MATH3431 Machine Learning and Neural Networks III — Epiphany term 2024

Homework 1

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As formative assessment, submit the solutions to all the Exercises

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Exercise 1. ($\star$) Let $f:\mathbb{R}^d\rightarrow\mathbb{R}$ be a convex and β-smooth function.

1. Show that for $v,w\in\mathbb{R}^d$

$$f(v)-f(w)\in\left(\langle\nabla f(w),v-w\rangle,\langle\nabla f(w),v-w\rangle+\frac{\beta}{2}\|v-w\|^2\right)$$

2. Show that for $v,w\in\mathbb{R}^d$ such that $v=w-\frac{1}{\beta}\nabla f(w)$, it is

$$\frac{1}{2\beta}\|\nabla f(w)\|^2\leq f(w)-f(v)$$

3. Additionally assume that $f(x)>0$ for all $x\in\mathbb{R}^d$. Show that for $w\in\mathbb{R}^d$,

$$\|\nabla f(w)\|\leq\sqrt{2\beta f(w)}$$

Exercise 2. ($\star$) Let $f:\mathbb{R}^d\rightarrow\mathbb{R}$ be a λ-strongly convex function. Assume that w^* is a minimizer of fi.e.

$$w^*=\arg\min_{w}\{f(w)\}$$

Show that for any $w\in\mathbb{R}^d$ it holds

$$f(w)-f(w^*)\geq\frac{\lambda}{2}\|w-w^*\|^2$$

Hint Use the definition of λ-strongly convex function, properly rearrange it, and let the coefficient $a\rightarrow 0$.