

Computer practical 1-cont: Topics in Statistics III/IV, Term 1

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Aim

- To check some descriptive statistics
-

Contingency table: data manipulation

Below we load a table where refers to a 1992 survey by the Wright State University School of Medicine and the United Health Services in Dayton, Ohio. The survey asked 2276 students in their final year of high school in a nonurban area near Dayton, Ohio whether they had ever used alcohol, cigarettes, or marijuana. Denote the variables in this $2 \times 2 \times 2$ table by A for alcohol use, C for cigarette use, and M for marijuana use.

Load the observed counts in a data frame `obs.frame` and print the result. Use commands :

- `data.frame()` : as in SC2
- `factor()` : to encode a vector as a factor (aka category)
- `expand.grid()` : to produce all combinations of the supplied vectors or factors.

```
# I will do this for you
```

```
## load the data
```

```
obs.frame<-data.frame(count=c(911,538,44,456,3,43,2,279),
                      expand.grid(
                        marijuana=factor(c("Yes","No"),levels=c("No","Yes")),
                        cigarette=factor(c("Yes","No"),levels=c("No","Yes")),
                        alcohol=factor(c("Yes","No"),levels=c("No","Yes")))
                      )
```

```
## print the obs.frame
```

```
obs.frame
```

```
##   count marijuana cigarette alcohol
## 1   911         Yes         Yes     Yes
## 2   538         No         Yes     Yes
## 3    44         Yes         No     Yes
## 4   456         No         No     Yes
## 5     3         Yes         Yes     No
## 6    43         No         Yes     No
## 7     2         Yes         No     No
## 8   279         No         No     No
```

Create 3 dimensional contingency table from `obs.frame`. Use command:

- `xtabs()`, to create a contingency table from cross-classifying factors in a `dara.frame`

```
# this is me again
obs.xtabs <- xtabs(count ~ marijuana+cigarette+alcohol, data=obs.frame)
## print
obs.xtabs
```

```
## , , alcohol = No
##
##      cigarette
## marijuana No Yes
##      No   279  43
##      Yes    2   3
##
## , , alcohol = Yes
##
##      cigarette
## marijuana No Yes
##      No   456 538
##      Yes   44 911
```

Compute the marginal contingency table of marijuana and cigarette.

- Use command `margin.table(, margin =)` and `obs.xtabs`.
- Save it in `obs.mc.xtabs`.

```
obs.mc.xtabs <- margin.table(obs.xtabs, margin=c(1,2))
obs.mc.xtabs
```

```
##      cigarette
## marijuana No Yes
##      No   735 581
##      Yes   46 914
```

Odds ratio calculations

Code an R function, named 'odds.ratio' with:

- Inputs:
 - `x` : a 2 by 2 matrix whose elements are the observed counts of a 2 by 2 contingency table
 - `conf.level` : with default input value 0.95 representing the confidence level
 - `theta0` : with default value 1 representing a null hypothesis value of the odds ratio test
- Outputs:
 - `estimator` : representing mle of odds ratio
 - `log.estimator` : representing the mle of log odds ratio
 - `asympt.SE` : representing the standard error / standard deviation of the mle of odds ratio
 - `conf.interval`: representing confidence interval of mle of odds ratio at sig level `conf.level` (from the inputs)
 - `conf.level` =representing confidence level
 - `Ztest` : representing the test statistic for the odds ratio test (2 tails)
 - `p.value` : representing the p value of the odds ratio test (2 tails)

- log.conf.interval : representing in log scale the confidence interval of mle of odds ratio at sig level conf.level (from the inputs)

```
odds.ratio <- function(x,conf.level=0.95,theta0=1)
{
  if (any(x==0)) x <- x+0.5
  theta <- x[1,1] *
    x[2,2]/(x[1,2] *
      x[2,1])
  SE <- sqrt(sum(1/x))
  Za2 <- qnorm(0.5 *
    (1+conf.level))
  Low <- exp(log(theta)-Za2 * SE)
  Up <- exp(log(theta)+Za2 *
    SE)
  CI <- c(Low,Up)
  Z=(log(theta)-log(theta0))/SE
  pv=2 *
    pnorm(-abs(Z))

  logCI <- log(CI)

  list (estimator=theta,
    log.estimator=log(theta),
    asympt.SE=SE,
    conf.interval=CI,
    conf.level=conf.level,
    Ztest=Z,
    p.value=pv,
    log.conf.interval=logCI
  )
}
```

For the marginal contingency table of marijuana and cigarette,

- compute the mle of the marginal odds ratio of marijuana and cigarette
- compute the 95% Confidence Interval of the marginal odds ratio of marijuana and cigarette
- perform a statistical hypothesis test that marijuana and cigarette are independent at sig level 0.05

```
obs.mc.xtabs <- margin.table(obs.xtabs, margin=c(1,2))

odds.ratio.marijuana.cigarette <- odds.ratio(obs.mc.xtabs, conf.level=0.95, theta0=1 )
odds.ratio.marijuana.cigarette

## $estimator
## [1] 25.1362
##
## $log.estimator
## [1] 3.224309
##
## $asympt.SE
## [1] 0.1609812
##
```

```
## $conf.interval
## [1] 18.33463 34.46093
##
## $conf.level
## [1] 0.95
##
## $Ztest
## [1] 20.02911
##
## $p.value
## [1] 3.071215e-89
##
## $log.conf.interval
## [1] 2.908792 3.539826
```

The MLE of the marginal odds ratio of marijuana and cigarette is 25.136197 .

The 95% confidence interval of the marginal odds ratio of marijuana and cigarette is [18.33463, 34.4609298] .

The hypothesis test with $H_0 : \theta = 1$ versus $H_1 : \theta \neq 1$ at sig. level 0.05 has a p-value $3.0712155 \times 10^{-89}$. Hence I reject $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$ at sig. level 0.05.

Fourfold Plots

You can draw Fourfold Plots

Tables 2 x 2

It is a graphical expression visualizing the odds ratio

$$\theta = \frac{n_{11}n_{22}}{n_{12}n_{21}}$$

in 2 x 2 contingency tables.

It shows the departure from independence as measured by the sample odds ratio,

Each cell n_{ij} is represented as a quarter-circle with radius proportional to $\sqrt{n_{ij}}$ and area proportional to n_{ij} .

- If there is no association $\theta = 1$ between classification variables, the quarter-circles should form a circle.
- If there is positive association $\theta > 1$ between classification variables, the diagonal areas are greater than the off-diagonal areas
- If there is negative association $\theta < 1$ between classification variables, the diagonal areas are smaller than the off-diagonal areas

R provides a function to draw this kind of plots by using the function `fourfoldplot` from the package `vcd`

- Install 'vcd' package and load it

```
# install.packages('vcd') # IF NOT ALREADY INSTALLED ON YOUR PC, THEN UNCOMMENT AND RUN THIS COMMAND
library(vcd)
```

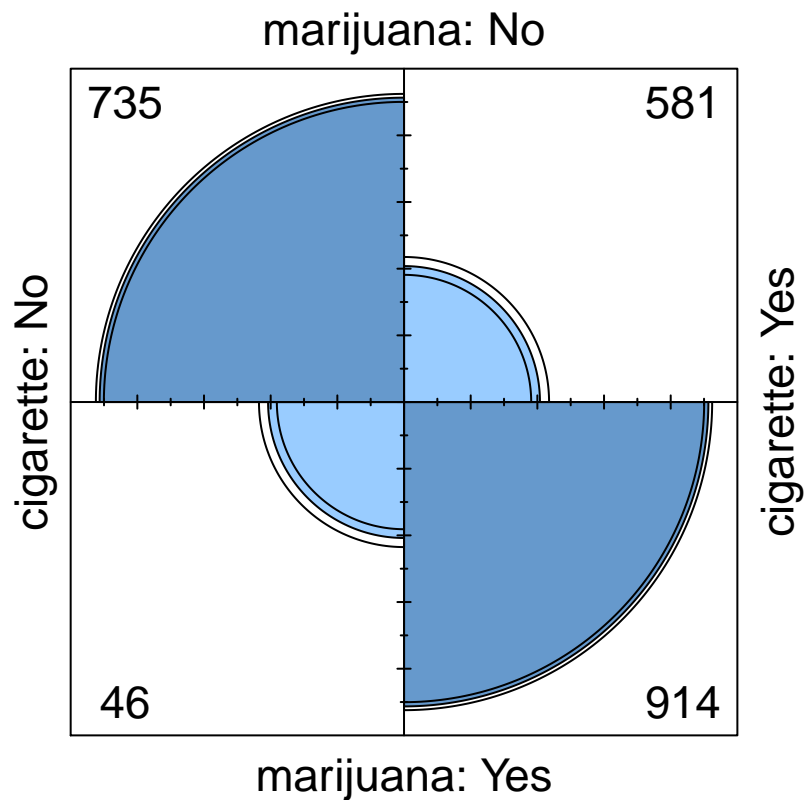
```
## Loading required package: grid
```

Check in help the function `fourfoldplot` by using the command `?fourfoldplot`

- Draw a Fourfold Plot for the marginal contingency table of marijuana and cigarette.

- Discuss what you can see

```
obs.mc.xtabs <- margin.table(obs.xtabs, margin=c(1,2))
fourfoldplot(obs.mc.xtabs)
```



Note that:

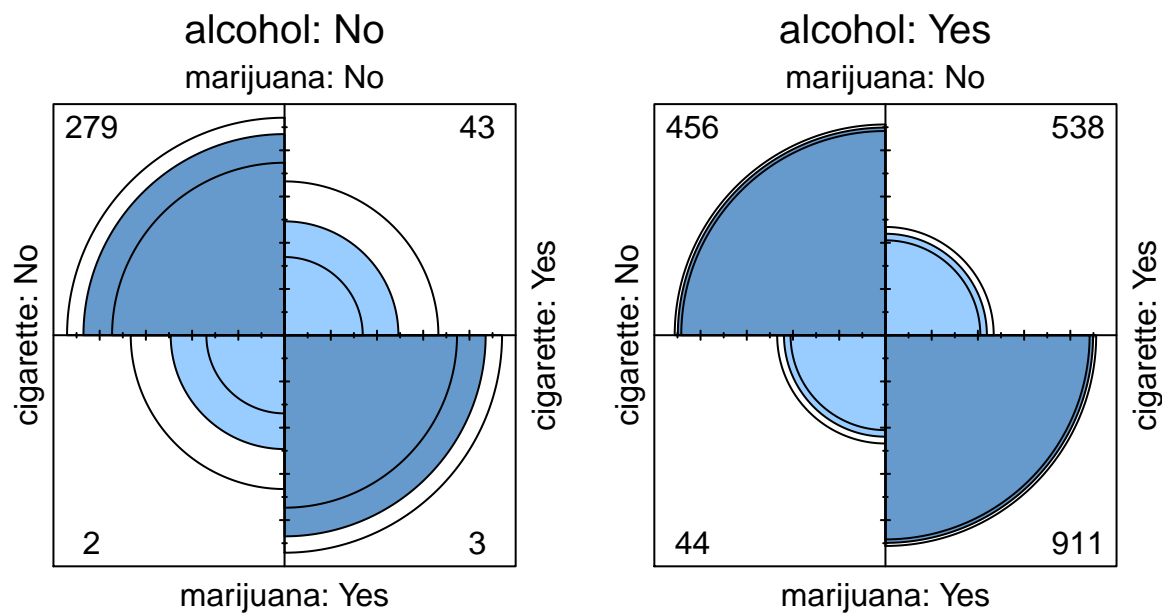
- * The area of each shaded quadrant shows the observed counts.
- * Circular arcs show the limits of confidence interval for the odds ratio.

Tables 2 x 2 x K

Fourfold Plots can be also used for 2 x 2 x K contingency tables

- Draw a Fourfold Plot for the contingency table of marijuana cigarette, and alcohol by controlling on the alcohol levels.
- inspect the plots

```
fourfoldplot(obs.xtabs,
             mfrow = c(1,2)
             )
```



Mosaic plot

Mosaic plot display graphically the cells of a contingency table as rectangular areas of size proportional to the corresponding observed frequencies.

When the classification variables are independent the areas tend to be perfectly aligned in rows and columns.

The greater the deviation is, the worse the aforesaid alignment is.

Furthermore, specific locations of the table that deviate from independence the most can be identified and thus the pattern of underlying association can be explained.

The strength of individual cells contribution to divergence from independence as well as the direction of the divergence are reflected in the magnitude and sign of the corresponding independence model's residuals that can be incorporated in a mosaic plot.

R provides a function to draw this kind of plots by using the function `mosaic` from the package `vcd`

- Install 'vcd' package and load it

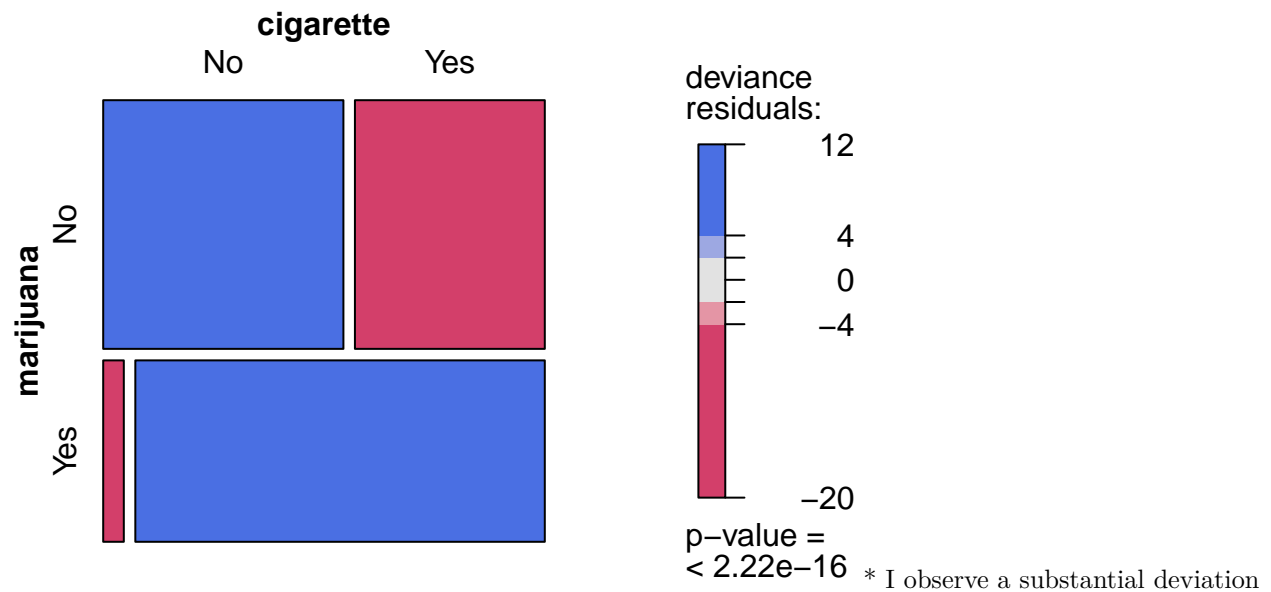
```
# install.packages('vcd') # IF NOT ALREADY INSTALLED ON YOUR PC, THEN UNCOMMENT AND RUN THIS COMMAND
library(vcd)
```

- Check the command `mosaic` in help by typing `?mosaic`

For the I x J case:

- Draw a Mosaic Plot for the marginal contingency table of marijuana cigarette.
- in particular use `mosaic(x,residuals_type="deviance",gp=shading_hcl)` where x is the contingency table of interest
- Interpretet the plots

```
mosaic(obs.mc.xtabs,
       residuals_type="deviance",
       gp=shading_hcl)
```



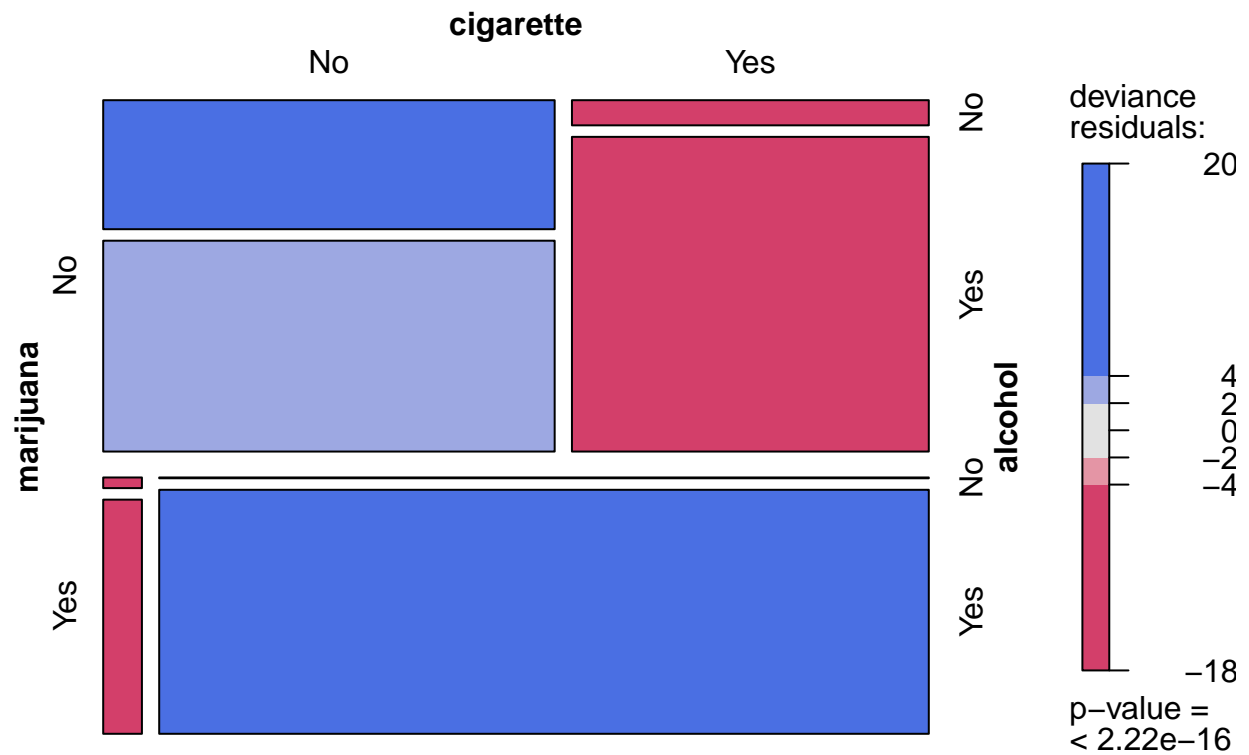
from the Independent association.

- The p-value of the GoF test based on the deviance for independence model is below the colorbar, and smaller than 0.05; hence I reject the hypothesis at sig. level 5%.

For the I x J x K case:

- Draw a Mosaic Plot for the contingency table of marijuana cigarette and alcohol.
- Interpretet the plots

```
mosaic(obs.xtabs,
       residuals_type="deviance",
       gp=shading_hcl)
```



- I observe a substantial deviation from the Independent association.
- The p-value of the GoF test based on the deviance for independence model is located below the colorbar, and smaller than 0.05; hence I reject the hypothesis at sig. level 5%.

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