Stochastic processes
Assignment-4

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1. $-H(p) = \frac{1}{2}\lg\frac{1}{2} + \frac{1}{4}\lg\frac{1}{4} + \frac{1}{8}\lg\frac{1}{8} + \frac{1}{16}\lg\frac{1}{16}$

$H(P) = 1.875$

$-H(q) = \frac{1}{2}\lg\frac{1}{2} + \left(\frac{1}{8}\lg\frac{1}{8}\right) \times 4$

$= 2$

$D(P\|q) = \sum_{x} p(x)\lg\left(p(x)/q(x)\right)$

$= \frac{1}{2}\lg\frac{1/2}{1/2} + \frac{1}{4}\lg\frac{1/4}{1/8} + \frac{1}{8}\lg\frac{1/8}{1/8} + \frac{1}{16}\lg\frac{1/16}{1/8} + \frac{1}{16}\lg\frac{1/16}{1/8}$

$= 0.125$

$D(q\|p) = \frac{1}{2}\lg\frac{1/2}{1/2} + \frac{1}{8}\lg\frac{1/8}{1/4} + \frac{1}{8}\lg\frac{1/8}{1/8} + \frac{1}{8}\lg\frac{1/8}{1/16} + \frac{1}{8}\lg\frac{1/8}{1/16}$

$= 0.125$

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2.
a) $-H(P) = \frac{1}{2}\lg \frac{1}{2} + \frac{1}{4}\lg\frac{1}{4} + \frac{1}{4}\lg\frac{1}{4}$

$H(P) = 1.5$

$H(q) = -\frac{1}{3}\lg\frac{1}{3} \times 3$

$= 1.585$

$D(p\|q) = \frac{1}{2}\lg\frac{1/2}{1/3} + \frac{1}{4}\lg\frac{1/4}{1/3} + \frac{1}{4}\lg\frac{1/4}{1/3}$

$= 0.085$

$D(q\|p) = \frac{1}{3}\left[\lg\frac{1/3}{1/2} + \lg\frac{1/3}{1/4} + \lg\frac{1/3}{1/4}\right]$

$= 0.0817$

$\Rightarrow D(p\|q) \neq D(q\|p)$

a) Finding K-L distance $D(P\|q)$ between the two ~~normal~~ distributions.

$$D(P\|q) = \int_{-\infty}^{\infty} P(x)\log\frac{P(x)}{q(x)}\,dx$$

$$= \underbrace{\int_{-\infty}^{\infty} P(x)\log(P(x))\,dx}_{①} - \underbrace{\int_{-\infty}^{\infty} P(x)\log(q(x))\,dx}_{②}$$

$$① = \int_{-\infty}^{\infty} P(x)\left[\frac{-1}{2}\log(2\pi\sigma^2) - \frac{(x-\mu_1)^2}{2\sigma^2}\right]dx$$

$$③ \leftarrow \quad = \frac{-1}{2}\log(\cancel{3}2\pi\sigma^2) - \frac{1}{2\sigma^2}\int_{-\infty}^{\infty} P(x)(x^2+\mu_1^2-2x\mu_1)\,dx$$

$$= -\frac{\log(2\pi\sigma^2)}{2} - \frac{1}{2\sigma^2}\left[E[x^2] + \mu_1^2 - 2\mu_1 E[x]\right]$$

$$= -\frac{\log(2\pi\sigma^2)}{2} - \frac{1}{2\sigma^2}\left[2\mu_1^2 - 2\mu_1^2 + \sigma^2\right]$$

$$= \frac{-1}{2} - \frac{\log(2\pi\sigma^2)}{2}$$

$$② = \frac{-1}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\int_{-\infty}^{\infty} p(x)(x^2+\mu_2^2-2x\mu_2)\,dx$$

(from ③)

$$= \frac{-1}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\left[\overset{E[x^2]}{\cancel{\mu_1^2}} + \mu_2^2 - 2\mu_1\mu_2\right]$$

$$= \frac{-1}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\left[\mu_1^2+\mu_2^2-2\mu_1\mu_2+\sigma^2\right]$$

$$\Rightarrow D(p\|q) = \cancel{\frac{-1}{2}\log(2\pi\sigma^2)} - \frac{1}{2}$$

$$+ \cancel{\frac{1}{2}\log(2\pi\sigma^2)} + \frac{(\mu_1-\mu_2)^2+\sigma^2}{2\sigma^2}$$

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$$= \cancel{\frac{-1}{2}} + \frac{(\mu_1 - \mu_2)^2}{2\sigma^2} + \cancel{\frac{\sigma^2}{2\sigma^2}}$$

$$= \frac{\mu_1 - \mu_2}{2\sigma^2}$$

3\. a) $$f_{XY}(x, y) = \frac{\partial^2 F_{XY}(x, y)}{\partial x \, \partial y}$$

$$= \frac{\partial}{\partial x}\left[+2e^{-2y} - 2e^{-(x+2y)}\right]$$

$$= 2e^{-(x+2y)}$$

b) $$P(X < 2Y) = \int_y \int_x f_{XY}(x, y)\, dx\, dy$$

$$= \int_0^\infty \int_0^{2y} 2e^{-(x+2y)}\, dx\, dy$$

$$= 2\int_0^\infty \left[e^{-(x+2y)}\right]_0^{2y} dy$$

$$= 2\int_0^\infty \left(e^{-2y} - e^{-4y}\right) dy$$

$$= 2\left[\frac{e^{-2y}}{2}\right]_0^\infty - 2\left[\frac{e^{-4y}}{4}\right]_0^\infty$$

$$= 1 - \frac{1}{2} = 0.5$$

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c) $f_X(x) = \int_Y f_{XY}(x,y)\, dx$

$= \int_0^{\infty} 2e^{-(x+2y)} dx$

$= \frac{2}{2}\left[e^{-(x+2y)}\right]_0^{\infty} = e^{-x}$

$f_Y(y) = \int_Y f_{XY}(x,y) dx$

$= 2\int_0^{\infty} e^{-(x+2y)} dx = 2\left[e^{-(x+2y)}\right]_0^{\infty}$

$= 2e^{-2y}$

$f_X(x) \cdot f_Y(y) = e^{-x} \cdot 2e^{-2y}$

$= 2e^{-(x+2y)} = f_{XY}(x,y)$

$\therefore$ they are independent.