{s^2+\omega^2}$
Phase-shifted sine wave	$\sin(\omega t + \phi)$		$\frac{\omega \cos \phi + s \sin \phi}{s^2+\omega^2}$
Exponentially damped sine wave	$\exp(-\alpha t)\sin \omega t$		$\frac{\omega}{(s+\alpha)^2+\omega^2}$
Ramp with exponential decay	$t\exp(-\alpha t)$		$\frac{1}{(s+\alpha)^2}$

Frequency Response

Definition: Response to a sinusoidal input at the steady state (i.e. $t \to \infty$).

$$\Delta O(ss) = \frac{k_2}{k_1}(\frac{A}{\sqrt{1+\tau^2\omega^2}} sin(\omega t + \phi_d)) \tag{11}$$

Amplitude, $|\Delta O(ss)| = \frac{k_2}{k_1}(\frac{A}{\sqrt{1+\tau^2\omega^2}})$ (input amplitude scaled by $\frac{k_2/k_1}{\sqrt{1+\tau^2\omega^2}}$), Phase lag $\phi_d = -tan^{-1}(\omega\tau)$.
Using transfer function approach,

$$|\Delta O(ss)| = |G(s = j\omega)|A = \frac{k_2/k_1}{\sqrt{1+\tau^2\omega^2}} A \tag{12}$$

$$\phi_d = \phi(G(s = j\omega)) = \phi(\frac{k_2/k_1}{j\omega\tau + 1}) = -tan^{-1}(\omega\tau) \tag{13}$$

Frequency Response

When $|\tau\omega| << 1$,

$$|\Delta O(ss)| = \frac{k_2}{k_1}A, \;\; \phi_d = 0 \tag{14}$$

When $|\tau\omega| >> 1$,

$$|\Delta O(ss)| = \frac{k_2}{k_1}\frac{A}{\tau\omega}, \;\; \phi_d = -tan^{-1}(\omega\tau) \tag{15}$$

When $|\tau\omega| = 1$,

$$|\Delta O(ss)| = \frac{k_2}{k_1}\frac{A}{\sqrt{2}}, \;\; \phi_d = -tan^{-1}(1) = -\frac{\pi}{4} \tag{16}$$

Bandwidth : $\omega = \frac{1}{\tau}$ (30% or 3 dB reduction in amplitude).

Time response of second order system

- Second order system:

$$\frac{d^2\Delta O(t)}{dt^2} + k_0\frac{d\Delta O(t)}{dt} + k_1\Delta O(t) = k_2\Delta I(t) \quad (17)$$

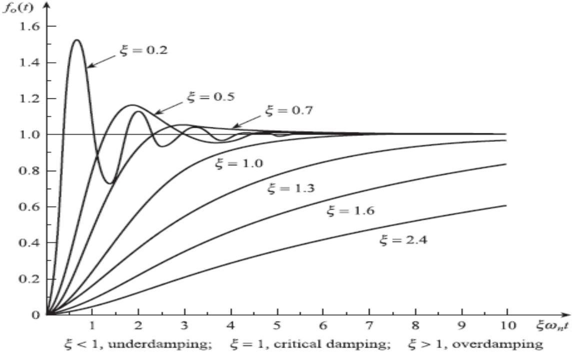
$$G(s) = \frac{\Delta O(s)}{\Delta I(s)} = \frac{k_2}{s^2 + k_0 s + k_1} = k\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (18)$$

$\omega_n = \sqrt{k_1}$, $\zeta = \frac{k_0}{2\sqrt{k_1}}$, $k = \frac{k_2}{k_1}$.
ω_n= natural frequency (rad/s), ζ = damping ratio.
Example: $\Delta I(t)$ is force and $\Delta O(t)$ is deflection.

Step Response

$f_o(t) = \Delta O(t) = \mathcal{L}^{-1}(\Delta O(s)) = \mathcal{L}^{-1}(G(s)\Delta I(s))$. For input $\Delta I(s) = \frac{1/k}{s}$,

$$\Delta O(s) = \frac{k_2}{s^2 + k_0 s + k_1}\frac{1/k}{s} = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \tag{19}$$



Critical parameters to look for in step response

- Peak overshoot: $e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$.
- Settling time (response settles within 1% of the steady state value): $\frac{5}{\zeta\omega_n}$.
- Damped frequency: $\omega_d = \omega_n\sqrt{1-\zeta^2}$ (only for $\zeta \leq 1$).

Frequency Response

$\Delta I(t) = A\, sin\omega t$

$$|\Delta O(ss)| = |G(s = j\omega)|A = k\frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}}A \tag{20}$$

$$\phi_d = \phi(G(s = j\omega)) = \phi(\frac{k\omega_n^2}{(\omega_n^2 - \omega^2) + j(2\zeta\omega_n\omega)}) = -tan^{-1}(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}) \tag{21}$$

when $\omega = \omega_n$,

$$|\Delta O(ss)| = \frac{kA}{2\zeta},\ \phi_d = -\frac{\pi}{2} \tag{22}$$

when $\omega = \omega_n\sqrt{1 - 2\zeta^2}$ (resonant frequency),

$$|\Delta O(ss)| = \frac{kA}{2\zeta\sqrt{1 - \zeta^2}},\ \phi_d = -\frac{\sqrt{1 - \zeta^2}}{\zeta} \tag{23}$$

Analysis of Cascaded Systems

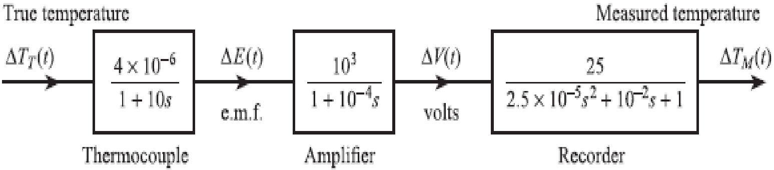


Figure: A Cascaded System

$$\frac{\Delta T_M(s)}{\Delta T_T(s)} = \frac{\Delta T_M(s)}{\Delta V(s)}\frac{\Delta V(s)}{\Delta E(s)}\frac{\Delta E(s)}{\Delta T_T(s)} \tag{24}$$

NB: Multiply the frequency response of individual systems to get the frequency response of the cascaded system.

$$\Delta T_M(t) = \mathcal{L}^{-1}(\Delta T_M(s)) = \mathcal{L}^{-1}([\frac{\Delta T_M(s)}{\Delta V(s)}\frac{\Delta V(s)}{\Delta E(s)}\frac{\Delta E(s)}{\Delta T_T(s)}]\Delta T_T(s)) \tag{25}$$

Response modification using compensators

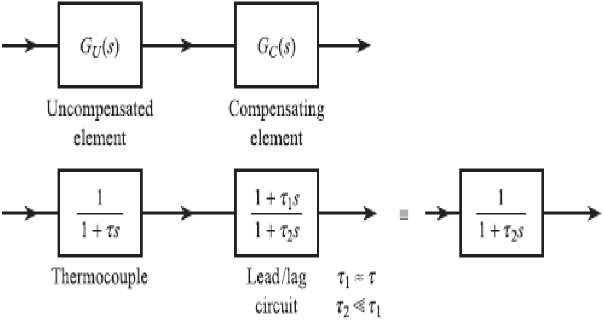


Figure: Cascading with a Compensator

THANK YOU