Mechatronics System Design

EC4.404 - S2023

Nagamanikandan Govindan

Robotics Research Center, IIIT Hyderabad.
nagamanikandan.g@iiit.ac.in

linear momentum

$P = m \cdot v$

Newton's Law:

$$\boldsymbol{F} = m\boldsymbol{a}$$

Principle of Linear Momentum: The rate of change of the linear momentum of a particle is equal to the applied force

$P = m\dot{r}$

$$\boldsymbol{F} = \frac{d\mathbf{p}}{dt} = m\ddot{\boldsymbol{r}} + \cancel{\dot{m}}\dot{\boldsymbol{r}}$$ happening w.r.to Inertial frame.

0

$$\boldsymbol{F} = m\ddot{\boldsymbol{r}} = \dot{\boldsymbol{p}}$$

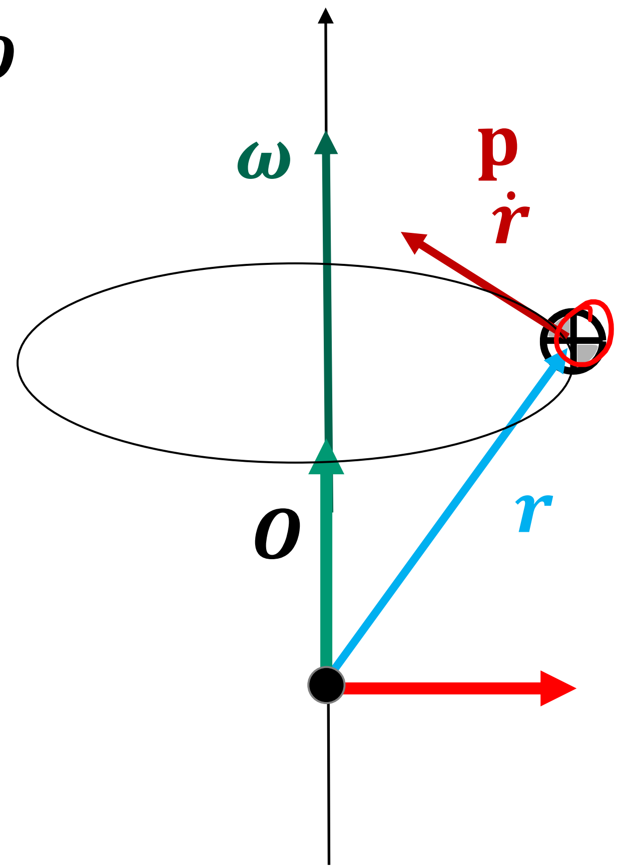
For the applied force $\boldsymbol{F}$ to a mass $\boldsymbol{m}$, the point mass exerts with an acceleration $\ddot{\boldsymbol{r}}$

Forces and Moments

- If the particle at $\boldsymbol{r}$ from $\boldsymbol{O}$ is rotating about $\boldsymbol{O}$

Then the angular momentum $\boldsymbol{H}$ about $\boldsymbol{O}$

$$\boldsymbol{H_o} = \boldsymbol{r} \times \mathbf{p} = \boldsymbol{r} \times m\dot{\boldsymbol{r}}$$



$\boldsymbol{r}$ = Position vector

m = Mass of the particle

$\boldsymbol{v} = \dot{\boldsymbol{r}}$ = Velocity of the particle

Principle of angular momentum: The rate of change of the angular momentum of particle about a fixed point $\boldsymbol{O}$ is equal to the resultant moment of forces acting on the particle about point $\boldsymbol{O}$

$H = r \times m\dot{r}$

Moment $\boldsymbol{M_o}$ about origin $\boldsymbol{O}$

$$\dot{\boldsymbol{H}}_{\boldsymbol{o}} = \dot{\boldsymbol{r}} \times m\dot{\boldsymbol{r}} + \boldsymbol{r} \times m\dot{\boldsymbol{v}} \rightarrow \ddot{r}$$

$$\dot{\boldsymbol{H}}_{\boldsymbol{o}} = 0 + \boldsymbol{r} \times \boldsymbol{F} = \boldsymbol{M}_{\boldsymbol{o}}$$

$\boldsymbol{M_o}$ = moment about $\boldsymbol{O}$

$\boldsymbol{r}$ = Position vector

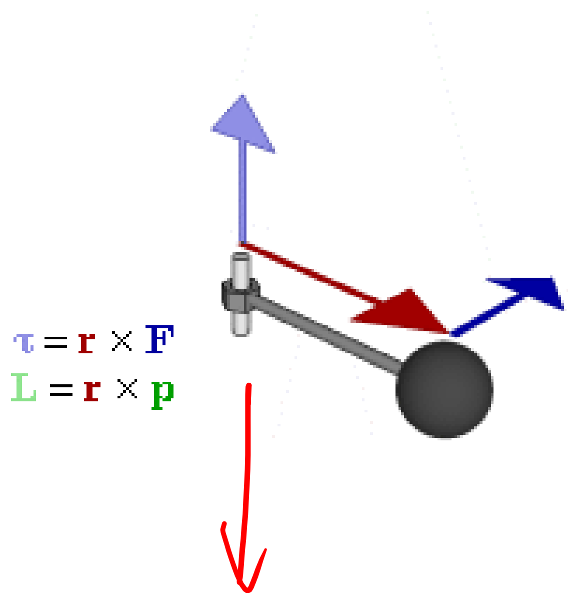
m = Mass of the particle

$\boldsymbol{v}$ = Velocity of the particle

Moment

- **Torque:** (a.k.a moment, moment of force, rotational force or turning effect)
  - Rotational equivalent of force

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$$



Power

Mechanical Power (P):

- is a measure of the rate at which **work** (W) is performed or energy is used over **time** (t).

$$P = \frac{W}{t}$$

$P = \frac{W}{t}$

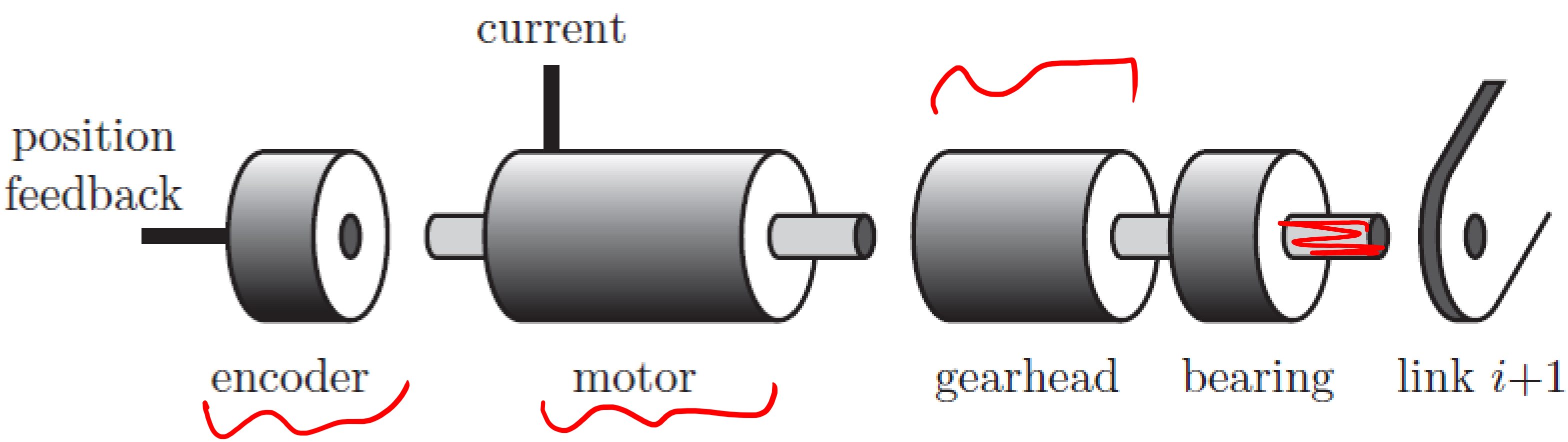
For linear motion

$$P = \frac{Fd}{t} = Fv$$

For rotary motion

$$P = \frac{\tau\theta}{t} = \tau\dot{\theta} = I\omega$$

DC motor



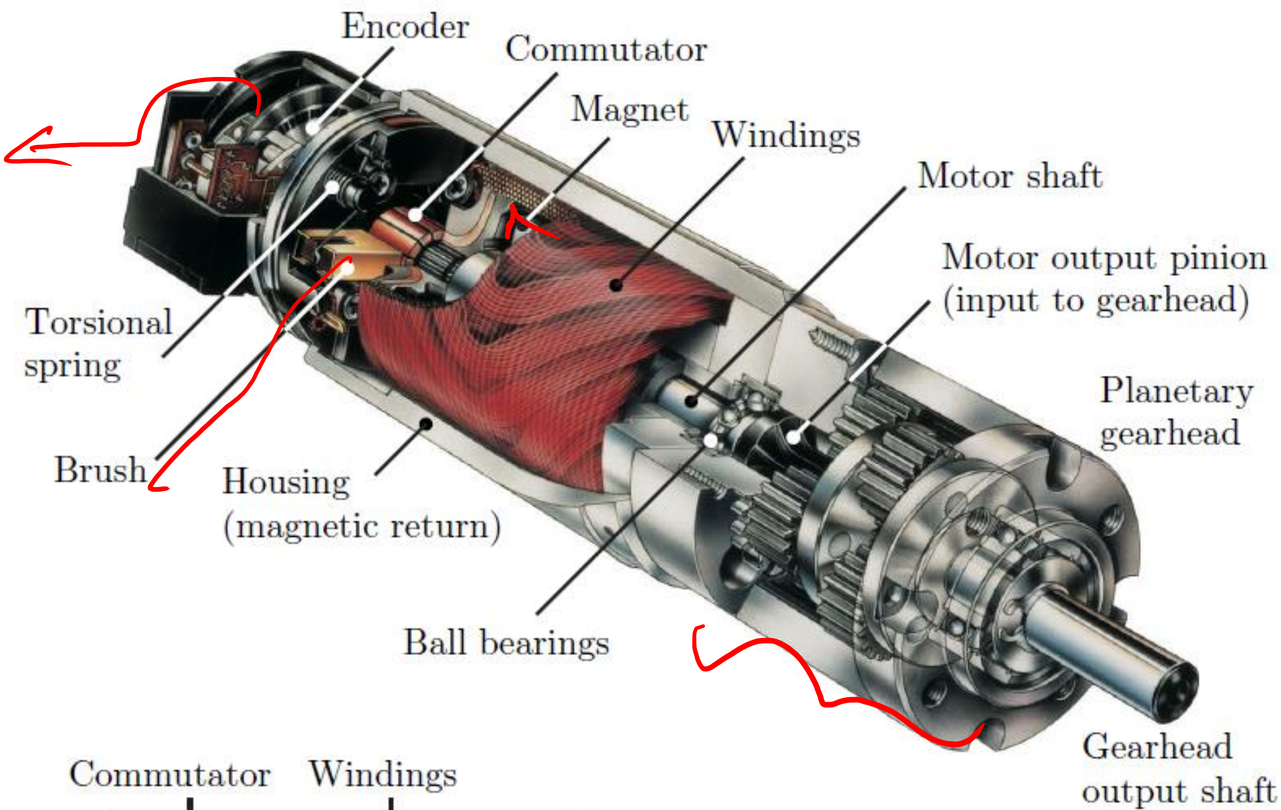
Conceptual representation of the motor and other components for a single axis

DC Motor modelling

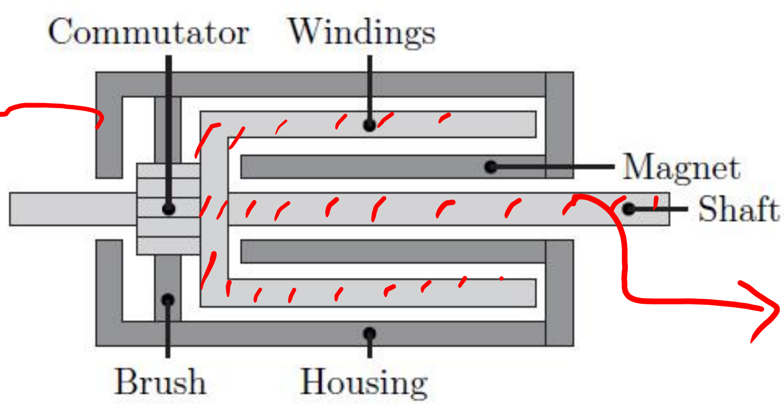
- A DC motor consists of a stator and a rotor that rotates relative to the stator.
  - Magnets are attached to the stator and
  - Windings are attached to the rotor, or vice versa.

- DC electric motors create torque by sending current through windings in a magnetic field created by permanent magnets

A cutaway view of a ~~Maxon~~ brushed DC motor with an encoder and gearhead



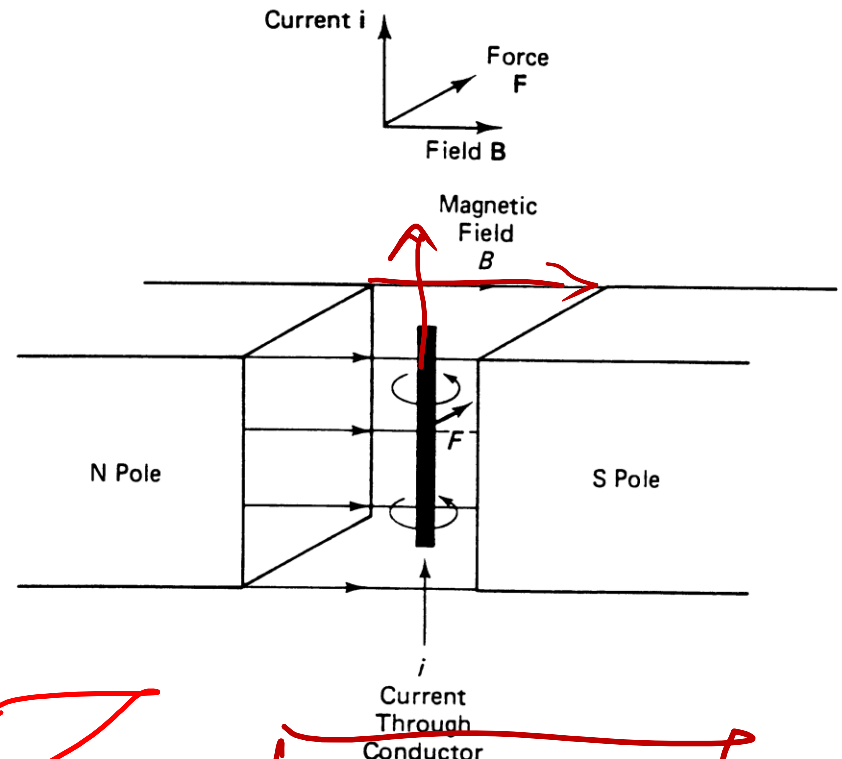
Stator



rotor

Lorentz Force Law

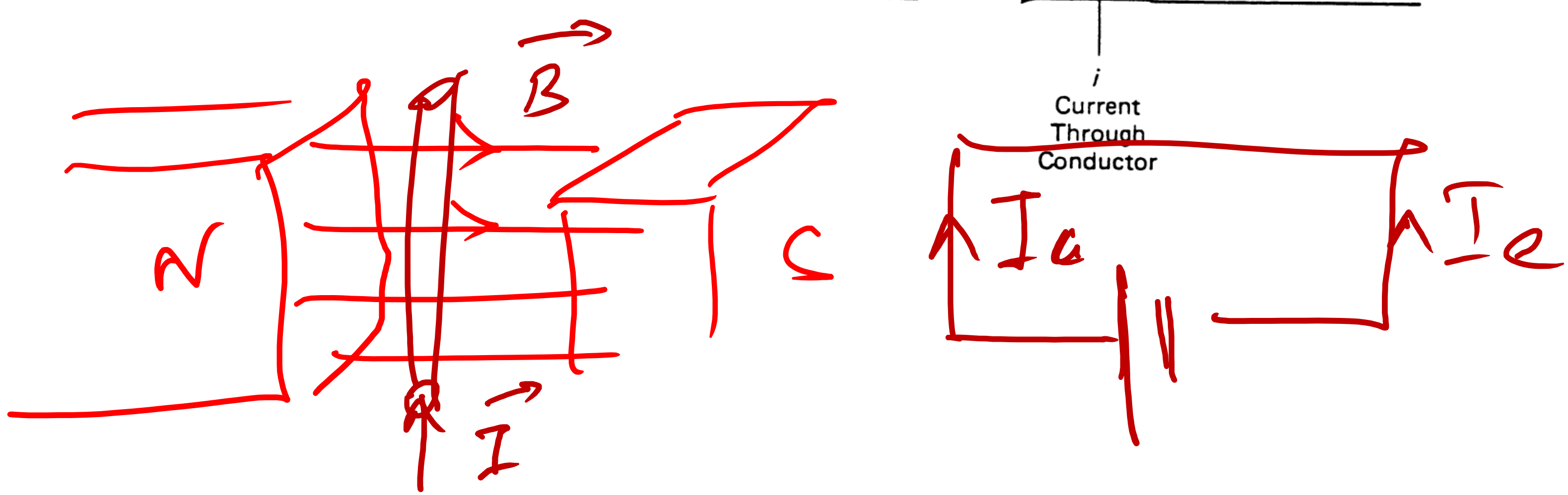
$$\boldsymbol{F} = I\boldsymbol{L} \times \boldsymbol{B}$$



B = magnetic flux density

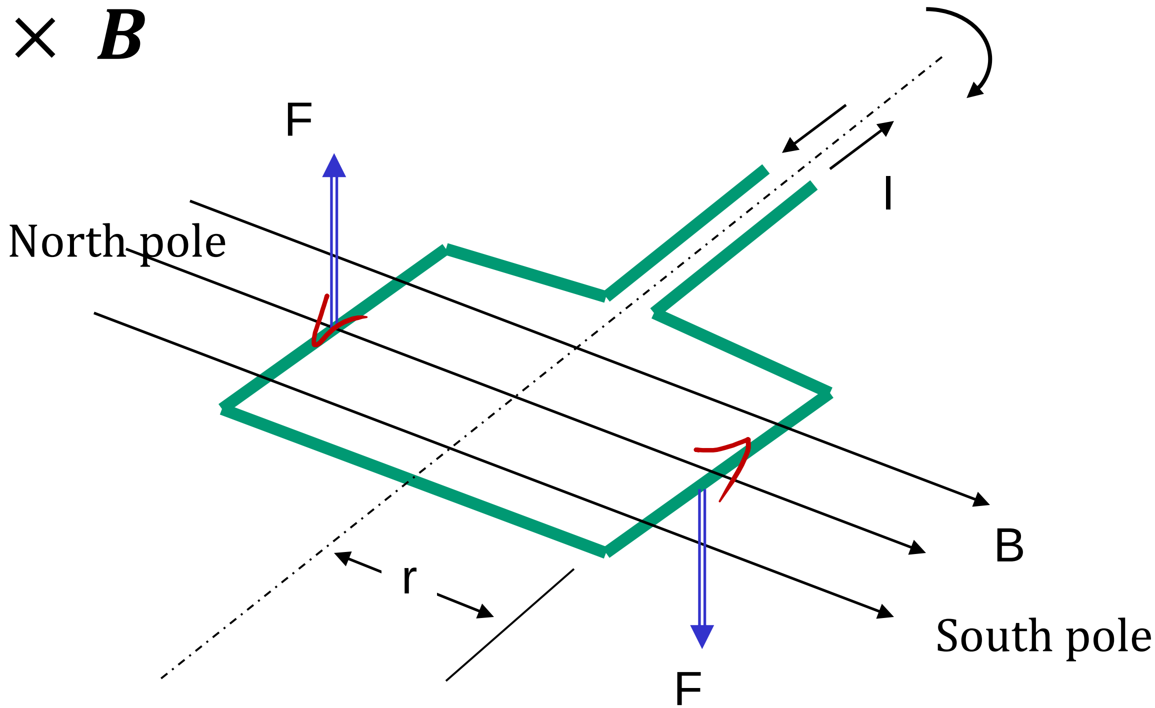
L = length of wire in the magnetic field

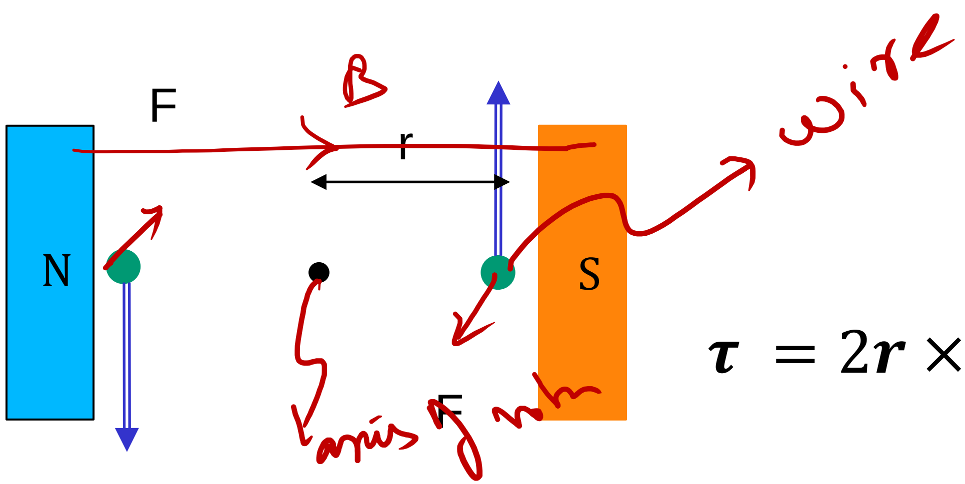
I = current



DC Motor

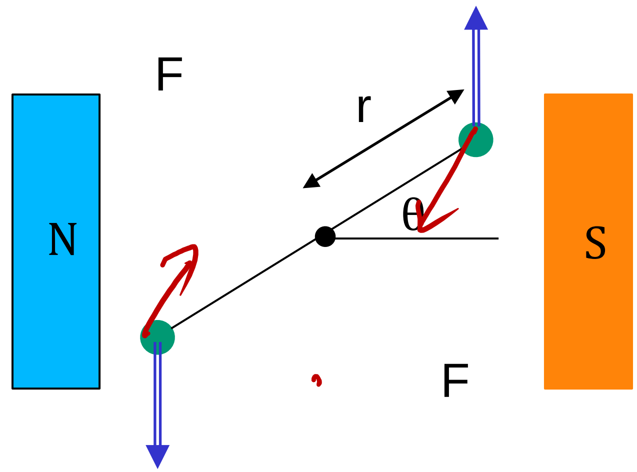
$$\boldsymbol{F} = L\,\boldsymbol{I} \times \boldsymbol{B}$$

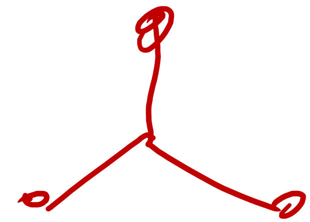




$$\boldsymbol{\tau} = 2\boldsymbol{r} \times \boldsymbol{F}$$

DC Motor

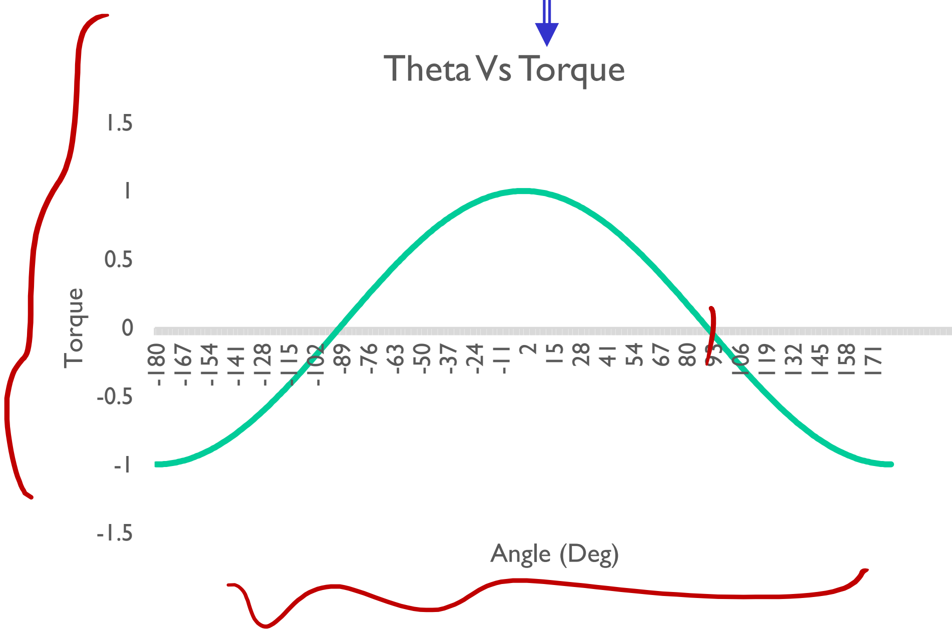


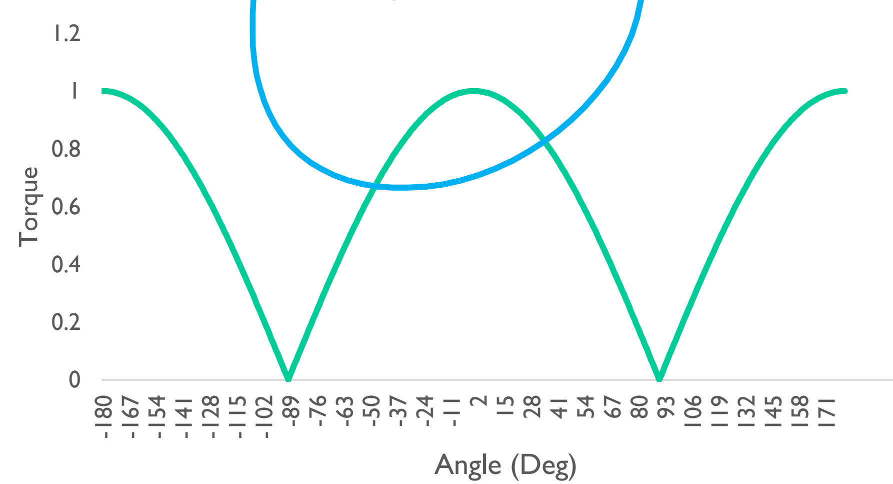


$$\tau = 2rF\,Cos\,\theta$$

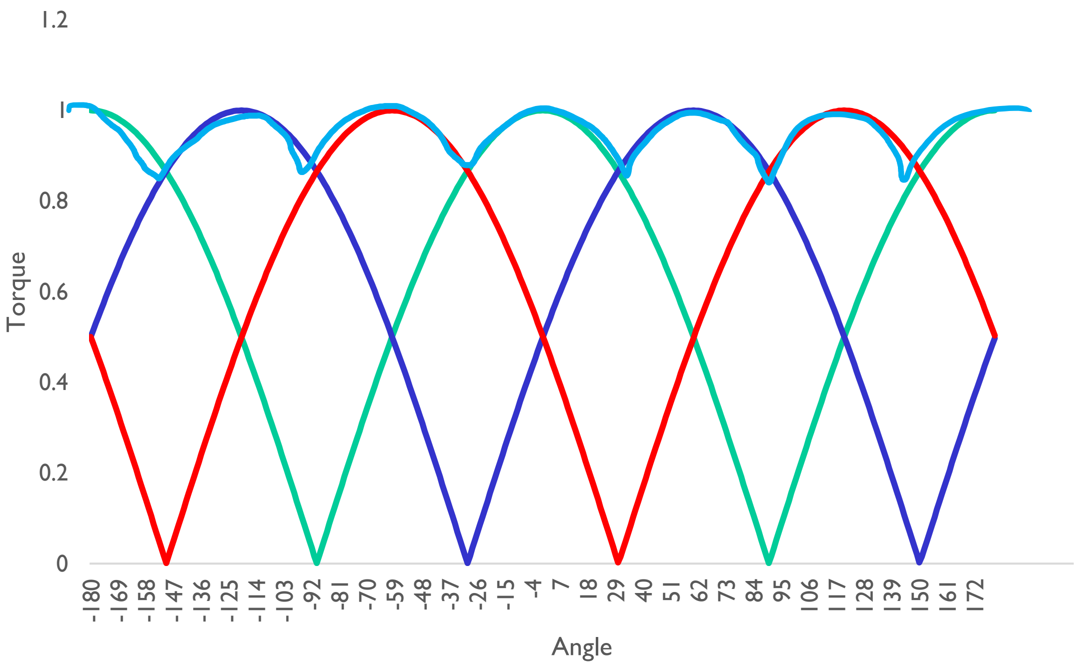
$I\,dl \times B$

pan cake
motor



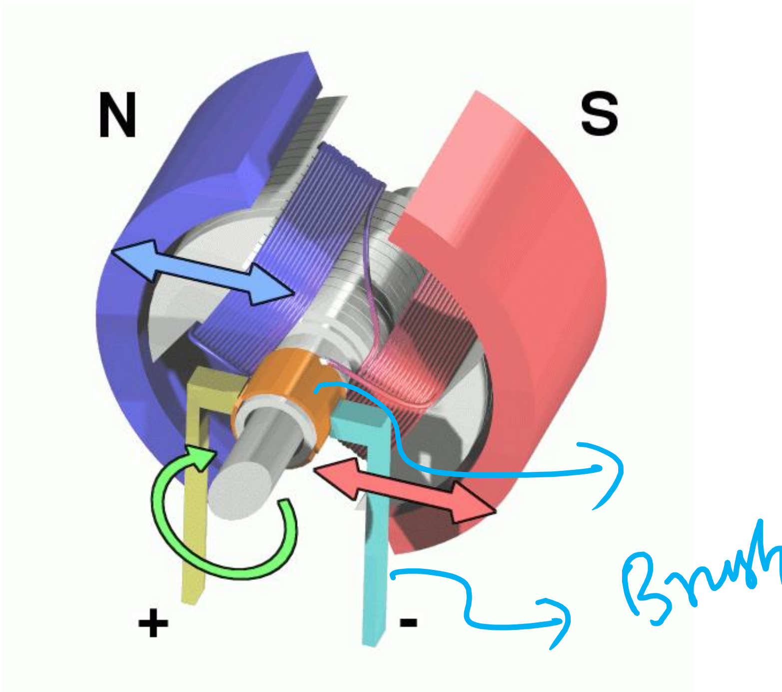


DC Motor


1.2
1
0.8
0.6
0.4
0.2
0
Torque
-180
-169
-158
-147
-136
-125
-114
-103
-92
-81
-70
-59
-48
-37
-26
-15
-4
7
18
29
40
51
62
73
84
95
106
117
128
139
150
161
172
Angle

If you want to get more torque out of motor:

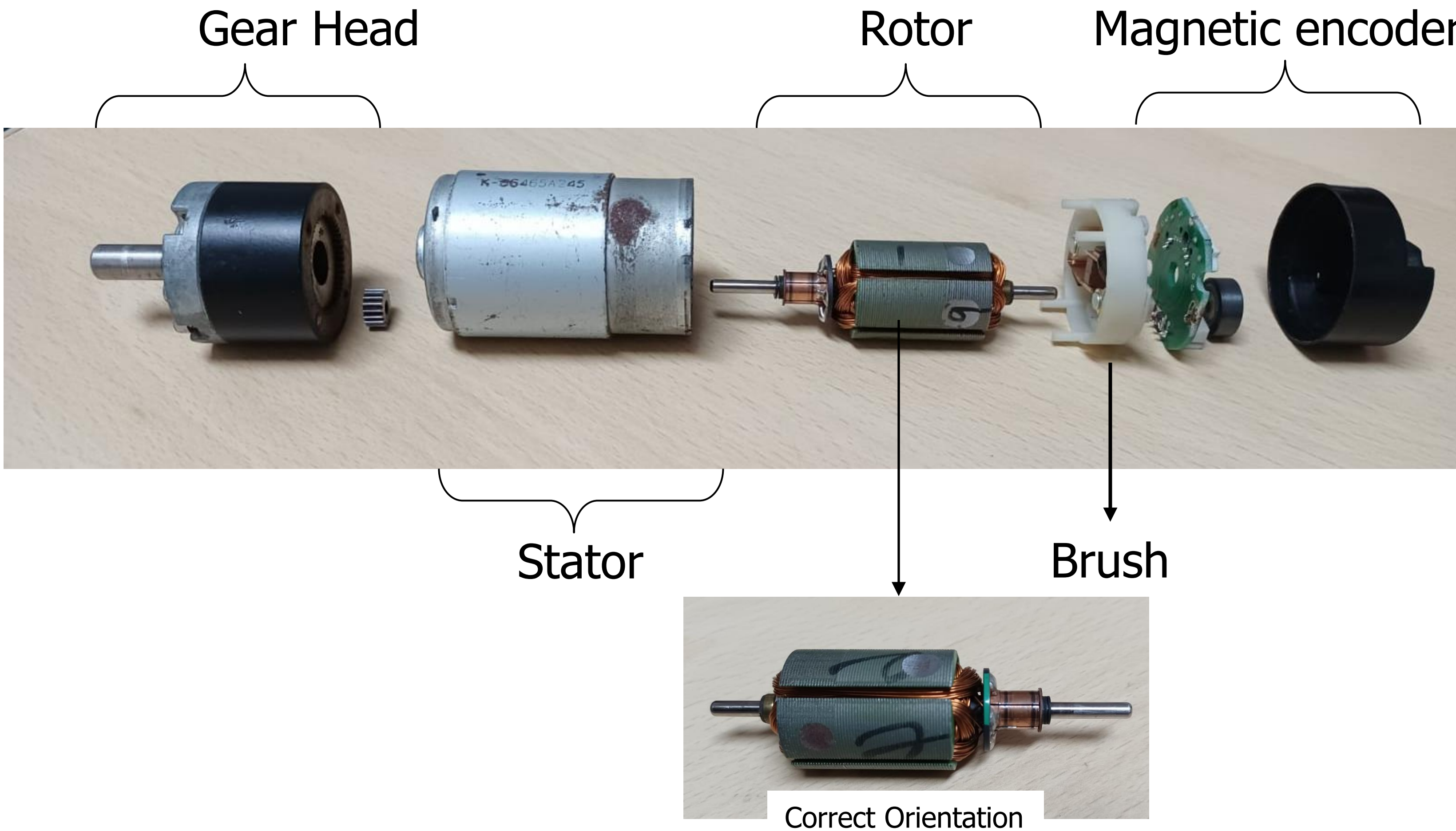
- Increase L – more coils, longer armature
- Stronger magnetic field (B) – use stronger magnets
- Increase current (I)
- Increase armature diameter, (r)



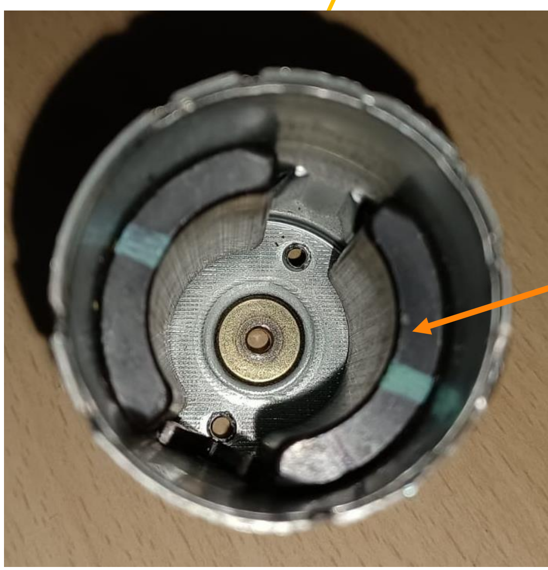
DC motor

100

DC Motor


Gear Head
Rotor
Magnetic encoder
Stator
Brush
Correct Orientation

DC Motor



Permanent Magnet

The Stator (stationary part)

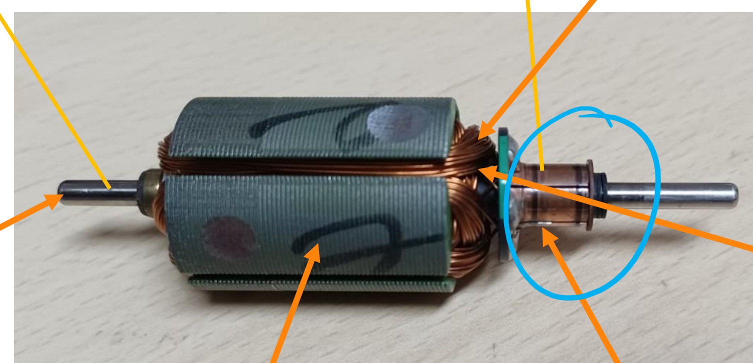
The frame

Supports the permanent magnets

DC Motor - Rotor

The Rotor (the spinning part)

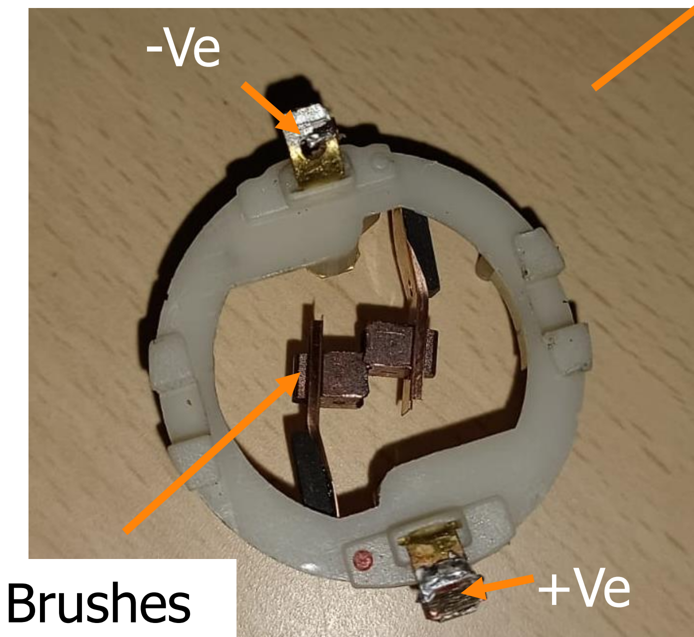
- Connected to an axle which is also the rotor shaft



The Commutator

- Switches the DC voltage polarity for continuous rotation (polarity has to switch every half rotation)

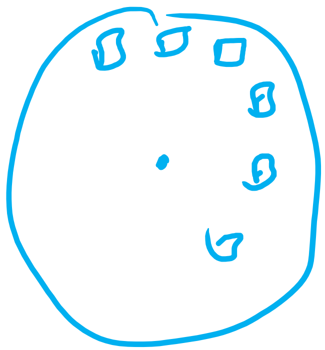
DC Motor



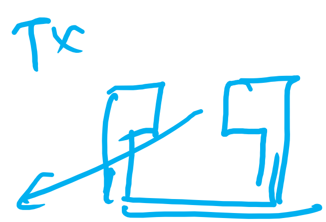
The Brushes

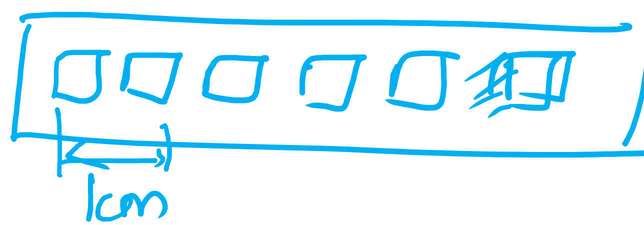
Gets electricity into the rotor

DC Motor

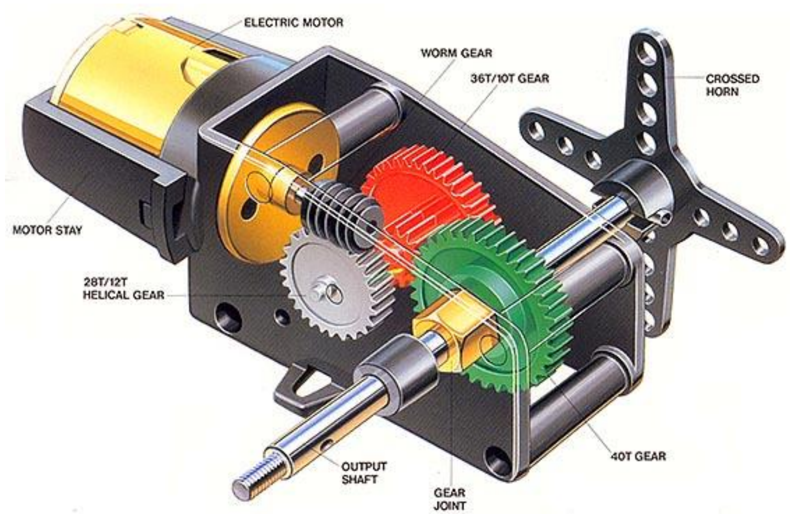


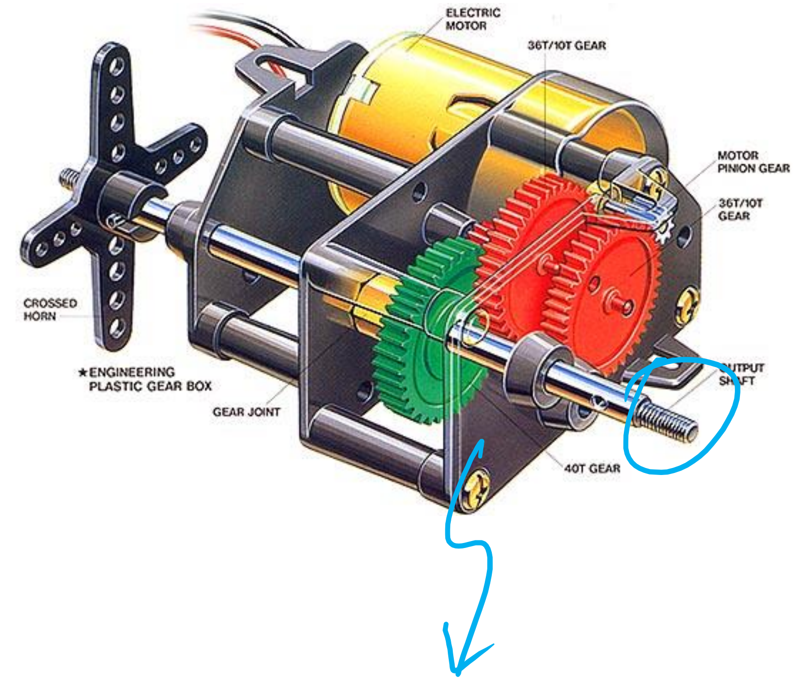
optical Encoder





Servo Motor


ELECTRIC MOTOR
WORM GEAR
36T/10T GEAR
CROSSED HORN
MOTOR STAY
28T/12T HELICAL GEAR
OUTPUT SHAFT
GEAR JOINT
40T GEAR


ELECTRIC MOTOR
36T/10T GEAR
MOTOR PINION GEAR
36T/10T GEAR
CROSSED HORN
★ENGINEERING PLASTIC GEAR BOX
GEAR JOINT
40T GEAR

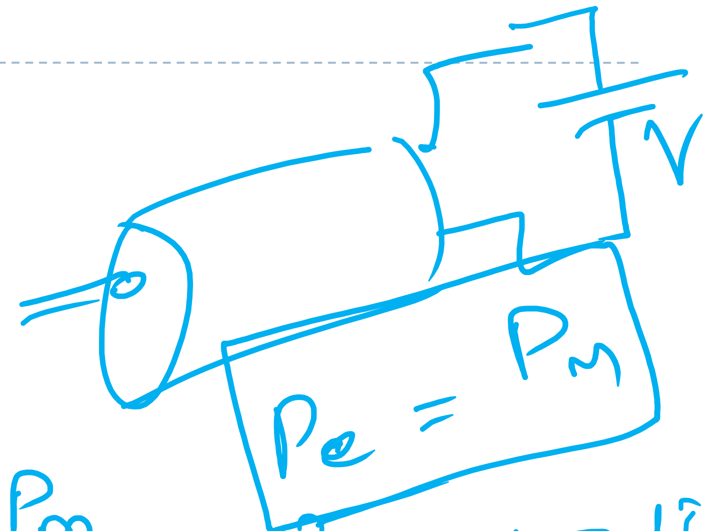
DC Motor modelling

The Torque created by the DC motor is

$\tau \propto I$

$\tau = k_t I$

$$\tau = k_t I$$



I = Current through the windings

k_t = Torque constant

$P_e = P_m$

$VI = \tau\omega + I^2R + LI\frac{dI}{dt}$

$E = \frac{1}{2}LI^2$

The power dissipated as heat by the windings is governed by

$$P_{heat} = I^2R$$

R is the resistance of the windings in ohms

To keep the motor windings from overheating, the continuous current owing through the motor must be limited.

DC Motor modelling

- A simplfied model of a DC motor can be derived by equating

$$P_{elec} = P_{mech} + P_{Others}$$

$$IV = \tau\omega + P_{Others}$$

$$IV = \underline{\tau\omega} + \underline{P_{heat}} + \underline{P_{inductor}} + friction\ and\ others$$

energy stored in an inductor is $\frac{1}{2}LI^2$, and power is the time derivative of energy

DC Motor modelling

$$IV = \overbrace{k_t I}^{\tau}\omega + I^2R + LI\frac{dI}{dt} + \cancel{friction\ and\ others}$$

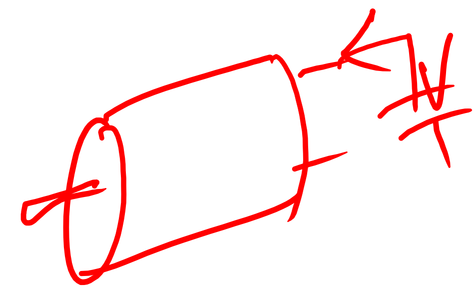
Divide by I on both sides and dropping the last term =>

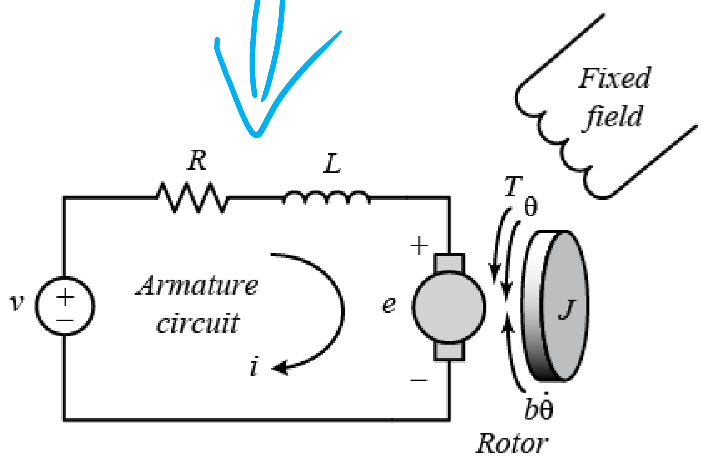
$\tau \propto I$

$\tau = k_t I$

$$\boxed{V = k_t\omega + IR + L\frac{dI}{dt}}$$

$V = K_t\omega$





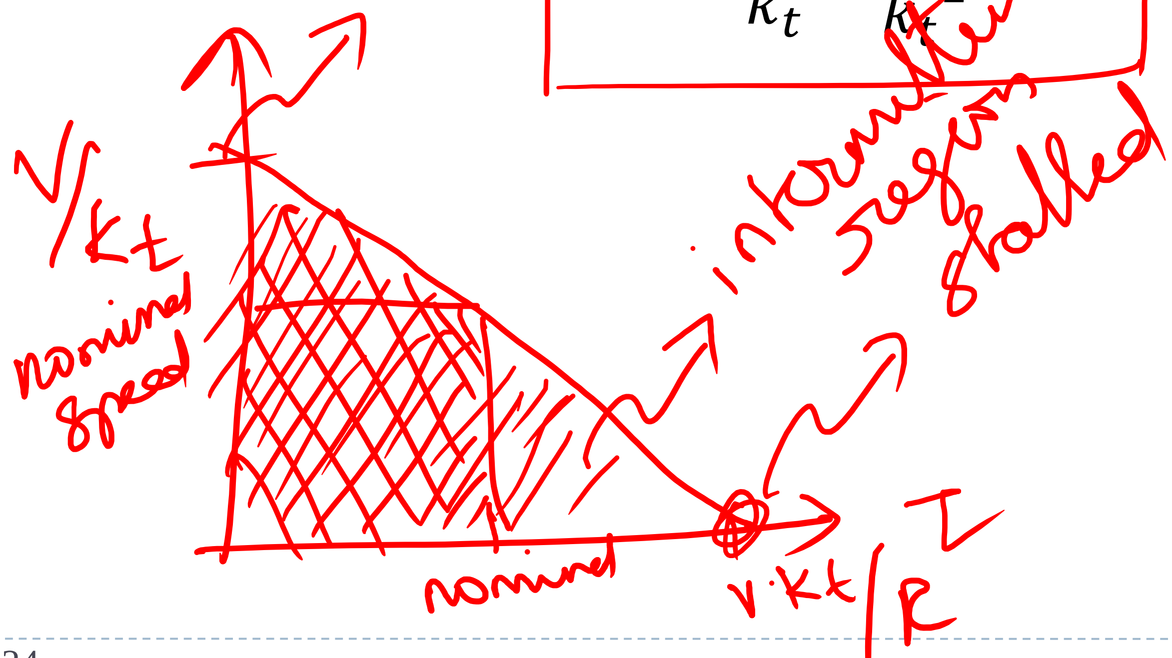
The electric equivalent circuit of the armature and the free-body diagram of the rotor

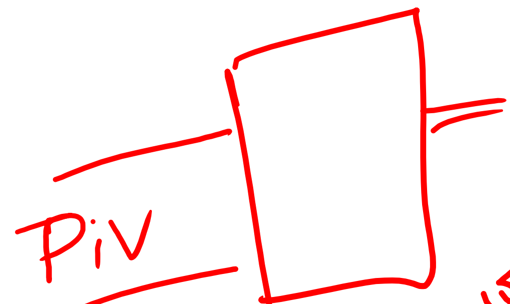
DC Motor modelling

- For simplicity, lets assume the motor is operating at a constant current => $\frac{dI}{dt} = 0$

$$\omega = \frac{1}{k_t}(V - IR)$$

$$\boxed{\omega = \frac{V}{k_t} - \frac{R}{k_t^{\,2}}\tau}$$

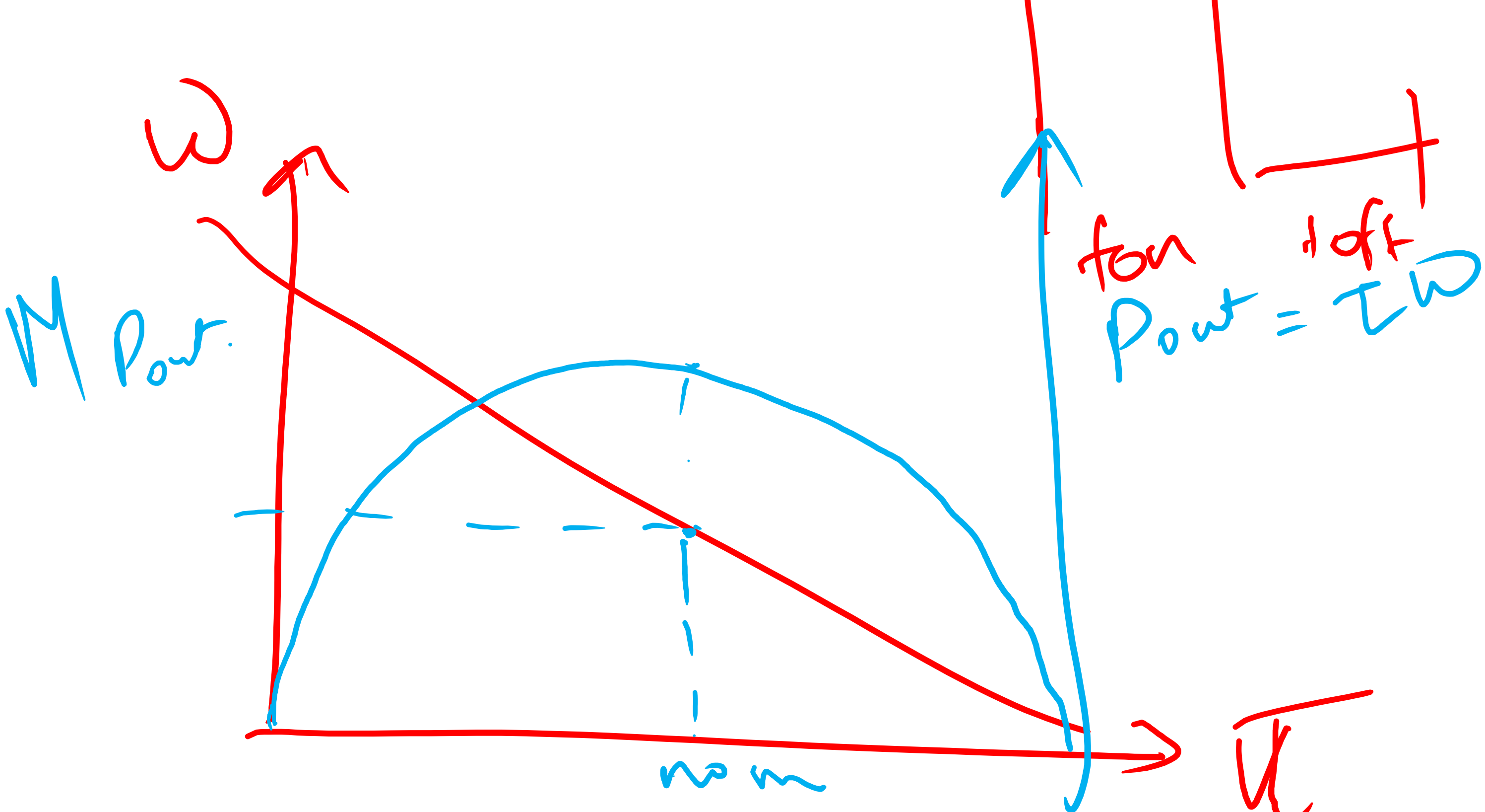




$$\varepsilon = \frac{P_m}{P_{in}} \times 100$$

DC Motor Speed

- A DC motor is a variable speed device.
- Speed is controlled by the amount of DC voltage applied:
  - As the voltage increases, the speed increases
  - As the voltage decreases, the speed decreases



Transmission

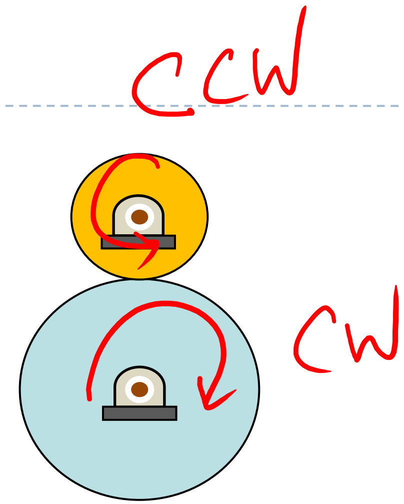
- Transfers/amplifies force/torque from motor

- Types:
  - Rolling cylinders
  - gears
  - belts/pulleys
  - capstan drive
  - none (direct drive)

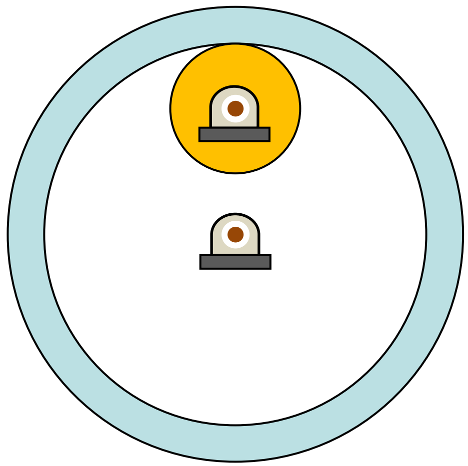
Rolling cylinders

- The simplest means of transferring rotary motion from one shaft to another is a pair of rolling cylinders.

- Provided that sufficient friction is available



External Set

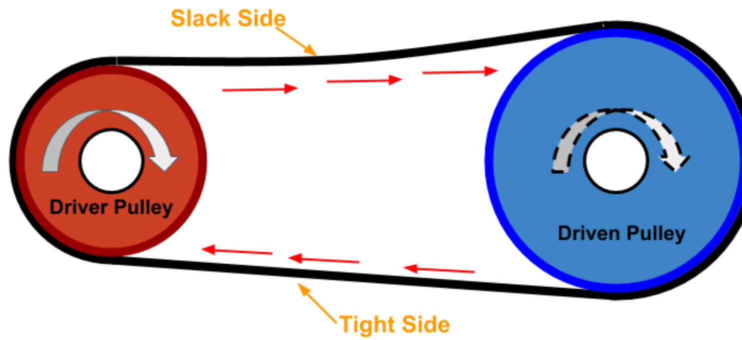


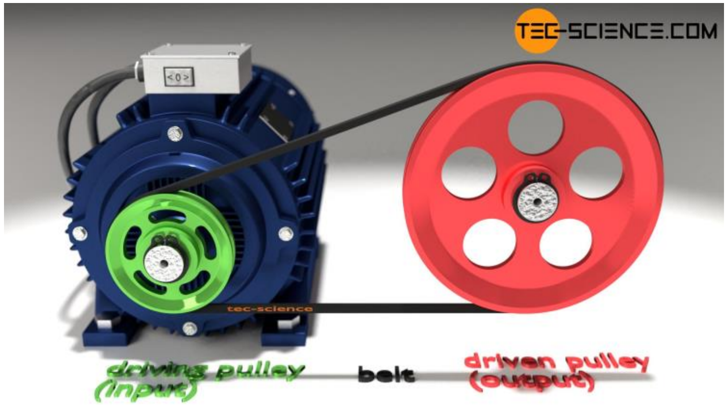
Internal Set

Belt drives

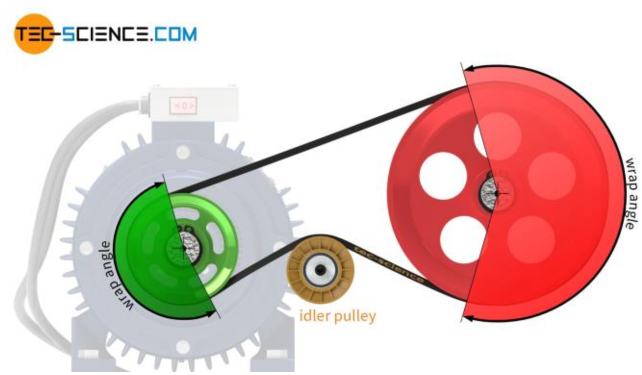
- A variant on the rolling cylinder drive is the flat or vee belt. This mechanism transfers power through friction and is capable of quite large power levels, provided enough belt cross section is provided.

- On the tight side, the belt moves towards the driving pulley; on the slack side, the belt moves towards the driven pulley!





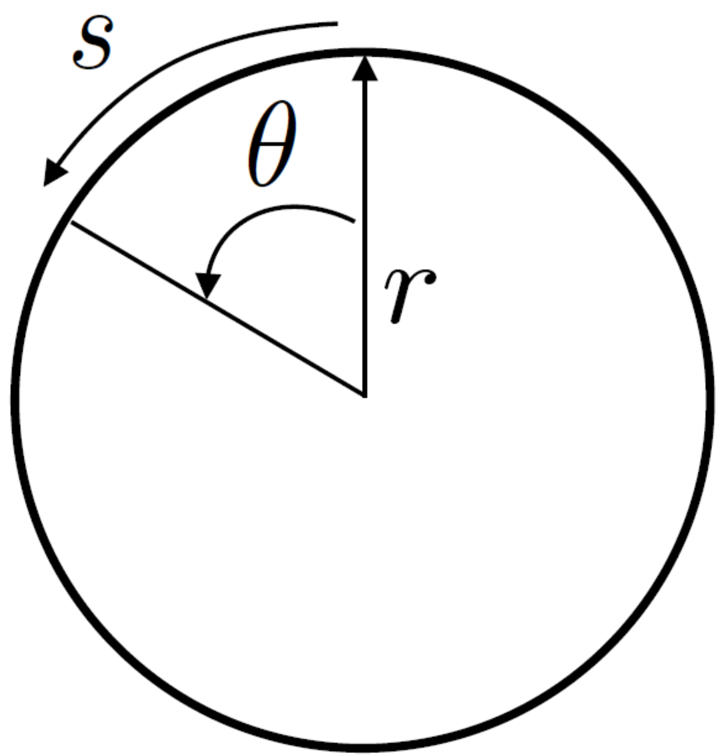
https://www.tec-science.com/



Kinematic relation

- A key kinematic relationship is

$$s = r\,\theta$$



Ref: https://web.stanford.edu/

Belt-on-pulleys

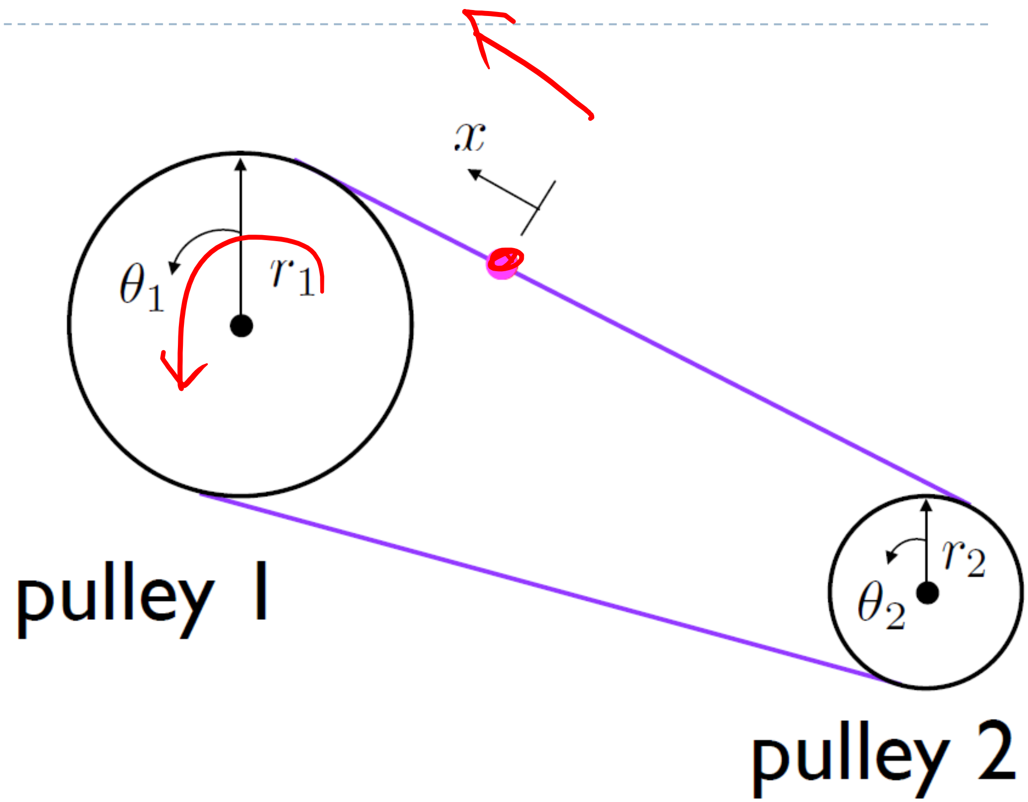
$$s_1 = r_1\theta_1$$

$$s_2 = r_2\theta_2$$

$$x = s_1 = s_2$$

Angular Displacement:

$$\theta_2 = \frac{r_1}{r_2}\theta_1$$



if $r_1 > r_2$, which pulley rotates more?

Ref: https://web.stanford.edu/

Belt-on-pulleys

Angular Displacement:

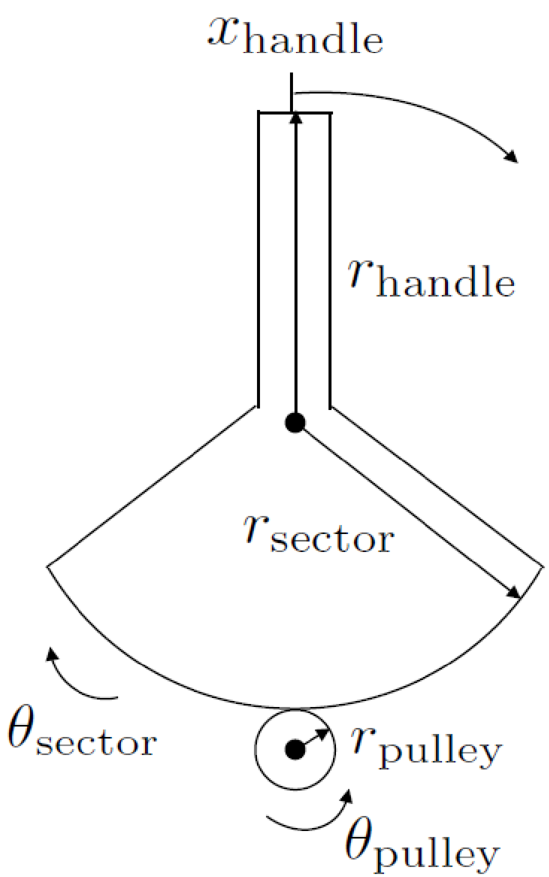
$$\theta_2 = \frac{r_1}{r_2}\theta_1$$

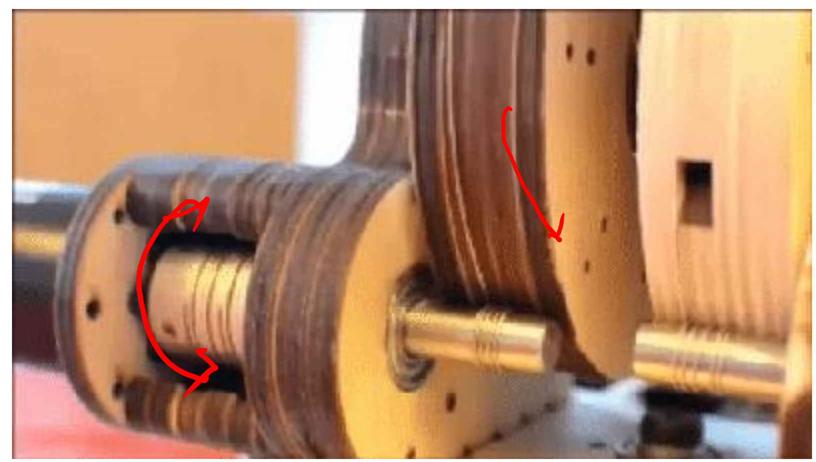
Angular Velocities:

$$r_1\dot{\theta}_1 = r_2\dot{\theta}_2 \rightsquigarrow \frac{d\theta}{dt}$$

Capstan Drive

- employ dual cables wrapped in a figure-eight shape around the input (sheave) and the output (pulley)





Ref: https://web.stanford.edu/

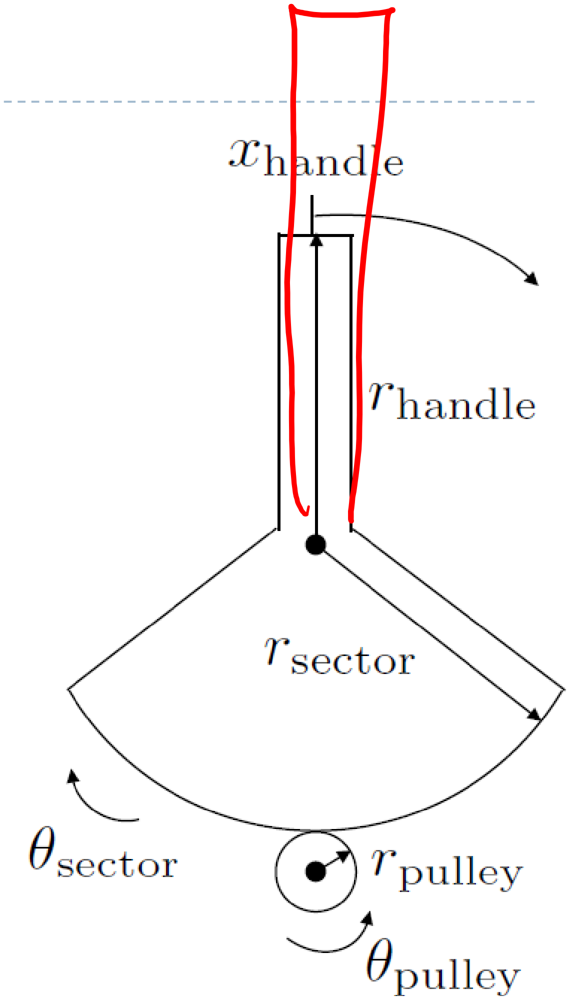
Capstan Drive

$$r_{pulley}\theta_{pulley} = r_{sector}\theta_{sector}$$

$$x_{handle} = r_{handle}\theta_{sector}$$

$$x_{handle} = \frac{r_{handle}r_{pulley}}{r_{sector}}\theta_{pulley}$$

The sensor measures θ_{pulley}.

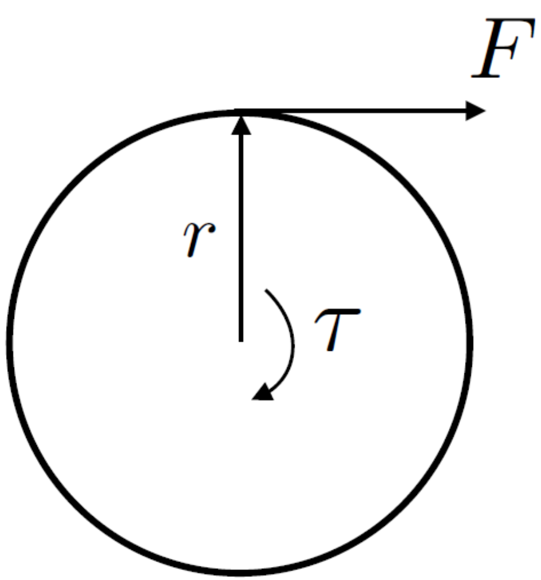


Ref: https://web.stanford.edu/

Force-Torque Relationships

- Torque, or moment, is the tendency of a force to rotate an object.
- If a force is perpendicular to r (the vector connecting the point about which the torque acts to the point at which the force is applied)

$$\tau = r \text{ x } F$$



Belt-on-pulleys

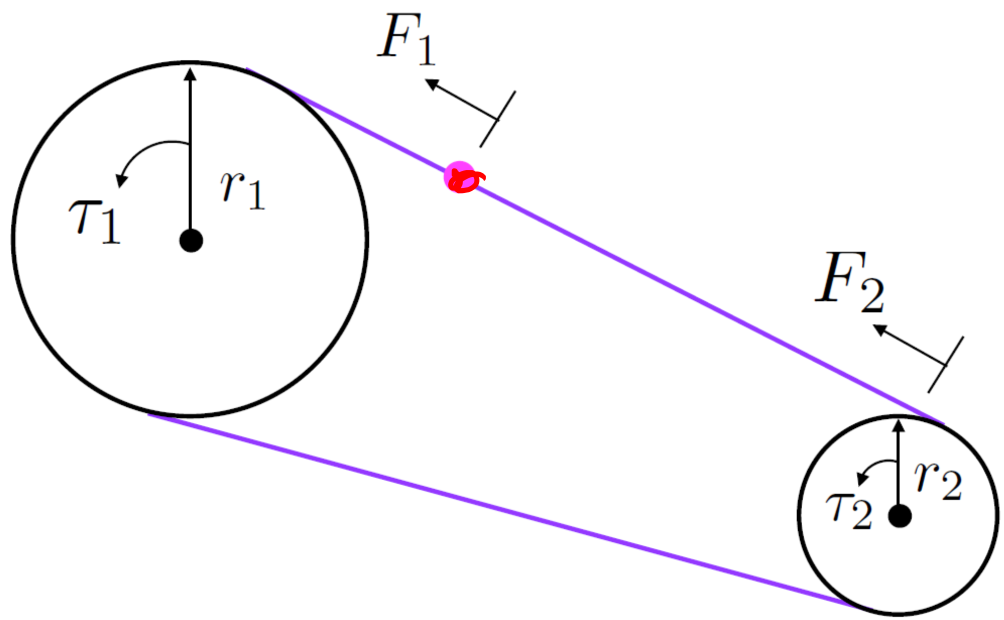
$\tau_1 = r_1 F_1$

$\tau_2 = r_2 F_2$

The belt tension will be
$F_1 = F_2$

Torque ratio:

$$\tau_2 = \frac{r_2}{r_1} \tau_1$$



Ref: https://web.stanford.edu/

Relation between the torque and speed.

Torque ratio:

$$\tau_2 = \frac{r_2}{r_1}\tau_1 \Rightarrow \frac{r_1}{r_2} = \frac{\tau_1}{\tau_2}$$

Angular Velocities:

$$\frac{r_1}{r_2} = \frac{\dot{\theta}_2}{\dot{\theta}_1}$$

Relation between the torque and speed.

$$\frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\tau_1}{\tau_2}$$

$\tau_2 \omega_2 = \tau_1 \omega$

Relation between the torque and speed.

- Conservation of power

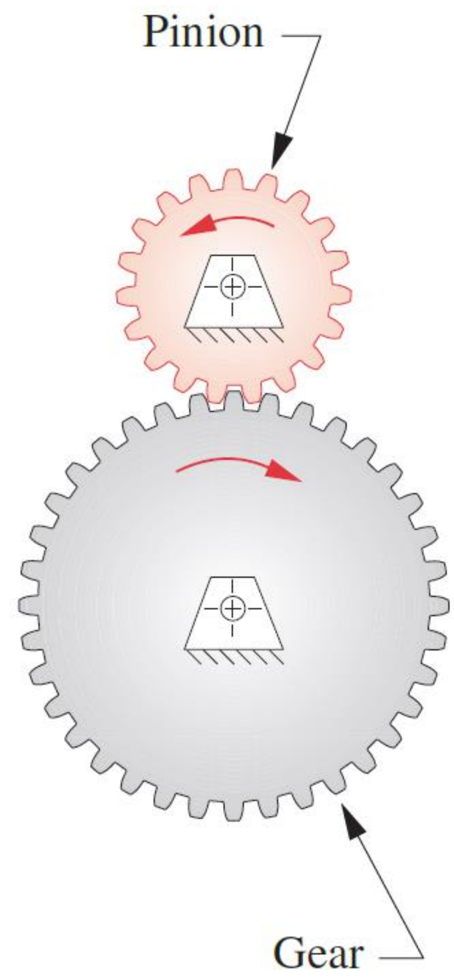
$$\tau_2 \dot{\theta}_2 = \tau_1 \dot{\theta}_1$$

$$Power_{out} = Power_{in}$$

Gears

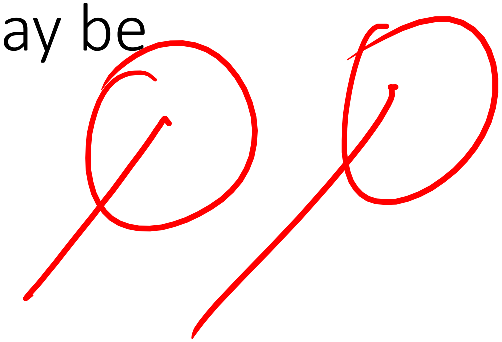
- One way of avoiding slip between the rolling cylinder is by adding some meshing teeth to the cylinders.

- It is conventional to refer to the smaller of the two gears as the pinion and to the other as the gear.



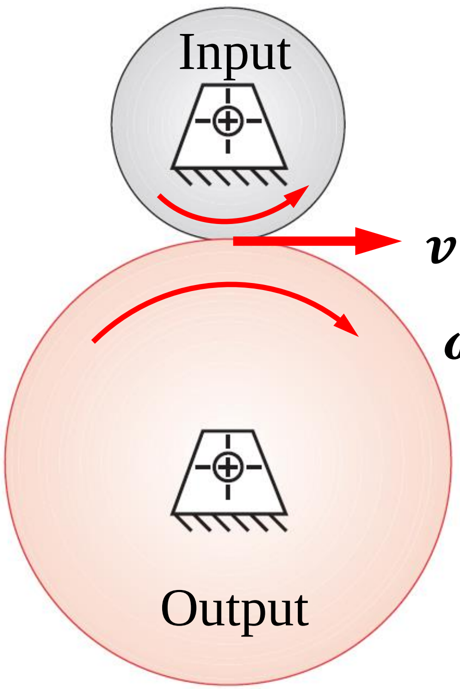
Gears

- Gears are machine elements that transmit motion by means of successively engaging teeth.
- Gears are generally used for one of four different reasons:
  - To reverse the direction of rotation
  - To increase or decrease the speed of rotation
  - To move rotational motion to a different axis
  - To keep the rotation of two axis synchronized
- Gears may be classified according to the relative position of the axes of revolution. The axes may be
  - parallel,
  - intersecting,
  - neither parallel nor intersecting.



Fundamental Law of Gearing

- The angular velocity ratio between 2 meshing gears remains constant throughout the mesh
- Angular velocity ratio (m_V)



$$\boldsymbol{v = \omega r}$$

$$\boldsymbol{\omega_{in} r_{in} = \omega_{out} r_{out}}$$

rolling cylinder drive

$$m_V = \frac{\omega_{out}}{\omega_{in}} = \frac{r_{in}}{r_{out}}$$

Drawbacks:

- low torque capability
- possibility of slip

Gears

N = no. of teeth

m = module $= \frac{D}{N}$

D = Pitch circle diameter

Module must be same for all the gears; otherwise, they won't mesh

At the contact point

$$v_{in} = v_{out}$$

$$\omega_{in} r_{in} = \omega_{out} r_{out}$$

$$\omega_{in} \frac{D_{in}}{2} = \omega_{out} \frac{D_{out}}{2}$$

$$\omega_{in} N_{in} = \omega_{out} N_{out}$$

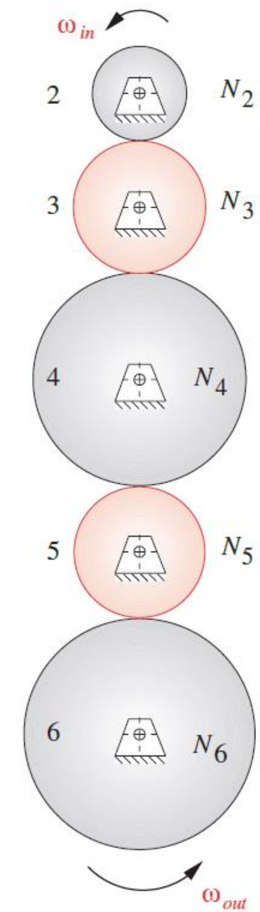
$$\omega_{out} = \frac{N_{in}}{N_{out}} \omega_{in}$$

Simple Gear Trains

- each shaft carries only one gear
- simple train's velocity ratio is

$$\omega_{out} = \frac{N_2}{\cancel{N_3}} \frac{\cancel{N_3}}{\cancel{N_4}} \frac{\cancel{N_4}}{\cancel{N_5}} \frac{\cancel{N_5}}{N_6} \omega_{in}$$

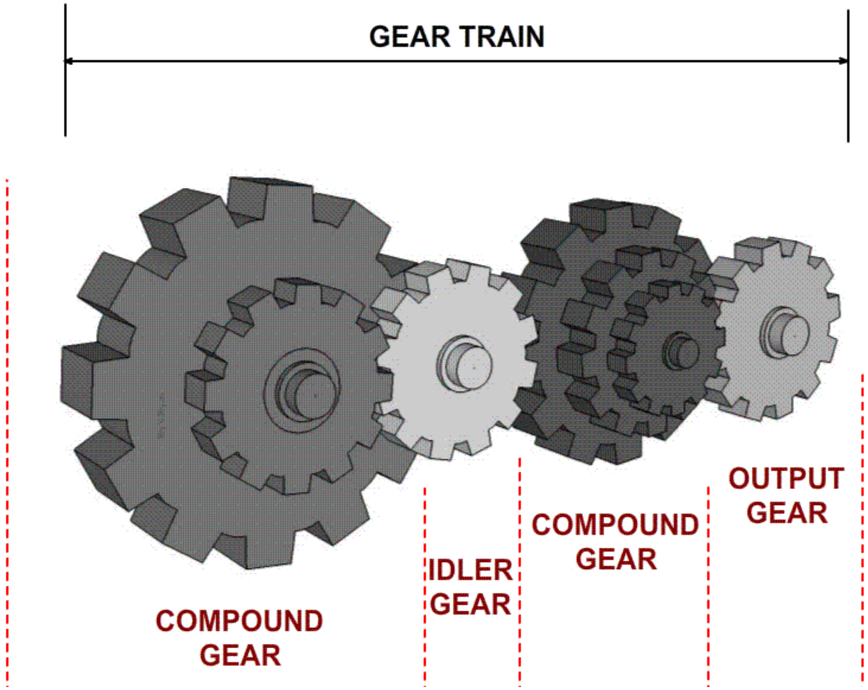
$$= \frac{N_2}{N_6} \omega_{in}$$



- Gear ratios cancel each other out
- Odd no. of gears = output direction will be ~~opposite~~ Same
- Even no. of gears = output direction will be the ~~same~~ opposite

Compound Train

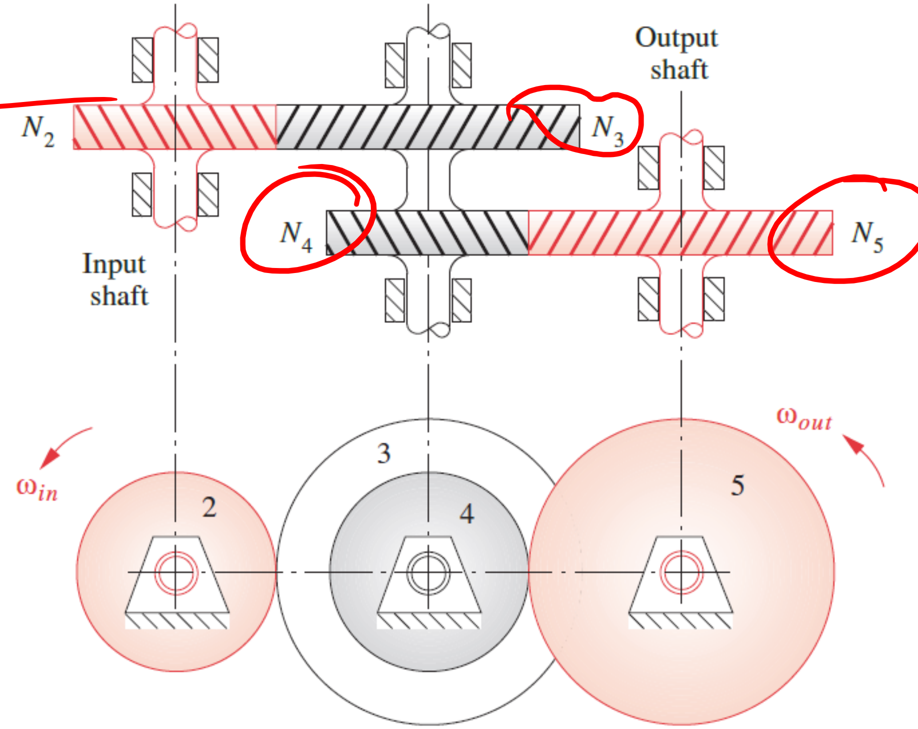
- at least one shaft carries more than one gear



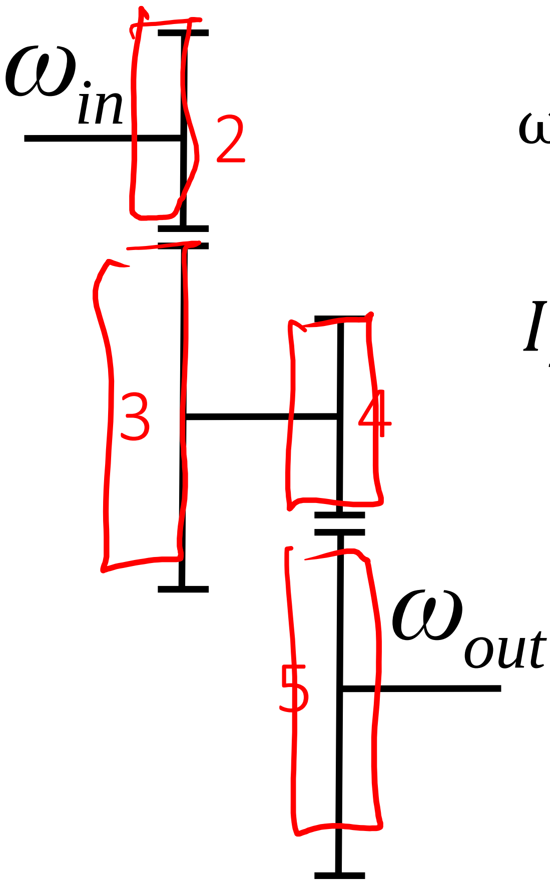
Compound Train

$$\omega_{out} = \left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right)\omega_{in}$$

Harmonic drives



Compound Train Design



2 stages

$$\omega_{in}\left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right) = \omega_{out}$$

N_2

N_3

$$If\ N_2 = N_4\ and\ N_3 = N_5$$

$$\omega_{in}\left(\frac{N_2}{N_3}\right)^2 = \omega_{out}$$

$$\frac{\omega_{in}}{\omega_{out}} = \left(\frac{N_3}{N_2}\right)^2$$ Reduction ratio

Will be used to determine the no. of stages given a reduction ratio