Bad Honnef Physics School Methods of Path Integration in Modern Physics, August 25–31, 2019

Gert-Ludwig Ingold: Introduction to Feynman path integrals

• https://github.com/gertingold/feynman-intro

Exercise: Verify the relation

$$\sum_{n=-\infty}^{+\infty} \delta(x - 2\pi n) = \frac{1}{2\pi} \sum_{\ell=-\infty}^{+\infty} e^{i\ell x}.$$
 (1)

Solution:

The left-hand side is a 2π -periodic function which can be expressed in terms of a Fourier series

$$f(x) = \sum_{\ell = -\infty}^{+\infty} c_{\ell} e^{i\ell x}$$
 (2)

with coefficients

$$c_{\ell} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{d}x f(x) \mathrm{e}^{-\mathrm{i}\ell x} \,. \tag{3}$$

In our specific case, we have

$$f(x) = \sum_{n = -\infty}^{+\infty} \delta(x - 2\pi n) \tag{4}$$

and obtain for the Fourier coefficients

$$c_{\ell} = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} \int_{-\pi}^{\pi} \mathrm{d}x \delta(x - 2\pi n) \mathrm{e}^{-\mathrm{i}\ell x}$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \mathrm{d}x \delta(x) \mathrm{e}^{-\mathrm{i}\ell x}$$

$$= \frac{1}{2\pi}.$$
(5)

The Fourier series representation of the δ -comb thus follows as

$$\sum_{n=-\infty}^{+\infty} \delta(x - 2\pi n) = \frac{1}{2\pi} \sum_{\ell=-\infty}^{+\infty} e^{i\ell x}.$$
 (6)